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Market Behavior With Demand Uncertainty

and Price Inflexibility

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Number 179 June 1976

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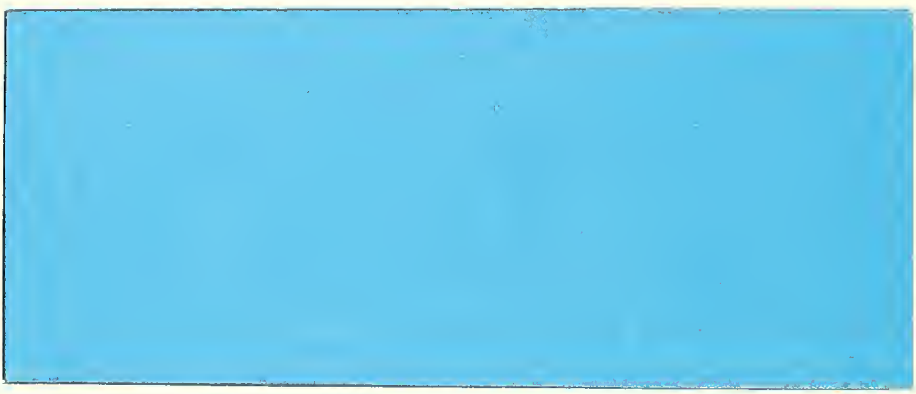

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I wish to thank Professors Franklin M. Fisher, Peter A. Diamond, and Robert E. Hall for valuable advice. This paper is based in part on my Ph.D. Thesis, M.I.T., September 1975. I thank Barbara Feldstein for her expert typing.

1. Introduction

Most economists would agree that the large majority of markets do not precisely fit the classical assumptions of competition. For many markets, prices do not adjust at each instant of the day to balance supply and demand. Moreover, firms often do not know how much of their product will be demanded each day.

There are good reasons why most markets depart from the strict classical assumptions. Changing prices frequently is time consuming and may be costly. More importantly, prices may have to remain in effect for some time if their "signal" is to be received. The demand that an individual firm sees is random because the number of customers that frequent the firm will generally vary from day to day. In formulating its operating policy, a firm must take into account the randomness of its demand. Firms do not feel that they can sell all they want at the going market price and are concerned with overproducing or having excess capacity. Firms are also concerned with underproducing or having too little capacity. In these markets, it is an outcome of the market process that occasionally some customers will be unable to purchase the good instantly.

For these uncertain markets, the amount that a firm is willing to supply depends not only on the going market price, but also on the entire stochastic structure of demand that it faces. In this environment, supply cannot be defined without first specifying the random structure of demand.

There will be three essential features of market operation that we will study; price inflexibility, demand uncertainty, and timing considerations. By price inflexibility, we do not mean that prices do not respond to permanent shifts in the underlying supply and demand factors, but only that prices cannot be adjusting at each instant of time. An important feature of the analysis will be to determine exactly how prices are endogenously determined by market forces. Demand uncertainty means that, at the beginning of any market period, after prices have been set, firms do not know for sure what their demand will be, although they do know what the random distribution of demand looks like. Demand is uncertain over the period for which prices are inflexible. Timing considerations refer to the need to have produced or to have made some prior commitment to production, such as the purchase of equipment, before the unknown customer demand is observed.

It is not immediately clear what the consequences of these three nonclassical features of market operation are, even though these three features would appear to be realistic characterizations of many market operations. In this paper, we address the following questions. How do firms compete in such markets? Can equilibrium be meaningfully defined and if so how does it compare to the classical equilibrium when the uncertainty is removed from the demand side? What are the properties of the competitive equilibrium as the size of the market increases? Will this equilibrium be Pareto-Optimal? Would society benefit if the government paid lump sum subsidies to firms so as to encourage them to expand their production of the good?

For the markets under study, it will be a natural feature to have some customers being unable to purchase the good, and some firms being unable to sell all of their stock, or equivalently use all their capacity. Each good will have two characteristics associated with it, namely its price and the probability that it can be purchased. Customers will have preferences not only for the price of the good, but also for the probability of obtaining it. Firms will compete amongst themselves until an equilibrium is reached. Market clearing will require equilibrium along the dimensions of both price and probability of obtaining the good. In equilibrium, supply will not, in general, equal demand and there will always be some customers who are unable to purchase the good. The "customers" can also be interpreted as being other firms who are trying to buy factor inputs for their production process. With this interpretation, we obtain a model where it is perfectly natural for firms to be concerned with obtaining an "assured" supply of the input, a concern that appears uppermost in the minds of businessmen.[1]

We show how the model of this paper is formally equivalent to models in the literature on peak load pricing under uncertainty. In the special case where social welfare is measured by expected surplus, we show that the competitive equilibrium is optimal. This result stands in sharp contrast to previous models in the literature on optimal pricing under uncertainty. However, in general, the competitive equilibrium will not

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[1] See for example A. Chandler (1964). See Carlton (1976a) for the analysis of vertical integration in these markets.

lead to the socially optimal point. The social optimum will, under a plausible set of assumptions, involve paying lump sum subsidies to encourage firms to expand their production.

The model is applicable to any market where availability of the good or of the means to produce the good is important. It does appear that in the private sector for many industries demand fluctuations are not always absorbed by price changes and that changes in the probability of whether or when the good can be obtained is often an important equilibrating mechanism. Some examples include retail stores, hotels, restaurants, and manufacturing. In the regulated and government sector too, the model also seems to have widespread application. For example, for airlines, railroads, public parks, and electric utilities, prices do not vary continuously and uncertainty in demand is absorbed by changes in rationing frequency.

2. Competitive Market Clearing

There is a large literature on the effects of uncertainty on firm behavior.[1] Analyses of competitive markets focus on the effect of having uncertainty in price and maintain the assumption that firms can always sell all they want at the future uncertain market price.[2] There are never any shortages in equilibrium. In his pioneering works, Mills [1959, 1962] has examined the effect of demand uncertainty and price inflexibility on the behavior of a monopolist who must decide what price

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[1] See Rothschild (1973) and McCall (1971) and the references cited therein.

[2] See, for example, E. Zabel (1967).

to charge and how much to produce before demand can be observed. Surprisingly, despite the realism of the assumptions of demand uncertainty, price inflexibility, and a lead time necessary for production, there has been no attempt to examine the implication of these assumptions within a competitive environment. The purpose of this section is to provide such an examination, and to derive and investigate the properties of an equilibrium in which it is natural to have supply not equal to demand. We now present a simple model which attempts to capture the essential features of the markets under study.

There are N identical firms who compete with each other, and L identical customers each of whom has a nonstochastic demand for the good given by x(p). To make the assumption of competition plausible, the number of firms N will be considered to be large enough to prevent firms from having any monopoly power. Individuals maximize expected utility and firms maximize expected profits.

At the beginning of each period, each firm sets price, which remains in effect for the entire period, and decides how much of the good to produce and stock for the period.[1] No deliveries of the good can occur during the period. The production cost per unit of the good is c, where c is strictly positive. To keep the model simple, we assume that the good is perishable so that it is impossible to hold inventories between periods. Provided holding inventory is a costly activity, the qualitative results derived below will be unchanged.

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[1]It is not necessary for the good to be produced at the very beginning of the market period. All that is required is that some commitment to production, such as the purchase of equipment or other inputs, be made prior to observing demand.

During each period, each of the L identical customers frequents a firm of his own choosing. The customer knows the price the firm charges, and through reputation, the probability that the good will be available at that firm. If a customer finds a firm out of the good, he simply leaves the firm and does not obtain the good for that period. He does not search at the other firms.[1] Buyers have preferences for not only how much they purchase and spend on the good, but also for the probability of being able to buy the good. Therefore, competition does not force firms to necessarily charge the same price but rather to offer price-shortage combinations which leave the consumer at the same level of expected utility.

Equilibrium in an uncertain market is said to exist when 1) consumers are indifferent as to which of the firms they shop at each period, and 2) no firm, behaving optimally, can offer a price-shortage combination which would leave consumers better off, and which would allow the firm to earn nonnegative expected profits. Before examining how market equilibrium is determined, let us first look at the incentives facing individual consumers and firms.

3. Consumer Behavior

In calculating his expected utility from going to any firm, a customer is concerned with both the probability, $1-\lambda$, of obtaining the good and the price, p, charged for the good. We can write his expected utility as $U(1-\lambda, p)$. The function U defines the isoutility contours between

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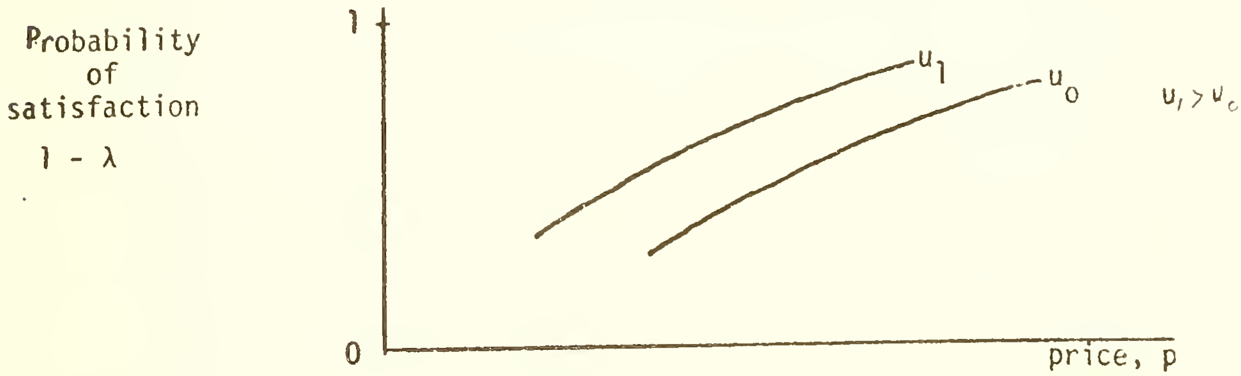
[1] Just as in the case of inventory holding, consumer search behavior, providing it is costly, would not alter any of the qualitative features of the model.

$1-\lambda$ and p that leave a consumer indifferent. Typical isoutility contours are drawn in Figure 1.

The diagram shows that along any isoutility curve, as price rises, the probability of satisfaction must rise if consumers are to remain indifferent. Also, for any fixed probability of satisfaction, consumers always prefer lower prices.

Figure 1 - Isoutility Contours



Consumers will always try to reach their highest isoutility contour, and will only go to a firm that they think will provide this highest isoutility level. If the buyer believes that several firms provide this highest utility level, then he will choose among them randomly.

In general, no strong conclusion about the shape of the isoutility curves seems justified. Because of the ambiguity in determining the shape of the isoutility curves, the subsequent results will not in general depend on any assumed convexity or concavity properties of isoutility curves. Instead we will only require the very weak assumptions that the isoutility curves exist over the relevant range[1] in $(1-\lambda, p)$ space,

[1] By this assumption we simply mean that there is some range of prices, which includes $p = c$, the cost of production, for which the consumer is interested in purchasing the good. In other words, if the consumer does not have positive demand for prices near c, then the market for the good will not exist, and there is nothing to analyze.

that they are continuous, and that they satisfy an upper and lower Lipschitz condition. This latter condition postulates that there exist two numbers, b and B, such that $0 < b < B < \infty$ and such that the slope along any isoutility curve always lies between them. This condition rules out horizontal and vertical segments for isoutility curves, which, as will be shown later, can result in pathological and uninteresting market behavior. The Lipschitz requirements insure that the consumer is never willing to make infinite trade-offs in either the p or $1-\lambda$ directions.

4. Behavior of the Firm

Since consumers will wind up going only to those firms that provide the highest utility level in the market, competition forces firms to take the utility level as given. (If instantaneous production were possible so that no shortages could occur, then each good would have only one characteristic, price, associated with it. In that case, utility-taking behavior is equivalent to price-taking behavior and this market would behave exactly as a classical supply and demand analysis would indicate.) At the beginning of each period, firms have to decide on a price and production policy so as to maximize their profits subject to the constraint that they provide at least the given level of utility to consumers. Firms know that if they remain competitive with the other firms, then they will randomly receive $\frac{1}{N}$th of the total population L.

We can write the total amount that the firm decides to produce at price p as $s \cdot x(p)$. The variable s can be interpreted as the maximum number of customers that a firm can satisfy that period. Henceforth, we will refer to s as customer capacity. Clearly, the amount that a firm decides to produce affects the probability that a customer will be able to obtain the good from that firm.

Let us examine the relation between the expected number of customers, M, who will find the firm out of the good, and the customer capacity, s, that the firm provides. Let pr(i) stand for the binomial probability that i customers from the L customers arrive at the firm. Then, we can write that

$$M(s) = \sum_{s+1}^{\infty} (i-s)pr(i)$$

If all N firms follow the same operating policies, then the total expected number of customers who will be dissatisfied is $N \cdot M$, and the fraction of dissatisfied customers will equal NM/L. The fraction $1-\lambda$ of customers who are able to obtain the good can be written as

$$1 - \lambda(s) = 1 - \frac{N \cdot M(s)}{L} \tag{1}$$

In the appendix, we show that using the normal distribution to approximate the discrete binomial process of customer arrival, the probability of satisfaction function, $1 - \lambda(s)$ can be written as:

$$1 - \lambda(s) = \frac{\sigma I(u) + s}{\sigma^2} \tag{2}$$

where $\sigma^2 = L/N$, $I(u) = \int_{-\infty}^{u} [t-u]f(u)du$, $f(u)$ = normal density function, and $u = \frac{s - \sigma^2}{\sigma}$.

Technically, as derived above, the $1 - \lambda(s)$ function applies to an individual firm only when all firms follow the same operating policies. However, if customers and firms calculate the probability of satisfaction that an individual firm offers as the expected shortage of that firm divided by the expected number of customers for that firm, then we can interpret (1) as applying to the individual firm. More importantly, since $M(s)$ and $1 - \lambda(s)$ are in one to one relation by (1) the entire analysis could be carried out in (M, p) space and not $(1-\lambda, p)$ space. Since M obviously applies to the individual firm, this approach would avoid any questions about whether the derived curves apply to individual firms. Because of the one to one relation between $1-\lambda$ and M in (1), the result of such an analysis will be identical to one in $(1-\lambda, p)$ space. However, it seems more natural to talk of consumers as having preferences for the probability of satisfaction, $1-\lambda$, and not the expected shortage, M. For these reasons, we carry out the analysis in $(1-\lambda, p)$ space, and regard the $1 - \lambda(s)$ curve as applying to individual firms.

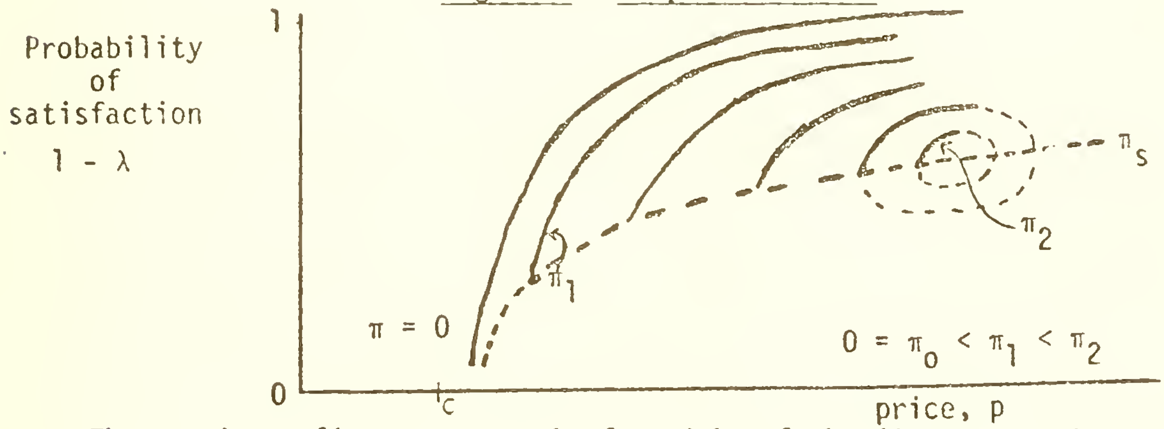
For a given level of utility, firms want to choose a price, p, and a customer capacity, s, so that profits are maximized and the consumer is able to achieve the given level of utility. When firms remain competitive by offering the given level of utility, they randomly receive their equal share of the L customers. Letting pr(i) stand once again for the probability that i of the L customers visit a firm this period, we can write that expected profits equal

$$\pi(s,p) = p\cdot x(p) \sum_{o}^{s} i\ pr(i) + px(p)s \sum_{s+1}^{L} pr(i) - csx(p) \qquad (3)$$

The first term in (3) represents expected sales revenue when $i \leq s$ customers come to the firm, while the second term represents expected sales revenue when more than s customers come to the firm. The last term (3) is the cost of being able to service s customers. Since (2) expresses a one to one relation between the probability of satisfaction $1-\lambda$ and the customer capacity s, we can interpret (3) as expressing profits as a function of $1-\lambda$ and p.

Regarding profits as a function of $1-\lambda$ and p, we can draw isoprofit curves in $(1-\lambda, p)$ space. A typical family of such curves is depicted below.

Figure 2 - Isoprofit Curves



The two isoprofit curves at the far right of the diagram are drawn to illustrate that each isoprofit curve involving positive profits "turns around" on itself as price rises sufficiently high to drive demand to zero. Since consumers always prefer to be on the northwest boundary of the isoprofit curves, competition will insure that the "dotted" segments of the isoprofit curves are never observed. The heavy dotted line in the diagram represents the π_s curve which is derived by setting $\frac{\partial \pi}{\partial s} = 0$

in (3). As the diagram illustrates, isoprofit curves cross the π_s curve vertically, and so the relevant portions of all isoprofit curves emanate from the π_s curve. For any fixed probability of satisfaction, profits increase as price increases. Hence, in the diagram $\pi_1 < \pi_2$. The curve on the far left of Figure 2 represents the zero profit curve.

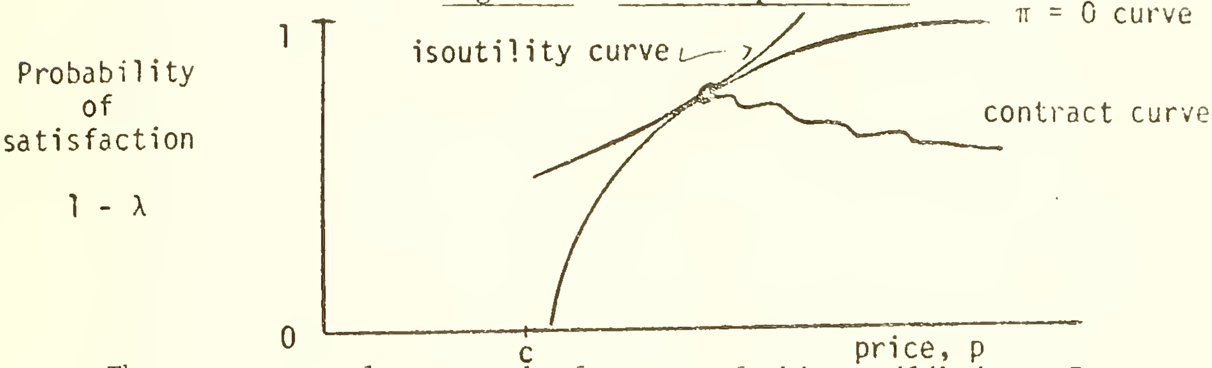
For any given isoutility level, $\bar{u}$, the firm will choose to operate at the point of tangency between the isoutility curve representing isoutility level, $\bar{u}$, and the highest isoprofit curve. No firm ever chooses to operate to the left of the $\pi = 0$ curve since that represents negative expected profits.

5. Market Equilibrium

In the diagram of the isoprofit curves, superimpose the isoutility curves of consumers. We can define a contract curve as the locus of tangencies between the isoutility and isoprofit curves. Firms always operate on this contract curve.

In a classical market, firms compete with each other by offering to consumers lower prices (i.e., higher utilities) than other firms. Prices (or consumer utilities) continue falling (rising) until firms have no incentive to lower price (raise utility) any more. Analogously, in this market, firms compete with each other by offering better (i.e., higher utility) combinations of price and probability of satisfaction to consumers. The utility level is "bid" up until there is no incentive for any firm to continue to alter its price-probability of satisfaction combination. This point occurs when the contract curve intersects the zero profit (π = 0) curve. At this point, firms would prefer to go out of business rather than offer a higher utility combination to consumers and earn negative expected profits. Hence, competition on the utility level forces the market equilibrium up the contract curve, until the zero profit curve is reached. Equilibrium can be regarded as the tangency[1] between the zero profit curve and the highest attainable isoutility curve.[2] This equilibrium is depicted below.

Figure 3 - Market Equilibrium



There are several noteworthy features of this equilibrium. In general, there will be a positive probability of being unable to obtain the good. Second, in equilibrium the price will exceed the constant cost of production. This occurs because the revenue from sold goods must compensate not only for the cost of producing those goods but also for the cost of producing the unsold goods. Equivalently, it is necessary to pay for available but unused capacity. Third, there is no reason why supply

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[1] We will soon argue that corner solutions are uninteresting, unlikely, and under the Lipschitz Assumptions on preferences, impossible.

[2] With instantaneous production, the model becomes identical to the classical supply and demand model. For that case, the π = 0 curve is a vertical line at p = c, and equilibrium as defined above, coincides with the classical equilibrium of price = c, probability of satisfaction = 1. We see then that the classical model is a special case of this model.

should equal demand even in expected values since equilibrium depends in part on consumers' willingness to take risk.

Before investigating the properties of the market equilibrium defined above, let us consider the competitive process in a little more detail. Firms are assumed to be "utility-level" takers, yet in the description of how a market reaches equilibrium, we stated that firms "compete" with each other on the offered utility level. If firms take the utility level as given, which firms are changing the utility level in the approach to market equilibrium? The problem here is identical to the one in pure competition. If all firms are price takers, who ever changes price to insure that price clears the market? Traditional explanations rely on a Walrasian auctioneer. More ambitious attempts at realistic adjustment mechanisms have met with little success. "Despite great and admirable efforts by many leading theorists, we have no...satisfactory theory of how equilibrium is reached." (Fisher, 1970, p. 195). It is possible (see Carlton (1975)) to tell a simple story of market operation that suggests that competition will result in a stable market operation that leads to the equilibrium discussed above. The story requires that firms wish to remain in business for more than one period, that consumers do not all instantly discover and go to the firm offering the best deal but will not return to firms who consistently provide poor deals. However, just as in the case of simple classical markets, a mathematically complete dynamic theory of adjustment to equilibrium remains an important, unsolved theoretical problem. For the remainder of this paper, the question of dynamic adjustment to equilibrium is not addressed.

6. Qualifications

In the previous section, we refer to equilibrium as the tangency between the highest isoutility curve and the zero profit. This statement requires some qualifications. If we want to examine the behavior of a particular market, it is useful to rule out certain types of behavior as unrealistic or uninteresting. Previously, we assumed that for prices near c, per capita demand was positive. Without this assumption, the market will not exist, hardly an interesting case to study.

There are two nontangency "equilibrium" points that are possible for the markets under study. Both these "equilibria" which correspond to "corner" solutions between the isoutility and zero profit curves are strange and/or uninteresting. If the isoutility curves are vertical, then equilibrium involves zero production - again the uninteresting case of the market not existing. If the isoutility curves are horizontal, then equilibrium involves a very high price and each firm stocking enough of the good to by itself satisfy the entire market. Both these corner equilibrium seem sufficiently uninteresting to exclude them from further analysis. (It can be shown (Carlton, 1975) that the zero profit curve has a very large slope at the low price end, and a flat slope at the high price end. Hence the Lipschitz conditions on consumer preferences are sufficient to rule out the uninteresting corner solutions.)

With no restrictions on the shape of the isoutility curves, it is possible to have tangencies with utility curves offering less than the maximum utility. This last consideration arises because of the possible nonconvexity of the isoutility curves. Whether the market reaches the tangency of maximum utility depends on the dynamic properties of the market. It is also possible to have multiple tangencies with the same isoutility curve. Once again the dynamic adjustment process would determine which point(s) the market winds up at. Since all firms and consumers are identical, we regard it as unlikely that identical firms will be operating differently in equilibrium, and exclude this case from further discussion.

We now want to examine how the market equilibrium under uncertainty behaves as the market size increases. This will clarify the relation between market clearing under certainty and under the uncertain conditions under study here.

7. The Zero Profit Curve

The properties of the zero profit curve play a key role in determining the behavior of equilibrium as the customer per firm ratio increases. In this section, we describe the relevant properties of the zero profit curve.

Since we have ruled out as uninteresting the case where the market vanishes (i.e., $x(p) = 0$), we can use (3) to write the condition for zero profits as

$$0 = \pi = p \cdot \sum_{0}^{s} i\, pr(i) + p \cdot s \sum_{s+1}^{L} pr(i) - cs$$

where all notation was defined previously. Notice that as a consequence of the assumption of a constant cost, c, the per capita demand x(p) does not appear in the zero profit condition. Using the normal distribution to approximate the binomial,[1] we obtain that the zero profit condition can be written as

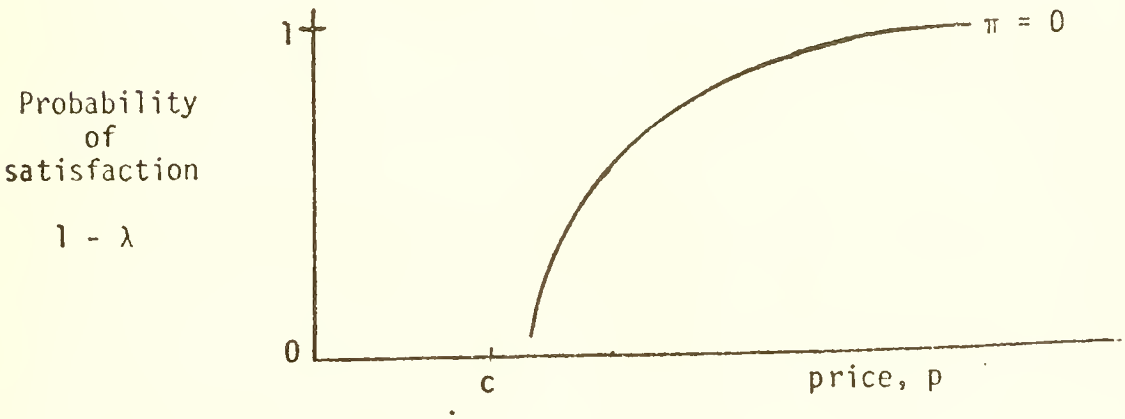
$$[\sigma I(u) + s] \cdot L \cdot p - c \cdot s \cdot N = 0 \qquad (4)$$

where all notation was defined previously beneath (1) and (2). Since the customer capacity, s, and the probability of satisfaction, $1-\lambda$, are in one to one correspondence by (2), we see that (4) can be interpreted as expressing a relation between $1-\lambda$ and p that must hold along the zero profit curve. We can write (4) as

$$\pi(1-\lambda, p) = 0 \qquad (5)$$

The general shape of the zero profit ($\pi = 0$) curve is depicted below.

Figure 4 - The Zero Profit Curve



[1] Since we are using a continuous random variable to approximate a discrete positive random variable, there is a slight error involved. By the Central Limit Theorem, we know that any such approximation errors become insignificant for even moderate (i.e., 15-20) values of the customer per store ratio, L/N. In the subsequent analysis, we shall ignore such approximation errors.

The $\pi = 0$ curve is concave[1] (i.e., $\frac{d^2(1-\lambda)}{dp^2} < 0$), starts off with a very large slope at a point a little to the right of $p = c$ on the horizontal axis, rises to 1 as price increases, and has a very flat slope for sufficiently high prices. The curve always lies to the right of the vertical line $p = c$, since price must cover not only production costs, but also the cost of unsold goods. As price rises, firms can afford to provide a larger customer capacity, s. Hence along the $\pi = 0$ curve the probability of satisfaction increases to 1, as price increases.

As the customer per firm ratio, L/N, increases, the $\pi = 0$ curve is affected is several ways. First, the entire curve shifts up, indicating that for fixed price as the number of customers per store increases firms can afford to increase their customer capacity in such a way that there is a higher probability of satisfying customers. Basically, this result occurs because there are economies of scale in servicing a stochastic market. The proportional risk of having unsold goods declines as the customer per firm ratio increases. In other words, to achieve a satisfaction probability of .5 in a market with 100 customers per store requires a $\frac{s}{100}$[2] figure that is larger than the $\frac{s}{1000}$ figure in a market with 1000 customers per store. As the customer per firm ratio continues to increase, the $\pi = 0$ curve shifts up to the $1-\lambda = 1$ line.

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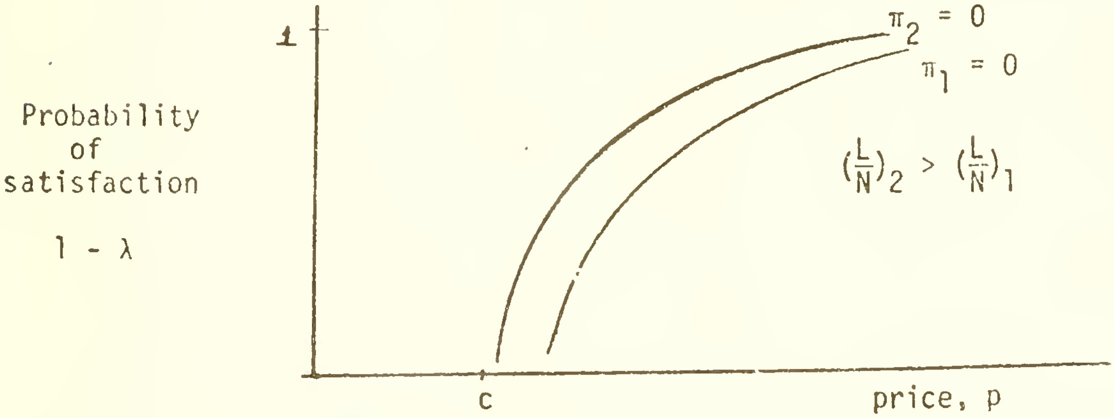
[1] All these geometric properties are proved in Carlton (1975). These proofs are available on request from the author.

[2] Recall that s refers to customer capacity.

How does the slope of the $\pi(1-\lambda,\ p) = 0$ curve behave as the customer per store ratio increases? It is possible to prove (Carlton, 1975) that for any fixed price, p, greater than c, the slope $(\frac{d(1-\lambda)}{dp})$ falls monotonically to zero as L/N increases. Furthermore, for any fixed probability of satisfaction, $\overline{1-\lambda}$, below 1, the slope $\frac{d\ 1-\lambda}{dp}\ \Big|\ \overline{1-\lambda}$ approaches infinity as L/N increases. These properties are illustrated in the diagram below.

Figure 5 - The Zero Profit Curve and the Customer Per Firm Ratio



8. Behavior of Market Equilibrium as the Customer Per Firm Ratio Increases

Armed with these properties of the $\pi = 0$ curve, we can now investigate the behavior of equilibrium as the customer per firm, L/N, ratio increases. It will be useful for the reader to recall from the discussion on consumer preferences that b and B are the lower and upper bounds on the slope of the isoutility curves, respectively.

Theorem 1: As the customer per firm ratio, L/N, increases, the equilibrium price associated with the market clearing point approaches the deterministic market clearing price c.

Proof: The method of proof will be to show that as L/N increases, the equilibrium point (p^*, $1-\lambda^*$) of the market clearing under uncertainty will eventually lie to the left of the vertical line $p = c + e$ for every positive e.

Choose the point $p = c + e$ for any positive e. Choose L/N large enough so that the $\pi = 0$ curve is defined by (4) for p equal $c + e$. Equilibrium in the uncertain market is defined as the point of tangency between the $\pi = 0$ curve and an isoutility curve.[1] Now, increase L/N. As L/N increases, the slope of the $\pi = 0$ curve declines to zero for any fixed $p > c$. Increase L/N so that the slope of the $\pi = 0$ curve is less than b at $p = c + e$. This implies that the slope of $\pi = 0$ is less than b for all $p \geq c + e$, since the $\pi = 0$ curve is concave. But then it is impossible for any isoutility curve to be tangent to the $\pi = 0$ curve at any price above $c + e$. Hence, the market equilibrium price p^* is less than $c + e$. Since p^* must be greater than c for any production to occur at all, and since p^* is less than $c + e$ for any positive e, it follows that $\lim_{\frac{L}{N} \to \infty} p^* \to c$. Q.E.D.

Theorem 2: As the customer per firm ratio, L/N, increases, the equilibrium probability of satisfaction approaches 1.

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[1] Recall that we are excluding the uninteresting case of boundary solutions. Actually, since for sufficiently large L/N the slope of the $\pi = 0$ curve is arbitrarily large for low prices, and arbitrarily small for high prices, the Lipschitz conditions on the isoutility curves rule out the possibility of boundary solutions.

Proof: The method of proof will be to show that as L/N increases, the equilibrium point (p^*, $1-\lambda^*$) lies above the horizontal line defined by probability of satisfaction = $1 - \bar{\lambda}$ for $1 - \bar{\lambda} < 1$.

As before, equilibrium is determined by the point of tangency between the $\pi = 0$ curve and an isoutility curve. Choose any $1 - \bar{\lambda} < 1$. Increase L/N. As L/N increases, the slope along $\pi = 0$ curve at the point associated with a probability of satisfaction equal to $1 - \bar{\lambda}$ becomes arbitrarily large. Continue increasing L/N until the slope at $1 - \bar{\lambda}$ on the $\pi = 0$ curve exceeds B. Because of the concavity of the $\pi = 0$ curve, the slope along the $\pi = 0$ curve exceeds B for all $1 - \lambda < 1 - \bar{\lambda}$. Hence, for sufficiently large L/N, it is impossible for any isoutility curve to be tangent to the $\pi = 0$ curve for a probability of satisfaction less than or equal to $1 - \bar{\lambda}$. Since the equilibrium probability of satisfaction is bounded above by 1, and lies above every $1 - \bar{\lambda}$ less than 1, it follows that

$$\lim_{\frac{L}{N} \to \infty} 1 - \lambda^* \to 1.$$ Q.E.D.

It immediately follows from Theorems 1 and 2 that the equilibrium level of expected utility achievable by consumers in equilibrium approaches the level of utility achievable in the deterministic market, where price equals c and the probability of satisfaction equal one.

Theorem 3: As the customer per firm ratio, L/N, increases, the percent discrepancy between the amount supplied and the amount demanded approaches zero.

Proof: The total amount demanded equals the number of customers times the per capita demand, $L \cdot X(p)$, while the total amount supplied equals the number of firms times the customer capacity per firm times the per capita demand, $N \cdot s \cdot x(p)$. To prove the Theorem it is sufficient to show that $\frac{N \cdot s}{L} \to 1$ as $\frac{L}{N}$ increases.

Using (2) and (4), we can write that the zero profit condition implies

$$(1-\lambda)p \cdot L = Nc \cdot s$$

From the previous two theorems we know that in equilibrium $p \to c$ and $1 - \lambda \to 1$ as L/N increases. Hence the Theorem follows immediately.

Q.E.D.

Theorem 3 dealt with the percent discrepancy between supply and demand. What about the absolute discrepancy, $[L - N \cdot s]x(p)$ - does that too approach zero as the customer per firm ratio, L/N, increases? The answer in general is no. Usually the absolute discrepancy will approach either plus or minus infinity as L/N increases. In other words, equilibrium is possible even though the number of dissatisfied customers is arbitrarily large.

The reason why the market equilibrium does not converge to the deterministic one in all respects as the customer per firm ratio L/N increases can be explained as follows. As L/N increases, the total uncertainty in the market increases, so that market operation under uncertainty differs considerably from that under certainty. On the other hand, by the law of large numbers, the proportional risks caused by the uncertainty vanish as L/N increases. Therefore, percentage-wise concepts (e.g., supply ÷ demand), or concepts that apply to individual units of the good (e.g., price) or individual customers (e.g., probability of satisfaction) approach their values in the corresponding deterministic market as L/N increases. However, aggregate concepts such as supply, demand, and total number of customers dissatisfied do not, in general, approach their values in the deterministic market as the customer per firm ratio increases.

9. Comparisons of Market Clearing Under Certainty and Under Uncertainty

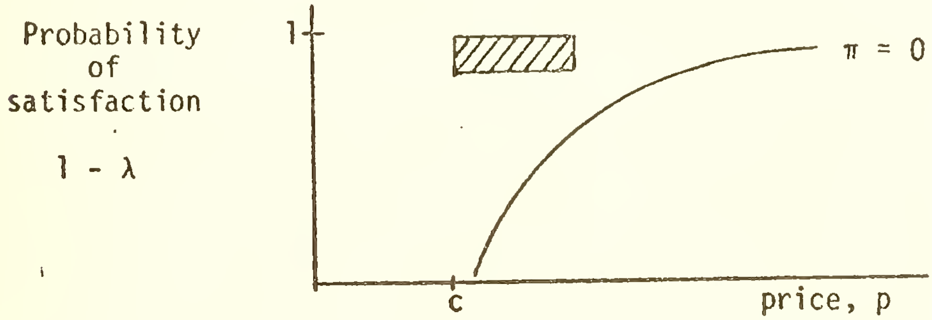
The reader might well be wondering just how important it is to examine market clearing under uncertainty by an analysis more complex than the simpler deterministic analysis that says price equals c, probability of satisfaction equals 1, and quantity supplied and demanded equals $L \cdot x(c)$. It is not possible to fully perceive the sharp differences between these uncertain markets and the traditional deterministic ones until the social welfare implications and the incentives facing interacting firms (Carlton, 1976a) are examined. Still, at this stage, it is possible to give a preliminary evaluation.

First, for "moderate" values of the customer per firm ratio, L/N, it is evident that the deterministic analyses could lead one totally astray. As seen above, equilibrium will usually involve having supply and demand out of balance, a price in excess of c, and a probability of satisfaction below 1. Certainly, for moderate values for L/N, the deterministic analysis is simply inadequate.

What about for "large" L/N - can the deterministic analysis suffice there? As seen above, even as L/N increases, the discrepancy in equilibrium between supply and demand need not vanish and will in general become arbitrarily large. A deterministic analysis will completely miss this potentially important feature of market equilibrium. On the other hand, Theorems 1 and 2 do assure us that for "sufficiently large" values of L/N, the deterministic analysis will predict correctly the price and probability of satisfaction.

The question naturally arises as to how large does L/N have to be before the deterministic analysis is not too far wrong in its predictions of price and probability of shortage. To answer this question, the exact shape of the isoutility curves would be needed. Alternatively, we can ask the somewhat simpler (and less informative) question of what is the smallest value for the customer per firm ratio such that it is even <u>possible</u> for the deterministic analysis to be "approximately" correct. The value of L/N that answers this question will <u>not</u> tell us that for all larger values of L/N the deterministic analysis will suffice. Indeed, the value of L/N for which the deterministic analysis does suffice will usually considerably exceed the value of L/N that answers the preceeding question. What the answer to the question does tell us is that if L/N is less than the calculated L/N, the deterministic analysis will definitely fail. The calculated value for L/N provides a lower bound on the value of L/N that is required if the deterministic analysis is to even have a chance of satisfying the desired tolerance limits. In terms of the diagram below, if the shaded box represents acceptable errors for the deterministic analysis, we want to answer the question of how large L/N has to be before the zero profit curve hits the lower right hand corner of the shaded box. Call this value of the $\frac{L}{N}$ the "critical" L/N value.

Figure 6 - Error Tolerances



Only for values of L/N larger than this critical value can the equilibrium possibly occur in the shaded region. Of course, this critical value of L/N depends on the size of the shaded region which reflects the size of the allowable errors.

For example, if we are not very demanding and are willing to tolerate a 10% error in price (i.e., the actual equilibrium price $\leq$ 1.1.c) and a 10% error in the probability of shortage (i.e., actual equilibrium probability of satisfaction $\geq$ .90), then the deterministic analysis will have a chance of succeeding only if L/N exceeds 60.[1] If we tighten our tolerance limits to a 2% error in price and a 2% error in the probability of satisfaction, L/N must rise to 1600 before equilibrium could

[1]This value for L/N is calculated using (4).

possibly fall in the shaded region. For more stringent requirements of only 1% errors in the price and probability of satisfaction, L/N must exceed 6500 before the deterministic analysis could even hope to meet the error standards. Considering that these figures are lower limits on the value of L/N needed if the deterministic analysis is to suffice, it seems that for most purposes in order to be sure the deterministic analysis will not make large errors in the price and probability of shortage, we must require what for most markets is an uncomfortably large customer per firm ratio.

In summary, for moderate values of the customer per firm ratio, L/N, the deterministic analysis is inadequate. It is only for very large, perhaps unrealistically large, values for the customer per firm ratio that the deterministic analysis will be able to yield some useful results. Even then, however, the deterministic analysis will be unable to detect arbitrarily large absolute discrepancies between supply and demand. The consequences of these markets on social welfare will make even clearer the differences between market clearing under certainty and under uncertainty.

10. Different Types of Customers

It is perfectly natural to imagine a market with two types of customers, who have different preferences between price and probability of satisfaction. In such a situation, it is possible to have an equilibrium in which two types of firms are established, each of which caters only to the preferences of one type of consumer. For example, suppose that there are two types of customers, and two types of firms. There are N_1 firms that cater to type 1 customers and N_2 ($N_2 < N_1$) firms that cater to type 2 customers. The equilibrium involving firm specialization is depicted in Figure 7.[1]

As Figure 8 illustrates, such specialized equilibrium may not always exist. The specialized equilibrium cannot exist because all the type 2 customers are better off at type 1 stores than at type 2 stores.

Figure 7 - Specialized Equilibrium

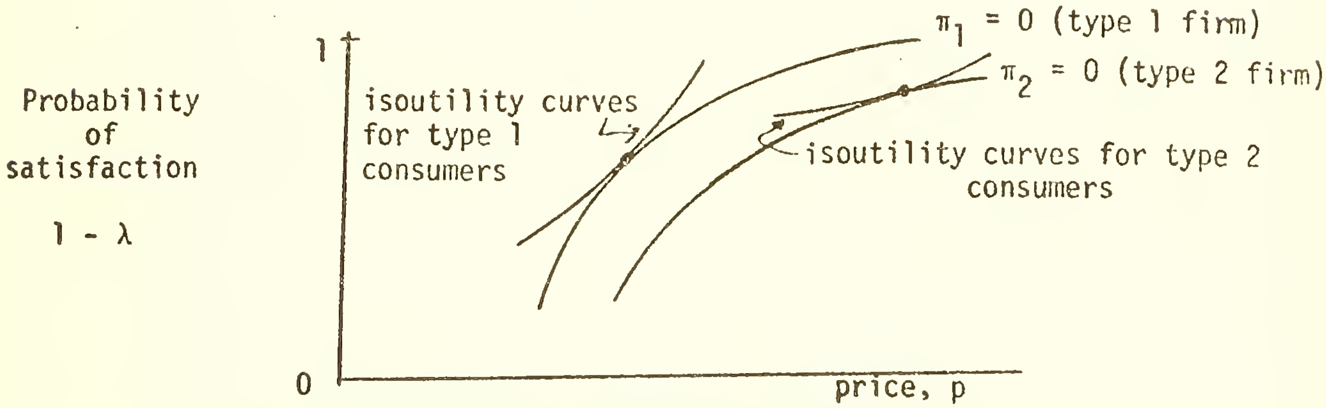
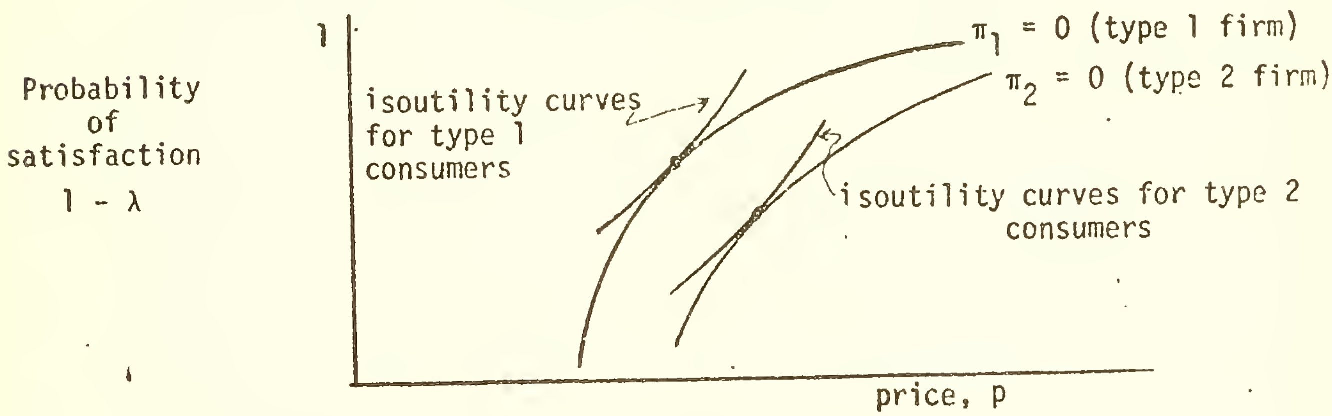


Figure 8 - Specialized Equilibrium Not Existing



[1] For the case of equilibrium involving firm specialization, an outside observer might incorrectly conclude that there was a distribution of prices for an identical good, and attribute it to consumer ignorance. As this paper emphasizes, since each type of firm offers a different probability of satisfaction, the "goods" at different types of firms are not identical.

When only one equilibrium can exist in the market, the question as to where it is established will be determined by the tastes of the majority. If any firm does not cater to the tastes of the majority, it will lose a majority of its business and will have to specialize in the minority's tastes. But, by assumption, specialized equilibrium is impossible, so the firm could not profitably attract just the minority types to its firm.

11. The Number of Firms

In the model, the number of firms, N, is exogenous and is taken to be greater than one. Although this is a perfectly reasonable assumption to make in order to investigate the short-run equilibrium for the markets under study, the question arises as to whether in the long run we expect the number of firms to exceed one, or whether we expect all N firms to merge into one giant firm. If two firms merge and are able to pool together their demand and their stocks, then the combined cost of operation of the merged firms will be lower than that of the unmerged firms. Because of the stochastic environment, there are economies of scale as demand increases. In this section, we suggest why the N firms may not have an incentive to merge together, and therefore why it is reasonable to expect there to be more than one firm in long-run equilibrium for the markets under study.

First, there might be congestions costs associated with an N firm merger. In other words, there may be some increasing costs associated with horizontal merger and expansion that are not in the model. Transaction costs might overwhelm any gain from merger and thereby prevent one firm from desiring to merge with other firms.

A second and very important reason has to do with spatial location. When we are talking about merger, we imagine two or more of the firms combining operations at the same location. However, if demand is assigned randomly on a geographic basis, then if two firms merge at the same location their combined total demand could fall. If two firms merge but maintain separate locations, then there are no gains to mergers unless the merged firms can ship goods back and forth amongst themselves. But, in the model, the reason why it was assumed that firms cannot receive delivery of the good during the market period was presumably because such delivery was costly and/or time consuming. In such cases, there may well be no gains to spatially separate mergers.

It is clear then that there are good reasons to expect that for the markets under study the number of firms will exceed one in the long-run equilibrium. For the remainder of this paper, we shall usually regard the number of firms as fixed and greater than one, and not distinguish between short- and long-run concepts of equilibrium.

12. Social Welfare Implications

The previous section examined how markets operate when the production decision must be made before the uncertain demand for the product can be observed, and when prices, once set, cannot vary over the market period.

An important question to ask is whether the competitive equilibrium involves a combination of price and probability of satisfaction that is optimal in the sense of maximizing some measure of social welfare. This is the issue that we now examine.

Throughout this examination, we do not allow insurance markets to develop. There are well-known reasons why such markets do not exist. For example, in the present case, there would be the problem of ascertaining that someone actually attempted to purchase the good. Such insurance markets rarely exist in practice. (If a customer finds a firm out of a good, or an airline booked up, there is not a market to compensate him.)

The first question we ask is when, if ever, will the market equilibrium maximize the expected value of the total consumer surplus. This question is motivated by two considerations. First, deterministic markets in competition maximize consumer surplus, so it is natural to see if uncertain markets do also. Second, expected consumer surplus is often used as an approximate measure of social welfare.[1] We will show that, in the special case where expected consumer surplus represents an individual's preferences between price and probability of satisfaction, the competitive equilibrium does indeed maximize the expected value of consumer surplus to society. This result contrasts with that of Brown and Johnson (1969) and Visscher (1973).

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[1] See for example Brown and Johnson (1969).

Consumer surplus as a measure of social welfare is known to suffer from several defects because of its partial equilibrium nature. Moreover, in an uncertain setting expected consumer surplus may not properly reflect consumer attitudes toward risk. To avoid these defects associated with consumer surplus, we examine the social welfare question in a simple two good model. We set up a two good model by introducing an alternative (nonrationed) good, and ask how a social planner who takes both markets into account would operate this economy so as to maximize the expected utility of a representative consumer. It will be shown that the socially optimal solution will usually be different from the competitive equilibrium. The socially optimal solution will, under a plausible assumption, involve paying lump sum subsidies to the firms that deal with the good that is subject to shortages. Compared to the social optimum, the competitive equilibrium will usually not devote sufficient resources to production of the good that is subject to shortages.

13. Maximizing Expected Consumer Surplus

As mentioned above, consumer surplus is not generally a good measure of social welfare for uncertain markets because, aside from the known problems associated with its partial equilibrium nature, it may not reflect consumer preferences between the probability of obtaining the good and the price of the good. For the special case where expected consumer surplus does reflect consumer attitudes toward risk, we want to examine whether the competitive equilibrium maximizes expected consumer surplus. The main result of this section is that for this special case, the competitive equilibrium does indeed maximize the expected consumer surplus to society.

The model is the same as before. Let us consider the expression for expected consumer surplus to society when all N firms follow the same price and stocking policy. Expected consumer surplus to society equals the per capita consumer surplus times the number of customers times the expected fraction of customers that are satisfied minus the cost of the goods. Expressed mathematically, we have that

$$\text{Expected Consumer Surplus to Society} \equiv CSS = (1 - \lambda(s)) \int_0^{x(p)} x^{-1}(q)dq \cdot L - c\, s\, N\, x(p), \qquad (6)$$

where, to refresh the reader's mind, we repeat the definitions:

L = number of customers in the market,

N = number of firms,

$x(q)$ = the per capita demand curve,

$x^{-1}(p)$ = the inverse per capita demand function,

p = price of the good,

c = cost per unit of the good,

s = the number of customers that can be serviced per firm, and

$1-\lambda$ = the probability of satisfaction as a function of s.

The government wishes to determine an operating policy, (i.e., s and p), so that expected consumer surplus to society (CSS) is maximized when all firms behave according to this operating policy. To maximize

CSS with respect to s and p, take derivatives of (6) to obtain the following first order conditions:

$$(1-\lambda)x'(p)pL - c\ s\ N\ x'(p) = 0,$$

or equivalently $$(1-\lambda)pL - c\ s\ N = 0 \qquad (7)$$

and $$\frac{d(1-\lambda)}{ds} \cdot L \int_0^{x(p)} x^{-1}(q)dq - c\ x(p)\ N = 0. \qquad (8)$$

Equations (7) and (8) determine the s and p of the operating policy for each firm that the government should follow to maximize the expected total consumer surplus to society.[1] Using the expression for profits derived earlier it can be seen that (7) is equivalent to the condition that expected profits per firm equal zero. Equation (8) determines the point along the zero profit (i.e., the $\pi = 0$) curve at which the government should operate.

The question then arises as to whether the competitive market equilibrium would maximize the expected value of consumer surplus to society if consumers' tradeoffs between the price of the good and the probability of obtaining the good were adequately represented by the expected value of their consumer surplus. At first glance, the answer to this question appears obvious. If expected consumer surplus reflects consumer preferences, then we know from the properties of equilibrium in an uncertain

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[1] As before we disregard the uninteresting case of boundary solutions and assume that (7) and (8) have a solution that represents an interior maximum.

market that the expected consumer surplus of each individual is maximized (barring the multiple tangencies discussed in Section 4). Hence, the social planner will maximize the consumer surplus to society at this point. This reasoning is faulty although the conclusion turns out to be correct. The sum of individual consumer surplus does not equal the consumer surplus to society for the markets under study. There are unsold goods at the end of each period which must enter the government's calculation of consumer surplus but not that of any individual.

More specifically, suppose each of the L consumers seeks to maximize

$$\text{Consumer Surplus to an Individual} \equiv \text{CSI} = (1-\lambda)\left[\int_0^{x(p)} x^{-1}(q)dq - p\, x(p)\right] \tag{9}$$

where the notion was defined previously beneath (1).[1] Summing CSI over all L consumers and comparing this sum to the objective function, CSS, of the government, we see that the two expressions differ by

$$(1-\lambda)pLx(p) - csx(p)N = x(p)[p(1-\lambda)L - sNc].$$

This last expression is the difference between the expected revenue to be received and the cost of all the goods, sold and unsold. In view of the differences in the objective functions between the individual and the government, it is interesting that the following theorem holds.

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[1] Consumers will maximize CSI if their Von Neumann utility functions are of the form $u(x_1,x_2) = g(x_1) + x_2$, where x_1 = the good under analysis, and x_2 = all other goods.

Theorem 4: Suppose expected consumer surplus to an individual, CSI, as defined in (9), represents consumer preferences between the price, p, and probability of satisfaction, $1-\lambda$. Then, as long as problems with multiple tangencies (see section 4) do not arise, the competitive equilibrium maximizes the expected value of consumer surplus to society (CSS).

Proof: If CSI reflects consumer preferences, then from the definition of competitive equilibrium, we know that barring multiple tangencies the competitive equilibrium occurs at that point along the zero profit curve that maximizes CSI. From (7), we know that the point that maximizes CSS also occurs along the zero profit curve.

The difference between consumer surplus to society, CSS, and the sum of consumer surplus to an individual, L·CSI, was derived above and equals $x(p)[(1-\lambda)pL - N\ s\ c]$. However, from (7), we see that along the zero profit curve, this difference equals zero. Therefore, along the zero profit curve, the two measures, CSS and L·CSI, attain their maximum values at the same point. Q.E.D.

We see then that if individual consumer preferences are represented by expected consumer surplus (CSI), then just as in deterministic markets, the competitive equilibrium (provided it occurs at the tangency with the highest isoutility curve) will maximize the expected value of the total consumer surplus to society (CSS). Notice that price exceeds c and firms earn zero expected profits when expected consumer surplus is maximized.[1] These results contrast sharply with those of other models that appear in the public finance literature (Brown and Johnson (1969), Visscher (1973)) and deal with a similar type of problem.[2] The results of those other models imply that to maximize expected consumer surplus to society, price should in general be less than c, and hence firms should operate at an expected loss.

The reason for the difference in results stems from the manner in which the randomness is introduced into the demand curve and the way goods are rationed. In the model under study, a firm's demand is multiplicative and equals $x(p) \cdot i$ where $x(p)$ equals per capita demand and i equals the random number of consumers who visit the firm. All customers face the same probability of being rationed. In Brown and Johnson (1969), rationing is done by willingness to pay with the demand for units that generate large consumer surplus being satisfied first. As Visscher (1973) points out, it is difficult to imagine how such a rationing scheme could be implemented without using a recontracting market. In Visscher's models, more realistic rationing schemes are introduced, however only the case of additive demand uncertainty is analyzed. Additive demand implies

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[1]It should be mentioned that the derivation of (7), and hence the result that expected profits equal zero at the social optimum does not depend on demand being normally distributed.

[2]To see the relation of the model of this paper to the peak load problem under uncertainty, reinterpret c as the fixed cost per unit. In the model the marginal variable cost β is taken as 0. If β is nonzero, prices would rise by β. The model above suggests that under certain assumptions $p = c + \beta$ is optimal. The previous models in the literature suggest that $p < c + \beta$ is optimal.

that the absolute variation in demand is independent of the level of expected demand. So, for example, a 100 unit demand deviation is regarded as equally likely whether expected aggregate demand is 200 or 2 million. Multiplicative uncertainty implies that the relative variation in demand is independent of the level of expected demand. So, for example, a 1% deviation in demand is regarded as equally likely whether expected aggregate demand is 200 or 2 million. For most purposes, the multiplicative formulation would appear more plausible.[1]

If expected consumer surplus, CSI, does not represent consumer preferences for the probability of satisfaction, $1-\lambda$, and the price, p, then Theorem 4 will not hold. However, if CSI does not represent consumers' preferences toward risk, then the expected consumer surplus is a very poor criteria to use as a measure of market performance in an uncertain environment.[2] In the next section, we allow the consumer to have quite general preferences for the probability of satisfaction and the price, and examine how the introduction of an alternative good affects the analysis of the social optimum.

14. The Social Optimum in a Simple Two Good Model

Let there be two goods on which each of the L consumers can spend their identical endowment Y. Good 1 is the good that is subject to shortages.

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[1] This is one reason why econometric equations are specified so often in log-log form.

[2] This point is not addressed by either Brown and Johnson (1969) or Visscher (1973).

Good 2 is always available from the outside world at a constant price. The price of good 1 is p, while the price of good 2 is 1. As before, each unit of good 1 uses up c units of resources and must be produced before any firm observes its random demand. Demand is random in the same manner as discussed previously. As usual, no firm can receive delivery of the good once a market period has begun. The government owns each of the N firms that dispense good 1, and wishes to choose the same tax policy and operating policy for each of the N firms so as to maximize the expected utility of a representative consumer. The government faces the budget constraint that the sum of the firms' expected profits plus the total taxes collected or dispersed must equal zero.

Let u(x,z) represent the Von Neumann utility function of each consumer where x denotes good 1 and z denotes good 2. When good 1 is obtainable at price p, the utility of each consumer is given by V(p,Y), the indirect utility function. When good 1 is not obtainable, the utility of each consumer is given by u(o,y). If $1-\lambda$ represents the probability of obtaining good 1, then the expected utility of a representative consumer can be written as

$$U(1-\lambda,\ p) = (1-\lambda)V(p,Y) + \lambda u(0,Y).$$

The government seeks to determine a transfer, T, for each individual,[1] a price, p, and a customer capacity s (recall that s refers to the maximum

[1] The variable T is the transfer from the firms to each consumer. Hence, if $T < 0$, consumers pay a tax, while firms receive a subsidy.

number of customers that can be serviced at any firm in any market period), so that the expected utility of any consumer is maximized. The government's problem can be written as

$$\max_{s,p,T} \quad (1 - \lambda(s))V(p,Y + T) + \lambda(s)u(0,Y + T) \tag{10}$$

subject to the budget constraint,

$$\pi(s,p) - \frac{L}{N} T = 0, \tag{11}$$

where $\pi(s,p)$ is the expression for expected profits per firm, which can be written as

$$\pi(s,p) = (1 - \lambda(s))p \frac{L}{N} x (p) - c\ s\ x(p) \tag{12}$$

where $1 - \lambda(s)$ is the expression for the probability of satisfaction as a function of customer capacity, s, and is given by (6).

From the statement of the problem, we see that if (and only if) the transfer, T, equals 0 in the socially optimal solution, then it follows that the competitive equilibrium will also be the socially optimal point since both points maximize expected utility subject to the constraint that expected profits are zero. In general, there is no reason to expect that the optimal solution to the above problem will have T = 0, so that the competitive equilibrium will usually not represent the social optimum. The social optimum will usually involve either taxes or subsidies for the firms who sell good 1, the good subject to shortages. In such cases government intervention into a competitive market will be called for.

In order to investigate the conditions under which either taxes or subsidies will be paid in the social optimum, it is necessary to make an assumption about consumers' preferences.

Assumption 1: The marginal utility of an extra dollar, when good 1 is obtainable, is higher than the corresponding marginal utility when good 1 is unobtainable. More precisely, $V_2(p,Y) > u_2(0,Y)$ for all p,Y, where the subscripts denote partial differentiation.

The assumption reflects the idea that the greater the variety of goods that can be purchased, the higher is the marginal utility of an extra dollar. (One sufficient condition for this assumption is that $u_{21} \geq 0$.) Given the above assumption, the following theorem holds.

Theorem 5: Under Assumption 1 and the assumption that per capita demand depends positively on income, the social optimum involves operating the N firms that sell good 1 at a loss and using lump sum taxes to subsidize their operation.

Proof: The Lagrangian for the government's maximization problem can be written as

$$\mathcal{L}(s,p,T,\mu) = (1-\lambda)V(p,Y+T) + \lambda u(0,Y+T) - \mu[((1-\lambda)p\cdot \frac{L}{N} - cs)x(p,Y+T) - \frac{L}{N}T], \qquad (13)$$

where μ is a Lagrange multiplier.[1] The first order conditions are:[2]

[1]The notation was defined previously. Recall that λ is not a Lagrange multiplier, but is the probability of disappointment which is a function of s given in (6).

[2]Subscripts denote partial differentiation.

$$(1-\lambda)V_1 = \mu[(1-\lambda)\frac{L}{N}x + [(1-\lambda)\frac{L}{N}p - sc]x_1], \tag{14}$$

$$(1-\lambda)V_2 + \lambda U_2 = \mu[(1-\lambda)\frac{L}{N}p - sc]x_2 - \frac{L}{N}], \tag{15}$$

$$(1-F)\frac{N}{L}[V-U] = \mu[(1-F)p - c]x, \tag{16}$$ and

$$[(1-\lambda)\frac{L}{N}p - cs]x = \frac{L}{N}T \tag{17}$$

Substituting (17) into (14) and (15), we obtain

$$(1-\lambda)V_1 = \mu[(1-\lambda)\frac{L}{N}\cdot x + \frac{L}{N}\frac{T}{x}x_1] \tag{18}$$

and

$$(1-\lambda)V_2 + \lambda U_2 = \mu[\frac{L}{N}\frac{T}{x}x_2 - \frac{L}{N}]. \tag{19}$$

Since V is an indirect utility function, we have that $x = -\frac{V_1}{V_2}$. Using this relation, rewrite (18) as

$$(1-\lambda)V_1 = \mu[(1-\lambda)\frac{L}{N} + \frac{L}{N}\frac{T}{x}\frac{x_1}{x}]\cdot(-\frac{V_1}{V_2}),$$

or

$$(1-\lambda)V_2 = (-\mu)[(1-\lambda)\frac{L}{N} + \frac{L}{N}\frac{T}{X}\cdot\frac{x_1}{x}],$$

or,

$$V_2 = (-\mu)[\frac{L}{N} + \frac{L}{N}\frac{T}{x}\cdot\frac{x_1}{x}\frac{1}{1-\lambda}]. \tag{20}$$

From (19) and Assumption (1), it follows that

$$\mu[\frac{L}{N}\frac{T}{x}x_2 - \frac{L}{N}] < V_2 \tag{21}$$

Substituting the expression for V_2 from (20) into (21), we have that

$$\mu[\frac{L}{N}\frac{T}{x}x_2 - \frac{L}{N}] < (-\mu)[\frac{L}{N} + \frac{L}{N}\frac{T}{x}\frac{x_1}{x}\frac{1}{1-\lambda}], \text{ or}$$

$$(-1)(-\mu)\frac{L}{N}\frac{T}{x}x_2 - \mu\frac{L}{N} < -\mu\frac{L}{N} - \mu\frac{L}{N}\frac{T}{x}\frac{x_1}{x}\frac{1}{1-\lambda}, \text{ or since } -\mu > 0,$$

$$(-1)\frac{Tx_2}{x} < \frac{T}{x}\frac{x_1}{x}\frac{1}{1-\lambda}, \text{ or}$$

$$(-1)T\,x_2(1-\lambda) < \frac{T}{x}x_1, \text{ or}$$

$$T(-\frac{x_1}{x \cdot x_2}) < T(1-\lambda). \tag{22}$$

If $T > 0$, then $\frac{-x_2}{x \cdot x_2} < 1 - \lambda < 1$, while if $T < 0$, then $\frac{-x_1}{x \cdot x_2} > 1 - \lambda$. But from the Slutsky equation, we know that $x_1 + x \cdot x_2 < 0$ or $\frac{-x_1}{x \cdot x_2} > 1$. Therefore if $T > 0$, we obtain a contradiction. Hence only $T < 0$ is possible in the optimal solution. Q.E.D.

Therefore, under Assumption 1, the socially optimal solution involves operating the N firms that produce good 1 at a loss, and using lump sum taxes to subsidize these firms' revenues. Since the competitive equilibrium involves zero profits, we see that government intervention into a competitive market will be necessary to achieve the social optimum.[1]

[1]As should be clear from the proof of the theorem, if we replace Assumption 1 with the (less plausible) assumption that the marginal utility of income declines as the variety of goods increases, then in the social optimum firms would be taxed and consumers subsidized.

The heuristic reason why, under Assumption 1, it is optimal to tax consumers and pay subsidies to firms can be seen as follows. There are two states in which the consumer can wind up, one where he can purchase the good at the market price and one where he cannot. Under Assumption 1, the last dollar is more valuable in the state in which the good is obtainable than in the state in which the good is unobtainable. A person could increase his utility if he could in some way transfer part of his income between the two possible states. Such transfers of income are impossible in the problem under examination. (Remember no insurance or recontracting markets exist.) However, what is possible is that the government can use taxes to reduce the income of a consumer in both states, and subsidize the operation of firms that produce the good and thereby reduce the price of the good subject to shortages. In this way, a transfer of purchasing power can occur between the two possible states in which the consumer can find himself. It turns out that this price reduction is always sufficient to overwhelm the decline in income, so that under Assumption 1 imposing some taxes always raises expected consumer utility.

Theorem 5 tells us that the competitive equilibrium will not achieve the social optimum. Can we say whether, under Assumption 1, the competitive equilibrium will devote too few resources to firms selling good 1 and/or will involve a higher price for the good than occurs in the social optimum? Without further restrictions, all that can be said is that in the social optimum either the probability of satisfaction, (or equivalently the customer capacity, s) will be higher and/or the price of good 1 will be lower[1] than in the competitive equilibrium. We expect the normal case to involve an increase in the probability of satisfaction $1-\lambda$, and a decrease in the price p. For this normal case it immediately follows that under Assumption 1 the competitive equilibrium will involve devoting too few resources (i.e., c s N x(p)) to the production of the good that is subject to shortages, when compared to the social optimum.

15. Summary

This paper has examined the behavior of markets characterized by price inflexibility, demand uncertainty, and production lags. It appears that many private and government regulated markets are better described by the model examined here than by the traditional supply and demand model. It was possible to prove that as the size of the market increased, that percentagewise the equilibrium under uncertainty approached that under certainty. Numerical calculations suggested that the customer per firm ratio might have to be unrealistically large for this convergence. The social welfare implications of markets under uncertainty differ from those under certainty. In the special case where expected consumer surplus reflects consumer's preferences, the competitive equilibrium is optimal. This result contrasts with previous results in the literature. In general though, the competitive equilibrium is not socially optimal, and the conditions under which subsidization would occur were derived.

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[1] In fact, in the social optimum, not only is it possible for the price to be below c, but it is even possible for the price to fall to zero. For example, if $V(p,Y) = \frac{Y}{p}$ and $u(0,Y) = 0$, then the social optimum involves the price falling to zero and the transfer going to $-Y$ in such a way that $\frac{(Y+T)}{p}$ remains finite.

In a subsequent paper, Carlton (1976b), the issues of regulation, monopoly behavior, and responses to increasing risk are discussed within the context of these uncertain markets. In Carlton (1976a), the question of how firms in different markets interact in an uncertain environment is examined. Incentives for vertical integration to assure a certain source of input supplies are analyzed. What this paper and the just cited papers make clear is that the behavior of markets characterized by price inflexibility, demand uncertainty, and production lags differs in important respects from that of traditional deterministic markets.

Appendix 1

Define the expected shortage, M, for one store with customer capacity s as

$$M(s) = \sum_{s+1}^{\infty} (i-s)\ pr(i), \text{ or}$$

$$M(s) = \sum_{s+1}^{\infty} (i-\bar{s})\ pr(i) + (\bar{s}-s)\ pr(i), \text{ or}$$

$$M(s) \doteq -\sigma N^{L}(u) - u\sigma(1 - F(u)),$$

where $u = \frac{s - \bar{s}}{\sigma}$, $\bar{s} = \frac{L}{N}$, $\sigma = \sqrt{\frac{L}{N}}$, $N^{L}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} te^{-\frac{1}{2}t^{2}} dt$, $\bar{s} = E(i)$, $pr(i) =$ binomial probability that i of the L customers come to one store.

Notice that $\bar{s}$ and σ^2 are the mean and approximate variance of this binomial process. Hence $\frac{i - \bar{s}}{\sigma}$ is approximately normally distributed with mean 0, and variance 1. If all firms follow the same operating policy then

$$1 - \lambda(s) = 1 - \frac{N \cdot M}{L}, \text{ or}$$

$$1 - \lambda(s) = 1 - \frac{M}{\sigma^2} \text{ or}$$

$$1 - \lambda(s) = \frac{\sigma^2 + \sigma N^{L}(u) + \sigma u[1 - F(u)]}{\sigma^2}, \text{ or}$$

defining $I(s) = N^{L}(u) - uF(u)$, we have

$$1 - \lambda(s) = (\sigma I + s)/\sigma^2$$

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