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ON THE IRRELEVANCE OF TRADE TIMING

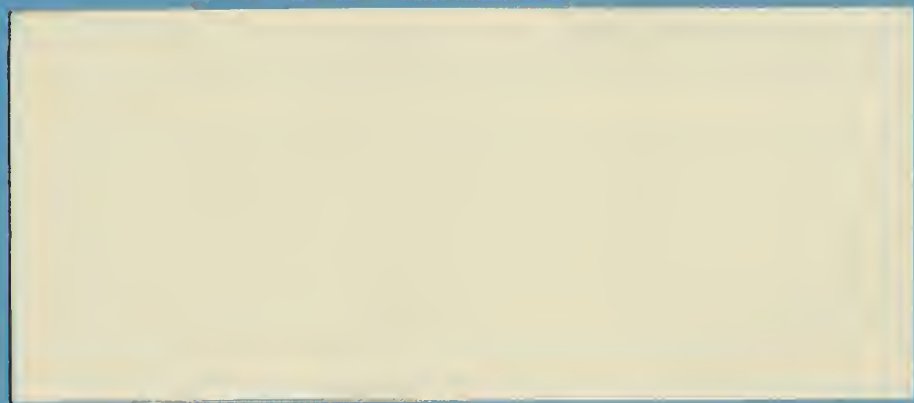
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Feb. 1996

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*On the Irrelevance of Trade Timing**

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February 4, 1996

Abstract

Given standard, transparent assumptions, this paper questions the Wall Street adage that ‘timing is everything’. I show that for an Arrow security, a ‘small’ risk-neutral trader with private information that is *conditionally independent* of the public information is exactly *indifferent* about the timing of his trade: His *expected return* per dollar invested is a *martingale*. This is true despite the fact that he expects the asset price itself to rise given favorable information and fall given unfavorable information.

I demonstrate the result in generality and point out that the Arrow security assumption cannot be relaxed: With compound securities paying on two states, there is a (generically strict) preference to trade immediately, while for more general assets, the result is ambivalent — one may even wish to *delay* trading!

*I owe special thanks to Massimiliano Amarante for proving a key inequality (\blacktriangle). I am also grateful to Glenn Ellison for inquiring of an economic interpretation of the ‘conditional martingale’ used in an early version of Smith and Sørensen (1996), and to Angel Serrat Tubert for helping me clarify my thoughts. I have likewise profited from comments on the usenet group `sci.math.research` — including Dan Cass (S.J.F.C. in Rochester), and especially Dražen Pantić (University of Belgrade) for extending the proof of the timing irrelevance proposition to nondiscrete measures. Finally, I thank Peter DeMarzo, Thomas Piketty, Paul Samuelson, Peter Sørensen, Jiang Wang, and Richard Zeckhauser for helpful feedback. I acknowledge the NSF for financial support. Any errors and opinions expressed are mine alone.

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1. INTRODUCTION

‘Timing is everything’ is a popular refrain on Wall Street. Yet the finance literature has remarkably little to say about a topic that is deemed of supreme importance among applied practitioners. This paper attempts to redress this anomaly, by inquiring of the *purely informational motives* for trade timing. As such, I consider the problem of a *small trader* with no strategic reasons to time his trade. When is his ‘informational edge’ over the market maximized?

To fix ideas, consider the following hypothetical scenario. A corporate insider of a biotech firm has gotten wind of an announcement next Friday of the unexpected FDA approval of his firm’s soon-to-be major drug. The insider naturally wishes to profit from this information.¹ Of course, in theory there are obscenely large arbitrage opportunities: Indeed, by buying on slim enough margins, the insider can expect to reap tremendous profits. So suppose for whatever reason that he cannot buy on margin — a restriction that will ring truer when I consider only imperfect information.² Let him have a given amount of money with which to invest, by buying stocks, and assume he is poor enough that his purchases do not affect the price. When should he purchase? Typical gut instincts argue in favor of buying immediately. Indeed, at one obvious level, delay only sees the depreciation of his informational advantage, as he expects the price to start rising. But that is not the issue here, as the trader cares not so much about the asset *price* as its reciprocal, the *return* (or his ‘bang per buck’).

By this yardstick, I now wish to make the point that under standard and transparent assumptions, so long as his timing is in advance of the public announcement, it is *completely irrelevant!* To illustrate this hopefully striking proposition, let’s consider the following simple two-period, two-state model. The asset pays off 1 or zero in the ‘high’ and ‘low’ states H, L , respectively, and the prior chance $\mathcal{P}(H) = 1 - \mathcal{P}(L) \in (0, 1)$ is given. Discounting is ignored, and traders are assumed risk neutral, so that $p_0 = \mathcal{P}(H)$ is also the initial asset price. Let the insider discover his private information that the payoff will be

1. He wishes buy as many shares as his wealth will permit, and being ‘small’, this activity

¹This example is for illustrative purposes only, and neither endorses such trading activity, nor suggests that it actually occurs.

²More generally, with unfavorable information, assume he likewise faces a short sale constraint.

will not affect the price. If he purchases immediately, his return per dollar will be $1/p_0$. If he waits until the next period's public information is revealed, namely the announcement of a public signal σ whose outcome in $\{\sigma_1, \dots, \sigma_n\}$ is correlated with the state of the world H or L , then the expected price is higher: $E[p_1 | H] = \sum_{k=1}^n \mathcal{P}(\sigma_k | H) \mathcal{P}(H | \sigma_k) \geq p_0$.³ But the insider's expected return is unchanged! For it equals the expected reciprocal of the next price p_1 , namely^{4,5}

$$E[1/p_1 | H] = \sum_{k=1}^n \frac{\mathcal{P}(\sigma_k | H)}{\mathcal{P}(H | \sigma_k)} = \sum_{k=1}^n \frac{\mathcal{P}(H \cap \sigma_k) / \mathcal{P}(H)}{\mathcal{P}(\sigma_k \cap H) / \mathcal{P}(\sigma_k)} = \sum_{k=1}^n \mathcal{P}(\sigma_k) / \mathcal{P}(H) = 1/p_0$$

In this paper, I wish to explore the generality of this observation. Obviously, trade timing is not always moot. For one, the insider here has perfect information. The paper formulates and proves the timing irrelevance proposition for any partially informed trader — *so long as his information is conditionally independent of the public information given the true state of the world*. It must be underscored that this is the standard regularity assumption in the literature. It so happens that the result turns on the asset being a pure 'Arrow security', paying a dividend of \$1 in one state of the world, and 0 elsewhere. Once one ventures outside this domain, I can show that timing irrelevance no longer prevails. In fact, with compound bundles of two Arrow securities, there is a (generically strict) preference to trade immediately on any private information, while the result is ambivalent for more general securities: I show by example that one may even wish to *delay* trading!

Let me briefly explore the economics of my main result. Why is the return constant in expectation, even though the price is expected to rise (with favorable information)? There is a simple perhaps well-known intuition that at least justifies why the return process is more favorable than the price process: Since the return is strictly convex in the price, it benefits from the riskiness of the stochastic price movements. For instance, if the current price is $1/3$, an equal chance of moving to $1/2$ or $1/4$ leaves the expected price at $3/8 > 1/3$ tomorrow, while the expected return is flat at 3. But why then does the benefit *exactly*

³This intuitive result follows from the Cauchy-Schwartz inequality $(\sum a_k b_k)^2 \leq (\sum a_k^2)(\sum b_k^2)$, if we put $a_k = \mathcal{P}(H \cap \sigma_k) / \sqrt{\mathcal{P}(\sigma_k)}$ and $b_k = \sqrt{\mathcal{P}(\sigma_k)}$.

⁴The identity sums over all 'possible' public signals σ_k , namely those with $\mathcal{P}(\sigma_k \cap H) > 0$.

⁵One might hazard that with a longer time horizon, the trader could learn from early public signals to better time his trade; but this conjecture is false, for the basic two-period logic can just as well be iterated into the future.

cancel out the expected rise in the price? Here, I have not found any deeper intuition than is suggested by a close reading of the above example: the vector of possible prices next period is proportional to the vector of quotients of conditional and unconditional signal probabilities, or $\mathcal{P}(H|\sigma_k) \propto \mathcal{P}(\sigma_k|H)/\mathcal{P}(\sigma_k)$. Thus, the expected return — the dot product of the return vector and the vector of conditional signal chances — is independent of the signal distribution; since the expected return is unchanged for signals that are pure noise, the result follows. This intuition extends to the case of partial information in the paper.

The finance literature has been very focused in what it has to say about trade timing.⁶ First, there is a large and well-established literature on option pricing, especially for the American call and put options. The exercise date of such an instrument is a choice variable, and one may optimally wish to exercise it before its expiration date. For my purposes, this is relevant only insofar as it represents a single-person decision problem dealing with timing a discrete trade. But in two absolutely crucial respects, the option pricing problem is a different beast: It is concerned not with returns but with prices, and more crucially, the interaction between *private* and *public information* is a non-issue for option pricing. For this reason, timing work in Ross (1989) on the phenomenon of ‘uncertainty resolution irrelevancy’, is also not relevant here — and my resolution date is assumed known, anyway.

Second, there is a large and growing literature on the optimal timing of trades when faced with *strategic* considerations. See, for instance, Kyle (1985) and (1989), Admati and Pfleiderer (1989), and more recently, Smith (1995). This line of work is clearly orthogonal to my thrust here — which concerns a ‘small trader’ effect.

Third, there has been work on market timing based on returns, but at a much more heuristic level than here. The best paper of this genre I have seen here is Merton (1981), which uses CAPM rather than the *arbitrage pricing theory* (APT) as I have. If anything, it is enlightening to read that literature knowing my main result.

The next section describes a plain vanilla model in which I can state and prove both the timing irrelevance proposition for Arrow securities and a timing impatience proposition for compound securities on two states. An example follows with a strict preference to *delay* trading for a three state security! I close with a conclusion and a mathematical appendix.

⁶See Duffie (1992) or Merton (1990) for references, a discussion of this point, and a glimpse at how little attention the mainstream finance literature has paid to issues of timing.

2. THE FORMAL MODEL AND RESULTS

★ **Information Structure.** To be deliberately uncontroversial at this stage, I shall build on the now standard (1982) model of Milgrom and Stokey (MS) and partition the state space Ω into two statistically-related components $\Omega = \Theta \times \Sigma$. Think of Θ as the payoff relevant *states*, and Σ as the space of payoff irrelevant *signals* realizations. The set Θ is assumed finite. While no harm is done by treating Σ also as discrete, my later analysis (unlike MS) will also admit general signal spaces. I shall therefore define \mathcal{S} , the sigma algebra on Σ — namely, the space of all ‘signal events’. Let $\omega = (\theta, \sigma) \in \Omega$. The space Ω is somewhat tedious, so I shall instead simply think of the signals as stochastic variables $\sigma : \Theta \rightarrow \mathcal{S}$ mapping the payoff-relevant state space into the signal events (or, signal outcomes). Bayesian traders observing a realization of σ will be able to make inferences about θ . Finally, let \mathcal{P} be the common prior probability measure on Ω .

The small trader with the starring role in this paper is initially endowed with a private informative signal τ , and observes in particular that $\tau \in T \in \mathcal{S}$. In periods $n = 1, 2, \dots, N$, a public signal $\sigma_n \in S \in \mathcal{S}$ about the state of the world is realized. Crucially, I assume that

(I) *All signals $\tau, \sigma_1, \sigma_2, \dots$ are independent, conditional on each state $\theta_k \in \Theta$*

Beyond the primary assertion that the private and public information is conditionally independent, this also precludes any distracting incentives the trader might have to try to learn from early public signal realizations and make inferences about future ones. Let \mathcal{S}_n be the sigma-field generated by the first n signals $(\sigma_1, \dots, \sigma_n)$. Thus, $\langle \mathcal{S}_n \rangle$ is a *filtration* within \mathcal{S} , or equivalently a progressively refined sub-sigma-field of \mathcal{S} . (So every event in \mathcal{S}_n is also an event in \mathcal{S}_{n+1} , for instance.)

★ **Payoffs and Prices.** Let the payoff function of the asset be $\pi : \Theta \mapsto \mathbb{R}$, realized and observed in period N (a known date), with no discounting until then. So the price of the asset at the end of period n is the expected asset payoff, or $p_n \equiv \mathcal{E}[\pi | \mathcal{S}_n]$. Embedded here is the standard assumption that the asset is efficiently priced given risk neutral traders. As usual, the random price process $\langle p_n \rangle$ corresponding to the filtration $\langle \mathcal{S}_n \rangle$ must be a *martingale* (when adapted to $\langle \mathcal{S}_n \rangle$). That is, $p_m = \mathcal{E}[p_n | \mathcal{S}_m]$ for all $n > m$. This implies that there are no arbitrage opportunities on the basis of public information alone. But given one’s private information, one anticipates a period by period erosion in one’s

informational edge. Indeed, the following result is not surprising (and its proof omitted).

Lemma (Prices) *Under assumption \mathcal{I} , prices are expected to move in the direction of private information:*

$$\mathcal{E}(\pi|\mathcal{S}_n, T) \lesseqgtr \mathcal{E}[\mathcal{E}(\pi|\mathcal{S}_{n+1}, T) | \mathcal{S}_n, T]$$

as the private information $\tau \in T$ is favorable or unfavorable, or $\mathcal{E}(\pi|\mathcal{S}_n, T) \gtrless \mathcal{E}(\pi|\mathcal{S}_n)$.

So on price considerations alone, the individual ought to trade right away on his private information, regardless of the informational structure. If the trader is ever more optimistic than the market, then he is always so, by the conditional independence of the signals. But the purchase price is expected to rise, so now is the anticipated *cheapest* time to buy.

★ **Timing Irrelevance.** As noted earlier, the preceding logic is not relevant if one is simply trying to invest a given amount. Rather, one cares about the *return* per dollar. This is true even for short-selling — namely, buying negative units of the share worth \$1 say, and later netting out by buying the shares back after the asset value is realized.

In this general setting, the most far-reaching timing irrelevance claim would hold that the return of an investment of \$1 in period n equals the expected payoff from deferring this investment one period, namely

$$\mathcal{E}[\pi|\mathcal{S}_n, T]/p_n = \mathcal{E}[\mathcal{E}[\pi|\mathcal{S}_{n+1}, T]/p_{n+1} | \mathcal{S}_n, T]$$

This conjecture is false. I shall soon explore the nature of its failure, and will point out that the inequality can go either way. The reason is that the range of the space of payoff functions π is simply too rich. What I must do is restrict focus to a pure *Arrow security* — namely, one in which $\pi = \mathbb{I}_{\theta_0}$, the ‘indicator function’ on some state $\theta_0 \in \Theta$. Such a simple security awards a payoff $\pi(\theta) = 1$ if $\theta = \theta_0$, and 0 otherwise. General *compound securities* are linear combinations of Arrow securities. With this motivation, we have

Proposition 1 (Timing Irrelevance) *Under assumption \mathcal{I} , a partially informed trader with a fixed amount to invest is indifferent about how he times his trades of an Arrow security:*

$$\mathcal{P}(\theta_0|\mathcal{S}_n, T)/\mathcal{P}(\theta_0|\mathcal{S}_n) = \mathcal{E}[\mathcal{P}(\theta_0|\mathcal{S}_{n+1}, T)/\mathcal{P}(\theta_0|\mathcal{S}_{n+1}) | \mathcal{S}_n, T]. \quad (1)$$

The proposition is proved in the appendix, but right here I shall assume a discrete signal space Σ , where the argument is less general but certainly more transparent. In particular, forget about the multiple periods, and simply denote $\mathfrak{S} = \{S_1, S_2, \dots, S_M\}$. Thus, \mathfrak{S} is a totally exhaustive and mutually exclusive partition of Σ , one of whose elements will be publicly observed next period. Then given information $\tau \in T$, the expected return next period for an Arrow security \mathbb{I}_H coincides with its current return, since

$$\begin{aligned}
\mathbb{E}[\mathcal{P}(H|S_m, T)/\mathcal{P}(H|S_m)|T] &= \sum_m \mathcal{P}(S_m|T)\mathcal{P}(H|S_m, T)/\mathcal{P}(H|S_m) \\
&= \sum_m \mathcal{P}(S_m|T) \frac{\mathcal{P}(S_m|H, T)\mathcal{P}(H|T)/\mathcal{P}(S_m|T)}{\mathcal{P}(S_m|H)\mathcal{P}(H)/\mathcal{P}(S_m)} \\
&= \sum_m \mathcal{P}(S_m|T) \frac{\mathcal{P}(S_m|H)\mathcal{P}(H|T)/\mathcal{P}(S_m|T)}{\mathcal{P}(S_m|H)\mathcal{P}(H)/\mathcal{P}(S_m)} \\
&= \mathcal{P}(H|T)/\mathcal{P}(H) \sum_m \mathcal{P}(S_m) \\
&= \mathcal{P}(H|T)/\mathcal{P}(H)
\end{aligned}$$

where all steps are simplifications except the middle equality, which is the statement of conditional independence, namely $\mathcal{P}(S_m|H, T) = \mathcal{P}(S_m|H)$.

The intuition in the introduction is now more involved. The public signal-dependent return $\mathcal{P}(H|T, S_m)/\mathcal{P}(H|S_m)$ next period is really a measure of an *informational premium* of T given S_m . The vector of such premia is proportional to the signal likelihood ratio of state H given information T divided by the unconditional signal likelihood ratio of state H , or $\mathcal{P}(S_m|H, T)/\mathcal{P}(S_m|T) \cdot \mathcal{P}(S_m)/\mathcal{P}(S_m|H)$. So this time, provided conditional independence holds, or $\mathcal{P}(S_m|H, T) = \mathcal{P}(S_m|H)$, this reduces to $\mathcal{P}(S_m)/\mathcal{P}(S_m|T)$ — and the dot product of the informational premium vector and the conditional likelihood vector $\langle \mathcal{P}(S_m|T) \rangle$ is independent of the signal distribution, as in the introduction.

★ **Trading Impatience with Two-State Securities.** One might argue that the conditional independence assumption ‘clearly’ drives the result. I now refute such logic, by showing that the Arrow security restriction is absolutely crucial. For two state securities, the natural intuition that traders prefer to trade at once is valid.

Proposition 2 (Trading Impatience) *For (generic) assets that bundle together two Arrow securities, one has a (strict) incentive to trade immediately on any information.*

Proof: Let the compound security pay out $\pi(\theta_\ell)$ in state θ_ℓ , with $\pi(\theta_1) \neq \pi(\theta_2)$.⁷

For simplicity, let's restrict focus to discrete signal spaces. Rephrasing the desired result, we wish to show that if the per dollar return exceeds one, it is highest immediately. This statement is simultaneously captured for both favorable information (stock purchase) and unfavorable information (short sale) by the twin inequalities,

$$\mathcal{E}[\pi|T]/p_0 \gtrless \mathcal{E}[\pi|T, \mathcal{S}]/p_1 \gtrless 1 \quad (2)$$

These initial and final returns are (resp.) $\mathcal{E}[\pi|T]/p_0 = \sum_\ell \mathcal{P}(\theta_\ell|T)\pi(\theta_\ell) / \sum_\ell \mathcal{P}(\theta_\ell)\pi(\theta_\ell)$ and

$$\begin{aligned} \mathcal{E}[\pi|T, \mathcal{S}]/p_1 &= \sum_m \mathcal{P}(S_m|T) \frac{\sum_\ell \mathcal{P}(\theta_\ell|T, S_m)\pi(\theta_\ell)}{\sum_\ell \mathcal{P}(\theta_\ell|S_m)\pi(\theta_\ell)} \\ &= \sum_m \frac{\sum_\ell \mathcal{P}(S_m, \theta_\ell|T)\pi(\theta_\ell)}{\sum_\ell \mathcal{P}(S_m|\theta_\ell)\mathcal{P}(\theta_\ell)\pi(\theta_\ell)/\mathcal{P}(S_m)} \\ &= \sum_m \frac{\sum_\ell \mathcal{P}(S_m|\theta_\ell)\mathcal{P}(\theta_\ell|T)\pi(\theta_\ell) \sum_\ell \mathcal{P}(S_m|\theta_\ell)\mathcal{P}(\theta_\ell)}{\sum_\ell \mathcal{P}(S_m|\theta_\ell)\mathcal{P}(\theta_\ell)\pi(\theta_\ell)} \end{aligned} \quad (3)$$

With only $M = 2$ states, simply define $p_m = \mathcal{P}(S_m|\theta_1)$, $q_m = \mathcal{P}(S_m|\theta_2)$, $x = \mathcal{P}(\theta_1)$, $y_\ell = \mathcal{P}(\theta_\ell|T)\pi(\theta_\ell)$, and $z_\ell = \mathcal{P}(\theta_\ell)\pi(\theta_\ell)$. Notice that $x(y_1/z_1) + (1-x)(y_2/z_2) = 1$. So to establish (2), it suffices to show that for *any* $x \in (0, 1)$, $y_i, z_i > 0$, and nonnegative weights $\{p_m\}$ and $\{q_m\}$ that each sum to 1, we have

$$\frac{y_1 + y_2}{z_1 + z_2} \gtrless \sum_m (xp_m + (1-x)q_m) \frac{p_my_1 + q_my_2}{p_mz_1 + q_mz_2} \gtrless x(y_1/z_1) + (1-x)(y_2/z_2) \quad (\blacktriangle)$$

This nifty dual inequality happens to be new, and is thus established in the appendix. \diamond

REMARK. The proof of (\blacktriangle) also shows that timing is irrelevant exactly when either:

- the two Arrow securities have the same initial return: $\mathcal{P}(\theta_1|T)/\mathcal{P}(\theta_1) = \mathcal{P}(\theta_2|T)/\mathcal{P}(\theta_2)$
- the two Arrow securities are informationally indistinguishable: $\mathcal{P}(S_m|\theta_1) \equiv \mathcal{P}(S_m|\theta_2)$

★ **Timing Ambivalence with General Compound Securities.** With Arrow securities, timing is irrelevant, and with two-state securities, impatience is the rule. With more general compound securities, the result may go either way. I now provide a minimal example showing that with three states, one may in fact wish to delay trading, pending

⁷Otherwise, the states can be merged.

observation of the public information. Let the payoffs in states⁸ θ_1 , θ_2 , and θ_3 be $\pi(\theta_1) = 96$, $\pi(\theta_2) = 48/5 = 9.6$, and $\pi(\theta_3) = 96/11 \doteq 8.73$, respectively. Common prior beliefs $\mathcal{P}(\theta_1) = 1/96$, $\mathcal{P}(\theta_2) = 40/96$, $\mathcal{P}(\theta_3) = 55/96$ are chosen to yield a neat initial asset price

$$p_0 = \mathcal{E}[\pi] = (1/96)96 + (40/96)9.6 + (55/96)(96/11) = 1 + 4 + 5 = 10$$

The trader is assumed to have information $\tau \in T$, resulting in posterior beliefs $\mathcal{P}(\theta_1|T) = 1/48$, $\mathcal{P}(\theta_2|T) = 25/48$, $\mathcal{P}(\theta_3|T) = 11/24$. This yields the neat expected return per dollar

$$\mathcal{E}[\pi|T]/p_0 = [(1/48)96 + (25/48)9.6 + (11/24)(96/11)]/10 = (2 + 5 + 4)/10 = 11/10$$

The public signal realization between the two periods is either S_0 or S_1 with assumed respective state-dependent chances $(1/2, 1/4, 1/2)$ and $(1/2, 3/4, 1/2)$. So observing S_1 tips public beliefs towards state θ_2 and away from θ_1 and θ_3 . Plugging the above data into (3) yields an expected return of $845/768$ which very slightly exceeds the initial return of 1.1, and so it is strictly optimal to wait until the public information is revealed before buying.

★ **A Link to the Arbitrage Theory of Pricing.** Beyond the key role played by Arrow securities, the timing irrelevance proposition is also related by lineage to the APT — either in Ross (1976) or the primal version of it in Samuelson (1965). Namely, in a risk-neutral economy, the sequence $\langle p_n \rangle$ of prices is a martingale, i.e. $p_m = \mathcal{E}[p_n | p_m]$ for all $n > m$. At a stylized level, the APT is a consequence of the *law of iterated expectations* applied to the ultimate asset payoff π : (★) If $\Pi_n = \mathcal{E}[\pi | S_0, \dots, S_n]$ then $\mathcal{E}[\Pi_n | \Pi_m] = \Pi_m$ for all $n > m$. A closely related and classic martingale stochastic process is the set of *likelihood ratios*, if one conditions on the true state.⁹ For instance, if $q_n = \mathcal{P}(H | S_0, \dots, S_n)$ and $\lambda_n = (1 - q_n)/q_n$ then $\lambda_m = \mathcal{E}[\lambda_n | \lambda_m, H]$ for all $n > m$. In a theoretical exploration of herding, Smith and Sørensen (1996) have suggestively dubbed this the *conditional martingale* and (★) the *unconditional martingale*. Since one plus the likelihood ratio is the return on the asset paying \$1 in state H , this latter martingale is formally identical to the timing irrelevance proposition. And this paper was indeed inspired by this conditional martingale.

⁸The slightly exotic nature of the numbers reflects the difficulty of constructing the example.

⁹See, for instance, Chow, Robbins and Siegmund (1971).

3. CONCLUDING REMARKS

★ **Apologetics.** Almost as firm as the belief that no free lunch exists, is economists' skepticism of simple new results that appear surprising. Reactions to earlier versions of this paper have thus been (i) disbelief; (ii) 'of course it must be true'; or (iii) yes, the proposition is nontrivially true, but the question posed is not relevant. I believe that the paper has addressed the first response by proof, and the second by showing how the result fails absent the Arrow security restriction. At any rate, I feel the question itself is novel.

For the third reaction, my claim is humble: This paper merely explores a transparent and very natural (*the most* natural, I feel) *thought experiment* in financial timing. Are there *purely informational* reasons that private information rationally be traded upon without haste, or for otherwise timing one's trade?

As with any economic model, this paper is meant to sharpen our understanding of the role played by the driving assumptions. Previous work on endogenous timing of trades has either focused on models of pure public information, or confounded strategic effects with pure timing considerations. I have sidestepped strategic effects by focusing on a 'small trader'. The key finite wealth assumption, which suggested the focus on returns rather than prices, is only a natural corollary of 'smallness'. But surely *any* timing model ought to be robust to the simple calculus of when best to invest a dollar. Since a small trader's information is not impacted in the price, this also precludes applying insights from Radner (1979), that the equilibrium will be fully revealing concerning the states for which the Arrow-Debreu securities are traded.¹⁰ This is reasonable as I am simply studying the *single-person* decision problem.

★ **Lessons of the Paper.** This paper has been a simple economic exploration of how the interaction of private with public information affects the timing of trades. Within the maintained small trader world, the critical assumption is that private and public signals are conditionally independent. This is almost universally posited, not only in most strategic timing papers but also in classics like Grossman and Stiglitz (1980). The tautological alternative is that private and public information is conditionally correlated. While by no means technically appealing, many plausible information structures produce just this

¹⁰But see Geanakoplos (1990) for further discussion.

result. For instance, if distinct traders learn of different components of a firm, and if the profitability of the firm is the *sum* of the profitability of the separate parts, then given the value of the firm, the separate signals are ‘conditionally correlated’.

A curious application of the paper is in the problem of optimal design of a security: One may wish to choose a noncompound Arrow security simply because it assures there are no perverse incentives for the delay of informational revelation.

Smith (1995) has recently studied endogenous trading ‘frenzies’ in a strategic setting. One lesson of this paper is that — correlated information or compound securities aside — strategic considerations (or other large trader effects) are the *only* rational explanations for attempts to ‘time one’s trade’.

Finally, I feel the inviting mathematical similarity between the arbitrage pricing theory and the timing irrelevance proposition and the pivotal role of Arrow securities in each may speak to a deeper underlying connection, and merits further investigation.

A. APPENDIX

★ **General Proof of Proposition 1.** This proof is for completeness, but also addresses concerns that my application of conditional independence may take special advantage of a discrete public signal space.

Let \mathcal{S}_k^T be the sigma-algebra $\{T \cap S | S \in \mathcal{S}_k\}$. Assumption \mathcal{I} justifies $\mathcal{P}(S \cap T \cap \theta_0 | \mathcal{S}_n) = \mathcal{P}(S \cap T | \theta_0, \mathcal{S}_n) \mathcal{P}(\theta_0 | \mathcal{S}_n) = \mathcal{P}(T | \theta_0, \mathcal{S}_n) \mathcal{P}(S | \theta_0, \mathcal{S}_n) \mathcal{P}(\theta_0 | \mathcal{S}_n) = \mathcal{P}(T | \theta_0, \mathcal{S}_n) \mathcal{P}(S \cap \theta_0 | \mathcal{S}_n)$. For $S \in \mathcal{S}_{n+1}$, we have $S \cap T \in \mathcal{S}_{n+1}$, and thus

$$\begin{aligned}
 \int_{S \cap T} \mathcal{P}(\theta_0 | \mathcal{S}_{n+1}^T) d\mathcal{P} &= \int_{S \cap T} \mathbb{I}_{\theta_0} d\mathcal{P} \\
 &= \mathcal{P}(S \cap T \cap \theta_0 | \mathcal{S}_n) \\
 &= \mathcal{P}(T | \theta_0, \mathcal{S}_n) \mathcal{P}(S \cap \theta_0 | \mathcal{S}_n) \\
 &= \mathcal{P}(T | \theta_0, \mathcal{S}_n) \int_S \mathcal{P}(\theta_0 | \mathcal{S}_n) d\mathcal{P} \\
 &= \mathcal{P}(T | \theta_0, \mathcal{S}_n) \int_S \mathcal{P}(\theta_0 | \mathcal{S}_{n+1}) d\mathcal{P},
 \end{aligned}$$

since $S \in \mathfrak{S}_{n+1}$. Hence

$$\int_S \mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T) \mathbb{I}_T d\mathcal{P} = \mathcal{P}(T | \theta_0, \mathfrak{S}_n) \int_S \mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}) d\mathcal{P}.$$

As $S \in \mathfrak{S}_{n+1}$ is arbitrary and $\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1})$ is an \mathfrak{S}_{n+1} -measurable function, this is equivalent to

$$\mathcal{E}[\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T) \mathbb{I}_T | \mathfrak{S}_{n+1}] = \mathcal{P}(T | \theta_0, \mathfrak{S}_n) \mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}).$$

by definition of a conditional expectation. Dividing both sides by $\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1})$ yields¹¹

$$\mathcal{P}(T | \theta_0, \mathfrak{S}_n) = \frac{\mathcal{E}[\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T) \mathbb{I}_T | \mathfrak{S}_n]}{\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1})} = \mathcal{E} \left[\frac{\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T) \mathbb{I}_T}{\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1})} \middle| \mathfrak{S}_{n+1} \right]$$

since $\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1})$ is a \mathfrak{S}_{n+1} -measurable function. Taking expectations, I have

$$\mathcal{P}(T | \theta_0, \mathfrak{S}_n) = \int_T \frac{\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T)}{\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1})} d\mathcal{P}$$

Finally, apply Bayes rule to the LHS, and use the definition of a conditional expectation on the RHS, to get $\mathcal{P}(\theta_0 | \mathfrak{S}_n, T) / \mathcal{P}(\theta_0 | \mathfrak{S}_n) = \mathcal{E}[\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T) / \mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}) | \mathfrak{S}_n, T]$, as required.

★ **Proof of Inequality (▲).** The proof is rather illustrative, as it shows when equality holds. Assume WLOG that $y_2/z_2 > y_1/z_1$, for otherwise double equality must hold in (▲). Next, set $\Delta \equiv y_2/z_2 - y_1/z_1$. If we define

$$A = \sum_m p_m \frac{p_m y_1 + q_m y_2}{p_m z_1 + q_m z_2} \quad \text{and} \quad B = \sum_m q_m \frac{p_m y_1 + q_m y_2}{p_m z_1 + q_m z_2}$$

then we wish to show that $(y_1 + y_2)/(z_1 + z_2) \gtrsim xA + (1-x)B \gtrsim x(y_1/z_1) + (1-x)(y_2/z_2)$.

The first step is to prove that $B \geq A$, with $B > A$ generically. To see this, write

$$A = \sum_m p_m \frac{p_m (y_1/z_1) z_1 + q_m y_2 (y_1/z_1 + \Delta) z_2}{p_m z_1 + q_m z_2} = y_1/z_1 + \Delta \sum_m \frac{p_m q_m z_2}{p_m z_1 + q_m z_2}$$

and likewise

$$B = y_1/z_1 + \Delta \sum_m \frac{q_m^2 z_2}{p_m z_1 + q_m z_2}$$

¹¹Note that $\mathcal{P}(\theta_0 | \mathfrak{S}_{n+1}^T) > 0$ since Θ is finite and includes no zero chance states.

Hence,

$$\begin{aligned}
B - A &= \Delta \sum_m \frac{(q_m - p_m)q_m z_2}{p_m z_1 + q_m z_2} = \Delta \sum_m (q_m - p_m) \left(\frac{z_2}{z_1 + z_2} + \frac{(q_m - p_m)z_1 z_2}{(z_1 + z_2)(p_m z_1 + q_m z_2)} \right) \\
&= \Delta \sum_m \frac{(q_m - p_m)^2 z_1 z_2}{(z_1 + z_2)(p_m z_1 + q_m z_2)}
\end{aligned}$$

where the final equality holds since $\sum_m p_m = \sum_m q_m$. So $B > A$, unless $p_m = q_m$ for all m .

Simple calculation reveals that the critical threshold $x = \bar{x} \equiv z_2/(z_1 + z_2)$ yields double equality in (\blacktriangle) . There are two immediate consequences: First, since we have just shown that $B > A$, we have

$$\frac{y_1 + y_2}{z_1 + z_2} \begin{matrix} \geq \\ < \end{matrix} xA + (1 - x)B$$

as $x \begin{matrix} \geq \\ < \end{matrix} \bar{x}$. Second, because $B > A$, and A and B are clearly each weighted averages of y_1/z_1 and y_2/z_2 , we must have $y_2/z_2 > B > A > y_1/z_1$. It is then intuitive, and also easily verified, that

$$xA + (1 - x)B \begin{matrix} \geq \\ < \end{matrix} x(y_1/z_1) + (1 - x)(y_2/z_2)$$

for $x \begin{matrix} \geq \\ < \end{matrix} \bar{x}$.

Finally, $x = \bar{x}$ corresponds to $\pi(\theta_1) = \pi(\theta_2)$, which has been ruled out. Thus, equality in (\blacktriangle) can only hold if $\Delta = 0$ or $p_m = q_m$ for all m . This reduces to $\mathcal{P}(\theta_1|T)/\mathcal{P}(\theta_1) = \mathcal{P}(\theta_2|T)/\mathcal{P}(\theta_2)$ or $\mathcal{P}(\mathcal{S}_m|\theta_1) \equiv \mathcal{P}(\mathcal{S}_m|\theta_2)$ for all m , as required.



References

- Admati, Anat and Paul Pfleiderer**, “A Theory of Intraday Patterns: Volume and Price Stability,” *Review of Financial Studies*, 1989, 1, 3–40.
- Chow, Y. S., Herbert Robbins, and David Siegmund**, *Great Expectations: The Theory of Optimal Stopping*, Boston: Houghton Mifflin Company, 1971.
- Duffie, Darrell**, *Dynamic Asset Pricing Theory*, Princeton, N.J.: Princeton University Press, 1992.
- Geanakoplos, John**, “An Introduction to General Equilibrium with Incomplete Markets,” *Journal of Mathematical Economics*, 1990, 19, 1–38.
- Grossman, Sanford J. and Joseph E. Stiglitz**, “On the Impossibility of Informationally Efficient Markets,” *American Economic Review*, 1980, 70, 393–408.
- Kyle, Albert S.**, “Continuous Auctions and Insider Trading,” *Econometrica*, 1985, 53, 1315–1335.
- , “Informed Speculation with Imperfect Competition,” *Review of Economic Studies*, 1989, 56, 317–356.
- Merton, Robert C.**, “On Market Timing and Investment Performance, I. An Equilibrium Theory of Value for Market Forecasts,” 1981, 54, 363–406.
- , *Continuous-time Finance*, 2nd ed., Cambridge, MA: Blackwell Publishers, 1990.
- Milgrom, Paul and Nancy Stokey**, “Information, Trade, and Common Knowledge,” *Journal of Economic Theory*, 1982, 26, 17–27.
- Radner, Roy**, “Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices,” *Econometrica*, 1979, 47, 655–678.
- Ross, Stephen A.**, “The Arbitrage Theory of Capital Asset Pricing,” *Journal of Economic Theory*, 1976, 3, 343–362.
- , “Information and Volatility: The No-Arbitrage Martingale Approach to Timing and Resolution Irrelevancy,” 1989, 44, 1–17.
- Samuelson, Paul A.**, “Proof that Properly Anticipated Prices Fluctuate Randomly,” 1965, 6, 41–63.
- Smith, Lones**, “Do Rational Traders Frenzy?,” 1995. MIT mimeo.
- and **Peter Sørensen**, “Pathological Outcomes of Observational Learning,” 1996. MIT Working Paper.

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