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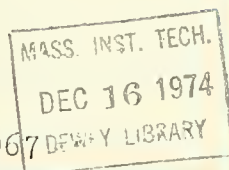
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An Aspect of the Theory of
Infant-Industry Protection"

by

Pranab K. Bardhan

Number 10 -- November 1967



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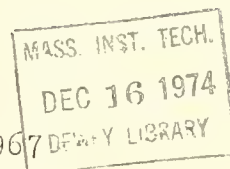


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The views expressed in this paper are the author's sole responsibility, and do not reflect those of the Department of Economics, nor of the Massachusetts Institute of Technology.

ON OPTIMUM SUBSIDY TO A LEARNING INDUSTRY:
AN ASPECT OF THE THEORY OF INFANT-INDUSTRY PROTECTION*

Pranab K. Bardhan

I

One of the earliest instances of the incorporation of the concept of 'learning by doing' in economic theory is the Hamilton-List infant-industry argument. It has been recognized in principle by John Stuart Mill and subsequent writers on international trade. But as any elaboration of this idea involves some explicitly dynamic analysis, it has hardly been integrated into the main corpus of trade theory which is mostly comparative-static in nature: until recently, it has received nothing more than nodding recognition as just one of the few 'exceptions' to the doctrine of free trade.

In this paper¹ we take a very simple dynamic model of 'learning by doing' in an open economy and work out the optimum extent and time-path of protection to the learning industry. (In contrast, usual analysis stops at merely pointing out the need for protection

*I have received useful comments from Tony Atkinson, Harry Johnson, Murray Kemp, Robert Mundell and Takashi Negishi on an earlier draft. Errors are, of course, mine.

¹The essential ideas in this paper may be found in my unpublished Ph.D. dissertation [3] at Cambridge University, England. Very recently I came across some unpublished work on a broadly similar subject by Harl Ryder and by Simone Clemhout and Henry Wan, Jr. Unlike the Ryder paper, I have here abstracted from the added complications of a capital accumulation model in order to bring out the essential learning effect in sharp focus. The Clemhout-Wan paper has a different set of assumptions about learning, production, and objective functions.

in such cases.) In a brief digression in Section II, we also mention some of the implications for the standard results of 'positive' trade theory (regarding patterns of factor prices, output, comparative advantage, etc.) when one introduces the learning effect.

We have two goods c and m that use capital, K , and labour, L , in production under constant returns to scale. The learning effect which increases productivity of factors depends on the cumulated volume of output in an industry.² Since the infant-industry argument is based (at least implicitly) on some kind of differential learning effect, we assume, for simplicity, that learning is operative only in one of the industries, say, in industry m . The production functions are:

$$Q_c = F_c(K_c, L_c) = L_c \cdot f_c(k_c) \quad (1)$$

$$Q_m = Q^n \cdot f_m(k_m, L_m) = Q^n \cdot L_m f_m(k_m) \quad (2)$$

where Q_i is the current rate of output and k_i the capital-labour ratio employed in i^{th} industry, $i = m, c$: Q is the cumulated volume of output of m so that $\dot{Q} = \frac{dQ}{dt} = Q_m$ (we shall, in Section III, alter this assumption slightly by introducing a term for depreciation of experience): Q^n incorporates the effect of learning or experience

²Arrow in his celebrated model of learning by doing [1] takes cumulated gross investment as the stock of experience affecting productivity. While this idea is useful in the context of an aggregative growth model like Arrow's, for studying the phenomenon of learning at the industry level it seems more appropriate to take the cumulated volume of industry output as the index of productivity-raising experience. At any rate, most of the empirical evidence of learning in production of airframes, machine tools, etc. relates to the cumulated volume of industry output. See, for example, Hirsch [6].

on productivity (we assume $1 > n > 0$)³; the way we have introduced the learning term also implies our simplifying assumption that the productivity-enhancing effect of learning is neutral with respect to the two factors of production. The stock of experience for the industry is under the control of no single firm; it comes as an irreversible external economy to it.⁴

With full employment of capital and labour,

$$K = K_m + K_c \quad (3)$$

$$L = L_m + L_c \quad (4)$$

³This is, of course, a very special kind of learning function that is similar to the empirically observed learning function for airframes. Most of our subsequent results, however, carry through (with one exception noted in footnote 6, if, instead, we take a more general learning function so that

$$Q_m = G(Q) \cdot F_m(K_m, L_m) \quad \text{with } G'(Q) > 0 \text{ and } G''(Q) < 0.$$

The assumption about the derivatives of the $G(Q)$ function means that learning enhances productivity but at a diminishing rate.

In formalizing the infant-industry concept, however, a more suitable learning function might be one where $G(Q)$ reaches an upper bound \bar{G} for a finite Q , where \bar{G} is a measure of the rest of the world's efficiency level (the thing to be "caught up"). In Section V of this paper we explore the implications of such a learning function.

⁴That the basic rationale for the infant-industry argument is provided by such irreversible external economies is now well recognized. See Meade [9], p. 256, Haberler [5], p. 56, Kemp [8], p. 187. For a number of concrete examples of such external economies, see Bardhan [2].

From (3) and (4),

$$\frac{L_c}{L} = \frac{k - k_m}{k_c - k_m} \quad (5)$$

$$\frac{L_m}{L} = \frac{k_c - k}{k_c - k_m} \quad (6)$$

$$\text{with } k = \frac{K}{L} \text{ and } k_c \neq k_m.$$

Since we want to concentrate on the operation of the learning effect, we shall assume that the total supply of labour and capital in the economy is given. Without further loss of generality we shall take $L = 1$. (1) and (2) may now be rewritten as

$$Q_c = f_c(k_c) \frac{[k - k_m]}{[k_c - k_m]} \quad (7)$$

$$Q_m = Q_c^n \cdot f_m(k_m) \frac{[k_c - k]}{[k_c - k_m]}. \quad (8)$$

Before we introduce in this model a planning authority that maximizes a social welfare function over time, let us for the time being analyze the behaviour of a stylized competitive economy in this model.

With pure competition in both factor and commodity markets and with no individual firm being able to control the amount of experience or learning, the price implications of the model will be like those of a competitive economy with exogeneous technical progress. Factors will be paid their marginal products valued at market prices, so that with both goods produced, with c as the numéraire good and the market price of m being P ,

the wage rate, $W = \frac{\partial Q_c}{\partial L_c} = f_c(k_c) - f'_c(k_c)k_c$

$$= P \cdot \frac{\partial Q_m}{\partial L_m} = P \cdot Q^n [f_m(k_m) - f'_m(k_m)k_m] \quad (9)$$

and the rental rate on capital, $R = \frac{\partial Q_c}{\partial K_c} = f'_c(k_c)$

$$= P \cdot \frac{\partial Q_m}{\partial K_m} = P \cdot Q^n \cdot f'_m(k_m). \quad (10)$$

From (9) and (10) the wage-rental ratio, $w = \frac{f_i(k_i)}{f'_i(k_i)} - k_i, i=c, m.$ (11)

With the usual assumptions of $f''_i(k_i) < 0$, etc., we can show k_i and w to be uniquely related from (11). From (10), therefore,

$$P = \frac{f'_c(k_c(w))}{f'_m(k_m(w)) \cdot Q^n} \quad (12)$$

which is equal to the ratio of the two marginal products of capital.

II

We shall consider the policy question of infant-industry protection in the next Section, but let us in this Section use equations (11) and (12) for our stylized competitive economy to analyze some of the implications of the phenomenon of learning by doing in a descriptive competitive model of international trade. We shall try

to be very brief here and only hint at the possibilities of unconventional results.

For illustrative purposes we shall assume in this Section that c is the more capital-intensive good, i.e., $k_c > k_m$ (the opposite factor-intensity case may be discussed with the same method of analysis).

$$\text{From (12), } P = \frac{H(w)}{Q^n} \quad (13)$$

$$\text{where } H(w) = \frac{f'_c(k_c(w))}{f'_m(k_m(w))}.$$

It can be checked with the help of (11) that $H'(w) > 0$ for $k_c > k_m$. It is self-evident from (13) that unlike in the usual two-sector incomplete specialization model, commodity prices are no longer uniquely determined by (or related to) factor prices alone. So the usual Stolper-Samuelson results about the relationship between commodity prices and factor prices and the Lerner-Samuelson result of factor-price equalization under free trade may no longer hold good.

Take, for example, the standard Stolper-Samuelson result that a fall in P should lead to a fall in the relative and absolute reward of the factor (labour) used more intensively in producing m . If one takes into account the learning effect this is no longer guaranteed. With positive current production of m , Q is ever-increasing and the fall in P may be outweighed by the rise in Q^n so that with $H'(w) > 0$, the relative and absolute reward of labour may still go up.

It also follows from (13) that if two trading countries have identical production and learning functions then with the same

commodity prices under free trade, the country with larger Q (say, the country with the 'earlier start' in producing m) will have the higher relative and absolute wage rate.

Can we say anything about the pattern of comparative advantage? For that we shall have to look at the pre-trade relative prices. Once again equation (13) is useful. Suppose again that the two countries have identical production and learning functions. It is possible for the country with more expensive labour (higher w) to have a comparative advantage in the more labour-intensive commodity m (i.e., P is lower), if Q is large enough (say, because of earlier start).

It is also not unexpected that the familiar Rybczynski result may no longer hold good. It can easily be checked that an increase in the capital stock with commodity prices constant does not necessarily bring about a drop in the production of the more labour-intensive commodity. Another interesting point to note is that with positive current production of m , i.e., rising Q , constant P means an increasing w : an increase in the wage-rentals ratio involves an increase in the sectoral capital-labour ratios and if the total stock of capital and labour is static, full employment necessitates a reallocation of resources in favour of the labour-intensive sector and against the capital-intensive sector: so if commodity prices are kept constant all this means declining production in the more capital-intensive sector.

The reader can easily think of other examples⁵ of changes in the usual comparative-static results of trade theory when the learning effect is introduced.

III

In Section I we introduced a stylized competitive economy in order to facilitate our analysis of the implications of learning for 'positive' trade theory. But let us now introduce a full optimizing model where the planning authority in maximizing social welfare over time takes due account of the productivity-increasing effects of society's experience in producing m .

Suppose the social objective is to maximize

$$\int_0^{\infty} U(A_c, A_m) e^{-\delta t} dt. \quad (14)$$

With given initial conditions and subject to the following constraints:

$$A_c = Q_c + X_c = f_c(k_c) \frac{[k - k_m]}{[k_c - k_m]} + X_c \quad (15)$$

⁵Some more examples (relating to the effect of tariffs) are given in Bardhan [2]. This is important because the usual results of tariff theory do not carry over to the case of infant-industry tariffs (since they affect production and income in a different way). The impact of an infant-industry tariff has been rather neglected in descriptive comparative-static models of protection.

$$A_m = Q_m + X_m = Q^n \cdot f_m(k_m) \frac{[k_c - k]}{[k_c - k_m]} + X_m \quad (16)$$

$$X_c + \bar{P}X_m = 0 \quad (17)$$

$$\dot{Q} = Q_m - \rho Q \quad (18)$$

A_i is the consumption of the i^{th} commodity, U is an instantaneous utility function that is concave and has positive marginal utilities, δ is the given positive social rate of discount, X_i is the amount imported of the i^{th} commodity, \bar{P} is the international price of m in terms of c and ρ is a constant rate of depreciation of experience.

In (18) we have slightly altered an assumption we made in Section I and II. Our stock of experience Q in m industry increases by the current rate of production in that industry net of a constant rate of depreciation (or 'forgetting').⁶ (17) gives us the balance of trade equation; i.e., imports are paid by exports. For simplicity, we shall assume that ours is a small country in a large world so that the international price, \bar{P} , is given (this helps one to isolate the considerations of infant-industry argument from those of

⁶Although this is assumed for mathematical convenience, some motivation may be given for this depreciation of experience: for example, in an underdeveloped country an important part of learning is adaptation to industrial employment on the part of the worker from rural areas and this is lost when he leaves the labour force. For those who find this assumption of depreciation of experience still not very appealing, we have worked out a case without this assumption in Section VI.

the standard 'optimum-tariff' argument). For the time being we are assuming the rest of the world as static. But a more realistic thing would be to have continuous learning going on in the rest of the world as well, possibly changing the international price level. We consider this case in Section VI of this paper.

For a full solution of the optimizing problem we have to discuss a number of patterns of specialization (specialization in consuming c, in consuming m, in producing c, in producing m, consumption of both m and c, production of both m and c, etc.). But in order to avoid tedium and economize space, we confine ourselves to only the 'interior' case--where we produce and consume both the commodities--the case which is usually the most interesting. This means

$$Q_c > 0 \quad \text{and} \quad Q_m > 0 \quad (19)$$

and
$$A_c > 0 \quad \text{and} \quad A_m > 0 \quad (20)$$

(17) and (20) imply that

$$-Q_m < X_m < \frac{Q_c}{P} \quad (21)$$

(5), (6), and (19) imply that

$$1 > \frac{L_c}{L} > 0 \quad \text{and} \quad 1 > \frac{L_m}{L} > 0 \quad (22)$$

The Hamiltonian H of the present problem is given by

$$\begin{aligned} He^{\delta t} = & U(A_c, A_m) + \lambda \left[f_c(k_c) \frac{(k - k_m)}{(k_c - k_m)} - \bar{P}X_m - A_c \right. \\ & \left. + \mu \left[f_m(k_m) \cdot \frac{(k_c - k)}{(k_c - k_m)} \cdot Q^n + X_m - A_m \right] + \left[\gamma f_m(k_m) \cdot \frac{(k_c - k)}{(k_c - k_m)} \cdot Q^n - \rho Q \right] \right] \quad (23) \end{aligned}$$

where λ , μ and γ are (positive) imputed prices of the respective constraints.

λ and μ are the demand prices for consumption of c and m respectively and γ is the imputed price of productivity-enhancing experience.

The conditions for maximum are as follows:

$$\lambda = \frac{\partial U}{\partial A_c} \quad (24)$$

$$\mu = \frac{\partial U}{\partial A_m} \quad (25)$$

$$\frac{\mu}{\lambda} = \bar{p} \quad (26)$$

(26) implies that the ratio of marginal utilities in consumption should be equalized to the given international price ratio. The marginal rate of domestic transformation is given by

$$\frac{\mu + \gamma}{\lambda} = \frac{f'_c(k_c(w))}{f'_m(k_m(w))Q^n} \quad (27)$$

where w is given, as before, by (11).

$$\dot{\gamma} = (\rho + \delta)\gamma - (\mu + \gamma) \left[f'_m(k_m) \frac{(k_c - k_m)}{(k_c - k_m)} n \cdot Q^{n-1} \right] \quad (28)$$

(28) gives the optimum rate of change in the shadow price of experience. The usual transversality condition is given by

$$\lim_{t \rightarrow \infty} \gamma Q e^{-\delta t} = 0 \quad (29)$$

In Section IV we analyze the implications of these conditions. But before that let us introduce another assumption to simplify the problem further. We shall assume that the instantaneous utility

function $U(A_c, A_m)$ is homogeneous of degree one (it implies that the income elasticity of demand for either good is unity). This means that in (24) and (25) λ and μ depend only on $\frac{A_c}{A_m}$, the ratio of consumption of the two goods. But, then, from (26), this ratio is constant so that λ and μ are really constants.

IV

In (27), let us define P^d as

$$\frac{\mu + \gamma}{\lambda} = \frac{f'_c(k_c(w))}{f'_m(k_m(w)) \cdot Q^n} = \frac{H(w)}{Q^n} = P^d \quad (30)$$

where $H(w)$ is defined, as in (13).

P^d is the marginal rate of domestic transformation, while \bar{P} is the marginal rate of transformation through foreign trade as well as the marginal rate of substitution in consumption. Unless the Government intervenes, competitive producers of m will produce according to the market price \bar{P} ($= \frac{\mu}{\lambda}$) and there will be underproduction of m from the social point of view. In order to attain the social optimum, the Government should assure the producers a price equal to P^d ($= \frac{\mu + \gamma}{\lambda}$). The optimum rate of subsidy to the learning industry⁷ is given by $\bar{P}(1 + \tau) = P^d$, or $\tau = \frac{\gamma}{\mu}$, where τ is the rate of subsidy per unit of output. As long as experience increases productivity its imputed price γ is positive, and so is the rate of subsidy τ . Two points are important to note here. (a) As is by now well recognized [4] [7], a subsidy is better than tariff in such cases because the latter

⁷It may be a mixture of tax on the non-learning industry and subsidy to the learning industry.

in restoring the equality between domestic and foreign marginal rates of transformation also drives a wedge between the marginal rate of consumer substitution and that of transformation. (b) It should be noted that our good m may be imported or exported, although we have assumed some limits on both sides as implied in (21). So our prescribed subsidization of learning may involve a subsidy to the import-competing industry or to the export industry, as the case may be.⁸

One of our main purposes in this paper is to find out the time-pattern of the optimum subsidy, i.e., the nature of \dot{i} over time.

Let us for this purpose take the two differential equations of our model given by (18) and (28).

$$\dot{Q} = Q^n \cdot f_m(k_m(w)) \frac{[k_c(w) - k]}{[k_c(w) - k_m(w)]} - \rho Q \quad (31)$$

$$\dot{Y} = (\rho + \delta)\gamma - (u + \gamma) \left[f_m(k_m(w)) \frac{\{k_c(w) - k\}}{\{k_c(w) - k_m(w)\}} nQ^{n-1} \right]. \quad (32)$$

Now, from (30), w can be written as a function of P^d and Q . So (31) can be rewritten as

$$\dot{Q} = \phi(Q, P^d). \quad (33)$$

Since u and λ are constant, from (30) and (32)

$$\dot{P}^d = \frac{\dot{Y}}{\lambda} = \frac{\gamma}{\lambda} \left[(\rho + \delta) - (1 + \frac{u}{\gamma}) nQ^{n-1} f_m(k_m(w)) \frac{\{k_c(w) - k\}}{\{k_c(w) - k_m(w)\}} \right]. \quad (34)$$

⁸cf. Haberler [5], p. 57: 'It is, a priori, probable that in many cases not a customs duty but an export bounty would be in order in as much as external economies may be realizable in the export rather than in import industries.'

Since from (30), w is a function of P^d and Q , and γ is a function of P^d alone, we can rewrite (34) as

$$\dot{P}_d = \Psi(Q, P^d)$$

As explained in detail in the Appendix A, under the sufficient condition of $\frac{\partial Q_m}{\partial Q} \frac{Q}{Q_m}$ (given P^d)--or what might be called the 'learning elasticity of output' of m --being less than unity, the stationary solution (Q_*, P_*^d) to the two differential equations (33) and (35) is unique and is also a saddle point, as is indicated in Figure 1. Under the same sufficient condition, the $\dot{Q} = 0$ curve is uniformly upward-sloping for the ('interior') region we are considering. Under a stronger sufficient condition of the elasticities of factor substitution being small enough,⁹ the $\dot{P}^d = 0$ curve is uniformly downward-sloping in the region we are considering.

Given our transversality condition (29), if $Q(0) = Q_*$, the unique optimum path is indicated by the singular solution (Q_*, P_*^d) and the optimum rate of subsidy to the learning industry, $\tau_* = \frac{P_*^d - \bar{P}}{\bar{P}} = \frac{n\rho}{\delta + (1-n)\rho}$, a constant. If, however, $Q(0) \neq Q_*$, the optimum path of (Q, P^d) lies along the stable branches of the saddle point given the transversality condition. It can be seen from Figure 1 that along the optimum path, if $Q(0) < Q_*$ (i.e., the initial stock of experience is small enough), Q steadily increases and P^d steadily decreases to asymptotically

⁹In the context of developing economies (where the infant-industry argument is supposed to apply with particular relevance) this assumption may not be inappropriate, since such economies are usually characterized by low substitution elasticities in production (low 'capacity to transform' in Kindleberger's terms).

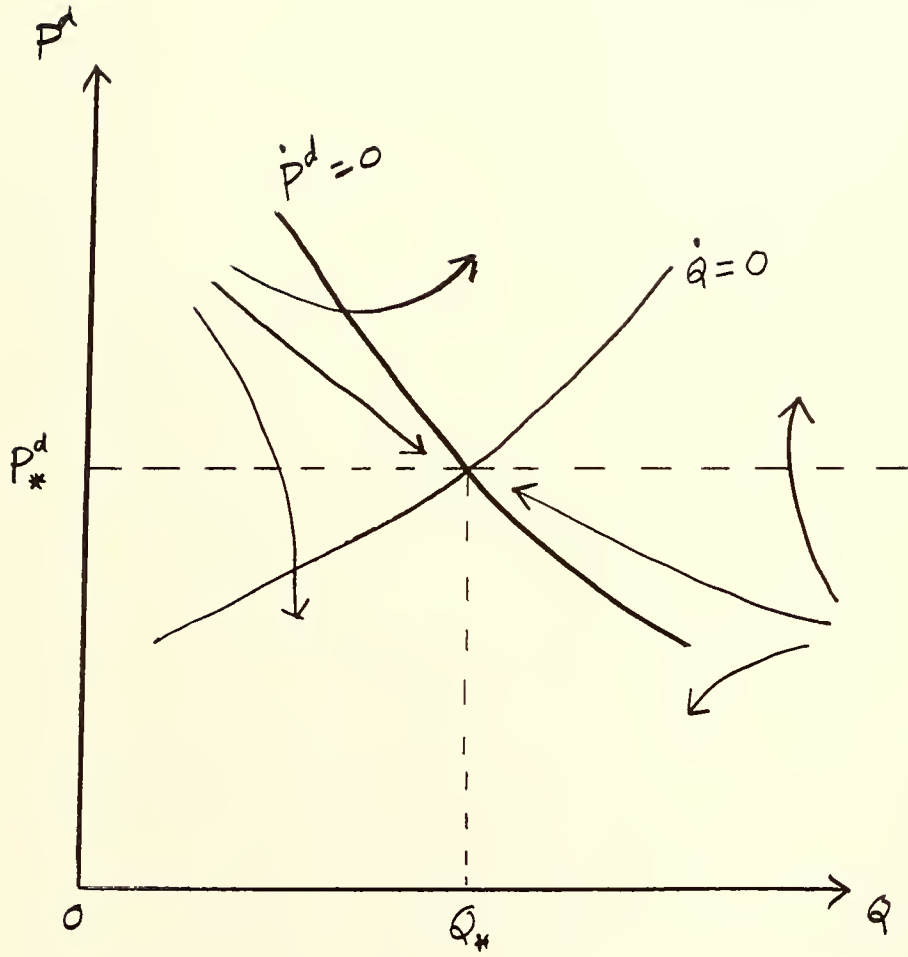


Figure 1

approach the stationary solution (Q_*, P_*^d) , and therefore, the optimum rate of subsidy, τ , steadily decreases to asymptotically approach the stationary rate τ_* . If, on the other hand, $Q(0) > Q_*$, by similar reasoning the optimum rate of subsidy steadily increases to asymptotically approach the stationary rate τ_* .

V

In Section IV we have characterized the time-path of the optimum rate of subsidy to the learning industry in this model: If the initial stock of experience is small, the optimum rate of subsidy steadily decreases over time to asymptotically approach a stationary rate that is positive. The subsidy always remains positive because with our learning function incorporated in equation (2), experience always enhances productivity and with the spill-over of benefits of the learning process to all firms¹⁰ there tends to be an underproduction of m from the social point of view if the industry is not subsidized. But much of the usual infant-industry argument is concerned with temporary protection, where the learning process is more in the nature of overcoming a historical handicap, a matter of catching up with a foreign country's efficiency level than that of a continuous productivity-enhancing phenomenon, and the subsidy is to be removed as soon as the 'infant' becomes an adult. The implications of this kind of a learning process can, however, be discussed retaining a large part of the analytic framework in earlier sections of this paper.

¹⁰There are some types of learning processes the benefits of which are specific to the learning firm, and in such cases, obviously, the case for subsidy should not arise.

Let us rewrite (2) as

$$Q_m = G(Q) \cdot F_m(K_m, L_m) \quad (36)$$

where G is the learning function, $G(0)$ is a constant, $G'(Q) > 0$, $G''(Q) < 0$ for Q less than a finite positive level of \bar{Q} , and for $Q \geq \bar{Q}$, $G(Q) = \bar{G}$, a constant.¹¹ This means that the stock of experience enhances productivity (at a diminishing rate) up to a point, and then at a certain level of experience, \bar{Q} , the country catches up with the foreign efficiency level, \bar{G} , and there is no more learning.

How will this affect our analysis of the time-path of the optimum rate of subsidy? As long as $Q < \bar{Q}$, both the $\dot{Q} = 0$ and $\dot{P}^d = 0$ curves will be of the same shape as before, if one makes the same assumptions. The imputed price of experience, γ , will be positive as long as $Q < \bar{Q}$, but for $Q \geq \bar{Q}$, $\gamma = 0$, implying the completion of the learning process. The rate of subsidy, τ , being equal to $\frac{\gamma}{\mu}$ is therefore zero for $Q \geq \bar{Q}$. So if the $\dot{Q} = 0$ and $\dot{P}^d = 0$ curves do not intersect for $Q < \bar{Q}$, then the optimum rate of subsidy, τ , steadily declines over time until it reaches zero at $Q = \bar{Q}$. If the $\dot{Q} = 0$ and $\dot{P}^d = 0$ curves intersect before Q reaches \bar{Q} , the conclusion of our Section IV remains valid for the case when the initial stock of experience is small enough.

VI

In this Section we revert to the case of continuous learning but consider the relaxation of two other assumptions in our model. One

¹¹We assume that at the point when $Q = \bar{Q}$, $G'(Q) = 0$ and $G''(Q) = 0$.

of our assumptions has been to take the rest of the world as static. A more interesting model might be one in which learning by doing is continuously going on abroad as well as in our home country. This means that even if ours is a small country in a large world we may no longer take the international price \bar{P} as static: \bar{P} may now change because of learning in the rest of the world. Once we introduce learning abroad, we shall, of course, have to assume something about the transferability of the benefits of that learning. In other words, we have to consider the international external economies of the learning process. In this Section we shall assume that an increase in the stock of experience in the large world outside will costlessly enhance the productivity¹² in our small country as well. In the process of working out the implications of this factor we shall also do away with another possibly restrictive assumption in our preceding three Sections, viz. that of positive depreciation of experience.

In order to simplify our calculations we shall assume that the stock of experience, \hat{Q} , in the rest of the world for industry m is growing at a constant rate α and also that the international price of m , \bar{P} , is, as a consequence, declining at the rate α . The production function of m in the home country is now given by

$$Q_m = G(Q, \hat{Q}) \cdot F_m(K_m, L_m) \quad (37)$$

where G is the neutral productivity function, which is increasing (at a diminishing rate) for both Q and \hat{Q} . For simplicity we shall

¹²This is, of course, an extreme assumption of free transferability of knowledge across countries (apart from problems of adaption of foreign technology to the specific pattern of available resources, factor prices, market possibilities, etc. in the home country).

assume that the $G(Q, \hat{Q})$ function is homogeneous of degree one so that

$$G(Q, \hat{Q}) = \hat{Q} \cdot g(x) \quad (38)$$

where $x = \frac{Q}{\hat{Q}}$.

We shall also make a simplifying assumption regarding the utility function $U(A_c, A_m)$. In Section III we took U as homogeneous of degree one: for simplification, we shall now take U to be in the more specific Cobb-Douglas form:

$$U(A_c, A_m) = A_c^\beta A_m^{1-\beta} \quad (39)$$

where β is a positive constant.

Armed with these simplifying assumptions, we are now ready to tackle our learning model which is much more general than in the preceding sections.

The international price of m , \bar{P} , is now declining at a constant rate α , so that in comparison to (26) we now have

$$\frac{\mu(t)}{\lambda(t)} = \bar{P}(0)e^{-\alpha t}. \quad (40)$$

As shown in Appendix B, it is easy to derive from (24), (25), (39), and (40) that

$$\lambda(t) = \lambda(0)e^{\alpha(1-\beta)t} \quad (41)$$

$$\text{and } \mu(t) = \mu(0)e^{-\alpha\beta t}. \quad (42)$$

Without loss of generality we shall take $\lambda(0) = \mu(0) = \bar{P}(0) = \hat{Q}(0) = 1$. Comparing with (30) the marginal rate of domestic transformation is now given by

$$p^d = \frac{\mu(t) + \gamma(t)}{\lambda(t)} = \frac{H(w)}{G(Q, \hat{Q})}. \quad (43)$$

Since Q is growing at rate α we may rewrite (43), with the help of (38) and (40), as

$$\frac{H(w)}{g(x)} = 1 + \gamma(t)e^{\alpha\beta t} = 1 + y(t) \quad (44)$$

where $y(t) = \gamma(t)e^{\alpha\beta t}$.

In Section IV, we have seen that $\tau = \frac{\gamma(t)}{\mu(t)}$. From (42), this means $\tau = y(t)$. We shall be interested in finding out the optimum time-path of $y(t)$.

Now let us take the differential equations in the system. Since we are no longer assuming depreciation of experience, (31) is to be rewritten, with the use of (37) and (38), as

$$\dot{Q} = Q_m = \hat{Q}g(x) \cdot f_m(k_m(w)) \frac{[k_c(w) - k]}{[k_c(w) - k_m(w)]}. \quad (45)$$

$$\text{Since } x = \frac{Q}{\hat{Q}}, \quad \dot{x} = \frac{\dot{Q}}{\hat{Q}} - \alpha x. \quad (46)$$

From (44), w is only a function of x and y . Using \dot{Q} from (45), (46) may now be rewritten as

$$\dot{x} = \phi(x, y). \quad (47)$$

With our new production function for m as denoted by (37) and (38), (32) has to be rewritten as

$$\dot{y} = \delta\gamma - (\mu + \gamma) g'(x) \cdot f_m(k_m(w)) \frac{[k_c(w) - k]}{[k_c(w) - k_m(w)]}. \quad (48)$$

From (44),

$$\frac{\dot{y}}{y} = \alpha\beta + \frac{\dot{Y}}{Y} . \quad (49)$$

Since $\frac{\mu + Y}{Y} = 1 + \frac{1}{y}$, (49) may be rewritten as

$$\frac{\dot{y}}{y} = \alpha\beta + \delta - g'(x) \left[1 + \frac{1}{y}\right] \cdot f_m(k_m(w)) \frac{[k_c(w) - k]}{[k_c(w) - k_m(w)]} \quad (50)$$

or, since w is a function of x and y alone,

$$\dot{y} = \Psi(x, y) . \quad (51)$$

As explained in Appendix B, under the sufficient condition of

$\frac{\partial Q_m}{\partial Q} \frac{Q}{Q_m}$ -- or what might be called the elasticity of output with respect to domestic learning--being less than unity, the stationary

solution (x_*, y_*) to the differential equations (47) and (51) is unique and is also a saddle point. Under the same sufficient condition, the

$\dot{x} = 0$ curve is uniformly upward-sloping, and under a stronger sufficient

condition of the elasticities of factor substitution being small enough, the

$\dot{y} = 0$ curve is uniformly downward-sloping for the ('interior')

region we are considering. The phase diagram looks exactly

as in Figure 1, when one replaces Q by x and P^d by y .¹³

If $x(0) = x_*$, the unique optimum path is indicated by the singular solution (x_*, y_*) and the optimum rate of subsidy to the

¹³We assume that the social rate of discount is high enough so that $\delta > (1-\beta)\alpha$. This is needed to satisfy the transversality condition (29).

learning industry, $\tau_{\#} = y_{\#} = \frac{\alpha g'(x_{\#})x_{\#}}{g(x_{\#})[\delta + \alpha\beta] - \alpha g'(x_{\#})x_{\#}}$.

If, however, $x(0) \neq x_{\#}$, the optimum path of (x, y) lies along the stable branches of the saddle point. If the home country's initial stock of experience is small enough, so that $x(0) < x_{\#}$, along the optimum path y , and therefore the optimum rate of subsidy τ , steadily decreases to asymptotically approach the stationary rate $\tau_{\#}$. If, on the other hand, $x(0) > x_{\#}$, the optimum rate of subsidy steadily increases to asymptotically approach the stationary rate $\tau_{\#}$.

¹⁴That the stationary rate $\tau_{\#}$ is positive can be shown as follows. Since under our assumption in f.n. 13 $\delta > (1-\beta)\alpha$, the denominator in the expression for $y_{\#}$ is larger than $\alpha[g(x_{\#}) - g'(x_{\#})x_{\#}]$, which is positive since the bracketed expression is equal to the marginal contribution to domestic output from a rise in the stock of experience abroad.

Appendix A

Take our two differential equations (33) and (35).

From (31) and (33),

$$\left(\frac{\partial \phi}{\partial Q}\right)_{\phi=0} = Q^n \cdot \left[\frac{dB(w)}{dw} \cdot \frac{\partial w}{\partial Q} \right] - Q^{n-1} \cdot B(w)(1-n) \quad (52)$$

where $B(w) = f'_m(k_m(w)) \frac{[k_c(w)-k]}{[k_c(w)-k_m(w)]}$

and $Q^n \cdot B = \rho Q$ when $\phi = 0$.

Now the 'learning elasticity of output' of m is

$$\frac{\partial Q_m}{\partial Q} \cdot \frac{Q}{Q_m} = n + \frac{Q}{B(w)} \cdot \left[\frac{dB(w)}{dw} \cdot \frac{\partial w}{\partial Q} \right]. \quad (53)$$

If we assume this elasticity to be less than unity,¹⁵ then we can easily check that (52) is negative.

From (31) and (33) again,

$$\frac{\partial \phi}{\partial P} d = Q^n \cdot \left[\frac{dB(w)}{dw} \cdot \frac{\partial w}{\partial P} d \right]. \quad (54)$$

Differentiating B with respect to w, and using (11) and that the elasticity of factor substitution $\sigma_i = \frac{dk_i}{dw} \cdot \frac{w}{k_i}$, $i = c, m$, we get

$$\frac{dB(w)}{dw} = \frac{f''_m}{w(k_c - k_m)^2} [\sigma_c \cdot k_c(w+k_m)(k-k_m) + \sigma_m k_m(w+k_c)(k_c-k)] \gtrless 0$$

as $k_c \gtrless k_m$. (55)

¹⁵A restriction on the learning elasticity is needed to ensure the concavity of the production function of m: since Q positively affects Q_m directly as well as indirectly through w, $n < 1$ is not enough to ensure concavity.

Appendix A (Cont'd)

From (30),
$$\frac{\partial w}{\partial P^d} = \frac{Q^n}{H'(w)} \gtrless 0 \text{ as } k_c \gtrless k_m \quad (56)$$

where $H(w)$ is as defined before.

So from (55) and (56), (54) is positive. That (54) is positive can, of course, be expressed in a more familiar way. Since $\frac{\partial \phi}{\partial P^d} = \frac{\partial Q_m}{\partial P^d}$,

that (54) is positive means the price elasticity of output m is positive.

Given that (52) is negative and (54) positive we can now say that

$$\left(\frac{dP^d}{dQ} \right)_{\phi = 0} = - \left[\begin{array}{cc} \frac{\partial \phi}{\partial Q} & \frac{\partial \phi}{\partial P^d} \end{array} \right]_{\phi = 0} > 0. \quad (57)$$

So that $\dot{Q} = 0$ curve is upward-sloping as in Figure 1. From (30), (34) and (35),

$$\left(\frac{\partial \Psi}{\partial Q} \right)_{\Psi = 0} = \left[nQ^{n-2}(n-1)B(w) + nQ^{n-1} \cdot \frac{dB(w)}{dw} \cdot \frac{\partial w}{\partial Q} \right] P^d. \quad (58)$$

From (53), once again, if the 'learning elasticity of output' of m is less than unity, (58) is positive.¹⁶

From (30), (34) and (35), and since μ and λ are constant,

$$\left(\frac{\partial \Psi}{\partial P^d} \right)_{\Psi = 0} = nQ^{n-1} \left[B \cdot \frac{\mu}{\gamma} - P^d \cdot \frac{dB(w)}{dw} \cdot \frac{\partial w}{\partial P^d} \right]. \quad (59)$$

¹⁶If instead of having the special learning function $G(Q) = Q^n$, $1 > n > 0$, we have a more general learning function $G(Q)$ with $G'(Q) > 0$, $G''(Q) < 0$, we have to assume the following for (58) to be positive:

$$\frac{G'(Q)Q}{G(Q)} - \frac{G''(Q)Q}{G'(Q)} = 1.$$

An alternative sufficient condition for (58) to be positive is to have elasticities of factor substitution to be small enough so that from (55), $B'(w)$ is very small and thus

$$[G''(Q)B + B'(w) \frac{\partial w}{\partial Q} G'(Q)] \text{ is negative.}$$

Appendix A (Cont'd)

Without some extra assumption it does not seem to be possible to be unambiguous about the sign of (59) in general. In view of our discussion above and of (55), we know that if the elasticities of factor substitution are small enough (or the price-elasticity of output is small enough), $\frac{dB(w)}{dw} \cdot \frac{w}{P^d}$ is small enough for (59) to be positive. In that case,

$$\left(\frac{dP^d}{dQ} \right)_{\Psi = 0} = - \left[\begin{array}{cc} \frac{\partial \Psi}{\partial Q} & \frac{\partial \Psi}{\partial P^d} \end{array} \right]_{\Psi = 0} < 0. \quad (60)$$

So under our assumptions the $\dot{P}^d = 0$ curve is downward-sloping as in Figure 1.

Given the shapes of our $\dot{Q} = Q$ and $\dot{P}^d = 0$ curves under our assumptions, the stationary solution is unique. But for proving uniqueness or the saddle-point property of our stationary solution, the last assumption we have just made, viz. the elasticities of factor substitution are small enough, is not really necessary. Let us show why.

Although we cannot be unambiguous about the sign of $\left(\frac{\partial \Psi}{\partial P^d} \right)_{\Psi = 0}$ in general without this extra assumption, we can show that around the stationary equilibrium point (Q_*, P_*^d) , we know its sign without that assumption. From (31) and (34), when $\dot{Q} = 0$ and $\dot{P}^d = 0$,

$$1 + \frac{\mu}{\gamma_*} = \frac{\rho + \delta}{B(w)_* \cdot nQ_*^{n-1}} = \frac{\rho + \delta}{n\rho} \quad (61)$$

¹⁷This implies that $\tau_* = \frac{\gamma_*}{\mu} = \frac{n\rho}{\delta + (1-n)\rho}$.

Appendix A (Cont'd)

Now from (30),

$$\frac{\partial w}{\partial Q} = \frac{nP^d \cdot Q^{n-1}}{H'(w)}. \quad (62)$$

Using (56), (61), and (62) in (59)

$$\left(\frac{\partial \Psi}{\partial P^d} \right)_{\substack{\psi = 0 \\ \phi = 0}} = B(w)_* \cdot Q_*^{n-1} \left[\frac{\delta}{\rho} + 1 - \left\{ n + \frac{Q_*}{B(w)_*} \cdot \frac{dB(w)_*}{dw_*} \cdot \frac{\partial w_*}{\partial Q_*} \right\} \right] \quad (63)$$

(63) is positive under our earlier assumption that the 'learning elasticity of output' of m is less than unity, as may be checked from (53). So around the stationary equilibrium $\dot{P}^d = 0$ curve must be downward-sloping and the stationary solution is unique, since $\dot{Q} = 0$ is always upward-sloping. Expanding the two differential equations (33) and (35) around the point (Q_*, P_*^d) and using (52), (53), (54), (58) and (63), we can see that under our assumption of the 'learning elasticity of output' of m being less than unity, the characteristic roots of the resulting linear system are real and opposite in sign indicating that the stationary solution is a saddle-point.

We need the extra assumption of elasticities of factor substitution being small enough to ensure that $\left(\frac{\partial \Psi}{\partial P^d} \right)_{\psi = 0} > 0$ not merely around the point (Q_*, P_*^d) but in general throughout the region we are considering. This ensures the result that for $Q(0) < Q_*$, τ steadily decreases.

Appendix B

From (39),

$$\lambda = \beta a^{\beta-1} \text{ and } \mu = (1-\beta)a^\beta \quad (64)$$

where $a = \frac{A_c}{A_m}$.

Using (64) in (40), $\frac{\dot{a}}{a} = -\alpha$, and that immediately implies (41) and (42). The analysis of the properties of the two differential equations (47) and (51) is nearly the same as that of our earlier differential equations (33) and (35) analyzed in Appendix A. We shall, therefore, be very brief here.

From (45) and (46)

$$\dot{x} = B(w)g(x) - \alpha x = \phi(x,y) \quad (65)$$

where $B(w)$ is as defined for (52).

As in (52), if the elasticity of output with respect to domestic learning, i.e., $\frac{\partial Q_m}{\partial Q} \frac{Q}{Q_m} = \frac{g'(x)x}{g(x)} + \frac{B'(w)}{B(w)} \frac{\partial w}{\partial x} \cdot x$ is less than unity, then $(\phi_x)_{x=0} < 0$.

From (13), (44), (55) and (65),

$$\phi_y = g(x) B'(w) \frac{\partial w}{\partial x} = g^2 \frac{B'(w)}{H'(w)} > 0. \quad (66)$$

This means that

$$\left(\frac{dy}{dx} \right)_{\dot{x}=0} = - \frac{\phi_x}{\phi_y} > 0 \quad (67)$$

or, the $\dot{x} = 0$ curve is upward-sloping in (x,y) space.

From (50),

$$\dot{y} = y(\delta + \alpha\beta) - [1 + y]g'(x) \cdot B(w) = \Psi(x, y) \quad (68)$$

$$\Psi_x = - [1 + y][B(w) \cdot g''(x) + g'(x) \cdot B'(w) \cdot \frac{\partial w}{\partial x}] \quad (69)$$

As in f.n. 16 of Appendix A, (69) is positive (since $g'' < 0$) if the elasticities of factor substitution are small enough to make $B'(w)$ very small, or alternatively if

$$\frac{g'(x)x}{g(x)} - \frac{g''(x)x}{g'(x)} \geq 1. \quad 18$$

From (68),

$$\left(\frac{\Psi_y}{\dot{y}=0}\right) = \frac{g'(x)}{y} [B(w) - y(1+y)B'(w)\frac{\partial w}{\partial y}]. \quad (70)$$

As for (59) in Appendix A, (70) is positive if the elasticities of factor substitution are small enough.

Thus

$$\left(\frac{dy}{dx}\right)_{\dot{y}=0} = - \frac{\Psi_x}{\Psi_y} < 0 \quad (71)$$

or, under our assumptions $\dot{y} = 0$ curve is downward-sloping in (x, y) space.

Given the shapes of our $\dot{x} = 0$ and $\dot{y} = 0$ curves the stationary solution is a unique saddle-point. But, as in Appendix A, for proving uniqueness or the saddle-point property of the stationary solution the

¹⁸If we assume a special form of the $G(Q, \hat{Q})$ function, viz.,

$$G(Q, \hat{Q}) = Q^n \hat{Q}^{1-n}, \quad 1 > n > 0.$$

Then it is easy to show that $\frac{g'(x)x}{g(x)} - \frac{g''(x)x}{g'(x)} = 1.$

assumption of elasticities of factor substitution being small enough for (70) to be positive is not really necessary. The proof of this statement follows exactly the same kind of proof for a similar statement about the stationary solution in Appendix A and hence is omitted here. We need the assumption about the elasticities of factor substitution for (71) to be positive in order to ensure that the $\dot{y} = 0$ curve is downward-sloping not merely around the stationary equilibrium point (x_*, y_*) but in general throughout the region we are considering. This ensures the result that for $x(0) < x_*$, y steadily decreases over time.

The stationary rate of subsidy, τ_* , can easily be calculated by putting (65) and (68) equal to zero. From (65) and (68)

$$\tau_* = y_* = \frac{\alpha g'(x_*)x_*}{g(x_*)[\delta + \alpha\beta] - \alpha g'(x_*)x_*} \quad .19$$

¹⁹If we take the special form of the $G(Q, \hat{Q})$ function as in f.n. 18,

then $\tau_* = \frac{\alpha n}{\delta + \alpha(\beta - n)}$.

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