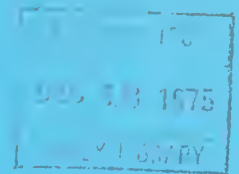


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OPTIMAL TRADE POLICY AND COMPENSATION
UNDER ENDOGENOUS UNCERTAINTY:
THE PHENOMENON OF MARKET DISRUPTION*

J. N. Bhagwati and T. N. Srinivasan

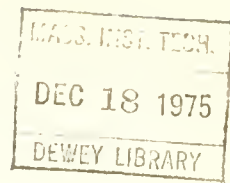
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The views expressed here are the authors' responsibility and do not reflect those of the Department of Economics or of the Massachusetts Institute of Technology.

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OPTIMAL TRADE POLICY AND COMPENSATION UNDER
ENDOGENOUS UNCERTAINTY: THE PHENOMENON OF MARKET DISRUPTION

Introduction

The fact that "market disruption" permits or prompts importing countries to invoke quantitative import restrictions (or, what is more fashionable in recent times, voluntary export restrictions by the exporting countries, at the urging of the importing countries) immediately implies that the exporting country faces a situation of endogenous uncertainty: where its own export level can affect the probability of such quantitative restrictions (QR's) being imposed. It simultaneously raises the following analytical questions which have obvious policy implications:

(1) What is the optimal trade policy for an exporting country which is faced by such potential QR-intervention?

(2) Since the possibility of such QR-intervention must restrict the trade opportunity set relative to that which would obtain in the absence of the QR-possibility, can one meaningfully define the loss that such a QR-possibility imposes on the exporting country and therefore the compensation that could be required to be paid to the exporting country under, say, a modified set of GATT rules?

I: Optimal Trade Policy: Two-Period
Model with Zero Adjustment Costs

To analyze the problem of optimal trade policy for the exporting country in the presence of market-disruption-induced possibility of

QR-intervention, we will deploy the usual trade-theoretic model of general equilibrium, but will extend it to a two-period framework in Sections I-IV. In Section III, we will also introduce adjustment costs, beginning with a simple formulation which has putty in period 1 and clay in period 2, and then extending the analysis in Section IV to lesser rigidity of redeployment of resources in period 2. In Section V, we will consider a steady state with an infinite time horizon rather than a two-period analysis, so that we can analyze the effects of continuous uncertainty (as against just period-1 uncertainty).

Thus, consider a 2-commodity model of international trade. We then assume a 2-period time horizon such that the level of exports E in the first period affects the probability $P(E)$ of a quota \bar{E} being imposed at the beginning of the next period.*

Let $U[C_1, C_2]$ be the standard social utility function defined in terms of the consumption C_i of commodity i ($i = 1, 2$). By assumption, it is known at the beginning of the next period whether the quota \bar{E} has been imposed or not. Thus, the policy in the next period will be to maximize U subject to the transformation function $F[X_1, X_2] = 0$ and the terms of trade function π if no quota is imposed and with an additional constraint $E \leq \bar{E}$ if the quota is imposed.

Let now the maximal welfare with and without the quota be \underline{U} and \bar{U} respectively. Clearly then, we have $\bar{U} > \underline{U}$ when the quota is binding.

* This method of introducing market disruption presupposes that the QR-level is prespecified but that the probability of its being imposed will be a function of how deeply the market is penetrated in the importing country and therefore how effective the import-competing industry's pressure for protection will be vis-a-vis the importing country's government.

The expected welfare in the second period is then clearly:

$$\underline{U} P(E) + \bar{U} [1 - P(E)].$$

The objective function for the first period therefore is:

$$\phi = U[X_1 - E, X_2 + \pi E] + \rho[\underline{U} P(E) + \bar{U}\{1 - P(E)\}]$$

where ρ is the discount rate. This is then to be maximized subject to the domestic transformation constraint, $F[X_1, X_2] = 0$. In doing this, assume that $P(E)$ is a convex function of E , i.e. the probability of a quota being imposed increases, at an increasing rate as E is increased, and that, in the case where π depends on E , πE is concave in E . Then, the first-order conditions for an interior maximum are:

$$\frac{\partial \phi}{\partial X_1} = U_1 - \lambda F_1 = 0 \quad (1)$$

$$\frac{\partial \phi}{\partial X_2} = U_2 - \lambda F_2 = 0 \quad (2)$$

$$\frac{\partial \phi}{\partial E} = -U_1 + U_2\{\pi + E\pi'\} - \rho(\bar{U} - \underline{U}) P'(E) = 0 \quad (3)$$

Now, Eqs. (1) and (2) yield the familiar result that the marginal rate of substitution in consumption equals the marginal rate of transformation. Eq. (3) moreover can be written as:

$$\frac{U_1}{U_2} = (\pi + \pi'E) - \frac{\rho\{\bar{U} - \underline{U}\}}{U_2} P'(E). \quad (3')$$

If (A) monopoly power is absent ($\pi' = 0$) and if (B) the first period's exports do not affect the probability of a quota being imposed in the second period, then (3') clearly reduces to the standard condition that the marginal rate of substitution in consumption equals the (average = marginal)

terms of trade. If (A) does not hold but (B) holds, then $\frac{U_1}{U_2}$ equals the marginal terms of trade ($\pi + \pi'E$), leading to the familiar optimum tariff. If both A and B are present, there is an additional tariff element: $\frac{\rho[\bar{U} - \underline{U}]}{U_2} P'(E)$. This term can be explained as follows: if an additional unit of exports takes place in period 1, the probability of a quota being imposed and hence a discounted loss in welfare of $\rho(\bar{U} - \underline{U})$ occurring, increases by $P'(E)$. Thus, at the margin, the expected loss in welfare is $\rho(\bar{U} - \underline{U})P'(E)$ since there is no loss in welfare if the quota is not imposed. Converted to numeraire terms, this equals $\frac{\rho(\bar{U} - \underline{U})P'(E)}{U_2}$, and must be subtracted from the marginal terms of trade ($\pi + \pi'E$), the effect of an additional unit of exports on the quantum of imports.

It is then clear that the market-disruption-induced QR-possibility requires optimal intervention in the form of a tariff (in period 1). It is also clear that, compared to the optimal situation without such a QR-possibility, the resource allocation in the QR-possibility case will shift against exportable production : i.e., comparative advantage, in the welfare sense, shifts away, at the margin, from exportable production. Moreover, denoting the utility level under the optimal policy intervention with quota possibility as ϕ_Q^{OPT} , that under laissez faire with the quota possibility as ϕ_Q^L , and that under laissez faire without this quota possibility as ϕ_{NQ}^L , we can argue that:

$$\phi_{NQ}^L > \phi_Q^{OPT} > \phi_Q^L$$

This result is set out, with the attendant periodwise utility levels achieved under each option, in Table 1 which is self-explanatory.

For the case of a small country, with no monopoly power in trade

Table 1: Alternative Outcomes under Different Policies

	Alternative Outcomes		
	Optimal Policy Intervention with Possible Quota	Laissez Faire with Possible Quota	Laissez Faire with no Quota Possibility
Period 1	U^*	\bar{U}	\bar{U}
Period 2	$\rho [U P^*(E) + \bar{U} (1 - P^*(E))]$	$\rho [U \bar{P}(E) + \bar{U} (1 - \bar{P}(E))]$	$\rho \bar{U}$
ϕ : social utility level	ϕ_Q^{OPT}	ϕ_Q^L	ϕ_{NQ}^L
	$\phi_{NQ}^L > \phi_Q^{OPT} > \phi_Q^L$		

- Notation:
- (1) \underline{U} is utility level if quota is imposed.
 - (2) \bar{U} is utility level if quota is not imposed.
 - (3) U^* is utility level with optimal policy intervention when quota can be imposed in second period.
 - (4) P is the probability of second-period quota of \bar{E} being imposed, as a function of the first-period exports, E . P^* is the probability in the optimal-policy case and \bar{P} in the case with laissez faire but possible quota.
 - (5) ρ is the rate of discount.

(except for the quota possibility), the equilibria under alternative policies are illustrated in Figure 1. Thus, \bar{U} represents the utility level in the absence of a quota, \underline{U} the utility level when the quota is imposed, and U^* the first-period utility level reached under the optimal policy intervention option. Note that equilibrium with U^* naturally requires that the export level is being restricted below the level that would be reached with non-intervention (at \bar{U}), while exceeding the level reached in equilibrium when the quota is invoked (at \underline{U}). Also, note that the optimal policy for restricting the first-period level of exports is a tariff: a conclusion that is, of course, familiar from the theory of optimal intervention under non-economic objectives as considered in Johnson (1965) and Bhagwati and Srinivasan (1969).

II: Defining the Loss from Market-Disruption-Induced QR-Possibility

Consider now the measure of the loss to the exporting country from this possibility of a market-disruption-induced QR. One can think of alternative ways in which this loss could be defined:

Measure I: Taking expected utilities, one can define the loss of welfare to the exporting country as the difference between ϕ_{NQ}^L and ϕ_Q^L : i.e. the loss in expected welfare that follows, in the absence of optimal intervention by the exporting country, from the QR-possibility.

This measure clearly is: $\rho \bar{P}(E) \{\bar{U} - \underline{U}\}$ and is, of course, nothing but the expected loss in period-2 from the possible imposition of the quota, discounted at rate ρ .

Now, it is also clear that this measure will lie between the ex-post period-2 loss if the quota is invoked (which loss, duly discounted, is

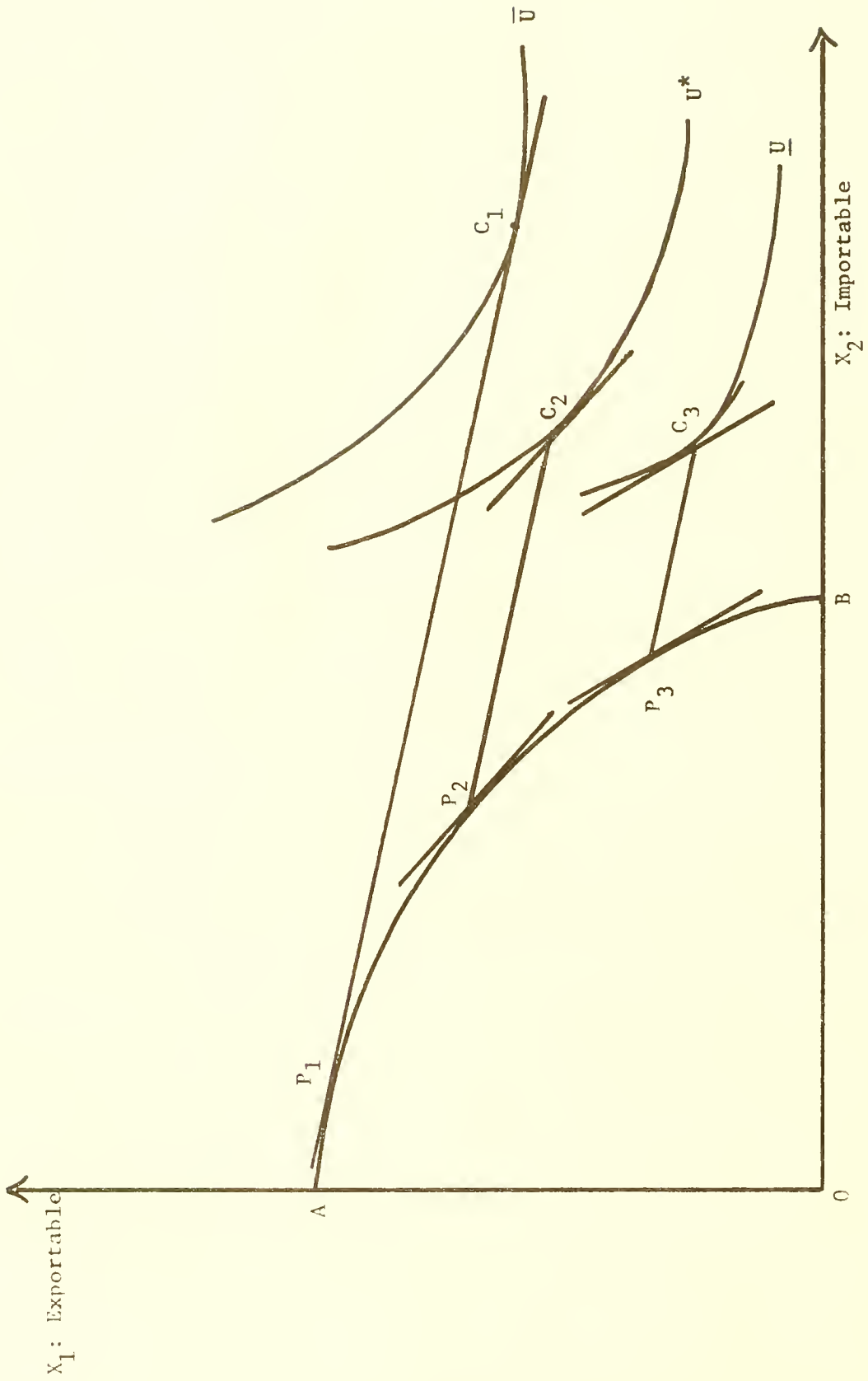


FIGURE 1

$\rho(\bar{U} - U)$) and the ex-post period-2 loss if the quota is not invoked (which loss is, of course, zero). Thus, one must regard the actual period-2 loss when the quota is invoked as an upper bound on the loss in this model.

It also follows that there is a welfare loss, measured as $\rho\bar{P}(E)\{\bar{U} - U\}$ even if the quota is not actually invoked in period 2 and, (in our 2-period model), the actual equilibrium allocations in each period are identical between the QR-possibility and the no-QR-possibility situations. This follows clearly from the fact that, in period 1, consumers face the prospect of uncertain forces in period 2, as the QR may or may not be invoked.

Measure II: Alternatively one may measure the loss to the exporting country as the difference between ϕ_{NQ}^L and ϕ_Q^{OPT} : the difference between expected welfare when there is no QR possibility and that when the government of the exporting country intervenes with optimal policy to maximize expected welfare when there is a QR-possibility. This alternative measure would be more meaningful for exporting countries with governmental trade agencies or exporters' associations with ability to regulate their overall export levels, whereas Measure I would be more meaningful for exporting countries with (only) atomistic exporters.

III: Adjusting for Adjustment Costs: A Putty-Clay Model

So far, our analysis was based on the assumption that the choice of optimal production in period 2 was not constrained by the choice of production in period 1. Thus, in Figure 1, the economy could move from P_1 or P_2 in period 1 to P_3 in period 2, along the (long-run) transformation curve AB.

However, this procedure fails to take into consideration possible adjustment costs: i.e., we were essentially dealing with a putty model.

However, this procedure eliminates an important aspect of the problem raised by market disruption. So, in this section, we modify our model and analysis to allow for adjustment costs. However, to simplify the analysis, we take initially the extreme polar case of a putty-clay model, where the production choice made in period 1 cannot be modified in any way in period 2.

With this modification, the choice variables now are: X_i , the production of commodity i in periods 1 and 2 ($i = 1, 2$); E_1 , the net exports of commodity 1 in period 1; and E_2 , the net exports of commodity 1 in period 2 when no quota is imposed. As before, \bar{E} is the net export of commodity 1 when the quota is imposed.

Clearly then, the expected welfare ϕ is now as follows:

$$\begin{aligned} \phi = & U[X_1 - E_1, X_2 + \pi E_1] + \rho P(E_1) \underline{U}[X_1 - \bar{E}, X_2 + \pi \bar{E}] \\ & + \rho \{1 - P(E_1)\} \bar{U}[X_1 - E_2, X_2 + \pi E_2] \end{aligned}$$

This is then maximized subject to the implicit transformation function, $F(X_1, X_2) = 0$, as before. The first-order conditions for an interior maximum then are:

$$\frac{\partial \phi}{\partial X_1} = U_1^1 + \rho P(E_1) \underline{U}_1^2 + \rho \{1 - P(E_1)\} \bar{U}_1^1 - \lambda F_1 = 0 \quad (4)$$

$$\frac{\partial \phi}{\partial X_2} = U_2^1 + \rho P(E_1) \underline{U}_2^2 + \rho \{1 - P(E_1)\} \bar{U}_2^2 - \lambda F_2 = 0 \quad (5)$$

$$\begin{aligned} \frac{\partial \phi}{\partial E_1} = & -U_1^1 + \{\pi(E_1) + E_1 \pi'(E_1)\} U_2^1 \\ & - \rho P'(E_1) \{\bar{U}_2^2 - \underline{U}_2^2\} = 0 \end{aligned} \quad (6)$$

$$\frac{\partial \phi}{\partial E_2} = \rho \left[-\bar{U}_1^2 + \{\pi(E_2) + E_2 \pi'(E_2)\} \bar{U}_2^2 \right] \{1 - P(E_1)\} = 0 \quad (7)$$

where

$$\bar{U}_j^1 = \frac{\partial U[X_1 - E_1, X_2 + \pi E_1]}{\partial X_j},$$

$$\bar{U}_j^2 = \frac{\partial U[X_1 - E_2, X_2 + \pi E_2]}{\partial X_j},$$

$$\underline{U}_j^2 = \frac{\partial U[X_1 - \bar{E}, X_2 + \pi \bar{E}]}{\partial X_j}, \text{ and}$$

λ = the Lagrangean multiplier associated with the constraint,

$$F(X_1, X_2) = 0.$$

The interpretation of these first-order conditions is straightforward.

Condition (7) states that, given the optimal production levels, the level of exports in period 2 when no quota is imposed must be such as to equate the marginal rate of substitution in consumption to the marginal terms of trade. Condition (6) is identical in form to the one obtained earlier: the optimal exports in period 1 must not equate the marginal rate of substitution in consumption in that period to the marginal terms of trade, but must instead also allow for the marginal change in expected welfare arising out of the change in probability of a quota being imposed: the latter equals $P'(E_1) (\bar{U}^2 - \underline{U}^2)$ where $\bar{U}^2 = U[X_1 - E_2, X_2 + \pi E_2]$ and $\underline{U}^2 = U[X_1 - \bar{E}, X_2 + \pi \bar{E}]$. Thus, condition (6) ensures the optimal choice of exports in period 1, given the production levels. Conditions (4) and (5) then relate to the optimal choice of production levels and, as we would expect, the introduction of adjustment costs does make a difference.

Writing (4) and (5) in the familiar ratio form, we get:

$$\frac{F_1}{F_2} = \frac{U_1^1 + \rho P(E_1) \underline{U}_1^2 + \rho \{1-P(E_1)\} \bar{U}_1^2}{U_2^1 + \rho P(E_1) \underline{U}_2^2 + \rho \{1-P(E_1)\} \bar{U}_2^2} \quad (3)$$

Clearly therefore the marginal rate of transformation in production (in periods 1 and 2, identically, as production in period 1 will carry over into period 2 by assumption), i.e. F_1/F_2 , must not equal the marginal rate of substitution in consumption in period 1, i.e. U_1^1/U_2^1 , (unlike our earlier analysis without adjustment costs in Sections I and II). Rather, F_1/F_2 should equal a term which properly takes into account the fact that production choices once made in period 1 cannot be changed in period 2 to suit the state (i.e. the imposition or absence of a quota) obtaining in period 2. Eq. (8) can be readily interpreted as follows.

The LHS is, of course, the marginal rate of transformation in production. The RHS represents the marginal rate of substitution in consumption, if re-interpreted in the following sense. Suppose that the output of commodity 1, the exportable, is increased by one unit in period 1 (and hence in period 2 as well, by assumption). Given an optimal trade policy, then, the impact of this on welfare can be examined by adding it to consumption in each period. Thus social utility is increased in period 1 by U_1^1 while in period 2 it will increase by \bar{U}_1^2 if no quota is imposed and by \underline{U}_1^2 if the quota is imposed. Thus, the discounted increase in period-2 welfare is given as: $\rho \left[\underline{U}_1^2 P(E_1) + \bar{U}_1^2 (1-P(E_1)) \right]$. Thus, the total expected welfare impact of a unit increase in the production of commodity 1 is :

$$U_1^1 + \rho \left[\underline{U}_1^2 P(E_1) + \bar{U}_1^2 (1-P(E_1)) \right].$$

Similarly, a decrease in the production of commodity 2 by a unit in period 1

(and hence in period 2 as well) reduces expected welfare by:

$$U_2^1 + \rho \left[\underline{U}_2^2 P(E_1) + \bar{U}_2^2 (1-P(E_1)) \right] .$$

Hence, the ratio of these two expressions, just derived, represents the "true" marginal rate of substitution, and this indeed is the RHS in Eq. (8) to which the marginal rate of transformation in production— F_1/F_2 , the LHS in Eq. (8)—is to be equated for optimality.

The optimal policy interventions in this modified model with adjustment costs are immediately evident from Eqs. (6) - (8) and the preceding analysis. Thus, in period 1, the ratio U_1^1/U_2^1 is clearly the relative price of commodity 1 (in terms of commodity 2) facing consumers, while $\pi(E_1)$ is the average terms of trade. Thus U_1^1/U_2^1 differs from $\pi(E_1)$ by $[\pi'(E_1) - \rho P'(E_1) \{\bar{U}^2 - \underline{U}^2\}]$ and this difference constitutes a consumption tax on the importable, commodity 2. An identical difference between F_1/F_2 , the relative price facing producers, and $\pi(E_1)$ would define a production tax on commodity 2 at the same rate, so that a tariff at this rate would constitute the appropriate intervention in the model with no adjustment costs. However, with adjustment costs, Eq. (8) defines, for period 1, the appropriate production tax-cum-subsidy which, in general, will diverge from the appropriate consumption tax: so that the optimal mix of policies in the model with adjustment costs will involve a tariff (reflecting both the monopoly power in trade and the QR possibility) plus a production tax-cum-subsidy in period 1. In period 2, in both the models (with and without adjustment costs), an appropriate intervention in the form of a tariff (to exploit monopoly power) would be called for; however, with production fixed at period-1 levels in the adjustment-cost

model, a consumption tax-cum-subsidy would equally suffice. Specifically, note that in period 2, with adjustment costs, the price-ratio facing consumers would be \bar{U}_1^2/\bar{U}_2^2 if no quota is imposed, with the average terms of trade at $\pi(E_2)$ and the producer's price-ratio (as defined along the putty-transformation frontier) would be F_1/F_2 ; on the other hand, if the quota is imposed, these values change to $\underline{U}_1^2/\underline{U}_2^2$, $\pi(\bar{E})$ and F_1/F_2 respectively. The consumption tax-cum-subsidy and the equivalent tariff (with no impact on production decision, already frozen at period-1 levels) are then defined by these divergences, depending on whether the quota obtains or not.

A tabular comparison of the characteristics of the optimal solution, with and without adjustment costs, is presented in Table 2 and should assist the reader.

Table 2: Characteristics of Optimal Solutions in Models With and Without Adjustment Costs¹

	No Adjustment Costs	Adjustment Costs
Period 1	$DRS_1 \neq FRT_1$ $DRS_1 = DRT_1$	$DRS_1 \neq FRT_1$ $DRS_1 \neq DRT_1$
Period 2	$DRS_2 = DRT_2$ $= FRT_2$	$DRS_2 = FRT_2$ (DRT ₂ not relevant as production is frozen at period-1 levels)

Notes: ¹DRS, DRT and FRT represent the marginal rates of substitution in consumption, domestic transformation, and foreign transformation respectively. For an earlier use of these abbreviations see Bhagwati, Ramaswami and Srinivasan (1969). Since we are considering an interior maximum, the inequalities do not include corner equilibria, of course. The subscripts refer to the periods, 1 and 2.

Note that the above results are quite consistent with the basic propositions of the theory of distortions, as developed in Bhagwati-Ramaswami (1963), Johnson (1965) and Bhagwati (1971): the first-best, optimal policy intervention for the case with adjustment costs requires a trade policy to adjust for the foreign distortion (represented by the effect of current exports on the period-2 probability of a quota being invoked)* and a production tax-cum-subsidy to adjust for the existence of adjustment costs in production. It also follows, from the equivalence propositions, that the combination of the optimal tariff and the optimal production tax-cum-subsidy can be reproduced identically by a tariff set at the "net" production tax-cum-subsidy required by the optimal solution plus a consumption tax-cum-subsidy. Similarly, while our analysis has been focussed on first-best policy intervention, the fundamental results of the theory of distortions and welfare on second-best policies also can be immediately applied to our problem. Thus, if there are zero adjustment costs so that there is only the foreign distortion in period 1, then clearly a production tax-cum-subsidy will improve (but not maximize) welfare. Similarly, if there are adjustment costs as well, then there will be two distortions and then we would now have applicable here the Bhagwati-Ramaswami-Srinivasan (1969) proposition that no feasible, welfare-improving form of intervention may exist if both of the policy measures that will secure optimal intervention cannot be used simultaneously.

IV: Adjustment Costs: A General Formulation

So far, we have considered only the extreme version of an adjustment-costs model, where the period-1 production levels are frozen in period 2.

* In addition, of course, to the usual optimal tariff if there is also monopoly power in trade.

We may now briefly consider however a more general formulation, (with basically the same results, of course, for optimal policy intervention), where the clay nature of period 1 allocation is partially relaxed: some reallocation is now permitted in period 2.

The simplest way to do this is to write out the period-2 implicit transformation function as $G[\underline{X}_1^2, \underline{X}_2^2, X_1^1, X_2^1] = 0$ for the with-quota case and as $G[\bar{X}_1^2, \bar{X}_2^2, X_1^1, X_2^1] = 0$ for the no-quota case, such that the feasible output levels in period 2 are explicitly made a function of the (allocation-cum-) output levels of period 1, X_1^1 and X_2^1 . Our welfare problem then becomes one of maximizing:*

$$\begin{aligned} \phi = & U[X_1^1 - E_1, X_2^1 + \pi E_1] \\ & + \rho P(E_1) \underline{U}[\underline{X}_1^2 - \underline{E}_2, \underline{X}_2^2 + \underline{E}_2 \pi] \\ & + \rho \{1 - P(E_1)\} \bar{U}[\bar{X}_1^2 - \bar{E}_2, \bar{X}_2^2 + \bar{E}_2 \pi] \end{aligned}$$

subject to:

$$F(X_1^1, X_2^1) = 0 \tag{9}$$

for period 1;

$$G[\underline{X}_1^2, \underline{X}_2^2, X_1^1, X_2^1] = 0 \tag{10}$$

for period 2, with quota imposed;

$$G[\bar{X}_1^2, \bar{X}_2^2, X_1^1, X_2^1] = 0 \tag{11}$$

for period 2, with no quota imposed; and

$$\underline{E}_2 \leq \bar{E} \tag{12}$$

where \bar{E} is the quota level, as before.

* The lower bar and the upper bar refer to the with-quota and without-quota values respectively.

The first-order conditions for an interior maximum then are:

$$\frac{\partial \phi}{\partial X_1^1} = U_1^1 - \lambda_1 F_1 - \lambda_2 G_3 - \lambda_3 \bar{G}_3 = 0 \quad (13)$$

$$\frac{\partial \phi}{\partial X_2^1} = U_2^1 - \lambda_1 F_2 - \lambda_2 G_4 - \lambda_3 \bar{G}_4 = 0 \quad (14)$$

$$\frac{\partial \phi}{\partial X_1^2} = \rho P(E_1) U_1^2 - \lambda_2 G_1 = 0 \quad (15)$$

$$\frac{\partial \phi}{\partial X_2^2} = \rho P(E_1) U_2^2 - \lambda_2 G_2 = 0 \quad (16)$$

$$\frac{\partial \phi}{\partial X_1^2} = \rho \{1 - P(E_1)\} \bar{U}_1^2 - \lambda_3 \bar{G}_1 = 0 \quad (17)$$

$$\frac{\partial \phi}{\partial X_2^2} = \rho \{1 - P(E_1)\} \bar{U}_2^2 - \lambda_3 \bar{G}_3 = 0 \quad (18)$$

$$\frac{\partial \phi}{\partial E_1} = -U_1^1 + (\pi + \pi' E_1) U_2^1 - \rho [\bar{U}^2 - U^2] P'(E_1) = 0 \quad (19)$$

$$\frac{\partial \phi}{\partial E_2} = \rho P(E_1) \left[-U_1^2 + (\pi + \pi' E_2) U_2^2 \right] - \gamma = 0 \quad (20)$$

$$\frac{\partial \phi}{\partial E_2} = \rho \{1 - P(E_1)\} \left[-\bar{U}_1^2 + (\pi + \pi' \bar{E}_2) \bar{U}_2^2 \right] = 0 \quad (21)$$

where λ_1 , λ_2 , λ_3 and γ are the Lagrangean multipliers associated with constraints (9) - (12) respectively, and G_i is the partial derivate with respect to the i^{th} argument.

It is then easy to see that, while $\underline{DRS}_2 = \underline{DRT}_2$ (because Eqs. (15) and (16) imply that $\underline{U}_1^2/\underline{U}_2^2 = \underline{G}_1/\underline{G}_2$) and $\overline{DRS}_2 = \overline{DRT}_2$ (because Eqs. (17) and (18) imply that $\overline{U}_1^2/\overline{U}_2^2 = \overline{G}_1/\overline{G}_2$), as before, one can see the effect of adjustment costs more readily, from Eqs. (13) and (14), i.e. $\underline{DRT}_1 \neq \underline{DRS}_1$,

as follows:

$$\frac{F_1}{F_2} = \frac{U_1^1 - \lambda_2 \underline{G}_3 - \lambda_3 \bar{G}_3}{U_1^2 - \lambda_2 \underline{G}_4 - \lambda_3 \bar{G}_4} \quad (22)$$

or, alternatively:

$$\frac{F_1}{F_2} = \frac{U_1^1 - \rho P(E_1) U_1^2 \left(\frac{\underline{G}_3}{\underline{G}_1} \right) - \rho \{1-P(E_1)\} \bar{U}_1^2 \left(\frac{\bar{G}_3}{\bar{G}_1} \right)}{U_1^2 - \rho P(E_1) U_2^2 \left(\frac{\underline{G}_4}{\underline{G}_2} \right) - \rho \{1-P(E_1)\} \bar{U}_2^2 \left(\frac{\bar{G}_4}{\bar{G}_2} \right)} \quad (22')$$

From either (22) or (22'), it is easy to see that, if we have the polar case with no reallocation possible in period 2 (the putty-clay model of Section III), the transformation curve in period 2 reduces to the single point (X_1^1, X_2^1) . As such, the partial derivatives $\underline{G}_i, \bar{G}_i$ ($i = 1, 2, 3, 4$) are not defined. However, one could define G in such a way that putty-clay is a limiting case and, in the limit, $\bar{G}_3 = \underline{G}_3 = -\underline{G}_1$ and $\bar{G}_4 = \underline{G}_4 = -\underline{G}_2$. This is analogous to obtaining the Leontief fixed coefficient production function as a limiting case of the CES production function. Therefore, Eq. (22') reduces to Eq. (8), as it should. If, however, we have no adjustment costs (as in Section I), then $\bar{G}_3 = \underline{G}_3 = \bar{G}_4 = \underline{G}_4 = 0$ and Eq. (22') will reduce to $\frac{U_1^1}{U_2^1} = \frac{F_1}{F_2}$ (which is what Eqs. (1) and (2) imply in Section I). For any situation with some, but not total, inflexibility of resource allocation in period 2, the ratios $-\underline{G}_3/\underline{G}_1, -\bar{G}_3/\bar{G}_1, -\underline{G}_4/\underline{G}_1$ and $-\bar{G}_4/\bar{G}_1$ will lie between 0 and 1.

The parametric values of these ratios will clearly reflect the "pattern of inflexibility" that one contends with. Thus, if one assumes total factor price flexibility but no resource mobility, as in Haberler (1950), then the putty-clay model is relevant. On the other hand, one might assume just the opposite, where factor prices are inflexible but resources

are fully mobile: this being the case systematically analyzed by Brecher (1974). Variations on these two polar possibilities include analyses such as that of Mayer (1974) which assumes an activity-specific factor with no mobility in the short-run (where, interpret "short-run" as period 2 for our purposes) but with factor price flexibility.

Whatever the source of adjustment costs in period 2, what they do imply is that the transformation curve of period 1 is not feasible in period 2. Hence the illustration of optimal-policy equilibrium in period 2 would be as in Figure 2, where AB is the (putty) period-1 transformation curve, P^1 the production point on it in period 1 - representing therefore (X_1^1, X_2^1) -, CPD the clay transformation curve for period 2 and QPR the (partial-clay) transformation curve when resources in period 2 are partially mobile. With equilibrium production at P^1 (with tangency in period 1 to AB) and consumption at C^1 , and assuming for simplicity that the international terms of trade are fixed at P^1C^1 , we can then illustrate that $F_1/F_2 \neq U_1^1/U_2^1$ (i.e. that the tangents to AB and to the social utility curve U^1 are not equal): as required by Eq. (22') for the case of adjustment costs.

V: Steady State Analysis: Infinite Time Horizon

So far, we have worked with a 2-period time horizon, where the uncertainty essentially obtains in period 1 and is resolved in period 2. However, it would be useful to consider an infinite-time-horizon model where each period can face unresolved uncertainty. In this section, therefore, we consider now an infinite-time-horizon steady state analysis of our basic model.

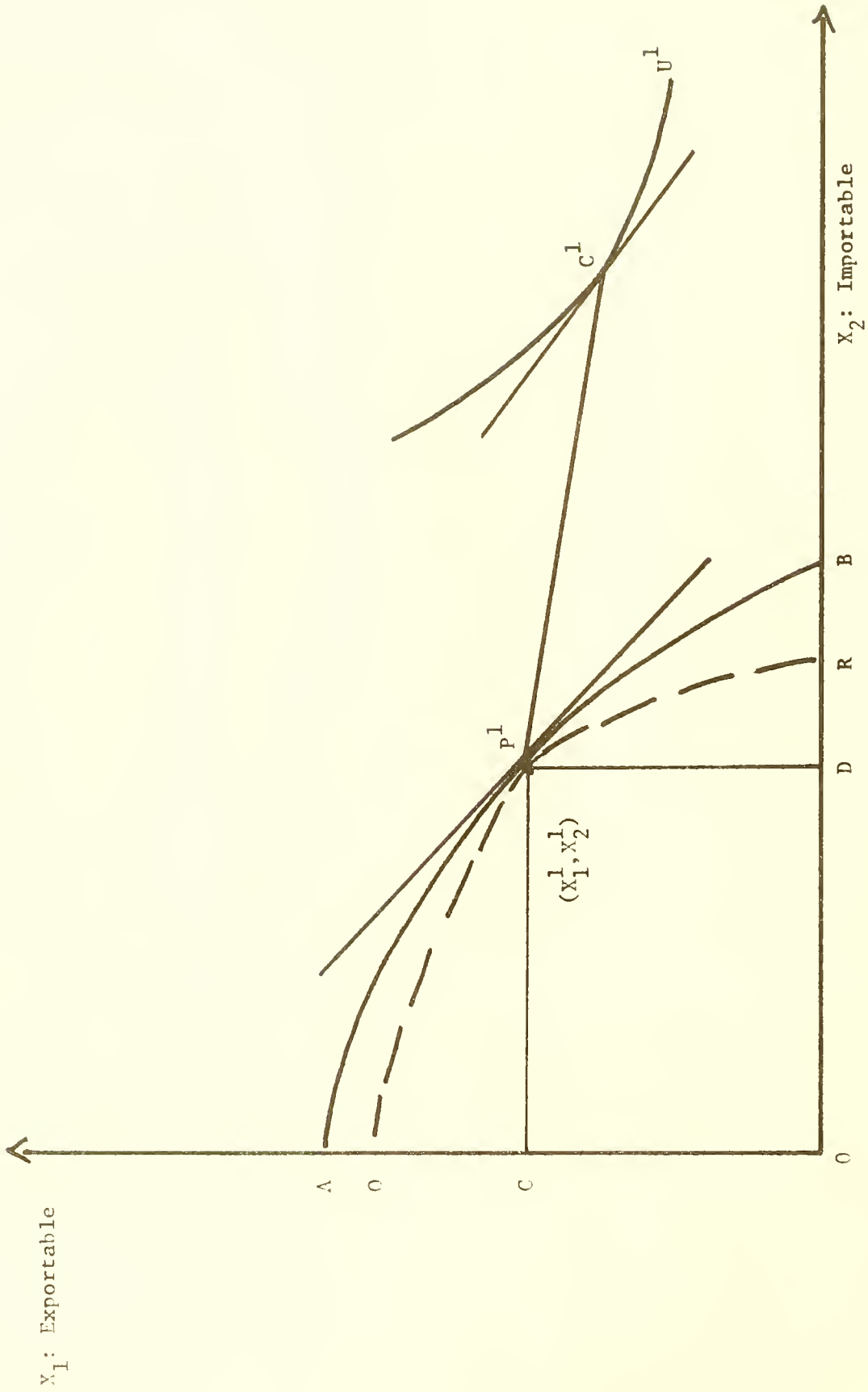


FIGURE 2

However, to simplify the analysis, we will assume that the quota, once imposed, will not be lifted. The analysis therefore applies to the case where the prospect of a quota being levied is not certain but the prospect that, once levied, it will persist is certain: a situation that is fairly approximated by commodities/items falling within the scope of, say, the long term agreement on textiles, and other similar commodities.

Thus, consider now that, aside from a quota persisting forever once invoked, the probability of a quota being invoked in any period depends only on the level of exports in the previous period, and that this relationship remains invariant over time. Further, assume that the chance of the event of a quota not being invoked in any period is independent of the same event in the previous periods. Then it is clear that, in the event that the quota is not imposed in any period, the optimal production and trade policies in that period will be the same regardless of the calendar time at which this event happens.

Let us then start from an initial period at which the quota is not in force, and let $W[X_1, X_2, E]$ denote the discounted sum of expected welfare levels at all future points, given that the production and export levels are X_1, X_2 and E respectively. In other words, W is the welfare associated with the stationary policy (X_1, X_2, E) in any period in which no quota has been imposed till then.

It then follows (as will be demonstrated below) that:

$$W = U[X_1 - E, X_2 + \pi E] + \rho\{1 - P(E)\}W + \frac{\rho}{1-\rho} P(E) \underline{U} \quad (23)$$

This is seen as follows. The policy (X_1, X_2, E) yields utility of

$U[X_1 - E, X_2 + \pi E]$ in the first period. In the second period, if the quota is not imposed (the probability of which event is $1 - P(E)$), the policy is again (X_1, X_2, E) , so that one can regard the welfare from that point on as $W[X_1, X_2, E]$ as in period 1. This W would however have to be discounted back to the first period, thus yielding the second term on the RHS of Eq. (23): $\rho\{1 - P(E)\}W$. However, if the quota is imposed in the second period, (the probability of which event is $P(E)$), the optimal policy from then on remains the same (as the quota persists forever by assumption) and yields welfare \underline{U} in each period. The discounted sum of this series is clearly $\underline{U}\rho/(1-\rho)$, so that the result is to yield the third term on the RHS of Eq. (23): $\frac{\rho}{1-\rho} P(E)\underline{U}$.

In the following analysis, note first that the maximizing procedure will, as before, be different for the cases with and without adjustment costs. In the case where adjustment costs are zero, \underline{U} will be obtained by maximizing $U[\hat{X}_1 - \bar{E}, \hat{X}_2 + \pi\bar{E}]$ with respect to \hat{X}_1, \hat{X}_2 subject to $F(\hat{X}_1, \hat{X}_2) = 0$, and where \bar{E} is the specified quota. In the case where there are adjustment costs, however, X_1 and X_2 cannot be altered (altogether, if we take the putty-clay model) once chosen; hence \underline{U} must be defined as $U[X_1 - \bar{E}, X_2 + \pi\bar{E}]$, the optimal X_1 and X_2 now being chosen so as to maximize $W[X_1, X_2, E]$.

(A): Zero Adjustment Costs: In this case, we must now maximize:

$$W[X_1, X_2, E] = \frac{U[X_1 - E, X_2 + \pi E] + \frac{\rho}{1-\rho} \underline{U} P(E)}{1 - \rho(1-P(E))}$$

subject to

$$F(X_1, X_2) = 0.$$

Recalling that, in this case, \underline{U} does not depend on X_1 , X_2 and E , we can derive the first-order conditions for an interior maximum:

$$\frac{\partial W}{\partial X_1} = \frac{U_1}{1-\rho\{1-P(E)\}} - \lambda F_1 = 0 \quad (24)$$

$$\frac{\partial W}{\partial X_2} = \frac{U_2}{1-\rho\{1-P(E)\}} - \lambda F_2 = 0 \quad (25)$$

$$\begin{aligned} \frac{\partial W}{\partial E} &= \frac{\{1-\rho(1-P(E))\} \left[-U_1 + (\pi + \pi'E)U_2 + \frac{\rho}{1-\rho} U P'(E) \right] - \rho P'(E) \left[U + \frac{\rho}{1-\rho} \underline{U} P(E) \right]}{\{1 - \rho(1-P(E))\}^2} \\ &= 0 \end{aligned} \quad (26)$$

As one would expect, Eqs. (24) and (25) imply, (given that $1-\rho(1-P(E)) > 0$), that $U_1/U_2 = F_1/F_2$, so that $DRS_1 = DRT_1$. And, rewriting Eq. (26) as follows:

$$-U_1 + (\pi + \pi'E)U_2 - \frac{\rho P'(E)(U-\underline{U})}{1-\rho(1-P(E))} = 0 \quad , \quad (26')$$

we can see that U_1/U_2 differs from the marginal terms of trade $(\pi + \pi'E)$ by the term $\frac{\rho P'(E)(U-\underline{U})}{1-\rho(1-P(E))}$ and hence the optimal policy intervention is a tariff that suitably corresponds to the difference between U_1/U_2 and π .

This result, of course, is identical to that derived in the 2-period model, except that the infinite time horizon model leads to a different tariff rate. In particular, this difference arising from the fact that a quota may be imposed at any time in the future, with probability $P(E)$, reflects itself in the tariff term in two ways: (i) the utility in a period in which no quota is imposed is now U , whereas \bar{U} ($>U$) is the maximum feasible utility and enters the tariff for the 2-period case in Eq. (3'); and (ii) the term $\{1 - \rho(1-P(E))\}$ now enters the denominator.

This difference is commented on, below.

(B): Non-zero Adjustment Costs: We now must maximize:

$$W[X_1, X_2, E] = \frac{U[X_1 - E, X_2 + \pi E] + \frac{\rho}{1-\rho} U[X_1 - \bar{E}, X_2 + \bar{E}]P(E)}{1 - \rho\{1 - P(E)\}}$$

subject to: $F(X_1, X_2) = 0$.

The first-order conditions for an interior maximum now are:

$$\frac{\partial W}{\partial X_1} = \frac{U_1 + \frac{\rho}{1-\rho} \underline{U}_1 P(E)}{1 - \rho\{1 - P(E)\}} - \lambda F_1 = 0 \quad (27)$$

$$\frac{\partial W}{\partial X_2} = \frac{U_2 + \frac{\rho}{1-\rho} \underline{U}_2 P(E)}{1 - \rho\{1 - P(E)\}} - \lambda F_2 = 0 \quad (28)$$

$$\begin{aligned} \frac{\partial W}{\partial E} &= \frac{\{-U_1 + U_2(\pi + \pi' E) + \frac{\rho}{1-\rho} \underline{U} P'(E)\} \{1 - \rho(1 - P(E))\} - \rho P'(E) \{U + \frac{\rho}{1-\rho} \underline{U} P(E)\}}{\{1 - \rho(1 - P(E))\}^2} \\ &= 0 \end{aligned} \quad (29)$$

Since $\{1 - \rho(1 - P(E))\} > 0$, we then get:

$$\frac{F_1}{F_2} = \frac{U_1 + \frac{\rho}{1-\rho} \underline{U}_1 P(E)}{U_2 + \frac{\rho}{1-\rho} \underline{U}_2 P(E)} = \frac{(1-\rho)U_1 + \rho \underline{U}_1 P(E)}{(1-\rho)U_2 + \rho \underline{U}_2 P(E)} \quad (30)$$

$$-U_1 + U_2(\pi + \pi' E) - \frac{\rho P'(E)}{\{1 - \rho(1 - P(E))\}} (U - \underline{U}) = 0 \quad (31)$$

Hence, it is evident from Eq. (30) that, as in the two-period model of Sections III and IV, the introduction of adjustment costs results in establishing a wedge between the marginal rate of transformation, F_1/F_2 , in any period and the marginal rate of substitution in consumption (U_1/U_2 or $\underline{U}_1/\underline{U}_2$, depending on whether the quota has not, or has, been imposed). And, Eq. (31) shows that the first-order condition relating to

exports (E) continues to be of the same essential form as in the case without adjustment costs.*

(C): Welfare Comparisons: Confining ourselves to the simpler case of zero adjustment costs, we can now see that, in the infinite time horizon model, laissez faire will lead to a welfare level (given that a quota may be imposed at any time), of:

$$W_Q^L = \frac{\bar{U} + \frac{\rho}{1-\rho} \underline{U} P(\hat{E})}{1 - \rho(1-P(\hat{E}))} \quad (32)$$

where $\bar{U} = \text{Max}_{X_1, X_2, E} U[X_1 - E, X_2 + \pi E]$ subject to $F(X_1, X_2) = 0$, and \hat{E} is the corresponding optimal export level; and $\underline{U} = \text{Max}_{X_1, X_2, E} U[X_1 - E, X_2 + \pi E]$ subject to $F(X_1, X_2) = 0$ and $E \leq \bar{E}$.

The same laissez faire policy, when the probability of a quota being imposed is zero, will clearly lead to welfare level:

$$W_{NQ}^L = \frac{\rho \bar{U}}{1-\rho} \quad (33)$$

Finally, the optimal-policy solution to the situation with the probability of a quota being imposed leads to the welfare level:

$$W_Q^{OPT} = \frac{U^* + \frac{\rho}{1-\rho} \underline{U} P(E^*)}{1 - \rho\{1-P(E^*)\}} \quad (34)$$

where $U^* = U[X_1^* - E^*, X_2^* + \pi E^*]$ and X_1^*, X_2^*, E^* maximize

$$\frac{U[X_1 - E, X_2 + \pi E] + \frac{\rho}{1-\rho} \underline{U} P(E)}{1 - \rho(1-P(E))}$$

subject to $F(X_1, X_2) = 0$.

* The \underline{U} values, in the two cases, will not of course be equal.

Clearly then, we have the ranking:

$$W_{NQ}^L > W_Q^{OPT} > W_Q^L .$$

(D): Interpreting the Difference Between Infinite-Time-Horizon and Two-Period Results: In concluding the analysis of Section V, it would be useful to comment on the difference in the optimal tariffs that obtains between the infinite time horizon and the 2-period models, that was noted explicitly above for the simpler case of zero adjustment costs.

For this purpose, it is best to take the two-period model and to turn it into an infinite time horizon model by assuming that, in the second period, the uncertainty is resolved fully forever: i.e. that, whether the quota is imposed in period 2 or not, that will also be the case thereafter. In this event, welfare will continue to be \underline{U} (or \bar{U}), depending on whether the quota is (or is not) imposed. And, discounted to the present, this yields a welfare of $\frac{\rho \underline{U}}{1-\rho}$ (or $\frac{\rho \bar{U}}{1-\rho}$). Thus the maximand becomes:

$$U[X_1 - E, X_2 + \pi E] + \frac{\rho}{1-\rho} \{ \underline{U} P(E) + \bar{U}(1-P(E)) \}$$

and yields the first-order conditions for an interior maximum as follows:

$$U_1 - \lambda F_1 = 0 \tag{35}$$

$$U_2 - \lambda F_2 = 0 \tag{36}$$

$$-U_1 + U_2 [\pi + \pi' E] - \frac{\rho}{1-\rho} [\bar{U} - \underline{U}] P'(E) = 0 \tag{37}$$

On the other hand, in the infinite time horizon model with continuing uncertainty as to whether a quota, still not imposed, will be imposed or not (as in this section), the equivalent conditions were derived as

Eqs. (24), (25) and (26'). To contrast this case with the preceding case of infinite time horizon with uncertainty resolved in period 2, we will use \sim to denote the present case of unresolved uncertainty.

This contrast, between Eqs. (35)-(37) and Eqs. (24)-(26'), shows that the same values of X_1 , X_2 , E and λ will solve both sets of equations if:

$$\frac{\tilde{\rho}(\tilde{U} - U)}{1 - \tilde{\rho}\{1-P(\tilde{E})\}} = \frac{\rho(\bar{U} - U)}{1 - \rho} \quad (38)$$

But Eq. (38), in turn, implies that :

$$1 - \frac{\rho}{\tilde{\rho}} = \frac{(1-\tilde{\rho}) \left[1 - \frac{(\tilde{U}-U)}{(\bar{U}-U)} \right] + \tilde{\rho}P(\tilde{E})}{\{1-\tilde{\rho}(1-P(\tilde{E}))\} \left\{ 1 + \tilde{\rho} \frac{(\tilde{U}-U)}{(\bar{U}-U)} \frac{1}{1-\tilde{\rho}(1-P(\tilde{E}))} \right\}} > 0$$

since $\tilde{U} > \bar{U}$

$$\text{and } 1 - \tilde{\rho}(1-P(\tilde{E})) > 0. \quad (38')$$

we must have

Thus, $\rho < \tilde{\rho}$, for the two infinite-time-horizon cases, with and without unresolved uncertainty, to yield identical results (i.e. values of X_1 , X_2 , E and λ). It is easy to see now that the residual-uncertainty model has a larger discount factor ($\tilde{\rho}$) than the resolved-uncertainty model (ρ): as one would expect, one would discount the future more heavily in the former case in view of the unresolved uncertainty.

Concluding Remarks

The preceding analysis of the phenomenon of market-disruption-induced QR-imposition can be shown both to have other applications and to be generalizable in many directions.

Thus, it is readily seen that the phenomenon of a trade embargo on a

country's imports can be analyzed in the same way as the market-disruption phenomenon. The analysis, and results, would in fact be identical if we were to assume that the probability of the imposition of an export embargo (e.g., by OPEC) by the exporting country was an increasing function of the import level by the importing country (e.g., import of oil by U.S.A.).*

In this case, the optimal policy intervention by the importing country, faced by such an (import-level-related) embargo-prospect of reduced (or eliminated) feasible import level, would be a trade tariff if there were no adjustment costs, and a trade tariff plus production tax-cum-subsidy if there were adjustment costs as well. The analysis would however have to be slightly modified if the embargo problem were modeled rather as one where the probability of the exporting country allowing reduced, permissible exports were made a function instead of the ratio of imports to domestic production (as this may be a better index of import dependence). In this case, since the probability of the quota being invoked is now a function of a ratio involving both trade and production levels, one should expect that the optimum tariff would now be replaced by a combination of a tariff and a production tax-cum-subsidy, on this account (even in the absence of adjustment costs). Finally, if one models the probability of an embargo imposition as independent of a country's trade level or import-to-production ratio, so that the uncertainty is exogenous, then clearly the optimal policy for a small country (with no monopoly power in trade) is free trade with zero adjustment costs and, if there are adjustment costs, it will consist of a production tax-cum-subsidy related to these adjustment costs.

As for the generalizations of our analysis in other directions, we

* The economic rationale for this assumption is that the probability of the exporter invoking an export embargo may be a function of the "import dependence" by the importer.

may indicate some. Thus, for example, an important extension would be to incorporate technical change as a source of export expansion and hence accentuated probability of a triggering of market-disruption-induced QR's: this would provide yet another instance of immiserizing growth, while also carrying implications for optimal imports of technology in developing countries, to mention only two possible analytically-interesting consequences. Again, our analysis has explicitly modelled only the exporting country as far as welfare implications of the market-disruption phenomenon are concerned. However, one could take a "world-welfare" approach and model the importing country also more explicitly. If this was done, then one could no longer meaningfully take the importing country's QR-imposition policy as "given"; and the basic model of this paper would have to be modified in an essential manner.

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