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RETURNS-TO-SCALE IN RESEARCH AND DEVELOPMENT:
WHAT DOES THE SCHUMPETERIAN HYPOTHESIS IMPLY?

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1. The Problem and the Literature

Joseph Schumpeter argued strongly in Capitalism, Socialism and Democracy that "the large-scale establishment or unit of control" was to be preferred to the small, competitive firm. He did not deny the static theory of competition; he argued that inefficiency at any given moment of time was more than offset by the increase in the rate of productivity growth promoted by the large firm. The attractiveness of this idea is obvious, and it has been discussed widely in the quarter-century since Schumpeter's book appeared. It has also been subjected to empirical tests, the most popular of which is to ask if the proportion of workers or expenditures in firms used in research and development activities increases with firm size. This proposition is not the same as Schumpeter's, and a theoretical argument is needed to get from one to the other.

We have attempted to provide such an argument and have discovered the surprising result that the test is in fact not a test either of Schumpeter's ideas or of the soundness of the policy prescriptions derived from them.

In addition, Schumpeter's own clearly implied conclusions that rates of productivity growth will be higher or that more innovations will be forthcoming if small firms are combined into big ones also do not follow from a reasonable formulation of his assumptions concerning returns-to-scale in research and development.

These conclusions have been obscured by the informal nature of the discussion, which has taken its cue from Schumpeter's own discussion. Schumpeter equated size with market power and then made two points. First, the "monopoly firm" will have greater demand for innovations because its market power will increase its ability to profit from the innovation. Second, the "monopoly firm" will generate a larger supply of innovations because

> there are advantages which, though not strictly unattainable on
> the competitive level of enterprise, are as a matter of fact
> secured only on the monopoly level, for instance, because mono-
> polization may increase the sphere of influence of the better,
> and decrease the sphere of influence of the inferior, brains,
> or because the monopoly enjoys a disproportionately higher
> financial standing. [10, p. 101]

The argument about the demand for innovations has been formalized and rejected by Kenneth Arrow who showed that the existence of market power before the innovation reduced the profits to be expected from a given change in costs. [1] The argument about the supply of innovations has never been formalized. Schumpeter's equating of size and market power has been criticized, but the argument has continued to be carried on variously with reference to one or the other.

Most of the discussion of the supply side, however, has concentrated on size. Two arguments have been given. The first argument relating scale to

the supply of innovations asserts that there are economies of scale in research and development (R&D, for short) expenditures. This has two parts. First, a larger R&D staff can operate more efficiently than a small one. Second, an R&D staff of a given size operates more efficiently in a larger firm.

A large R&D staff is presumed to be more efficient because it allows room for more specialized personnel. With a large staff, engineers can concentrate on particular areas of a firm's activities and develop expertise in them. There will be more engineers who have been with the firm a long time and whose accumulated experience is at the service of the firm. And there will be room for pure scientists, supervisors, technical writers or other communications personnel. A large R&D staff also helps a firm hire research firms to work for it, because the in-house R&D personnel can formulate the research project to be done and communicate the firm's needs to the outside research organization.

A given R&D staff will operate more efficiently in a larger firm, according to this argument, because of the risks underlying any research. Since it is impossible to predict the nature of new knowledge, a given R&D expenditure may or may not produce knowledge useful to a particular line of activity. If a firm is engaged in only one activity, it takes the risk of producing knowledge it cannot use. The more activities undertaken by the firm, the smaller this risk is. A large firm tends to have more diversified activities than a small one, and it therefore stands to gain more from a given R&D expenditure.

This argument assumes that there is no market for new knowledge per se.

If there were, a firm that discovered something of no use to itself could sell it to another firm, and there would be no risk to the firm deriving from the unknown applicability of the knowledge. Pure knowledge, however, is not a widely traded commodity; it is linked to personnel or machines or organizations. Consequently the diversified firm stands to gain more from a given volume of R&D than a smaller less diversified firm.

The second argument relating firm scale to innovation asserts that there are economies of scale in the financial market. Large firms have access to more financial markets than small firms. Their credit standing is more widely known. They can borrow more cheaply than small firms, and they can borrow more money before they reach the point where each dollar borrowed costs more than the last one.

As a result, large firms can support larger R&D staffs relative to their size than small firms, i.e., a firm that is twice as large as another can have an R&D staff that is more than twice as large. This will increase the flow of innovations per worker in large firms if the R&D staffs are not less efficient. In addition, larger firms will be more willing to take risks because the greater ability to borrow reduces the risk that any specific loss or failure will result in bankruptcy. A large firm, therefore, can afford to make more mistakes or to lose more gambles than a small firm before it succeeds in a risky endeavor.[1]

Neither of these arguments is directly testable; economies of scale are

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[1] On average, of course, the big firm will have to do as well as the small one to remain profitable. This argument refers not to the average, but to the variation around the average.

very hard to measure, particularly in R&D. Investigators therefore have tried to test these arguments by examining a supposed consequence of them, namely that the inputs into R&D activities, measured by the size of the R&D staff or by R&D expenditures, increase more than proportionately with firm size. D. Hamberg [4], Frederic Scherer [7] and [8], Edwin Mansfield [5], Jabob Schmookler [9], Henry Villard [11], and James Worley [12] have all investigated this question -- with varying results. Jesse Markham has summarized and criticized these studies [6], while Grabowski and Mueller recently have asserted that the data are not good enough to run a serious test [2].

These articles are critical of each other and of the results reached, but none of them suggests that the test itself is inappropriate.[1] Yet it is clear that it is only an indirect test of the existence of economies of scale and that its relevance to the existence of such economies has to be demonstrated. It should also be clear that this test also is not a direct test of the policy prescription usually derived from Schumpeter's arguments. That policy is one of favoring the growth of large firms, presumably because a large firm will innovate more than a collection of small firms of the same aggregate size. This rationale is identical neither with the assertion of economies of scale nor with an assertion about the allocation of resources in large firms.

There are, then, three separate sets of propositions at issue: Schum-

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[1] But Griliches [3, p. 353] casts a skeptical eye on the theoretical dis-
cussion as well as the empirical evidence and suggests that the whole matter
is of second-order importance as a determinant of the rate of growth of
inventive activity.

peter's, those about the effect of firm size on the allocation of the firm's resources, and those about the effect of firm size on the volume of innovations. These different propositions can be formalized as follows. (For ease of exposition, we use the allocation of the labor force as an index of the firm's resources, but the results do not depend on this interpretation.) Let R equal the number of R&D workers in a firm and S equal the total number of workers in the firm, which we will take as our index of firm size. Let $N = S - R$. Let $F(R, N)$ be the dollar value of the <u>per-worker</u> output of the R&D staff, or the average labor-productivity of research and development. It is made a function of the size of the R&D staff and of the operating staff of the firm.

We implicitly assume throughout later sections that inputs other than labor are combined efficiently with the labor inputs and that there is complete certainty about the output of R&D. These assumptions enable us to come to grips with how little can be proved in the area of interest even with the most stringent of models. (To link this discussion directly with the empirical discussion of expenditures as well as with the discussion of the labor force, S, R, and N can be interpreted directly as expenditures, -- making the wage rates introduced below into costs of capital.)

The most obvious way to formalize Schumpeter's arguments, and the way which seems most in agreement with the literature[1], is in terms of inequalities on the partial derivatives of $F(R, N)$. Thus, if a larger R&D staff is

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[1] Schumpeter did not say and the subsequent literature also has not specified whether he was talking about the effects of size on average or marginal products. It is more natural when thinking of increasing returns to interpret Schumpeter and the literature in terms of average products than in terms of marginal products as we have done here. In addition, there are logical difficulties with the marginal interpretation (which we note below) that preclude its easy acceptance. But we consider that version later, in any case.

supposed to be more efficient than a smaller one (for given operating firm size), we would expect average R&D productivity to rise with R:

(1a) $F_1 > 0$

(where subscripts denote differentiation in the obvious way). Similarly, if a given R&D staff is more efficient in a larger firm, average R&D product should increase with N:

(1b) $F_2 > 0$.

We shall refer to inequalities (1a) and (1b) as Schumpeter's hypotheses. The test of these hypotheses used in the literature is to attempt to see whether R rises more rapidly than S when S changes over firms, i.e., to see whether:

(2) $\eta \equiv \dfrac{S}{R} \dfrac{dR}{dS} > 1$.

On the other hand, the rationale for combining small firms into large ones is the Schumpeterian conclusion that the total R&D output of the firm, RF, rises more than proportionately with S, or that:

(3) $\varepsilon \equiv \dfrac{S}{RF} \dfrac{d(RF)}{dS} = \eta + \dfrac{S}{F} \dfrac{dF}{dS} > 1$.

Schumpeter's own argument would appear to be that inequalities (1a) and (1b) imply inequality (3). The empirical literature seems to have asserted that inequalities (1a) and (1b) imply inequality (2) which then in turn implies inequality (3).

We shall show that most of this is incorrect. Indeed, we shall show that, without additional special assumptions, about the only interesting true implication that can be drawn here is that inequalities (1a), (1b), and (2) plus a marginal version of Schumpeter's hypothesis together imply inequality (3). Since it turns out, however, that inequality (2) is neither necessary nor sufficient for inequalities (1a) and (1b) and also neither necessary nor sufficient for inequality (3), empirical analysis of inequality (2) is of very little interest. Moreover, the Schumpeterian conclusion (3) follows neither from the empirically testable (2) nor from the Schumpeterian hypotheses (1a) and (1b) taken separately.

2. Formal Analysis

Every firm hires R&D workers at a wage, w, and operating workers at a wage, v, which may or may not be the same.[1] Its revenues consist of the dollar value of its R&D output, RF(R, N), and the revenues from its operating division which we shall denote as G(N, α), where α is a shift parameter whose use will be explained in a moment. For convenience, we shall denote the net revenues of the operating division by $H(N, α) \equiv G(N, α) - vN$. The firm chooses R and N to maximize profits, Π, given by:

$$(4) \qquad Π = RF(R, N) - wR + H(N, α) \quad .$$

Before proceeding, some comments are in order. First, as already remarked, we assume, for simplicity, that the dollar return to R&D is known with certainty (this may be taken as the present value of a future stream of returns). Moreover, we have not allowed the output of R&D explicitly to influence the returns to the operating division, although such influence may be taken as implicitly included in F(R, N). We have done this because our purpose is to show that the standard conclusions fail to follow in simple models embodying the standard assumptions; the burden of proof that they follow in more complex models with additional features is then on those who believe that they follow at all.

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[1] As remarked above, we have chosen to simplify matters by working with a single explicit factor input. If other factors are implicitly included, their costs per worker are included in w and v, which may then not be constants. To allow for such non-constancy would merely complicate the analysis without changing its conclusions.

Second, it is important to bear in mind that $H(N, \alpha)$ is not merely a production relationship but also includes the effects of demand conditions and wages. In particular, increasing or decreasing returns to scale in production affect $H(N, \alpha)$ but are not equivalent to statements about H_1.

Finally, we come to the role of the shift parameter, α. If there were no shift parameter in the model, all firms would be identical and there would be nothing to compare between large and small firms. Hence we must allow firms to differ in some respects. There are really only two general ways to allow for such differences. One of these is to allow the functions $H(N, \alpha)$ to be different for different firms; this is what we have done. The shift parameter, α, may be thought of as indexing a difference in production functions or in demand conditions (or, for that matter, in the wage of production workers, v). Except for notational convenience, there is no reason why α cannot be a vector.

The other way to allow for differences between firms would be to have a shift parameter appearing in $F(R, N)$ (or affecting w). This would be pointless for the present analysis, however, since it is clear that all results of interest would turn on exactly how such a shift parameter entered. If this area of analysis means anything at all, it must be on the presumption that small and large firms differ only as to size in their R&D opportunities and that were they the same size (in both R and N), those opportunities would be the same.

We have thus placed our shift parameter only in $H(N, \alpha)$. Our technique of analysis is as follows. Since the firm chooses R and N to maximize (4),

R and N, and, therefore, S, are implicitly determined as functions of α. By obtaining $dR/d\alpha$, $dN/d\alpha$, and $dS/d\alpha$, we will be able to examine dR/dS, dN/dS, and the remaining derivatives of interest, where the derivatives are all taken parametrically on α, the only exogenous variable which shifts. It will turn out that the precise way in which α enters matters not at all.

We begin then by maximizing Π with respect to R and N. The first-order conditions are:

(5a) $F + RF_1 - w = 0$

and

(5b) $RF_2 + H_1 = 0$.

The second-order conditions are:

(6a) $2F_1 + RF_{11} < 0$

and

(6b) $(2F_1 + RF_{11})(RF_{22} + H_{11}) > (F_2 + RF_{12})^2$.

Differentiating (5a) totally with respect to α, yields:

(7) $(2F_1 + RF_{11}) \dfrac{dR}{d\alpha} + (F_2 + RF_{12}) \dfrac{dN}{d\alpha} = 0$,

or

(8) $\dfrac{dR}{d\alpha} \Big/ \dfrac{dN}{d\alpha} = - \dfrac{F_2 + RF_{12}}{2F_1 + RF_{11}}$.

We now wish to evaluate dR/dS, where the derivative is taken along a curve, movements along which are determined by changes in α. Recalling that $S = R + N$, and using (8), we obtain:

$$(9) \qquad \frac{dR}{dS} = \frac{dR/d\alpha}{dS/d\alpha} = \frac{dR/d\alpha}{dR/d\alpha + dN/d\alpha} = \frac{\frac{dR}{d\alpha}/\frac{dN}{d\alpha}}{\frac{dR}{d\alpha}/\frac{dN}{d\alpha} + 1} = \frac{F_2 + RF_{12}}{F_2 + RF_{12} - (2F_1 + RF_{11})}$$

Note that neither α nor H enters this expression directly.

Recalling that $\eta = (dR/dS)(S/R)$, the first thing we can immediately say about the relations between Schumpeter's hypotheses (1a) and (1b) and their supposed implication (2) is:

Theorem 1:   The inequalities, $F_1 > 0$ and $F_2 > 0$, do not even suffice to determine the sign of η, let alone implying that $\eta > 1$.

Proof: It is obvious that the sign of $(F_2 + RF_{12})$ is entirely unrestricted by the first-and second-order conditions and the two Schumpeterian inequalities.

In other words, even though the average product of research and development may rise both with the size of the R&D staff and with the size of the operating division, larger firms may find it profitable to have not merely relatively smaller but absolutely smaller R&D staffs than smaller ones. The reason for this is not hard to find.[1] $\eta < 0$ only if $(F_2 + RF_{12}) < 0$, in view of (6a). The latter expression, however, gives the effect of an increase in the size of the operating division on the marginal product of the R&D staff. If this is negative, then a firm with a larger operating division will find that it has to cut back on its R&D staff in order to make the latter's marginal product equal to the wage.

It is perhaps natural to assume that this does not happen, and we shall

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[1] We are indebted to Robert Hall for discussion of this point.

do so in a moment. It is considerably less natural to argue that such an assumption is really what is meant by the Schumpeterian argument that increasing the size of the operating division increases the returns to R&D. That argument seems to us to be embodied in the statement about average product, $F_2 > 0$, and it seems stretching to claim that it means that the marginal product of the R&D staff rises with the size of the operating division. Indeed, our view is reinforced by the fact that it is hard to see how one could argue that this half of the Schumpeterian argument refers to increasing marginal products without also arguing that the other half does also. The latter view, however, would lead to the observation that the Schumpeterian hypothesis is in direct contradiction to the second-order condition (6a). For firms in equilibrium, it cannot be the case that the marginal product of R&D staff is increasing with the size of the staff itself (although it can be the case at uneconomically low staff sizes), since otherwise there would be nothing to limit the size of the staff. Thus, it seems difficult to argue that Schumpeter's hypothesis as to the effect of the size of the operating division should be interpreted as a statement about marginal product without also being led into a view that another part of his hypothesis cannot be true for observed firm sizes.

We shall ignore Theorem 1 in what follows, however, and assume that it is indeed true that dR/dS is positive. In other words, we henceforth assume:

(10) $F_2 + RF_{12} > 0$.

With this assumption, it is possible to obtain some (not very strong) positive

results. We begin by showing:

__Theorem 2__: If $F_2 + RF_{12} > 0$, then $0 < dR/dS < 1$ and $0 < dN/dS < 1$.

__Proof__: The first statement follows directly from (9), (10), and the second-order condition (6a). The second statement follows from the first and the fact that $S = R + N$.

In other words, larger firms will have both absolutely larger R&D staffs and absolutely larger operating staffs than smaller ones. Note that only the marginal-product hypothesis (10) is involved in this; the average-product hypotheses (1a) and (1b) play no role.

Those hypotheses do play a role, however, in:

__Theorem 3__: If $F_1 > 0$, $F_2 > 0$, and $F_2 + RF_{12} > 0$, then $\epsilon > \eta$.

__Proof__:

$$(11) \qquad \epsilon = \eta + \frac{S}{F} \frac{dF}{dS} = \eta + \frac{S}{F} \left(F_1 \frac{dR}{dS} + F_2 \frac{dN}{dS} \right) \quad .$$

The theorem now follows from Theorem 2.

__Corollary__: $F_1 > 0$, $F_2 > 0$, $F_2 + RF_{12} > 0$, __and__ $\eta > 1$ imply $\epsilon > 1$.

Thus, our expanded version of Schumpeter's hypotheses __together with__ their supposed consequence as to the relative size of R&D staffs in large and small firms suffice to imply the Schumpeterian conclusion about the relative output of R&D from large and small firms. Unfortunately, as we shall see below, $\eta > 1$ is required for that conclusion and does not itself follow from Schumpeter's hypotheses even with the addition of the marginal

product hypothesis (10).[1]

Indeed, without further assumptions, no further positive results seem available. One such further assumption would be to restrict F to be homogeneous of some degree. If one is willing to do so, very strong results can be obtained, as follows:

Theorem 4: If F is homogeneous of degree m, then $\eta \gtreqless 1$ according as $m \gtreqless 0$.

Proof: By (10), the denominator of the right-hand expression in (9) is positive, so $\eta \gtreqless 1$ according as

$$(12) \qquad (S - R)(F_2 + RF_{12}) + R(2F_1 + RF_{11}) \gtreqless 0 \quad .$$

However,

$$(13) \qquad (S - R)(F_2 + RF_{12}) + R(2F_1 + RF_{11}) = R(RF_{11} + NF_{12}) + 2RF_1 + NF_2$$

$$= (m - 1)RF_1 + 2RF_1 + NF_2$$

$$= m(RF_1 + F) = mw \quad ,$$

where we have used, successively, Euler's theorem applied to F_1, Euler's theorem applied to F, and the first-order condition (5a). The theorem now follows immediately, since the wage is positive.

Corollary 1: If F is homogeneous of some degree, then $F_1 > 0$, $F_2 > 0$, and $F_2 + RF_{12} > 0$ imply both $\eta > 1$ and $\epsilon > 1$.

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[1] Remark: It might be thought that (10) need not be separately assumed in order to obtain the corollary, since $\eta > 1$ already implies that $dR/dS > 0$. This is not true, since without (10) it would be possible to have $dN/dS < 0$ if η were sufficiently great. (10) is a sufficient condition for $dR/dS > 0$, not a necessary one.

Proof: By Euler's theorem, $F_1 > 0$ and $F_2 > 0$ imply that F is homogeneous of positive degree. The corollary now follows from Theorem 4 and the corollary to Theorem 3. Note that (10) must still be assumed for this result.[1]

Corollary 2: If F is homogeneous of some degree, then $F_1 > 0$, $\eta > 1$, and $F_2 + RF_{12} > 0$ imply $\epsilon > 1$.

Proof: If $F_2 > 0$, then the result follows from Corollary 1, so we may as well assume $F_2 \leq 0$. Using (11) and the fact that $S \equiv R + N$,

$$(14) \qquad \epsilon = \eta + \frac{S}{F}\left(F_1 \frac{dR}{dS} + F_2\left(1 - \frac{dR}{dS}\right)\right) = \eta + \eta(F_1 - F_2)\frac{R}{F} + \frac{S}{F}F_2$$

$$= \eta + \eta\left(\frac{RF_1 + NF_2}{F}\right) + (1 - \eta)\frac{S}{F}F_2$$

$$= \eta(1 + m) + (1 - \eta)\frac{S}{F}F_2 \quad ,$$

where m is the degree of homogeneity of F and the last equality follows from Euler's theorem. Since $\eta > 1$, the term multiplying F_2 is negative and, since $F_2 \leq 0$, $\epsilon \geq \eta(1 + m)$. By Theorem 4, however, $\eta > 1$ implies $m > 0$, proving the Corollary.

Thus, if we could assume F homogeneous of any degree, Corollary 1 assures us that all the results which are supposed to flow from Schumpeter's hypotheses (plus (10)) would indeed do so. Moreover, Corollary 2 states that, with that assumption, observing that $\eta > 1$ would make the Schumpeterian hypotheses that $F_2 > 0$ irrelevant in concluding that $\epsilon > 1$, provided we are

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[1] Remark: If F is homogeneous of degree greater than or equal to unity (which does not follow from $F_1 > 0$ and $F_2 > 0$), then (10) need not be separately assumed. In that case, $F_1 > 0$ implies $F_{11} < 0$, from the second-order condition (6a). Euler's theorem applied to F_1 then shows $F_{12} > 0$, which, together with $F_2 > 0$, yields (10).

still willing to assume $F_1 > 0$ <u>and</u> (10). Even here, however, $\eta > 1$ does not

imply $F_1 > 0$ and $F_2 > 0$ (although it certainly does imply that at least one

of these are true by Theorem 4). Moreover, it does not imply (10) or that

$\epsilon > 1$.[1]

Even so, it would clearly be very helpful -- particularly in view of

Corollary 1 -- to be able to assume homogeneity. Unfortunately, however

familiar to economists, there does not seem to be any reason to suppose that

F has such a property.[2] Without it, the desired results simply do not

follow, as we now show.

<u>Theorem 5</u>: $F_1 > 0$, $F_2 > 0$, and $F_2 + RF_{12} > 0$ imply neither $\eta > 1$ nor $\epsilon > 1$.

<u>Proof</u>: It is obvious that even (10) is not enough to bound η away from zero,

since $F_2 + RF_{12}$ can be made as small as desired. This clearly makes it very

unlikely that $F_1 > 0$, $F_2 > 0$, and (10) imply $\epsilon > 1$, but there seems no way to

prove this save by providing a counter-example (which, by the Corollary to

Theorem 3, will also exhibit $\eta < 1$). We do so by choosing a form for F which

has all the appropriate properties in a region surrounding the equilibrium

point; far away from equilibrium, the function would have to be altered, but

this presents no difficulty.

Accordingly, in the relevant region, we choose:

(15) $F(R, N) = 40(2^{20}) R^{.1} N^{.1} - R^3 N^2$.

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[1] Even assuming (10), $\epsilon > 1$ cannot be deduced from homogeneity and $\eta > 1$ without assuming $F_1 > 0$. Examing (14), (9), and the calculation in the proof of Theorem 4, it can be shown that there are cases in which η is close to unity and m close to zero and the sign of $\epsilon - 1$ is determined by appropriate choice of the term in $(-SF_2/F)$. Consideration of the examples in the proof of Theorems 5 and 6 below reveals that the latter term is essentially unrestricted in magnitude.

[2] F is not a production function in two factors in any reasonable sense. The two variables represent the effects of one direct input and a kind of externality, and the usual arguments for homogeneity do not apply.

For appropriate choice of $H(N, \alpha)$ and w, we shall show that $R = 1$, $N = 2^{10} = 1024$ is an equilibrium. Since $H_1(2^{10}, \alpha)$ and $H_{11}(2^{10}, \alpha)$ are essentially at our disposal, this amounts to verifying a number of inequalities.

(16) $F(1, 2^{10}) = 79(2^{20}) > 0$.

(17) $F_1(1, 2^{10}) = 5(2^{20}) > 0$.

(18) $F_2(1, 2^{10}) = 6(2^{10}) > 0$.

Hence both F and its two first partials are positive. It follows that (5a) will be satisfied at the point in question for an appropriately chosen positive wage ($w = 84(2^{20})$).[1] Further, (5b) will be satisfied for $H_1(2^{10}, \alpha) = -6(2^{10})$. Thus $R = 1$ and $N = 2^{10}$ can satisfy the first-order conditions for positive w and appropriate H.

Turning to the second-order conditions:

(19) $2F_1(1, 2^{10}) + F_{11}(1, 2^{10}) = -3.2(2^{20}) < 0$,

satisfying (6a). Since H_{11} can be taken as negative as desired, (6b) can also be satisfied.

Checking (10), we have:

(20) $F_2(1, 2^{10}) + F_{12}(1, 2^{10}) = .8(2^{10}) > 0$.

Hence, the function and equilibrium point chosen satisfies all the assumptions. We now compute η and ϵ at the point in question. Using (9),

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[1] The units are irrelevant, of course. We could count our workers, for example, in groups of 100, so the firm analyzed would have a total work force of 100,000 and an R&D staff of 100.

(19), and (20):

(21) $\quad \eta = \dfrac{S}{R} \dfrac{dR}{dS} = (2^{10} + 1)(\dfrac{.8(2^{10})}{.8(2^{10}) + 3.2(2^{20})}) = .25 \quad ,$

approximately. Using this, (14), and our other numerical results:

(22) $\quad \varepsilon = \eta + \eta(F_1 - F_2)\dfrac{R}{F} + \dfrac{S}{F} F_2$

$\qquad = .25 + \dfrac{.25(5(2^{20}) - 6(2^{10})) + 1025(6)(2^{10})}{79(2^{20})} = .34 \quad ,$

approximately. This completes the proof.

There remain two questions of interest. First, were empirical work to show that $\eta > 1$, would one then be entitled to conclude that at least one of F_1 and F_2 was positive, or entitled to conclude that $\varepsilon > 1$? Second, even though $F_1 > 0$ and $F_2 > 0$ do not imply either $\eta > 1$ or $\varepsilon > 1$, do $F_1 < 0$ and $F_2 < 0$ imply either $\eta < 1$ or $\varepsilon < 1$? In the latter case the Schumpeterian hypothesis would at least be necessary for the Schumpeterian conclusion, even though we know that it is not sufficient.[1] We dispose of these possibilities with another counter-example.

Theorem 6: (a) $\eta > 1$ does not imply $F_1 > 0$ or $F_2 > 0$;

(b) $\eta > 1$ does not imply $\varepsilon > 0$, let alone $\varepsilon > 1$;

(c) $F_1 < 0$ and $F_2 < 0$ do not imply $\eta < 1$ or $\varepsilon < 1$.

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[1] Obviously, the proof of Theorem 3 shows that $F_1 < 0$, $F_2 < 0$ and $\eta < 1$ imply $\varepsilon < 1$. This is not of much interest, since we know that $\eta < 1$ does not imply $F_1 < 0$ and $F_2 < 0$. The only other possibility that might be of some interest is that $\eta < 1$ implies $\varepsilon < 1$, which is clearly false, as consideration of Theorem 3 reveals.

Proof: Choose

(23) $F(R, N) = (2 - \lambda)(10^{-2})RN^4 - R^2N^2 - 60N + k$,

where $\lambda > 0$ and $k > 0$ are parameters which we will specify later. We will show that $R = 1$, $N = 10$ can be an equilibrium point.

   In the first place, by choosing k sufficiently large, $F(1, 10) > 0$. Evaluating F_1 and F_2:

(24) $F_1(1, 10) = - 100\lambda < 0$; $F_2(1, 10) = - 40\lambda < 0$.

For large enough k, $F(1, 10) + F_1(1, 10)$ will be positive, so that (5a) can be satisfied at a positive wage. Further, since H is at our disposal, (5b) can also be satisfied.

   Turning to the second-order conditions:

(25) $2F_1(1, 10) + F_{11}(1, 10) = - 200(1 + \lambda) < 0$,

so that (6a) is satisfied, while (6b) will be satisfied for appropriate choice of H_{11}.

   Checking (10):

(26) $F_2(1, 10) + F_{12}(1, 10) = 40 - 80\lambda$

which will be positive for $0 < \lambda < .5$.

   We now evaluate η at the equilibrium point.

(27) $\eta = \dfrac{S}{R}\dfrac{dR}{dS} = 11\left(\dfrac{40 - 80\lambda}{40 - 80\lambda + 200(1 + \lambda)}\right) = 11\left(\dfrac{1 - 2\lambda}{6 + 3\lambda}\right)$.

This will be greater than unity for λ sufficiently small, proving (a) and the first part of (c).

Turning to the evaluation of ε at the equilibrium point, we have:

$$(28) \qquad \varepsilon = \eta + \eta(F_1 - F_2)\,\frac{R}{F} + \frac{S}{F}\,F_2$$

$$= 11\left(\frac{1 - 2\lambda}{6 + 3\lambda}\right) - 11\left(\frac{1 - 2\lambda}{6 + 3\lambda}\right)(60\lambda)(1/F) - 11(40\lambda)(1/F) \quad .$$

By choosing k sufficiently large and λ sufficiently small, the last two terms can be made as small as desired, and ε made to approach $\eta > 1$. This proves the second part of (c). On the other hand, by making k small enough, F can be made close to zero and, if λ is also taken small enough, this can be done so as to keep (5a) satisfied at positive wage. Indeed, it is clear that the operative restriction is the latter one, so that, using (24), k must be chosen so that $F(1, 10) > 100\lambda$. By choosing k close to this bound, however, we leave everything else unaffected and ε can be made to approach:

$$(29) \qquad \varepsilon^* = 4.4\left(\frac{1 - 2\lambda}{6 + 3\lambda} - 1\right) \quad ,$$

and for λ sufficiently small, this approaches -3.67, approximately, Hence ε can also be made to approach -3.67 and (b) is proved.

This completes our analysis. We have shown the rather disappointing result that the Schumpeterian hypotheses and conclusions have relatively little to do with each other without further assumptions. Moreover, the empirical test which has been attempted in the literature has relatively little to do with either. One cannot do comparative statics without second derivatives.

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