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PRICE LEADERSHIP

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I. INTRODUCTION

In many industries a price change is announced by one of the firms in advance of the date at which the new price will take effect and the new price is swiftly matched by the other firms in the industry. This form of price leadership is the focus of this paper. It has two striking features: price changes are usually matched to the penny even though the products produced by the firms are often somewhat differentiated, and a long time elapses between price changes (often a year or more).

Examples of this pattern of pricing behavior abound. Perhaps the best-known example is that of the cigarette industry in the late 1920's and early 1930's. For example on October 4, 1929 Reynolds announced an increase in its price from $6.00 to $6.40 per thousand (effective the October 5) and was followed the next day by both of its major competitors, Liggett and Meyers and American Tobacco. That price was in effect for almost two years before Reynolds led a further increase (to $6.85).[1] Similar pricing behavior has been documented in the steel, dynamite, anthracite and airline industries.[2]

The Industrial Organization literature distiguishes between three forms of price leadership. The first, collusive price leadership, was described by Markham in 1951 as "price leadership in lieu of overt collusion". However, the method by which price leadership could achieve the coordination necessary to sustain monopolistic outcomes has not been spelled out.

The second, "barometric" price leadership, refers to a situation in which the price leader merely announces the price that would prevail anyway

[1]See Nicholls (1951) for a detailed discussion of pricing in the cigarette industry during this period.

[2]See Stigler (1947) for a discussion and further references.

under competition. In contrast to the collusive price leader, the barometric price leader has no power to (substantially) affect the price that is charged generally in the industry. Indeed the actual price being charged may soon diverge from that announced by the barometric firm, which in turn is unable to exert any disciplining influence to prevent this from occuring.

The third form of price leadership, and the one which has been the focus of most formal modelling, is the one that results from the existence of a dominant firm. Models of this type (see Gaskins (1971), Judd and Petersen (1985)) assume that the dominant firm sets the price of a homogeneous product. This price is then taken as given by a competitive fringe of firms. Unfortunately, this model cannot explain the behavior of oligopolies in which there are several large firms. Such large firms cannot be assumed to take as given the price set by any one firm. Rather, they should be expected to act strategically.[3]

The goal of this paper is to develop a model of collusive price leadership. We study markets in which duopolists attempt to achieve monopolistic outcomes by exploiting the fact that they have repeated encounters over time. In particular, they attempt to collude by threatening to revert to noncooperative behavior if any firm deviates from implicitly agreed upon behavior.[4] The key feature of the markets that we model is that

[3]The identical criticism can be applied to the model of d'Aspremont et al (1983). There, a group of equal-sized firms collude to set the price; the remaining firms, which are assumed to be of the same size, treat this price parametrically. Since this explicitly assumes that the fringe firms are large, the above criticism is especially relevant.

[4]This model is thus in the spirit of the supergame model of Friedman (1971).

the firms may be asymmetrically informed about industry demand conditions. In an ideal world (from the duopolists' point of view) the firms would meet to aggregate their information and then fix prices which would hold until their next meeting. In practice, however, such behavior would run foul of the antitrust laws. Moreover if the firms are not unanimous about the prices that should be charged, they may have an incentive to misreport their private information to each other.

What then ought an asymmetrically informed duopoly do to ensure outcomes better than the noncooperative ones? It could allow the better informed firm to announce its price first and let the second firm pick a price as a function of the first firm's announcement. To prevent the second firm from simply undercutting the first firm, the precise function must be understood by both firms, and the second firm must be punished (by a period of noncooperation) if it deviates from this function. Typically, where the products are differentiated, an optimal function of this type would lead to different prices across firms. In practice, the function that gives the second firm's price as a function of the first would probably have to be quite complicated for the duopoly to do as well as it can. Recall that the second firm must make an inference about the state of demand and costs from the first firm's price. But a high price by the first firm can mean that demand for both firms is high, that relative demand has shifted in favor of the first firm, or that the first firm has had an idiosyncratic increase in costs. Thus the optimal pricing function will have to take into account the probabilities of these various events. For punishments not to be triggered by accident, both firms must understand the exact form of this function. Furthermore, changes in the environment such as the development of new products, the advent of new competition and technology, and modifications to the nature of government interventions, require changes in this function over time. Again, the way in which the function changes must be understood by both firms without explicit communication. This is further complicated by the fact that, typically, firms do not have similar perceptions about environmental changes; nor do they fully understand each other's perceptions.

As Grossman and Hart (1984) have pointed out, even in a world in which enforceable explicit contracts are feasible, one cannot expect firms to take into account, as would be necessary for full optimality, the myriad contingencies that can arise. One reason for this is the bounded rationality of the participants and another is that more complicated contracts are more costly to develop than simple ones.

In such circumstances, and especially when one of the firms has superior information to the other, price leadership emerges as a natural collusive scheme. The scheme is simplicity itself: pricing decisions are delegated to the better informed firm (the "leader") who announces pricing decisions ahead of time. For its part, the follower is expected to match these prices exactly. This scheme has a number of positive attributes from the point of view of the duopoly: it is extremely easy to implement; defining adherence to the scheme is trivial in the sense that there is no ambiguity as to the desired response of the follower; no overt collusion (either through information transfer or price-fixing) is required and, while the scheme is generally not optimal, both firms enjoy responsiveness to demand conditions since prices embody the leader's superior information.

It seems natural that the better informed firm would be the price leader since aggregate profits are higher in this case. However, if the firms produce differentiated products and a common price is charged, they will typically have differing preferences about what that price should be. Since the firm that is the designated price leader typically earns higher profits under these circumstances, it might be expected that both firms would vie for the leadership position. We show, however, that when one of the firms is much better informed than the other, the latter may prefer to follow than to lead. Thus the industry leader may emerge endogenously.

There may be circumstances, however, when the disparity in profits between the leader and the follower is great. We ahow that this disparity in profits can be reduced (and a more "equitable" distribution of profits achieved) if the price leader keeps its price constant for some time. In this case, the leader faces a tradeoff. On the one hand, it would like to exploit the follower by responding strongly to current relative demand. On the other hand, if relative demand conditions are expected to revert to normalcy in a little while, such a strong response reduces future profits. Therefore, rigid prices reduce the response of the leader's price to current relative demand. Generally, however, the follower will not be in favor of completely rigid prices. Counterbalancing the profit-sharing benefit of inflexible prices is the advantage of letting the leader respond to common (and not merely relative) demand fluctuations. Somewhat (but not completely) rigid prices should therefore be expected to be a feature of a price leadership regime.

While the duopolists are able to achieve a somewhat collusive outcome using price leadership, they clearly do not do as well as they would if they were completely unconstrained i.e. if they could sign a binding contract and could enforce honest revelation of private information. Consumers, on the other hand, are generally better off under a price leadership regime than they are when faced with overt collusion. When demand is relatively strong for the leader's product, the leader sets a high price, and thus consumer surplus from each product is low compared with that under full collusion. The opposite is true when demand for the leader's product is relatively weak. However, the increase in consumer surplus when demand for the leader's product is relatively weak exceeds the reduction in consumer surplus when it is relatively strong. This is because consumer surplus is a convex function of price.

Evaluating welfare as the simple sum of consumer and producer surplus yields the result that overall welfare is even lower under price leadership than under overt collusion! This suggests that price leadership, where it occurs, warrants the scrutiny of the antitrust authorities. The U.S. Courts, however, take a very accomodating attitude towards price leadership. As early as 1927 in U.S. v. International Harvester, the U.S. Supreme Court stated:

> "[International Harvester] has not .. attempted to dominate or in fact controlled or dominated the harvesting machinery industry by the compulsory regulation of prices. The most that can be said as to this, is that many of its competitors have been accustomed, independently and as a matter of business expediency, to follow approximately the prices at which it has sold its harvesting machines....And the fact that competitors may see proper, in the exercise of their own judgement, to follow the prices of another manufacturer, does not establish any suppression of competition or show any sinister domination."[5]

The reluctance of the courts to find price leaders guilty of price fixing stems in part from the possibility that a perfectly innocent firm

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[5] U.S. v. International Harvester Co., 274 U.S. 693, 708-709 (1927). See also Scherer (1980) p. 520 ff.

could find itself in violation of the antitrust laws simply because its rivals follow it and in part because of the belief that it is generally difficult to distinguish collusive price leadership from its benign barometric counterpart. We argue, however, that competitive pricing is highly unlikely to lead to the pattern of pricing associated with price leadership described above if the firms produce differentiated products. In that case when there are sequential price announcements by the firms there are two forces that tend to drive prices apart. First, the differentiation itself tends to lead to different noncooperative prices, and, second, the last firm to move usually has an incentive to undercut the price of its predecessor. We examine these forces in a variety of noncooperative settings including simultaneous and sequential move games. In the case of the sequential move game we also study the noncooperative signalling model in which the follower attempts to extract the leader's information from its price. In brief, we find that in all of the settings we study at least one of the above forces is present and it is only by the sheerest coincidence that the firms' prices are identical in equilibrium. We investigate the implications of this finding for policy towards price leadership in our concluding remarks.

The paper treats the subjects in the following order: the noncooperative simultaneous move game with asymmetric information (Section II), the price leadership model with the leader perfectly informed and the follower completely uninformed (Section III), price rigidity (Section IV), the noncooperative sequential move games (Section V), the welfare analysis (Section VI), and the price leadership model (incorporating a model of endogenous choice of a price leader) with both firms imperfectly informed

(Section VII). Section VIII provides concluding remarks and some suggested policy guidelines.

II THE SETTING, AND A NONCOOPERATIVE SOLUTION

We consider a duopoly in which firm 1 has better information about demand conditions than firm 2. Both goods are produced with constant marginal cost c. The demands for goods are assumed to be given by:

$$Q_1 = a + e - bP_1 + d(P_2 - P_1)$$

$$Q_2 = a - e - bP_2 + d(P_1 - P_2) \qquad (1)$$

where Q_i and P_i are the quantities demanded and price of good i respectively. The parameters a and e vary over time while b and d are positive constants. The constant b gives the response of the quantity demanded of each good to a fall in the price of both goods while d gives the response to a relative increase in the other good's price. Note that d is zero if the demand for both goods is independent while it is infinite if the goods are perfect substitutes. The only lack of symmetry is given by e which raises the demand for good 1 relative to that of good 2. This asymmetry has essentially identical consequences to differences in marginal costs.

For simplicity, we initially assume that firm 1 knows both a and e while firm 2 is totally uninformed. In particular firm 2 knows only the mean of a ($\bar{a}$) and the mean of e (zero). (In section V we relax this restriction by letting both firms receive imperfect signals of a and e.) Moreover, we assume that there is no credible way for firm 1 to communicate its information. Note that firm 1 has an incentive to lie about its information.

If it could convince firm 2 that its demand is large, it could induce it to charge a high price thus increasing demand for its own product.

In principle there are a variety of ways of modelling the outcome of noncooperation between these two firms. We assume firms compete by announcing prices. These announcements commit the firm to sell whatever is demanded in the current period at that price. One can imagine both simultaneous and sequential announcements. Neither firm however, wants to be the first to announce a price since its competitor can then undercut it. Thus we focus first on simultaneous announcements. In section IV we study sequential announcements which might be thought to have some of the properties of price leadership.[6]

We now study the equilibrium that emerges if both firms must annouce their prices simultaneously. Since firm 2 has no state dependent information it always announces the same price P_2. Thus firm 1 maximizes:

$$\Pi_1 = (P_1 - c)(a + e - bP_1 + d(P_2 - P_1))$$

which leads to a price P_1 equal to $(a+e+dP_2+bc)/2(b+d)$. Firm 2, on the other hand, maximizes:

$$\Pi_2 = E(P_2 - c)(a - e - bP_2 + d(P_1 - P_2))$$

where E is the expectations operator and Π_2 is the expected profit of firm 2. Thus firm 2 charges:

$$P_2 = (\bar{a} + bc + dEP_1)/2(b+d) = (\bar{a} + (b+d)c)/(2b+d). \qquad (2)$$

This implies that firm 1 charges:

[6]Both of these equilibrium concepts are somewhat unsatisfactory because, in both cases, at least one firm would like to change its price after it has heard the other firm's price announcement. This is always true when prices are announced sequentially. Here it is also true when they announce their prices simultaneously because firm 2 learns some combination of a and e from firm 1's announcement.

$$P_1 = (\bar{a} + (b+d)c)/(2b + d) + (a - \bar{a} + e)/2(b+d). \tag{3}$$

Thus, unless $d=\infty$ so that the products are perfect substitutes, it is only by coincidence that the prices charged by the two firms are the same. For d finite this occurs only when $(\bar{a}-a)$ equals e.

For future reference we calculate Π_2 in this equilibrium:

$$\Pi_2 = [(\bar{a} + (b+d)c)/(2b + d) - c][\bar{a} - b\ (\bar{a} + (b+d)c)/(2b+d)]$$
$$= b[(\bar{a} - bc)/(2b + d)]^2.$$

Note that in the first equality the first term in brackets represents the difference between price and marginal cost while the second represents the average output of firm 2. For both of these magnitudes to be positive $\bar{a}$ must exceed bc, which, in turn, must be positive if coordinated price increases are to reduce industry sales.

III. PRICE LEADERSHIP IN SUPERGAMES

In this section we show that price leadership can be a natural outcome in oligopolies that try to sustain collusion by threatening to revert to competition. These threats are credible because, even in the repeated game, the noncooperative outcome studied in the previous section (or the sequential move noncooperative equilibrium we study in section V) is an equilibrium. In other words, the equilibria which embody these threats are subgame perfect.

We consider only punishments which consist of reversions to noncooperative outcomes for some time. (In what follows, we assume, for simplicity, that the reversion to noncooperation lasts forever).[7] The actual

[7]As Abreu (1982) has shown such punishments are not generally optimal. Larger punishments can be achieved by punishing deviations from the punishment strategy. Making this modification would have no material affect on our results.

value of these punishments is computed below, where we also give a condition which ensures that this punishment is sufficiently large that no deviations take place in equilibrium.[8]

We consider at first only two possible arrangements: either firm 1 or firm 2 must announce its price first. Then the other firm sets its price. Unless it is punished for doing so, the second firm will almost always desire to undercut the price charged by the first firm. Thus, if collusion is to be maintained, the follower must be penalized if it chooses a price lower than the leader's.

Suppose that firm 1 is the leader and it can be sure that firm 2 will follow its price P for the current period. Then firm 1 chooses P to maximize:

$$R_1 = (P - c)(a + e - bP). \tag{4}$$

Thus:

$$P = c/2 + (a + e)/2b, \tag{5}$$

and firm 1's profits are given by:

$$R_1 = [a + e - bc]^2/4b. \tag{6}$$

Now consider firm 2. Should it follow firm 1? Firm 2 has two incentives to deviate. First, when e is zero, it can increase its current profits by undercutting firm 1. Second, when e is nonzero, it prefers both P_1 and P_2 to differ from the price given by (5). Thus, to ensure that our form of price leadership is an equilibrium we must ensure that the benefits from continued cooperation outweigh the benefits from charging a price different from P.

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[8]In this sense this paper differs from Rotemberg and Saloner (1984a,b) in which these punishments were insufficient to generate collusion so that price cutting (1984a) and inventories (1984b) were also employed.

The expected profits from matching firm 1's price (R_2) are given by:

$$R_2 = E[(a+e)/2b-c/2][a-e - (a+e)/2 - bc/2]$$
$$= E[a - bc]^2/4b - 3Ee^2/4b, \qquad (7)$$

which is lower than R_1. If firm 2 fails to match firm 1's price, it triggers noncooperation from the following period on. This means that, if a and e are i.i.d., it loses $R_2-\Pi_2$ in every period starting with the next one. If δ is the discount factor the discounted present value of these losses equals:

$$\Delta' = \{[E(a-\bar{a})^2 - 3Ee^2]/4b + (\bar{a} - bc)^2 d^2/[4(2b+d)^2 b]\}\delta/(1-\delta).$$

As we will see below, the first term in brackets actually represents the advantage of letting firm 1 be the leader instead of firm 2. The second term gives the excess of collusive over competitive profits when e is zero and a equals $\bar{a}$. Δ' is the difference in profits between being a follower in our price leadership model and refusing to cooperate. It must be positive for price leadership to be in both firms' interests.

Now we consider the profits made during the period in which firm 2 deviates. After observing P, firm 2 becomes somewhat informed about $(a-\bar{a})$ and e since it has an indirect observation of $(a-\bar{a}+e)$. Indeed, it knows that $[2bP- bc-\bar{a}] \equiv x$ is equal to $(a-\bar{a}+e)$. Let s_a be the variance of a while s_e is the variance of e. Then, firm 2's expectation of $(a-\bar{a})$ is equal to $xs_a/(s_a+s_e)$ while its expectation of e is $xs_e/(s_a+s_e)$. If it were to charge P_2 after firm 1 irrevocably announced the price given by (5) its expected profits would be:

$$(P_2 - c)\{\bar{a} + [2bP-bc-\bar{a}]\,(s_a-s_e)/(s_a + s_e)-bP_2+d(P - P_2)\}. \qquad (8)$$

If it deviates, Firm 2 maximizes (8) which gives a price $\tilde{P}_2$:

$$\tilde{P}_2 = \{\bar{a} + [2bP-bc-\bar{a}](s_a-s_e)/(s_a+s_e) + dP + (b+d)c\}/2(b+d)$$
$$= \{2s_e\bar{a} + [bs_a+(b+d)(s_a-s_e)]P + [ds_a+(2b+d)s_e]c\}/2(b+d)(s_a+s_e)$$

and expected profits of $b\tilde{P}_2^2$.

On the other hand, by not deviating, firm 2 earns the expectation of (8) evaluated at P. This is the expectation of (P-c)(a-e-bP) conditional on P, which equals:

$$(P-c)[\bar{a}s_e+b(s_a-3s_e)P-bc(s_a-s_e)]/(s_a+s_e). \tag{8}$$

So, for deviations not to take place, i.e. for price leadership to be an equilibrium, the following condition must be satisfied:

<u>Condition A</u>: $(b+d)(\tilde{P}_2-c)^2 - \dfrac{(P-c)[\bar{a}s_e + b(S_a - 3S_e)P-bc(S_a - S_e)]}{S_a + S_e} - \Delta' < 0.$

The above equilibrium assumes that firm 2 would be punished for downward <u>and</u> upwards deviations in its price. This, however, seems unreasonable since firm 1 always prefers firm 2 to charge a higher price. Suppose then, that a higher price by firm 2 would go unpunished. We now derive a condition under which firm 2 would nonetheless never choose to raise its price above that given by (5).

Taking the derivative of (8) with respect to P_2 and evaluating at P, one obtains:

$$\Delta \equiv [d-2(b+d)+2b(s_a-s_e)/(s_a+s_e)]P +(b+d)c -(\bar{a} + bc)(s_a-s_e)/(s_a+s_e)$$

which implies:

$$\Delta = \{-[(4b+d)s_e+ds_a]P - [\bar{a}+cb](s_a-s_e)\}/(s_a+s_e)+ (b+d)c.$$

If Δ is positive firm 2 wants to raise its price above P. As P rises,

Δ falls since this makes undercutting more desirable. So we want to compute this derivative for the smallest of all conceivable P's, namely P=c. Then Δ becomes:

$$\Delta = [-\bar{a}s_a - (2bc - \bar{a})s_e]/(s_a+s_e). \tag{9}$$

If the variance of e is zero this expression is always negative. It is only when low prices signal low values of e that raising the price above P would conceivably be in firm 2's interest. Also, for an increase in the price to be worthwhile $\bar{a}$ must exceed 2bc so that monopoly profits are large. Even then, the variance of e must exceed $s_a/[1-2bc/\bar{a}]$. A necessary condition for this to be true is that the variance of e exceed that of a. So, under fairly weak conditions we can rule out the possibility that the follower raises its price above the leader's. So, if Condition A is satisfied and (9) is negative leadership by firm 1 is an equilibrium.

However that does not imply that this equilibrium is desired by firm 2. If, however, the profits of firm 2 at this equilibrium exceed those it would earn if it were the price leader itself, it is natural to expect firm 1 to be the leader. If firm 2 is the price leader, it sets a price equal to $(\bar{a} + bc)/2b$ which leads to expected profits given by:

$$L_2 = [\bar{a} - bc]^2/4b. \tag{10}$$

Thus the difference between R_2 and L_2 is simply

$$[E(a-\bar{a})^2 - 3Ee^2]/4b. \tag{11}$$

Firm 2 prefers to be a follower if the variance of a exceeds three times the variance of e. A high variance of a makes firm 2 want to be a follower since movements in a are incorporated into firm 1's prices. On the other hand variations in e are also incorporated in firm 1's prices, to firm

2's detriment. Thus, a high variance of e makes firm 2 prefer to be a leader.

It might seem at this point that a price leadership scheme is a natural way of organizing a duopoly with asymmetric information. Not only does the leader have absolutely no incentive to deviate from his equilibrium strategy, but the detection of cheating by the follower is extremely simple. Moreover, in some circumstances the firms are unanimous in their choice of a leader.

However, we have not established that the duopoly cannot improve on the profits garnered by price leadership. Indeed, we now show that, under full information, the equilibrium achieved by our scheme is dominated (from the point of view of the duopoly) by other pricing arrangements. In figure 1 we show isoprofit lines for both firms in the space of P_1 and P_2. These lines are drawn for $e=\bar{e}>0$. Therefore the tangency of an isoprofit line for firm 1 and the 45° line occurs at prices higher than the tangency of an isoprofit line of firm 2 and the 45° line. In our model of price leadership firm 1 picks the point at which one of its isoprofit lines is tangent to the 45° line. It is immediately apparent from the figure that both firms can be made better off if they both lower their prices with firm 2 lowering its price by more than firm 1 lowers its own. Furthermore, from the point of tangency of firm 1's isoprofit line with the 45° line a small reduction in both prices by the same amount raises total profits. This occurs because there is only a second order deleterious effect on firm 1's profits while the beneficial effect on firm 2 is of first order.[9]

[9]On the other hand the tangency of an isoprofit line of firm 2 with the 45° line can be interpreted as the price picked by firm 1 as the leader when e is equal to $-\bar{e}$. There, the prices are too low. Both firms would benefit from price increases with the follower increasing its price more than the leader.

To see whether the duopoly can in fact improve upon its performance with price leadership there are three issues to consider. The first is whether given a fixed rule that picks firm 2's price as a function of firm 1's price, it is possible to induce firm 1 to choose a price other than the one that maximizes its own profits given this rule. For instance, we know that, even when firm 2 matches firm 1's price the total profits of the duopoly can be increased from those that are achieved when firm 1 announces prices given by (5). Yet, is it possible to induce firm 1 to announce a price (say P') different from the one given by (5)? We now argue that this is impossible unless a and e become common knowledge at some point.

If a and e do become common knowledge firm 2 can threaten to punish firm 1 if it fails to announce P'. In other words, there exists an equilibrium in which, as soon as a and e become common knowledge and it becomes known P' was not picked, both firms revert to noncooperation because each expects the other firm to revert to noncooperation.

Suppose that, instead, a and e never become common knowledge, i.e. either firm 2 doesn't know them precisely or firm 1 doesn't know that firm 2 knows them or firm 2 doesn't know that firm 1 knows that it knows etc. Then what is to ensure that the punishment for deviations on the part of firm 1 is actually meted out? One firm, say firm 2, must reveal that is now sure that firm 1 has cheated in order for the noncooperative period to begin. Yet, firm 2 can always pretend that it is unaware of firm 1's deviations. By doing so it ensures that no punishment will take place. This is better for firm 2, even when firm 1 has announced the price given by (5), as long as unilateral deviations by firm 2 from the price leadership equilibrium are not profitable i.e. as long as Condition A is satisfied.[10]

This analysis does not apply only to the case in which firm 2 must match firm 1's price to avoid punishment. Instead, for any rule that picks firm 2's price as a function of firm 1's, we have shown that, unless a and e become common knowledge, firm 1 must be allowed to pick the price it likes best given the rule.

The second issue that arises, therefore, is whether it wouldn't be better to let the follower pick a price different from the leader's. In our analysis of figure 1 we showed that, when e is high, the follower should undercut the leader while the opposite is true when e is low. In general we therefore cannot rule out the possibility that complicated rules that make firm 2's price a (possibly) nonlinear function of firm 1's make the duopoly better off. Yet, even when fluctuations in a are ignored the first best (with sidepayments) cannot be achieved by such rules. This can be seen as follows. Maximum duopoly profits with zero marginal cost are achieved when firm 1 charges $a/2b+e/2(b+2d)$ while firm 2 charges $a/2b-e/2(b+2d)$. Thus if firm 1 charges this maximizing price, firm 2 infers that $e/2(b+2d)$ equals $P_1 - a/2b$ and charges $a/b-P_1$. But then, firm 1 knowing this maximizes:

$$P_1[a + e - bP_1+d(a/b-2P_1)],$$

which gives:

$$P_1 = ab/2b(b+2d) + e/2(b+2d).$$

This price is below that which maximizes duopoly profits. Since firm 2 infers a low value of e (which leads it to charge a high price) from a low

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[10]Further implications of this analysis are sketched in the concluding section.

price by firm 1, there is an incentive for firm 1 to lower its price.[11]

On the other hand variations in a unaccompanied by changes in e make it optimal for both prices to move in tandem. Indeed if e is always zero, our form of price leadership always brings about the first best allocation from the duopoly's point of view. Thus it is only when fluctuations in e are large relative to fluctuations in a that rules of the kind we are considering here offer important benefits to the duopoly. Even then, price leadership offers an advantage: the equilibrium behavior of the follower is related to the leader's action in the simplest possible way. This both simplifies the task of the follower and makes monitoring trivial. On the other hand, as discussed in the introduction, the rules that improve upon price leadership require a degree of sophistication, fine-tuning and implicit understanding on the part of both firms, that strains plausibility. We therefore limit our attention to price leadership.

The third issue which we want to consider is whether simultaneous announcements can somehow improve upon price leadership. With simultaneous announcements and firm 2 uninformed, firm 2's price must be state independent. On the other hand, if we allow firm 1 to state prices which are state dependent, it will always want to undercut the price firm 2 will announce. Thus, firm 1 must also charge a state independent price for the duopoly to achieve some measure of collusion.

The constant prices which maximize ex ante industry profits are natural candidates for such a collusive outcome. They are on the profit frontier;

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[11]This analysis neglects the fact that if firm 1 behaves in this manner consistently firm 2 will ultimately learn this and punish firm 1 (see Radner (1980) for an analysis along these lines). However, in the presence of even moderate discounting and nonstationarities in the distribution of a, long-term monitoring of this kind will be insufficient to keep firm 1 in line.

there are no other constant prices that make both firms better off ex ante. Moreover, they treat both firms symmetrically.

The price that maximize both firms' ex ante profits is given by $[c/2 + \bar{a}/2b]$ which is also the price that firm 2 would set if it were a price leader. This price leads to expected profits of L_2 for both firms. Thus this scheme is strictly dominated by price leadership when R_2 exceeds L_2. Much more generally, it produces total profits which fall far short of those produced by price leadership.

IV. PRICE STICKINESS AND PRICE LEADERSHIP

The problem, from firm 2's perspective, in allowing firm 1 to be the leader, is that firm 1 takes advantage of this and thus picks a price that rises proportionately to e, the difference in the two demand curves. This exploitation comes about not only when demands differ, but also when costs differ, as when firm 1 faces a strike by its workers. One way of mitigating this effect, particularly when the firms possess information of similar quality, is to let the firms alternate the leadership role.[12]
An alternative way, and one that is more applicable when the firms' quality of information differs substantially and when temporary fluctuations in e are important, is to make prices relatively rigid. In other words, the leader is threatened with reversion to noncooperation if he changes his price too often. We study this role for rigid prices here.

If the leader must keep his price fixed for some time, it will make its price a function of current and expected future e's. The longer the

[12]Alternation of this kind has been observed in a variety of industries, including steel and cigarettes.

period of price rigidity, the more important are the expected future e's when firm 1 sets its price. Accordingly, if the expectation of future e's is relatively insensitive to current demand conditions, the presence of rigid prices dampens the effect of current e on price.

We illustrate this advantage of price rigidity with a simple example. In particular, we assume that e_t, i.e. the value of e at time t, is given by:

$$e_t = \beta e_{t-1} + \varepsilon_t \tag{12}$$

where β is a number between zero and one while ε_t is an i.i.d. random variable with zero mean. The intercept has two components; the first of which is $\bar{a}$, a constant, while the second, a_t, moves over time according to the law of motion:

$$a_t = \phi_{t-1} + \alpha_t \tag{13}$$

where ϕ is a number between zero and one while α is an i.i.d. random variable with zero mean. Thus the intecept, $\bar{a}+a_t$, tends to return to its normal value as well. To accomodate the existence of inflation we write the demand curve at t as:

$$\begin{aligned} Q_{1t} &= \bar{a} + a_t + e_t - bP_{1t}/S_t + d(P_{2t} - P_{1t})/S_t \\ Q_{2t} &= \bar{a} + a_t - e_t - bP_{2t}/S_t + d(P_{1t} - P_{2t})/S_t \end{aligned} \tag{14}$$

where S_t is the price level at t. This price level is given by:

$$S_t = \lambda^t. \qquad (\lambda > 1) \tag{15}$$

The difference between λ and one is the general rate of inflation. If the price leader must set a price that will be in force for n periods starting at time zero, it will pick a price that maximizes:

$$E_{10} \sum_{t=0}^{n} \delta^t \left(\frac{P}{S_t} - c \right) \left(a_t + \bar{a} + e_t - b \frac{P}{S_t} \right) \tag{16}$$

$$= \sum_{t=0}^{n} \delta^t \left(\frac{P_t}{\lambda^t} - c \right) \left(\phi^t a_0 + \bar{a} + \beta^t e_0 - b \frac{P}{\lambda^t} \right)$$

where E_{10} is the expectation conditional on information avaliable at time 0 to firm one and δ is the real discount rate. This price, $P(n)$ is given by:

$$P(n) = \frac{\frac{a_0}{2b} \frac{1-(\delta\phi/\lambda)^{n+1}}{1-\delta\phi/\lambda} + \frac{e_0}{2b} \frac{1-(\delta\beta/\lambda)^{n+1}}{1-\delta\beta/\lambda} + \left(\frac{\bar{a}}{2b} + \frac{c}{2} \right) \frac{1-(\delta/\lambda)^{n+1}}{1-\delta/\lambda}}{\frac{(1-\delta/\lambda^2)^{n+1}}{1-\delta/\lambda^2}} \tag{17}$$

Note that this price is increasing in n if there is inflation (λ is greater than one). It is also increasing in λ and e.

The expectation of the present discounted value of profits of the follower ($W_2(n)$) is given by the expectation of the present value of profits for the n periods during which the leader keeps his price fixed ($\Pi(n)$) divided by $(1-\delta^{n+1})$. $\Pi(n)$ is given by:

$$\Pi(n) = E_0 \sum_{t=0}^{n} \delta^t \left(\frac{P(n)}{\lambda^t} - c \right) \left(\bar{a} + \phi^t a_0 - \beta^t e_0 - b \frac{P(n)}{\lambda^t} \right) \tag{18}$$

where E_0 takes unconditional expectations. The unconditional expectation of both a_0 and e_0 is zero while their unconditional variance is $\text{var}(\alpha)/(1-\phi)$ and $\text{var}(\varepsilon)/(1-\beta)$ respectively. This focus on unconditional expectations is warranted if the follower is completely uncertain about the state of demand at the moment he implicitly agrees to be a follower.[13]

We can now write $W_2(n)$ as:

[13]The analysis is essentially unchanged if the expectations used to evaluate $\Pi(n)$ are conditional on somewhat more information as in section VII.

$$W_2(n) = \frac{[\bar{a} + c^2b]^2[1-\delta/\lambda^2][1-(\delta/\lambda)^{n+1}]^2}{4b[1-\delta/\lambda]^2\ [1-(\delta/\lambda^2)^{n+1}[1-\delta^{n+1}]} - \frac{\bar{a}c}{1-\delta}$$

$$+ \frac{var(\delta)[1-\delta/\lambda^2][1-(\delta\phi/\lambda)^{n+1}]^2}{4b(1-\phi)[1-\delta\phi/\lambda]^2\ [1-(\delta/\lambda^2)^{n+1}][1-\delta^{n1}]}$$

$$- \frac{3var(\varepsilon)\ [1-\delta/\lambda^2][1-(\delta\beta/\lambda)^{n+1}]^2}{4b(1-\beta)[1-\delta\beta/\lambda]^2[1-\beta/\lambda^2)^{n+1}[1-\delta^{n+1}]} .$$

To evaluate the benefits of price rigidity we consider two special cases. In the first special case inflation is positive but the variance of α is zero so that demand for the sum of the two products is deterministic[14] while in the second special case it is one (so that there is no inflation). In both cases there is an incentive to prolong the duration of prices, because this prolongation reduces the deleterious effect of var(ε) on W_2. In both cases there is also a cost to long price durations. In the first case this cost is the erosion in prices caused by inflation while in the second it is the insufficient response to changes in a. We analyze these special cases as follows. First, we give the conditions under which the follower would prefer some price rigidity, i.e. under which $W_2(2) > W_2(1)$. Then we study the numerical properties of $W_2(n)$ for certain parameters.

Consider first the case in which inflation is zero. Assume also that $var(\alpha)/[(1-\phi)(1-\delta\phi)^2]$, which we denote σ_a, equals $3var(\varepsilon)/[(1-\beta)(1-\delta\beta)^2]$, which we denote σ_e. Recall that this is the condition under which firm 2 is just indifferent between being a follower and a leader. Then, the follower prefers prices to be constant for two periods if:

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[14]Of course, in a model in which the variance of α is literally zero, our rationale for price leadership disappears. However, this example is only intended to illustrate the effects of inflation on price rigidity.

$$\left[\frac{1-(\delta\phi)^2}{1-\delta^2}\right]^2 - \left[\frac{1-(\delta\beta)^2}{1-\delta^2}\right]^2 > \left[\frac{1-\delta\phi}{1-\delta}\right]^2 - \left[\frac{1-\delta\beta}{1-\delta}\right]^2. \qquad (19)$$

As shown in the appendix this inequality is satisfied as long as β is less than ϕ. Thus as long as the decay towards zero of the difference in demands is more rapid than the decay of the absolute level of demand towards its normal value, the follower prefers the leader to maintain some price rigidity. The intuition for this is that since a rapid decay of e towards zero means that the leader will be relatively inattentive to e when setting a price for a relatively long horizon. On the other hand, if a decays slowly, he will still make his price fairly responsive to the current value of a.

What we must show now is that the follower may prefer a finite period of price rigidity to an infinite one. This is plausible since, if e decays rapidly, the benefit from continued price rigidity, namely the loss in responsiveness to e becomes unimportant as the horizon becomes longer. We provide a numerical example in which the follower does indeed prefer a finite period of price rigidity.[15] Figure 2 shows the value of $W_2(n)$ when σ_a equals σ_e while k is .98, β is .6 and ϕ is .9. The follower's welfare is maximized when n is equal to five. If, instead, σ_a is made to equal only $.8\sigma_e$ then the maximum occurs at n = 7. Clearly, an increase in the variance of the difference in demands warrants a longer period of price rigidity to reduce further the effect of e on price.

Now consider the special case in which the var(α) is zero. Assume further that, σ_e is equal to $[(c^2b+\bar{a})/(1-\delta/\lambda)]^2/4b$ which we denote σ_λ.[16] This case reveals a slightly different advantage of price rigidity.

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[15]This preference for finite periods of price rigidity is a feature of every numerical example we have studied.

[16]This is probably unrealistic since it requires a very large variance of ε.

Price rigidity makes it impossible for the leader to fully respond in every period to increases in e even if these are permanent. Even when β is one, the follower benefits from this lack of responsiveness to e. This can be seen by noting that, under the current assumptions, when β is one, W_2 is independent of both n and the rate of inflation. In this case the loss in the responsiveness to e just offsets the loss from responding to inflation more generally. If, instead, β is less than one, it can be shown that the follower always prefers some price rigidity. Indeed, it can be shown then that $W_2(2)>W_2(1)$. This requires that:

$$\frac{[1-(\delta/\lambda)^2]^2}{(1-\delta^2/\lambda^4)(1-\delta^2)} - \frac{[1-\delta\beta/\lambda]^2}{(1-\delta^2/\lambda^4)(1-\delta^2)} > \frac{(1-\delta/\lambda)^2}{(1-\delta/\lambda^2)(1-\delta)} - \frac{(1-\delta\beta/\lambda)^2}{(1-\delta/\lambda^2)(1-\delta)} \qquad (20)$$

which is proved in the appendix. Thus, if the variance of ε is sufficiently big while e decays even slightly towards its mean, firm 2 prefers some rigidity to complete flexibility. Once again, a decay of e over time induces the leader to make its price unresponsive to e if it is to keep a relatively rigid price. The follower benefits from this. We again consider some numerical examples to show that the follower may prefer a finite period of price rigidity. Figure 3 shows $W_2(n)$ for δ of .98, β of .7, λ of 1.02 and σ_e equal to one fourth of σ_λ. The maximum is given by n=18. Reductions in inflation that make λ equal to 1.01 raise the optimal n, from the follower's perspective, to 25. Increases in σ_e also tend to raise this optimal n.

While we have rationalized price rigidity by arguing that it leads to a more equitable distribution of the profits from implicit collusion one must be careful in drawing the implications of this analysis for macroeconomics. In particular, we have only shown that the follower wants to reduce the ability of the leader to respond to temporary shifts in relative demand (or cost). Thus, for instance, the follower does not want the leader to be able to take advantage of temporary consessions by the leader's suppliers. This argues for limits on the ability of the leader to change prices at will. However, it does not argue for complete nominal rigidity. Rather it argues for rules that make prices respond to aggregate shocks in a mechanical rather than a discretionary way at least for some time. It argues for some forms of indexation. It is only if such indexation is infeasible (because, for example, the antitrust authorities might become suspicious at such preannounced indexation or because customers prefer to be quoted a price rather than a price rule) that fixed nominal prices become the preferred solution to the oligopoly's problem.

V. The Tendency for Noncooperative Prices to Differ

We have shown that when implicitly colluding oligopolists operate in the presence of asymmetric information, they may find a form of price leadership to be an effective strategy for both price-setting and profit-sharing. We now turn our attention to the converse: should price leadership be considered to be evidence of implicit collusion? We argue below that if the oligopolists are selling differentiated products, the answer is "yes". We already saw in section II that if firms act noncooperatively and announce their prices simultaneously, their prices tend to differ. This occurs unless they are perfectly symmetric in the full information case or their information is very perculiar in the asymmetric information case. However, simultaneous price announcements cannot possibly capture the dynamics of price leadership. In this section we explore the possibility of noncooperative price leadership by letting firms announce their prices sequentially. This seems to be what is popularly referred to as "barometric" price leadership. We show that sequential price announcements, if anything, make it less plausible that the firms will charge the same price. Even if the firms are symmetric the follower tends to undercut the leader's price. We show that prices tend to differ not only when there is symmetric information, but also when there is asymmetric information of the kind modeled in Section II.

(a) Symmetric Information

Suppose that demand is as in (1) and that both a and e are common knowledge. Let firm 1 announce P_1 first and firm 2 announce P_2 after learning P_1. This is the analog in this setting of the Stackelberg model.

Given a price, P_1, of firm 1, firm 2 maximizes $(P_2 - c)(a-e-bP_2 + d(P_1 - P_2))$. Its optimal P_2 is therefore $(a- e+(b+d)c+dP_1)/2(b+d)$. Knowing this, firm 1 maximizes:

$$(P_1-c)(a+e-(b+d)P_1+d[(a-e-(b+d)c+dP_1)/2(b+d)]. \tag{21}$$

Thus firm 1 charges

$$P_1 = [a(2b+3d)+e(2b+d) + c(2b+d)(b+2d)]/2(2(b+d)^2-d^2). \tag{22}$$

Therefore the difference in the equilibrium prices of the two firms is:

$$P_1 - P_2 = \frac{[ad^2+e(8b^2+3d^2+12bd) - cbd^2]}{4(b+d)(2b^2+d^2+2bd)}. \tag{23}$$

If e=0, $P_1-P_2 > 0$ provided $a > bc$ which is a necessary condition for outputs to be positive. Thus whenever the firm's positions are "symmetric", firm 2 undercuts firm 1's price. If e is positive so that firm 2's demand is weaker than firm 1's, firm 2 has an added incentive to lower its price and P_2 is again below P_1. If, on the other hand, e is negative then firm 2's demand is stronger than firm 1's. Although firm 2 still exploits its followership position and "undercuts" firm 1's price, since P_2 is decreasing in e (and P_1 is increasing in e) as e decreases the two equilibrium prices achieve equality momentarily and then P_2 becomes larger than P_1. Although in general either firm's price can be the higher one, it would be sheer coincidence for them to be exactly equal.

(b) Asymmetric Information

The argument for differing equilibrium prices in the previous subsection may not be convincing in the presence of asymmetric information, which is after all the basis of our price leadership model. Suppose, for example, that firm 1 has superior information about demand. Suppose further that e=0 so that the firms face symmetric demands. If the firm's are behaving noncooperatively, might firm 2 not believe that firm 1 is setting the industry noncooperative price and, in the absence of any precise demand information of its own, simply choose to match firm 1's price? This cannot constitute equilibrium behavior, however. If firm 1 believed that it was going to be matched in this simple way it would have an incentive to raise its price to the monopoly level. Knowing that, however, firm 2 would have an incentive to undercut firm 1's price. Thus, there is no simple equilibrium matching strategy.

In what follows we analyze the equilibrium of this noncooperative game in which the firms are asymmetrically informed and which firm 1's price may signal some (or all) of its information to firm 2. As in the previous subsection we find that firm 2's equilibrium price is generally lower than that of firm 1.

We suppose that demand is as in (1) and consider the symmetric case, $e=0$. For simplicity we assume that $c=0$ and that $a \varepsilon [0,\infty]$. We analyze the separating, pooling and partially pooling equilibria in turn.

(i) Separating Equilibrium

In a separating equilibrium firm 2 is able to infer a exactly from P_1. Thus firm 2 believes that there is a one-to-one relationship between a and P_1 which we denote by $a(P_1)$. After observing P_1, firm 2 maximizes $P_2(a(P_1) - (b+d)P_2 + dP_1)$ which requires setting $P_2 = [a(P_1) + dP_1)]/2(b+d)$. Knowing this, firm 1 maximizes

$$P_1(a-(b+d)P_1+d[a(P_1)+dP_1]/2(b+d) \tag{24}$$

which gives the f.o.c:

$$2a(b+d)/d - (4(b+d)^2-2d^2)P_1/d + a(P_1) + P_1a'(P_1)=0. \tag{25}$$

But in equilibrium firm 2's beliefs must be correct so that $a=a(P_1)$. Substituting this in (25) gives

$$\lambda a(P_1) + \beta P_1 + P_1a'(P_1) = 0 \tag{26}$$

where $\lambda\equiv(2b+3d)/d$ and $\beta\equiv-(4(b+d)^2-2d^2)/d$. Equation (26) is a first-order ordinary differential equation with solution

$$\beta P_1^{\lambda+1} + a(\lambda+1)P_1^{\lambda} + k = 0. \tag{27}$$

Consider the case where $a=0$. Clearly, P_1 cannot be negative for this case. Suppose then that P_1 is positive. In equilibrium firm 2 is able to infer that $a=0$ and charges its best-response, namely $P_2 = dP_1/2(b+d) < P_1$. But then $Q_1= -(b+d)P_1 + d^2P_1/2(b+d) < 0$. Therefore, in this case P_1 must be zero, which from (27) implies that $k=0$ since $\beta\neq0$. Thus (26) has the unique solution $\beta P_1 + a(\lambda+1)=0$ or, rewriting,

$$P_1 = \frac{a(b+2d)}{2(b+d)^2-d^2}. \tag{28}$$

Thus firm 2's equilibrium beliefs are given by $a(P_1) = P_1(2(b+d)^2-d^2)/(b+2d)$. Since $P_2 = [a(P_1) + dP_1]/2(b+d)$, we have

$$P_2 = \frac{a(2b+3d)}{2(2(b+d)^2-d^2)}. \tag{29}$$

If $d \neq 0$ and $a \neq 0$ then $P_2 < P_1$. In equilibrium firm 2 infers the actual value of a from firm 1's price and then undercuts that price.

In the symmetric information case of the previous subsection, we had $P_1 = a(2b+3d)/2(2(b+d)^2-d^2)$ for the case in which $e=c=0$. Thus with asymmetric information if $a\neq 0$ then the equilibrium price is strictly higher than with symmetric information. This is the familiar "overinvestment" characteristic of signaling models. Firm 1 has an incentive to convince firm 2 that a is high since P_2 is increasing in a. Since $a(P_1)$ is increasing in P_1 in equlibrium, this leads firm 1 to charge a higher price than it otherwise would. Firm 1 is worse off, and firm 2 better off, in the separating equilibrium than with complete information.

Note that as the products become increasingly good substitutes ($d\to\infty$) the prices tend to zero, the competitive price, both when information is symmetric and when it isn't. Similarly, if $d=0$, they charge the monopoly price $a/2b$ in both cases. However, unless the products are perfect substitutes or completely independent in demand, the prices charged by the two firms differ if the firms behave noncooperatively. This occurs even if firm 1's price is acting as a "barometer" of industry demand.

(ii) Pooling Equilibrium

In a pooling equilibrium, firm 1 sets the same price, say $\tilde{P}_1$, regardless of the true value of a. Firm 2 maximizes $P_2(\bar{a} - (b+d)P_2 + d\tilde{P}_1)$, which implies $P_2 = (\bar{a} + (b+d)\tilde{P}_1)/2(b+d)$. Here the firms charge equal prices if (and only if) $P_2 = \tilde{P}_1 = (\bar{a}+d\tilde{P}_1)/2b+d)$, or $\tilde{P}_1 = \bar{a}/(2b+d)$.

Thus there may exist a pooling equilibrium in which the firms charge the same price. Furthermore, this price would be unresponsive to fluctuations in demand. However, we show below, that within the class of pooling equilibria, <u>both</u> firms would prefer one in which $\tilde{P}_1$ is higher: Consider firm 1's profits when it charges $\tilde{P}_1$:

$$\Pi'(\tilde{P}_1) = \tilde{P}_1(a-(b+d)\tilde{P}_1 + d(\bar{a} + d\tilde{P}_1)/2(b+d).$$

$$\frac{d\Pi'}{d\tilde{P}_1}\Bigg|_{\tilde{P}_1 = \bar{a}/(2b+d)} = \bar{a}d^2/2(b+d)(2b+d) > 0.$$

Thus firm 1 prefers a higher pooled price. Since firm 2's profits are increasing in $\tilde{P}_1$, it too prefers a higher price. Thus the pooling equilibrium with a common price is Pareto dominated by another pooling equilibrium. But, in the equilibria in which $\tilde{P}_1$ is higher than $\bar{a}/(2b+d)$, firm 2's price is lower than firm 1's. Thus even in the case of pooling equilibria, equal prices are unlikely.

(iii) Partial-Pooling Equilibria

There may also exist equilibria that are hybrids of the separating and pooling equilibria. In equilibria of this kind firm 1 partitions a into a finite number of intervals and charges the same price for all a's within an interval, but different prices for a's from different intervals. Thus there is pooling within intervals but separation across intervals. It is easy to see that in general the firms' prices will differ in partial-pooling equilibria as well. This is because the analysis for each interval is the same as that for the pooling equilibria above. An equilibrium in which the firms have identical prices in some interval will be Pareto dominated by one that has higher and unequal prices in that interval.

VI WELFARE CONSEQUENCES OF PRICE LEADERSHIP

In the light of the previous section we should view price leadership as a collusive device. Thus it is natural to examine how the perfomance of price leadership compares with that of overt collusion. We already know, from section III, that the former has lower aggregate profits. Here we study consumer welfare and the aggregate of consumer and producer surplus.

In principle the analysis of consumer welfare should be carried out using the preferences of consumers whose aggregate demands are given by (1). In practice this is very difficult, in part because different individuals will be affected differently thus mandating interpersonal comparisons of utility, and in part because modelling the individual consumers whose demands aggregate to (1) is nontrivial. Therefore we compare instead the expected consumer surplus under the two regimes.

This is done as follows. For each value of e we consider the excess willingness to pay for good i above the price paid by consumers. Consumer surplus in the market for good i is the sum across consumers who buy good i of this excess willingness to pay. It is simply the area under firm i's demand curve. We then add the consumer surplus for both goods to obtain the industry's level of consumer surplus.[17] We then compute the expected value of consumer surplus across variations in e for each regime[18]. Finally we compare these two values.

To simplify this welfare comparison we let c equal zero and we ignore fluctuations in the sum of the demands of the two firms so that a is constant.[19] Initially we also assume that e takes only two values with equal probability: a high one, $\bar{e}$ and a low one, $-\bar{e}$. Since our results depend only on $\bar{e}^2$, they are valid for any symmetric distribution of e.

Under price leadership the price charged by both firms when e is high is $(a+\bar{e})/2b$. This leads firm 1 to sell $(a+\bar{e})/2$ and firm 2 to sell $(a-3\bar{e})/2$. On the other hand when e is low, the price is $(a-\bar{e})/2b$, the outpu of firm 1 is $(a-\bar{e})/2$ and the output of firm 2 is $(a+3\bar{e})/2$. The level of consumer surplus in the market for good 1 when e is high is half the productt of the output of firm 1 and the difference between the intercept of the inverse demand curve of good 1, $[a/b+e/(b+2d)]$, and the price. An analogous computation gives the level of consumer surplus in the other cases.

This gives a level of consumer surplus under price leadership of:

$$a^2/2b + \bar{e}^2(5b+2d)/2b(b+2d). \qquad (30)$$

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[17]This gives, ignoring income effects, results identical to those using the procedure of Diamond and McFadden (1974).

[18]This might be viewed as somewhat problematic if the different regimes provide different degrees of insurance against fluctuations in e. Yet, it measures accurately, (ignoring income effects) the expected disbursements of a social planner who wishes to make consumers equally well off under both regimes. These expected disbursements are relevant if the planner is risk neutral with respect to fluctuations in e.

[19]As discussed in footnote 20 below, this latter simplification has no material effects on our results.

With joint profit maximization the price $(a/2b + \bar{e}/2(b+2d))$, and output $(a/2 + \bar{e}(b+4d)/2(b+2d))$, of firm 1 when e is high are respectively equal to the price and output of firm 2 when e is low. Also, the price $(a/2b- \bar{e}/2(b+2d))$ and output $(a/2 -\bar{e}(b+4d)/2(b+2d))$ of firm 1 when e is low are respectively equal to the price and output of firm 2 when e is high. Thus total consumer surplus is given by twice the consumer surplus from either good. This equals:

$$a^2/2b + \bar{e}^2(b+4d)/2(b+2d)^2. \tag{31}$$

Thus the difference in consumer surplus depends only on $\underline{e}^2$.[20] Indeed the difference between (30) and (31) is given by $2(b+d)^2\bar{e}^2/[(b+2d)^2b]$ which is clearly positive so that consumers prefer price leadership. From the point of view of consumers, price leadership has the disadvantage that, when e is high, both prices are high so that there is little surplus either in the market for the preferred good (good 1) or the other good. On the other hand when e is low, both prices are low. This gives a very large surplus particularly because the price for the preferred good (good 2) is much lower than under joint profit maximization. This second effect dominates the first because as prices are higher, the loss in consumer surplus from an additional price increase are smaller simply because there are fewer consumers in the market.

On the other hand, the gain in consumer surplus from price leadership over joint profit maximization is less than the loss in profits from the former arrangement. This is not surprising since price leadership is quite inefficient for the duopoly when e is nonzero. Thus price leadership is a less efficient arrangement from the point of view of the risk neutral

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[20]The lack of dependence on a demonstrates that this comparison is independent of the distribution of a.

planner. This can be seen by computing expected profits under both arrangements.

With joint profit maximization, expected profits are just twice consumer surplus or:

$$a^2/b + \bar{e}^2/(b+4d)/(b+2d)^2.$$

With price leadership the leader earns $(a^2 + \bar{e}^2)/2b$ while the follower earns $(a^2 - 3\bar{e}^2)/2b$. So profits under joint profit maximization exceed leadership profits by:

$$\bar{e}^2[(b+4d)/(b+2d)^2 + 1/b] = \bar{e}^2\ 2(b^2 + 2d^2 + 4bd)/[(b+2d)^2 b]$$

which exceeds the gain to consumers by $2\bar{e}^2 d(2b+d)/[(b+2d)^2 b]$.

VII ENDOGENOUS LEADERSHIP WHEN BOTH FIRMS RECEIVE SIGNALS

Up to this point we have treated firm 1 as fully informed about the current state of demand while firm 2 was completely uninformed. This is much too restrictive and stacks the deck in favor of firm 1 emerging as the leader. Also, it means that it is relatively straightforward for firm 2 to infer the information available to firm 1 from observations of output and price. Thus it is much easier for deviations of firm 1 from an arbitrary pricing path to become common knowledge than is intuitively plausible. In this section we remedy these deficiencies by letting both firms receive signals a_i and e_i which are unbiased indicators of a and e. In particular we assume that:

$$a_i = a + \alpha_i$$

$$e_i = e + \varepsilon_i$$

where the α's and ε are forecast errors uncorrelated with information available to each individual firm. Thus if firm i can be sure that firm j will follow its price, it sets a price equal to P_i:

$$P_i = c/2 + [a_i - (-1)^i e_i] / 2b.$$

We now consider the expected profit of firm 2 both when firm 1 is a leader (R_2) and when firm 2 is a leader (L_2). The former is given by:

$$R_2 = E[(a_1 + e_1)/2b - c/2][a - e - (a_1 + e_1)/2 - bc/2]$$

$$= E[(a - bc)^2 - \alpha_1^{\ 2} - 3e^2 - \varepsilon_1^{\ 2}/- 4e\varepsilon_1]/4b.$$

The latter is given by:

$$L_2 = E[(a_2 - e_2)/2b - c/2][a - e - (a_2 - e_2)/2 - bc/2]$$

$$= E[(a - bc)^2 - \alpha_2^{\ 2} + e^2 - \varepsilon_2^{\ 2}]/4b.$$

Thus, firm 2 prefers to let firm 1 be the leader if:

$$E[(\alpha_2^{\ 2} - \alpha_1^{\ 2}) + (\varepsilon_2^{\ 2} - \varepsilon_1^{\ 2}) - 4(e^2 + e\varepsilon_1)]$$

is positive.

The three terms inside the square brackets are easily interpretable. The first term is positive as long as firm 1 makes smaller forecast errors when forecasting a while the second term is positive when firm 1 makes smaller forecasting errors as it forecasts e. Such higher precision on the part of firm 1 makes it a natural leader. On the other hand, if e varies a lot then leaving the leadership to firm 1 hurts firm 2 since the leader exploits the movements of e to its opponent's disadvantage. Finally, the more correlated is firm 1's signal of e with e itself, the worse the outcome for firm 2 since firm 1 will charge a high price precisely when firm 2's demand is low. Note that if we view firm 1 as updating from its prior mean of zero every time it observes a variable related to e we expect a high value for e to be generally associated with a negative value of ε_1. In other words we expect $Ee\varepsilon_1$ to be negative. In the extreme case in which firm 1 always has the same signal for e, $Ee\varepsilon_1$ is equal to Ee^2 and the last term cancels. Here, firm 1 is uninformed about e so that it does not exploit the differences in demand.

Now we can ask whether in this case, where the follower is much more informed than before, we can improve from simple price leadership in a straightforward way. For instance, can the follower succeed in making the leader's price less sensitive to e_1 by threatening to revert to noncooperation? Once again, these threats of reversions to noncooperation are only credible if the reversions are triggered by variables which are common knowledge. Otherwise firm 2, who does not prefer the punishment period <u>ex post</u> will prefer to pretend that it has not discovered the deviation by firm 1.

In a setup such as this, it is extremely unlikely that a_1 and e_1 ever become common knowledge. While in section III knowledge of a and e was sufficient to infer a_1 and e_1 because firm 1 was known to be perfectly informed, this knowledge is insufficient when firm 1 receives noisy signals.

However, suppose a and e ultimately become common knowledge. Then firm 2 can threaten to revert to competition if the price firm 1 posts proves <u>ex post</u> to be out of line with a desirable price $P'(a,e)$. Such a price might for example be the common price that maximizes industry profits ($c/2 + a/2(b-d)$). However, in this case reversions to competition will also be triggered by simple mistakes on the part of firm 1.[21] If firm 1 thinks

[21]Such mistakes are analogous to those that trigger price wars in Green and Porter (1983).

incorrectly that demand is high while e also happens to be high, then the high price it announces must eventually lead firm two to punish it. Thus, in situations in which the *ex post* outcome conveys relatively poor information about firm 1's *ex ante* knowledge, we would not expect the follower to punish the leader for picking prices which are too responsive to changes in e.

VIII. CONCLUDING REMARKS

This paper is concerned with explaining the behavior of firms in industries in which price setting is accomplished through the announcements of a price leader. The model we have developed gives an explanation for the presence of price rigidity, provides an argument for exact price matching even in differentiated products markets, and is capable of determining endogenously which firm will be the price leader. It is thus capable of explaining the pattern of pricing in a wide variety of industries.

While the price leadership scheme has many advantages from the point of view of the duopoly, it does not usually implement outcomes on the duopoly's profit frontier. In the paper we argued that this is due in part to the inability of the follower to punish the leader (in a subgame perfect equilibrium) in the absence of common knowledge about whether the leader has charged the appropriate price given its private information. This point has ramifications, which we are currently investigating, that extend beyond this paper to many settings in which cooperative behavior is sustained through the threat of later punishments.

By enriching the model in one of two ways the role for credible punishments can be restored. First, even absent *ex post* common knowledge about the leader's private information, there may be relevant indexes of demand conditions that become available *ex post* and that are common knowledge. For example, trade associations and the press publish estimates of industry output. There is no reason why punishment strategies should not be made conditional on these. Then, the duopoly may be able to do better than under simple price leadership by implementing a scheme of the type proposed by Green and Porter (1983). There, if the index of demand conditions suggests that the leader has deviated from the collusive scheme (i.e. that it is unlikely that its private signal is consistent with the price it in fact charged) a price war ensues. Since, in general, the leader will have only an imperfect signal of demand and the index is also an imperfect measure of demand, in equilibrium price wars will sometimes be triggered by mistake. Even though this will result in some losses to the duopoly, it is nonetheless conceivable that the duopoly would be better off with such a scheme than with price leadership.

Second, one can envisage a reputations model *a la* Kreps *et al.* (1983) that would restore a role for punishments. Suppose that there is no common knowledge index of demand conditions, but rather that the leader believes that there is (very small but) positive probability that the follower is "irrational". Here irrationality implies that if the follower believes with high probability that the leader has cheated it will revert to noncooperation. Although the development of a model of this type is beyond the scope of this paper, we conjecture that in such a setting credible punishments would play a role. Not only would the follower be willing to mete out the punishment if it is truly "irrational", but even a rational follower may be willing to masquerade as its irrational counterpart so as not to reveal its rationality, thereby keeping open the possibility of enjoying cooperative behavior again in the future.

Let us turn now to the policy implications of our analysis. We have argued that it is highly unlikely that the pattern of pricing under investigation could arise in a reasonable model in which the firms behave noncooperatively. Furthermore, our welfare analysis shows that while consumers prefer price leadership to overt collusion, overall welfare is lower. This suggests that a much less sanguine view of this practice is warranted than that which has traditionally been taken by the courts.

However, anyone who would render price leadership a criminal act must be sensitive to the possibility that there is a benign explanation for the existence of identical prices, even for somewhat differentiated products. In particular, if one is to condemn a firm simply because its rivals choose to charge the same price as it does, one must be fairly convinced that the price matching is a collusive practice. Certainly, the mere fact that firms, even with differentiated products, charge identical prices cannot be deemed (on the basis of this paper alone) to be proof of collusion. Yet, differences in prices among competitively provided differentiated products seem pervasive, thus creating a presumption of collusion when the prices are the same. Tempering this conclusion, however, is the observation that brokers in geographically unrelated markets (who presumably face different demand conditions) charge the same commission for their services. For example, residential realtors in the U.S. charge a commission of six percent regardless of the part of the country in which they operate. Absent an explanation of this pricing behavior, per se condemnation of identical prices for firms with differentiated products facing differing relative demands seems premature.

Rather, we suggest an approach that asks whether the market is one in which the exertion of monopoly power is feasible and, second, whether there is evidence of a consistent pattern of price leadership. The first question corresponds to the first tier of the two-tier approach for diagnosing predatory pricing advocated by Klevorick and Joskow (1979). It entails examining profitability, the conditions of entry, and the number and size distribution of firms. The second question asks whether the hallmarks of price leadership are present: advance announcement of price changes, swift matching of these prices by rivals and relatively inflexible prices. The presence of significant product differentiation should be considered to be strong supporting evidence.

The framing of an exact legal rule is beyond the scope of this paper. Such a rule would, for example, have to contend with situations in which prices always differ by one penny. While we do not provide such a rule here, our analysis suggests that one is called for.

Firms, and some practitioners, may consider this approach to be unduly harsh. After all, should firms risk prosecution simply for doing what others are doing? However, firms in differentiated products markets will seldom have to worry about prosecution. Aggressive firms that vigorously compete with their rivals for the business of their customers will typically find that the workings of the noncooperative process alone will lead their prices to differ from those of their competitiors.

APPENDIX I: PROOF OF INEQUALITIES 19 AND 20

Since, δ, β and ϕ are less than one inequality (19) can be written as follows:

$$\frac{(\delta\beta)^2 - (\delta\phi)^2}{1-\delta^2} > \frac{\delta\beta - \delta\phi}{1-\delta} X$$

where

$$X = \frac{(2 - \delta\phi - \delta\beta)(1 + \delta)}{2-(\delta\phi)^2 - (\delta\beta)^2}$$

This is equivalent to:

$$\frac{\delta(\beta-\phi)}{1-\delta} \left[\frac{\delta\beta + \delta\phi}{1+\delta} - X\right] > 0.$$

So, if $\beta<\phi$ the inequality is satisfied as long as X is bigger than $(\delta\beta+\delta\phi)/(1+\delta)$ which is obviously less than one. Yet X is bigger than one since by subtracting the denominator of X from its numerator one obtains:

$$2 - \delta\phi - \delta\beta + 2\delta-\delta^2\phi-\delta^2\beta - [2-(\delta\phi)^2-(\delta\beta)^2] =$$

$$\delta(1-\phi)\ (1-\delta\phi) + \delta(1-\beta)\ (1-\delta\beta) > 0$$

This completes the proof.

To prove the inequality given by (20) we first note that, since δ is smaller than one while $1/\lambda$ and β are at most equal to one, (20) can be written as:

$$\frac{(\delta\beta/\lambda)^2-(\delta/\lambda)^2}{1 + \delta/\lambda^2} > (\delta\beta/\lambda - \delta/\lambda)X'$$

where:

$$X' = \frac{(2 - \delta\beta/\lambda - \delta/\lambda)(1+\delta)}{2 - (\delta\beta/\lambda)^2 - (\delta/\lambda)^2}$$

This is equivalent to:

$$(\delta\beta/\lambda - \delta/\lambda)\left[\frac{\delta/\lambda + \delta\beta/\lambda}{1 + \delta/\lambda^2} - X'\right] > 0$$

So, if β is smaller than one the inequality is satisfied as long as X' is bigger than $(\delta/\lambda+\delta\beta/\lambda)/(1+\delta/\lambda^2)$. This latter expression is smaller than one since by subtracting the denominator from the numerator one obtains:

$$(\delta/\lambda)(\beta-1) + 2\delta/\lambda - [1-\delta + \delta + \delta/\lambda^2]$$

$$= (\delta/\lambda)(\beta-1) - (1-\delta) - \delta(1-1/\lambda)^2.$$

Moreover X' is greater than one since, by subtracting its denominator from its numerator one obtains:

$$(2-\delta\beta/\lambda-\delta/\lambda)(1+\delta) - (2-(\delta B/\lambda)^2-(\delta/\lambda)^2) =$$

$$(1-\beta/\lambda)(\delta-\delta^2\beta/\lambda) + (1-1/\lambda)(\delta-\delta^2/\lambda) > 0$$

This completes the proof.

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FIGURE 1: The Suboptimality of Price Leadership Under Full Information

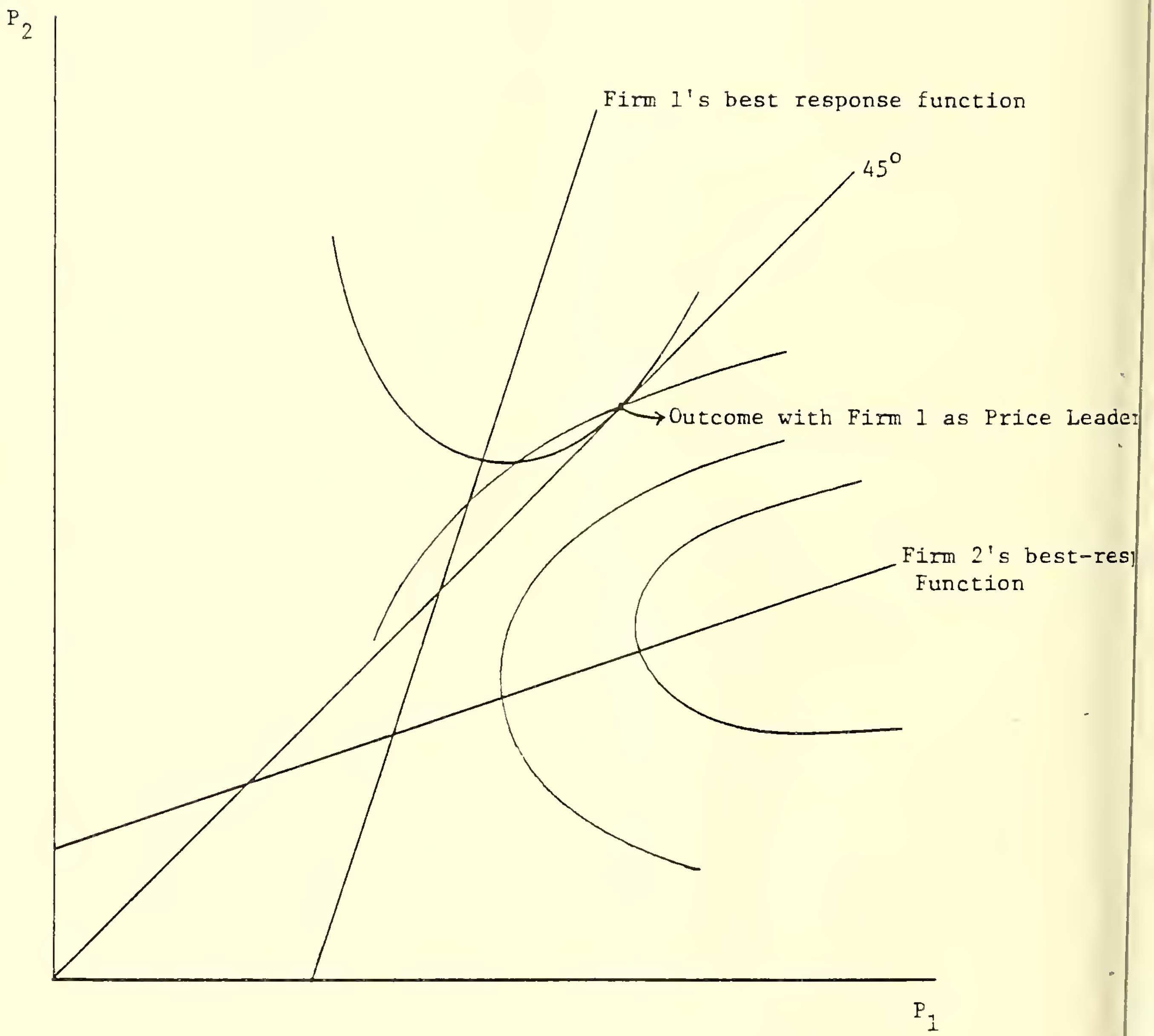


FIGURE 2: Profitability of Price Stickiness for the Follower - The Case of Stochastic Aggregate Demand

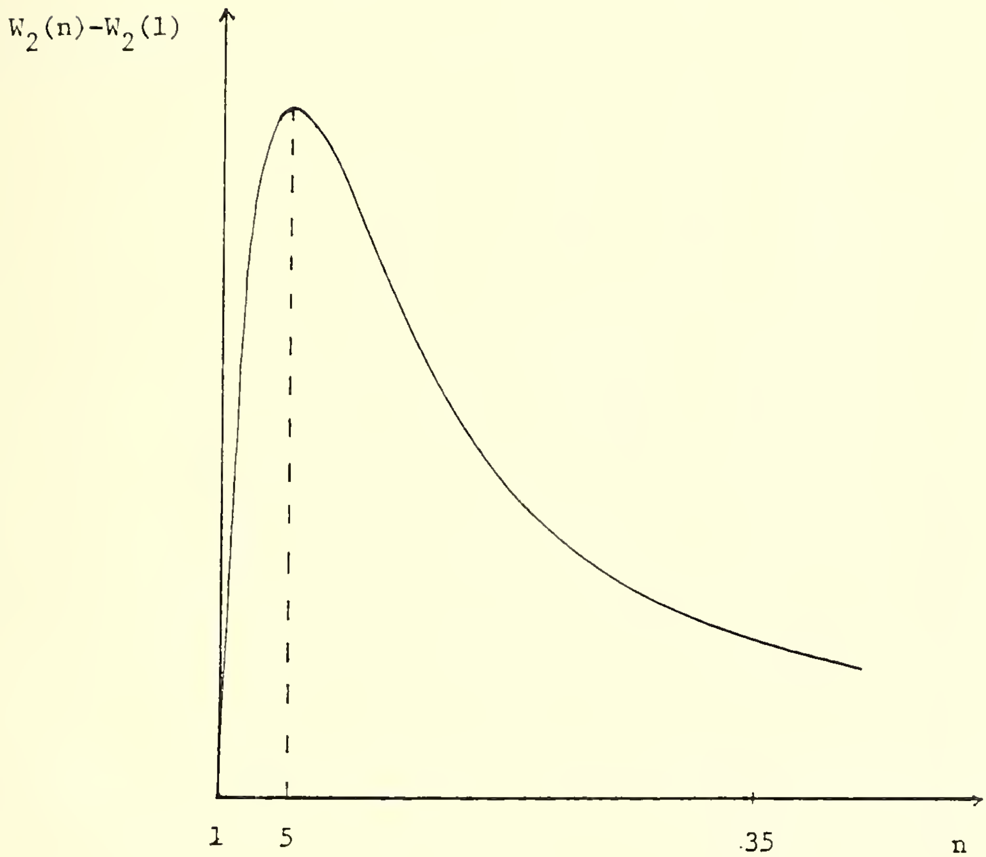
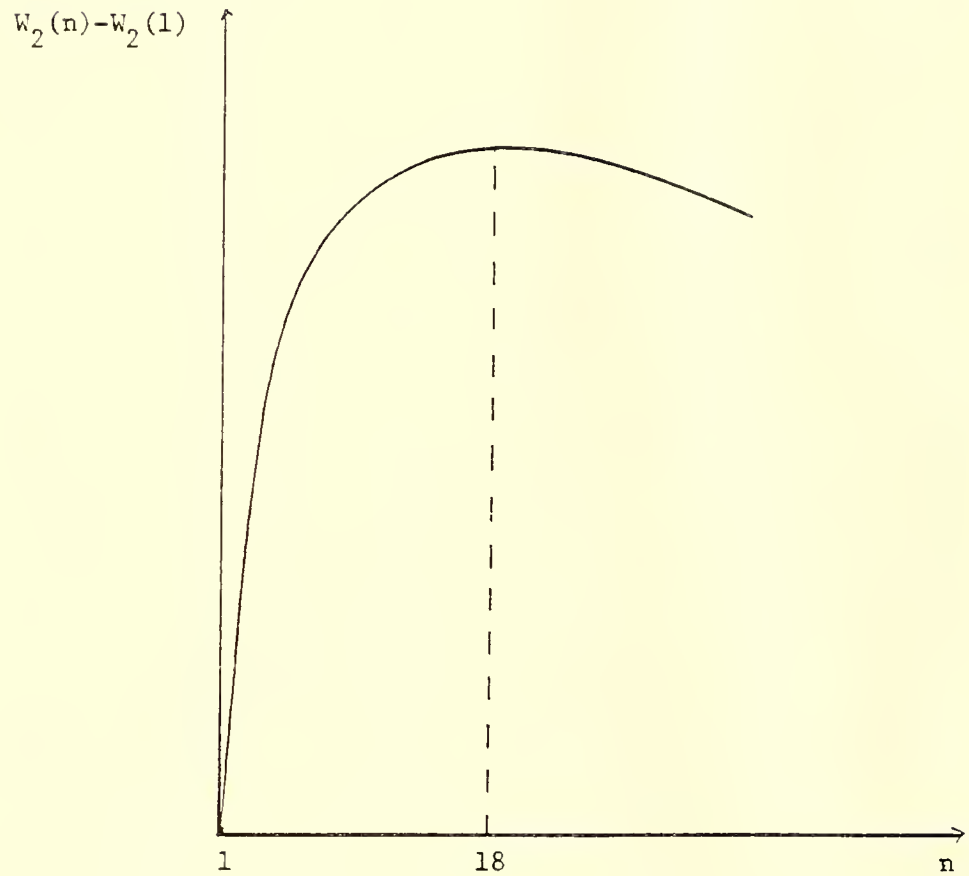


FIGURE 3: Profitability of Price Stickiness for the Follower - The Case of Inflation



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