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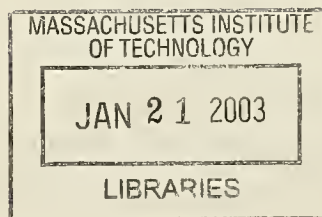
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Sequential Decision Making: How Prior Choices Affect Subsequent Valuations

Elie Ofek, Muhamet Yildiz and Ernan Haruvy*

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Abstract

This paper develops and tests a model of sequential decision making where a first stage of ranking a set of alternatives is followed by a second stage of determining the value of these same alternatives. The model assumes a boundedly rational Bayesian decision maker who is uncertain about his/her underlying preferences over the relevant attributes, and who has to exert costly cognitive effort to resolve this uncertainty. Compared to when only valuation takes place, the analysis reveals that ranking a set of alternatives prior to determining their value has three primary effects: a) the spread (or dispersion) of valuations between most and least preferred alternatives increases, b) decision makers will, on expectation, exert more effort in the valuation phase, and c) the more each attribute contributes to overall utility the greater the relative impact of ranking is on valuation spread. The analysis also sheds light on how prior ranking impacts the demand for a product. These results are then corroborated in a series of controlled lab experiments with actual prizes. The findings have implications for many real life decision making situations ranging from auctions, where there is a tendency to prioritize items before determining a bid, to the ranking of job candidates prior to determining wages and benefits to be offered. More generally, the results bear on our understanding of how past decisions can affect future related decisions.

1 Introduction

Past decisions are often used as input to guide future related decisions. This is regarded as beneficial when the information conveyed in a previous decision is expected to shed light on dimensions associated with the decision at hand, even if these decisions are not identical. In the context of such dynamic decision-making, we ask how previous choices would affect the subsequent determination of maximum willingness to pay for a given good. For example, consider an individual who has chosen a job offer with certain wages in a big metropolitan city over an identical job that pays more wages but is located in a small suburban town. Imagine that several months later, this same individual is considering buying a small house downtown or a big house in the suburbs. How would the willingness to pay (and thus bid) for a house located in downtown or suburbia change if the individual is reminded of the fact that she has chosen the lower-paying urban job over the higher paying suburban job? A similar question may arise even within a single decision if the decision maker divides the complex decision problem into multiple sub-decisions. Examples of such procedures are common: an employer might first rank the set of candidates interviewed before determining the details of each offer to be made (Roth 1984), a consumer at an auction site with many similar items might have to repeatedly choose among sellers before determining how much to bid until an item is secured (Peters and Severinov 2001), and a management team might first rank order product development projects before deciding how much R&D resources to devote to each one (Keefer 2001).

In an ideal world where individuals know their preferences with certainty or can figure them out effortlessly, Individual's previous choices would not have an informational value for subsequent decisions. In reality, however, individuals seldom know their own preference structure with full confidence (for example, the exact trade-off between two product attributes) or may need to anticipate the likelihood of future usage/consumption contingencies. Hence, most decision making tasks regarding multiple alternatives entail costly effort, in the form of cognitive thinking or time-consuming research, and will render past choices potentially useful input. How then should we expect the effort expended in determining valuations to be impacted by the knowledge of a prior choice or rank ordering of the alternatives? How do previous choices impact final valuations, compared to when only valuation takes place? The goal of this paper is to provide an answer to these questions both theoretically and empirically.

Social-science literature has examined the implications of various decision tasks on preference elicitation. The focus has been on how performing various tasks, such as

choice, rating and matching can yield different outcomes when performed *separately* (e.g., Tversky et al. 1988, Montgomery et al. 1994, Bazerman et al. 1992, Huber et al. 2002). However, the implications of intertemporally *combining* a set of tasks on final elicited preferences have not been studied. In this context, our paper focuses on how a previous ranking task, where the output required is an ordinal relationship between the alternatives, affects a subsequent valuation task, where the output required is a measure of willingness to pay for each of the alternatives.

To examine this prevalent sequence of evaluating alternatives, we construct a model of individual decision-making over a set of two alternatives defined over two attributes that need to be traded-off. The central features of the model that make it relevant for examining the above decision sequence are: a) individuals are uncertain about their preferences (much in the vein of March 1978, and Keeny and Raiffa 1976), b) through costly cognitive effort they can resolve part of this uncertainty, and c) though individuals may be forgetful of specific details emerging from cognitive effort during ranking, the outcome of this decision phase (i.e., the rank ordering of alternatives) can be incorporated in the subsequent determination of willingness to pay. As such, our boundedly rational agents use their own previous choices as a source of information about their own utility structure, and may be perceived to have a preference for consistency (similar to the agents in Yariv (2002)). That said, our agents also exhibit other specific patterns of behavior (also observed in our experimental setting) that cannot be explained by such preference for consistency.

The analysis of the model reveals three interesting findings. First, it is shown that the spread of valuations for the two alternatives is, on expectation, greater when ranking precedes valuation. This is not only because of the extra information embodied in the rankings, but, interestingly, also because rankings may induce the agent to think more when the information is perceived to be more valuable. Second, this increased spread as a result of ranking is more pronounced when the contribution of each attribute to overall utility is higher. Lastly, we find that the effort expended in valuing alternatives is (canonically) expected to be higher when ranking information is present, even though previous effort has obviously already been expended in the ranking stage. We also examine how prior ranking affects the likelihood of purchase (with any given distribution of prices). In particular, we show through a canonical example that prior ranking increases the probability of a sale when prices are either very low or very high (but not when they are centered around the mean expected value).

A series of experiments designed to allow comparison of valuation and ranking deci-

sions (and their combined effect) were carried out. The experiments used actual prizes for future consumption of a familiar product category, namely dining at local restaurants, and were constructed to induce truth-telling through a Becker-DeGroot-Marschak (1964) mechanism. The empirical results strongly confirm the implications of the theory, and were designed to rule out possible alternative explanations (such as learning or task familiarity). In particular, we confirm that the effect of ranking on valuation spread becomes more pronounced as the stakes involved in the task increase (i.e., the average value of the prizes is higher). This provides an example for a general possibility that certain effects that are generated by bounded rationality (Rubenstein 1998) may get even larger as the decisions themselves become more important.

The rest of the paper is organized as follows: In the next section we develop theory that allows the modeling of a sequence of decisions regarding the same alternatives, in particular, ranking and subsequent monetary valuation. We formulate the central findings of the model as hypotheses, which we then test through a series of controlled experiments. The paper ends with concluding remarks.

2 A Model of Sequential Decision Making

To shed light on our questions of interest, in this section we develop a simple model of utility with uncertain preferences. We then analyze the utility maximizing behavior of an agent confronted with two different alternatives that she either needs to 1) rank in order of preference, 2) provide an exact monetary value for each, or 3) first rank and then value each of the alternatives.

Notation: We will use the following standard notation throughout. Given any random variable x , $E[x]$ is the expected value of x and $Var(x)$ the variance of x . We write $E[x|G]$ and $Var(x|G)$ for the conditional expected value and the conditional variance of x given event G . We also write $\Pr(G)$ for the probability of event G and let $E[x : G] = E[x|G] \Pr(G)$. The sets of real and non-negative real numbers are denoted by \mathcal{R} and \mathcal{R}_+ , respectively. Given any three-times differentiable function $f : \mathcal{R} \rightarrow \mathcal{R}$, we write f' , f'' , and f''' for the first, second, and third derivatives, respectively.

2.1 Set-Up

Consider a boundedly rational agent who wishes to maximize the expected value of a utility function u , the parameters of which she does not know with certainty. The

agent is rational in the sense that she wishes to make a normatively optimal decision but at the same time is constrained by the costliness of effort needed to resolve the uncertainty. This principle is prevalent in models of bounded rationality (e.g., Simon 1982 and Gabaix and Laibson 2000) and has been generally accepted as a factor not to be ignored in real or laboratory settings (Smith 1985).² Thus, it is possible for the agent to become more informed in making her decision by introspectively accessing information associated with the alternatives at hand. This introspection requires cognitive effort and can be thought of as a mental cost that is accompanied by disutility.³ Ergin (2002) uses a few plausible axiomatic assumptions regarding decision makers that are consistent with our characterization above. In addition, while the agent is forgetful about specific details arising from cognitive effort expended in any previous related decision tasks, the outcome of such decisions can be taken into account by the agent (the updating process will become clear in what follows).

Assume our agent is presented with two indivisible goods, X and Y , each defined in terms of two attributes a and b :

	a	b
X	a_X	b_X
Y	a_Y	b_Y

Here, $a_X, a_Y, b_X, b_Y \in \mathcal{R}$. The value the agent attaches to obtaining good i is:

$$w_a(a_i) + \delta w_b(b_i)$$

where $w_a : \mathcal{R} \rightarrow \mathcal{R}$ and $w_b : \mathcal{R} \rightarrow \mathcal{R}$ are increasing functions, δ is a non-negative real random variable and $i \in (X, Y)$. With this notation, we write $u(X) = w_a(a_X) + \delta w_b(b_X)$ and $u(Y) = w_a(a_Y) + \delta w_b(b_Y)$. Thus, while the agent knows $w_a(\cdot)$ and $w_b(\cdot)$ she does not with certainty know δ , whose expected value $E[\delta]$ is taken to be 1.⁴ For the decision to be non-trivial, we are concerned with cases where no good dominates the other.⁵

²This principle is also related to what Marschak (1968) termed as the *cost of thinking, calculating, deciding, and acting*.

³It may also be possible in some cases for the agent to expend resources in obtaining information by seeking out costly external sources. For example, speaking to friends, getting an expert opinion, conducting a survey of relevant literature, etc.

⁴Our results generalize to the cases where there are more than two attributes and where the agent's value for obtaining good i is: $\delta_a w_a(a_i) + \delta_b w_b(b_i)$, where δ_a, δ_b are independent. For expositional ease we focus on the case where only $\delta_b (= \delta)$ is random, reflecting the situation whereby the contribution of one attribute to overall utility is *ex ante* known with less certainty.

⁵It is worth noting that the *attribute conflict hypothesis* of Fischer et. al (2000) predicts that such a tradeoff in attributes will lead to preference uncertainty.

Hence, without loss of generality, we assume $\Delta_a \equiv w_a(a_X) - w_a(a_Y) > 0$ and $\Delta_b \equiv w_b(b_Y) - w_b(b_X) > 0$. We write $\rho = \frac{\Delta_a}{\Delta_b} > 0$. Note that if our agent knew her true utility structure, she would be indifferent between X and Y if and only if $\delta = \rho$, and strictly prefer good X to good Y if and only if $\delta < \rho$.

To reduce the uncertainty associated with δ , the agent may expend effort c (measured in terms of utility). In our model, the agent has the following simple, non-adaptive way of acquiring information about δ . For some $\bar{c} > 0$, consider a function $F : \mathcal{R}_+ \times [0, \bar{c}] \rightarrow [0, 1]$ such that for each $c \in [0, \bar{c}]$, $F(\cdot; c)$ is a cumulative distribution function (CDF) with mean 1 and variance $V(c)$, where $V(\bar{c}) = Var(\delta)$. By expending $c \in [0, \bar{c}]$ units of effort, our agent can get an estimate $\hat{\delta}$ of δ such that

$$\delta = \hat{\delta}\varepsilon, \tag{1}$$

where ε is the remaining error associated with the estimator $\hat{\delta}$, ε and $\hat{\delta}$ are stochastically independent, and $\hat{\delta}$ has CDF $F(\cdot; c)$. We emphasize that $\hat{\delta}$ is treated as an estimator (i.e., a random variable) before actual thinking has been expended. After thinking, the agent obviously obtains an estimate (i.e., a number) that she can then use to establish her utility for each of the goods. The net utility from each good with estimator $\hat{\delta}$ can now be expressed as:

$$w_a(a_i) + \hat{\delta}w_b(b_i) - c.$$

It can easily be shown that $Var(\varepsilon) = (Var(\delta) - Var(\hat{\delta}))/Var(\hat{\delta})$. Hence, the variance of the remaining uncertainty is decreasing with $Var(\hat{\delta})$, which measures the precision of $\hat{\delta}$.

In subsequent analysis, determining the variance of the estimators plays a central role. We will assume that V satisfies the following condition:

Assumption 1 *The function V is strictly increasing, strictly concave, and three-times continuously differentiable.*

The condition that V is increasing (i.e., $V' > 0$) implies that as the agent thinks more she gets a more precise idea about δ (i.e., with lower noise, measured by the variance of the error term ε). The concavity of V (i.e., $V'' < 0$) ensures that the first order condition is sufficient for optimization, and more importantly, is consistent with the agent having an incentive to learn only part of the information about δ , leaving the variance $Var(\varepsilon)$ of the noise term positive when making her decision. Since $V'' < 0$, the first derivative V' has an inverse, which will be denoted by $h(\cdot)$.

2.1.1 Subsequent Effort and the Updating of Estimators

In order to be able to trace the impact of a previous decision regarding the two alternatives on a subsequent decision, we need to specify how the agent would incorporate information from the former into the latter. Through initial effort c_0 in thinking about the relative importance of attribute b , the agent obtains an estimate $\hat{\delta}_0$ of δ (with $\delta = \hat{\delta}_0 \varepsilon_0$). Given this estimate, if the agent then expends c_1 units thinking about the residual uncertainty ε_0 , she gets an estimate $\hat{\delta}_1$ such that

$$\delta = \hat{\delta}_0 \hat{\delta}_1 \varepsilon_1,$$

where (again) the estimators $\hat{\delta}_0$, $\hat{\delta}_1$, and ε_1 are stochastically independent, and $\hat{\delta}_1$ has CDF $F(\cdot; c_1)$. (Here c_1 is constrained so that the variance of the error term is not rendered negative). Note that $E[\hat{\delta}_0] = E[\hat{\delta}_1] = 1$, hence $E[\varepsilon_0] = E[\varepsilon_1] = 1$, yielding unbiased estimators. Note also that $Var(\hat{\delta}_0) = V(c_0)$ and $Var(\hat{\delta}_1) = V(c_1)$.

Using the model described above, we will analyze how our agent's previous rankings will affect her subsequent valuation of goods. Before this, we present a canonical example to illustrate how information is processed in accordance with the model.

2.1.2 A Canonical Example

Consider the stochastic process $D_t = e^{Z_t}$ ($t \in [0, 1]$), where Z_t is given by the stochastic differential equation

$$dZ_t = -\frac{1}{2}\sigma^2 dt + \sigma dB_t, \quad Z_0 = 0$$

and B_t is a standard Brownian motion. Thus defined, D_t is a martingale, which implies that the expected future change in D is nil, and $E(D_t) = 1$ for any t . D_t can be thought of as a random walk the agent goes through in her mind to obtain relevant information for the task at hand. In this context, t should be thought of as an abstract point along the continuum of information she can potentially consider. We can now express δ as D_1 so that $E[\delta] = 1$ and $\log \delta \sim N(-\frac{1}{2}\sigma^2, \sigma^2)$. That is, δ can be completely determined by the aggregate effect of *all* associations, which are assumed to be stochastically independent of each other. Upon thinking c units, she learns incremental information in an interval $[t_0, t_c]$ of length $(t_c - t_0) = \alpha \log(1 + c)$; the resulting estimate is given by $\hat{\delta} = \frac{D_{t_c}}{D_{t_0}}$, a log-normal random variable with mean $E(\hat{\delta}) = 1$ and variance

$$Var(\hat{\delta}) = V(c) = \exp[\alpha \log(1 + c) \sigma^2] - 1 = (1 + c)^{\alpha \sigma^2} - 1.$$

For $(\alpha \sigma^2) < 1$, V satisfies Assumption 1. The error term $\varepsilon = \delta / \hat{\delta}$ is also a log-normal random variable (with mean $E(\varepsilon) = 1$ and variance $Var(\varepsilon) = \exp[(1 - \alpha \log(1 + c))\sigma^2] - 1$).

$1 = \exp[\sigma^2](1+c)^{-\alpha\sigma^2} - 1$. Note that in this example, $\bar{c} = \exp(1/\alpha) - 1$, so that by thinking \bar{c} units, the agent would learn all the information relevant for determining δ .

In the above example, the support of the estimators was unbounded. For simplicity, and without qualitatively affecting any of our findings, we will assume in what follows that estimators are uniformly bounded:

Assumption 2 *There exists some $\bar{D} > 0$ such that $F(\bar{D}, c) = 1$ for each $c \in [0, \bar{c}]$.*

2.2 Ranking

We now describe how the agent ranks goods X and Y in order of preference. We explicitly define the agent's optimization problem and present a basic comparative static about the choice of introspection length.

When asked to rank goods X and Y in terms of which good she would prefer to receive, the agent first decides on the length c_r of her thought process. Upon thinking c_r units, she obtains an estimate $\hat{\delta}_r$, and based on this estimate makes a preference ordering of goods X and Y . It is clear that she prefers X if and only if $E[u(X)|\hat{\delta}_r] \geq E[u(Y)|\hat{\delta}_r]$, i.e., $\hat{\delta}_r \leq \rho$. Therefore, her expected utility from choosing c_r is

$$U_r(c_r) = E[w_a(a_X) + \delta w_b(b_X) : \hat{\delta}_r \leq \rho] + E[w_a(a_Y) + \delta w_b(b_Y) : \hat{\delta}_r \geq \rho] - c_r. \quad (2)$$

Note that the expectation operator, E , depends on c_r (through $F(\cdot; c_r)$). We compute that

$$U_r(c_r) = \Delta_b E[\rho - \hat{\delta}_r : \hat{\delta}_r \leq \rho] + E[u(Y)] - c_r. \quad (3)$$

Using (3), we obtain the following proposition, which states a basic comparative static.

Proposition 1 *Assume that F is continuously differentiable, $\frac{\partial F(d;c)}{\partial c} \geq 0$ at each $d < 1$, and $\frac{\partial F(d;c)}{\partial c} \leq 0$ at each $d > 1$. Then,*

1. $\hat{c}_r \equiv \arg \max_{c \in [0, \bar{c}]} U_r(c; \rho)$ decreases with $|\rho - 1|$, and
2. \hat{c}_r increases whenever both w_a and w_b are multiplied by a constant $\lambda > 1$.

The condition assumed in the proposition states that as the agent thinks longer, she obtains a more precise estimator in the sense of second order stochastic dominance. (This is somewhat stronger than the condition that V is increasing.) Under this condition, the optimal choice of introspection length before the ranking decision decreases with $|\rho - 1|$,

which measures the degree by which one good *ex ante* dominates the other.⁶ In addition, all else equal, introspection length increases when the stakes involved (i.e., the unit contribution of each attribute to overall utility) are higher. The proof of this proposition is relegated to the Appendix.

In the next two subsections, we analyze our agent's behavior when confronted with valuation decisions. To simplify our expressions and ensure an interior solution for the effort expended, we will assume two additional regularity conditions. The first condition is:

$$V'(0) > 1/(A(E[\delta|\hat{\delta}_r \leq \rho])^2), \quad (4)$$

where A is a constant (that will be defined later). This condition guarantees that the agent will always think some positive amount in a subsequent valuation task, even when she knows the outcome from a previous ranking of the two alternative goods. The second condition is:

$$V'(c) < 1/(A(E[\delta|\hat{\delta}_r > \rho])^2) \quad (5)$$

where c is given by $V(c) = Var(\varepsilon_r) = \left(Var(\delta) - Var(\hat{\delta}_r)\right) / \left(1 + Var(\hat{\delta}_r)\right)$. This condition guarantees that some uncertainty will remain unresolved even after the valuation decision.

2.3 Valuation

We now analyze how the agent decides on the value of X and Y , when required to provide the highest amount that she would be willing to pay for each of the goods. In a valuation decision, the agent must again first decide on the length of thought, c_v , obtain an estimate $\hat{\delta}_v$ for the relative importance of attribute b , and finally determine estimates \hat{u}_X and \hat{u}_Y for $u(X)$ and $u(Y)$, respectively. Given that thinking is costly, the exact amount of effort to be expended prior to making a decision depends on how the valuation estimates will be used to determine the agent's payoffs. In this paper, we consider the following mechanism, proposed by Becker et al. (1964). The agent provides estimates \hat{u}_X and \hat{u}_Y for goods X and Y , respectively. One of the goods (say good X) is then selected with some known probability π .⁷ Subsequently, we draw a monetary prize (say p_X) from a uniform distribution on the interval $[0, m]$ for some large integer m . If the estimate for

⁶ For instance, let $\rho > 1$, in which case *ex ante* X is better than Y . The assumption $\frac{\partial F(d;c)}{\partial c} < 0$, leads to $\frac{\partial^2 V_r}{\partial c_r \partial \rho} < 0$. By Milgrom and Roberts (1994), this shows that the optimal choice of introspection length decreases as ρ increases.

⁷That is, each good is selected with probability π ; and the events that X is selected and that Y is selected are mutually exclusive.

the good is higher than the monetary prize ($\hat{u}_X \geq p_X$), the agent receives the good (X); otherwise, she receives the monetary prize (p_X). Consistent with Assumption 2, we take

$$m \geq \max \{w_a(a_X) + \bar{D}w_b(b_X), w_a(a_Y) + \bar{D}w_b(b_Y)\} \quad (6)$$

so that $m \geq \max \{\hat{u}_X, \hat{u}_Y\}$ with probability 1. Under this mechanism, truth-telling is a dominant strategy. That is, the agent submits $\hat{u}_X = w_a(a_X) + \hat{\delta}_v w_b(b_X)$ and $\hat{u}_Y = w_a(a_Y) + \hat{\delta}_v w_b(b_Y)$ where $\hat{\delta}_v$ is the expectation of δ conditional on all the information the agent has by the end of the valuation phase.

Note that in reality when the agent faces a price p_X for good X , she buys the good (i.e., pays $p(X)$) if and only if $\hat{u}_X \geq p_X$. In that case, she would get what she gets under our mechanism minus p_X . Thus, her payoffs will be the same as here minus a constant, reflecting the same preferences.

Our next proposition states that the agent maximizes the variance of her estimator, multiplied by a constant that is determined by the specifics of the mechanism, minus the cost of thinking. We will write $U_v(c_v)$ for the agent's expected utility when she thinks c_v units and submits valuations \hat{u}_X and \hat{u}_Y (as defined above), to be followed by the mechanism described above to determine her payoff.

Proposition 2 *Under Assumption 2, take m as in (6). Then, given any $c_v \in [0, \bar{c}]$, we have*

$$U_v(c_v) = AVar(\hat{\delta}_v) + B - c_v = AV(c_v) + B - c_v,$$

where

$$\begin{aligned} A &= \frac{\pi}{2m} [w_b^2(b_X) + w_b^2(b_Y)], \\ B &= \frac{\pi}{2m} [E[u^2(X)] + E[u^2(Y)]] + \pi m - AVar(\delta), \end{aligned}$$

and $\hat{\delta}_v$ is the estimator she obtains upon thinking c_v units.

The proof is relegated to the Appendix. From Proposition 2 we have,

$$U'_v(c_v) = AV'(c_v) - 1. \quad (7)$$

Since V is increasing, it follows that the optimal length of thought in valuation (prior to providing utility estimates), $\hat{c}_v = \arg \max_{c \in [0, \bar{c}]} U_v(c; A)$, is increasing with A . The other parameters affect \hat{c}_v only through A . Note that A is increasing with π (i.e., the probability that the estimate will be used for a given good), with $w_b^2(b_X)$ and $w_b^2(b_Y)$

(i.e., the coefficients that translate the variance of $\hat{\delta}_v$ to the variances of \hat{u}_X and \hat{u}_Y , respectively) and with $1/m$ (i.e., the probability that the price will be in an interval of unit length, measuring the need for precision).

Finally, since V is concave, using the first order condition (7), we obtain

$$\hat{c}_v \equiv \arg \max_{c \in [0, \bar{c}]} U_v(c) = h(1/A), \quad (8)$$

where $h = (V')^{-1}$. From conditions (4) and (5) it is evident that $\hat{c}_v \in (0, \bar{c})$.

2.4 Ranking and Valuation

We now describe how the agent determines the value of goods *that she has previously ranked*. Our boundedly rational agent knows which good she prefers but does not remember the details of her introspection. In other words, by the nature of the ranking decision she knows which good is preferred and the optimal amount of effort she must have expended to rank the alternatives (c_r), but would need to go through the same introspection (expending the same effort again) to recapture the pertinent details regarding the relative importance of attribute b .

During her ranking decision, the agent obtained an estimator $\hat{\delta}_r$ with

$$\delta = \hat{\delta}_r \varepsilon_r, \quad (9)$$

where $\hat{\delta}_r$ and ε_r are stochastically independent, and $\hat{\delta}_r$ has CDF $F(\cdot; c_r)$ with $c_r = \arg \max_{c \in [0, \bar{c}]} U_r$ (as explained in §2.2). Let us first examine the case whereby the agent knows that she has ranked X at least as high as Y . This implies she must have found $\hat{\delta}_r \leq \rho$. Thus, while the agent has some information on $\hat{\delta}_r$, she has no information about ε_r (the residual error associated with $\hat{\delta}_r$). Given the concavity of V and the fact that the cost structure for resolving uncertainty regarding ε_r and $\hat{\delta}_r$ is the same, in any decision task subsequent to ranking, it is optimal for the agent to think about ε_r . Furthermore, our regularity condition (5) guarantees that the variance of ε_r is sufficiently high so that the agent will choose to resolve only part of the uncertainty in ε_r . Hence, based on 2.1.1, subsequent introspection for purposes of valuation yields an estimator of ε_r that is independent of $\hat{\delta}_r$ and satisfies

$$\delta = \hat{\delta}_r \hat{\delta}_{\varepsilon_r} \varepsilon_{rv}, \quad (10)$$

where $\hat{\delta}_r$, $\hat{\delta}_{\varepsilon_r}$, and ε_{rv} are stochastically independent and $\hat{\delta}_{\varepsilon_r}$ is the estimator of ε_r , obtained through an introspection of length c_{rv} . Her new estimator for δ is $\hat{\delta}_{rv} = E[\delta | \hat{\delta}_r \leq \rho, \hat{\delta}_{\varepsilon_r}] = \hat{\delta}_{\varepsilon_r} E[\delta | \hat{\delta}_r \leq \rho]$.

Similar to the proof of Proposition 2, we compute the expected utility from thinking c_{rv} units, given the ranking information, to be

$$U_{rv}(c_{rv}|\hat{\delta}_r \leq \rho) = AVar(\hat{\delta}_{rv}|\hat{\delta}_r \leq \rho) + B_{[\hat{\delta}_r \leq \rho]} - c_{rv},$$

where $B_{[\hat{\delta}_r \leq \rho]}$ is the same as B defined in Proposition 2, except that the expectations and variances are conditional on $\{\hat{\delta}_r \leq \rho\}$. Since $Var(\hat{\delta}_{rv}|\hat{\delta}_r \leq \rho) = (E[\delta|\hat{\delta}_r \leq \rho])^2V(c_{rv})$,⁸ this becomes

$$U_{rv}(c_{rv}|\hat{\delta}_r \leq \rho) = A(E[\delta|\hat{\delta}_r \leq \rho])^2V(c_{rv}) + B_{[\hat{\delta}_r \leq \rho]} - c_{rv}. \quad (11)$$

Since $(E[\delta|\hat{\delta}_r \leq \rho])^2 \leq 1$, we must have

$$\hat{c}_{rv}[\hat{\delta}_r \leq \rho] \equiv \arg \max_{c \in [0, \bar{c}]} U_{rv}(c_{rv}|\hat{\delta}_r \leq \rho) \leq \arg \max_{c \in [0, \bar{c}]} U_{v_0}(c_{v_0}) \equiv \hat{c}_{v_0}, \quad (12)$$

where c_{v_0} and $U_{v_0}(c_{v_0})$ are introspection effort and expected utility of valuation without previous ranking. The implication of the inequality (12) is as follows: knowing that she has ranked X at least as high as Y , the agent infers that she must have found attribute b not so important. Thus she chooses to think less in resolving the remaining uncertainty associated with its relative weight. Together with (4) and (5), (11) implies that

$$c_{rv}[\hat{\delta}_r \leq \rho] = h(1/(A(E[\delta|\hat{\delta}_r \leq \rho])^2)). \quad (13)$$

In similar fashion, if the agent were to know she previously ranked good Y higher than good X , then she would infer that she must have found $\hat{\delta}_r > \rho$, and the utility from introspection of length c_{rv} would be

$$U_{rv}(c_{rv}|\hat{\delta}_r > \rho) = A(E[\delta|\hat{\delta}_r > \rho])^2V(c_{rv}) + B_{[\hat{\delta}_r > \rho]} - c_{rv}, \quad (14)$$

where $B_{[\hat{\delta}_r > \rho]}$ is the same as B in Proposition 2, except that the expectations and variances are now conditional on $\{\hat{\delta}_r > \rho\}$. In this case, the agent chooses to expend thinking effort

$$c_{rv}[\hat{\delta}_r > \rho] = h(1/(A(E[\delta|\hat{\delta}_r > \rho])^2)). \quad (15)$$

Analogously to (12), it is straightforward to establish that $c_{rv}[\hat{\delta}_r > \rho] \geq c_{v_0}$. This is because the agent now knows she must have found attribute b relatively important,

⁸Note that $Var(\hat{\delta}_{rv}|\hat{\delta}_r \leq \rho) = Var(\hat{\delta}_{\epsilon_r}E[\delta|\hat{\delta}_r \leq \rho]|\hat{\delta}_r \leq \rho) = (E[\delta|\hat{\delta}_r \leq \rho])^2Var(\hat{\delta}_{\epsilon_r}|\hat{\delta}_r \leq \rho) = (E[\delta|\hat{\delta}_r \leq \rho])^2V(c_{rv})$, where the penultimate equality is due to the fact that $\hat{\delta}_r$ and $\hat{\delta}_{\epsilon_r}$ are stochastically independent.

and has a greater incentive to expend effort to make a more informed decision. In the Appendix we show that $0 < c_{rv} < \bar{c}$ and $V(0) < V(c_{rv}) < Var(\varepsilon)$. Hence the optimal effort is positive, yet some uncertainty still remains unresolved after the valuation phase (satisfying the regularity conditions (4) and (5)).

Clearly, the presence of ranking information impacts the optimal effort in the valuation stage. What is the direction of this impact in expectation? The *ex ante* expected effort with ranking information is:

$$E[c_{rv}] = \Pr(\hat{\delta}_r \leq \rho) h(1/(A(E[\delta|\hat{\delta}_r \leq \rho])^2)) + \Pr(\hat{\delta}_r > \rho) h(1/(A(E[\delta|\hat{\delta}_r > \rho])^2)).$$

Since $\Pr(\hat{\delta}_r \leq \rho) E[\delta|\hat{\delta}_r \leq \rho] + \Pr(\hat{\delta}_r > \rho) E[\delta|\hat{\delta}_r > \rho] = E[\delta] = 1$ and $\hat{c}_{v0} = h(1/A)$, we have the following comparative statics result:

Proposition 3 *Under Assumption 2 and conditions (4) and (5), $E[c_{rv}] \geq \hat{c}_{v0}$ (resp., $E[c_{rv}] \leq \hat{c}_{v0}$) whenever $h(1/x^2)$ is a convex (resp., concave) function of x .*

In words, Proposition 3 implies that rank ordering a set of goods makes the agent expend more effort (in expectation) in subsequently generating valuations for these same goods, compared to the case when no ranking has taken place and provided that $h(1/x^2)$ is convex. Note that $h(1/x^2)$ is a convex function of x if and only if $-\frac{V''(h(1/x^2))}{V'(h(1/x^2))}x$ is a non-increasing function of x . As we will discuss later, $-V''/V'$ is typically decreasing in x ; we need it to decline faster than $1/x$ for convexity. In the canonical case described in §2.1.2, $h(1/x^2)$ is indeed a convex function of x ,⁹ hence $E[c_{rv}] \geq \hat{c}_{v0}$ and the presence of ranking information *increases* the optimal effort during the valuation stage (in expectation). The proposition reflects the fact that when $h(1/x^2)$ is convex, the greater returns to effort when attribute b is found important (i.e., $\hat{\delta}_r > \rho$) overshadow the lowered incentives to expend effort when $\hat{\delta}_r \leq \rho$.

2.5 How Ranking Affects Subsequent Valuation

Having established how the agent incorporates her knowledge of previous rankings when selecting the optimal effort to allocate in the valuation decision, we now turn to examine how the valuations themselves are impacted by such a sequence of decisions. To do so, we will first establish when the findings from introspection in the valuation phase leave the findings from the ranking phase intact, and alternatively, when they can lead to

⁹If $V(c) = c^\gamma$ for some $\gamma \in (0, 1)$, $V'(c) = \gamma c^{\gamma-1}$ and $h(z) = (z/\gamma)^{1/(\gamma-1)}$. Hence, $h(1/x^2) = \gamma^{1/(1-\gamma)}x^{2/(1-\gamma)}$, is indeed a convex function of x .

a preference reversal (i.e., when the good found to have a higher value was previously ranked lower). Then, confining ourselves to a very large class of canonical functions, we will show that knowledge of the previous ranking decision leads to more accurate valuations, as measured by the variance of the final estimator for δ . This occurs not only because of the extra information embodied in the rankings, but also because the agent is induced to think more when the information is perceived to be more valuable. Using this result, we will show that knowledge of a previous ranking decision increases the spread of valuations, as measured by the mean squared difference between the utility estimators for the goods, and that this effect becomes more pronounced as the stakes involved increase, as measured by the contribution of each attribute to overall utility.

Using the relationship in (10), we write

$$\hat{\delta}_{rv} = \begin{cases} E[\delta|\hat{\delta}_r \leq \rho] \hat{\delta}_{\varepsilon_r}[\hat{\delta}_r \leq \rho] & \text{if } \hat{\delta}_r \leq \rho \\ E[\delta|\hat{\delta}_r > \rho] \hat{\delta}_{\varepsilon_r}[\hat{\delta}_r > \rho] & \text{if } \hat{\delta}_r > \rho \end{cases} \quad (16)$$

for the estimator for δ when the agent needs to provide monetary values for the goods that she has already ranked, and where $\hat{\delta}_{\varepsilon_r}[\hat{\delta}_r \leq \rho]$ and $\hat{\delta}_{\varepsilon_r}[\hat{\delta}_r > \rho]$ have CDFs $F(\cdot; c_{rv}[\hat{\delta}_r \leq \rho])$ and $F(\cdot; c_{rv}[\hat{\delta}_r > \rho])$, respectively. We also write $\hat{u}_X^{rv} = w_a(a_X) + \hat{\delta}_{rv} w_b(b_X)$ and $\hat{u}_Y^{rv} = w_a(a_Y) + \hat{\delta}_{rv} w_b(b_Y)$ when the agent incorporates ranking information in her estimators for the values of $u(X)$ and $u(Y)$. Similarly, we write $\hat{u}_X^{v_0}$, $\hat{u}_Y^{v_0}$ (with estimator $\hat{\delta}_{v_0}$) when no previous ranking information is available. It will also prove useful in subsequent derivations to define $\hat{u}_X^{\varepsilon_r}$, $\hat{u}_Y^{\varepsilon_r}$ as the agent's valuation estimates if she were to use the estimator $\hat{\delta}_{\varepsilon_r}$, i.e., when she (hypothetically) ignores the ranking decision outcome (see §2.4). Note that we have $\hat{u}_X^{rv} - \hat{u}_Y^{rv} = \Delta_b(\rho - \hat{\delta}_{rv})$, $\hat{u}_X^{\varepsilon_r} - \hat{u}_Y^{\varepsilon_r} = \Delta_b(\rho - \hat{\delta}_{\varepsilon_r})$, and $\hat{u}_X^{v_0} - \hat{u}_Y^{v_0} = \Delta_b(\rho - \hat{\delta}_{v_0})$.¹⁰

For purpose of exposition, let us focus on the case where the agent knows she has ranked good X as least as high as good Y . When introspection in the valuation phase yields $\hat{\delta}_{\varepsilon_r} \leq \rho$, the new findings are aligned with the ranking information, hence the agent clearly keeps valuing X higher than Y . When $\rho < \hat{\delta}_{\varepsilon_r} \leq \rho/E[\hat{\delta}_r|\hat{\delta}_r \leq \rho]$, the new findings no longer favor X . But, since the difference is small, the agent still gives X a higher value than Y , rendering the previous ranking intact. When the result of introspection is $\hat{\delta}_{\varepsilon_r} > \rho/E[\hat{\delta}_r|\hat{\delta}_r \leq \rho]$, the new findings strongly favor Y , and the agent in effect reverses her rankings by giving Y a higher value than X (albeit with a decreased difference due to the ranking information). In this last case, we observe a preference reversal between the two decision tasks.

¹⁰To see why, take for example $\hat{u}_X^{rv} - \hat{u}_Y^{rv}$. One can easily establish that $\hat{u}_X^{rv} - \hat{u}_Y^{rv} = [w_a(a_x) + \hat{\delta}_{rv} w_b(b_x)] - [w_a(a_y) + \hat{\delta}_{rv} w_b(b_y)] = (w_a(a_x) - w_a(a_y)) - \hat{\delta}_{rv} (w_b(b_y) - w_b(b_x)) = \Delta_b(\rho - \hat{\delta}_{rv})$.

It is also worth pointing out that when the agent knows she has ranked X higher than Y (i.e., $\hat{\delta}_r \leq \rho$), she incorporates the ranking information into her new findings by multiplying $\hat{\delta}_{\varepsilon_r}$ with $E[\hat{\delta}_r | \hat{\delta}_r \leq \rho] \leq 1$ (see (16)). By doing this, she *lowers her valuations for both X and Y* , i.e., $\hat{u}_X^{r_v} \leq \hat{u}_X^{r_r}$ and $\hat{u}_Y^{r_v} \leq \hat{u}_Y^{r_r}$.¹¹ The reverse is true when $\hat{\delta}_r > \rho$.

2.5.1 The Impact of Rankings on the Variance of the Estimators and the Likelihood of Purchase

In a valuation decision, the variance of the estimator measures how informed the agent is when she provides her estimates. Thus, it is critical to understand how information from previous rankings affects the variance of the estimator for δ . In this discussion, we will focus on the case when $-V''/V'$ is non-increasing.¹²

To express the variances of estimators for δ , with CDF $F(\cdot; h(\frac{1}{Ax^2}))$, one can define a function $\Psi: \mathcal{R}_+ \rightarrow \mathcal{R}$ by

$$\Psi(x) = x^2 V \left(h \left(\frac{1}{Ax^2} \right) \right), \forall x > 0. \quad (17)$$

When no prior ranking has taken place, we have $E[\delta] = 1$ (before valuation) and by (8) the agent expends effort $\hat{c}_{v_0} = h(1/A)$ in the valuation phase. This results in estimator $\hat{\delta}_{v_0}$ with variance $Var(\hat{\delta}_{v_0}) = \Psi(1)$. Analogously, if prior ranking has taken place we have $\Psi(E[\delta | \hat{\delta}_r \leq \rho]) = Var(\hat{\delta}_{rv} | \hat{\delta}_r \leq \rho)$ when good X is ranked higher than good Y , and $\Psi(E[\delta | \hat{\delta}_r > \rho]) = Var(\hat{\delta}_{rv} | \hat{\delta}_r > \rho)$ otherwise. The following Lemma describes the shape of Ψ under the assumptions of our model, and sheds light on how information from a ranking decision affects the variance of the final estimator for δ .

Lemma 1 *Under Assumption 1, Ψ is increasing. Moreover, Ψ is convex whenever $-V''/V'$ is non-increasing.*

Based on the above lemma, previous ranking affects the variance of valuation estimators as follows. When $\hat{\delta}_r \leq \rho$, ranking leads the agent to think less compared to

¹¹One might think that the information that X has been ranked higher than Y would lead her to increase her estimate for X . This is not true. The answer depends on the parameters as well as the source of uncertainty. In our case, when the agent knows the value of attribute a , but is uncertain about the relative value of attribute b , it will lead her to decrease her estimate.

¹²When V is a utility function, $-V''/V'$ measures the absolute risk aversion. Canonically, the absolute risk aversion ($-V''/V'$) is non-increasing. In Economics, much work has focused on the family of utility functions with constant (CARA) and decreasing (DARA) absolute risk aversion, such as $1 - e^{-\alpha c}$, $\log(1 + c)$, and c^μ , which satisfy the above property.

the case when no ranking information is available. Given that the agent obtains less additional information in the valuation phase, her ultimate decision is in a sense *less informed*, i.e., $Var(\hat{\delta}_{rv}|\hat{\delta}_r \leq \rho) = \Psi(E[\hat{\delta}_r|\hat{\delta}_r \leq \rho]) \leq \Psi(1) = Var(\hat{\delta}_{v_0})$. When $\hat{\delta}_r > \rho$, the agent is prompted to think more, and in this case $Var(\hat{\delta}_{rv}|\hat{\delta}_r \geq \rho) \geq Var(\hat{\delta}_{v_0})$. Recall also that ranking information is incorporated into $\hat{\delta}_{rv}$ according to (16). Which of these effects dominates is determined by the shape of Ψ . Since Ψ is convex, *ex anté*, ranking information will lead to a more informed decision, as stated by our next Proposition.

Proposition 4 *Under Assumptions 1 and 2 and the regularity conditions (4), (5), and (6), we have*

$$Var(\hat{\delta}_{rv}) \geq Var(\hat{\delta}_{v_0}) \quad (18)$$

whenever $-V''/V'$ is non-increasing.

Proof. Under our hypothesis, the conditional variances are determined by Ψ , and by Lemma 1, Ψ is convex whenever $-V''/V'$ is non-increasing. In that case, we have

$$\begin{aligned} Var(\hat{\delta}_{rv}) &= \Pr(\hat{\delta}_r \leq \rho)Var(\hat{\delta}_{rv}|\hat{\delta}_r \leq \rho) + \Pr(\hat{\delta}_r > \rho)Var(\hat{\delta}_{rv}|\hat{\delta}_r > \rho) \quad (19) \\ &= \Pr(\hat{\delta}_r \leq \rho)\Psi(E[\hat{\delta}_r|\hat{\delta}_r \leq \rho]) + \Pr(\hat{\delta}_r > \rho)\Psi(E[\hat{\delta}_r|\hat{\delta}_r > \rho]) \\ &\geq \Psi\left(\Pr(\hat{\delta}_r \leq \rho)E[\hat{\delta}_r|\hat{\delta}_r \leq \rho] + \Pr(\hat{\delta}_r > \rho)E[\hat{\delta}_r|\hat{\delta}_r > \rho]\right) \\ &= \Psi(1) = Var(\hat{\delta}_{v_0}). \end{aligned}$$

The inequality holds due to the facts that Ψ is convex and that $\Pr(\hat{\delta}_r > \rho) = 1 - \Pr(\hat{\delta}_r \leq \rho)$; the penultimate equality is due to the fact that $E[\hat{\delta}_r] = 1$. ■

Proposition 4 establishes that, *ex anté*, prior rankings lead to more informed decisions. As we discussed earlier, this is not only because of the information contained in the rankings but also because such information leads the agent to think longer when the decision is more important.

We now turn to examine the implications of Proposition 4 for the *ex anté* likelihood of the agent purchasing a good. Consider good X , with price p_X . Given an estimate $\hat{\delta}$, the agent is willing to buy X if and only if $\hat{u}_X = w_a(a_X) + \hat{\delta}w_b(b_X) \geq p_X$, i.e., $\hat{\delta} \geq q \equiv (p_X - w_a(a_X))/w_b(b_X)$. Let us write x for the conditional expectation of δ prior to the valuation decision (before expending cognitive effort). Then $x = 1$ when there is no prior ranking, $x = E[\hat{\delta}_r|\hat{\delta}_r \leq \rho]$ when the agent knows that he ranked X higher than Y , and $x = E[\hat{\delta}_r|\hat{\delta}_r \geq \rho]$ otherwise. The probability that the agent is willing to buy the good is

$$\Pi(x; q) = 1 - F\left(\frac{q}{x}; h\left(\frac{1}{Ax^2}\right)\right).$$

We will focus on the canonical example described in §2.1.2. In that example, we have

$$\Pi(x; q) = 1 - \Phi\left(\frac{\log(q/x)}{\sqrt{v}} + \sqrt{v}/2\right),$$

where Φ is the standard normal CDF, $v = \frac{\gamma}{1-\gamma} \log(\gamma Ax^2)$ is the variance, and $\gamma = \alpha\sigma^2$. The behavior of Π depends on whether γ is greater than or less than $1/2$. When γ is large, the function V is approximately linear. With non-decreasing marginal returns to effort, the agent has an incentive to invest in cognitive effort until virtually all uncertainty is resolved (if she invests at all). As it is highly likely we will end up in a corner solution when $\gamma > 1/2$, we believe our theory is more relevant for $\gamma < 1/2$ and focus on this case.

To understand the impact of prior ranking on the probability of sale occurring, we examine how $\Pi(x; q)$ changes as x increases (in the region $\gamma < 1/2$). On the one hand, q/x decreases, causing $\Pi(x; q)$ to increase. On the other hand, the variance v also increases as x increases, as the agent now thinks longer. From the properties of the log function, this may initially increase $\Pi(x; q)$ (provided $q > x$) but will eventually decrease $\Pi(x; q)$ to 0 because $\hat{\delta}$ converges to 0 in probability as its variance goes to infinity. When $\gamma < 1/2$ the latter effect from increasing variance will be small. In fact, it is easy to verify that

$$\lim_{x \rightarrow \infty} \Pi(x; q) = \begin{cases} 0 & \text{if } \gamma > 1/2 \\ 1/2 & \text{if } \gamma = 1/2 \\ 1 & \text{if } \gamma < 1/2 \end{cases}$$

for each q . We also check that $\Pi(x; q)$ is increasing if and only if

$$q \geq (\gamma A)^{\frac{3\gamma-2}{2(1-\gamma)}} x^{\frac{2\gamma-1}{1-\gamma}}.$$

Hence, as plotted in Figure 1, when $\gamma < 1/2$, $\Pi(\cdot; q)$ is a U-shaped function minimized at $x = q^{\frac{1-\gamma}{1-2\gamma}} / (\gamma A)^{\frac{3\gamma-2}{2(1-2\gamma)}}$ and approaches 1 as $x \rightarrow \infty$. Since $\gamma Ax^2 \geq 1$, $\Pi(\cdot; q)$ is an increasing function in the allowable region whenever $q \geq (\gamma A)^{(5\gamma-3)/2}$.

What is the impact of prior rankings on the *ex ante* probability of sale taking place? Recall that x can take the values 1, $E[\hat{\delta}_r | \hat{\delta}_r \leq \rho]$, and $E[\hat{\delta}_r | \hat{\delta}_r > \rho]$. Therefore, the *ex ante* probability of sale is higher with ranking if and only if

$$\Pr(\hat{\delta}_r \leq \rho) \Pi(E[\hat{\delta}_r | \hat{\delta}_r \leq \rho]; q) + \Pr(\hat{\delta}_r > \rho) \Pi(E[\hat{\delta}_r | \hat{\delta}_r > \rho]; q) > \Pi(1; q). \quad (20)$$

Since

$$\Pr(\hat{\delta}_r \leq \rho) E[\hat{\delta}_r | \hat{\delta}_r \leq \rho] + \Pr(\hat{\delta}_r > \rho) E[\hat{\delta}_r | \hat{\delta}_r > \rho] = 1,$$

(20) holds if and only if $\Pi(\cdot; q)$ is convex with respect to these three points ($E[\hat{\delta}_r | \hat{\delta}_r \leq \rho]$, $E[\hat{\delta}_r | \hat{\delta}_r > \rho]$, and 1). When $\Pi(\cdot; q)$ is convex (resp., concave) with respect to these

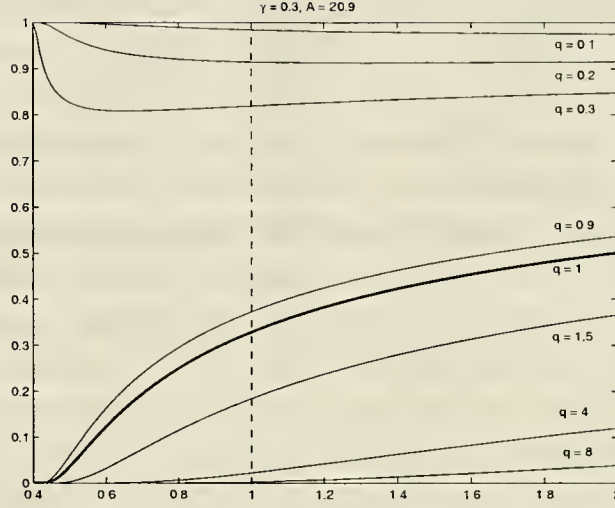


Figure 1: Π as a function of x for $\gamma < 0.5$. ($A = 1/(0.16\gamma)$).

three points, prior rankings increase (resp., decrease) the *ex ante* probability of sale. In particular, assuming that both $E[\hat{\delta}_r | \hat{\delta}_r \leq \rho]$ and $E[\hat{\delta}_r | \hat{\delta}_r > \rho]$ are close to 1, we need to check whether $\Pi(\cdot; q)$ is convex or concave around 1. We refer to Figure 1. First, consider the case that the price (and hence q) is very small (e.g., when $q \leq 0.3$ in the figure). In that case, $\Pi(\cdot; q)$ is convex at $x = 1$, hence prior rankings *increase* the *ex ante* probability of sale. When the price of the good is around the *ex ante* value of the good (e.g., when $0.9 < q < 1.5$ in the figure), $\Pi(\cdot; q)$ is concave at $x = 1$, hence prior rankings *decrease* the *ex ante* probability of sale. When the price is very high (e.g., when $q \geq 4$ in the figure), $\Pi(\cdot; q)$ is again convex at $x = 1$, hence prior rankings again *increase* the *ex ante* probability of sale. In summation, prior ranking tends to increase the probability of a sale occurring at extreme prices (that are relatively high or low) in our canonical example.

2.5.2 The Impact of Rankings on the Dispersion of Valuations

Having established in Proposition 4 that prior rankings increase the variance of the estimator for δ , we can derive a testable hypothesis about the spread of valuations submitted for the two goods. We measure the spread by the mean squared difference between the valuations for the two goods. Hence, $E[(\hat{u}_X^{rv} - \hat{u}_Y^{rv})^2]$ and $E[(\hat{u}_X^{v0} - \hat{u}_Y^{v0})^2]$ are the spreads of the valuations with and without information from a previous ranking decision, respec-

tively. Since $\hat{u}_X^{rv} - \hat{u}_Y^{rv} = \Delta_b(\rho - \hat{\delta}_{rv})$ and $\hat{u}_X^{v_0} - \hat{u}_Y^{v_0} = \Delta_b(\rho - \hat{\delta}_{v_0})$, we have

$$E [(\hat{u}_X^{rv} - \hat{u}_Y^{rv})^2] = \Delta_b^2 E[(\rho - \hat{\delta}_{rv})^2] = \Delta_b^2 \left[\text{Var}(\hat{\delta}_{rv}) + (\rho - 1)^2 \right]$$

and

$$E [(\hat{u}_X^{v_0} - \hat{u}_Y^{v_0})^2] = \Delta_b^2 E[(\rho - \hat{\delta}_{v_0})^2] = \Delta_b^2 \left[\text{Var}(\hat{\delta}_{v_0}) + (\rho - 1)^2 \right].$$

The following proposition is an immediate corollary to Proposition 4. It states that under our usual regularity conditions, knowledge of previous rankings increases the *ex ante* spread between the maximum amounts the agent is willing to pay for the two goods.

Proposition 5 *Under Assumptions 1 and 2, and the regularity conditions (4), (5), and (6), we have*

$$E [(\hat{u}_X^{rv} - \hat{u}_Y^{rv})^2] \geq E [(\hat{u}_X^{v_0} - \hat{u}_Y^{v_0})^2] \quad (21)$$

whenever $-V''/V'$ is non-increasing.

The above proposition establishes that by first ranking a set of goods, the subsequent valuation spread between them is higher than if valuation alone were to be performed. The next proposition examines how the effect of ranking on valuation dispersion depends on the relative contribution to overall utility of each unit of the attributes (a_i and b_i). We first define some notation.

Notation Given any $\lambda > 0$, multiply w_a and w_b by λ so that the agent's utility from obtaining good i is $\lambda(w_a(a_i) + \delta w_b(b_i))$. Use superscript λ to indicate that the payoffs are multiplied with λ .

Proposition 6 *Assume: (i) $F(\cdot; \cdot)$ is such that $E[\hat{\delta}_r | \hat{\delta} > \rho]$ increases and $E[\hat{\delta}_r | \hat{\delta} < \rho]$ decreases when we increase the amount of effort c to obtain the estimator $\hat{\delta}$, (ii) $-V''/V'$ is non-increasing, and (iii) $-V'(h(1/x^2))/V''(h(1/x^2))$ is a convex function of x . Then, under the same assumptions of Propositions 1 and 5,*

$$\text{Var}(\hat{\delta}_{rv}^\lambda) - \text{Var}(\hat{\delta}_{v_0}^\lambda) \quad (22)$$

is increasing in λ .

The first assumption (i) states that as the agent exerts more effort she gets a better estimate in a sense that is stronger than the second-order stochastic dominance. Given (ii), and the fact that h is decreasing, (iii) can be replaced by the assumptions that V'/V''

is (weakly) concave and $h(1/x^2)$ is a convex function of x . These two assumptions are clearly satisfied when $V(c) = c^\gamma$ and $\gamma \in (0, 1)$, as in our canonical example.¹³

An increase in λ increases $[Var(\hat{\delta}_{rv}^\lambda) - Var(\hat{\delta}_{v_0}^\lambda)]$ in two ways. First, as λ gets larger then from Proposition 1 the agent exerts more effort in the ranking decision. Hence, by (i), $E[\hat{\delta}_r | \hat{\delta}_r > \rho]$ becomes larger while $E[\hat{\delta}_r | \hat{\delta}_r < \rho]$ becomes smaller. This increases $Var(\hat{\delta}_{rv}^\lambda)$, as Ψ is convex by (ii) — see (19). Since $Var(\hat{\delta}_{v_0}^\lambda)$ is not affected, this increases $[Var(\hat{\delta}_{rv}^\lambda) - Var(\hat{\delta}_{v_0}^\lambda)]$. Second, an increase in λ increases A , and hence leads the agent to exert more effort in the valuation stage as well. This increases both $Var(\hat{\delta}_{rv}^\lambda)$ and $Var(\hat{\delta}_{v_0}^\lambda)$. Under (iii), it impacts $Var(\hat{\delta}_{rv}^\lambda)$ more and thereby further increases the difference $[Var(\hat{\delta}_{rv}^\lambda) - Var(\hat{\delta}_{v_0}^\lambda)]$.

Proposition 6 also implies that the greater the contribution of each additional unit of the attributes, the more pronounced the effect of ranking on the spread of valuations. That is, $E[(\hat{u}_X^{rv\lambda} - \hat{u}_Y^{rv\lambda})^2] - E[(\hat{u}_X^{v_0\lambda} - \hat{u}_Y^{v_0\lambda})^2] = \lambda^2 \Delta_b^2 [Var(\hat{\delta}_{rv}^\lambda) - Var(\hat{\delta}_{v_0}^\lambda)]$ is increasing in λ . (This statement is true even without condition (iii).) Finally, $E[(\hat{u}_X^{rv\lambda} - \hat{u}_Y^{rv\lambda})^2] / \lambda^2 - E[(\hat{u}_X^{v_0\lambda} - \hat{u}_Y^{v_0\lambda})^2] / \lambda^2$ is also increasing in λ , an implication we will test in our experiments.

3 Testing Model Implications

We now present the results of a series of experiments designed to examine the model findings in actual decision making situations. Given the interest in understanding the impact of ranking on subsequent valuation (compared to when valuation alone is conducted) we focus on the findings of §2.4-2.5. In particular, one can state the primary testable implications in the form of three hypotheses as follows:

Hypothesis 1 When valuation comes after ranking and the rank ordering of alternatives is known, the squared (or absolute) difference between the monetary values stated for the goods is higher than if valuation alone takes place.

Hypothesis 2 The relative impact of ranking on valuation is increasing in the contribution of a unit of each product attribute to overall utility.

Hypothesis 3 When valuation comes after ranking and the rank ordering of alternatives is known, the effort expended in determining valuations for all goods is higher than if

¹³The assumptions (i-iii) of the proposition are all satisfied by the canonical case described in §2.1.2.

valuation alone takes place.

In addition, we will examine the implications of our analysis in §2.5.1 for the effects of prior ranking on the likelihood of purchase in a given price range.

3.1 Experimental Design and Method

To test the above hypotheses, we conducted three experiments. All three presented individuals with decisions regarding pairs of restaurants that were described in terms of two or three attributes. The set of possible attributes included Location (different areas in Cambridge, MA), Type of Food (Asian, Indian, Seafood), Service Level, Food Quality and Decor; see Appendix B (Table B1) for a more detailed description. Subjects were recruited from the general Cambridge and Boston (MA) areas and included both students (undergraduate and graduate) and non-students. Subjects were promised a minimum of \$10 for their willingness to participate and a chance to earn more in cash or prizes as explained below. The main goal of Experiment 1 was to test Hypothesis 1 regarding valuation spread, and together with Experiment 2 to test Hypothesis 2. Experiment 2 also allowed a test of Hypothesis 3 regarding effort expended. Experiment 3 was designed to replicate the results of Experiment 2 and rule out alternative explanations (we describe these issues in greater detail below).

3.2 Experiment 1

Participants were presented with eight separate decisions, each describing two restaurants (one pair at a time). Table B1 (Appendix B) lists the set of alternatives. Stimuli were presented using a computer interface and responses were recorded through the same interface. Subjects were randomly assigned into one of two conditions- a “valuation only” condition (VO) and a “ranking and valuation” condition (RV). In the VO condition (N=44), subjects were asked to state *their* dollar value for a dinner-for-two (which included an appetizer, main course and dessert) at each restaurant. Subjects were told up-front that at the end of the experiment prizes would be awarded as follows: one of the sixteen (2x8) restaurants for which they provided a dollar value would be selected with equal probability. In addition, the computer would randomly draw a number between 0-50. If the dollar value the individual stated for the dinner-for-two at the selected restaurant was greater than the randomly drawn number, a dinner-for-two voucher (non-transferable) at that specific restaurant would be granted to the individual.

Otherwise, the individual would get the random number in actual dollars. It is easy to see that this mechanism induces truth-telling and is consistent with Becker, Degroot and Marschak (1964). Participants were presented with several examples before beginning the experiment to help familiarize them with the above mechanism.

In the RV condition (N=44), subjects faced the same eight pairs of restaurants as in the VO condition in *two separate stages*. In the first stage, participants were asked to rank the two restaurants in each decision in order of dining preference (again for a dinner-for-two), by designating with a '1' their most preferred and a '2' their least preferred alternative. In the second stage, subjects were sequentially shown the same eight pairs of restaurants and were asked to state their dollar value for a dinner-for-two at each alternative. Subjects were *reminded of their first stage ranking designations*. The two stages were separated by an explanation of the mechanism by which prizes would be awarded (with examples). In the RV condition, subjects were informed that the computer would first randomly select either a stage I or a stage II decision. If a stage I decision was selected, they would receive a dinner-for-two voucher at their most preferred restaurant (the restaurant they designated with a '1'). If a stage II decision was selected, a mechanism identical to that described above for the VO condition was invoked. On average, 20 minutes elapsed between a particular ranking decision in stage I and the need to value the same alternatives in stage II. This process provided us with a eight observations per individual and most likely reduced the recollection of any specific details from introspection in a particular ranking decision.¹⁴

3.2.1 Results of Experiment 1

Given the valuations supplied by subjects for each of the decisions, we could test whether or not prior ranking affected the squared spread of valuations using the following regression model:

$$(u_A - u_B)^2 = \alpha_0 + \alpha_j d_j + \alpha_r RANK + e, \quad (23)$$

where α_0 is an intercept term, $d_j, j \in [1, 2, \dots, 8]$ are dummy variables to control for the specific decision, $RANK$ is a dummy variable denoting whether ranking had taken place prior to the valuation phase, and e is a standard normal error term. The results of the regression analysis are presented in Table 1.

¹⁴Note that the same attributes appear in only two cases that are separated by several intervening decisions. In the decisions used in Experiment 2, there were no two exactly overlapping attributes per decision.

Table 1: Effect of Prior Ranking on Valuation Spread

Parameter	Estimate	t-stat	p-value
Intercept	69.71	6.01	<0.01
Decision 1	11.34	0.73	0.46
Decision 2	-19.93	-1.29	0.20
Decision 3	-32.72	-2.05	0.04
Decision 4	-25.61	-1.66	0.10
Decision 5	-40.11	-2.59	0.01
Decision 6	-22.53	-1.46	0.15
Decision 7	-37.80	-2.44	0.02
RANK	30.526	3.95	<0.01

As can be seen from the above table, ranking alternatives prior to determining willingness to pay significantly increases the spread of valuations for any two alternatives. Experiment 1 thus strongly supports Hypothesis 1. This result also holds if in the regression (23) we use the absolute difference between valuations instead of the squared difference. The average absolute valuation difference between first and second ranked alternatives (across all decision pairs) was \$5.2 in the VO condition and \$7.1 in the RV condition (see Table B2).

3.3 Experiment 2

In this experiment we had subjects consider dinner-for-one dining alternatives (N=37 and N=39 in the VO and RV conditions, respectively). Given that the contribution of each attribute to overall willingness to pay (or utility) is expected to be lower than when the same alternatives are considered for a dinner-for-two,¹⁵ using responses from both experiments enables us to test Hypothesis 2. The restaurant pairs used in this experiment were a subset of those used in Experiment 1 (see Table B1). To examine Hypothesis 3, we wanted to compare the time subjects spent thinking in the valuation phase when

¹⁵ It could be the case that a dinner-for-two prize introduces considerations not present with a dinner-for-one, but since willingness to pay given the same restaurant alternatives (and hence same attribute levels) is higher between Experiments 1 and 2 (comparing same decisions and same conditions, see Table B4 in Appendix B), we believe that we are capturing a formulation consistent with Proposition 6. Vouchers for dinner-for-two cost exactly twice as much as dinner-for-one vouchers for the same restaurant.

prior ranking was or was not performed. In order to do so, we equated the number of decisions performed in the VO and RV conditions to avoid confounding effects associated with overall time spent in the experiment and/or learning effects.¹⁶ Thus, in the VO condition we had subjects state dollar values for ten pairs of restaurants, while in the RV condition we had subjects rank five pairs of restaurants and then state dollar values for these same five pairs (ten decisions altogether). The construction was such that the last five pairs in the VO condition corresponded to the five pairs in the RV condition (the first five pairs of the VO condition were ignored). The reward mechanism was identical to that of Experiment 1, with subjects able to receive dinner-for-one vouchers or cash (and the computer randomly drawing a number between 0-35).

3.3.1 Results of Experiment 2

The average absolute valuation difference between first and second ranked alternatives (across all decision pairs) with dinner-for-one options was \$3.3 in the VO condition and \$4.1 in the RV condition (see Table B3). A similar regression to (23), again confirms that prior ranking increases the spread of valuations (significant at the 10% level). To examine how increasing/decreasing the relative contribution of each attribute to overall utility affects the spread of valuations between the two conditions, we compute the following variable:

$$y = \frac{(u_A - u_B)^2}{(u_A + u_B)^2/2} \quad (24)$$

This variable “normalizes” the valuation spread by an average measure of the utility (or willingness to pay) from the two alternative restaurants. Thus, a comparison of the relative impact of ranking on valuation between the dinner-for-one and dinner-for-two scenarios is possible. To do so, we estimate the following regression equation:

$$y = \beta_0 + \beta_j d_j + \beta_{RV_1} RV * D1 + \beta_{RV_2} RV * D2 + \beta_{V0_2} V0 * D2 + e, \quad (25)$$

¹⁶ In Experiment 1 we observed that on average subjects monotonically decreased the amount of time they spent with each subsequent decision (perhaps due to boredom with the experiment or learning effects). Hence, because subjects in the RV condition had eight ranking decisions prior to the valuation phase while those in the VO condition immediately valued the eight pairs, there would be a confound between the number of decisions previously encountered and whether more or less time was being spent as a result of having ranking information available. Theoretically, since the number of decisions is doubled in RV condition, the probability that a particular decision will be chosen for prize is divided by two, substantially decreasing the optimal effort in RV condition by (7). The set-up in Experiment 2 was designed to overcome these concerns. In addition, in this experiment all relevant decisions involved trade-offs between different attributes (see Table B1).

where β_0 is an intercept term, $d_j, j \in [1, 2, \dots, 8]$ are dummy variables to control for the specific decision, RV and VO are dummies for the experimental condition, and $D1, D2$ are dinner-for-one and dinner-for-two indicators, respectively. The results from estimating the above regression model are given in the table below.

Table 2: Effect of Prior Ranking and Attribute Importance on Valuation Spread

Parameter	Estimate	t-stat	p-value
Intercept	0.15	4.65	<0.01
Decision 1	-0.08	-2.20	0.03
Decision 2	-0.124	-3.15	<0.01
Decision 3	-0.01	-0.58	0.56
Decision 4	-0.02	-2.47	0.014
RV*D1	0.04	0.94	0.35
RV*D2	0.18	4.35	<0.01
VO*D2	0.001	0.02	0.98

As the results indicate, the higher the “stakes” of the decision at hand (i.e., the more each attribute is expected to increase overall utility) the more a prior ranking impacts valuation spread. Thus Hypothesis 2 is strongly confirmed.¹⁷ This result also derives validity from a pilot study we conducted whereby we assigned respondents to VO and RV conditions with *no incentive* mechanism in place (i.e., subjects were paid for their participation in the study, but there were no prizes allocated that depended on their responses). In this pilot, where the “stakes” for the valuations supplied were in a sense nil, there was no significant overall effect for ranking. In fact, in five of the eight decisions the spread was actually greater in the VO condition. It is also noteworthy that in both Experiments 1 and 2, when examining the average amounts subjects were willing to pay in each decision for their most preferred vs. least preferred alternative, there was no common pattern of divergence for the increased spread across conditions. In some cases both most and least preferred alternatives were given a higher value in the RV condition compared to the VO condition (though the spread still increased), in some cases both valuations were lower, and in other cases the most preferred alternative was given a higher value in the RV than in the VO condition and the least preferred a lower value. Though

¹⁷ Once again, these results were corroborated if in (24) we used absolute values instead of squares.

we realize our experiments are between subjects, we believe these results are consistent with our model (see §2.5, on how valuations for both goods can increase or decrease as a result of a prior ranking).¹⁸

As we measured the duration of time subjects spent making each of their decisions, we could also determine the effect of prior ranking on the effort expended in valuating pairs of dinner-for-one alternatives. We estimated the following regression model:

$$T = \gamma_0 + \gamma_j d_j + \gamma_r RANK, \quad (26)$$

where T is the time the subject spent determining valuations for any given decision pair, γ_0 is an intercept term, d_j are as in (23) and $RANK$ designates if a ranking decision had preceded valuation. Estimation results are given in the table below

Table 3: Effect of Prior Ranking on Time Spent in Valuation Phase

Parameter	Estimate	t-stat	p-value
Intercept	15.09	5.54	<0.01
Decision 1	20.09	5.74	<0.01
Decision 2	6.00	1.71	0.087
Decision 3	1.41	0.40	0.69
Decision 4	5.63	1.61	0.11
RANK	10.90	4.92	<0.01

As clearly evident from Table 3, having performed a prior ranking of alternatives significantly increases the time spent on determining willingness to pay for the same alternatives (compared to when no such ranking takes place). In fact, the average amount of time spent (across all decisions) determining valuations for a decision pair in the RV condition was almost 50% higher than that in the VO condition (32.16 and 21.72 seconds, respectively; see Table B5 in Appendix B). This result is consistent with Hypothesis 3.¹⁹

¹⁸Given that related behavioral decision theory has mainly been concerned with how different decision tasks, when examined separately, would yield different outcomes, it is not clear they would entirely bear on the focus of our study (explicitly combining two decision tasks). For example, it is not clear that ‘anchoring and adjustment’ (Tversky and Kahneman 1974) is relevant in this case, because a ranking is a different output measure altogether than a valuation (or rating) measure. Even if subjects were to ‘anchor’ on the rankings, one might expect the most preferred alternative to be ‘adjusted’ to have a higher (average) value in the RV condition than in the VO condition, and the opposite be true for the least preferred alternative. As indicated, this pattern was not prevalent in our experiments.

¹⁹ We believe that in the last decision subjects were perhaps hurried to conclude the experiment, and

One additional point regarding the connection between the experimental results and the theory developed is worth stressing. In our model, the *ex ante* mean value of the estimates for a given alternative should remain the same regardless of whether ranking has taken place, since $E[\delta] = E[\hat{\delta}_{vo}] = E[\hat{\delta}_{rv}] = 1$. We find strong, between subject, support for this property in Experiment 2. Comparing the mean valuation for each restaurant across subjects in each condition (see Table B4 in Appendix B), reveals a statistically significant difference for only one restaurant out of ten (namely, alternative B in decision pair 3).

3.3.2 The Impact of Prior Ranking on Likelihood of sales

We also examined how the demand for each restaurant, as a function of price, would be affected by prior ranking. Recall from §2.5.1 that, under certain conditions, ranking is expected to increase demand when prices are extreme, i.e. either very low or very high. To examine whether this holds in our experimental setting, we looked at three regions defined by price points: a) p^m = the mean value for a restaurant in any decision (see Table B4), b) p^{m-} = one standard deviation below the mean, c) p^{m+} = one standard deviation above the mean. We then separately counted the number of individuals in each treatment (RV, VO) that would be willing to pay for a dinner-for-one at a given restaurant, when $p^{m-} < v_k^i$, $p^m \leq v_k^i$, and $p^{m+} < v_k^i$, where v_k^i is individual i 's valuation for restaurant k (there are 10 restaurants). In Table 4 below, we present the difference between the two treatments in the percent of individuals willing to buy at each price (denoted $\Delta\%$). As Table 4 shows, our experimental results support a pattern whereby sales are more likely to take place at low or high prices as a result of alternatives first being rank ordered and then valued. Around the mean, willingness to pay is relatively unaffected by a prior ranking.

hence the times are less divergent across the two conditions. See also §3.4 where the result on effort levels is again strongly confirmed. Note that this is also consistent with the discussion after Proposition 3 that $h(1/x^2)$ is likely to be a convex function.

Table 4: Effect of Prior Ranking on Demand at Different Price Points

		$\Delta\%$		
		p^{m-}	p^m	p^{m+}
Decision	Restaurant			
1	A	5.1%	0%	7.7%
	B	5.4	0	2.7
2	A	2.6	0	5.1
	B	2.7	0	2.7
3	A	12.8	0	0
	B	0	0	21.6
4	A	0	0	7.7
	B	8.1	0	5.4
5	A	0	7.7	10.3
	B	8.1	2.7	8.1
Average		4.5%	1%	7.1 %

3.4 Experiment 3

While in Experiment 2 we controlled for the overall number of decisions each subject faced by having five "filler" valuation decisions in the VO condition (in lieu of the five ranking decisions in the RV condition), one cannot entirely rule out alternative explanations for the results presented in Table 3 based on a difference in the nature of tasks in the two conditions. In particular, there may be greater 'task familiarity' or learning specificity about valuations in the last five decisions of the VO condition (given that five valuation decisions had already taken place). To account for such alternative explanations, we used a set-up similar to that in the VO condition of Experiment 2 (i.e., dinner-for-one with five relevant valuation decisions) except that subjects (N=49) first rank ordered five irrelevant restaurant pairs.²⁰ This way, in both the RV and VO conditions, subjects faced ten decisions, five ranking and five subsequent valuation decisions.

²⁰The first five ranked pairs used in the VO condition were different restaurants than those in the last five valuation decisions. We thank Al Roth for pointing out the potential problem arising in Experiment 2 and for suggesting Experiment 3 to us.

3.4.1 Results of Experiment 3

Table B6 (in Appendix B) presents the amount of time subjects spent in each of the last five valuation decisions. For convenience, we juxtapose these results with those from the corresponding RV dinner-for-one condition (using RV data from Experiment 2). We also re-estimated equation (26) with the new data. The parameter estimate for prior ranking, γ_r , was 10.2 and significant at the 1% level. The results once again strongly corroborate Hypothesis 3.

In addition to replicating the results on effort levels, re-estimating regression equation (25) with the new data yielded a parameter estimate $\beta_{RV_2} = 0.15$ (significant at the 1% level), while the parameters β_{RV_1} and β_{VO_2} were not significantly different from 0. Thus supporting Hypothesis 2. As for the likelihood of sales, the results from Experiment 3 support higher demand in the RV condition when price is one standard deviation below the mean (in analogy to column 3 of Table 4) and no difference in demand when price is at the mean (in analogy to column 4 of Table 4). When price is one standard deviation above the mean, no difference between the conditions was found (as opposed to expected higher demand in the RV condition, see column 5 of Table 4).²¹

4 Conclusion

Whether implicit or explicit, the tendency to first obtain a relative sense of ranking among alternatives prior to attaching a specific value to each seems pervasive in many decision making contexts. In this paper, we set out to examine how this sequence of decisions affects the final valuation of alternatives and the amount of effort expended in arriving at these valuations. The model we have proposed incorporates aspects of both rational optimizing behavior and bounded rationality.

Several interesting findings, supported through a series of laboratory experiments, have emerged. We find that, on expectation, first ranking a set of alternatives increases the spread of values generated in a later stage and increases the amount of effort expended in the valuation phase; this is in addition to the effort already expended in first ranking

²¹That said, our Experiment 3 was conducted almost a year after Experiment 2. Because we are comparing VO data from the former with RV data from the latter, there was the potential for inflationary and/or seasonal effects. Comparing the mean valuation for each restaurant in the VO condition between the two experiments reveals that the new valuations are in fact slightly higher (by 7%). If we deflate the new valuations for each subject by this factor, the demand at one standard deviation above the mean is in fact higher (by 10% on average) with prior ranking (in analogy to column 5 of Table 4).

alternatives. Furthermore, we have shown that prior ranking can affect the likelihood of sales taking place. In particular, using a canonical example, we predict that prior ranking tends to increase sales when prices are either very low or very high. Given that the ranking of alternatives is often viewed as a way to simplify valuation decisions, we believe that these results bear significance for our understanding of many decision-making contexts.

At a practical level, our results have implications for real life situations where the tendency to rank alternatives prior to determining valuation is encountered. Sellers may attempt to use this two-stage process to their advantage. For example, car dealers will often refuse to discuss the price of any vehicle before they have taken the customer through the lot and made him/her rank the set of relevant cars. The dealers' reasoning, it would seem, is that encouraging a ranking of alternatives (mainly on non-price attributes), will work to their advantage in subsequent price negotiations. In addition, the growing pervasiveness of Internet commerce and digital information dissemination in recent years has made this two-stage process far more pronounced. For instance, in consumer electronic auctions, consumers are often given a listing of several relevant products in the desired category. For each alternative, attribute levels are specified in the description of the product. After reading all descriptions, consumers must decide which item to bid on first (or if multiple items are required a complete rank ordering of all). They then must decide on a specific valuation of the alternatives in order to enter a bid.

At a theoretical level, our paper provides a framework for capturing the impact of previous choices on current related decisions. We do so in the context of a model that assumes boundedly rational agents with uncertain preferences. Though it may be plausible to model these agents' behavior using some anomalous preference relation, such as a preference for consistency, we show that there will be other details in their expected pattern of behavior that could not possibly be the result of such an anomalous preference characterization. Subjects in our controlled lab experiments did, in fact, exhibit such patterns of behavior.

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A Appendix – Omitted Proofs

Proof. (Proposition 1) Under our model set-up, we write (3) as

$$U_r(c; \rho) = \Delta_b \int_0^\rho (r - \hat{\delta}_r) f(\hat{\delta}_r; c) d\hat{\delta}_0 + E[u(Y)] - c,$$

where $f = \partial F / \partial c$. Then,

$$\begin{aligned} \frac{\partial U_r(c; \rho)}{\partial \rho} &= \Delta_b \left[\left[\frac{d\rho}{d\rho} (\rho - \hat{\delta}_r) f(\hat{\delta}_r; c) \right]_{\hat{\delta}_r=\rho} + \int_0^\rho \frac{\partial}{\partial \rho} (\rho - \hat{\delta}_r) f(\hat{\delta}_r; c) d\hat{\delta}_r \right] + \frac{\partial}{\partial \rho} E[u(Y)] - \frac{\partial}{\partial \rho} c \\ &= \Delta_b \left[(\rho - \rho) f(\rho; c) + \int_0^\rho f(\hat{\delta}_r; c) d\hat{\delta}_r \right] = \Delta_b F(\rho; c). \end{aligned}$$

Thus,

$$\frac{\partial^2 U_r(\rho; c)}{\partial c \partial \rho} = \frac{\partial}{\partial c} [\Delta_b F(\rho; c)] = \Delta_b \frac{\partial F(\rho; c)}{\partial c}.$$

Then, by hypothesis, for any $\rho > 1$, we have $\frac{\partial^2 U_r(\rho; c)}{\partial c \partial \rho} = \Delta_b \frac{\partial F(\rho; c)}{\partial c} < 1$, hence $\arg \max_{c \in [0, \bar{c}]} U_r(c; \rho)$ is decreasing with ρ , and so is with $|\rho - 1| = \rho - 1$. Similarly, for any $\rho < 1$, $\frac{\partial^2 U_r(\rho; c)}{\partial c \partial \rho} > 1$, hence $\arg \max_{c \in [0, \bar{c}]} U_r(c; \rho)$ is increasing with ρ , and hence decreasing with $|\rho - 1| = -\rho + 1$.

To prove, part 2, note that when w_a and w_b is multiplied with λ , we have

$$U_r(c; \lambda) = \lambda \Delta_b \int_0^\rho (r - \hat{\delta}_r) f(\hat{\delta}_r; c) d\hat{\delta}_0 + \lambda E[u(Y)] - c.$$

Hence,

$$\frac{\partial U_r(c; \lambda)}{\partial \lambda} = \Delta_b \int_0^\rho (r - \hat{\delta}_r) f(\hat{\delta}_r; c) d\hat{\delta}_0 + E[u(Y)],$$

which is increasing in c under the hypothesis of the proposition. ■

Proof. (Proposition 2) Given $u(X)$, and given that X is selected, the value of having estimate \hat{u}_X is

$$\begin{aligned} U_v(\hat{u}_X) &= \frac{1}{m} \int_0^{\hat{u}_X} u(X) dp_X + \frac{1}{m} \int_{\hat{u}_X}^m p_X dp_X = \frac{1}{m} \left(u(X) \hat{u}_X - \frac{1}{2} \hat{u}_X^2 \right) + \frac{m}{2} \\ &= -\frac{1}{2m} (\hat{u}_X - u(X))^2 + \frac{1}{2m} u(X)^2 + \frac{m}{2}. \end{aligned}$$

Here, the first equality expresses the fact that the agents will receive X if $p_X \leq \hat{u}_X$, and p_X otherwise, while p_X is distributed uniformly on $[0, m]$. We obtain the second equality by simple calculation, and the final one by adding and subtracting $\frac{1}{2m} u(X)^2$. Similarly, Given $u(Y)$, and

given that Y is selected, the value of having \hat{u}_Y is $U_v(\hat{u}_Y) = -\frac{1}{2m}(\hat{u}_Y - u(Y))^2 + \frac{1}{2m}u(Y)^2 + m/2$. Therefore, given the true value δ , the utility of having an estimate $\hat{\delta}_v$ is

$$\begin{aligned} U(\hat{\delta}_v) &= \pi U_v(\hat{u}_X) + \pi U_v(\hat{u}_Y) \\ &= -\frac{\pi}{2m}(\hat{u}_X - u(X))^2 - \frac{\pi}{2m}(\hat{u}_Y - u(Y))^2 + \frac{\pi}{2m}[u(X)^2 + u(Y)^2] + \pi m \\ &= -\frac{\pi}{2m}w_b(b_X)^2(\hat{\delta}_v - \delta)^2 - \frac{\pi}{2m}w_b(b_Y)^2(\hat{\delta}_v - \delta)^2 + \frac{\pi}{2m}[u(X)^2 + u(Y)^2] + \pi m \\ &= -\frac{\pi}{2m}[w_b(b_X)^2 + w_b(b_Y)^2](\hat{\delta}_v - \delta)^2 + \frac{\pi}{2m}[u(X)^2 + u(Y)^2] + \pi m. \end{aligned}$$

Therefore, the value of introspection of length c is

$$U_v(c_v) = E[U(\hat{\delta}_v)] - c_v = -AE[(\hat{\delta} - \delta)^2] + B' - c, \quad (27)$$

where $A = \frac{\pi}{2m}[w_b(b_X)^2 + w_b(b_Y)^2]$ and $B' = \frac{\pi}{2m}[E[u(X)^2] + E[u(Y)^2]] + \pi m$. Now,

$$E[(\hat{\delta}_v - \delta)^2] = E[(\hat{\delta}\varepsilon_v - \delta)^2] = E[\hat{\delta}_v^2(\varepsilon - 1)^2] = E[\hat{\delta}_v^2]E[(\varepsilon_v - 1)^2] = E[\hat{\delta}_v^2]Var(\varepsilon_v).$$

On the other hand,

$$\begin{aligned} Var(\delta) &= E[(\hat{\delta}_v\varepsilon_v)^2] - 1 = E[\hat{\delta}_v^2]E[\varepsilon_v^2] - 1 = E[\hat{\delta}_v^2](E[\varepsilon_v^2] - 1) + E[\hat{\delta}_v^2] - 1 \\ &= E[\hat{\delta}_v^2]Var(\varepsilon_v) + Var(\hat{\delta}_v). \end{aligned}$$

Substituting the previous equation in this one, we obtain

$$E[(\hat{\delta}_v - \delta)^2] = Var(\delta) - Var(\hat{\delta}_v).$$

Substituting this in (27), we obtain

$$U_v(c_v) = -A[Var(\delta) - Var(\hat{\delta}_v)] + B' - c_v = AVar(\hat{\delta}_v) + B - c_v = AV(c_v) + B - c_v,$$

where $B = B' - AVar(\delta)$. ■

When $\hat{u}_X > m$, the agent gets $u(X)$ independent of her estimate \hat{u}_X . Hence, when $\hat{\delta}_v$ is not bounded, we need to correct U_v , by adding a term in the order of

$$\frac{1}{2m}[E[\hat{u}_X^2 : \hat{u}_X > m] + E[\hat{u}_Y^2 : \hat{u}_Y > m]].$$

Therefore, when the tails of the distributions are sufficiently thin, and m is sufficiently large, $U_v(c)$ and $\arg \max U_v(c_v)$ will be sufficiently close to the ones computed through Proposition 2.

Proof. (Boundary Conditions on c_{rv} and $V(c_{rv})$) The boundary conditions (which are omitted) are not binding: By (4), $V'(c_{rv}[\hat{\delta}_r \leq \rho]) < V'(0)$, hence $c_{rv}[\hat{\delta}_r \leq \rho] > 0$. From (5), by $V(c_{rv}[\hat{\delta}_r > \rho]) < Var(\varepsilon)$, hence $c_{rv}[\hat{\delta}_r > \rho] < c_r \leq \bar{c}$. Since $c_{rv}[\hat{\delta}_r \leq \rho] \leq c_{v_0} \leq c_{rv}[\hat{\delta}_r > \rho]$, it follows that the boundary conditions are not binding. Moreover, since V is increasing, we have $V(0) < V(c_{rv}[\hat{\delta}_r \leq \rho]) \leq V(c_{v_0}) \leq V(c_{rv}[\hat{\delta}_r > \rho]) < Var(\varepsilon)$. Hence, some uncertainty remains unresolved in each decision. ■

Proof. (Lemma 1) In this proof we will omit the arguments of the functions V , V' , V'' , and V''' , which will always be $h(1/(Ax^2))$. Since $V'(h(1/(Ax^2))) = 1/(Ax^2)$, we will write V' and $1/(Ax^2)$, interchangeably. To show that Ψ is increasing, we first compute that

$$\Psi'(x) = 2xV + x^2(V') \cdot (h(1/(Ax^2)))' = 2xV + \frac{1}{A}(h(1/(Ax^2)))',$$

where

$$(h(1/(Ax^2)))' = h'(1/(Ax^2))(-2/(Ax^3)) = -\frac{2}{Ax^3V''}. \quad (28)$$

Since $V'' < 0$, we have $(h(1/(Ax^2)))' > 0$. Hence, $\Psi'(x) > 0$ at each $x > 0$, showing that Ψ is increasing.

Towards showing Ψ is convex, we further compute that

$$\begin{aligned} \Psi''(x) &= 2V + 2x(V') \cdot (h(1/(Ax^2)))' + \frac{1}{A}(h(1/(Ax^2)))'' \\ &= 2V + \frac{2}{Ax}(h(1/(Ax^2)))' + \frac{1}{A}(h(1/(Ax^2)))''. \end{aligned} \quad (29)$$

By (28), we have

$$\frac{2}{Ax}(h(1/(Ax^2)))' = -\frac{4}{(Ax^2)^2V''} = -\frac{4(V')^2}{V''} \quad (30)$$

Using (28), we further compute that

$$\begin{aligned} \frac{1}{A}(h(1/(Ax^2)))'' &= \frac{1}{A}\left(-\frac{2}{Ax^3V''}\right)' = \frac{2}{A^2x^6(V'')^2}\left[3x^2V'' + x^3V''' \cdot \left(-\frac{2}{Ax^3V''}\right)\right] \\ &= \frac{6}{(Ax^2)^2V''} - \frac{4V'''}{(Ax^2)^3(V'')^3} = \frac{6(V')^2}{V''} - \frac{4V''' \cdot (V')^3}{(V'')^3}. \end{aligned} \quad (31)$$

Substituting (30) and (31) in (29), we obtain

$$\Psi''(x) = 2V - \frac{4(V')^2}{V''} + \frac{6(V')^2}{V''} - \frac{4V''' \cdot (V')^3}{(V'')^3} = 2\left[V + \frac{(V')^2}{V''}\left(1 - \frac{2V''' \cdot V'}{(V'')^2}\right)\right].$$

Since $V \geq 0$ and $V'' < 0$, this implies that $\Psi'' > 0$ whenever $1 - 2V'''V'/(V'')^2 \leq 0$, i.e., $V''' \geq \frac{(V'')^2}{2V'}$. But, when $-V''/V'$ is non-increasing, V'/V'' is also non-increasing, hence we have $(V'/V'')' = 1 - 2V'''V'/(V'')^2 \leq 0$, showing that $V''' \geq \frac{(V'')^2}{V'}$ and completing the proof. ■

We will now prove Proposition 6. Firstly,

Fact 7 Given any convex function f , any $x, x', y, y' \in \mathbb{R}$, and any $\theta, \theta' \in [0, 1]$ with $x' \leq x \leq y \leq y'$ and with $\theta x + (1 - \theta) y = \theta' x' + (1 - \theta') y'$, we have

$$\theta f(x) + (1 - \theta) f(y) \leq \theta' f(x') + (1 - \theta') f(y').$$

Proof. (Proposition 6) First note that

$$\frac{\partial \Psi(x; A)}{\partial A} = -\frac{\theta V'(h(1/(Ax^2)))}{A^2 V''(h(1/(Ax^2)))}. \quad (32)$$

Given any x, y , and θ with $\theta x + (1 - \theta) y$, define

$$\phi(x, y, \theta; A) = \theta \Psi(x; A) + (1 - \theta) \Psi(y; A) - \Psi(1; A).$$

By (32),

$$\frac{\partial \phi}{\partial A} = -\frac{1}{A^2} \left[\theta \frac{V'(h(1/(Ax^2)))}{V''(h(1/(Ax^2)))} + (1 - \theta) \frac{V'(h(1/(Ay^2)))}{V''(h(1/(Ay^2)))} - \frac{V'(h(1/A))}{V''(h(1/A))} \right].$$

Since $\frac{V'(h(1/z^2))}{V''(h(1/z^2))}$ is concave in z , we have $\partial \phi / \partial A \geq 0$, hence ϕ is increasing in A .

Now, given any λ and λ' with $\lambda \leq \lambda'$, we have

$$\begin{aligned} \text{Var}(\hat{\delta}_{rv}^{\lambda'}) - \text{Var}(\hat{\delta}_{v_0}^{\lambda'}) &= \phi \left(E[\hat{\delta}_r^{\lambda'} | \hat{\delta}_r^{\lambda'} \leq \rho], E[\hat{\delta}_r^{\lambda'} | \hat{\delta}_r^{\lambda'} > \rho], \Pr(\hat{\delta}_r^{\lambda'} \leq \rho); A^{\lambda'} \right) \\ &\geq \phi \left(E[\hat{\delta}_r^\lambda | \hat{\delta}_r^\lambda \leq \rho], E[\hat{\delta}_r^\lambda | \hat{\delta}_r^\lambda > \rho], \Pr(\hat{\delta}_r^\lambda \leq \rho); A^{\lambda'} \right) \\ &\geq \phi \left(E[\hat{\delta}_r^\lambda | \hat{\delta}_r^\lambda \leq \rho], E[\hat{\delta}_r^\lambda | \hat{\delta}_r^\lambda > \rho], \Pr(\hat{\delta}_r^\lambda \leq \rho); A^\lambda \right) \\ &= \text{Var}(\hat{\delta}_{rv}^\lambda) - \text{Var}(\hat{\delta}_{v_0}^\lambda). \end{aligned}$$

By assumption (i) in the hypothesis and by Proposition 1, $E[\hat{\delta}_r^{\lambda'} | \hat{\delta}_r^{\lambda'} \leq \rho] \leq E[\hat{\delta}_r^\lambda | \hat{\delta}_r^\lambda \leq \rho] \leq E[\hat{\delta}_r^\lambda | \hat{\delta}_r^\lambda > \rho] \leq E[\hat{\delta}_r^{\lambda'} | \hat{\delta}_r^{\lambda'} > \rho]$. Together with Fact 7, this gives the first inequality. The second inequality holds because ϕ is increasing in A , which is increasing in λ . Equalities are by definition. ■

B Appendix – Supplement to Experimental Findings

Table B1: Restaurant Pairs

Decision ^a	Alternative ^b	Type ^c	Location ^d	Food Quality ^e	Service Quality ^f	Decor ^g
1	A	Indian	Kendall Sq.			
	B	Seafood	Inman Sq.			
2 (1)	A		Harvard Sq.	12	13	
	B		Central Sq.	17	16	
3 (2)	A			20		14
	B			17		16
4	A				21	16
	B				17	19
5	A		Kendall Sq.	15	16	
	B		Harvard Sq.	17	13	
6 (3)	A	Indian			17	
	B	Asian			12	
7 (4)	A			16		13
	B			20		6
8 (5)	A	Seafood				15
	B	Indian				18

^aNumber in parenthesis denotes decision used in Experiment 2 (which only had 5 restaurant pairs).

^bIn each decision, subjects were presented with two options designated “A” or “B”. Restaurants were not identified by name to reduce chances of personal biases.

^cDenotes the type of cuisine served at each of the alternatives (Asian, Indian or Seafood).

^dAll restaurants are in Cambridge, Massachusetts (at either one of the following squares: Central, Harvard, Inman, Kendall). This allowed prizes to be relevant to the subject pool recruited from the general Boston area.

^eDenotes the quality of food at the given restaurant based on the restaurant guide/critic “Zagat” (see www.zagat.com). The scale for food grade ranges from 0-30, where 30 in the Zagat scale implies highest possible food quality. Subjects were informed of the scale and its meaning prior to the beginning of the experiment.

^fDenotes the quality of service at the given restaurant based on the restaurant guide/critic “Zagat” (see www.zagat.com). The scale for service grade ranges from 0-30, where 30 in the Zagat scale implies highest possible quality of service.

^gDenotes the state of interior furnishings and the emphasis on decorative style at the given restaurant based on the restaurant guide/critic “Zagat” (see www.zagat.com). The scale for decor ranges from 0-30, where 30 in the Zagat scale implies an excellent/extraordinary level.

Table B2: Mean Absolute Difference in Valuation (Experiment 1)

Decision Pair	VO Condition (N=44)	RV Condition (N=44)
1	7.71	8.98
2	4.02	7.75
3	4.59	6.91
4	5.20	4.98
5	4.36	5.96
6	4.84	7.59
7	4.36	6.23
8	6.11	8.48
Average	5.15	7.11

Table B3: Mean Absolute Difference in Valuation (Experiment 2)

Decision Pair	VO Condition (N=37)	RV Condition (N=39)
1	2.97	4.10
2	2.97	4.00
3	3.23	4.31
4	3.40	3.54
5	3.75	4.31
Average	3.26	4.05

Tale B4: Average Willingness to Pay For Each Restaurant²²

Decision ²³	Restaurant	Experiment 1		Experiment 2	
		VO	RV	VO	RV
2(1)	A	23.25	20.52	11.94	12.90
	B	25.73	24.01	14.64	15.72
3(2)	A	31.64	27.71	17.94	18.08
	B	27.14	22.89	15.13	15.05
6(3)	A	24.61	23.32	13.45	14.46
	B	21.73	21.68	10.76	12.82
7(4)	A	24.23	24.05	14.16	14.15
	B	25.23	23.91	14.97	15.80
8(5)	A	29.64	25.43	15.30	15.74
	B	27.75	24.36	14.35	14.05

²²Table shows the average willingness to pay (across all respondents in a particular study and condition). Only the five common decisions across experiments are presented.

²³Number in parenthesis is the corresponding Experiment 2 decision number.

Table B5: Time Spent in Valuation Phase (Experiment 2)

Decision Pair	VO Condition (N=37)	RV Condition (N=39)	p-value
1	27.41	53.46	<0.01
2	17.78	35.13	0.02
3	20.78	23.33	0.42
4	20.89	31.46	0.02
5	21.73	19.70	0.45
Average	21.72	32.16	

Table B6: Time Spent in Valuation Phase (Experiment 3)

Decision Pair	VO Condition (N=49)	RV Condition (N=39)	p-value
1	35.27	53.46	<0.01
2	21.56	35.13	0.04
3	17.78	23.33	0.03
4	19.98	31.46	<0.01
5	17.49	19.69	0.26
Average	22.4	32.16	

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