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STOCHASTIC CREDIT IN SEARCH EQUILIBRIUM, II

Peter Diamond

No. 493

April 1988

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Stochastic Credit in Search Equilibrium, II

P. Diamond*

The factors determining the extent of liquidity in an economy can be divided into two groups. One group relates to the characteristics of the investment opportunities in the economy, including the costs of observation and verification of the characteristics of the investments and the extent of uncertainty about investment returns. The second group has to do with the organization of the availability of credit. The small but very interesting recent literature on the characteristics of credit markets and their properties has focused in particular on the presence of multiple equilibria in such markets. These papers have concentrated either on the workings of conventional financial markets or on the workings of financial intermediaries. The Chatterjee [1986] and Pagano [1986] papers considered fixed costs of entering markets as one of the determinants of the thickness of the markets and found multiple equilibria.[1] In a recent paper, [1986] I considered the availability of credit where all credit provision was pair-wise, associated with pair-wise trade. In that model debt positions were lumpy associated with the lumpy transactions they were financing. I found that multiple equilibria were a common phenomenon in the sense that they held for large parts of parameter space for which examples were calculated. This paper considers a smoothed version of my previous paper where lotteries on both the provision and repayment of debt are used to generate a smooth credit limit rather than a lumpy one. In the examples I have calculated I have not found multiple equilibria. This contrast highlights the importance of lumpiness or fixed costs in the possibility of multiple equilibria. Fixed costs associated with credit transactions or the arrangement of lines of credit are realistic phenomena. This suggests that multiple equilibria might be reintroduced into this model from a further extension which added costs of arranging credit as a substitute for the arbitrarily lumpy debt positions allowed in my earlier paper.

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*This paper differs from my (1986) paper by the introduction of smooth stochastic trade, a suggestion made by Kevin Murphy. I am indebted to L. Felli for research assistance, P. Howitt for valuable discussions, and to the National Science Foundation for financial support.

[1] The illiquidity of the production technology relative to short run changes in demand plays a key role in the literature following the Diamond-Dybvig paper [1983]. In contrast, this paper explores the effects of credit and trading limitation without such a difference.

I start with a model of trade with trading frictions and then introduce credit to examine its effects. Since my mathematical techniques do not permit me to analyze money and credit simultaneously, this is a barter model. I proceed by first presenting a barter model which is a simplification of a model that I have published earlier [1982]. I do not claim that it is a particularly good model for this purpose, but it was readily at hand. The model is set up in a world where not only is there no money because, let us say, nobody has thought of the idea, but also there is no credit because no one has thought of that idea. If someone thinks up the idea of credit, will credit be introduced into this economy? With no fixed setup costs for arranging credit, credit is always introduced to the economy (except at the knife edge where equilibrium with production is just sustainable). Interestingly, for some parameters, credit is also introduced in the equilibrium with no production. Next I consider the economy where credit is readily available, calculating comparative steady state examples.

1. Basic Model Without Credit

In order to have a model with both continuous time and discrete transactions, one needs to have a complicated purchase and storage technology or a preference structure that is different from the standard integral of discounted utility of instantaneous consumption. The alternative preferences I work with have the consumption good in an indivisible unit, which is consumed from time to time. I denote by y the utility that comes whenever one of these units is consumed. This is an instantaneous utility from a discrete consumption at an instant of time. That is a mathematically convenient approximation to the fact that it does take a while to consume goods. But we also do not go around consuming (or purchasing) nondurable goods continuously through the day. Similarly, production of consumer goods takes time but is modeled as an instantaneous process. (Modeling the length of time to complete production as a Poisson process permits a straightforward generalization of this class of models.) After production, the good is carried in inventory until it can be traded. Denote by c the labor disutility of instantly producing one unit of this good. All opportunities involve the same cost. Instantaneous utility thus satisfies

$$U = y - c \tag{1-1}$$

For viability of the economy we assume $0<c<y$. Over time there is a sequence of dates, t_i, at which one will have opportunities either to acquire a unit to consume or to produce a unit for trade. The preferences of the individual (identical for all agents) are representable as the expected discounted sum of the utilities associated with this random stream of discrete events as given in equation (1-2).

$$V = \sum_{i=1}^{\infty} e^{-rt_i} U_{t_i} \tag{1-2}$$

The focus of this analysis is on trade, so it will not do to have this economy collapse into autarchy, with people producing and promptly consuming what they produce themselves. Therefore we add some restrictions. The first restriction is that individuals never consume what they produce themselves. You can think of it as a physical impossibility or an element of preferences--people just do not like the good that they themselves produce. On producing a unit, agents look for someone else who also has one unit with whom to barter. The other restriction that will keep the model simple is that the inventory carrying costs are such that one never carries more than one unit of good available for trade. Thus an individual in this economy is in one of two positions. Either he has no goods in inventory and is unable to trade or he has one unit of good in inventory and is available to trade. In the former case the agent is looking for an opportunity to produce. I spread opportunities out smoothly in time by assuming a Poisson process with arrival rate a for the opportunity to give up the labor disutility c and add one unit to inventory. This process goes on continuously; there is no cost in being available to produce, there is merely a cost in actually carrying out production. Of course once one has an opportunity to produce, one still has a choice. One does not have to produce. If one has a unit in inventory, one does not produce because one can not carry the good in inventory. Without a unit in inventory, one looks ahead to the length of time it will take to trade a unit if produced. The utility y obtained when the good is traded one-for-one and consumed will happen some time in the future and will be discounted by the utility discount rate r. Therefore it will only be worthwhile to produce for trade if the process of carrying out a trade is fast enough relative to the utility discount rate and to the gap between the utility of consumption and the disutility of production.

I denote by e the fraction of people with inventory for sale. If every opportunity is carried out, and that will be my first assumption, then e is growing as all the people without goods for sale, the fraction 1-e, carry out all of their opportunities. (Thus I normalize the implicit continuum of the population to one.) In addition, people will be meeting each other. They will carry out a trade whenever they have the opportunity. In a barter economy with no money and no credit, such a trade can be carried out only when both of the people meeting have inventory to trade. We are not concerned here with the double coincidence of their liking each other's goods. That is assumed to happen automatically. But we are concerned with a double coincidence in timing. Two people must come together at a time when they both have goods in inventory. They do not have the ability, the communications technology, to keep track of lots of potential trading partners and so instantly trade on completing production. The underlying idea here is that for many goods consumers are not searching for the good, they are searching for the good in the right size, color, and design. So retailers stock large quantities of goods that are held for consumers who do a great deal of shopping, not because it is hard to find out who is a supplier but because it may take some time to find one that has available precisely what is wanted.

We assume that this meeting process takes the simplest possible stochastic form of random meetings between individuals. These meetings are going on all of the time. Any individual experiences a Poisson arrival of people at rate b. This is again a Poisson process with an exogenous technological parameter. But some of the people met have no inventory and can not be traded with. Some of the people met have inventory and can be traded with. So the rate at which goods can be traded is be, an endogenous variable depending on the stock of inventories in the economy. An economy with a high level of production will have strong incentives to produce for inventory because it is easy to meet people to trade with. Equation (1-3) is the differential equation for the behavior of inventories over time assuming that all production opportunities are carried out. (Below we give a sufficient condition for this behavior to be consistent.)

$$\dot{e} = a(1-e) - be^2 \tag{1-3}$$

That is each of the fraction e with inventories faces the probability be of having a successful trade meeting and being freed to seek a new opportunity. Each of the 1-e without inventories has the flow probability a of learning of an opportunity. With all opportunities taken, the employment rate converges to e_0, the solution to $\dot{e} = 0$ in (1-3).

$$2be_0 = (a^2 + 4ab)^{1/2} - a \tag{1-4}$$

Note that e_0 is homogeneous of degree zero in (a,b). Note also that as b/a varies from 0 to + ∞ so does be_0/a. Equation (1-4) describes the steady state equilibrium at which I will evaluate the possiblity of credit when I come to the next step.

In this steady state equilibrium we can calculate the expected discounted value of lifetime utility for those with and without inventory (W_e and W_u respectively) assuming that production opportunities are worth carrying out. (If they are not, W_u is zero.) For each value, the utility rate of discount times value equals the expected dividend plus the expected capital gain.

$$rW_e = be(y - W_e + W_u) \tag{1-5}$$

$$rW_u = a(W_e - W_u - c) \tag{1-6}$$

Those with inventory wait for the utility from consumption plus a change in status to being without inventory. Those without inventory wait for the disutility of labor plus a change in status. Note that the value equations are homogenous of degree one in (c,y) and homogeneous of degree zero in (a,be,r) and so in (a,b,r) given (1-4).

All projects will be taken if the capital gain from production, $W_e - W_u$, exceeds the cost of a project. To have an equilibrium at e_o, the economy must be productive enough to satisfy this condition, which I will call the breakeven constraint and denote by (B_o). Subtracting (1-6) from (1-5), we can write this breakeven condition as

$$(B_o): \quad c \leq W_e - W_u = \frac{bey + ac}{r + a + be} \tag{1-7}$$

Solving (1-7) we see that willingness to produce for sale can be written as

$$(B_o): \quad \frac{c}{y} \leq \frac{be}{r + be} \tag{1-8}$$

For later use we note that

$$rW_u = a(W_e - W_u - c) = \frac{a(be(y-c) - rc)}{r + a + be} \tag{1-9}$$

$W_u > 0$ is equivalent to $c < W_e - W_u$.

In Figure 1 we plot the breakeven condition relating c/y to b/a for given values of r/a where e in (1-9) is set equal to e_o, given in (1-4)and dependent on b/a. We have an equilibrium below the curve B_o. That is, projects are worth undertaking if the arrival rate of trade opportunities is sufficiently large relative to the ratio of cost to value of a good. There are five parameters in this economy but with two normalizations there are really only three. There is the utility of consumption and the disutility of labor. All we are really interested in is their relative size, c/y which is on the vertical axis. There are three flow rates per unit time, the utility discount rate, the arrival rate of production opportunities, the arrival rate of trading partners. Since we are free to measure time any way we want, again we have a normalization. I divide through by a so b/a is on the horizontal axis and I've drawn the curves for three different utility discount rates.

That completes the picture of the economy. It is simpler than my 1982 paper by having all of these projects cost the same. Of course there is another uniform equilibrium in this economy. If nobody ever produces anything then it is obviously not worthwhile to produce for trade. Even with no trade there is a possibility of introducing credit. We will return to that equilibrium below.

2. A Single Credit Transaction

We now wish to consider the introduction of credit to this barter economy, preserving the details of the search-trade technology and the simplicity of uniform inventory holdings. To do this we introduce two assumptions. The first is that repayment of a loan involves no transactions cost and represents consumable output. (It would be straightforward to add a transaction cost (in labor units) paid by either the borrower or the lender.) That is, individuals have sufficient memory to costlessly find each other to complete the (delayed) barter transaction but this memory (or perhaps taste for variety) does not permit a new transaction at the same time, nor the opening of a regular channel of trade. Nor do two individuals without inventory enter into contracts for two future deliveries.[2]

The second assumption is that credit terms are smoothly varied by changing the probabilities in lotteries for delivery of present and future goods. Let us consider a pair of individuals who have come together in this no credit steady state equilibrium. One of them has a unit of the good to trade and the other one does not. The proposed trade begins with realization of a random variable. With probability p, the inventory on hand is delivered for immediate consumption. Independent of the outcome of the random variable, the debtor promises that at his next opportunity to produce he will carry out production and with independent probability q will deliver that good to the creditor.

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[2] I suspect that costs of completing transactions could be used to justify the value of one delayed payment but not two. A need to inspect goods, plus symmetry in evaluations is an alternative route to justification.

Unless the borrower is known to be totally honest, the lender must check whether it is in the borrower's interest to produce and engage in this lottery. (A more complicated argument would consider subjective probabilities of total honesty.) That is, the lender must ask whether the borrower has an incentive to repay this loan if made. The answer depends on the structure of penalties available for enforcing contracts. I assume one particular example of penalty. I would be unhappy if the results depended critically on the particular choice of penalties for refusal to pay since penalties vary enormously with institutional structure; that is, they are very sensitive to the way the model is set up. I assume that it is observable to everybody whenever a production opportunity is carried out and that the legal system is available to enforce probabilistic delivery to the lender if one is carried out. But I assume that no one can observe whether there is in fact an opportunity which is not taken. So if a lender chooses not to pay back, he does that by ceasing production. In other words, not repaying a loan implies dropping out of the economy, going to the autarchic state which I have implicitly modeled as the origin. Thus a loan is a form of equity, being a claim on future production at whatever level occurs. These rules do not conform with modern bankruptcy law. They do preserve the characteristic that use of bankruptcy decreases the value of trading opportunities and they have the advantages of simplicity and of easy construction of a consistent equilibrium.[3]

Debtors will repay if it is worthwhile to pay the cost of production to remain in the economy. Thus, the debtor will repay if q is sufficiently small that it is worth paying c for the lottery of being in position W_u with probability q and position W_e with probability (1-q). Since this credit transaction is mutually advantageous the pair will choose the largest possible q. Provided q is less than one the maximal promise to repay which is credible satisfies:

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[3] For a more extensive discussion of this model relative to bankruptcy law see Diamond [1986].

$$qW_u + (1-q)W_e = c \tag{2-1}$$

or

$$q = \frac{W_e - c}{W_e - W_u}. \tag{2-2}$$

In order to have $q<1$, we need $W_u<c$. We restrict analysis below to parameters that yield a solution to (2-1) with $q<1$ at equilibrium with credit. Otherwise we would need to examine lending to a debtor. For $q < 1$ at the no credit equilibrium we need

$$\frac{c}{y} > abe(r^2+2ar+rbe+abe)^{-1}. \tag{2-3}$$

With repayment assured we need to determine p which, together with q, determines the implicit interest rate. To calculate the lender's gain from this trade, we compare the probabilistic dynamic programing cost of giving up a unit of inventory with the probabilistic utility gain from consumption adjusted for the expected waiting time. The trade is advantageous to the lender if

$$p(W_e - W_u) \leq \frac{qay}{r + a}. \tag{2-4}$$

The condition is that the probability of delivering the good times the value of a unit of inventory be less than the expected value of delayed payment. Delayed payment is realized as a Poisson process with arrival rate a. The utility payoff y is discounted at rate r.

The borrower needs to compare the cost of being in debt to the probabilistic gain of current consumption. The trade is advantageous to the borrower if

$$W_u - \left(\frac{a}{r + a}\right)(qW_u + (1-q)W_e - c) \leq py. \tag{2-5}$$

If he enters the trade, the debtor switches from the status of waiting for production (with value W_u) to waiting for the opportunity to repay his debt (at cost c) which will then restore him either to the status of waiting for production or to having a unit of inventory available for trade. Setting q as large as is credible reduces (2-5) to

$$W_u \leq py. \tag{2-6}$$

Combining (2-4) and (2-6) there is a mutually advantageous trade if there is a value of p, $0 < p \leq 1$, satisfying

$$\frac{W_u}{y} \leq p \leq \left(\frac{a}{r+a}\right)\frac{qy}{W_e-W_u}. \tag{2-7}$$

Substituting for q from (2-2) and for W_e-c from (1-9), we can write this as

$$\frac{W_u}{y} \leq p \leq \left(\frac{yW_u}{(W_e-W_u)^2}\right). \tag{2-8}$$

Since $y \geq W_e-W_u \geq c$ when production is worthwhile (c.f. (1-7)), there is always an interval of values of p that can satisfy this condition. From the assumption that $q \leq 1$,

$W_u \leq c < y$. Thus the lower bound on this interval is between zero and one. We note that if the economy is on the knife edge of just satisfying the break even condition (1-7) so that W_u=0 and W_e=c, then q=0 in (2-2) and (2-4) requires p=0 as well.

It is natural to think of the implicit interest rate for this credit transaction. The claim on future stochastic delivery of the consumer good trades at a "price" p in terms of the current consumer good. Given the stationary character of the Poisson process determining the date of repayment, the price does not change over time. Thus the implicit interest rate on this transaction times the "price" is equal to the flow probability of a repayment, a, times the expected return on repayment, which is the probability of delivery, q, less the loss in value of the asset, p, which becomes zero on repayment. Thus we have

$$i/a = (q-p)/p \tag{2-9}$$

3. Basic Model With Credit

We turn now to equilibrium with credit. We assume that if you have no goods in inventory and if you are not in debt, then somebody with inventory is willing to lend to you, willing to provide you (stochastically) consumption in return for (stochastic) future delivery of goods. However, I will not consider the network of being willing to lend to someone because he is a creditor of someone else. Also we restrict analysis to parameters for which the (endogenous) credit limit q is less than or equal to one. Thus there are three possible positions an individual can be in. (1) He can have a unit of good available for trade. He may or may not also be a creditor, but that is just future consumption, it does not affect his trading abilities. As before, e is the fraction of the population in this position. (2) He may not have a unit available to trade and also not be a debtor. We denote by u the fraction of the population in that position. Or, (3) he may be a debtor. The fraction of the population in that position is denoted by d. These people cannot borrow any more; they are up against their credit limit. We will need to examine the breakeven condition to check whether the economy is indeed in equilibrium.

We now consider dynamics where credit is given by those with inventory to finance all potential transactions with nondebtors but no transactions with debtors. The fraction with inventory, e, drops by any contact with someone with inventory and drops with the probability p from a contact with a nondebtor. The number with inventory rises from any production by a nondebtor. The latter lowers the fraction of nondebtors without inventory. This fraction also rises whenever two agents with inventory trade and whenever a debtor produces. There is an expected change of (p-1) in the number of nondebtors without inventory from a trade involving credit. The number of debtors, d, falls from production and rises from the acceptance of credit. Production by a debtor raises nondebtors by q and those with inventory by 1-q. Thus we have the differential equations

$$\begin{aligned} \dot{e} &= -be(e+up) + au + ad(1-q), \\ \dot{u} &= -au + be^2 + adq + beu(p-1), \qquad (3\text{-}1) \\ \dot{d} &= -ad + beu. \end{aligned}$$

It is convenient to eliminate d from these equations, giving us:

$$\begin{aligned} \dot{e} &= a - be^2 - beup + a(1-e-u)(1-q), \\ \dot{u} &= -au + be^2 + a(1-e-u)q + beu(p-1). \end{aligned} \qquad (3\text{-}2)$$

Since $\dot{e} + \dot{u} = a(1-e-u)-beu$ any intersection of $\dot{e} = 0$ and $\dot{u} = 0$ with $e > 0$ and $u > 0$ must have $e + u < 1$.

Setting $\dot{e} = 0$ and solving for u we have

$$u = \frac{be^2-a(1-e)(1-q)}{aq - bep}. \qquad (3\text{-}3)$$

Setting $\dot{u} = 0$ and solving for u we have

$$u = \frac{be^2 + a(1-e)q}{a(1+q) + be(1-p)}. \qquad (3\text{-}4)$$

Equating (3-3) and (3-4) we have a cubic equation for e. Defining $a' = a/b$ and $s = p+q-1$ we can write the cubic equation as

$$e^3 + a'(1-s)e^2 + (a'^2 + a's)e - a'^2 = 0. \qquad (3\text{-}5)$$

This equation has a unique root between zero and one. For all values of a' and s with $0 < a'$ and $-1 \leq s \leq 1$, the cubic is negative at $e = 0$ and positive at $e = 1$. If there is an extremum between zero and one it is a minimum. Thus there is precisely one root between zero and one. Diagramming (3-3) and (3-4) one sees that this root occurs where

$$aq > bep. \tag{3-6}$$

Define

$$x = (3(a'^2 + a's) - a'^2(1-s)^2)/3,$$

$$z = (2a'^3(1-s)^3 - 9a'(1-s)(a'^2 + a's) - 27a'^2)/27,$$

$$A = (-z/2 + (z^2/4 + x^3/27)^{1/2})^{1/3}, \tag{3-7a}$$

$$B = (-z/2 - (z^2/4 + x^3/27)^{1/2})^{1/3}.$$

Then, we can write the equilibrium level, e_c as

$$e_c = A + B - a'(1-s)/3. \tag{3-7b}$$

Implicitly differentiating (3-5), we see that e_c decreases with s and so decreases with p and q; that is, the greater the probability of delivery in a stochastic credit transaction the lower the steady state stock of inventory.

Eliminating aq-bep from (3-3) and (3-4) we have

$$u = \frac{a(1-e)}{a + be}. \tag{3-8}$$

Thus u_c is decreasing in e_c and so increasing in p and q.

Next we examine wealths under the assumption that all projects are carried out and credit is extended up to a credit limit of q<1. As above, the utility discount rate times the value of being in a position equals the expected flow of utility dividends and capital gains.

$$rW_e = be(y - W_e + W_u) + bu(\frac{aqy}{r+a} - pW_e + pW_u), \tag{3-9}$$

$$rW_u = be(yp - W_u + W_d) + a(W_e - W_u - c), \tag{3-10}$$

$$rW_d = a(qW_u + (1-q)W_e - W_d - c). \tag{3-11}$$

We write the cost of a project just worth taking to shift status from positions u to position e as:

$$c^* = W_e - W_u. \tag{3-12}$$

As above, q is set as large as possible consistent with a willingness to pay back. Thus $W_d = 0$. Subtracting (3-10) from (3-9) we have

$$(r + a + be + pbu)c^* = be W_u + ac + bu \frac{aqy}{r+a} + bey(1-p), \qquad (3\text{-}13)$$

$$(r + be)W_u = bepy + ac^* - ac. \qquad (3\text{-}14)$$

Solving from (3-13) and (3-14) we have

$$(r + a + be + pbu - \frac{abe}{r+be})\, c^* = \frac{r(ac-bepy)}{r + be} + bu \frac{aqy}{r+a} + bey. \qquad (3\text{-}15)$$

4. Terms of Credit

With q set as large as possible consistent with a willingness to pay back, we have $W_d = 0$ or

$$q = \frac{W_u + c^* - c}{c^*}. \qquad (4\text{-}1)$$

Positive production ($c^* \geq c$) implies $q \geq 0$. For $q \leq 1$, we need $W_u \leq c$.

In order to determine the remaining term of credit, (the value of p) we use the Nash bargaining solution for a single credit transaction, assuming all other credit transactions occur with delivery probability p; i.e., that position values satisfy (3-9) to (3-11). We want a fixed point in p so that the condition for the Nash bargaining solution is satisfied. Without this credit transaction the pair of agents who have met have values (W_e, W_u). With a credit transaction with delivery probability p, their values become

$$(W_e + aqy/(r + a) - p(W_e - W_u),\ py). \qquad (4\text{-}2)$$

Using the willingness to produce, (3-12), the gains from trade can be written as

$$(\frac{aqy}{r+a} - pc^*,\ py - W_u). \qquad (4\text{-}3)$$

The Nash bargaining solution satisfies the maximization problem

$$\text{Max } (py - W_u)(\frac{aqy}{r+a} - pc^*) \qquad (4\text{-}4)$$

Calculating the first order condition and solving for p we have

$$p = \frac{c^* W_u + aqy^2/(r+a)}{2yc^*} \tag{4-5}$$

We note that $q \geq 0$ implies $p \geq 0$. For there to be a mutually advantageous trade the gains to trade to both parties must be nonnegative. Thus p must satisfy

$$\frac{W_u}{y} \leq p \leq \frac{aqy}{(r+a)c^*} \tag{4-6}$$

The Nash bargaining solution value of p is the mean of the two limits in (4-6). Thus we have a mutually advantageous trade provided

$$c^* W_u \leq \frac{aqy^2}{(r+a)} \tag{4-7}$$

or

$$c^{*2} W_u \leq \left(\frac{a}{r+a}\right) y^2 (W_u + c^* - c)$$

5. Equilibrium with Credit

For an equilibrium with a credit limit of q, $0 < q < 1$, and a probability of delivery p, $0 < p < 1$, we need to have a solution to the six equations (3-7), (3-8), (3-14), (3-15), (4-1) and (4-5), yield probability values for p and q between zero and one and a willingness to produce, c^*, satisfying $c^* > c$. To distinguish this breakeven condition from that above we drop the subscript and write it as (B). Since (B) implies that both p and q are nonnegative, we have an equilibrium with credit if the three conditions are met:

$$p \leq 1, \quad q \leq 1, \quad (B): \quad c^* \geq c \tag{5-1}$$

Using (3-15) the breakeven constraint can be written as

$$(B): \quad \left(r + be + pbu - \frac{abeq}{r+a+be}\right) \frac{c}{y} \leq \frac{b^2 e^2 p}{r+a+be} + \frac{buaq}{r+a} + be(1-p) \tag{5-2}$$

We evaluate this condition at e_c, satisfying (3-7) and u_c satisfying (3-8).

To examine the curve on which the breakeven condition is just satisfied, we note from (4-1) and (3-14) that when $c^* = c$, we have

$$q = \frac{W_u}{c} = \frac{bepy}{(r+be)c} \tag{5-3}$$

With c* = c, substituting for q from (4-1) in (4-5) we have

$$p = \left[c+\left(\frac{a}{r+a}\right)\frac{y^2}{c}\right]\frac{W_u}{2yc}. \tag{5-4}$$

Substituting for p/W_u from (3-14) we have be/a as a function of c/y and r/a

$$\frac{r+be}{be} = \frac{1}{2}\left(1+\left(\frac{a}{r+a}\right)\frac{y^2}{c^2}\right) \tag{5-5}$$

Using (5-3) to eliminate q, (3-5) can be solved for p as a function of b/a, be/a, and c/y. Similarly, using (3-8) to eliminate u and (5-3) to eliminate q, (3-15) evaluated at c* = c can be solved for p as a function of the same variables. Equating these expressions for p, and recognizing that be/a is a function of c/y and r/a in (5-5) we get an expression for c* = c which is quadratic in b/a. Using the normalizations y=a=1, this expression is

$$\begin{aligned} &\left[\frac{be}{1+r} - c^2(r+be)\right] b^2 \\ &+ \left[(be)^2\left(\frac{rbe-1}{1+r}\right) - berc(1+be) + bec^2(r+be-rbe-r^2)\right]b \\ &+ \left[b^2e^2(r+be)^2c^2 + (b^3e^3+b^2e^2)(r+be)c+b^3e^3\left(\frac{1-rbe}{1+r}\right)\right] = 0. \end{aligned} \tag{5-6}$$

In Figure 2, for r/a=.1, we show (5-6) along with the no credit breakeven curve (B_o) which is an additional locus on which c* = c (with p=q=0).

The equations are only valid when $q \leq 1$. Thus we also plot the locus q=1, which is the locus W_u=c. The constraint $p \leq 1$ was not binding. The relevant parts of the c*=c locus are to the right of B_o and above W_u=c. Thus the shaded region in Figure 3 shows parameters which yield an equilibrium credit limit between zero and one.

With 6 equations in 6 unknowns, (e, u, p, q, c*, W_u), the search for multiple equilibria was by calculated example, not analysis of the equations. No multiple equilibria were found.

6. Comparative Statics

In Figure 4, we show e, u, p, and q as functions of b/a for r/a = .1 and c/y = .8. The Figure shows the values only for the parameter values for which there exists an equilibrium. In Figure 5 we show the implicit interest rate, i, (given in (2-9)) as a function of b/a for the same values. Figures 6 and 7 show the same variables as functions of c/y for r/a=.1 and b/a = 1.27. Again, the curves are drawn only for values for which there exists an equilibrium. In these calculations, the implicit interest rate is positive.[4] The monotonicity properties shown in the figures are present on figures drawn for a number of other parameter values.

A more efficient trade technology (increase in b/a) directly reduces the stock of inventory. In turn, this increases the rate of production in the economy, a(1-e). More efficient trade enhances the value of being in the trade network, tending to raise q and so p. These increases, in turn, also contribute to the decline in inventories. The figures suggest that the indirect effects of e and u on p and q do not offset these direct effects.

7. No-Trade Equilibrium

Above, we saw that credit would be introduced to the no credit equilibrium with positive production, except at the knife edge where the break even condition was just satisfied. In this section we examine the same question for the barter equilibrium with no production and no trade. If everyone believes that future credit transactions will not occur, then they will not occur, since there is no cost associated with being excluded from future trading opportunities. If agents believe that future credit transactions will occur if individually rational

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[4] Assuming equal gains from trade rather than the Nash bargaining solution I was able to prove i>0. With the Nash bargaining solution, the ratio of marginal utilities of probability changes, c*/y, enters the formula, complicating analysis.

(given belief in their occurrence), then they may occur. To explore this possibility, we first derive a condition such that naive extrapolation of the meeting probabilities justifies production for a credit transaction; i.e., destroys the no production equilibrium. However this naive extrapolation may have a Ponzi character to it. (In fact, this calculation may seem worthwhile even when $c>y$.) We then add the condition that after a single credit transaction the economy converges back to the no production equilibrium, implying that the myopic forecast is correct. This results in a more stringent condition that is sufficient for the introduction of credit.

Consider an equilibrium with no inventories and no production. A single individual considers bearing the cost c to produce a unit. We denote by V the dynamic programming value of this unit to the producer. With arrival rate b the individual experiences the arrival of an individual to whom to propose a credit transaction. As above a credit transaction is described by a pair of probabilities (p,q) of delivery of goods immediately and after future production. Both parties to the credit transaction believe that in the event of nondelivery of the good, that good can be the basis of a future credit transaction (which arrives at rate b) on the same terms as this one. In order for this credit transaction to occur, three conditions must be satisfied. First, the initial production must seem worthwhile

$$V \geq c. \tag{7-1}$$

Second, later production to satisfy the debt must seem worthwhile. The debtor will have a unit of the good as a basis for a future credit transaction with probability (1-q). Thus the later production constraint is

$$(1-q)V \geq c. \tag{7-2}$$

Since (7-2) is more stringent than (7-1), we can ignore (7-1). Third, the credit transaction must seem worthwhile to the debtor. With production for repayment worthwhile, the credit transaction is worthwhile with any nonnegative value of p. This peculiar result follows from the lack of alternative activities since production for barter is not profitable in this equilibrium and we have assumed that the debtor can't take the idea of credit and produce to become a creditor rather than becoming a debtor. We will review that assumption below.

To examine the condition such that a pair of probabilities (p,q) can be found which satisfy (7-2), we need to derive the value of a unit of inventory V. The dynamic programming equation for V is that the utility discount rate times V is equal to the arrival rate of a credit partner times the value of a credit transaction to the lender. The value of the credit transaction is the expected value of later consumption $(\frac{a}{r+a})qy$ less the expected cost of delivering the good, pV. Thus we have

$$rV = b((\frac{a}{r+a})qy-pV). \qquad (7\text{-}3)$$

Solving for V we have

$$V = abqy(r+a)^{-1}(r+bp)^{-1} \qquad (7\text{-}4)$$

We can now write the condition for the introduction of credit from (7-2) as

$$(1-q)q > \frac{(r+a)}{a}\frac{(r+bp)}{b}\frac{c}{y} \qquad (7\text{-}5)$$

The product q(1-q) varies between 0 and .25 as q varies between 0 and 1. Thus we can find a satisfactory value of q provided the right hand side of (7-5) is less than .25. The smallest value of this expression is achieved by setting p=0. That is, the most favorable case for the possibility of credit comes from marketing the idea of credit; there is no need to actually deliver any goods to initiate the credit contract. This strongly suggests the potential Ponzi nature of the introduction of credit. Note that with p less than one and r small relative to b, (7-6) can be satisfied with c>y.

If goods are never delivered at the initiation of a credit transaction (p=0) then the stock of inventory in the economy will grow, never returning to the zero stock initial position. This makes the naive forecast of an arrival rate b of potential debtors wrong. Thus we add a second condition that the stock of inventory returns to zero at the probabilities (p,q). This ensures that the naive forecast is correct.

Denote the number of individuals with goods to trade by e and the number of debtors by d. Then, in the neighborhood of zero these numbers satisfy

$$\dot{e} = a(1-q)d - bpe, \qquad \dot{d} = be - ad. \tag{7-6}$$

For the origin to be locally stable, we need

$$p > (1-q). \tag{7-7}$$

For the introduction of credit without Ponzi expectations we need to satisfy (7-5) and (7-7). That is, we need to find a value of q, $\frac{1}{2} < q < 1$, such that

$$q(1-q) > \frac{(r+a)}{a}\frac{(r+b-bq)}{b}\frac{c}{y} \tag{7-8}$$

The left hand side of (7-8) is quadratic in q while the right hand side is linear. To solve for parameter values for which we can find values of q satisfying (7-8), we solve for parameter values so that the two curves are tangent. Calculating the values for which this condition holds, the no production equilibrium is not sustainable in the presence of credit possibilities with rules as modeled here when

$$\frac{r+b}{b} < \left[1+2\left(\frac{r+a}{a}\right)\left(\frac{c}{y}\right) + \left(\frac{r+a}{a}\right)^2\left(\frac{c}{y}\right)^2\right]\left[4\left(\frac{r+a}{a}\right)\left(\frac{c}{y}\right)\right]^{-1} \tag{7-9}$$

When r is small relative to both a and b, (7-9) can be satisfied for nearly all c/y less than one.

In assuming that the offer of credit is accepted if profitable, we have preserved the assumption that autarchy is the only alternative. However the introduction of the idea of credit introduces a second possibility unless the initiator of the idea has contractually bound the would-be debtor not to use the idea except in contract with the initiator. This is a common sort of contract with intellectual property. Without such a contract, credit is accepted only if it is more valuable than waiting to produce for a future credit contract. Assuming initiation of only one credit contract is contemplated, this requires

$$p > \left(\frac{a}{r+a}\right)^2 \left(\frac{b}{r+bp}\right) q^2. \tag{7-10}$$

REFERENCES

Chatterjee, Satyajit, "Market Participation and Macroeconomic Equilibrium," working paper, University of Iowa, June 1986.

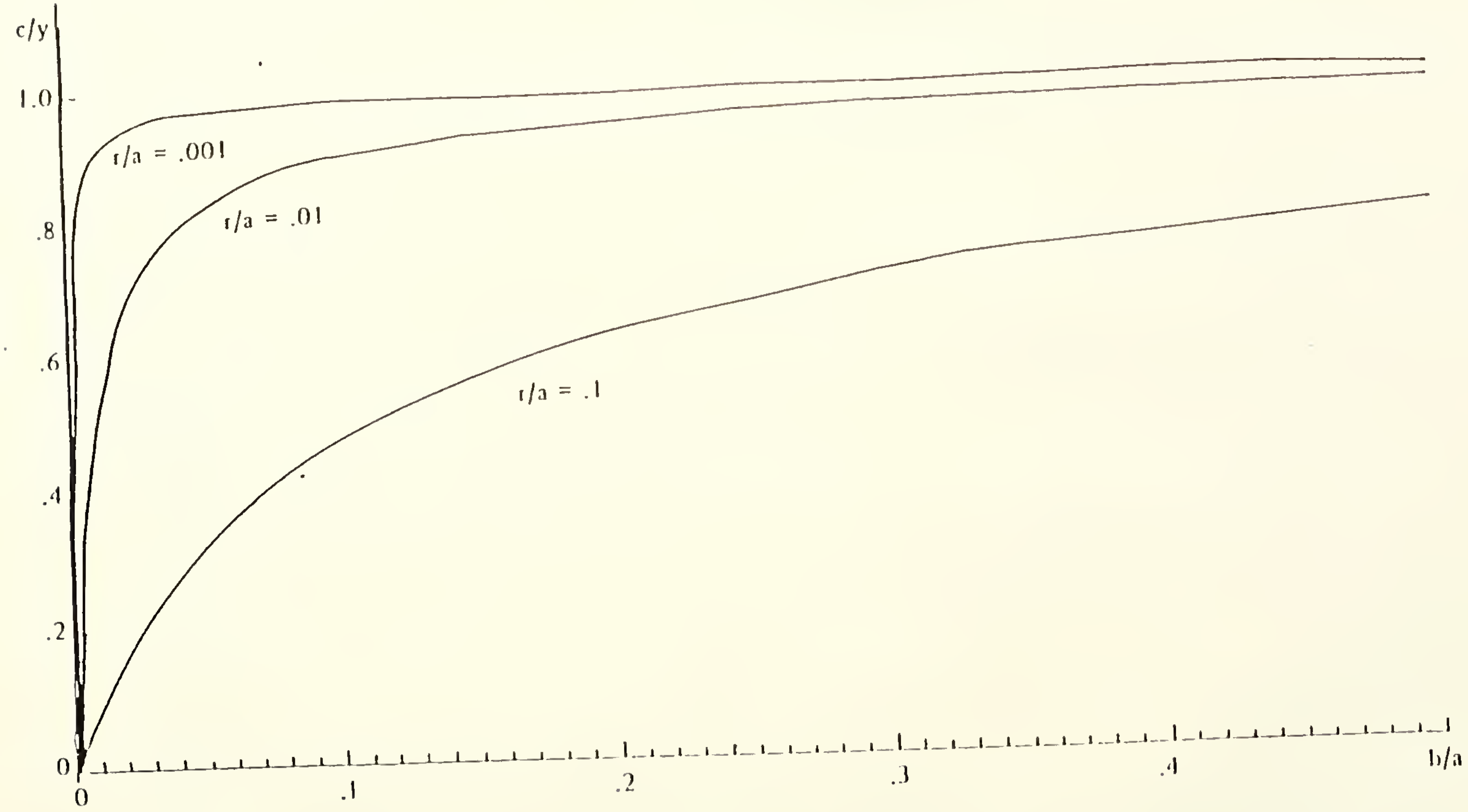
Diamond, Douglas W. and Dybvig, Philip H., "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy, 91 (1983) 401-419.

Diamond, Peter, "Aggregate Demand Management in Search Equilibrium," Journal of Political Economy, 90 (1982) 881-894.

———, "Stochastic Credit in Search Equilibrium," MIT Working Paper 421, June 1986.

Pagano, Marco, "Endogenous Stock Market Size and Asset Price Volatility," unpublished, MIT, March 1986.

Figure 1. Breakeven Condition



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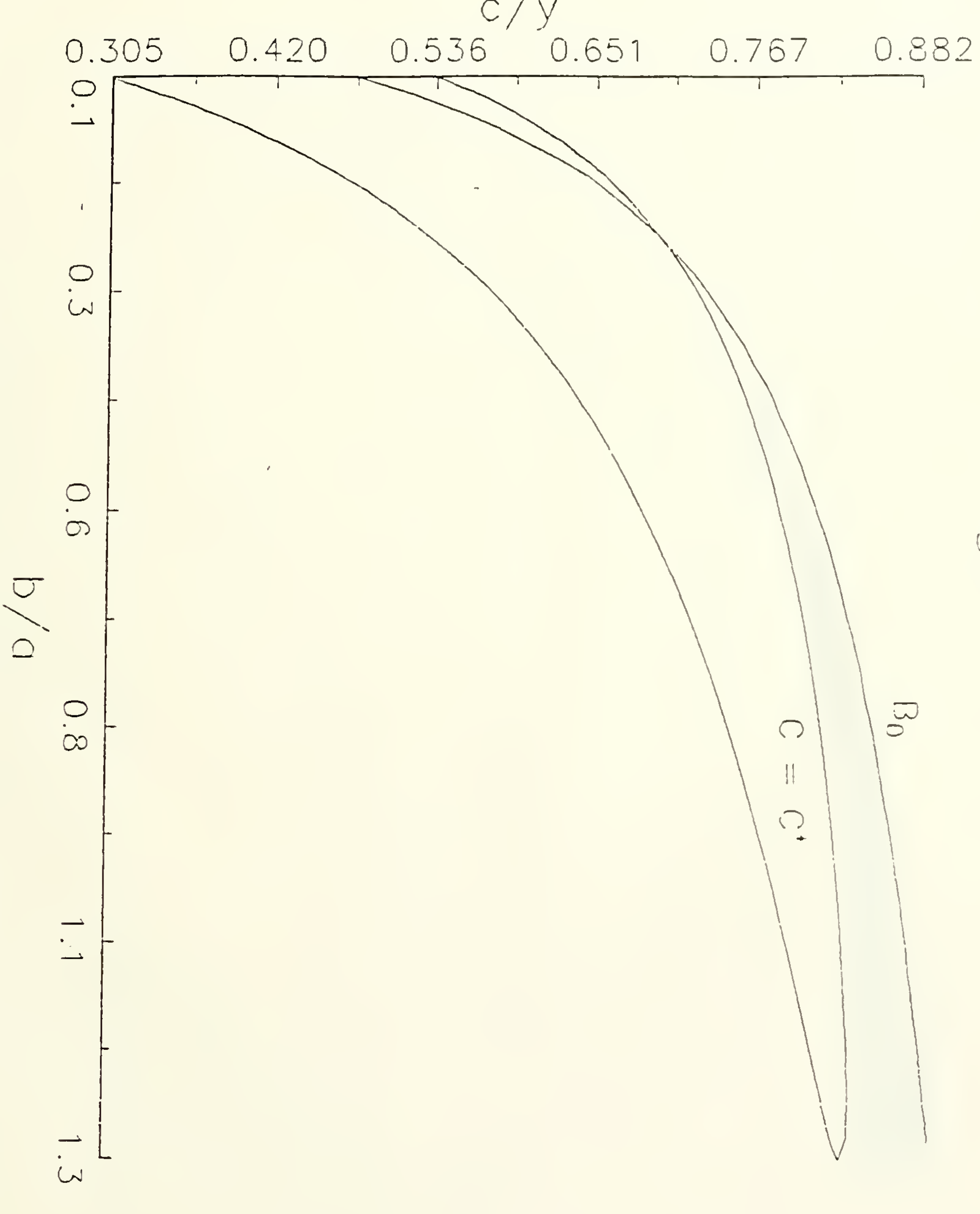

figure 2

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figure 3

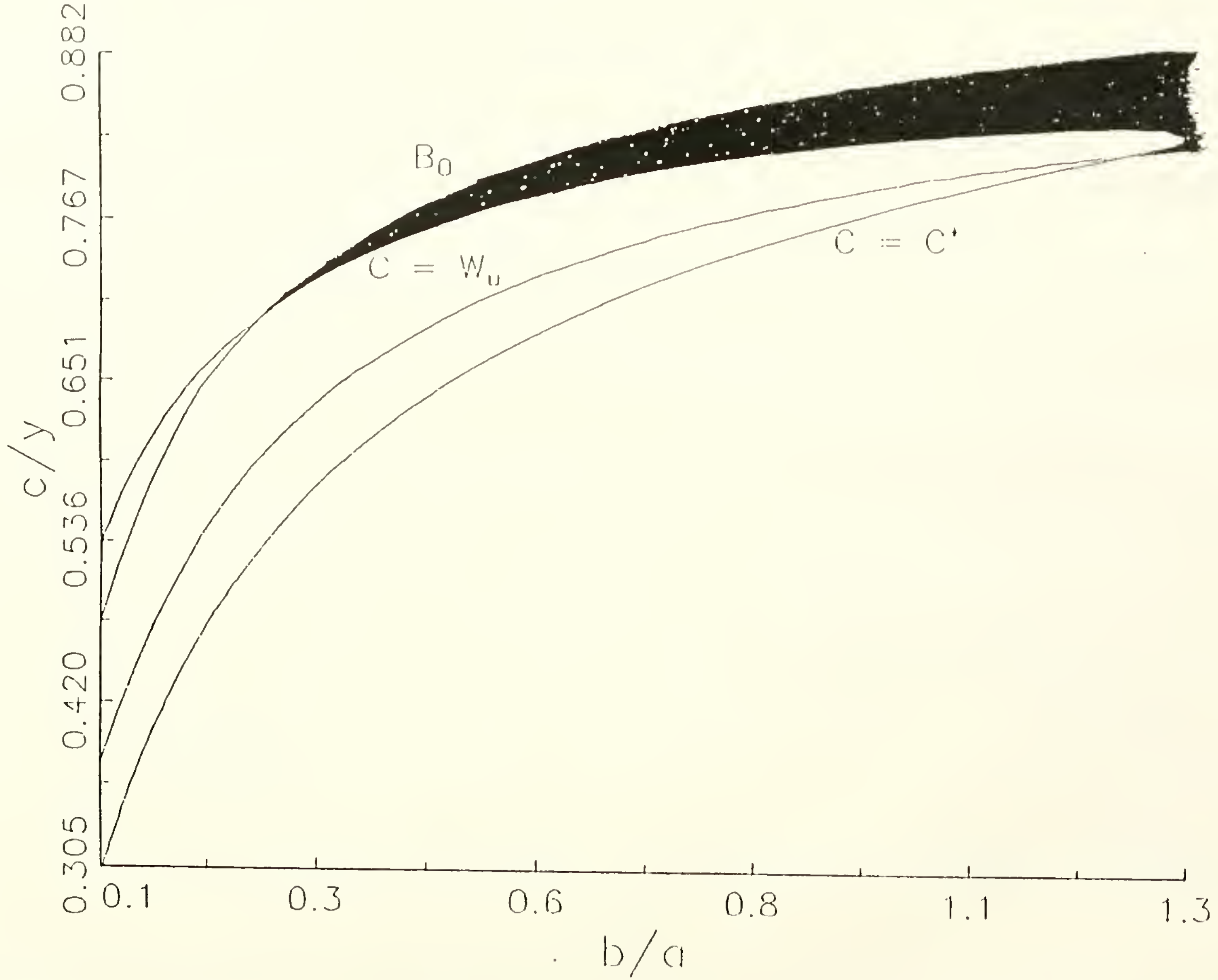


fig4 Apr. 1, 1988 5:16:20 PM

figure 4, (c/y) = 0.8, r = .1

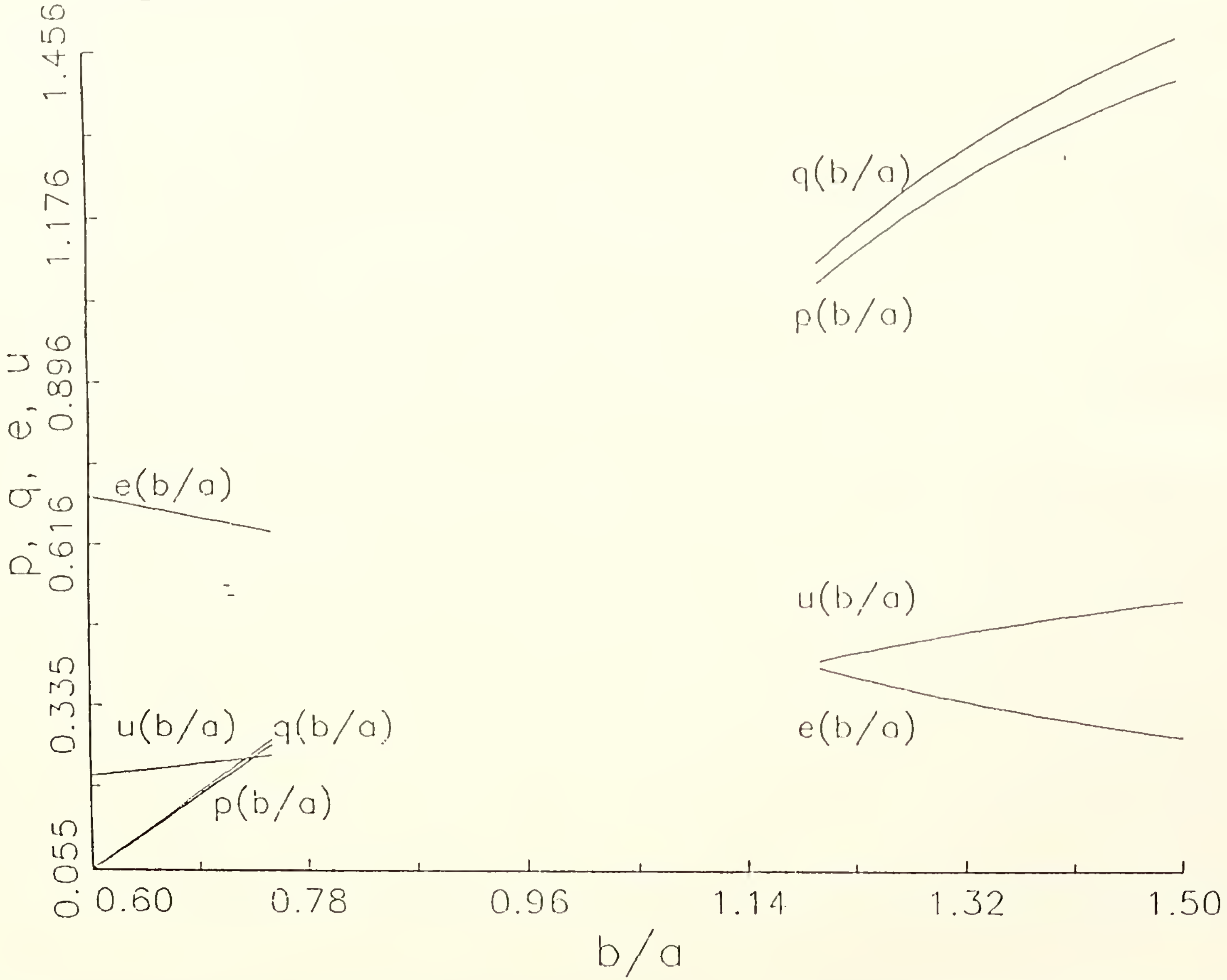


fig5 Apr. 1, 1988 5:23:45 PM

figure 5, (c/y) = 0.8, r = .1

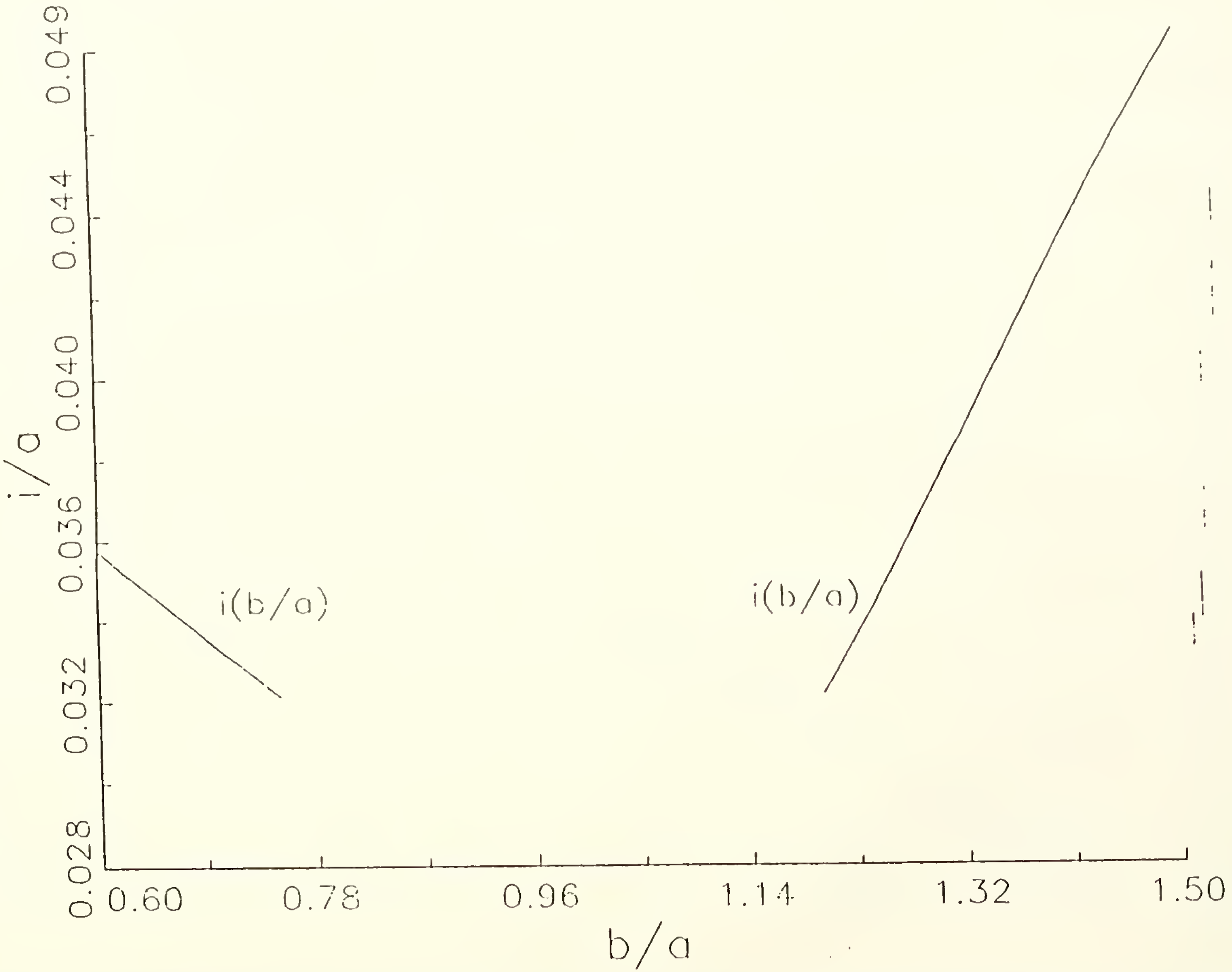


fig6 Apr. 1, 1988 6:31:48 PM

fig. 6, (b/a) = 1.27, r = .1

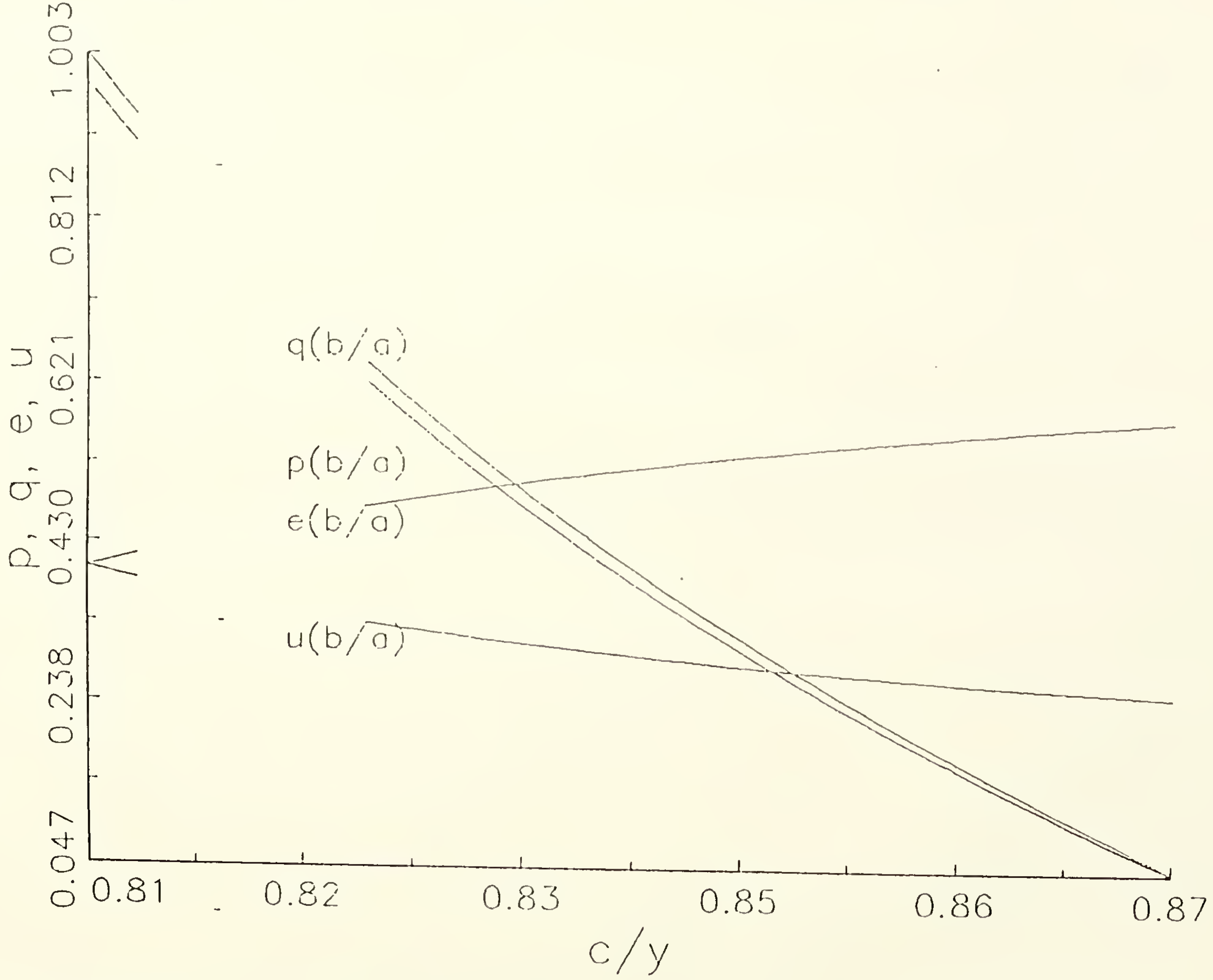
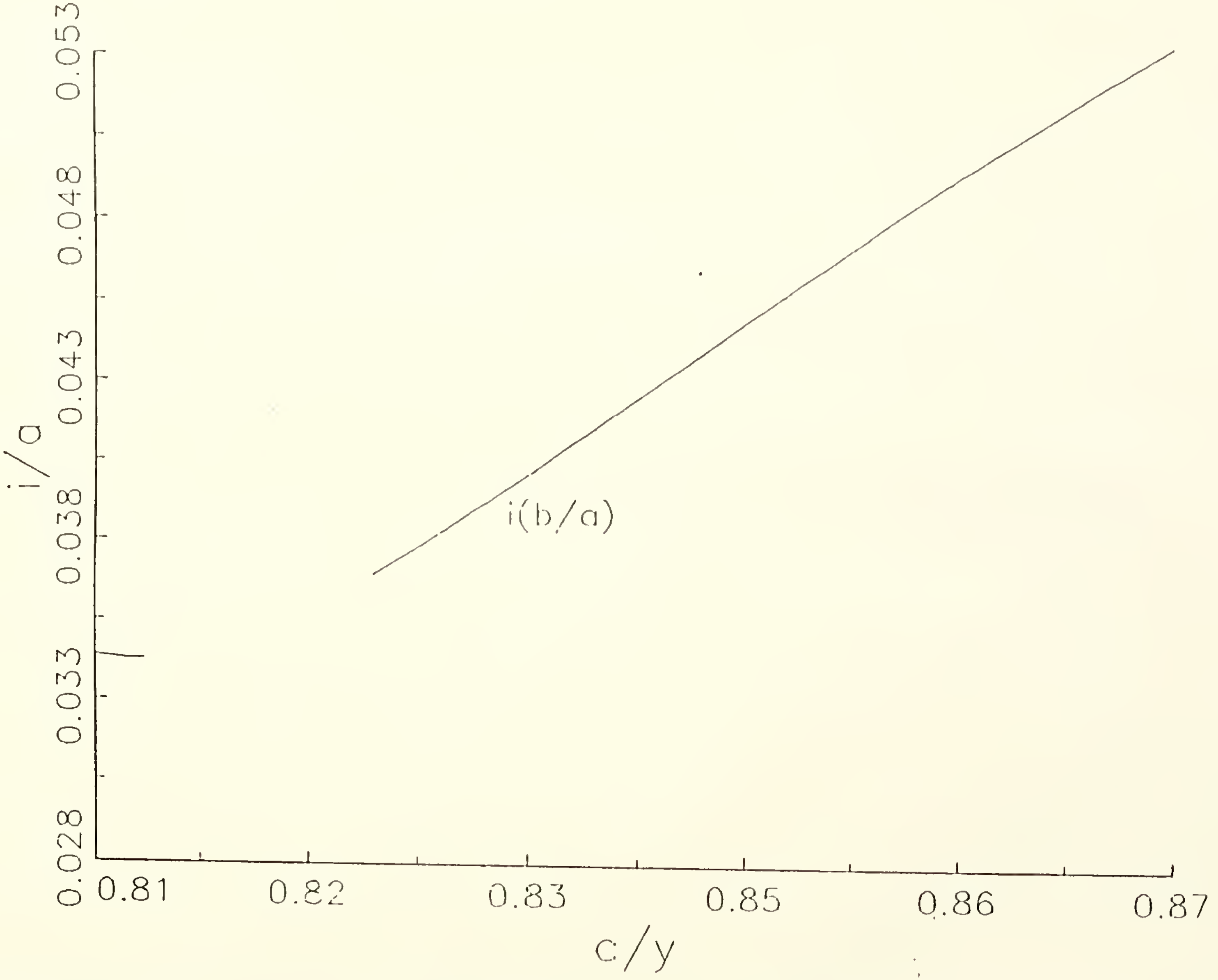


fig7 Apr. 1, 1988 6 36:23 PM

fig. 7, (b/a) = 1.27, r = .1



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