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A THEORY OF DEBT BASED ON THE INALIENABILITY OF HUMAN CAPITAL

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ABSTRACT

Consider an entrepreneur who needs to raise funds from an investor, but cannot commit not to withdraw his human capital from the project. The possibility of a default or quit puts an upper bound on the total future indebtedness from the entrepreneur to the investor at any date. We characterize the optimal repayment path and show how it is affected both by the maturity structure of the project return stream and by the durability and specificity of project assets. Our results are consistent with the conventional wisdom about what determines the maturity structure of (long-term) debt contracts.

1. Introduction

Consider an individual entrepreneur who has access to a profitable investment project, but does not have the funds to finance it. In an ideal world, the entrepreneur would raise the capital from an outside investor (e.g. a bank), promising in return a sufficient fraction of future cash flows that the investor breaks even. Such an ex-ante division of surplus may be unenforceable, however, if the entrepreneur always has the option to repudiate the contract by withdrawing his human capital from the project. The possibility of repudiation puts an upper bound on the total amount of future indebtedness from the entrepreneur to the investor at any date. We show that the result of this constraint is that some profitable projects will not be financed. In addition, we characterize the optimal repayment path between the entrepreneur and the investor when the project is financed, and show how this repayment path is affected both by the maturity structure of the project return stream and by the durability and specificity of the project assets. Our results are consistent with the conventional wisdom offered by practitioners about what determines the maturity structure of (long-term) debt contracts.

Throughout the paper, we will focus on the case where the project's return stream and its liquidation value (which we take to correspond to the best alternative use of the project assets) are perfectly certain. The assumption of perfect certainty is very restrictive, but it turns out that this case is already quite rich. We also suppose that, if the entrepreneur repudiates a debt contract, he loses control of the project's physical assets and the investor obtains the right to liquidate them. At this point, the parties can renegotiate the debt contract. However, renegotiation does not necessarily lead to an efficient outcome: in particular, the investor may liquidate even though gross returns exceed liquidation value, because the debtor cannot credibly promise to pay enough of the gross returns back.

It turns out that the assumption of perfect certainty, combined with that of (costless) renegotiation, implies that there is a continuum of optimal debt contracts. Among other things, the parties can write a succession of short-term contracts which are renegotiated, or a long-term contract that is never renegotiated along the equilibrium path. Because of this indeterminacy, we will concentrate on the flow of debt repayments on the equilibrium path. In fact, in the case where the entrepreneur and investor have a common discount rate, and the entrepreneur consumes only at the end of the project, even this flow exhibits considerable indeterminacy. In particular, we will show that there is a slowest repayment path (this involves the largest initial loan from the investor to the entrepreneur, and the largest indebtedness from the entrepreneur to the investor at every date thereafter) and a fastest repayment path (this involves the smallest initial loan from the investor and the smallest indebtedness from the entrepreneur at every date thereafter); and the set of optimal repayment paths consists of everything between these two extremes. The reason for the indeterminacy is that the constraint that the entrepreneur not repudiate puts an upper bound only on total future indebtedness at each date; thus, within limits imposed by the project cash flows, the constraint can still be satisfied if debt repayments are moved forward in time.

This indeterminacy in debt repayments disappears if we relax the assumption of a common discount rate. We will see that, if the entrepreneur's discount rate exceeds the investor's (more precisely, if the entrepreneur has profitable reinvestment opportunities), the unique optimal path is the slowest one; while if the investor's discount rate exceeds the entrepreneur's (more precisely, if the investor has profitable reinvestment opportunities), the unique optimal path is the fastest one.

As well as characterizing those projects that will be undertaken and the slowest and fastest repayment paths, we will analyze how the repayment path varies with the underlying parameters of the model. Among other things, we show that debt repayments will be pushed into the future as the project's assets become more durable; as project returns are earned later; or as the project's assets become less replaceable. These results correspond to the advice practitioners often give: for example, "lend long if the loan is supported by durable collateral"; and "match assets with liabilities". We are not aware of other theoretical models that deliver results of this type.

There is a vast literature on debt, and we do not have space to review it here. There are a few recent papers which, like this one, use the idea that creditors may liquidate in the event of default, because the debtor cannot pay enough of the project returns back. Aghion and Bolton (1991) analyse debt in terms of control over assets; theirs is a one-period model, in which inefficient liquidation arises because the debtor has nonpecuniary benefits of control and, owing to a wealth constraint, cannot bribe the creditor not to liquidate. Bolton and Scharfstein (1990) develop a model in which the threat of the witholding of future investment funds encourages an entrepeneur to repay a loan. They are more concerned with how debt can be used strategically to influence competition in product markets than with the nature of dynamic debt contracts. A series of papers by Diamond -- see, e.g., Diamond (1990, 1991a and b) -- assumes that borrowers cannot use the full value of future returns as backing for financial claims. His focus is rather different from ours: he wishes to explore the relationship between the unobservable quality of a borrower's project and the maturity/seniority structure of debt, as well as to ask whether loans should be placed through a financial intermediary or directly through the market.

Probably the paper in the literature closest to ours is that by Bulow and Rogoff (1989). Theirs is a model of sovereign debt in which a debtor country borrows from a creditor country for current consumption, but cannot commit to repay the loan out of future production. The creditor country has some bargaining power, however: if the debtor repudiates the loan, the creditor can retaliate by blockading the debtor country's trade. There are two main differences between the Bulow-Rogoff paper and ours. First, in their model there is nothing corresponding to irreversible liquidation, and, as a result, there is never any ex-post inefficiency, in the sense that no blockade occurs in equilibrium. In contrast, in our model, if an inappropriate contract were signed (or if the returns were uncertain), there could be inefficient liquidation ex post. Second, because of their concern with sovereign debt, Bulow and Rogoff do not study the role of legally enforceable, long-term contracts in sustaining optimal repayment paths, or obtain comparative statics properties concerning how optimal repayment paths vary with the underlying parameters of the model.

Given that so much has been written on debt, it may be worth rehearsing in what ways our model is new, and why we believe it is simpler than what has gone before. First, ours is a deterministic model, with full information throughout. Second, returns are exogenous in the sense that what the entrepreneur can produce with the assets, and what the assets alone can produce, are fixed. Third, contracts are "complete", in that there are no missing contingencies. The only potential for distortion is that the entrepreneur can, at any time, walk away from the contract; and the punishment for withdrawing his labour is that he loses access to the assets.[1] It turns out that these rudimentary ingredients are enough to generate a theory of debt which is surprisingly rich. The richness comes from the intertemporal structure: what the entrepreneur -- or indeed the investor, should she decide to liquidate and not to renegotiate -- potentially forfeits today from repudiating the contract is sensitive to what he anticipates he can obtain from tomorrow onwards, which in turn will be sensitive to whether or not he can usefully repudiate the contract again tomorrow, etc.

The paper is organized as follows. We set out a discrete time model in Sections 2-3, and in Section 4 find the condition under which the project will be undertaken (Proposition 1). We characterize the set of optimal repayment paths in Section 5 (Proposition 2); and we also extend the model by introducing the possibility of reinvestment. The results for the discrete case are somewhat difficult to interpret, and so in Section 6 we take limits as the number of periods tends to infinity and the length of each period tends to zero. For the limiting continuous time model, we are able to obtain simple formulae for the condition under which the project will be undertaken, and for the set of optimal repayment paths (Proposition 3). Section 7 examines a number of comparative statics properties -- e.g., how the maturity structure of the debt repayment path varies with the model's underlying parameters (Propositions 4A-4D). Finally, in Section 8, we discuss what light the model throws on actual debt contracts, and conclude with some suggestions for further research.[2] Proofs of certain results are gathered in an Appendix.

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[1] In this sense, the model resembles recent incomplete contracting theories of the firm which stress that physical assets are alienable, while human assets are inalienable; see Grossman and Hart (1986) and Hart and Moore (1990).

[2] One important extension of our model is to introduce the possibility of consumption during the lifetime of the project. This will be the subject of a companion paper.

2. The Model

Consider an entrepreneur who has access to a profitable investment project. Suppose that the initial cost of the project is K and the entrepreneur's initial wealth is $w_0 < K$. We will assume that the project has a finite horizon and that time is divided into periods $1,2,\ldots,n$, which are of equal length (however, below we will take limits as $n \to \infty$ and the period length tends to zero). The project's physical assets in combination with the entrepreneur's human capital yield a perfectly certain flow of nonnegative returns $r_1,r_2,\ldots,r_n$ at dates $1,2,\ldots,n$ -- the end of periods $1,2,\ldots,n$, respectively. The project's physical assets also have an alternative use. At any date i ($0 \le i \le n-1$), they can be "liquidated". Liquidation at date i, which is supposed to be irreversible, yields a perfectly certain flow of nonnegative returns $\ell_{i+1},\ell_{i+2},\ldots,\ell_n$ at the end of periods $i+1,i+2,\ldots,n$. These returns can be interpreted as spot rentals in the used capital market; the important point is that liquidation avoids the use of the entrepreneur's human capital.[3] Both return streams are illustrated in Figure 1.

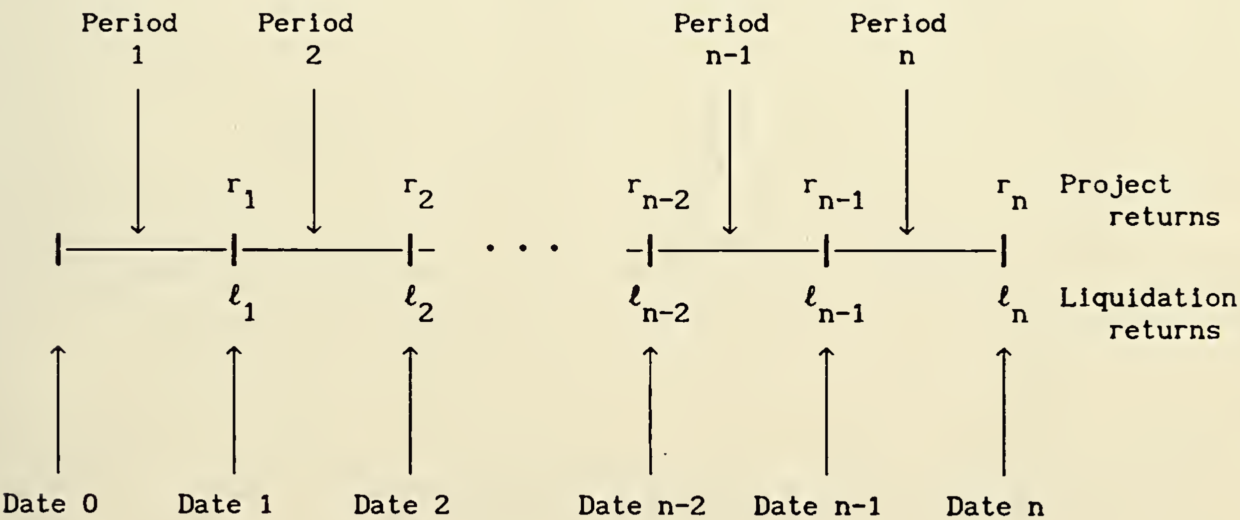

Figure 1

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[3]"Liquidation" could involve a new entrepreneur being brought in, with whom a new debt contract may need to be arranged. In this case, $\ell_{i+1}, \ell_{i+2}, \ldots, \ell_n$ denote the returns net of the new entrepreneur's wages (or, equivalently, the repayment stream which he makes under the terms of the new debt contract).

Two additional points should be noted: the assets are assumed to collapse at date n; and, prior to that date, the entrepreneur cannot do anything without the assets (he has a zero outside wage).

We will normalize the rate of interest to be zero. This implies that the present value at date i of continuing the project until date n is

$$R_i \equiv \sum_{j=i+1}^{n} r_j,$$

while the present value of liquidating the project at date i is

$$L_i \equiv \sum_{j=i+1}^{n} \ell_j.$$

Since L_i is decreasing in i, our formulation captures the idea that the project's physical assets depreciate over time.

We assume:

(A.1) $$R_0 > K \geq L_0;$$

i.e. the project has a positive net present value at date 0 and the present value of its alternative use at date 0 does not exceed its capital cost K. ($K - L_0$ can be interpreted as the sunk cost of installing the capital for specific use in the project.)

We also make the following simplifying assumption:

(A.2) $$r_i \geq \ell_i \quad \Rightarrow \quad r_{i+1} \geq \ell_{i+1} \qquad \text{for all } 1 \leq i \leq n-1.$$

In other words, although in early periods, project returns may be less than the corresponding liquidation rentals, once project returns exceed liquidation rentals (which they must eventually by (A.1)), they exceed them in all subsequent periods. (A.2) allows for projects which are unprofitable initially and then become profitable, but rules out projects whose profitability (relative to alternative use) cycles. (Notice that (A.2) implies that $R_i > L_i$ for all $0 \le i \le n-1$; i.e., once the project is under way it is efficient to continue it.)

In an ideal world, the entrepreneur would raise the funds for the project from a single (rich) investor, offering the investor a contract which divides up the net present value of the project $R_0 - K$ in such a way that both of them at least break even. We will assume, however, that such an ex-ante division may not be enforceable because the entrepreneur cannot commit himself not to withdraw his human capital from the project at a future date. Moreover, the most extreme penalty that can be exercised against the entrepreneur if he quits is that control of the project's physical capital passes to the investor. In other words, we suppose that the entrepreneur has no other assets that can be seized and that a breach of contract (e.g. a refusal to work) does not lead to any criminal penalties, such as jail or physical punishment.

This suggests consideration of the following contract written by the entrepreneur and the investor. The investor (henceforth known as the creditor C) agrees to put up an initial sum of money I prior to date 0 and to give the entrepreneur (henceforth known as the debtor D) control of the assets. In return, D promises a payment p_i to C at the end of each period i = 1,2,...,n. If D makes the payment, he has the right to continue using the assets for at least one more period, i.e. until p_{i+1} is due. However, if D quits or defaults, control of the assets switches to C, who can then decide whether to liquidate them.[4]

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[4]More general contracts could be considered. For example, the contract might specify that D keeps control of the assets only for the first m < n periods, and control automatically switches to C at date m+1 (unless there has been a prior default or breach, in which case C gets control earlier). In the equilibrium of our deterministic model, it turns out that such contracts need not be used: either D keeps control for the entire horizon, or the project never goes ahead.

We also rule out more complex arrangements such as "partial default" -- e.g., control of the assets might be decided by lottery, the terms of which are sensitive to the amount that D repays. In addition, we ignore the possibility of "partial liquidation": in our model, the assets are assumed to be indivisible, but in principle C might be required to pay D some of the liquidation receipts. It turns out that in equilibrium none of these possible arrangements helps.

We now consider D's decision to quit or default in more detail. In fact both decisions are examples of contract repudiation (i.e. D walks away from the contract); a quit corresponds to repudiation at the beginning of a period, and a default to repudiation at the end of a period.

Repudiation at the beginning of a period

At the beginning of each period i ($i = 1,\ldots,n$), D can choose to repudiate the contract, saying to C: "Future debt payments are so large that I cannot make a reasonable profit out of this venture. I therefore quit." In this event, control of the project's physical assets switches to C. It is important to note that she gets only these assets. That is, we assume that D's savings -- which for $i > 1$ include his previous (net) earnings, $\sum_{j=1}^{i-1}(r_j - p_j)$ -- cannot be seized (they are in a private savings account).[5]

Repudiation at the end of a period

We suppose that during any period i ($i = 1,\ldots,n$) of the project's operations, the cash flow r_i accrues to D in the first instance. At the end of the period, D owes C the amount p_i. D can use r_i to make this payment -- and thereby earn himself $r_i - p_i$, which is added to his private savings. (In the event that $r_i < p_i$, D would have to make up the shortfall, $p_i - r_i$, from his own savings if they are sufficient.[6]) Alternatively, D can choose to repudiate the contract and not pay p_i -- i.e. he can default. In this case

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[5] In fact, a considerable part of our analysis extends to the case where they can be seized. Our results concerning when the project is undertaken continue to hold, as do all our results concerning the fastest repayment path. Also note that a special case of our model is where D contributes the whole of his w_0 to the initial investment, and $p_i = r_i$ for all $1 \le i \le n$ -- in which case there are no private savings to be seized.

[6] If they are not sufficient, repudiation at the end of period i would be involuntary.

the creditor can seize the project's assets, which are assumed to comprise not only the project's physical capital, but also the current period's cash flow r_i.[7] Again, it is important to note that this is all that C gets. D's private savings, including his previous earnings, r_1-p_1, ..., $r_{i-1}-p_{i-1}$, cannot be seized.

Thus, at any date i, D has the choice not only whether to repudiate, but also when: at the end of period i, or at the beginning of period i+1. Fortunately, we can simplify matters. We claim that if $r_i \geq p_i$, repudiation at the beginning of period i+1 dominates repudiation at the end of period i; and vice versa if $r_i < p_i$.

To see this, suppose $r_i \geq p_i$. Then if D repudiates at the end of period i, he loses the physical assets and r_i; whereas if he pays the p_i owed and repudiates at the beginning of period i+1, he loses the physical assets but can pocket r_i-p_i, i.e. he loses only p_i. On the other hand, if $r_i < p_i$, then it is better for D to repudiate at the end of period i than to repudiate at the beginning of period i+1 since in the first case he avoids having to put in p_i-r_i from his own savings before he loses control of the project's physical assets.

Given this observation, the relevant moves for D are as summarized in Figure 2.

Note that the above formulation captures the idea that D can commit his human capital for just one period in advance. Suppose that we are at the beginning of period i and $r_i = 10$. (Take the liquidation value L_i to be zero.) Then D could commit himself to work during period i for a low wage of 2, say, by signing a debt contract specifying $p_i = 8$. If D tried to renege on this contract by repudiating at the end of period i, he would be unsuccessful since C could seize D's assets, including the whole of $r_i = 10$. On the other hand, D could not commit to work during periods i and i+1 for a wage of 2 by signing a debt contract specifying $(p_i, p_{i+1}) = (8,8)$. The reason

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[7] In other words, although the project's cash flow accrues to D in the first instance, we are assuming that he cannot divert or steal any part of it.

is that he would repudiate this contract at the beginning of period i+1; as we will see, by repudiating, he would be able to renegotiate the second period wage up to 5.

Repudiation leads to a shift in control of the assets to C. At this stage she may liquidate the assets (irreversibly) or negotiate a new debt contract with D. The next step is to describe the renegotiation procedure.

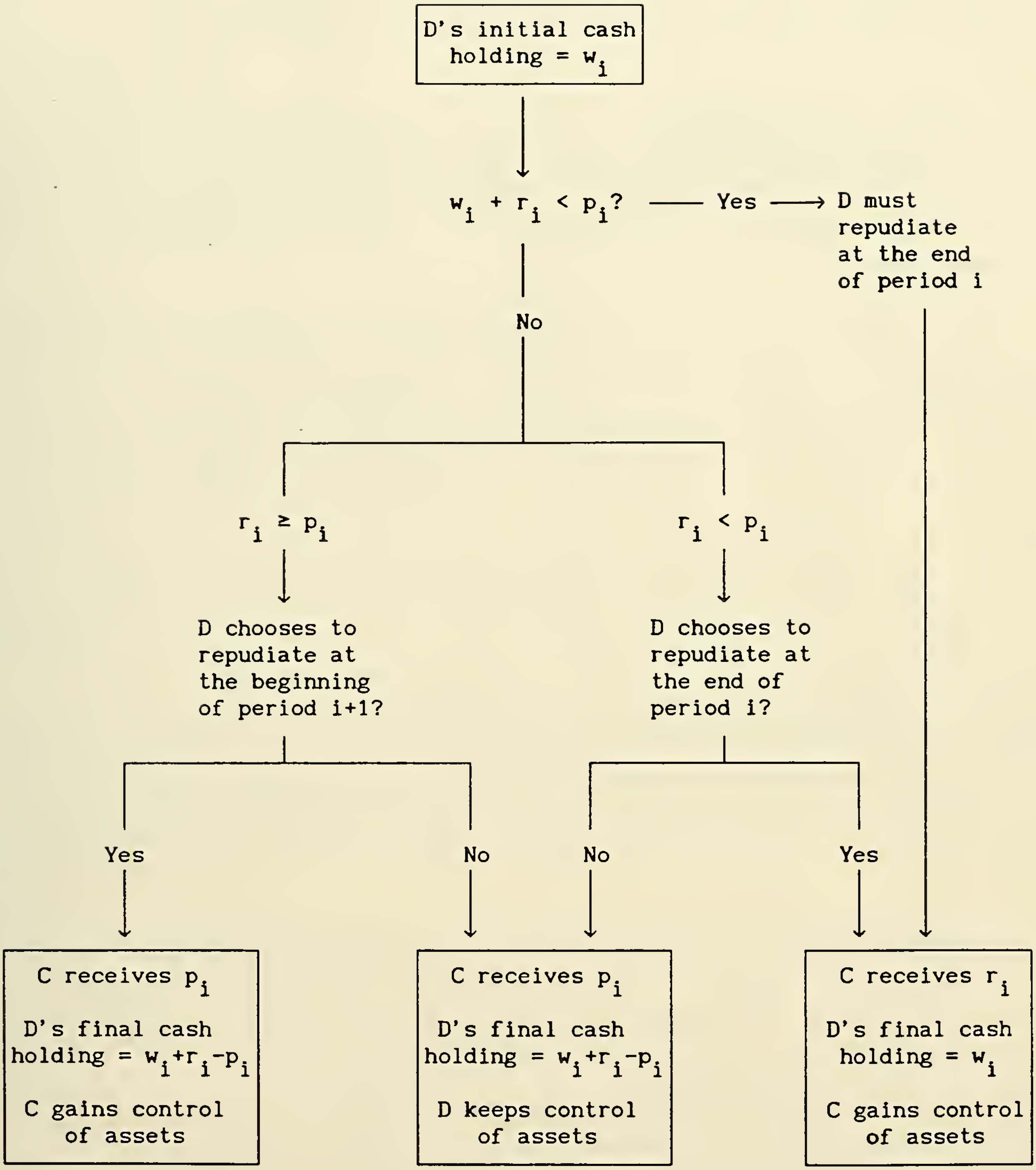
D's initial cash holding = w_i
$w_i + r_i < p_i$?
Yes
D must repudiate at the end of period i
No
$r_i \geq p_i$
$r_i < p_i$
D chooses to repudiate at the beginning of period i+1?
D chooses to repudiate at the end of period i?
Yes
No
No
Yes
C receives p_i
D's final cash holding = $w_i+r_i-p_i$
C gains control of assets
C receives p_i
D's final cash holding = $w_i+r_i-p_i$
D keeps control of assets
C receives r_i
D's final cash holding = w_i
C gains control of assets

Figure 2

3. Renegotiation

Suppose D repudiates either at the end of period i (if $r_i \le p_i$) or at the beginning of period i+1 (if $r_i > p_i$). Let his private savings or wealth at this point be w. C now has control of the assets and must decide what to do with them. We suppose that the following (very stylized) renegotiation game takes place.

First, at the beginning of period i+1, D makes C an offer of a new debt contract $(p'_i, p'_{i+1}, \ldots, p'_n)$, i.e. in return for having the assets back D promises to pay p'_i immediately and p'_j at the end of period j, for all $j \ge i+1$. C can either accept or reject this offer. If C accepts, the renegotiation game is over. If C rejects, C must now decide whether to liquidate (irreversibly) or to postpone this decision for at least one more period. Liquidation again ends the game. Postponement leads to a second offer of a new debt contract, made with equal probability (1/2) by D or C. If this second offer $(p''_i, p''_{i+1}, \ldots, p''_n)$, say, is accepted, the game ends (this second offer is made sufficiently close to the beginning of period i+1 that, if it is accepted, there is still time for the assets to earn r_{i+1}). On the other hand, if this second offer is rejected, the assets lie unused for a period (i.e. neither r_{i+1} nor ℓ_{i+1} is earned) and the whole process is then repeated in period i+2.[8]

Before we analyze the implications of this renegotiation game, some comments are in order. The game may seem ad hoc, but we believe that almost any extensive form bargaining game is subject to this criticism. We have chosen to work with this extensive form for three reasons: first, it is tractable; second, it gives each party some share of the surplus from renegotiation; and, third, it is close in spirit to much of the recent

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[8]Note that we assume that C cannot liquidate after this second round of offers and earn the period i+1 rental ℓ_{i+1}; liquidation at this stage earns the same rental stream $\ell_{i+2}, \ldots, \ell_n$ as liquidation at the beginning of period i+2.

literature on bargaining with outside options.[9] In addition, we suspect that, at least in qualitative terms, our results are reasonably robust to the extensive form used.

It is also important to note that we have deliberately chosen the renegotiation extensive form to be exogenously given and not manipulable via the initial debt contract.[10]

We now make the following assumption (closely related to (A.2)), which simplifies the analysis of the renegotiation game.

$$\text{(A.3)} \qquad \frac{1}{2} r_i \geq \ell_i \quad \Rightarrow \quad \frac{1}{2} r_{i+1} \geq \ell_{i+1} \qquad \text{for all } 1 \leq i \leq n-1.$$

We also suppose that C and D are both risk neutral.

Lemma 1 describes the (subgame perfect) equilibrium of the renegotiation game.

Lemma 1. Take $1 \leq i < n$. Suppose D repudiates either at the end of period i or at the beginning of period i+1, and assume that his private wealth (cash holding) at this point is w. Then in any equilibrium of the renegotiation game, C's payoff from date i onwards is given by

$$\text{(3.1)} \qquad U_C^i = \text{Max } (L_i, \tfrac{1}{2}R_i),$$

and D's payoff from date i onwards (including his wealth) is given by

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[9]Noncooperative bargaining with outside options was first investigated by Ken Binmore, Avner Shaked and John Sutton. For a recent overview, see Section 3.12 of Osborne and Rubinstein (1990).

[10]In the present context, the possibilities for manipulation seem particularly limited. For example, suppose, as an extreme case, that the initial contract tries to limit D's bargaining power by banning offers from him. Then there is nothing to stop D from defying this ban; since he has lost all his seizable assets there is no penalty that can be exercised against him.

$$(3.2) \qquad U_D^i = \begin{cases} w + R_i - U_C^i & \text{if condition (*) holds} \\ w & \text{if condition (*) does not hold} \end{cases}$$

where condition (*) is as follows:

<u>Condition (*)</u>

There exist $p_i, p_{i+1}, \ldots, p_n$ such that

$$(3.3) \qquad \text{Max}\,(p_k - r_k, 0) + p_{k+1} + \ldots + p_n$$

$$\leq U_C^k \equiv \text{Max}\,(L_k, \tfrac{1}{2}R_k) \qquad \text{for all } i+1 \leq k \leq n-1;$$

$$(3.4) \qquad p_i + \ldots + p_n \geq U_C^i;$$

$$(3.5) \qquad w - p_i + \sum_{j=i+1}^{k} (r_j - p_j) \geq 0 \qquad \text{for all } i+1 \leq k \leq n.$$

Moreover, if (*) holds, an equilibrium consists of a contract $(p_i', \ldots, p_n')$ offered by D and accepted by C, where $(p_i', \ldots, p_n')$ is any vector of prices satisfying (3.3)-(3.5), and (3.4) holds with equality. In this case the project continues until date n. On the other hand, if (*) does not hold, every equilibrium involves immediate liquidation at date i.

Lemma 1 is quite intuitive. The formula for C's payoff corresponds to that found in much of the outside option literature: C receives 50% of the surplus from bargaining unless this falls below C's outside option, in which case C receives the latter. In contrast to the outside option literature, however, bargaining does not guarantee a first-best efficient outcome, since D is liquidity-constrained. In fact, Lemma 1 tells us that the first-best is achieved if and only if (*) holds. Note also that C's payoff U_C^i is independent of D's wealth w; this is a feature of our particular bargaining model, but it would not be true of every model of efficient bargaining, given that D is wealth-constrained.

Condition (*) says that there is a contract $(p_i,\ldots,p_n)$ which is such that (a) C receives at least as much as the project's date i liquidation value ((3.4)); (b) D never runs out of money ((3.5)); and (c) D has no incentive to repudiate in the future ((3.3)). To understand (3.3), note that if $p_k \le r_k$, (3.3) says that D has no incentive to repudiate at the beginning of period k+1 (doing so would make C's utility U_C^k as opposed to $p_{k+1} +\ldots+ p_n$); while, if $p_k > r_k$, (3.3) says that D has no incentive to repudiate at the end of period k (doing so would make C's utility $U_C^k + r_k$ as opposed to $p_k +\ldots+ p_n$).

Lemma 1 is proved in the Appendix. It is worth indicating the main steps of the proof, however. First, the formula for U_C^i is established by induction. Next, it is shown that in equilibrium the parties can restrict attention to contracts that are repudiation-proof, i.e. that are such that the debtor never wishes to repudiate along the equilibrium path. The reason is that, if repudiation occurs, the parties can always, given the assumption of perfect certainty, anticipate what will happen next (renegotiation or liquidation) and include this as part of the initial contract.

The third step is to show that D and C will never sign a contract that leads to early termination of the project, i.e. prior to date n. The reason is straightforward. Either the project is losing money when it is terminated (in the sense that $r < \ell$) -- in which case, by assumption (A.2), it has been losing money all along and the parties should have terminated it earlier, at date i. Or it is making money ($r \ge \ell$), and, again by (A.2), it will continue to do so until date n. But in this case, D can bribe C not to liquidate on a period-by-period basis. (Recall our earlier observation that D can in effect commit his human capital for one period in advance.)

Finally, then, given these observations, D and C's decision at date i is a simple one: liquidate immediately unless there exists a repudiation-proof contract which involves continuation of the project until date n and which provides the creditor with at least U_C^i. But this is precisely the content of condition (*).

We illustrate Lemma 1 with an example, in which $n = i+2$ -- i.e., there are two remaining periods.

<u>Example 1</u>

$r_{i+1}=?$ $\quad$ $r_{i+2}=26$

$\ell_{i+1}=8$ $\quad$ $\ell_{i+2}=8$

Date i $\quad$ Date i+1 $\quad$ Date i+2=n

The example deliberately leaves r_{i+1} unspecified. Suppose r_{i+1} exceeds 6 -- say it equals 8. In this case, the parties will agree to continue the project whatever D's wealth w. To see why, notice that, by (3.1), C's payoff is $17 = \frac{1}{2}(r_{i+1}+r_{i+2}) > \ell_{i+1}+\ell_{i+2}$. Then D can simply offer C half the cash flow in each subsequent period: the contract $p_i' = 0$, $p_{i+1}' = 4$, $p_{i+2}' = 13$ satisfies (*).

Now suppose r_{i+1} is less than 6. C's payoff is $16 = \ell_{i+1}+\ell_{i+2} > \frac{1}{2}(r_{i+1}+r_{i+2})$. Provided that r_{i+1} is at least 3, D can offer C half the cash flow in the last period, and L_i minus this amount in the previous period: the contract $p_i' = 0$, $p_{i+1}'= 3$, $p_{i+2}' = 13$ satisfies (*). (This is feasible since, given that $r_{i+1} \geq 3$, D has no incentive to repudiate at the end of period i+1.) Again, the project is continued whatever D's wealth w.

However, if r_{i+1} is lower than 3 -- say equal to 2 -- then the parties cannot necessarily agree to continue the project. The most that D can credibly offer C in the last period is $p_{i+2}' = 13$ (if $p_{i+2}' > 13$, D will repudiate at the beginning of period i+2). This means that D must offer C at least 3 in the previous period to persuade her not to liquidate at date i. But since $r_{i+1} = 2$, this is possible only if $w \geq 1$. In particular, if $w \geq 1$, it is easy to see that the contract $p_i' = 1$, $p_{i+2}' = 2$, $p_{i+2}' = 13$ satisfies (*) and so the parties agree to continue the project. On the other hand, if $w < 1$, (*) is not satisfied and so liquidation occurs at date i.

4. Is the project undertaken?

Having analyzed the renegotiation game, we now consider whether the project will be undertaken at all. Consider the negotiation between D and C prior to date 0. For simplicity, suppose that D has all the bargaining power in this initial negotiation, i.e. C's net (present value of) profit is zero (this is reasonable if there are many creditors who can potentially finance the project prior to date 0).

Much the same logic applies to the date 0 negotiation as applied to the date i renegotiation, and we can therefore use Lemma 1 directly. In particular, for the project to go ahead there must exist a debt contract p_1, p_2, ..., p_n which is repudiation-proof at each date $i \geq 1$; that is, as in (3.3),

$$\text{(4.1)} \quad \operatorname{Max}(p_i - r_i, 0) + p_{i+1} + \ldots + p_n \leq U_C^i \quad \text{for all } 1 \leq i \leq n-1.$$

In addition, D must not have an incentive to repudiate at date 0 the instant after the sunk cost $K-L_0$ has been incurred but before the period 1 cash flows have been realized:

$$\text{(4.2)} \quad p_1 + p_2 + \ldots + p_n \leq U_C^0.$$ [11]

C's zero-profit condition means that D intially borrows $p_1+\ldots+p_n$. If this loan is to be enough to buy the assets at date 0, then

$$\text{(4.3)} \quad w_0 + p_1 + \ldots + p_n \geq K.$$

And if the project is to go ahead, it will never be terminated; hence for D subsequently not to run out of cash,

$$\text{(4.4)} \quad w_0 + p_1 + \ldots + p_n - K + \sum_{j=1}^{i} (r_j - p_j) \geq 0 \quad \text{for all } 1 \leq i \leq n.$$

---

[11] This is a new condition since repudiation directly after renegotiation at the beginning of period i+1 was not an issue in Lemma 1.

In sum, the project will be undertaken if and only if there exist $p_1,\ldots,p_n$ satisfying (4.1) - (4.4). These sets of inequalities can be written more compactly. Inequalities (4.1) and (4.2) can be combined to give

$$(4.5) \qquad p_i + \ldots + p_n \le \mathrm{Min}\,(U_C^{i-1},\ U_C^i + r_i) \quad \text{for all } 1 \le i \le n.$$ [12]

And inequalities (4.3) and (4.4) can be combined to give

$$(4.6) \qquad p_i + \ldots + p_n \ge K - w_0 - R_0 + R_{i-1} \quad \text{for all } 1 \le i \le n.$$ [13]

Hence we have proved:

<u>Proposition 1</u> The project will be undertaken if and only if there exist $p_1,\ldots,p_n$ satisfying (4.5) and (4.6) -- that is, if and only if

$$(4.7) \qquad K \le w_0 + R_0 - R_{i-1} + \mathrm{Min}\,(U_C^{i-1},\ U_C^i + r_i) \quad \text{for all } 1 \le i \le n$$

where $U_C^i \equiv \mathrm{Max}\,(L_i, \frac{1}{2}R_i)$.

---

[12] (4.1) represents <u>pairs</u> of inequalities, which, together with (4.2), have been combined in different pairings to give (4.5). Our choice of U_C^n is unimportant (provided it is nonnegative): notice that $U_C^{n-1} = \mathrm{Max}\,(\ell_n, r_n/2) \le r_n \le U_C^n + r_n$.

[13] Assumption (A.1) ensures that (4.4) is automatically satisfied when $i = n$.

<u>Corollary 1</u> The project is always undertaken if $K = L_0$ and $r_i \geq \ell_i$ for all $1 \leq i \leq n$ -- regardless of the size of D's initial wealth w_0.

<u>Proof</u> Given $r_i \geq \ell_i$, it follows that $U_C^{i-1} \leq U_C^i + r_i$ for all $1 \leq i \leq n$. Hence for each i, the RHS of (4.7) is no less than

$$R_0 - R_{i-1} + \text{Max}\,((L_{i-1}, \tfrac{1}{2}R_{i-1})$$

$$\geq R_0 - R_{i-1} + L_{i-1}$$

-- which in turn is no less than $L_0 = K$ (since $r_j \geq \ell_j$ for $1 \leq j \leq i-1$). Hence (4.7) is satisfied for all $1 \leq i \leq n$, and by Proposition 1 the project is undertaken.

Q.E.D.

Corollary 1 tells us that inefficiency arises only if either (a) there is an intial sunk cost of investment ($K > L_0$), and/or (b) the project's initial returns are smaller than the returns from the assets' alternative use (in particular, $r_1 < \ell_1$). The intuition behind Corollary 1 is that D can always commit himself to work for one period at a wage less than or equal to that period's return r -- and, given that r always exceeds ℓ, this means that D can in effect bribe C each period not to liquidate.

5. Equilibrium Repayment Paths

From now on we shall assume that the condition for the project to go ahead, (4.7), is satisfied. The question arises: what kinds of debt contract work? We will concentrate attention on the set of possible equilibrium repayment paths $p_1,\ldots,p_n$ rather than on the debt contracts themselves. Or to put it another way, we will ignore debt contracts that are renegotiated in equilibrium, and concentrate on those which are repudiation-proof (i.e. (4.5) is satisfied) and feasible (i.e. (4.6) is satisfied).

It turns out that there is a continuum of repayment paths $p_1,\ldots p_n$ which satisfy (4.5) and (4.6).

The slowest repayment path, $\bar{p}_1,\ldots,\bar{p}_n$, is the one in which at each date the total outstanding debt is maximized. It is found by setting $p_i+\ldots+p_n$ equal to its upper bound, the RHS of (4.5), to obtain:

$$
\begin{aligned}
\bar{p}_n &= U_C^{n-1}, \\
\bar{p}_{n-1} &= \text{Min}\left[U_C^{n-2},\ U_C^{n-1} + r_{n-1}\right] - \bar{p}_n, \\
\bar{p}_{n-2} &= \text{Min}\left[U_C^{n-3},\ U_C^{n-2} + r_{n-2}\right] - \bar{p}_n - \bar{p}_{n-1}, \\
&\vdots \\
\bar{p}_1 &= \text{Min}\left[U_C^{0},\ U_C^{1} + r_1\right] - \sum_{j=2}^{n} \bar{p}_j.
\end{aligned}
\tag{5.1}
$$

In this contract, the total outstanding debt at each date i is set equal to the most that D can credibly promise to repay -- viz., Min $(U_C^{i-1},\ U_C^{i} + r_i)$. In particular, this allows D initially to borrow the largest amount $\bar{p}_1+\ldots+\bar{p}_n$ that he can: Min $(U_C^{0},\ U_C^{1} + r_1)$. Notice that, for the project to be undertaken, this initial loan must be enough both to cover D's shortfall $K-w_0$ (i.e. satisfy (4.3)) and to ensure that he never runs out of cash at any subsequent date (i.e. satisfy (4.4)).

In other words, one way to understand condition (4.7) in Proposition 1 is to see that it is the condition guaranteeing that along the slowest repayment path D is always solvent.

It is straightforward to write out $\bar{p}_1, \ldots, \bar{p}_n$ in terms of $r_1, \ldots, r_n$ and $\ell_1, \ldots, \ell_n$, but the formulae are intricate and rather uninformative. If, however, we let the length of each period tend to zero and the number of periods tend to infinity, then the form of the slowest repayment path becomes quite simple; see Section 6.

The <u>fastest</u> repayment path, $\underline{p}_1, \ldots, \underline{p}_n$, is the one in which at each date the total outstanding debt is minimized. It is found by setting $p_i + \ldots + p_n$ equal to its lower bound, the RHS of (4.6), to obtain:

$$\underline{p}_i = r_i \qquad \text{for all } 1 \le i < n,$$

(5.2)

$$\underline{p}_n = K - w_0 - (r_1 + \ldots + r_{n-1}).$$

Here, D initially borrows the smallest amount that he needs to finance the project, viz. $K - w_0$. Thereafter, D repays at the maximum feasible rate for the first n-1 dates: $\underline{p}_i = r_i$. Notice that at the last date, D's repayment $\underline{p}_n = K - w_0 - (r_1 + \ldots + r_{n-1})$ may be negative, i.e. D may receive a payment from C at date n. In other words, D uses C like a savings bank -- depositing all project cash flows with C from date 1 to date n-1, and then receiving all accumulated interest at the end.

The fastest repayment path provides another way of understanding condition (4.7) in Proposition 1. At each date i, D has the smallest amount of total outstanding debt commensurate with his solvency constraint -- viz., $K - w_0 - (r_1 + \ldots + r_{i-1})$. This must not exceed what he can credibly promise to repay: Min $(U_C^{i-1}, U_C^i + r_i)$. In other words, condition (4.7) guarantees that along the fastest repayment path D never wishes to repudiate.

All other repayment paths lie between the fastest and the slowest:

Proposition 2 Assume that (4.7) holds. Then the set of all equilibrium repayment paths $p_1,\ldots,p_n$ satisfies

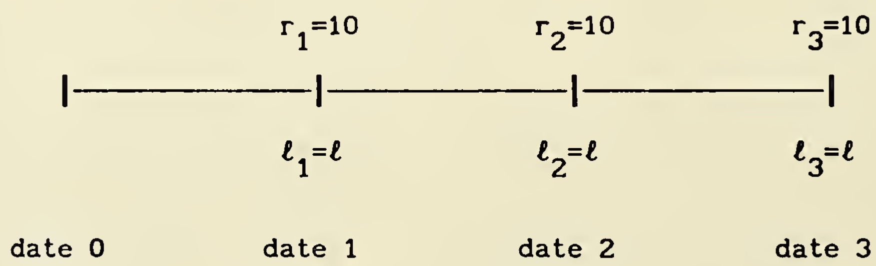
$$\sum_{j=i}^{n} \underline{p}_j \le \sum_{j=i}^{n} p_j \le \sum_{j=i}^{n} \bar{p}_j \quad \text{for all } 1 \le i \le n, \tag{5.3}$$

where $\underline{p}_1,\ldots,\underline{p}_n$ and $\bar{p}_1,\ldots,\bar{p}_n$ are defined in (5.1) and (5.2) respectively.

We now present two examples to illustrate Propositions 1 and 2.

Example 2

Suppose $n = 3$; and $r_i \equiv 10$, $\ell_i \equiv \ell$ for all i, for some constant $\ell < 10$:



Also assume that $K = L_0 = 3\ell$. Corollary 1 tells us that the project will be undertaken, regardless of D's initial wealth w_0.

We consider two possible values for ℓ: $\ell = 4$ and $\ell = 6$.

First, suppose $\ell = 4$. Then $U_C^0 = 15$, $U_C^1 = 10$, and $U_C^2 = 5$. From (5.1), the slowest repayment path is $\bar{p}_1 = \bar{p}_2 = \bar{p}_3 = 5$. And from (5.2), the fastest repayment path is $\underline{p}_1 = 10$, $\underline{p}_2 = 10$, and $\underline{p}_3 = -w_0-8$ (which is less than zero).

The intuition here is clear. Any slower path than (5,5,5) would involve $p_3 > 5$ or $p_2 + p_3 > 10$ or $p_1 + p_2 + p_3 > 15$. But none of these is feasible: D would repudiate at the beginning of periods 3, 2 or 1, respectively. On the other hand, there is no faster path than $(\underline{p}_1, \underline{p}_2, \underline{p}_3)$: D cannot borrow less than $K - w_0$ and still finance the project, and the project cash flows do not allow D to pay back more than 10 at date 1 or more than 20 at dates 1 and 2 combined.

Now suppose $\ell = 6$. Then $U_C^0 = 18$, $U_C^1 = 12$, and $U_C^2 = 6$. From (5.1), the slowest repayment path is $\bar{p}_1 = \bar{p}_2 = \bar{p}_3 = 6$. And from (5.2), the fastest repayment path is $\underline{p}_1 = 10$, $\underline{p}_2 = 10$, and $\underline{p}_3 = -w_0-2$.

Note that the slowest repayment path when $\ell = 6$ is "slower" than when $\ell = 4$; i.e. D borrows more initially, and owes more at each date. The reason is that, given a higher L_i at each date, it is credible for D to make payments later. In particular, when $\ell = 6$, $\bar{p}_3 = 6$ does not lead to repudiation at the beginning of period 3 since C is guaranteed a payment of 6 from liquidation at that date. For the same reason, D has no incentive to repudiate at the beginning of period 2 given $\bar{p}_2 + \bar{p}_3 = 12$, or at the beginning of period 1 given $\bar{p}_1 + \bar{p}_2 + \bar{p}_3 = 18$.

Example 3

Suppose n = 4 and the returns are as follows:

	$r_1=0$	$r_2=1$	$r_3=10$	$r_4=26$
	$\ell_1=8$	$\ell_2=8$	$\ell_3=8$	$\ell_4=8$
date 0	date 1	date 2	date 3	date 4

Here $U_C^0 = 32$, $U_C^1 = 24$, $U_C^2 = 18$, and $U_C^3 = 13$. From (5.1), the slowest repayment path is $\bar{p}_1 = 5$, $\bar{p}_2 = 1$, $\bar{p}_3 = 5$, and $\bar{p}_4 = 13$. And from (5.2), the fastest repayment path is $\underline{p}_1 = 0$, $\underline{p}_2 = 1$, $\underline{p}_3 = 10$, and $\underline{p}_4 = K-w_0-11$ (which may or may not be less than zero).

To understand why (5, 1, 5, 13) is the slowest path, note that D cannot credibly offer more than $\frac{1}{2}r_4 = 13$ at date 4 since this would give him an incentive to repudiate at the beginning of period 4. For similar reasons, D cannot credibly offer more than $\frac{1}{2}r_3 = 5$ at date 3. At the end of period 2, it is credible for D to pay 1 since this makes him indifferent between repudiating and not (if he repudiates, he pays $r_2 + U_C^2 = 1 + 18 = 19$; if he does not repudiate, he pays $\bar{p}_2 + \bar{p}_3 + \bar{p}_4 = 19$) -- but he would repudiate given any payment above 1. Finally, at the end of period 1, it is credible for D to pay $\bar{p}_1 = 5$, since $\bar{p}_1+\bar{p}_2+\bar{p}_3+\bar{p}_4 = 24$, whereas if he repudiates he is going to have to pay $U_C^1 = 24$ anyway.

Knowing that the slowest path is (5, 1, 5, 13), we can easily find the condition for the project to go ahead. Although D can borrow up to 5+1+5+13 = 24, he cannot use all of this to finance the project initially, because he needs to retain a "cash cushion" of 5 in order to cover the date 1 debt repayment $\bar{p}_1$ (given that $r_1 = 0$). In other words, the maximum amount, M, that that D can borrow from C for the purpose of buying assets is M = 19. We shall call M the <u>debt capacity</u> of the project; the project can go ahead only if $K \le w_0+M$. In this example, it is straightforward to confirm that condition (4.7) from Proposition 1 does indeed reduce to $K \le w_0+19$.

One can also arrive at the condition for the project to go ahead by checking that the fastest path, $(0, 1, 10, K-w_0-11)$, is repudiation-proof. In this example, the binding constraint concerns repudiation at the beginning of period 3. This constraint is satisfied as long as $\underline{p}_3 + \underline{p}_4 \le U_C^2$ -- i.e., provided $K - w_0 - 1 \le 18$, which is just (4.7).

<u>Resolving the indeterminacy of the repayment path: two extensions</u>

The indeterminacy in the repayment paths exhibited in Proposition 2 disappears if we vary the basic model a little. We close this section with two simple extensions, (E1) and (E2). The first extension selects out the slowest repayment path, $\bar{p}_1,\ldots,\bar{p}_n$, as the unique optimum; and the second extension selects the fastest, $\underline{p}_1,\ldots,\underline{p}_n$.[14]

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[14]Another way to reduce the indeterminacy is by introducing an optimal

Extension E1: Reinvestment Opportunities for the Debtor

Suppose D can reinvest cash flows from the project and receive a positive rate of return. To make matters very simple, assume that a dollar invested at the end of period i yields $1+\mu > 1$ dollars at the end of period i+1 ($i = 0,1,\ldots,n-1$). In contrast, any cash flows reinvested by the creditor yield the going rate of interest of zero.[15]

With this reinvestment opportunity for D, it is straightforward to show that (4.5) is still a necessary condition for a contract to be repudiation-proof.[16] Moreover, it is easy to show that the formula for U_C^k in (3.1) is unchanged by the presence of a reinvestment opportunity.[17] However, (4.6) -- which combined (4.3) and (4.4) -- does change, since D's private savings now earn a positive rate of return. In particular, although (4.3) does not change, condition (4.4) becomes:

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consumption decision by D during the lifetime of the project. We will study this in a companion paper. (Also, if the returns r and/or ℓ were stochastic, then we conjecture that this would typically lead to a unique repayment path. On this, see Hart and Moore (1989).)

[15] We continue to assume that reinvested cash is part of D's private savings, which cannot be seized by C in the event of repudiation.

[16] To see this, note first that the argument that D never repudiates at the end of period i if $r_i \geq p_i$, and never repudiates at the beginning of period i+1 if $r_i < p_i$, remains unchanged. Second, suppose that (4.5) is violated for at least one i. Choose the largest such i. Then D would be better off repudiating either at the beginning of period i (if $p_i+\ldots+p_n > U_C^{i-1}$) or at the end of period i (if $p_i+\ldots+p_n > U_C^i + r_i$).

[17] This is a simple matter of checking that each step of the induction argument in the proof of Lemma 1 still applies.

$$w_{i+1} = \left(w_0 + \sum_{j=1}^{n} p_j - K\right)(1+\mu)^i + \sum_{j=1}^{i} (r_j - p_j)(1+\mu)^{i-j} \geq 0$$

or

$$\frac{w_{i+1}}{(1+\mu)^i} = w_0 - K + \sum_{j=1}^{i} \left(1 - \frac{1}{(1+\mu)^j}\right) p_j + \sum_{j=i+1}^{n} p_j + \sum_{j=1}^{i} \frac{r_j}{(1+\mu)^j}$$

$$(4.4') \qquad \geq 0 \quad \text{for all } 1 \leq i \leq n.$$

-- where, recall, w_{i+1} is D's cash holding at the start of period i+1.

Since D has all the bargaining power in the negotiation prior to date 0, D will choose a solution $p_1,\ldots,p_n$ to (4.3), (4.4') and (4.5) which maximizes his objective function w_{n+1} (his final wealth after date n). Notice that the weights on p_j in (4.4') are increasing in j. Therefore by standard arguments, any first-order stochastic dominance shift in $p_1,\ldots,p_n$ (i.e., any shift that increases $\sum_{j=1}^{n} p_j$ for all i) will increase w_i for all i. Hence it follows from (5.3) that D's wealth at date n will be maximized if he chooses $\bar{p}_1,\ldots,\bar{p}_n$.

In other words, when D has a period by period reinvestment opportunity, there is a uniquely optimal path of debt repayments and it is given by the slowest path in Proposition 2. This is very intuitive -- the more slowly D repays C the greater D's opportunity to make profitable reinvestments.

Note that, although the magnitude of μ has no effect on the optimal repayment path $\bar{p}_1,\ldots,\bar{p}_n$, it does affect which projects will be undertaken (the higher μ is, the more likely the project will be undertaken since the constraint (4.4') is relaxed). As $\mu \to 0$, however, the conditions for the project to be undertaken become the same as in Proposition 1.

Extension E2: Reinvestment Opportunities for the Creditor

Suppose now, in contrast to Extension (E1), that C, rather than D, has a profitable reinvestment opportunity. The easiest way to formalize this is to suppose that C continues to earn a zero rate of interest on wealth, while D loses money, i.e. some fraction ξ of D's savings disappear every period. We assume, however, that D always has the option to earn the same return as C by investing his savings with C.

This modification in the model means that, before date n, it is never efficient for D to hold on to any of his cash. So in equilibrium, assuming that the project goes ahead, D always borrows the smallest amount possible at date 0, and pays C the full project returns until date n-1 -- whatever debt contract has been signed. The only possible variation across equilibrium repayment paths could be in the amount that C repays D at date n. But we know that C must break even overall, and so there can only be one equilibrium repayment path in which the project goes ahead -- viz., the fastest path $\underline{p}_1, \ldots, \underline{p}_n$ (see (5.2)).

Thus, when C has a profitable reinvestment opportunity, there is a uniquely optimal flow of net payments from D to C, and it is given by the fastest repayment path in Proposition 2. Since none of D's savings disappear in equilibrium, the introduction of ξ has no effect on which projects will be undertaken; these are still given by Proposition 1.

Remark

Let us return to our basic model in which D and C have the same discount rate. As we have noted, there are many different contracts which support a particular repayment path $p_1, \ldots, p_n$. As well as the long-term contract which specifies the repayments in question and which never has to be renegotiated, there are short-term contracts which trigger repudiation and which are renegotiated along the equilibrium path -- for example, the contract which specifies that D owes C $\sum_{i=1}^{n} p_i$ at date 1 will support the repayment path $p_1,...,p_n$.

Two points are worth noting. First, in the case of a short-term contract, it may be important that the debtor has the option to refinance the loan with an outside creditor. (This possibility is not important in the case of a repudiation-proof long-term contract.) For instance, take Example 3, with $K-w_0 = 19$ (i.e. the project can just go ahead). The fastest repayment path is (0,1,10,8), which allows D to borrow 19 initially. Consider now the short-term debt contract which specifies that, having borrowed 19, D must pay the entire amount back at date 1. In the absence of outside financing, D is forced to repudiate at the end of period 1. Notice that C's return from liquidation at this point, 24 (= U_C^1), is strictly greater than the most D can credibly promise to repay later, $\bar{p}_2+\bar{p}_3+\bar{p}_4 = 19$. Hence, C will liquidate. However, if D has access to a competitive captial market, then he can raise 19 at the end of period 1 by (credibly) promising a new creditor the repayment path $(p_2,p_3,p_4) = (1,5,13)$, and thereby avoid repudiation.[18]

Second, although there are many contracts which support a particular repayment path, not every contract will do. Again, take Example 3, with $K-w_0 = 19$. On the one hand, a contract which specifies too high a repayment early on -- e.g. D borrows 19 initially, and promises to repay $p_1 > 19$ at date 1 -- will lead to premature liquidation regardless of whether outside financing is possible. On the other hand, a contract which specifies repayments too late -- e.g. the contract $(p_1,p_2,p_3,p_4) = (0,0,0,19)$ -- will prevent C from breaking even.

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[18] If "partial liquidation" were admitted (see footnote 4), then the debt contract could specify that, in the event of liquidation at date 1, C must pay D an amount 5 from the liquidation receipts. In these circumstances, the short-term contract given in the text would work, even without assuming that D has access to a perfect capital market.

6. The Continuous Time Model

We now consider a continuous time version of the model in Sections 2-4. We continue to assume that the project has a finite horizon, but now suppose that the project return r(t) is a piecewise continuous function defined on [0,T]. On the other hand, liquidation yields a flow of returns $\ell(t)$, where $\ell(t)$ is a piecewise continuous function on [0,T]. As before, we take the interest rate to be zero, so that the present value at date t of continuing the project to the end is

$$R(t) \equiv \int_t^T r(\tau)\, d\tau;$$

and the present value of liquidating the project at date t is

$$L(t) \equiv \int_t^T \ell(\tau)\, d\tau.$$

We continue to denote the initial cost of the project by K and D's initial wealth by w_0.

We make the natural generalizations of (A.1)-(A.3):

(A.1′) $\quad R(0) > K \geq L(0).$

(A.2′) $\quad r(t) \geq \ell(t) \quad \Rightarrow \quad r(\tau) > \ell(\tau) \quad$ for all $\tau > t$.

(A.3′) $\quad \frac{1}{2}\, r(t) \geq \ell(t) \quad \Rightarrow \quad \frac{1}{2}\, r(\tau) > \ell(\tau) \quad$ for all $\tau > t$.

Two particular times will turn out to be important in what follows:

<u>Definition</u>: $t = \hat{t}$ is the time at which $\frac{1}{2}R(t) = L(t)$
-- unless either $\frac{1}{2}R(t)$ always exceeds L(t), in which case $\hat{t} = 0$;
or L(t) always exceeds $\frac{1}{2}R(t)$, in which case $\hat{t} = T$.

(Note that assumption (A.3′) ensures that $\hat{t}$ is uniquely defined.)

Definition: $t = \underline{t}$ is the earliest time at which $r(t) \geq \ell(t)$.

(More precisely, $\underline{t} = \inf \{ \tau \mid r(\tau) \geq \ell(\tau) \}$.)

(Assumptions (A.1′) and (A.2′) ensure that $\underline{t} < T$; and assumption (A.2′) ensures that $\underline{t}$ is uniquely defined.)

We now introduce two examples, to which we will return later:

Example 4

Suppose $T = 16$; $K = 80$; $\ell(t) \equiv 5$; and $r(t) \equiv t$.

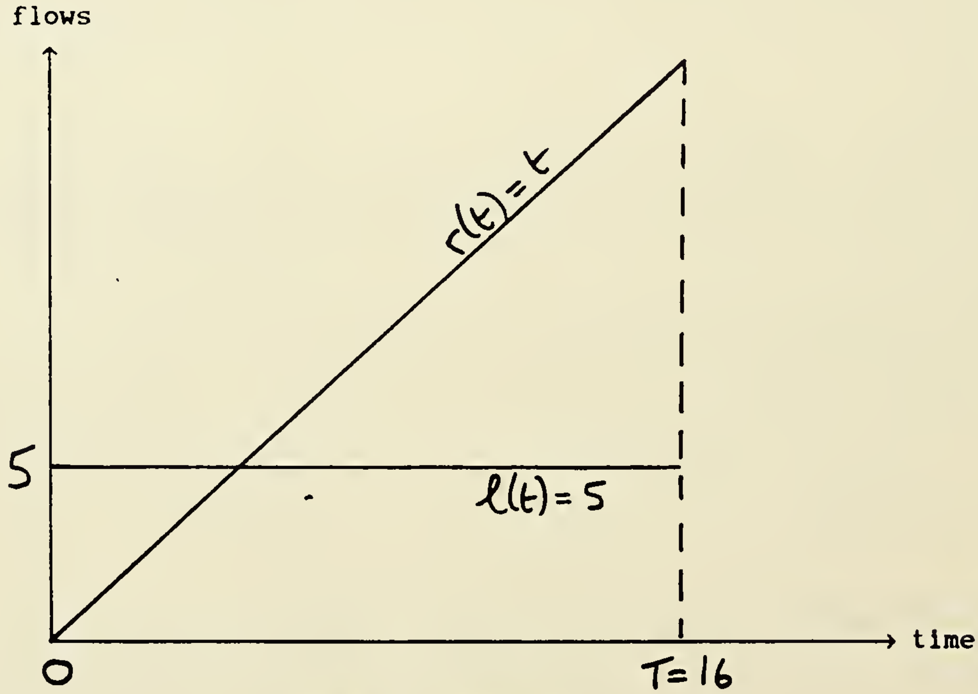

Figure 3: Example 4

Note that this example satisfies Assumptions (A.1′)-(A.3′); In particular,

$$R(0) = \int_0^{16} \tau d\tau = 128 \text{ exceeds } L(0) = \int_0^{16} 5 d\tau = 80 = K.$$

In this example $\hat{t} = 4$ -- since $\frac{1}{2}R(4) = \frac{1}{2}\int_4^{16} \tau d\tau = 60$, and $L(4) = \int_4^{16} 5d\tau = 60$.

And $\underline{t} = 5$.

Example 5

Suppose $T = 80$; $K = 480$; $\ell(t) \equiv 6$; and $r(t) = \begin{cases} 0 & \text{for } 0 \le t < 20 \\ 10 & \text{for } 20 \le t < 60 \\ 7 & \text{for } 60 \le t \le 80. \end{cases}$

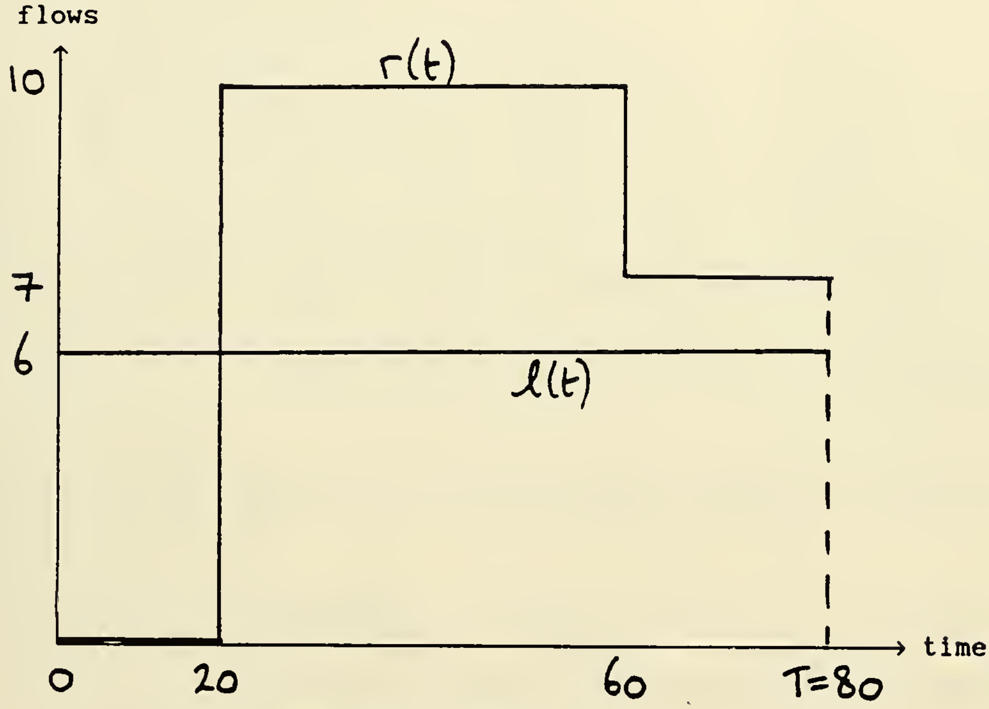

Figure 4: Example 5

Note that this example satisfies Assumptions (A.1′)-(A.3′); In particular, $R(0) = 40\times10 + 20\times7 = 540$ exceeds $L(0) = 80\times6 = 480 = K$.

In this example $\hat{t} = 80$ -- since $L(t)$ always exceeds $\frac{1}{2}R(t)$.
And $\underline{t} = 20$.

Examples 4 and 5 show that $\hat{t}$ does not necessarily come before $\underline{t}$ or vice versa. (In Example 4, $\hat{t} < \underline{t}$; in Example 5, $\underline{t} < \hat{t}$.)

Given the continuous model, consider the following discrete approximation. Divide the interval [0,T] into n subintervals:

$$\left(0, \frac{T}{n}\right], \quad \left(\frac{T}{n}, 2\frac{T}{n}\right], \quad \ldots, \quad \text{and } \left((n-1)\frac{T}{n}, T\right].$$

Define r_i, ℓ_i as the total project and liquidation return in the i^{th} sub-interval:

$$r_i = \int_{(i-1)\frac{T}{n}}^{i\frac{T}{n}} r(\tau)\, d\tau \qquad \text{and} \qquad \ell_i = \int_{(i-1)\frac{T}{n}}^{i\frac{T}{n}} \ell(\tau)\, d\tau.$$

For each $t > 0$, define $i^n(t)$ to be the i such that $t \in \left((i-1)\frac{T}{n}, i\frac{T}{n}\right]$. (The small superscript n on $i^n(t)$ denotes that we are working with the n^{th} discrete approximation.)

We will apply the analysis of Sections 2-5 to this discrete approximation, and then take limits as $n \to \infty$.

It is convenient to work with the total outstanding debt owed by D at any time, rather than current debt payments. Given a debt contract $(p_1,\ldots,p_n)$ in the discrete case, for each $0 < t \le T$ define the <u>outstanding debt</u>

$$P^n(t) \equiv p_i + \ldots + p_n \qquad \text{where } i = i^n(t)$$

to be the total due to be repaid by D on or after time t.

Proposition 3

(1) Consider the n^{th} discrete approximation to the continuous case. For sufficiently large n, a necessary (resp. sufficient) condition for the project to be undertaken is that $K \leq w_0 + M$ (resp. $K < w_0 + M$) -- where, putting $t^* = \min(\hat{t}, \underline{t})$,

$$\text{(6.1)} \qquad M = \text{Max}\left(\frac{1}{2}\int_0^T r(\tau)d\tau \,,\ \int_0^{t^*} r(\tau)d\tau + \int_{t^*}^T \ell(\tau)d\tau \right)$$

$$= \text{ the } \underline{\text{debt capacity}} \text{ of the project.}$$

(2a) Consider the slowest repayment path in the n^{th} discrete approximation. As $n \to \infty$, the initial amount that D borrows from C converges to

$$\text{Max}\left(\frac{1}{2}R(0),\ L(0) \right).$$

And at any subsequent time $0 < t \leq T$, the outstanding debt that D owes C converges to

$$\text{(6.2)} \qquad \bar{P}(t) = \begin{cases} \int_t^T \ell(\tau)d\tau & \text{if } t < \hat{t} \\ \frac{1}{2}\int_t^T r(\tau)d\tau & \text{if } t \geq \hat{t}. \end{cases}$$

That is, in the limit the rate, $\bar{p}(t) \equiv -\frac{d^+}{dt}\bar{P}(t)$, at which D repays C is given by $\bar{p}(t) = \ell(t)$ if $0 < t < \hat{t}$, and by $\bar{p}(t) = \frac{1}{2}r(t)$ if $\hat{t} \leq t \leq T$.

(2b) Consider the fastest repayment path in the n^{th} discrete approximation. As $n \to \infty$, the initial amount that D borrows from C converges to $K-w_0$. And at any subsequent time $0 < t \le T$, the outstanding debt that D owes C converges to

$$(6.3) \qquad \underline{P}(t) = K - w_0 - \int_0^t r(\tau)d\tau.$$

That is, in the limit the rate, $\underline{p}(t)$, at which D repays C is given by $\underline{p}(t) = r(t)$, for $0 < t < T$. And at $t = T$, C pays D the amount $w_0+R(0)-K$ -- which is positive, by Assumption (A.1').

Proposition 3 is proved in the Appendix. The interpretation of it is as follows. For simplicity, consider only the case $\hat{t} > 0$; i.e., $L(0) > \frac{1}{2}R(0)$. (The interpretation of the other case, $\hat{t} = 0$, is similar, but simpler.) In the slowest repayment path contract, D borrows $L(0)$ initially and pays at the rate of $\ell(t)$ for $t < \hat{t}$ and $\frac{1}{2}r(t)$ for $t > \hat{t}$. In other words, D compensates C for the depreciation of her collateral before date $\hat{t}$ and afterwards gives C half the project cash flows. Note that D has no incentive to repudiate at any date since D's future debt obligations (given by $L(t)$ for $t < \hat{t}$, and by $\frac{1}{2}R(t)$ for $t \ge \hat{t}$) exactly equal $U_C(t) = \max\left(L(t), \frac{1}{2}R(t)\right)$, the amount that C would obtain in liquidation or renegotiation. (This argument shows that a higher debt burden would not be sustainable; D would have an incentive to repudiate -- that is, the above really is the slowest path.)

Note that initially D's cash flows fall short of current debt repayments in the slowest path. In fact, this happens over the range where D pays $\ell(t)$ and $r(t) < \ell(t)$ (i.e. $t \le \hat{t}$ and $t \le \underline{t}$); that is, in the interval $(0,t^*)$, where $t^* = \min(\hat{t}, \underline{t})$. How does D cover these repayments? The answer is that D needs to put aside part of the initial sum borrowed $L(0)$ in a savings account, out of which he makes interest payments. The size of the necessary savings account is

$$\int_0^{t^*} \Big(\ell(\tau) - r(\tau)\Big) d\tau,$$

implying that the amount left of L(0) which can be used to buy project assets -- the debt capacity -- is

$$L(0) - \int_0^{t^*} \Big(\ell(\tau) - r(\tau)\Big) d\tau \quad = \quad \int_0^{t^*} r(\tau)\, d\tau \quad + \quad \int_{t^*}^{T} \ell(\tau)\, d\tau.$$

But this is precisely the formula for M in Proposition 3.

The intuition for the fastest path is as in Section 5. D borrows the smallest amount that he needs to finance the project, deposits all project cash flows with C before date T and then receives all accumulated interest from C at date T (net of the initial amount borrowed).

We now return to Examples 4 and 5, and apply Proposition 3.

Example 4

$$\hat{t} = 4;\ \underline{t} = 5;\ t^* = 4;\ \text{and } M \ = \ \text{Max}\left(\frac{1}{2}\int_0^{16} \tau d\tau,\quad \int_0^{4} \tau d\tau + \int_4^{16} 5 d\tau \right) \ = \ 68.$$

Hence by Proposition 3 part (1), w_0 must equal at least K-M = 12 in order for the project to be undertaken.

Part (2a) of the proposition tells us that the slowest repayment path has D borrowing Max $(\frac{1}{2}R(0),\ L(0)) = 80 = K$ initially, and then repaying at a rate of $\ell(t) = 5$ before time 4, and at a rate of $\frac{1}{2}r(t) = t/2$ thereafter. Notice that D has an initial cash cushion of w_0, of which $\int_0^4 (5-\tau)d\tau = 12$ is used to sustain the repayments up until time 4.

According to Proposition 3 part (2b), the fastest repayment path has D borrowing the minimum amount of $80-w_0$ initially, and then repaying at the maximum feasible rate of $r(t) = t$ until time $T = 16$. At the end, C pays D an amount $w_0+R(0)-K = w_0+48$ -- which comprises D's equity share (i.e. his initial contribution w_0) plus the profit from the project (i.e. $R(0)-K = 48$): C breaks even on her investment.

Example 5

$$\hat{t} = 80;\ \underline{t} = 20;\ t^* = 20;\ \text{and } M = \text{Max}\left(\frac{1}{2}\int_{20}^{60} 10d\tau + \frac{1}{2}\int_{60}^{80} 7d\tau,\ \int_{20}^{80} 6d\tau\right) = 360.$$

Hence by Proposition 3 part (1), w_0 must equal at least $K-M = 120$ in order for the project to be undertaken.

Part (2a) of the proposition tells us that the slowest repayment path has D borrowing Max $(\frac{1}{2}R(0), L(0)) = 480 = K$ initially, and then repaying at a rate of $\ell = 6$ until the end. Notice that D has an initial cash cushion of w_0, of which $\int_0^{20} 6d\tau = 120$ is used to sustain the repayments up until time 6.

According to Proposition 3 part (2b), the fastest repayment path has D borrowing the minimum amount of $480-w_0$ initially, and then repaying at the maximum feasible rates of $r(t) = 10$ for $20 \le t < 60$, followed by $r(t) = 7$ until time $T = 80$. At the end, C pays D an amount $w_0+R(0)-K = w_0+60$ -- which comprises D's equity share (i.e. his initial contribution w_0) plus the profit from the project (i.e. $R(0)-K = 60$): C breaks even on her investment.

7. Comparative Statics Properties

We now investigate some comparative statics properties. For simplicity, we shall work with the continuous time version of the model given in the last section. We want to know how shifts in the model's exogenous parameters affect:

(1) the condition for the project to be undertaken (that is, the debt capacity M in (6.1));

and (2) the form of the repayment paths.

Concerning (2), since there is a continuum of equilibrium repayment paths, clear-cut comparative statics results for any given path are obviously ruled out. Instead, we look for comparative statics results for the extreme paths -- viz., the slowest, $\overline{P}(t)$, from (6.2); and the fastest, $\underline{P}(t)$, from (6.3). The behaviour of these particular paths is of independent interest, since, as we saw at the end of Section 5, $\overline{P}(t)$ and $\underline{P}(t)$ respectively turn out to be uniquely optimal in two reasonable extensions of the basic model.

We investigate four comparative statics experiments: (A) the assets become longer-lived (or more durable); (B) the project returns become more front-loaded; (C) the assets become more replaceable; and (D) the debtor's initial wealth rises. In the course of answering (C) and (D), we shall develop the model a little further.

(A) The assets become more durable

We say that the assets become longer-lived, or more durable, if L(t) rises for all $0 \le t \le T$.[19] Note that this definition is consistent

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[19] Here, "rises" should be understood to mean weakly rises. This convention applies throughout this section, whenever we use "rises", "falls", "faster", "slower", "more likely", "less likely".

with L(0) staying the same -- i.e., the initial liquidation value of the assets may not change as the assets become longer-lived. In the Appendix we prove:

Proposition 4A In the continuous time model, if the assets become more durable:

(1) The project is more likely to be undertaken: M rises.

(2a) The slowest repayment path becomes slower: $\bar{P}(t)$ rises for all $0 \leq t \leq T$.

(2b) The fastest repayment path, $\underline{P}(t)$, is unaffected (assuming that the initial cost of the project, K, is the same).

Intuitively, as the assets become more durable, they provide the creditor with the security to wait longer before being repaid. This means that the slowest repayment path can be slower (part 2a). And hence the debtor need not set aside as much of his initial borrowing to finance early debt repayments, leaving more to finance the initial investment (part 1). The fastest repayment path is determined entirely by the return r(t), which is being kept fixed (see Proposition 3, part 2b).

(B) The project returns become more front-loaded

We say that the project returns become more front-loaded if R(0)-R(t) rises for all $0 \leq t \leq T$. Note that this definition is consistent with R(0) staying the same -- i.e., although the returns arrive faster, in total they may stay the same. In the Appendix we prove:

<u>Proposition 4B</u> In the continuous time model, if the project returns become more front-loaded:

(1) The project is more likely to be undertaken: M rises.

(2a) If R(0) does not change, the slowest repayment path becomes faster: $\overline{P}(t)$ falls for all $0<t\leq T$. In this case, $\overline{P}(0)$ is unchanged at $\text{Max}\left(L(0), \frac{1}{2}R(0)\right)$, and so the cumulative repayment $\overline{P}(0)-\overline{P}(t)$ rises for all $0<t\leq T$.

(2b) The fastest repayment path becomes faster: $\underline{P}(t)$ falls for all $0<t\leq T$. $\underline{P}(0)$ is unchanged at $K-w_0$, and so the cumulative repayment $\underline{P}(0)-\underline{P}(t)$ rises for all $0<t\leq T$.

As the project returns become more front-loaded, clearly the fastest repayment path can be faster (part 2b). But equally, the creditor has less to bargain over later on, and so the slowest repayment path also has to be faster (part 2a). The fact that the project returns come in earlier implies that the liquidation value of the assets is high at the very time when the creditor most needs leverage: this additional security means that the project is more likely to be undertaken (part 1).

<u>(C) The assets become more replaceable</u>

Consider next what happens if we drop the assumption that D cannot replace the assets if C seizes them (i.e. D's outside wage is zero). The ideal way to represent this would be by giving D an outside option, i.e. allowing D to terminate the relationship with C, and to start a relationship with a new creditor and new assets. One would then consider how the optimal repayment path varies with the attractiveness of this outside option. Such an analysis is unfortunately outside the scope of the present paper: the problem is that what D can achieve with a new creditor and new assets depends on what D can achieve if he replaces the new creditor with an even newer creditor and even newer assets, and so on. Instead, therefore, we proceed in a much more indirect (and rudimentary) fashion by considering the following simple extension to the basic model:

Extension E3: Alternative divisions of surplus

In the basic model, our renegotiation process in the event of repudiation implies that D and C split the surplus from bargaining 50:50 (unless C's payoff falls below her outside option). We can easily allow for different divisions of surplus by assuming that if D's initial offer is turned down and C elects not to liquidate, D makes a second offer with probability $(1-\theta)$ and C with probability θ, where $0 \le \theta \le 1$. (2.1) then becomes, in the continuous case, $U_C(t) = \text{Max}\ (L(t),\ \theta R(t))$. The rest of the analysis extends in a natural way.[20]

We claim that a decrease in θ, i.e. a reduction in C's bargaining power, corresponds to an increase in asset replaceability. To see why, consider the limiting case where the assets are completely replaceable: there is a frictionless second-hand market for the project's physical capital. D can always repudiate, replace the assets at their liquidation value $L(t)$ and take out a new loan (on competitive terms) to finance this. Under these conditions, C's payoff can never exceed $L(t)$ at any date, and so $U_C(t) = L(t)$ for all t; i.e. it is as if $\theta = 0$.

In short, we will treat the parameter θ as a proxy for the degree of asset replaceability. In the Appendix we prove:

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[20] In particular, Assumption (A.3′) becomes

$$\theta r(t) \ge \ell(t) \quad \Rightarrow \quad \theta r(\tau) > \ell(\tau) \quad \text{for all } \tau > t,$$

which we assume holds for all θ in the region of interest. $t = \hat{t}(\theta)$ is defined as the earliest time at which $\theta R(t) \ge L(t)$. In Proposition 3(1), the debt capacity M becomes $\text{Max}\left(\theta R(0),\ R(0) - R(t^*(\theta)) + L(t^*(\theta))\right)$, where $t^*(\theta) \equiv \min\ (\hat{t}(\theta),\ \underline{t})$. In Proposition 3(2a), $\overline{P}(t)$ becomes $\theta R(t)$ if $t \ge \hat{t}(\theta)$.

Proposition 4C In Extension E3 of the continuous time model, if the assets become more replaceable:

(1) The project is less likely to be undertaken: M falls.

(2a) The slowest repayment path becomes faster: $\bar{P}(t)$ falls for all $0 \le t \le T$.

(2b) The fastest repayment path, $\underline{P}(t)$, is unaffected.

The intuition for Propostion 4C is much the same as that for Proposition 4A. As the assets become more replaceable, they provide the creditor with less security, which means that not only must the slowest repayment path be faster (part 2a), but also the debtor must set aside more of his initial borrowing to finance early debt repayments, leaving less to finance the initial investment (part 1). Again, the fastest repayment path is determined entirely by the return r(t), which is being kept fixed.

(D) The debtor's initial wealth rises

It follows immediately from Proposition 3 that as the debtor's initial wealth w_0 rises, the project is more likely to be undertaken, the slowest repayment path, $\bar{P}(t)$, is unaffected, and the fastest repayment path becomes faster in that $\underline{P}(t)$ falls for all $0 \le t \le T$.

It is of interest to introduce another simple extension of the basic model:

Extension E4: Choice of project scale

We have assumed up to now that the project is of fixed size. Suppose, in contrast, that the scale of the project is a choice variable λ, and that the project exhibits constant returns to scale. Given Assumption (A1), the optimal first-best scale is infinite. In the second-best, the scale will be pushed to the point λ^* (say) where the condition for the project to be undertaken holds with equality -- i.e., from part 1 of Proposition 3, $\lambda^*K = w_0 + \lambda^*M$[21] or $\lambda^* = w_0/(K-M)$, provided $K > M$. If $K \leq M$, then λ^* will be infinite and the first-best can be attained -- for example, if $r(t) \geq \ell(t)$ for all $0 \leq t \leq T$, and $K = L(0)$ (c.f. Corollary 1). Let us assume for now that $K > M$.

The slowest and fastest repayment paths are given by parts 2a and 2b of Proposition 3, where $\overline{P}(t)$ and $\underline{P}(t)$ are now both multiples of w_0. In particular, for the slowest path initial borrowing is given by

$$\lambda^* \text{ Max } \left(\frac{1}{2}R(0), L(0)\right) = \frac{w_0}{(K-M)} \text{ Max } \left(\frac{1}{2}R(0), L(0)\right);$$

and for the fastest path initial borrowing is given by $\lambda^*K - w_0 = w_0M/(K-M)$.

We have therefore shown:

<u>Proposition 4D</u> In Extension E4 of the continuous time model, if the debtor's initial wealth increases, then all of the following rise proportionately: (1) the equilibrium scale of the project; (2a) the slowest repayment path, and the corresponding amount intially borrowed; and (2b) the fastest repayment path, and the corresponding amount intially borrowed.

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[21] Note that, given the constant returns to scale assumption, the project's debt capacity M and capital cost K are both multiplied by λ^*.

8. Discussion

It would be very desirable to see how our theory fits the facts. Unfortunately, we are not aware of any systematic empirical study relating, say, the maturity structure of debt with the intertemporal profiles of project returns and asset values; and we ourselves have not conducted such an enquiry. Nonetheless, we have tried to discover what might be construed as the "received wisdom" in the world of making and taking out loans.[22] It turns out that certain apparently quite robust rules of behaviour conform well with the predictions of our theory.

A basic truth, which is worth rehearsing, is that banks are more likely to lend if the project is a good one, and/or the collateralized assets are valuable, and/or the entrepreneur can supply a large fraction of the initial investment.[23] These conclusions marry with the comparative statics results from Section 7: see Propositions 4B(1), 4A(1) and 4D, which say that a

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[22] The evidence we looked at was of four kinds. First, in 1980, 1982, 1984 and 1987, the National Federation of Independent Business (NFIB) gathered data on the small business/commercial bank relationship from surveys of random samples of their 500,000 members. The results are contained in their 1985 and 1988 reports, Credit, Banks and Small Businesses: 1980-1984 and Small Businesses and Banks: The United States. See also Leeth and Scott (1989). Second, there are guides on how to borrow: see, for example, Smollen, Rollinson, and Rubel, Sourceguide for Borrowing Capital (1977). Also, INC. Magazine has run a regular feature called "Anatomy of a Start-Up", which describes, among other details, how the start-up of a particular new business was financed. Third, there are publications offering advice on how to lend. The Journal of Commercial Bank Lending regularly features "Lending to ..." articles giving advice to commercial lenders about how to lend to a particular industry. Among other things, the articles explain the financing needs of the industry, suggest sources of collateral, and provide statistics reflecting the historical ability of the industry to repay its loans (e.g., ratios of cash flow to current maturities of long-term debt). Fourth, we looked at examples of firms which had defaulted and renegotiated their loans: see, for example, "How to Renegotiate Your Loan" INC. (November 1988); and "Heartbreak Hill" INC. (April 1988).

[23] See JCBL publications, "Lending to .."; the 1985 NFIB report, Tables 1 and 2 (pp. 4-5), Tables 12 and 13 (pp. 24-29); the 1988 NFIB report, Table 3.7 (p. 23); and Smollen, Rollinson and Rubel's 1977 Sourceguide for Borrowing, p. 21. Also, for more formal empirical work on the determinants of debt levels, see Long and Malitz (1985) and Titman and Wessels (1988).

project is more likely to be undertaken if, respectively, the total returns R(0) are high, the (initial resale) value L(0) of the assets is high, or the entrepreneur's wealth w_0 is high.

More interesting, and equally striking, is the evidence relating the maturity of debt with the purpose of the loan and the nature of the assets.[24] Long-term loans are usually used for fixed-asset aquisition -- of property, leasehold improvements, machinery, and the like. (Debt with the longest term is typically on property: real-estate mortgages.) Short-term loans, on the other hand, tend to be used for working capital purposes -- e.g., for payroll needs, for financing inventory, for smoothing seasonal imbalances -- and the collateral is usually made up of such things as the inventories or the accounts receivable.

All this squares well with our model. Proposition 4A(2) predicts that if assets are longer-lived (keeping the initial resale value L(0) fixed), then this will not only allow more projects to be undertaken, but will also support longer-term debt. Conversely, if the assets are short-lived, as in the case of inventories (which may not retain their value, or which can relatively easily be disposed of), or accounts receivable, then the debt is likely to be short-term.

The evidence concerning short-term financing also supports Proposition 4B(2), which says that the faster the returns arrive (keeping the total return R(0) fixed), the shorter will be the maturity of the debt. A firm which is raising money for payroll needs, for purchasing inventory, or for smoothing seasonal imbalances, is typically the kind of firm whose returns will be coming in soon. Our Proposition 4B(2) suggests that, ceteris paribus, it is just this kind of firm which will be taking out short-term loans.

There is evidence that firms borrow more than they need strictly need to

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[24]See the 1985 NFIB report, Tables 6 and 7 (pp. 16-20), Table 10 (pp. 22-23), and Tables 12 and 13 (pp. 24-29). See also the 1988 NFIB report, Table 3.11 (p. 30).

cover the cost of their investment projects, in order to provide themselves with a "financial cushion".[25] This fits in with our prediction in Propostion 3(2a) about the nature of the slowest equilibrium repayment path -- indeed, it is true of most paths.

Another conventional wisdom is that general, non-specific assets are good for debt; and that specific or intangible assets are good for equity financing.[26] (Although our model does not have room for equity per se, equity can be interpreted as long term debt, in the sense that minority equity holders have weak control rights.) One way of capturing the intangibility of assets is to suppose that intangible assets are hard to replace.[27] Under this interpretation, Propositions 4C(1) and 4C(2) do indeed imply that, as the degree of intangibility rises, more projects will be undertaken, more debt can be raised, and the debt will be longer-term.[28]

As far as we know, ours is the first model to deliver these kinds of predictions. We would particularly like to stress that we have provided an explanation for the fundamental maxim: "assets should be matched with liabilities". To be precise, in Proposition 3(2) we have shown that liabilities (viz., the debt repayments $p(t)$ in our model) should be matched either with the return stream $r(t)$ (in the case of the fastest repayment path), or with the rate of depreciation $\ell(t)$ of the collateral (in the case of the slowest repayment path).

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[25]See the 1988 NFIB report, Table 3.11, page 30.

[26]See Titman and Wessels (1988) and Williamson (1988).

[27]For example, a successful retail store whose location is the intangible might find it hard to move.

[28]However, this is not the only interpretation of intangibility or specificity. An increase in asset specificity might correspond to a reduction in asset durability, in the sense that if D's assets are very specific to his needs, then these assets will have low resale value. Under this alternative interpretation, Propositions 4A(1) and 4A(2) imply that, as the degree of specificity rises, fewer projects will be undertaken, less debt can be raised, and the debt will be shorter-term.

There is one important caveat. We have concentrated on the equilibrium repayment paths. As was pointed out in Sections 3-5, in a deterministic model such as ours, we can without loss of generality focus on debt contracts that are repudiation proof -- that is, on contracts that are actually executed in full. However, other contracts, which are renegotiated, could have yielded the same intertemporal allocation. For example, the optimal repayment paths that we have identified could have been implemented using a sequence of (renegotiated) shorter-term debt contracts, provided that the debtor always has access to the credit market.

The indeterminacy in optimal debt contracts disappears if there is uncertainty about project returns or liquidation values. The uncertainty case was the subject of our earlier paper, Hart and Moore (1989). The analysis in that paper was intricate; we were unable to go beyond a three-stage model, and there were relatively few clear-cut results.[29] There were some general findings from the uncertainty model, however, which we believe would apply broadly to any intertemporal model of debt based on control. We found that a key tension between short-term and long-term debt is the following. On the one hand, short-term debt gives the creditor early leverage over the project's return stream, which is good because it can keep the total indebtedness low. On the other hand, short-term debt may give too much control to the creditor in certain states, and lead to premature liquidation -- that is, the creditor may liquidate early because the debtor cannot credibly promise to repay later.[30] In this sense, long-term debt contracts protect the debtor from the creditor. An important next step in the research is to formulate a tractable, multi-period model of debt with

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[29] Indeed, it was because of our disatisfaction on this score that we reformulated our model, and turned to the deterministic case. There are some significant other differences between the two papers. In the earlier paper, the debtor could steal the returns; it was only the threat of losing future returns that forced him to disgorge money to the creditor. We now believe that the focus on stealing was a little misplaced, and that it is the inalienability of the debtor's human capital -- the fact that he can always quit -- which is the root of the commitment problem.

[30] Notice that this explains an important cost of default. Hitherto, the costs of default were usually either seen as legal costs and the like, or were left simply as an unexplained loss of value.

uncertainty.

We end by mentioning two other directions for further research. First, our analysis has focussed on a single creditor. If there were multiple creditors, then presumably the process of renegotiating a delinquent debt contract would be more difficult. This may bring benefits as well as costs. For example, in the present model, had the creditor(s) been able to precommit never to renegotiate and always to liquidate (possibly on account of there being a large number of creditors, each of them individually unwilling to forgive debt), then it would have been possible to attain first-best: D would never repudiate for fear of losing his future share of the returns r(t).[31] This benefit of having multiple creditors would, in a world of uncertainty, be offset by a cost: inefficient liquidation may occur when the debtor is unable to meet his debt obligations, even though the future prospects may be good.

Second, it is fruitful to introduce the possibility that the borrower consumes during the lifetime of the project. One leading application is to lifetime consumption. People may be unable to borrow enough from their future income fully to smooth their lifetime consumption, because they cannot necessarily commit to repay in later life. They can always save, and so the distortion from first-best will be in the direction of too small a level of consumption early on. Preliminary work suggests that the debt contracts that support such (second-best) consumption paths will be types of mortgages, where the maturity of the loan is typically fixed at somewhat less than the length of the borrower's working life. This seems consonant with actual practice, and will be the subject of a companion paper.

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[31] There are counterarguments, however. It is common for delinquent debtors to buy the assets back at a liquidation sale. Alternatively, a new (large) creditor could buy the assets at price L(t) and then go back to D in order to generate (and share) the future return R(t). Either way, the debtor's original disincentive to repudiate is removed.

APPENDIX

In the Appendix, we gather together the proofs of Lemma 1, Proposition 3, and Propositions 4A-4C.

Proof of Lemma 1

We first establish (3.1) by an induction argument. Suppose that (3.1) holds from date i+1 onwards, where $i \leq n-2$. We show that it holds at date i too.

Consider the extensive form renegotiation game. C always has the option to turn down D's initial offer and decline to liquidate at date i. Then with probability (1/2) C gets to make an offer. Her best offer is: $p_i = 0$, $p_{i+1} = r_{i+1}$, with control reverting to C at the beginning of period i+2; i.e., she lets D have the assets for another period for the amount r_{i+1}, but then forces a default (c.f. footnote 4). D is indifferent between accepting this offer and rejecting it, since, if he accepts, D is forced to hand over all the period i cash flow r_{i+1} plus the assets to C, whereas if D rejects the offer the assets lie unused for a period (there is no period i+1 cash flow) and C again has possession of the assets at i+1. (It follows that any offer more favourable to C, e.g., $p_i > 0$, $p_{i+1} = r_{i+1}, \ldots$, would be rejected by D.) C's payoff from the above offer is $r_{i+1} + U_C^{i+1}$, by the induction hypothesis.

On the other hand, with probability (1/2) D makes a second offer. D's best offer is $p_i = 0$, $p_{i+1} = 0$, with control reverting to C at the beginning of period i+2, i.e., D pays nothing for use of the assets during period i+1.

C is indifferent between accepting and rejecting this offer, since, if she rejects it, the assets lie unused for the period anyway (i.e. they generate no return). C's payoff, given this offer, is U_C^{i+1}.

It follows that C's expected payoff from turning down D's initial offer and declining to liquidate at date i is $\frac{1}{2}\left(U_C^{i+1} + r_{i+1}\right) + \frac{1}{2}U_C^{i+1} = U_C^{i+1} + \frac{1}{2}r_{i+1}$. Since C also has the option to liquidate at date i, it follows that D must offer C at least Max $\left(U_C^{i+1} + \frac{1}{2}r_{i+1}, L_i\right)$ at the beginning of period i+1 to get her to accept his initial offer. Since D has no incentive to offer more, it follows that

$$U_C^i = \text{Max } (U_C^{i+1} + \frac{1}{2} r_{i+1}, L_i). \qquad (\dagger)$$

To see that (†) implies (3.1), note that this is immediate if $U_C^{i+1} = \frac{1}{2}(r_{i+1} + \ldots + r_n)$. On the other hand, suppose $U_C^{i+1} = L_{i+1} > \frac{1}{2}(r_{i+1} + \ldots + r_n)$. According to Assumption (A.2) this can happen only if $\frac{1}{2}r_i < \ell_i$, which implies that $L_{i+1} + \frac{1}{2} r_i < L_{i+1} + \ell_i \equiv L_i$. But then (†) again implies (3.1).

The final, trivial step of the induction hypothesis is to check that (3.1) holds at i = n-1.

We now complete the proof of Lemma 1. First, suppose that (*) holds. Then $p_1, \ldots, p_n$ can be chosen to satisfy (3.3), (3.4), and (3.5) with equality. Let D offer this contract at the beginning of the renegotiation game. C will accept since (3.3) - (3.5) imply that the contract is feasible, gives D no incentive to repudiate at a future date, and yields a payoff to C of U_1^C (her reservation utility). Moreover, this contract yields a first-best efficient outcome (the project continues until date n), and so there is no better contract offer that D can make.

Next, suppose that (*) does not hold. To show that immediate liquidation is the only subgame perfect equilibrium outcome, suppose in contrast that there is a subgame perfect equilibrium which involves continuation of the project until date m, where $i < m \le n$, at which time liquidation occurs if $m < n$. Let the stream of payments from D to C along the equilibrium path be $p_i, \ldots, p_j, \ldots, p_m$. (If, along the equilibrium path, D does not repudiate at date j, p_j is simply D's payment to C at the end of period j. On the other hand, if D repudiates at the beginning of period j+1, having made a payment to C at the end of period j, and then, as a result of renegotiation, makes a further payment to C at the beginning of period j+1, p_j is the sum of these two payments. Finally, if D repudiates at the end of period j, thus losing r_j to C, and D then makes a further payment to C at the beginning of period j+1 after renegotiation, p_j is r_j plus this further payment.)

We start by supposing that $m = n$. It is immediate that the payment stream $p_i, \ldots, p_n$ must yield C at least her reservation utility U_C^i (otherwise she would not have agreed to the renegotiation); i.e. (3.4) is satisfied. Also $p_i, \ldots, p_n$ must satisfy D's budget constraint at each date (otherwise D couldn't have made the payments in equilibrium); i.e. (3.5) is satisfied. It is also easy to see that the payment stream satisfies (3.3). Suppose (3.3) is violated for some $i+1 \le k \le n-1$, and $p_k \le r_k$. Then D will never make the payments $p_{k+1}, \ldots, p_n$; instead he will repudiate the contract at the beginning of period k+1 and give C a total of U_C^{k+1} instead. On the other hand, suppose (3.3) is violated for some $i+1 \le k \le n-1$, and $p_k > r_k$. Then D will never make the payments $p_k, \ldots, p_n$; instead he will repudiate at the end of period k and give C a total of U_C^k instead.

We have thus found a payment steam satisfying (3.3) - (3.5), which violates our assumption that (*) does not hold. It follows that, contrary to our hypothesis, m = n is impossible (if (*) does not hold).

It remains to consider the case m < n. In this case, control of the assets passes to C at the beginning of period m+1 and C chooses to liquidate. Suppose first that $r_{m+1} \geq \ell_{m+1}$. Then, by Assumption (A.1), $r_k \geq \ell_k$ for all $m+1 \leq k \leq n$. But, given this, it is easy to show that (*) is satisfied at date m. (Simply choose $p_n = U_C^{n-1}$, $p_{n-1} = U_C^{n-2} - U_C^{n-1}$, $p_{n-2} = U_C^{n-3} - U_C^{n-2}$, ..., $p_{m+1} = U_C^m - U_C^{m+1}$, $p_m = 0$. It is easy to check that Assumptions (A.1) and (A.2) imply that $p_k \leq r_k$ for all $m + 1 \leq k \leq n$, and so (3.3) - (3.5) are automatically satisfied by $p_m, \ldots, p_n$.) But this means that D and C will renegotiate rather than liquidate at date m, which contradicts the maintained hypothesis that liquidation occurs at date m.

It follows that $r_{m+1} < \ell_{m+1}$. Hence, by Assumption (A.1), $r_j < \ell_j$ for all $j \leq m$. This means that the sum, $U_C^i + U_D^i = \left(w + \sum_{j=i+1}^{m} r_j + L_m\right)$, of C's and D's payoffs from date i to date m (including D's wealth w) is less than $w + L_i$. But we know that $U_C^i \geq L_i$. Therefore D is worse off than if he had simply kept his w. In conclusion, D would never offer C a new contract at date i which involved liquidation at date m < n; he would prefer immediate liquidation at date i.

Q.E.D.

Proof of Proposition 3

Take the n^{th} discrete approximation, and for $0 < t \le T$ define

$$U_C^n(t) \equiv \text{Max}\left(\sum_{j=i+1}^{n} \ell_j, \ \frac{1}{2} \sum_{j=i+1}^{n} r_j \right) \quad \text{where } i = i^n(t).^1$$

Notice that for $0 < t < \hat{t}$ and for n sufficiently large, $U_C^n(t) = \sum_{j=i+1}^{n} \ell_j$ where $i = i^n(t)$. And for $\hat{t} \le t \le T$, $U_C^n(t) = \frac{1}{2} \sum_{j=i+1}^{n} r_j$ where $i = i^n(t)$. Hence the RHS of (4.7) for $i = i^n(t)$ converges to $w_0 + M(t)$, where

$$M(t) = \begin{cases} R(0) - R(t) + L(t) & \text{if } \ 0 < t < \hat{t} \\ R(0) - \frac{1}{2}R(t) & \text{if } \ \hat{t} \le t \le T. \end{cases}$$

By Proposition 1, we therefore have that, for sufficiently large n, a necessary (resp. sufficient) condition for the project to be undertaken is that $K \le w_0 + M$ (resp. $K < w_0 + M$) where $M = \underset{0<t\le T}{\text{minimum}}\, M(t)$.

We distinguish two cases: $\hat{t} > 0$ and $\hat{t} = 0$.

First, suppose $\hat{t} > 0$. By Assumption (A.2′), $R(t) - L(t)$ is maximized at $t = \underline{t}$, and is decreasing in t for $t > \underline{t}$. Also, $\frac{1}{2}R(t)$ is decreasing in t. Hence M = minimum (X,Y) -- where

[1] If $i^n(t) = n$, let $U_C^n(t) = 0$ -- c.f. the latter part of footnote 12.

$$X = \begin{cases} R(0) - R(\underline{t}) + L(\underline{t}) & \text{if } \underline{t} < \hat{t} \\ R(0) - R(\hat{t}) + L(\hat{t}) & \text{if } \underline{t} \geq \hat{t}, \end{cases}$$

and $Y = R(0) - \frac{1}{2}R(\hat{t})$. Notice X never exceeds $R(0) - R(\hat{t}) + L(\hat{t})$, which in turn equals Y (since $L(\hat{t}) = \frac{1}{2}R(\hat{t})$). Hence $M = X = R(0) - R(t^*) + L(t^*)$, where $t^* = \min(\hat{t}, \underline{t})$. Notice that, since $\hat{t} > 0$ and $t^* \leq \hat{t}$, Assumption (A.3′) implies $L(t^*) \geq \frac{1}{2}R(t^*)$; and so $M \geq R(0) - \frac{1}{2}R(t^*) \geq \frac{1}{2}R(0)$. Our expression for M, $R(0) - R(t^*) + L(t^*)$, thus agrees with the expression given in part (1) of Proposition 3.

Second, suppose $\hat{t} = 0$. $\frac{1}{2}R(t)$ is decreasing in t, and so M equals $\frac{1}{2}R(0)$, which, from the definition of $\hat{t}$, is no less than L(0). Thus, using the fact that $\hat{t} = 0$ implies $t^* = 0$, we obtain the expression for M given in part (1) of Proposition 3.

To prove part (2a), we appeal to Proposition 2 and (5.1) -- or, equivalently, to (4.5) as an equality. We have just (indirectly) shown that the RHS of (4.5) for $i = i^n(t)$ converges to

$$\begin{cases} L(t) & \text{if } 0 < t < \hat{t} \\ \frac{1}{2}R(t) & \text{if } \hat{t} \leq t \leq T \end{cases}$$

-- which is the expression for $\bar{P}(t)$ in Proposition 3. Such a repayment path allows D initially to borrow from C an amount Max $(\frac{1}{2}R(0), L(0))$.

To prove part (2b), we appeal to Proposition 2 and (5.2) -- or, equivalently, to (4.6) as an equality. Take any $t > 0$. The RHS of (4.6) for $i = i^n(t)$ converges to $K - w_0 - R(0) + R(t)$ -- the expression for $\underline{P}(t)$ in Proposition 3. Such a repayment path allows D initially to borrow from C an amount $K-w_0$.

Q.E.D.

Proof of Proposition 4A

Let a subscript 1 (resp. 2) denote the value of a variable before (resp. after) the assets become more durable. In particular, then, we are assuming that

(a) $$L_1(t) \le L_2(t) \qquad \text{for all } 0 \le t \le T.$$

We first prove part (1) of Proposition 4A. Appealing to Proposition 3(1), we need to show that $M_1 \le M_2$. It is useful to define, for $0 \le t \le T$,

$$X_i(t) \equiv \underset{0\le\tilde{t}\le t}{\text{minimum}} \left(\int_0^{\tilde{t}} r(\tau)d\tau + \int_{\tilde{t}}^{T} \ell_i(\tau)d\tau \right) \qquad \text{for } i = 1,2.$$

Notice that, by (A.2′), M_i equals the maximum of $\frac{1}{2}R(0)$ and

$$R(0) - R(t_i^*) + L_i(t_i^*) = X_i(t_i^*) = X_i(\hat{t}_i).$$

We divide the proof into cases:

Case 1: $\hat{t}_1 \ge t_2^*$. This implies $X_1(\hat{t}_1) \le X_1(t_2^*)$ -- which, by assumption (a), is no more than $X_2(t_2^*)$. Hence $M_1 \le M_2$.

Case 2: $\hat{t}_1 < t_2^*$.

Case 2A: $t_1^* = \underline{t}_1 \le \hat{t}_1$. By (A.2′), this implies $X_1(t_1^*) = X_1(T)$. But $X_1(T)$ is no more than $X_1(t_2^*)$ -- which, by assumption (a), is no more than $X_2(t_2^*)$. Hence $M_1 \le M_2$.

Case 2B: $t_1^* = \hat{t}_1 < \underline{t}_1$. Here, by the definition of $\hat{t}_1$, $X_1(\hat{t}_1) = R(0) - \frac{1}{2}R(\hat{t}_1)$, which is no more than $R(0) - \frac{1}{2}R(t_2^*)$ since $\hat{t}_1 < t_2^*$. By (A.3′), since $t_2^* \le \hat{t}_2$, $R(0) - \frac{1}{2}R(t_2^*) \le R(0) - R(t_2^*) + L_2(t_2^*)$. Hence $M_1 \le M_2$.

Part (2a) of Proposition 4A follows from the fact that, by Proposition 3(2a), $\overline{P}_i(t) \equiv \text{Max}\left(L_i(t), \frac{1}{2}R(t) \right)$, which increases as i goes from 1 to 2.

Part (2b) of Proposition 4B is immediate from Proposition 3(2b), since R(•) is unchanged.

Q.E.D.

Proof of Proposition 4B

Let a subscript 1 (resp. 2) denote the value of a variable before (resp. after) the returns become more front-loaded. In particuiar, then, we are assuming that

(b) $$R_1(0) - R_1(t) \le R_2(0) - R_2(t) \qquad \text{for all } 0 \le t \le T.$$

We first prove part (1) of Proposition 4B. Appealing to Proposition 3(1), we need to show that $M_1 \le M_2$. It is useful to define, for $0 \le t \le T$,

$$Y_i(t) \equiv \underset{0\le\tilde{t}\le t}{\text{minimum}} \left(\int_0^{\tilde{t}} r_i(\tau)d\tau + \int_{\tilde{t}}^{T} \ell(\tau)d\tau \right) \qquad \text{for } i = 1,2.$$

Notice that, by (A.2′), M_i equals the maximum of $\frac{1}{2}R_i(0)$ and

$$R_i(0) - R_i(t_i^*) + L(t_i^*) = Y_i(t_i^*) = Y_i(\hat{t}_i).$$

We divide the proof into cases:

Case 1: $\hat{t}_1 \ge t_2^*$. This implies $Y_1(\hat{t}_1) \le Y_1(t_2^*)$ -- which, by assumption (b), is no more than $Y_2(t_2^*)$. Hence $M_1 \le M_2$.

Case 2: $\hat{t}_1 < t_2^*$.

Case 2A: $t_1^* = \underline{t}_1 \le \hat{t}_1$. By (A.2′), this implies $Y_1(t_1^*) = Y_1(T)$. But $Y_1(T)$ is no more than $Y_1(t_2^*)$ -- which, by assumption (b), is no more than $Y_2(t_2^*)$. Hence $M_1 \le M_2$.

Case 2B: $t_1^* = \hat{t}_1 < \underline{t}_1$. Here, by the definition of $\hat{t}_1$, $Y_1(\hat{t}_1) = \frac{1}{2}R_1(0) + \frac{1}{2}\left(R_1(0) - R_1(\hat{t}_1)\right)$, which, since $\hat{t}_1 < t_2^*$, is no more than $\frac{1}{2}R_1(0) + \frac{1}{2}\left(R_1(0) - R_1(t_2^*)\right)$. By assumption (b), this is no more than $\frac{1}{2}R_2(0) + \frac{1}{2}\left(R_2(0) - R_2(t_2^*)\right)$ -- which, by (A.3′), is in turn no more than $R_2(0) - R_2(t_2^*) + L(t_2^*)$, since $t_2^* \le \hat{t}_2$. Hence $M_1 \le M_2$.

To prove part (2a) of Proposition 4B, notice that, if $R_1(0) = R_2(0)$, $R_1(t) \ge R_2(t)$ for all $0 < t \le T$. The result then follows from the fact that, by Proposition 3(2a), $\overline{P}_i(t) \equiv \text{Max}\left(L(t), \frac{1}{2}R_i(t) \right)$, which decreases as i goes from 1 to 2.

Part (2b) of Proposition 4B is immediate from Proposition 3(2b).

Q.E.D.

Proof of Proposition 4C

Let $t = \hat{t}(\theta)$ be the earliest time at which $\theta R(t) \ge L(t)$. Let $t^*(\theta) \equiv \min\,(\hat{t}(\theta), \underline{t})$. From (the modified) Proposition 3(1), the debt capacity equals $\text{Max}\left(\theta R(0),\ R(0) - R(t^*(\theta)) + L(t^*(\theta)) \right) \equiv M(\theta)$, say. To prove part (1) of Proposition 4C, we need to show that $M(\theta)$ is increasing in θ.

If $\underline{t} \le \hat{t}(\theta)$ then $M(\theta) = \text{Max}\left(\theta R(0),\ R(0) - R(\underline{t}) + L(\underline{t})\right)$, which is increasing in θ (maintaining the inequality $\underline{t} \le \hat{t}(\theta)$). If $\hat{t}(\theta) < \underline{t}$ then $M(\theta) = \text{Max}\left(\theta R(0),\ R(0) - R(\hat{t}(\theta)) + L(\hat{t}(\theta))\right)$, which is also increasing in θ --

using the facts that $\hat{t}(\theta)$ is decreasing in θ, and $r(\hat{t}(\theta)) < \ell(\hat{t}(\theta))$ because $\hat{t}(\theta) < \underline{t}$. $\hat{t}(\theta)$ and $M(\theta)$ are continuous in θ; and so part (1) of Proposition 4C is proved.

To prove part (2a) of Proposition 4C, note that from (the modified) proposition 3(2a), in the slowest repayment path the outstanding debt at any time $0 \le t \le T$ is $\text{Max}\left(\theta R(t),\ L(t)\right) \equiv \bar{P}(t|\theta)$, say. Clearly, $\bar{P}(t|\theta)$ is increasing in θ.

Part (2b) of Proposition 4C simply reflects the fact that, in (the modified) Proposition 3(2b), $\underline{P}(t)$ is independent of θ.

Q.E.D.

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