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SEARCH, STICKY PRICES, AND INFLATION

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No. 509 December 1988

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Abstract

This paper examines equilibrium in a market with free entry where consumers search and firms set prices on individual units of the commodity. The prices attached to newly produced goods are continuously adjusted. Prices attached to previously produced goods can only be changed at a cost. Inflation cuts into the market power created by the need to search for the good. Thus consumer welfare is u-shaped in inflation.

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Search, Sticky Prices, and Inflation

Peter A. Diamond

Search theory has been developed in response to the observation that resource allocation is a time consuming, costly process and the possibility that explicit modelling of the resource allocation process would result in a somewhat different picture of the workings of the economy. By and large, search theoretic models have been real models. Yet money exists as a transactions medium precisely to economize on transactions costs. Moreover, there are costs to selecting and adjusting nominal prices. Just as transaction costs are a necessary part of the coordination of trade, some degree of price stickiness is a necessary part of a realistic transactions technology. These costs of price adjustments have been recognized in the literature on (S, s) pricing, that was initiated by the Eytan Sheshinski - Yoram Weiss (1977) paper. In a recent paper, Roland Benabou (1988) combined consumer search with (S, s) pricing policy by firms. Benabou followed standard practice in the sticky price literature by assuming that a change in price by a firm affects all transactions by that firm after that date. This paper explores an alternative simple assumption: that nominal prices are attached to individual units of commodities. The prices attached to newly produced goods are continuously adjusted. Prices attached to previously produced commodities can be changed at a cost. This alternative reflects actual practice for some commodities where there is a wide distribution of units of inventory available for inspection with prices attached. Moreover this assumption avoids a difficult problem in equilibrium modelling with the standard alternative assumption - the relative timing of price changes of different firms. By assuming a constant cost per commodity for which the price is changed, all

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firms will behave the same, continuously repricing the lowest price goods in inventory. This paper explores the comparative statics of steady economy wide inflation in a market with consumer search and optimal price setting by firms. The first part of the paper examines the case where the cost of adjusting prices is sufficiently large that adjustments do not happen. In sections 6 and 7, the model is extended to (S, s) pricing.

The model has continuous time with a continuous flow of identical new consumers into the market, each of whom seeks to purchase one unit provided the real price does not exceed the utility value of the good. There is utility discounting, but no explicit cost of search. On the firm side there is free entry with identical firms and optimal price setting. The optimal price for a newly produced good is the maximum that consumers searching in an inflationary environment are willing to pay. Inflation produces the possibility of bargains from finding previously priced goods that have not yet been sold or repriced. The model assumes that the nominal interest rate rises one for one with the inflation rate. This assumption, appropriate for credit card purchases, is in contrast with a situation in which no interest is earned on the purchasing power being carried during the search process. It is assumed that the rate of meeting between consumers and commodities is a constant returns to scale function of the stocks of customers and inventory, with the probability of a contact being the same for each individual. In steady state equilibrium the flow of newly produced goods equals the exogenous flow of new customers. However, the stocks of goods in inventory and of searching customers adjust in response to the zero expected profit condition arising from free entry. With no repricing, the greater the inflation rate the greater the stock of customers and the smaller the stock of inventories (the smaller the meeting rate for customers and the greater the meeting rate for commodities). The real price placed on a newly produced good is not monotonic in the inflation rate. Since utility of consumption minus this price is also the expected utility of consumers, consumers are better off with some inflation than with none. The gain from moderate inflation comes from the dilution of the market power created by the costs of search. When inflation becomes large enough, the decrease in entry balances the gain from reducing market power. Calculations are presented giving the price of newly produced goods as a function of the inflation rate.

1. Matching Technology

It is assumed that there is a continuous flow of size x of new customers into this market. Each customer seeks to purchase one unit of the commodity as long as the real price does not exceed u. We denote by X the stock of customers actively searching in the market. Similarly, we denote by y the flow of newly produced commodities into inventory, and by Y the stock of goods available in inventory. There is a matching technology which determines the flow rate of matches as a function of the stocks of customers and inventory, m(X,Y). We assume that m has constant returns to scale with a strictly positive marginal contribution by each factor, $m_1 > 0$, $m_2 > 0$.

In steady state equilibrium the rate of matching equals the exogenous rate of arrival of new customers (since there is no reason for a meeting not to result in a purchase). Taste differences which would endogenize the purchase probability below one are not examined. Thus we have:

$$x = m(X,Y) \tag{1}$$

We assume that each individual experiences the same flow probability of a match and so experiences the arrival of a transaction opportunity as a Poisson process. We denote these arrival rates for customers and inventory by a and b. In steady state equilibrium these must satisfy

$$a = x/X \tag{2}$$
$$b = x/Y$$

With constant returns to scale, we have

$$1 = m(a^{-1}, b^{-1}) \tag{3}$$

2. The Distribution of Prices

As we will note below, firms will price newly produced goods at the maximum willingness of customers to pay. There is no reason for a distribution of prices of newly produced goods. Thus the distribution of prices on goods currently in the market reflects the constant arrival rate of goods whose real prices decay exponentially at the inflation rate, π ($\pi > 0$), with the quantity of goods still remaining on the market at any given price also declining exponentially at the arrival rate of customers, b. Thus at any time the distribution of real prices in the market has positive density between 0 and p, the price set on newly produced goods. Consider any real price, s, in this interval. Purchases reduce the fraction of goods with prices below s at the rate $bF(s)$ where F is the distribution of prices. Inflation adds to the stock of goods with real prices below s at the rate $\pi sf(s)$ where f is the density of prices. Equating these two flows, the steady state density of commodities with real prices satisfies

$$f(s) = \left(\frac{b}{\pi p}\right) \left(\frac{s}{p}\right)^{b/\pi - 1} \quad 0 \le s \le p \tag{4}$$

This distribution is homogeneous of degree 0 in b and π since proportional changes in both variables are equivalent to a change in the units in which time is measured. The mean price of goods on the market, $\bar{p}$, (and so of transactions) satisfies

$$\bar{p} = \frac{bp}{b + \pi} \tag{5}$$

3. Consumer Search

We assume that the purchasing power held by customers while searching is earning the going rate of interest in the economy and that the real rate of interest in the economy is constant. Thus we assume that the nominal rate increases point for point with the inflation rate:

$$i = \pi + r \tag{6}$$

where i and r are the nominal and real interest rates, respectively. This assumption fits with payment by check or credit card rather than currency. We denote by V the asset value of being a customer in the search market. We assume that the real rate of utility discount on the utility from consuming this good is equal to the real rate of interest in the economy earned on purchasing power. We also assume that utility is linear in income available to spend on other goods. Thus we can use the standard dynamic programing framework for describing consumer search. We denote by p* the maximum willingness to pay by a consumer. Thus we have

$$rV = a\int_0^{p^*}[u - V - s]f(s)ds \tag{7}$$

The maximum willingness to pay is equal to the utility from consuming the commodity less the value of continuing to search for later consumption:

$$p^* = u - V \tag{8}$$

Since, as will be argued below, firms never set real prices above real willingness to pay, p* coincides with p, the price of newly produced commodities. Thus, we can write (7) as:

$$rV = a(u - V - \bar{p}) \tag{9}$$

Substituting for the mean price from (5), and dropping the distinction between the price and the maximum willingness to pay, we have one equation for the price from consumer search behavior.

$$r(u - p) = a\ p\left(\frac{\pi}{b + \pi}\right) \tag{10}$$

The combination of a positive inflation rate and the only cost of search being delay in gratification implies that the maximum willingness to pay is strictly less than the utility value of the good, u. The addition of an explicit search cost would raise the possibility that u is the maximum willingness to pay, rather than a value derived from the comparison of purchasing today with purchasing in the future.

4. Pricing and Entry

We assume that the real cost of producing the unit for sale is c. This cost, like the utility value of consumption, is rising in nominal terms at the rate, π. We assume that there is no setup cost to entering this market. With free entry and identical firms, the expected real discounted profit from producing a good for sale in this market will equal c. When the firm produces a unit of the good it attaches a nominal price to the unit and is not allowed to revise that price in the future. No firm will set a price higher than the current maximum willingness to pay. To do so would simply introduce a period when a good was sitting in inventory, not available for sale. This would lose the real interest rate on the real cost of production that has already taken place, even though there was no loss from inflation while waiting for the willingness to pay to rise to the level of price that has been set on the commodity. Given this fixed nominal price we can calculate the expected present discounted value of profit from the sale of this commodity using the usual dynamic programming approach. For this commodity the profit opportunities given that the commodity has not yet been sold, are stationary in nominal terms. Thus the equation is stated in nominal terms. W is used to denote the value of a newly produced commodity for sale.

$$iW = b(p - W) \tag{11}$$

With free entry W must equal the cost of production, c. Converting the nominal interest rate into a real rate plus the inflation rate, the zero profit condition can be written as:

$$bp = (r + \pi + b)c \tag{12}$$

The markup over cost depends upon the real interest rate, the inflation rate, and the arrival rate of customers. Combining (5) and (11), we see that the mean real transactions price satisfies

$$\bar{p} = (1 + \frac{r}{b + \pi})c = \left[\frac{b + i}{b + \pi}\right]c \tag{13}$$

5. Equilibrium

The model has five endogenous variables: a, b, X, Y, and p. The variables are determined by the three equations describing the search technology, (1) and (2), and the two pricing equations coming from consumer search, (10), and zero profits, (12). Without restriction on the search technology there is no assurance that there will be an equilibrium with positive production, even with the cost of production less than the utility of consumption ($c < u$). What is needed is a search technology that permits b to be sufficiently large that firms can cover costs even with a high inflation rate. For example, Cobb-Douglas search technology ensures existence. Without attempting to mark out the range of inflation values for which there exists an equilibrium for a particular search technology, we continue comparative steady state analysis of alternative inflation rates assuming that the equilibrium exists.

Eliminating the price from (10) and (12) we get the condition:

$$\frac{u}{c} = (1 + \frac{r + \pi}{b})\ (1+ \frac{a\pi}{rb + r\pi}) \tag{14}$$

We can now solve for a and b from (3) and (14). The left-hand side of (14) is independent of the inflation rate. The right-hand side is increasing in the inflation rate. With constant return to scale in search, a is decreasing in b. Thus, an increase in the inflation rate will increase b and decrease a. From (13) we conclude that the mean price falls with the inflation rate. Given the constancy of the flow of new customers into the market, (2) implies that a rise in the inflation rate raises the stock of searching customers and lowers the steady state stock of inventory. (A more complicated model would endogenize the flow of new customers, depending on the value of becoming a searching customer in this market.)

The relationship between the real price of newly produced goods (and expected utility of consumers, (8)) and the inflation rate depends upon the nature of the search technology. This can be seen by using (10) and (12) to eliminate a and b from (3):

$$1 = (p - c)m\left[\frac{\pi p}{r(u - p)\ (rc + \pi p)}\ ,\ \frac{1}{(r + \pi)c}\right] \tag{15}$$

Equation (15) can be differentiated implicitly to examine how p varies with different parameters. For this calculation, it is useful to have a symbol for the share of customers in the marginal value of contributions to matching. Let α satisfy

$$\alpha = \frac{m_1}{am} \tag{16}$$

Differentiating (15) with respect to u, we have

$$\frac{dp}{du} = \alpha\left(\frac{u - p}{p - c} + \alpha\ \frac{urc + \pi p^2}{prc + \pi p^2}\right)^{-1} \tag{17}$$

For p closer to c than u, dp/du is smaller than $\alpha/(1 + \alpha)$. This is in sharp contrast with the no inflation case where $p = u$.

Differentiating (15) implicitly with respect to π, we have

$$\frac{dp}{d\pi} = \frac{-(p-c)\left[\left(\frac{prc}{r(u - p)\ (rc + \pi p)^2}\right)m_1 - \left(\frac{1}{c(r + \pi)^2}\right)m_2\right]}{m + (p - c)\ \left(\frac{\pi(urc + \pi\ p^2)}{r(u - p)^2\ (rc + \pi p)^2}\right)m_1} \tag{18}$$

The denominator is positive. Using (16) the sign of $\frac{dp}{d\pi}$ can be written as

$$\text{sign}\ \frac{dp}{d\pi} = \text{sign}\ \left(\frac{1 - \alpha}{\alpha} - \frac{rc(r + \pi)}{\pi(rc + p\pi)}\right) \tag{19}$$

Thus p is decreasing in π at an infinite rate at $\pi = 0$. The rate of inflation which maximizes consumer expected utility is positive. This result can be seen from the fact that with no inflation, this model results in a price equal to the reservation utility of consumers, $p = u$, (Diamond (1971)) and so no consumer surplus from this good. Thus inflation can only help. Similarly deflation will raise expected utility. To examine the behavior of welfare around zero inflation in a more interesting setting, one would want differing reservation utilities across consumers or demands which vary with price, taking on more than just two different values.

For the Cobb-Douglas case, we note from (19) that the inflation rate that minimizes p (and so maximizes expected utility) is less than $\frac{\alpha}{1 - \alpha}$ times the real rate of interest. Since p is endogenous, the optimal inflation rate cannot be read directly from (19). Using (15), the equilibrium price as a function of the inflation rate has been calculated for the Cobb-Douglas search technology, $m = A^{-1}X^{\alpha}Y^{1-\alpha}$. Some of the results are shown in Figures 1-3. The figures show the u-shaped pattern of real price as a function of the inflation rate, the effect of greater search speed in lowering price, and the small impact of the utility of the good on the equilibrium price.

6. Price Adjustment

We now generalize the model by assuming that the price of a single unit of the commodity can be changed at a cost c'. For $c' \geq c$, no one would ever bother to change the price since it is cheaper to produce a new unit. For $c' < c$, some price adjustment will take place. We denote by p' the real price at which the price change is made. The price will be changed to p, the maximal willingness to pay of consumers. The price change occurs if the unit remains on the market for length of time t equal to $(\ln(p/p'))/\pi$. A good stays on the market this long with probability

$$e^{-bt} = \left(\frac{p'}{p}\right)^{b/\pi}. \tag{20}$$

The density of prices in the market is a truncated (and proportionally increased) adjustment of the density in (4).

$$f(s) = \frac{b}{\pi} s^{b/\pi - 1} \left(p^{b/\pi} - p'^{\,b/\pi} \right)^{-1} \qquad p' \leq s \leq p \tag{21}$$

The mean price of goods in the market now satisfies

$$\bar{p} = \left(\frac{b}{b + \pi} \right) \left[\frac{p^{b/\pi + 1} - p'^{\,b/\pi + 1}}{p^{b/\pi} - p'^{\,b/\pi}} \right] \tag{22}$$

Consumer search gives us the value of search

$$rV = a \int_{p'}^{p} (u - V - s) f(s) ds \tag{23}$$

$$= a(u - V - \bar{p})$$

Solving for the maximal willingness to pay, p, (= u - V), we have

$$p = \frac{ru + a\bar{p}}{r + a} \tag{24}$$

Turning to the supply side of the market, we need to evaluate the real value, W, of a good with real price p that will be repriced to p when the price falls to p′. From the analysis above, (11), a good priced at p and left on the market indefinitely is worth $bp/(r + \pi + b)$. With probability e^{-bt}, at time t, the good is repriced, foregoing the expected profit $bp'/(r + \pi + b)$, bearing the cost c′, and restoring value. Thus

$$W = \frac{bp}{r + \pi + b} - e^{-bt} e^{-rt} \left[\frac{bp'}{r + \pi + b} + c' - W \right] \tag{25}$$

$$= \frac{bp}{r + \pi + b} - \left(\frac{p'}{p} \right)^{\frac{b + r}{\pi}} \left[\frac{bp'}{r + \pi + b} + c' - W \right]$$

Solving we have

$$W = \frac{\dfrac{bp}{r + \pi + b} - \left(\dfrac{p'}{p}\right)^{\frac{b + r}{\pi}} \left(\dfrac{bp'}{r + \pi + b} + c'\right)}{1 - \left(\dfrac{p'}{p}\right)^{\frac{b + r}{\pi}}} \tag{26}$$

Equation (26) gives W for any p'. To find the optimal p', we maximize (26) with respect to p'. Implicitly differentiating (25) and setting $\partial W/\partial p'$ equal to zero, we have

$$W - c' = \frac{bp'}{b + r} . \tag{27}$$

As before, free entry sets the value of a newly priced good equal to the cost of production

$$W = c \tag{28}$$

The model can now be described in terms of the endogenous variables a, b, X, Y, W, p, p', and $\bar{p}$. These variables solve equations (1), (2a), (2b), (22), (24), (26), (27) and (28). Eliminating W, X, and Y, the equations can be written as

$$1 = m(a^{-1}, b^{-1})$$

$$a = \frac{r(u - p)}{p - \bar{p}}$$

$$b = \frac{r(c - c')}{p' + c' - c} \tag{29}$$

$$\bar{p} = \left(\frac{b}{b + \pi}\right) \left(\frac{p - (p'/p)^{b/\pi} p'}{1 - (p'/p)^{b/\pi}}\right)$$

$$c = \frac{bp}{r + b + \pi} - \left(\frac{p'}{p}\right)^{\frac{b + r}{\pi}} \left(\frac{bp'}{r + b + \pi} + c' - c\right)$$

Solving out for a, b, and $\bar{p}$, (29) becomes a two equation system in p and p′. For the Cobb-Douglas case $m = A^{-1}X^{\alpha}Y^{1-\alpha}$, some calculated values are shown in Figures 4-6. Figure 4 shows minimum, mean, and maximum prices as a function of the inflation rate. Figure 5 shows the relation of price to inflation for different costs of price adjustment. Greater adjustment costs give lower prices. Also, the welfare gain from higher inflation extends to higher values of inflation with lower costs of adjustment. Figure 6 shows the equilibrium arrival rates for both sides of the market. The greater the inflation rate the more rapid the rate of sales and the slower the rate of purchase.

It is straightforward to add a real carrying cost, z, for holding the good in inventory. Since the probability of sale is independent of price, we can distinguish a gross of carrying cost value of a unit of inventory, W, and a net value, W′:

$$W - W' = \frac{z}{b + r} \tag{30}$$

The zero profit condition, (28), now becomes

$$W' = c$$

or

$$W = c + \frac{z}{b + r} \tag{31}$$

Since (28) is the only equation containing c in the set of equations determining equilibrium, we can calculate the equilibrium values by replacing c in (29) with $c + \frac{z}{b + r}$. Doing this, the changed equations in (29) become

$$b = \frac{r(c - c') + z}{p' + c' - c} \tag{32}$$

$$c = \frac{bp}{r + b + \pi} - \left(\frac{p'}{p}\right)^{\frac{b + r}{\pi}} \left[\frac{bp'}{r + b + \pi} + c' - c\right] - \left(1 - \left(\frac{p'}{p}\right)^{\frac{b + r}{\pi}}\right) \left(\frac{z}{b + r}\right)$$

For the Cobb-Douglas case, some calculated values are shown in Figures 7-10, Figure 7 relates the price of newly priced goods to the inflation rate for different levels of carrying cost. Note that the horizontal scale has been doubled to show the wide range over which welfare is rising with inflation for high z values. Figures 8 and 9 relate the price and the expected time in inventory to the carrying cost for .03 and .09 inflation rates. Figure 10 relates the length of time before repricing to the inflation rate for different costs of repricing. As noted in the derivation of (20), this time satisfies

$$t = \frac{\ln(p/p')}{\pi} \tag{33}$$

It would be interesting to explore analytically the monotonocities that have shown up in the calculated examples.

Search theory has been developed to explore the implications for trade coordination of the fact that there does not exist a costless instantaneous trade coordination mechanism. Money is used as a method of holding down transactions costs. In the absence of a costless and perfect indexing mechanism, nominal rigidities are a necessary part of realistic descriptions of trade coordination. Nominal rigidities come in a variety of forms associated with different technologies for arranging trades. This paper adds to the ongoing literature by examining the implications of one such nominal rigidity for the allocation process.

<u>Reference</u>

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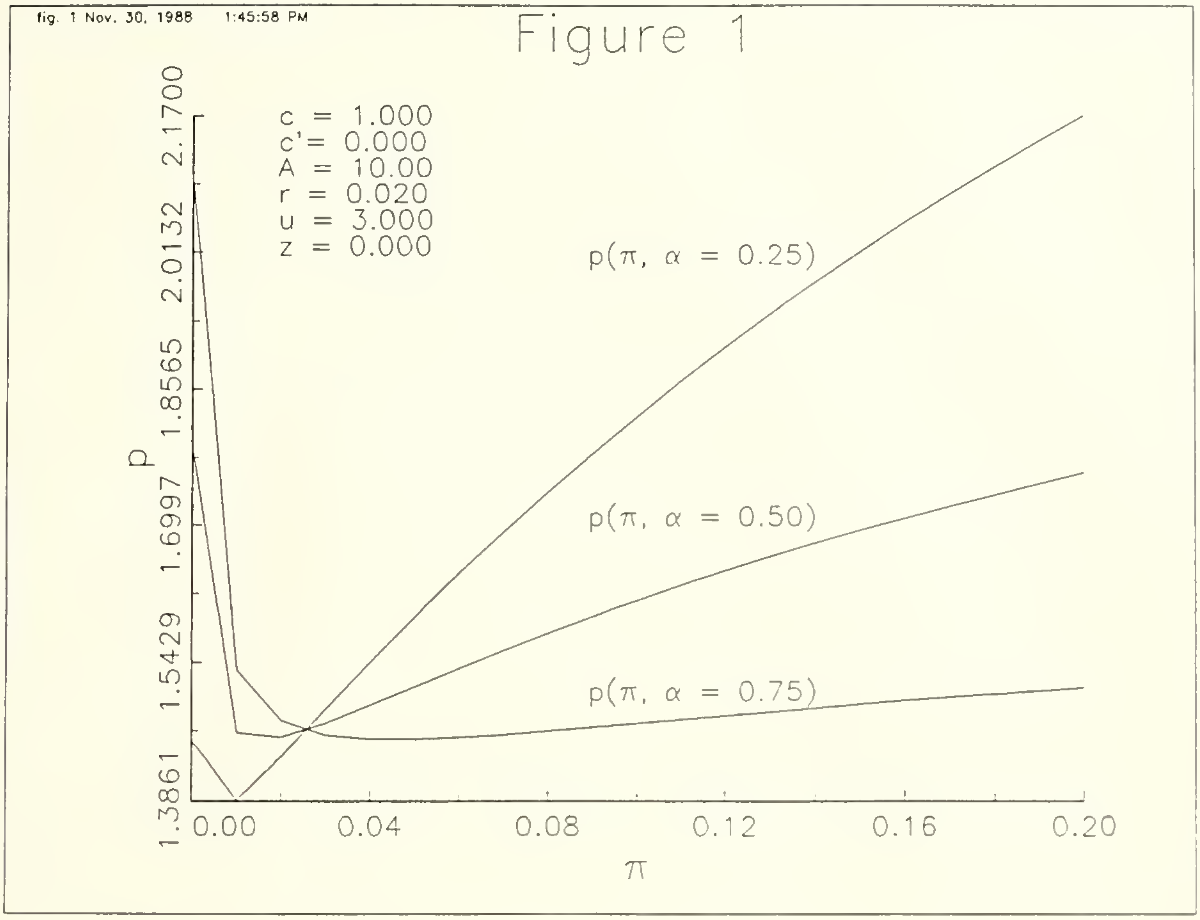
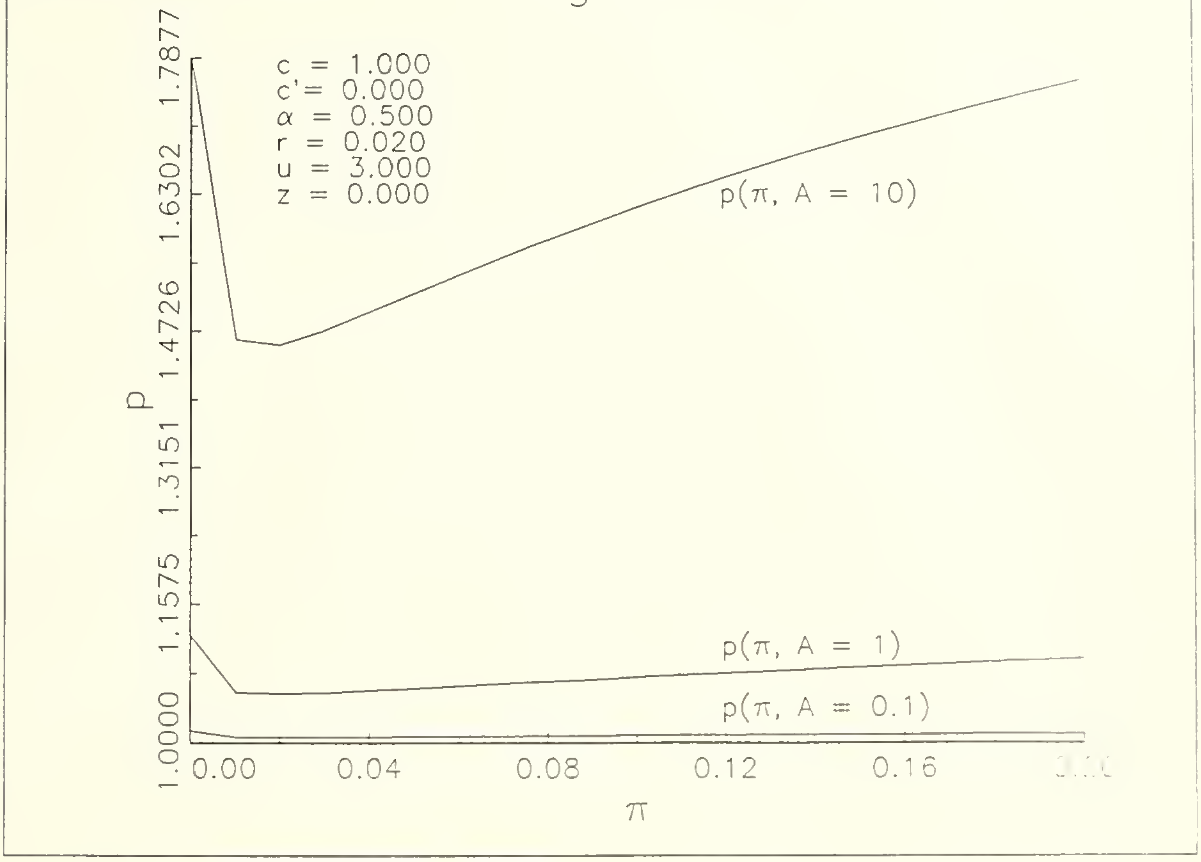


fig. 1 Nov. 30, 1988 1:45:58 PM
Figure 1
c = 1.000
c'= 0.000
A = 10.00
r = 0.020
u = 3.000
z = 0.000
p(π, α = 0.25)
p(π, α = 0.50)
p(π, α = 0.75)
p
1.3861
1.5429
1.6997
1.8565
2.0132
2.1700
0.00
0.04
0.08
0.12
0.16
0.20
π

fig. 2 Dec. 16, 1988 10:33:22 PM

Figure 2



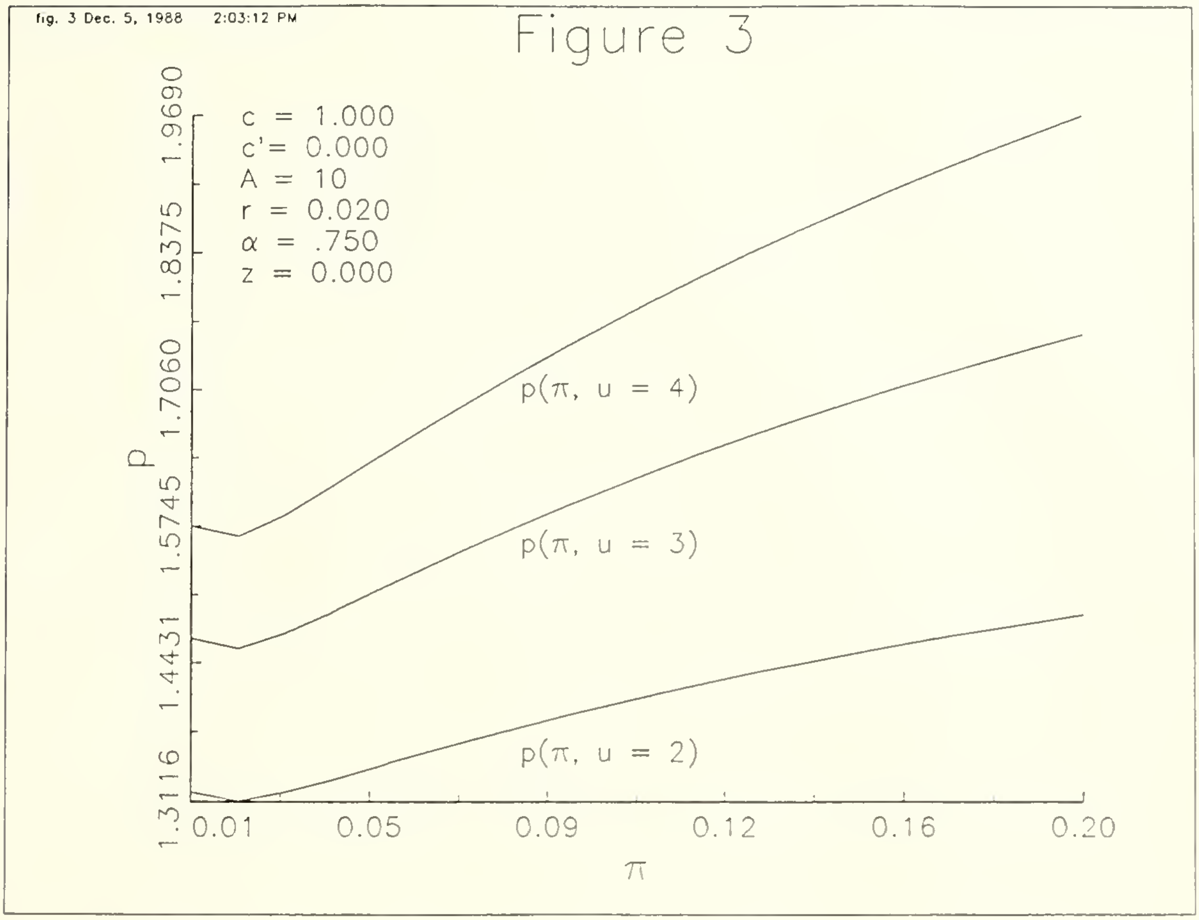


fig. 3 Dec. 5, 1988 2:03:12 PM
Figure 3
c = 1.000
c'= 0.000
A = 10
r = 0.020
α = .750
z = 0.000
p
1.3116
1.4431
1.5745
1.7060
1.8375
1.9690
p(π, u = 4)
p(π, u = 3)
p(π, u = 2)
0.01
0.05
0.09
0.12
0.16
0.20
π

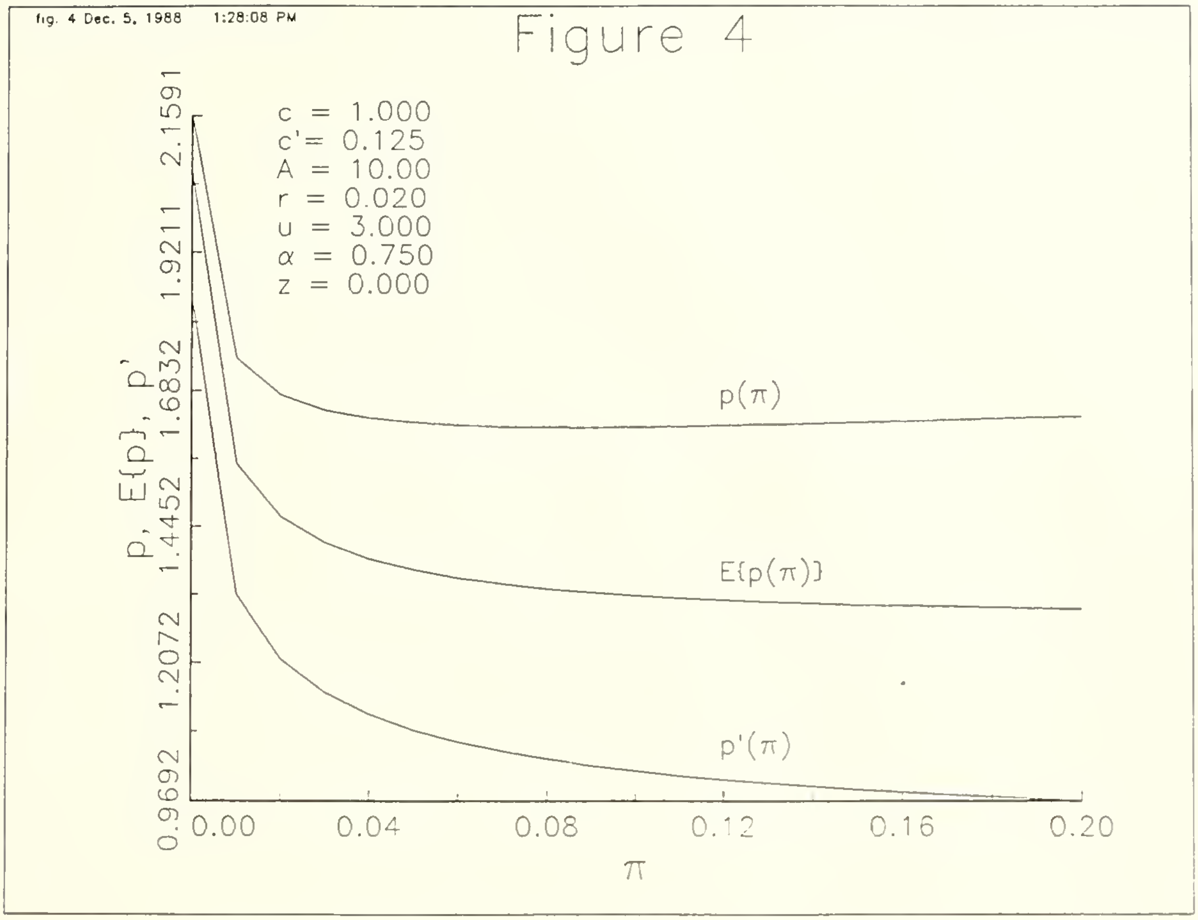

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Figure 4
c = 1.000
c'= 0.125
A = 10.00
r = 0.020
u = 3.000
α = 0.750
z = 0.000
p(π)
E{p(π)}
p'(π)
p, E{p}, p'
2.1591
1.9211
1.6832
1.4452
1.2072
0.9692
0.00
0.04
0.08
0.12
0.16
0.20
π

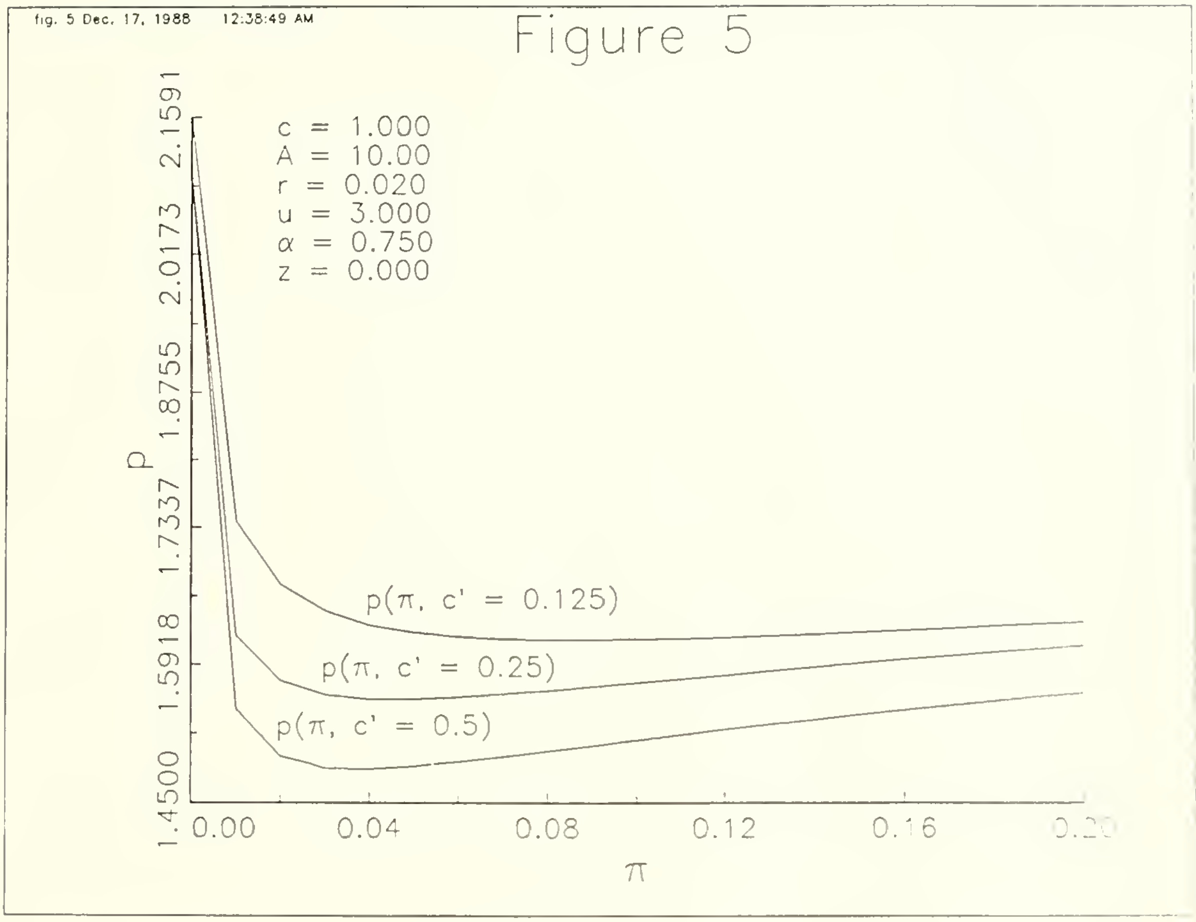
fig. 5 Dec. 17, 1988 12:38:49 AM
Figure 5
c = 1.000
A = 10.00
r = 0.020
u = 3.000
α = 0.750
z = 0.000
p(π, c' = 0.125)
p(π, c' = 0.25)
p(π, c' = 0.5)
p
2.1591
2.0173
1.8755
1.7337
1.5918
1.4500
0.00
0.04
0.08
0.12
0.16
π

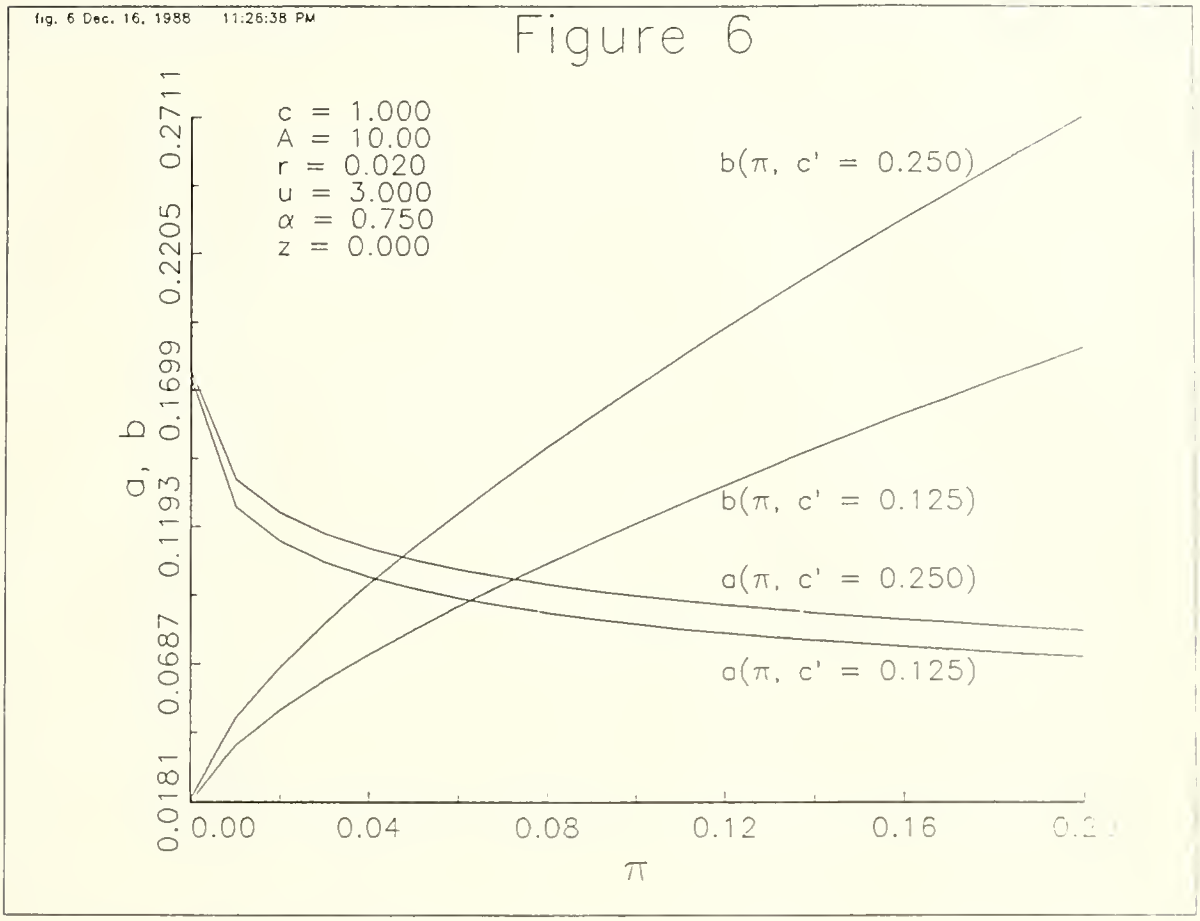
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Figure 6
c = 1.000
A = 10.00
r = 0.020
u = 3.000
α = 0.750
z = 0.000
b(π, c' = 0.250)
b(π, c' = 0.125)
a(π, c' = 0.250)
a(π, c' = 0.125)
a, b
0.0181
0.0687
0.1193
0.1699
0.2205
0.2711
0.00
0.04
0.08
0.12
0.16
π

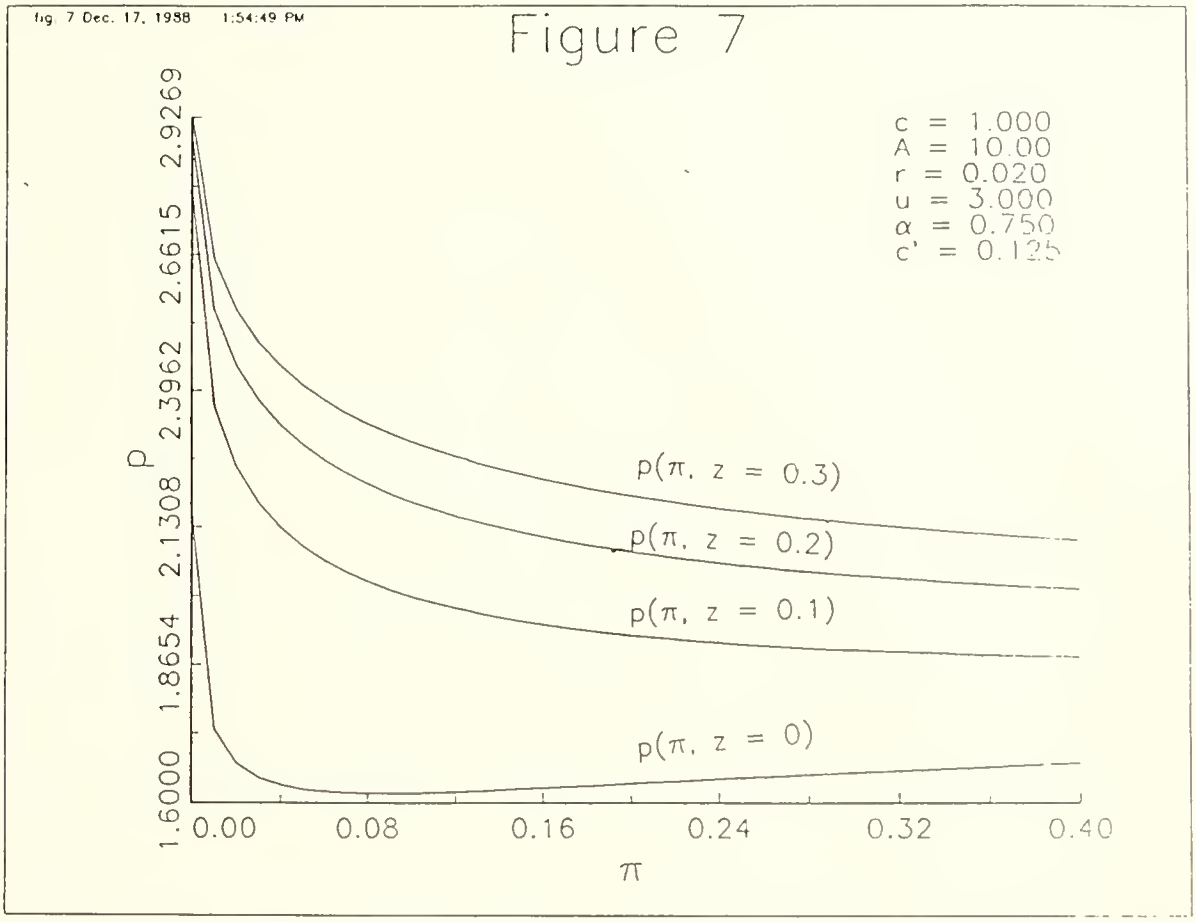
fig 7 Dec. 17, 1988 1:54:49 PM
Figure 7
c = 1.000
A = 10.00
r = 0.020
u = 3.000
α = 0.750
c' = 0.125
p
2.9269
2.6615
2.3962
2.1308
1.8654
1.6000
p(π, z = 0.3)
p(π, z = 0.2)
p(π, z = 0.1)
p(π, z = 0)
0.00
0.08
0.16
0.24
0.32
0.40
π

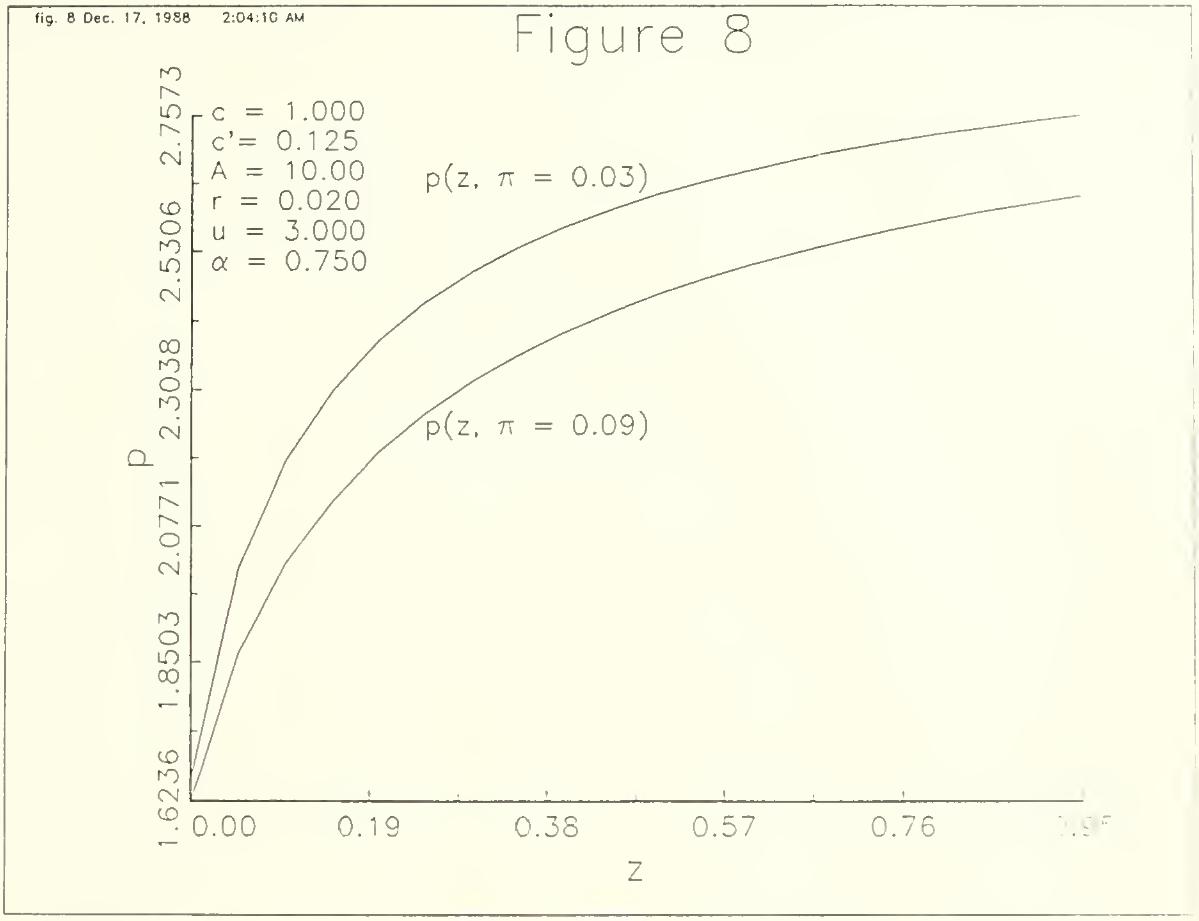

fig. 8 Dec. 17, 1988 2:04:10 AM
Figure 8
c = 1.000
c'= 0.125
A = 10.00
r = 0.020
u = 3.000
α = 0.750
p(z, π = 0.03)
p(z, π = 0.09)
p
1.6236
1.8503
2.0771
2.3038
2.5306
2.7573
0.00
0.19
0.38
0.57
0.76
z

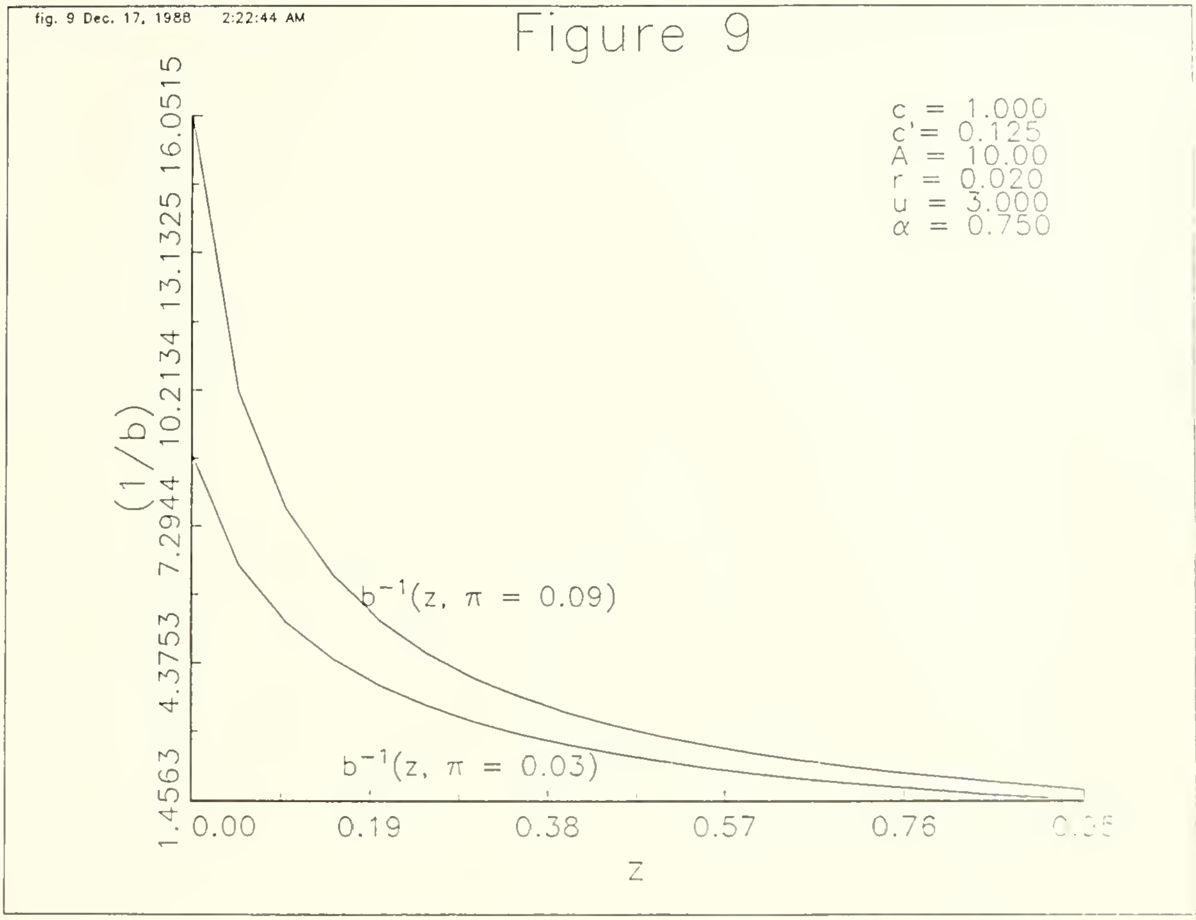
fig. 9 Dec. 17, 1988 2:22:44 AM
Figure 9
c = 1.000
c'= 0.125
A = 10.00
r = 0.020
u = 3.000
α = 0.750
(1/b)
16.0515
13.1325
10.2134
7.2944
4.3753
1.4563
b⁻¹(z, π = 0.09)
b⁻¹(z, π = 0.03)
0.00
0.19
0.38
0.57
0.76
z

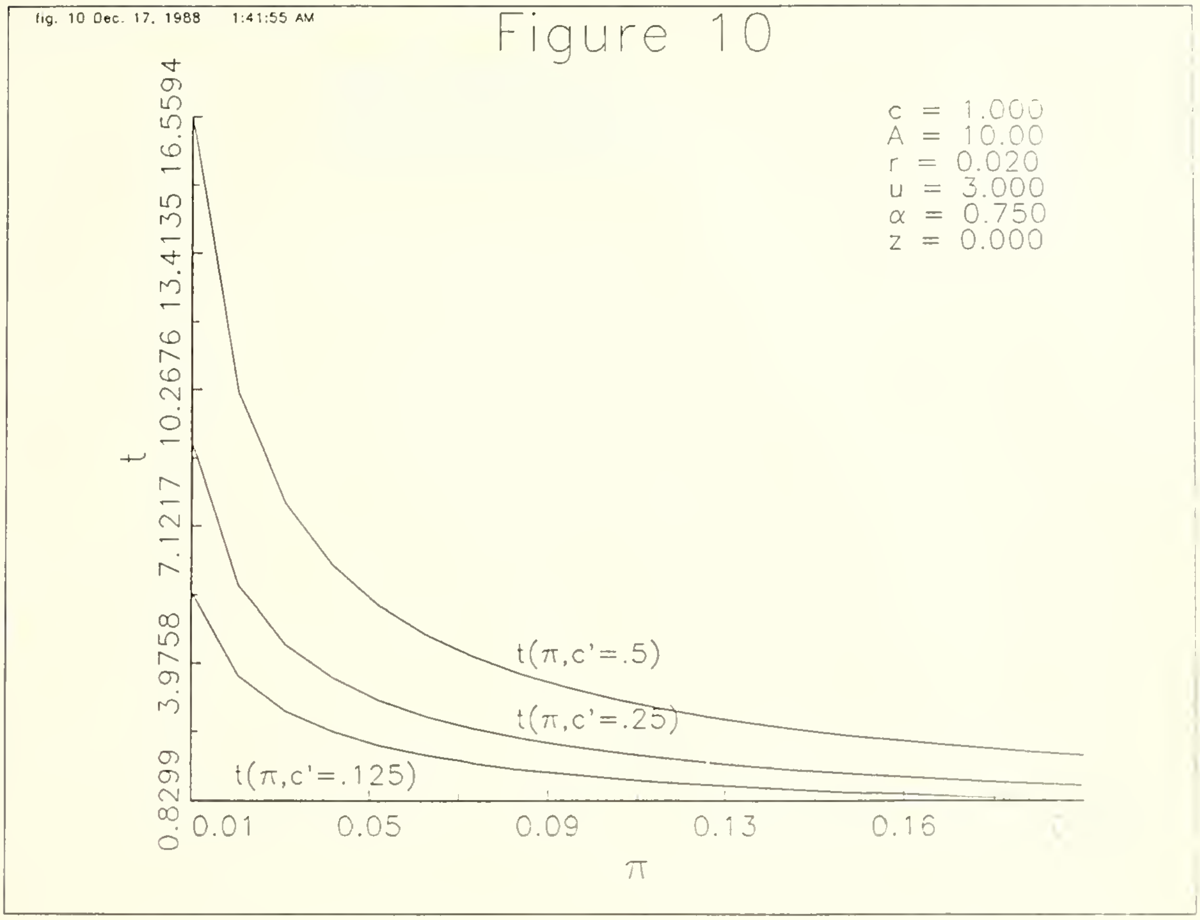
fig. 10 Dec. 17, 1988 1:41:55 AM
Figure 10
c = 1.000
A = 10.00
r = 0.020
u = 3.000
α = 0.750
z = 0.000
t
0.8299
3.9758
7.1217
10.2676
13.4135
16.5594
t(π,c'=.5)
t(π,c'=.25)
t(π,c'=.125)
0.01
0.05
0.09
0.13
0.16
π