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A SUPPLY FUNCTION MODEL OF
AGGREGATE INVESTMENT

by

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Number 89 August 1972

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In a recent book and articles, one of the present authors and Miguel Sidrauski (1970; 1969) have studied a macroeconomic growth model which explicitly includes monetary and fiscal policy tools as distinct parameters. One of the basic ideas of this work was to follow out rigorously an account of the volume of investment originated by Keynes (1936), Witte (1963) and others. In this paper we present an attempt to measure econometrically some of the key parameters of that model for the American economy in the period 1953-1970.

This paper consists of a review and extension of the theory of the determination of the volume of investment mentioned above, an account of our use of a spectral smoothing technique to handle an errors-in-variables problem inherent in our basic approach, and a report of our estimated equations for producer structures and durables over the period. We believe that these equations perform as well or better than those based on other theoretical constructs in terms of coefficient size and significance, goodness of fit, and reasonableness of lag structure. To facilitate comparison we have estimated our equations for the same period and date that Bischoff (1971) used in his study of investment functions.

*We are deeply indebted to Kanitta Meesook for her careful and tireless work on this project. Foley's work was partially supported by NSF grant # GS2966 Additional support for research assistance and computer time was provided by the Cambridge Project.

I - Rising Social Cost Determines the Rate of Investment

The model begins with a strictly concave production possibility frontier between investment producing and other sectors. (See Figure 1.) Given the relative price of capital to consumption goods, p_k, profit maximizing entrepreneurs will move to the point on the p.p.f. tangent to a line with slope $-p_k$. Clearly, the higher is p_k, the larger the proportion of resources producers find it profitable to devote to producing investment goods.

The position and shape of the p.p.f. between consumption and investment depend on the existing labor and capital endowment of the economy, and on the rates of utilization of capital and labor. Recent work on unemployment of labor* suggests that unemployment be treated as a third sector of the economy that draws labor (and possibly capital) away from the productive sectors. We will avoid in this paper discussing what mechanisms permit this third sector (unemployment or underemployment) to exist, and whether the resources devoted to it at any moment are being wasted or are being used to produce information. What is important to us is the idea that the effective production frontier between investment and consumption may shift as a result of underutilization of resources. The flow supply of consumption Q_C and investment Q_I, then, depend on the endowments of capital K, labor L, the relative price p_k, and the rate of unemployment of both capital and labor, u.

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*Phelps et.al.(1970)

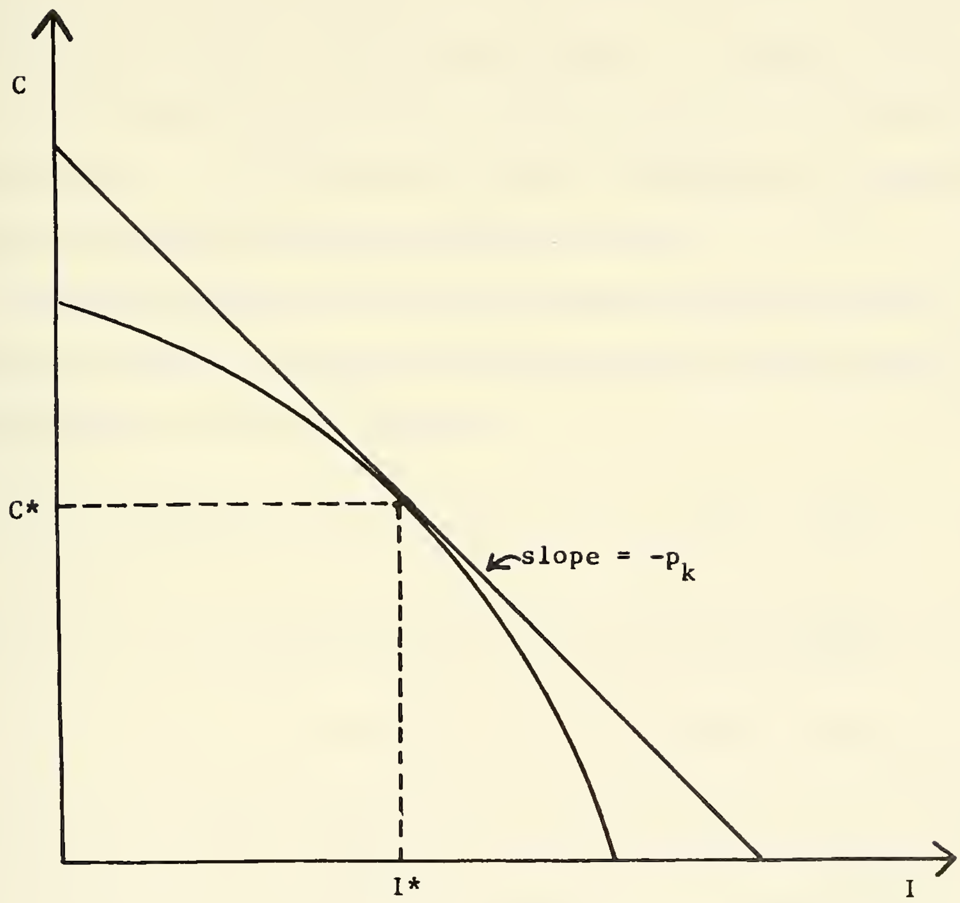

FIGURE 1

(1) $Q_C = Q_C(K, L, p_k, u)$

(2) $Q_I = Q_I(K, L, p_k, u)$

The price of capital, in addition to regulating the flow supply of consumer and investment goods, also influences the rate of return to existing capital goods viewed as assets in portfolios. We suppose that the higher is the price of capital, the lower is the rate of return to capital as an asset, other things, i.e., the degree of capital unemployment and expectations of capital gain or loss on holding capital, equal.

In the skeletal form of the model presented in Foley (1970), there are three assets only, money, bonds and capital, and the asset market equilibrium can be written in three equations:

(3) $p_m m = L(a, q(k, p_k, u), \pi_m, i + \pi_m, r(p_k, u)/p_k + \pi_k)$

(4) $p_m b = H(a, q(k, p_k, u), \pi_m, i + \pi_m, r(p_k, u)/p_k + \pi_k)$

(5) $p_k k = J(a, q(k, p_k, u), \pi_m, i + \pi_m, r(p_k, u)/p_k + \pi_k)$

where p_m is the value of money in terms of consumption goods, m is money, $a = p_m(m + b) + p_k k$ is wealth, q income, π_m the expected rate of deflation (the expected rate of return to money), i the interest rate, ($i + \pi_m$ the expected real return on bonds), r profits per unit of capital, π_k expected capital gains on real capital, b the quantity of bonds, and k the quantity of capital.

Monetary policy is modeled as a change in m and b holding the sum, m + b, constant. If we assume that the price of money does not adjust instantaneously to maintain full employment, the change in m and b alters the combinations of unemployment and price of capital compatible with asset market clearance. The schedule of such combinations (called the aa schedule) takes the place of and generalizes the LM curve of traditional models.

The static model is closed by considering the flow market for consumption

$$Q_C(k, L, p_k, u) = C(a, y^d(u)) + G$$

where y^d is disposable income, which depends on taxes, and G is government purchases (assumed for simplicity to be concentrated on consumption goods). This equation determines another schedule of combinations of u and p_k that are compatible with flow market equilibrium, the cc schedule which generalizes the IS curve.

The slope of the aa curve depends on the relative strength of the "Keynes effect" (the tendency as utilization falls for credit to become easier, and rates of return to decline forcing up p_k) and the "static accelerator" (as utilization falls the chance of capital being unemployed increases, which reduces the expected profitability of holding capital as an asset and drives down the price of capital). The slope of the cc depends on the assumption that higher unemployment reduces the supply of consumption goods more than it reduces demand through the consequent fall in disposable income.*

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*A more complete account of this model can be found in Foley (1971).

Figure 2 illustrates instantaneous equilibrium in which u and p_k are determined simultaneously.

The picture of the determination of the volume of investment in this

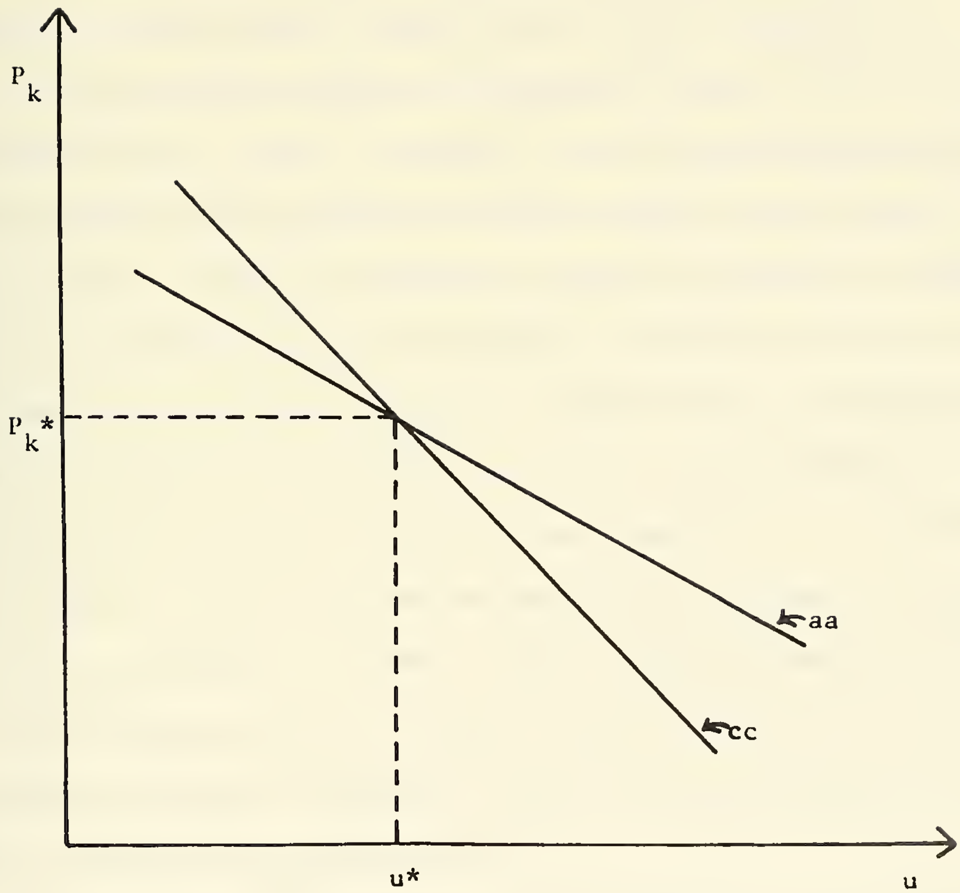

FIGURE 2

account eliminates any "demand for investment" and also avoids any explicit treatment of intertemporal optimization of accumulation by the firm by carefully separating the demand for capital as an asset and the demand for rental of capital services. It emphasizes the rising costs to investing faster as the whole economy shifts resources toward investment.

There are three attractive theoretical features of this view that seem relevant here. First, this model gives a clear and consistent picture of the mechanism by which monetary policy influences real spending streams in the economy. Second, it explicitly takes into account the problem of inflationary expectations in its treatment of interest rates, and avoids the fallacy of identifying movements of interest rates with changes in the incentives to invest. For example, in times of rising inflationary expectations or when the total supply of government debt is increasing rapidly, rising nominal interest rates may accompany a rising price of capital and hence rising incentives to invest. Third, this static model converts itself into a growth model without any change of specifications, and retains in this guise its explicit treatment of monetary and fiscal policy tools. For details see Foley and Sidrauski (1971).

These theoretical advantages are accompanied by certain practical disadvantages in trying to use this theory as a basis for empirical work. The chief of these is the central role played by the price of capital. In the American economy, households do not hold much real capital directly in their portfolios, but own indirect claims on it in the form of corporate bonds and shares. The process envisioned by the model, in which the price of capital is determined by direct competition with other assets, must be replaced by a more complex and indirect process, involving corporate liabilities. We cannot, in fact, even observe what the model calls the price of capital, that is, the marginal cost of investment. Reported price and cost series for capital goods have several deficiencies for this purpose. They tend to be list prices, and thereby miss some cyclical variance in terms and discounts. They generally are prices of individual machines, not of capacity actually put in place, and we expect that the rising costs we are looking for will be much more evident in the process of ordering, installing, and debugging capacity than in the individual components. Finally, construction cost indices weight materials very heavily, while again we expect materials to be relatively transferrable between the investment and other sectors and thus not very subject to rising costs due to scarcity as investment increases. Quasi-rents to skilled workmen and management seem more likely to rise, and these costs will not be fully reflected in construction cost indices.

Looking at the problem from the asset market side, we see that while the valuation of indirect claims to capital must bear some relation to valuation of the underlying capital, this relation is unlikely to be very stable over time. The indirect claims also represent monopoly power and organizational capital of various kinds which may be changing in value. The retention of earnings to finance acquisition of assets by corporations raises equity values when there has been no change in the per unit valuation of capital capacity. The markets for indirect claims, especially shares, are volatile, moving from day to day. It is implausible that the underlying valuation of capacity and incentives to add to it follow every daily gyration of stock prices.

When all this is said, it still seems that financial market data is the best hope for getting some information about changing incentives to invest.

But we must use it with care and with an understanding of the possible slippages between what we can easily observe and the entity, p_k, that our theory suggests we want to watch.

Thus, the major empirical problem faced in testing this theory is deducing the unobservable price of capital from the observable asset market data. This problem is addressed in Section II.

A second modification of the simple model is also necessary to take account of the fact that resources are not instantaneously shiftable among sectors. We expect that for short periods of time the p.p.f. is actually much more kinked at the point of production and that if the relative price of capital changes it will induce a gradual alteration of the p.p.f. as it moves out to a new long-run p.p.f. point. Figure 3 illustrates this point.

When the economy has been for some time producing at point A with the relative price of capital labeled p_k^A and p_k falls to p_k^B, the effect at first is very small in production. As time passes the economy adapts to the new p_k^B and the p.p.f. moves out to the situation labeled B. The dotted p.p.f. is the long-run envelope of short run p.p.f.'s.

Our picture of investment, then, is this. At a given price of capital, the volume of investment grows as the total resources of the economy grow, but

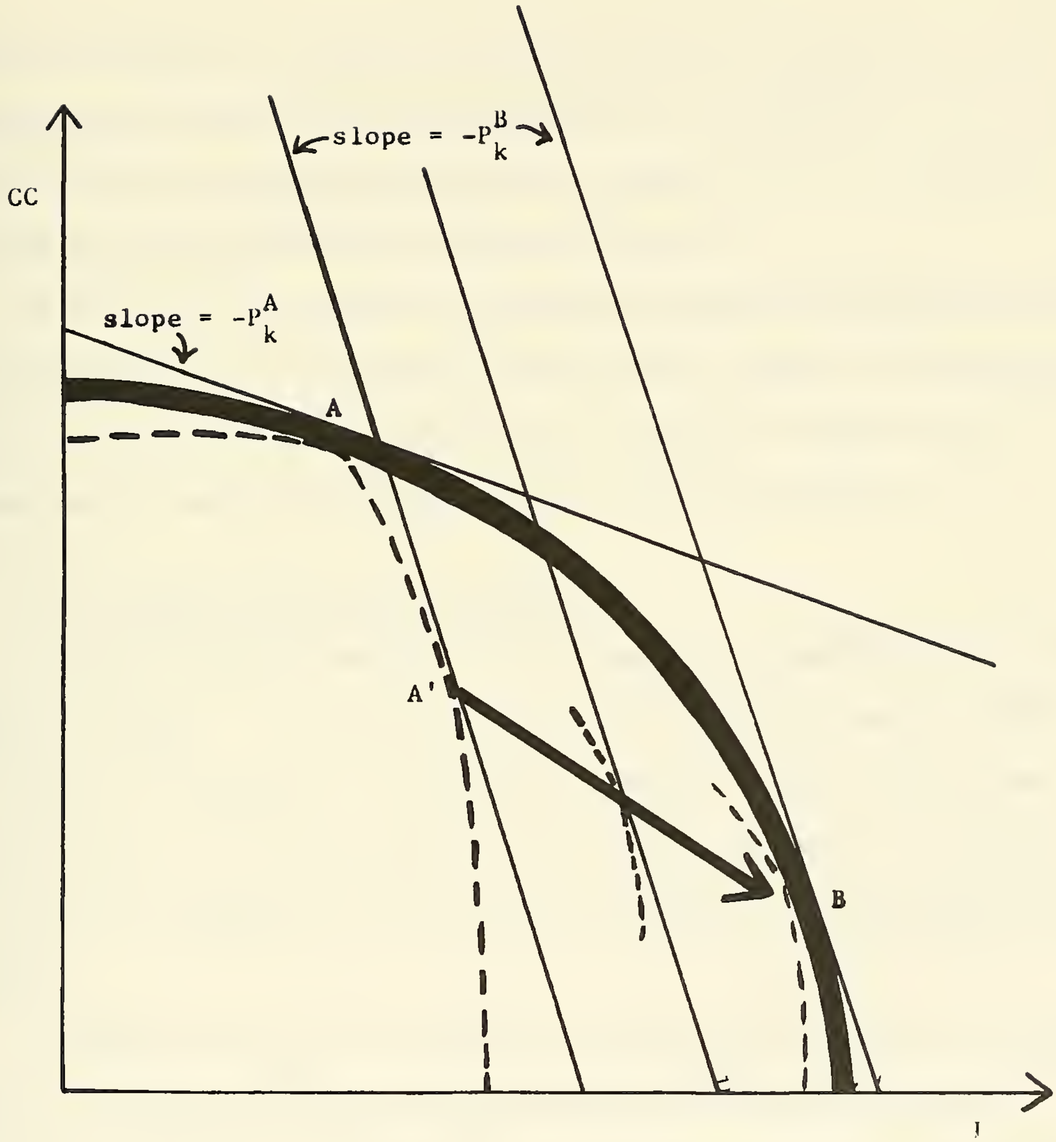

FIGURE 3

may also be altered by changes in the utilization of resources. A rise in the price of capital will raise this volume of investment, but the effect at first may be small and gradually cumulate due to the gradual shifting of resources toward the investment sector.

This picture of investment contains an important theoretical restriction (or hypothesis) for our econometric specification. There is no overshooting as the economy moves toward its new short-run p.p.f. We expect this movement to be cumulative and not reverse itself at any point.

II - Data for the Price of Capital

The most important market in which capital is bought and sold is the market for equity. In the gyrations of the stock market must be hidden the investor's evaluation of the stock of capital of the society at any particular time. The obscuring factors, however, are myriad, and efforts to sort out the important effects using conventional techniques have generally failed.

Systematic factors of particular concern are the effects of retained earnings, management expertise, debt ratios, and various individual firm market positions. In addition, we believe that there is a component of stock market prices that is best treated as noise in that speculation, incomplete information or the vicissitudes of individual investors overwhelm the information about the price of capital. Our strategy was to eliminate as many of the systematic effects as possible and then filter the series to diminish the impact of noise.

In order to eliminate as many of the individual firm effects as possible, we initially chose a broad based index of stock prices (Standard and Poor's 500 stock average) as our basic series.

We chose to ignore in this study the effects of changes in debt-equity ratios on stock prices (despite the fact that systematic refinancing by many of the firms in our sample could change stock prices with no corresponding change in per unit valuation of capital capacity). A better procedure might be to use total market valuation (including debt and equity of firms), an approach used by Tobin and others.

We did choose to eliminate the effect of retained earnings, which increase the amount of assets per unit of equity and thus change equity prices even if the price of capital stays constant. We construct an artificial series of numbers of shares S' and price per share P'. The constructed series give the same total value of outstanding equity as the existing data, but assumes that changes in retained earnings are reflected in number of shares rather than in prices.

$$(6) \qquad P'_n S'_n = P_n S_o$$

Any retained earnings, earnings minus dividends (E-D), give rise to a proportional increase in the number of shares.

$$(7) \qquad S'_{n+1} = S'_n(1 + (E_n - D_n)/P_n S_o)$$

A recursive solution to these relations generates P'

(8) $$P'_{n+1} = P_{n+1} / \prod_{i=0}^{n} (1 + (E_i - D_i)/P_i S_o)$$

This procedure assumes that there are no new issues of equity by the firms in the stock index. This is not, of course, precisely true, but new issues tend to be small in relation to the volume of outstanding shares.

The net effect of applying this correction to the Standard and Poor series from the first month of 1947 to the last month of 1970 was the elimination of most of the overall trend and some change in shape. The resultant series, deflated by the commodities component of the consumer price index in order to obtain measures of the relative price of investment goods to consumption goods, is plotted in Figure 4, together with the Standard and Poor 500 index itself.

Corrections of this series for management effects and changes in overall market position were judged impossible to make. However, these and other factors could clearly lead to a slow evolution in the relation between the price of capital and the value of the adjusted stock price index.

The second major alteration made to the stock price series was to attempt to eliminate the effects of short duration random movements in the series. It seemed reasonable to assume that there was still a lot of random noise in the series but that its importance is much greater at higher frequencies than at low frequencies. That is, if we think of the price of capital as the signal, the signal to noise ratio is much lower at high frequencies than at low frequencies. If this hypothesis is valid, then the appropriate procedure is to filter the series to eliminate high frequencies. This technique is exactly similar to Friedman's use of a moving average, which

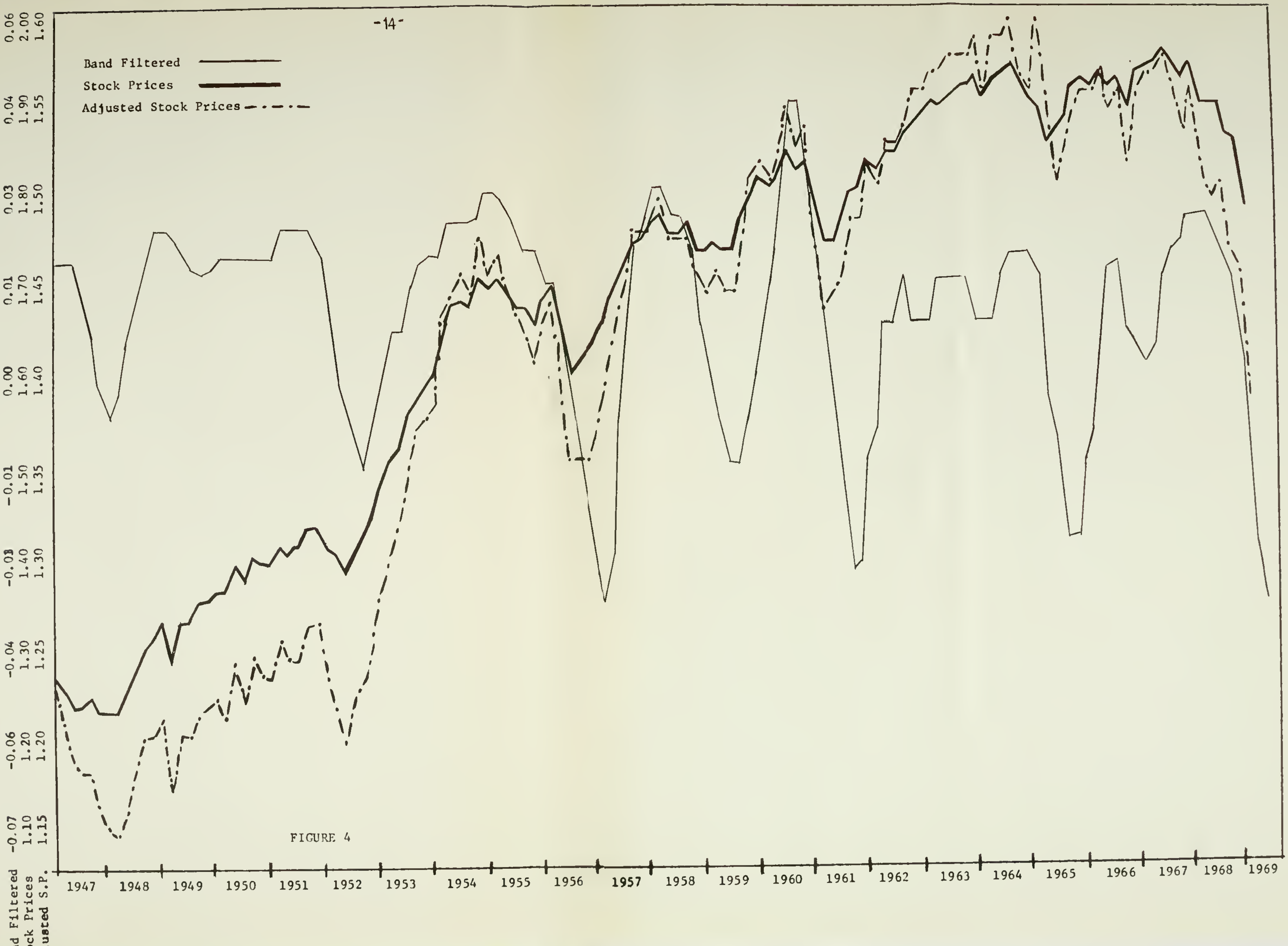
Band Filtered
Stock Prices
Adjusted Stock Prices
0.06 2.00 1.60
0.04 1.90 1.55
0.03 1.80 1.50
0.01 1.70 1.45
0.00 1.60 1.40
-0.01 1.50 1.35
-0.03 1.40 1.30
-0.04 1.30 1.25
-0.06 1.20 1.20
-0.07 1.10 1.15
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
nd Filtered
ock Prices
justed S.P.

FIGURE 4

is a low-pass filter, to eliminate the transient effects from observed income and obtain an estimate of permanent income and hence the marginal propensity to consume out of permanent income.

To examine the validity of this hypothesis, we obtained monthly data seasonally unadjusted on machine tool orders and non-residential construction, two series which roughly corresponded to national income account measures of investment. We estimated the cross-spectrum between our stock price series and each of these components (detrended) from 1947 1 through 1970 12. The coherency squared is plotted in Figures 5 and 6, with the critical level for rejection of the hypothesis that the coherence is zero at the 95% level.*

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*This test is described in Jenkins and Watts, p. 433.

This is a test which applies for each spectral point. However, points are independent only when separated by more than the bandwidth of the spectral window used in estimation. In this case, the Tukey-Hemming window was used with 24 lags and therefore the effective bandwidth covers 1 1/3 estimated points implying that there are approximately 18 independent estimates.

Examination of Figure 5 suggests that the coherence is small for very long period oscillations, rises to a peak at periods of several years, and then falls for oscillations shorter than one year. The estimates for non-residential construction are likely to be less reliable because the series have the peak in their cross-covariance function at a several month lag**

**See Jenkins and Watts, p. 399.

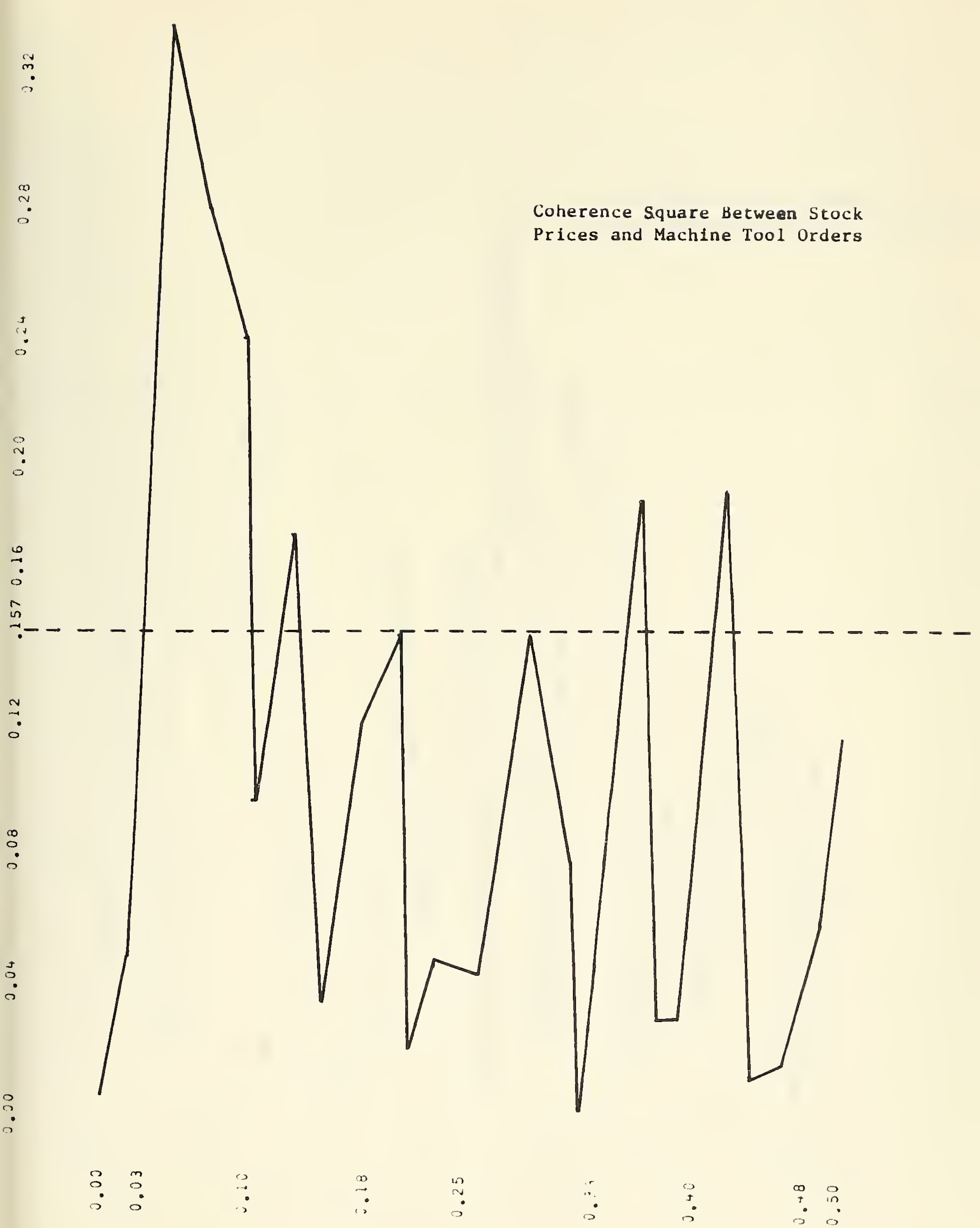
Coherence Square Between Stock Prices and Machine Tool Orders
0.32
0.28
0.20
0.16
.157
0.12
0.08
0.03
0.18
0.25
0.50

FIGURE 5


Coherence Square Between Stock
Prices and Non-residential Construction
0.00
0.03
0.10
0.18
0.25
0.33
0.40
0.48
0.50

FIGURE 6

and therefore are not "aligned." Both data series suffer from being only crude approximations to national income account data; this may explain some of the noise in the coherence results. Presumably the low coherence at low frequencies is a result of slow structural shifts between the price of capital and stock price series as described above.

This evidence, though not compelling, did suggest that the data should be filtered with a filter designed to attenuate frequencies shorter than a year and very low frequencies. Conventional smoothing filters which have traditionally been used in economics attenuate only high frequencies and therefore a different filter is required. One version is easily generated by following a procedure described by Granger and Hatanaka (1964) as demodulation, low-pass filtering, and remodulation.

Let the original series be x_t. Construct two series

$$y_{1t} = x_t \sin \omega t \qquad (9)$$

$$y_{2t} = x_t \cos \omega t$$

where ω is the frequency which is to be at the center of the filter. These series are then filtered with a low pass filter F such as a moving average to obtain

$$z_{1t} = F(y_{1t}) \qquad (10)$$

$$z_{2t} = F(y_{2t})$$

The final series is then reclaimed by constructing

$$x_t^* = 2z_{1t} \sin \omega t + 2z_{2t} \cos \omega t \qquad (11)$$

It is easily shown that this filter leaves series of the form A $\sin(\omega t + \phi)$ unchanged while attenuating to zero other frequency components.

For our purposes, we chose the center of the filter to correspond to a two year oscillation. The low-pass filter was a pair of simple moving averages of 24 and 10 months where the lengths were chosen specifically to attenuate to zero, frequencies $\omega \pm 2\pi/24$. In this case where ω itself is $2\pi/24$ these are the zero frequency and the annual frequency which is the principle seasonal frequency. The moving averages were taken symmetrically so that there would be no shift in the timing of the effects. This meant that we lost observations at both ends of the series. The filtering was performed on the log of the stock price series and the mean set to zero.

The final band filtered series of stock prices is plotted in Figure 4. The plots of the two series reveal that they move roughly together but that the filtered series misses many of the fluctuations of the original series. In addition, oscillations which have roughly a two year period appear to have an increased amplitude. The overall variance of the series is decreased to about 1/8 of the original value. This filtered series, SPADJ, is our basic estimate of the price of capital.

For readers who feel somewhat uncomfortable with the language of spectral analysis (as does one of the authors) a more intuitive account of this procedure may be helpful.

Ideally we would like to have a series of the price of capital but that is unobservable. We think, however, that <u>some</u> of the factors that affect the price of capital also affect stock prices, because capital and common stocks are linked through the balance sheets of corporations. We also believe that there are factors which affect stock prices that have no impact on the price of capital. These are of two kinds: slow drifts in management quality and practices, monopoly power, and similar phenomena that affect common stock values, and relatively short-lived and quickly reversed movements in expectations, fashion and so on, which move stock prices day-to-day. We thus expect that stock prices will faithfully reflect only certain kinds of movements of the price of capital, those with a period of about two years; we also hope that a substantial part of the variation of the price of capital takes place with the same period.

In no sense are we attributing any causal significance to the series we have constructed. We believe, however, that it reflects certain important features of the underlying movements of the price of capital to which we do attribute causal significance. It seems to us that the results we report below lend some credibility to this chain of assumptions.

III - Estimation

A linearized version of the investment supply function suitable for testing the theory is

$$I_t = \beta_0 + \beta_1 \, GNPPOT_t + \beta_2 \, GAP_t + \sum_i w_i \, P_{t-1} + \varepsilon_t \qquad (3\text{-}1)$$

where GNPPOT is potential GNP which represents resource endowments, GAP is the difference between actual and potential GNP and P is the price of capital series as described in the previous section. The gap is a more sensitive measure of what we called in the first section "unemployment" of labor and capital. It reflects not only actual unemployment, but underemployment of resources in slack periods as well. Measuring potential GNP as the fitted values of a regression of GNP on an exponential growth path and the GAP as the residuals, equation (3-1) is an estimable supply equation.

There are, however, several estimation problems. As in all macroeconomic models there is a simultaneous equation problem. The degree of utilization may alter the shape of the effective p.p.f., and therefore the supply curve for investment (3-1). But the degree of utilization is itself partly determined through the multiplier effects of investment on income, as Figure 2 indicates. The coefficient of the GAP in the supply equation when estimated by ordinary least squares will be asymptotically biased downward. The importance of this bias depends upon the strength of the feedback through the multiplier effect from investment to utilization. We acknowledge the existence of this problem, but we have not attempted to correct it by using any more sophisticated estimation techniques.

A second estimation problem which has received wide attention in the investment literature is the procedure for estimating the lag structure. In this study, it was assumed that the lag parameters followed a third order polynomial with restrictions on the endpoints. Several lengths were tried and the best chosen although the lag coefficients were relatively insensitive to this parameter. The final choice was to use a three year lag distribution for all equations. For each case, the unconstrained ordinary least squares estimate of the model was also made. These coefficients showed a great amount of instability and yet resulted in roughly similar unexplained variations. The hypothesis that the linear restrictions imposed by the Almon polynomial estimation procedure were inoperative was never rejected or even close.

A third estimation problem is that the model embodied in Dhrymes*

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*Dhrymes, p. 227

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showed considerable serial correlation, thus biasing the estimated standard errors. Assuming that the disturbance was generated by the first order Markov process,

$$\text{(13)} \qquad \varepsilon_t = \rho_{t-1} + n_t$$

where n_t is serially uncorrelated, the structural equation can be estimated by searching over the acceptable range of ρ and choosing the estimate with the minimum sum of squared residuals. This procedure is described in detail by Hildreth and Lu (1960). In each case, the Durbin Watson statistic of the residuals after correcting for the first order process indicated that little serial correlation was left.

Equation (12) was estimated using quarterly seasonally adjusted investment data in constant dollars for both producers' durable equipment and business construction. The primary estimates are for the period 1953 I to 1968 IV where the data is taken from the July 1971 Survey of Current Business and earlier editions. Potential GNP and the GAP were also measured in constant dollars. The estimated coefficients and their t-statistics are presented in column 1 of tables I and II. In column 2 of these tables the restriction is imposed that the coefficient of potential GNP is the negative of the GAP and therefore only GNP should enter as a variable.

These results show that the marginal propensity to invest in producers' durables out of potential GNP is about 10% with a rising average due to the negative constant while for structures this propensity is about 3%, also

TABLE I

PRODUCER'S DURABLE EQUIPMENT

Constant	-66.4	-65.81	-68.7
	(-2.61)	(-3.11)	(-3.32)
GNP		.114	
		(11.66)	
POT. GNP		.109	
		(10.44)	
GAP		-.127	
		(-4.86)	
$(GNP - GNP_{-1})$			-.037
			-1.79
Lagged Stock Prices			
0	.40	.71	1.22
	(2.69)	(2.51)	(2.47)
1	3.13	2.99	3.27
	(1.93)	(2.18)	(1.89)
2	4.68	4.71	4.59
	(1.92)	(2.18)	(1.88)
3	5.48	5.69	5.29
	(2.03)	(2.66)	(1.99)
4	5.67*	6.04*	5.46*
	(2.09)	(2.28)	(2.04)
5.	5.37	5.87	5.21
	(2.10)	(2.23)	(2.05)
6	4.69	5.29	4.63
	(2.09)	(2.17)	(2.04)
7	3.76	4.43	3.81
	(2.06)	(2.11)	(2.01)
8	2.70	3.38	2.86
	(1.98)	(2.08)	(1.94)
9	1.63	2.27	1.88
	(1.85)	(1.88)	(1.81)
10	1.20	.68	.96
	(1.91)	(1.85)	(1.81)
11	-.03	.30	.21
	(2.47)	(2.56)	(2.41)
Sum Lag Coefficient	42.58	38.45	39.40
L.R. Elasticity			
1968 Values	.78	.87	.80
Mean Lag	4.56	4.90	4.63
R^2	.9944	.9944	.9948
$\bar{R}^2$ †	.9930	.9928	.9933
SSE †	.855	.855	.836
ρ	.8	.9	.9
D.W.	2.19	2.02	2.37

* Modal lag

† Five extra degrees of freedom are extracted to compensate for the testing of various length lag distributions and the estimation of serial correlations.

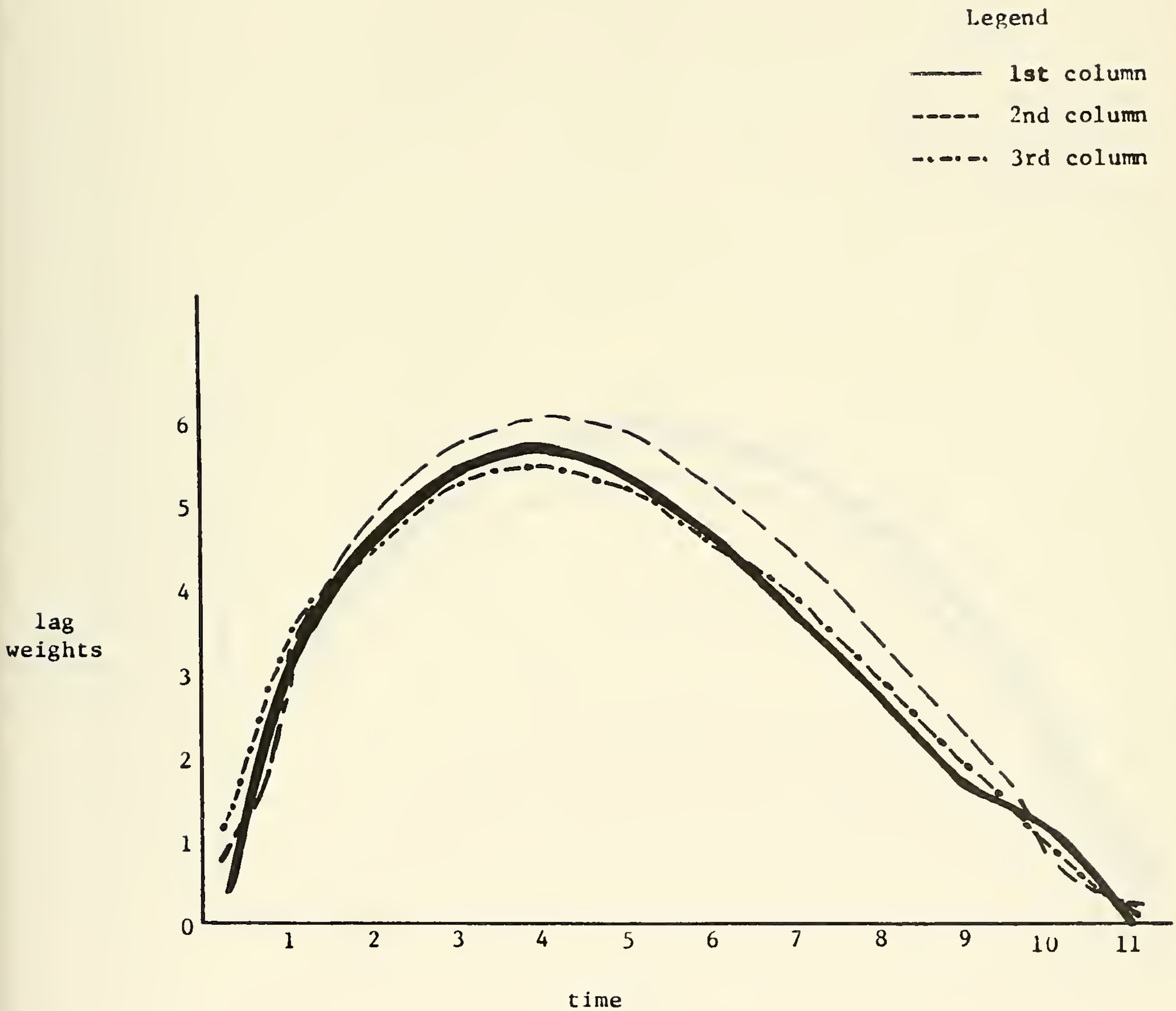
TABLE II

NONRESIDENTIAL CONSTRUCTION

Constant	-38.60	-47.18	-47.26
	(-2.38)	(-3.53)	(-3.50)
GNP		0.0329	
		(10.53)	
POT. GNP		0.028	
		(5.59)	
GAP		-0.049	
		(-3.23)	
$(GNP - GNP_{-1})$			-0.0050
			(0.30)
Lagged Stock Prices			
0	1.44	1.78	1.88
	(0.78)	(1.02)	(1.04)
1	3.31	2.76	3.34
	(2.64)	(1.90)	(2.63)
2	4.47	3.83	4.47
	(3.69)	(2.66)	(3.65)
3	5.30	4.63	5.28
	(4.14)	(3.13)	(4.07)
4	5.79	5.15	5.76
	(4.42)	(3.50)	(4.33)
5	5.95*	5.38*	5.92*
	(4.54)	(3.77)	(4.46)
6	5.80	5.31	5.77
	(4.42)	(3.85)	(4.35)
7	5.33	4.92	5.32
	(4.07)	(3.68)	(4.02)
8	4.57	4.21	4.57
	(3.57)	(3.29)	(3.53)
9	3.51	3.16	3.52
	(2.90)	(2.64)	(2.88)
10	2.17	1.76	2.19
	(1.74)	(1.44)	(1.73)
11	0.56	0.0036	0.57
	(0.32)	(.0021)	(0.32)
Sum Lag Coefficient	42.55	48.54	48.58
Elasticity	2.3	2.0	2.4
Mean Lag	5.13	5.12	5.12
R^2	.9613	.9616	.9614
$\overline{R}^2$ +	.9517	.9507	.9506
SSE +	.632	.626	.635
ρ	.7	.6	.7
D.W.	2.36	2.21	2.33

* Modal Lag

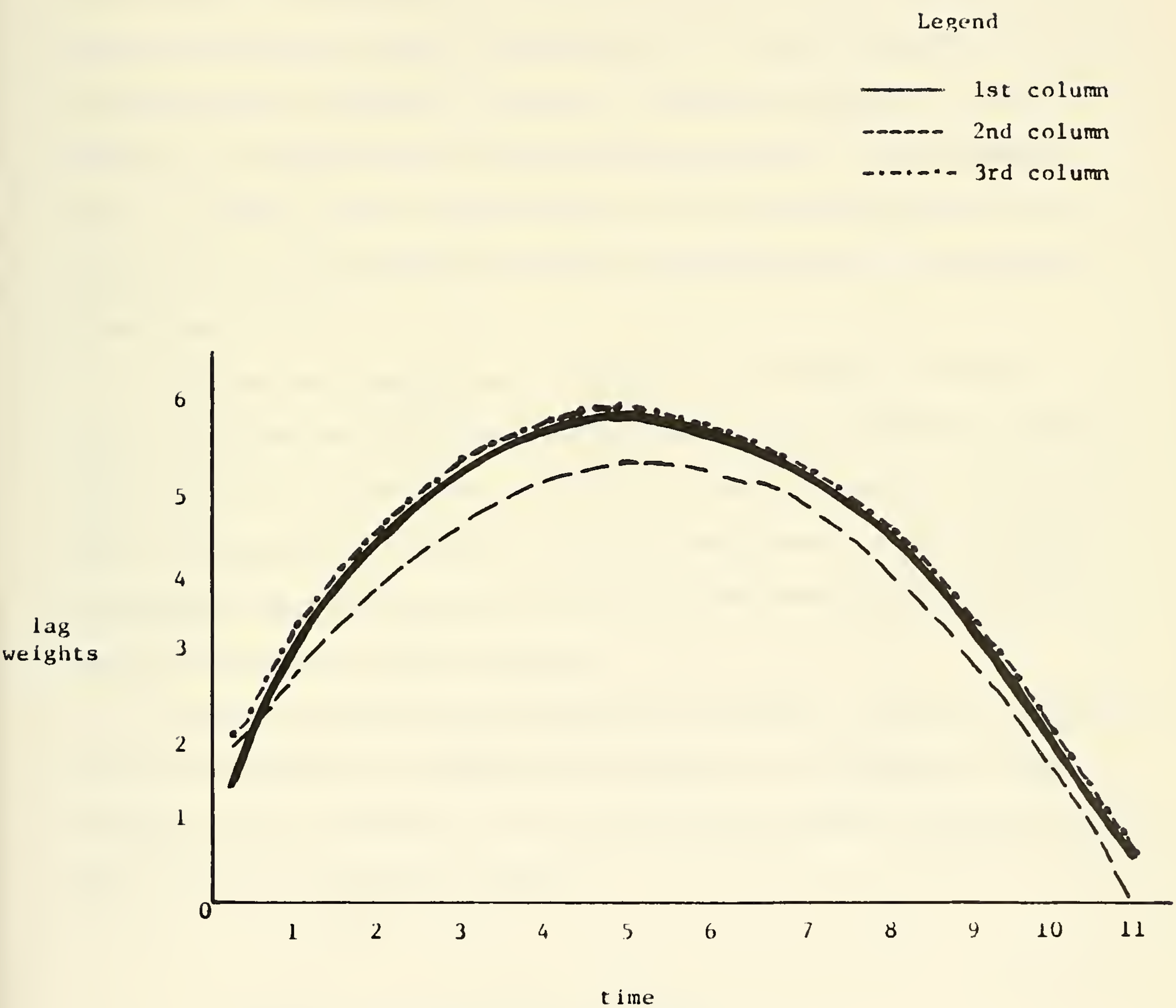
\+ Five extra degrees of freedom are extracted to compensate for the testing of various length lag distribution and the estimation of serial correlation.



PRODUCER'S DURABLE EQUIPMENT

LAG STRUCTURE

FIGURE 7



NONRESIDENTIAL CONSTRUCTION
LAG STRUCTURE

FIGURE 8

rising. The lag distributions for all four forms are surprisingly similar with a modal lag of four quarters for durables and five for structures. As the lag distributions are roughly symmetrical, this is approximately also the mean and median of the lag pattern. That is, a change in the price of capital will have its total effect in three years although half of the response will be realized in four or five quarters. The lags do exhibit the smooth increase and subsequent decline without overshooting which is anticipated by the theory.

The long run elasticities of investment with respect to the price of capital calculated at 1968 IV are quite substantial. For durable equipment this elasticity is slightly below 1 while for business construction, it is 2. Thus a 10% increase in the price of capital should result in an 8% increase in equipment and a 20% increase in structures once these prices are fully reflected in producer's supply decisions.

In order to compare this explanation with other theories of investment, a simple accelerator was included in the equation. The results in column 3 of tables I and II show that it is not significant and that it has very little effect on the lag behavior of prices.

VII - Comparison with Alternative Models

A specific comparison with several other models of investment over the same sample period using the same estimation techniques and reporting is afforded by the Bischoff (1971) survey. This survey compared five investment models for each of the fixed business investment components. These included the generalized accelerator, cash flow, securities value, standard neoclassical and Federal Reserve-MIT-Penn models with several variations.

The results obtained by Bischoff for both equipment and structures all show extraordinary fits which are comparable to our results in tables I and II. For example, his $\bar{R}^2$ for equipment range from .987 to .993. On the basis of $\bar{R}^2$ and standard error, our estimates of this equation are one of the best whereas for the structures equation, our fits are not quite as good as his which range from .958 to .973. The differences between the fits of the equations over the sample are however quite small and provide little basis for choosing a model.

The differences are far more pronounced in examining the size and timing of the elasticities and in forecasting. One very notable difference is that for many of the models, the best fitting Almon polynomial used more than a five year lag distribution. The shortest among all models was 13 quarters for the securities value model which is closest to our model in spirit while we found that 12 was the optimum length. The use of a 23 quarter lag distribution in many of the Bischoff models means that the first observation on the dependent variable is in the first quarter of 1959. The path of investment from this point to the end of 1968 has few turning points to explain and leads to an impressive $\bar{R}^2$.

Another obvious difference between the various forms of investment equations is the variety of shapes of the lag distributions. Many of the Bischoff models exhibit a peak effect in the first period which leads after several quarters to overshooting the desired capital stock and a decline in investment. In many cases, the coefficients become negative after 6 quarters and in all but 3 this has occurred by 10 quarters. Our model postulates a smooth transition from initial to final levels of investment which is the

TABLE III-A

FORECAST VALUES FOR 1969-1970
A COMPARISON WITH BISCHOFF

EQUIPMENT

			Generalized Accelerator	Cash Flow	Standard Neoclassical	Standard Neoclassical Alternate	Federal Reserve MIT-Penn	Securities Value	Supply Function Model 1	Supply Function Model 2
'69	I	54.4	54.4	52.3	54.3	56.1	55.5	58.7	54.8	54.7
	II	55.5	54.8	52.6	54.7	57.5	56.6	60.3	55.5	55.6
	III	56.0	55.2	52.8*	55.0	58.7	56.9	61.1	56.3*	56.3*
	IV	56.8*	55.4*	52.3	55.1*	60.1	57.4*	60.9	56.2	56.2
'70	I	54.7	55.3	51.7	54.1	61.1	56.5	61.2*	55.6	55.5
	II	55.0	54.5	50.5	54.0	62.3	54.8	61.1	55.5	55.3
	III	55.9	53.8	49.5	53.6	63.4*	53.9	58.7	55.1	54.9
	IV	52.0	52.9	46.6	53.9	62.6	52.9	56.0	53.4	53.0
Mean Error 7 OBS			.7	3.7	.957	-4.41	.471	-4.8	-.036	-.16
R Mean Squared Error			1.0	3.80	1.39	4.97	1.24	5.25	.57	.64

TABLE III-B

FORECAST VALUES FOR 1969-1970
A COMPARISON WITH BISCHOFF

CONSTRUCTION

			Generalized Accelerator	Cash Flow	Standard Neoclassical	Standard Neoclassical Alternate	Federal Reserve MIT-Penn	Federal Reserve MIT-Penn Alternate	Securities Value	Supply Function Model	
'69	I	24.3	23.5	22.2	24.0	23.1	24.0*	23.2	24.8	24.4	24.5
	II	23.6	23.7	21.9*	24.1*	23.8	23.9	24.0	25.8	25.1	24.9
	III	25.1*	23.8	21.6	24.0	24.4	23.7	24.9	26.4	25.7*	25.38*
	IV	24.8	23.9*	21.2	23.8	25.2	23.3	25.9	26.5*	25.4	25.36
'70	I	24.6	23.8	20.8	23.5	25.8*	22.7	26.9*	26.2	25.7	25.0
	II	24.4	23.5	20.3	23.1	24.7	21.8	25.6	25.6	25.5	24.7
	III	24.2	23.2	19.8	22.6	24.0	20.5	24.2	24.4	25.0	24.2
	IV	23.5	23.1	19.4	21.9	23.6	19.4	23.4	22.7	23.9	23.0
Mean Error		8 OBS	.75	3.4	.942	.0125	1.925	-.55	-1.0	-.79	-.48
R Mean Squared Error			.85	3.62	1.15	.68	2.4	1.1	1.34	.88	.57

behavior observed regardless of the length of the lag and with no constraints on either endpoint of the distribution. The same behavior was observed by Bischoff for the securities value equation.

A major comparison between alternative models was afforded by forecasting 1969 and 1970 investment data. In general Bischoff found that the models performed worse in the forecast period than in the sample period, although he did not extract serial correlation when forecasting and thus the comparison is not strictly appropriate. Several of the models were accurate in forecasting both turning points and levels while others were wildly erratic. Shortly after Bischoff's paper appeared major revisions in the investment data for 1968 through 1970 were published. Thus it is necessary to reexamine both Bischoff's forecasts and our own in the light of this new evidence.

In Table III, the forecasts for 1969 and 1970 for our equations 1, 2 as well as for all of Bischoff's models are tabulated along with the mean error and the root mean square error of the forecasts. As the investment data are revised from those used by Bischoff, the statistics are not the same as his. However, the models which he found superior, namely the FRB-MIT-PENN Standard Neoclassical, and Generalized Accelerator, are even better with the revised data although some of the rankings change. Our models for durables, however, are superior to all others in terms of rms error and mean bias. For structures, our basic model is best in rms error and very close in mean bias. In business construction, our model alone correctly picks the turning point; although, in equipment we are one period early.

Although these comparisons are very encouraging, they are not strictly appropriate as tests of forecasting ability. The price variable we used, while reflecting only tne current behavior of the stock market requires future values for the computation. Thus, our model in this form is not useful for forecasting into the future. The comparison in Table III is therefore best interpreted as a test of whether structural shifts appear to have taken place in 1969 and 1970. Only our basic model shows a smaller rms error in the forecast period than the standard error of the regression. A more formalized test of this hypothesis is the conventional Chow test which in this case, because of the correction for serial correlation, is only an asymptotic test. Nevertheless, all of our models pass the test indicating that 1969 and 1970 should indeed be included in the sample period.

In future work a forecasting form of the price variable will be constructed by no longer imposing the symmetry condition on the low pass filter. At this point, a further test of the theory will be available.

VIII - Summary and Conclusions

This work has consequences for econometric method, economic policy, and economic theory. We have estimated one important link between monetary policy and real spending streams. Our conclusions depend on viewing asset prices, particularly the price of capital, rather than interest rates as the proximate determinant of business investment.

Because our results are couched in a somewhat indirect way, it may be useful to interpret them again here. The results of our regressions indicate that if the authorities controlled asset prices so that the two-year component had an amplitude equal to 10% of the average level, producer durable expenditures would have a two-year component with an amplitude of about $4 billion dollars in 1958 prices, and non-residential structures a two-year component with an amplitude of about $4.5 billion dollars. A special case would be the situation where asset prices exactly followed a sine wave with a period of two years around a constant mean, swinging 10% above and below that mean value. In this case, all the other components would be zero, and we would expect producer durables and structures to swing by $4 and $4.5 billion respectively around their long-run growth path.

If we are willing to assume that the economy would exhibit the same response at lower frequencies even though we cannot measure low frequency response directly because stock prices may not reflect the price of capital faithfully at low frequencies, then a step of 10% in asset prices, by our estimates, would produce after 11 quarters rises of $4 and $4.5 billion in producer durables and structures respectively, about half of the rise occurring in the first 4 quarters.

The difficulty in using these estimates is that we do not observe the price of capital directly, but can only infer its movements from other observable asset prices, which have movements of their own, especially in the short run, that are uncorrelated with the price of capital. If these estimates are to be used in making or judging policy, they cannot be used mechanically. The policy maker must supplement them with his own judgment as to which market movements reflect an underlying change in asset valuations and which can be ignored as temporary aberrations.

A modest innovation is our use of spectral analysis. Our basic difficulty was the errors-in-variables problem inherent in the unobservability of the price of capital. Because we had reason to believe that these errors were concentrated at certain frequencies, spectral smoothing allowed us to handle the problem. Our results suggest that we were successful and that the method has a wider applicability in macroeconomic estimation.

The main impact of our results in macroeconomic theory relates to two views of the determination of the rate of investment. The majority of writers in this field (cf. Jorgenson [1971], Bischoff [1971]) speak of a "demand for investment" and base their econometric work on an analysis of a firm's decision to accumulate capital. Through the years a group of scholars (mentioned in Part 1 of this paper) have pressed the view that the aggregate rate of investment is determined by rising marginal costs to investment. This view argues that the rate of aggregate investment is supply-determined, and that the notion of a flow demand for investment is ill-formed or irrelevant in determining the rate of investment for the whole economy. We are not aware of any econometric study based on the supply view before the present paper (though there may be some that we have missed). Our results in this paper show that the supply-oriented view of investment can perform about as well econometrically as the demand view, so that no easy empirical resolution of the disagreement is possible.

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