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USING COST OBSERVATION  
TO REGULATE FIRMS\*

by  
Jean-Jacques LAFFONT\*\*  
and  
Jean TIROLE\*\*\*

No. 368

February 1985

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## 1. Introduction

The literature on the control of public firms or private monopolies can be divided into a literature studying the properties of given incentive schemes, and a few recent papers designed at characterizing optimal control mechanisms. The major interest of the earlier literature<sup>1</sup> stems from the simplicity of the schemes studied, which are easily related to what can be observed in planned economies or in large corporations. However, they are ad-hoc. There is clearly a need for a normative theory which will derive optimal incentive schemes, study the performance of these ad-hoc schemes and test the soundness of the intuitions on which they are based.

Recently, a normative theory has emerged from the non linear pricing literature and the more abstract incentive theory developed to deal with the free rider problem. In this approach the regulator/planner is viewed as a Bayesian statistician who has a prior knowledge about cost and demand conditions. The optimization problem of the regulator is to maximize the expected social welfare under the constraint of the decentralization of information. The outcome of the analysis is the characterization of optimal incentive schemes given the objective functions and the observations made by the regulator.

Papers in this tradition (Baron and Myerson (1982), Loeb-Magat (1979), Sappington (1982)), study the control of a private monopoly when the demand function is common knowledge and the cost function can be parameterized by one real number<sup>2</sup>. The optimal incentive mechanism in general entails a welfare loss compared to what could be achieved under perfect information.

Costs are easy to observe, at least at the firm's level. The value of cost observation to the planner depends on what he attempts to control. If he moni-

tors a single project in a multiproject firm, the latter can shift expenses to and from the particular project, both at real and accounting levels. In a first approximation it is reasonable to assume that the planner does not perfectly observe the firm's cost for the project<sup>3</sup>. When the planner controls the entire firm, aggregate cost information becomes very valuable. If cost observability is introduced into the Baron-Myerson model, it is then possible to infer the true value of the cost parameter and to reach the first best with appropriate penalties.

In this paper we introduce (possibly noisy) cost observability as well as an unobservable effort variable. Section 2 describes the model. A regulated firm produces a public good. The planner observes the firm's output and cost, but not its efficiency parameter, its effort and the cost disturbance. The firm knows its efficiency before contracting. After contracting, it chooses an output and a level of effort, which together with an additive uncertainty, result in a cost level. Its reward depends on output and cost (see section 4 for other interpretations of the model). Both parties are risk-neutral, and the firm can reject the contract if it is not guaranteed a minimum payoff. Section 3 gives a complete technical analysis of the firm's and the planner's optimization problems. We suggest that this section be skipped in a first reading by readers who are mainly interested in the regulatory implications of the model. Section 4, the main section of the paper, summarizes the properties of the optimal incentive scheme and of the firm's performance. The optimal scheme is linear in ex-post cost: the planner pays a fixed sum (which can be determined at the date of contracting), and then reimburses a fraction of costs. This fraction is inversely related to the fixed transfer, and decreases with the firm's output (or efficiency; or increases with the firm's announced expected cost, in another interpretation). Some implications are drawn when the optimal scheme resembles cost-plus-fixed-fee or fixed-price contracts. In particular it is shown that the



more concerned about output the regulator is, the more the optimal contract resembles a fixed-price contract. Section 4 also gives an alternative interpretation of the model that embodies the choice of a quality level. Section 5 introduces a choice of technology. The firm can trade off variable and fixed costs. Our assumption that accounting data are (at least partially) observable allows us to study the efficiency properties of rate-of-return regulations. In our model capital accumulation is insufficient when investment is not directly observable (i.e., only total cost is), but an Averch-Johnson rule does not increase welfare when investment is observable. Section 6 discusses the case of a risk-averse firm. Section 7 applies the model to the case of a marketed good, and briefly analyzes the average-cost-pricing rule. Section 8 compares our work to related contributions and concludes.

## 2. The model

A firm produces a single output  $q$  at (monetary) cost:

$$C = (\beta - e)q + \varepsilon.$$

$e > 0$  is a level of effort, which decreases the initial marginal cost  $\beta$ . The efficiency parameter  $\beta$  belongs to  $[\underline{\beta}, \bar{\beta}]$  where  $\underline{\beta} > 0$ ;  $\varepsilon$  is a random variable with zero mean and denotes an ex-post cost disturbance. We will interpret  $\varepsilon$  as a forecast error, unknown to the firm when it chooses its output and effort levels. We assume that  $\varepsilon$  is independent of the parameters and choice variables of the model. Alternatively,  $\varepsilon$  could denote an independent observation (accounting) error on cost with absolutely no change in our results.

The effort in principle could also influence the fixed cost ( $C = (\beta - e)q + \alpha - ke + \varepsilon$ ). The technical analysis then becomes more complex, but the same qualitative results hold if one assumes that the optimal incentive scheme is differentiable and that the various second-order conditions are satisfied (pro-

perties that are proved in the simpler case where effort influences the marginal cost only, which is considered in this paper): see Laffont-Tirole [1984].

The output is not marketed by the firm<sup>5</sup>; it is for example a public good which provides a consumer surplus  $S(q)$  ( $S' > 0$ ,  $S'' < 0$ ). The planner observes and reimburses the cost incurred by the firm and pays in addition a net monetary transfer  $t$ . The utility level of the firm's manager is then  $U = Et - \phi(e)$ , where  $\phi(e)$  stands for the disutility of effort. We assume  $\phi'(e) > 0$ ,  $\phi''(e) > 0$  for any  $e > 0$ . In the whole paper, expectations are taken with respect to  $\varepsilon$ .

The gross payment made by the planner to the firm is  $(t+C)$ . We assume that the planner can raise this amount only through a distortionary mechanism (excise taxes for example) so that the social cost of one unit raised is  $(1+\lambda)^6$ .

Consumer's welfare resulting from the activity of the firm is then:

$$S(q) - (1+\lambda)E(t+C).$$

If an utilitarian planner was able to observe the parameters of the cost function as well as the level of effort, he would solve

$$(2.1) \quad \begin{aligned} \text{Max } \{ & S(q) - (1+\lambda)E(t+C) + U \} = \{ S(q) - (1+\lambda)(\phi(e) + (\beta - e)q) - \lambda U \} \\ & (q, e, t) \\ \text{s.t.} \end{aligned}$$

$$(2.2) \quad U > 0.$$

The constraint (2.2), called the individual rationality constraint, says that the utility level of the firm's manager must be non-negative to obtain his participation.

The first-order conditions of problem (2.1) are

$$(2.3) \quad U = 0$$

$$(2.4) \quad S'(q) = (1+\lambda)(\beta - e)$$

$$(2.5) \quad \phi'(e) = q.$$

The individual rationality constraint is binding. The marginal utility of the commodity  $S'(q)$  is equated to its social marginal cost  $(1+\lambda)(\beta-e)$ . The marginal disutility of effort  $\psi'(e)$  is equated to its marginal utility, that is the marginal decrease in cost  $q$ .

We now make an assumption that ensures that the full information solution exists and is unique:

- A1)     i)  $S'(0) > (1+\lambda)(\bar{\beta} - \psi'^{-1}(0))$   
          ii)  $\psi'(\underline{\beta}) > \bar{q}$  where  $\bar{q}$  defined by  $S'(\bar{q}) = 0$   
          iii)  $-S''(q)(\psi''(\psi'^{-1}(q))) > (1+\lambda)$ .

A1i) says that the marginal surplus at no production is not too small. A1 ii) says that it is too costly (in terms of effort) to reduce marginal cost to zero, whatever the initial marginal cost. And A1 iii) requires enough convexity in the full information problem.

The task of this paper is to characterize and study the control mechanisms based on the observability of the output level  $q$  and the total cost  $C$ . The planner does not know  $\beta$  and cannot observe the level of effort  $e$ . He has a uniform prior on the range  $[\underline{\beta}, \bar{\beta}]$  of  $\beta$ ; moreover he knows the objective function of the firm (a more general distribution could be assumed; the uniform distribution saves notation, and simplifies the technical analysis since it satisfies the monotonic hazard rate property, which prevents bunching).

### 3. The optimal incentive scheme

The firm chooses output and effort. Once cost is realized and observed, the planner rewards the firm according to the two observables  $q$  and  $C$ . Equivalently (from the revelation principle), the planner can ask the firm to reveal its true productivity parameter, denoted  $\hat{\beta}$ . The reward then depends on the announcement  $\beta$

and on the ex-post cost:  $t(\beta, C)$ ; and output is imposed by the planner:  $q(\beta)$ . As is well-known, we can restrict ourselves to a truth-telling mechanism, so that the firm's optimal strategy includes  $\beta = \hat{\beta}$ . Let  $e(\beta)$  denote the optimal effort function for the truthful mechanism  $\{q(\beta), t(\beta, C)\}$ . We will characterize implementable allocations, i.e., allocations that induce the firm to tell the truth and such that the level of effort is (voluntarily) chosen by the firm. We will then treat the effort as a control variable for the regulator and check that one can find a transfer function  $t(\beta, C)$  that leads the firm to choose the corresponding level of effort.

Let

$$C(\beta) \equiv (\beta - e(\beta))q(\beta)$$

be the resulting expected cost, and let

$$s(\beta) \equiv Et(\beta, C(\beta) + \epsilon)$$

denote the expected net transfer (the expectation is taken with respect to the disturbance term  $\epsilon$ ).

a) The firm's optimization problem: In equilibrium it must be the case that the firm's decision variables  $(\beta = \hat{\beta}, e = e(\hat{\beta}))$  maximize  $\{Et(\beta, (\hat{\beta} - e)q(\beta) + \epsilon) - \psi(e)\}$ .

For the moment let us only consider a restricted class of possible deviations from the optimal strategy  $(\hat{\beta}, e(\hat{\beta}))$ . We will show that ruling out deviations in this class completely determines the output and effort functions. We will then exhibit a mechanism that implements this allocation; in particular other types of deviations are not optimal for the firm when it faces this mechanism. Lastly we will argue that this mechanism is optimal for any distribution for the disturbance.

Consider the following class of deviations from equilibrium behavior



$(\hat{\beta}, e(\hat{\beta}))$  for firm  $\hat{\beta}$ : it announces  $\beta$ , and makes effort  $\tilde{e}(\beta|\hat{\beta}) \equiv e(\beta) + \hat{\beta} - \beta$ . The set of such deviations  $(\beta, \tilde{e}(\beta|\hat{\beta}))$  will be called the concealment set for firm  $\hat{\beta}$ . Note that, when there is no uncertainty, any deviations outside the concealment set can be detected by the planner. Note also that the concealment set includes  $(\beta, e(\beta))$ ; and that if firm  $\hat{\beta}$  announces  $\beta$  and makes effort  $\tilde{e}(\beta|\hat{\beta})$ , the cost distribution is the same as for firm  $\beta$ , and therefore the expected transfer is  $s(\beta)$ . So ruling out deviations in the concealment set amounts to requiring that

$$(3-1) \quad \hat{\beta} \text{ maximizes } U(\beta|\hat{\beta}) \equiv s(\beta) - \psi(\tilde{e}(\beta|\hat{\beta})).$$

Appendix 1 shows that if  $s$  and  $\tilde{e}$  are such that (3-1) is satisfied, then these two functions, as well as the effort function  $e$ , are differentiable almost everywhere. So the first-order condition is

$$(3-2) \quad \dot{s}(\beta) - \psi'(\tilde{e}(\beta|\hat{\beta})) \dot{\tilde{e}}(\beta|\hat{\beta}) = 0 \quad \text{a.e., where a dot denotes a derivative with respect to } \beta.$$

Using the definition of  $\tilde{e}$  and truth-telling, we obtain (we delete the qualifier "a.e." from now on for notational simplicity):

$$(3-3) \quad \dot{s}(\beta) - \psi'(e(\beta))(\dot{e}(\beta) - 1) = 0$$

The local second-order condition can be written, using the first-order condition:

$$(3-4) \quad \frac{\partial^2 U}{\partial \beta^2} (\beta|\beta) \Big|_{\beta=\hat{\beta}} = - \frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} (\beta|\beta) \Big|_{\beta=\hat{\beta}} \leq 0 .$$

$$(3-5) \quad \dot{e}(\beta) < 1.$$

Note that (3-5) can be given a simple interpretation: the firm's expected average

cost is decreasing. Appendix 2 shows that if the local second-order condition is satisfied, then the global second-order condition is also satisfied.

Lastly, letting  $U(\beta) (= s(\beta) - \psi(e(\beta)))$  denote firm  $\beta$ 's (equilibrium) utility, we notice that the first-order condition (3-3) is equivalent to:

$$(3-6) \quad \dot{U}(\beta) = -\psi'(e(\beta)).$$

In other words the increase in the firm's utility for a unit decrease in "intrinsic cost"  $\beta$  is equal to the marginal disutility of effort (since the firm can reduce its effort by an amount equal to its increase in efficiency). We summarize these results in:

Proposition 1 (Firm's optimization problem). If deviations in the firm's concealment set are not profitable, then the effort, transfer and utility functions are differentiable almost everywhere. The first-order incentive compatibility constraint is given by (3-6). This necessary condition is also sufficient if the effort function satisfies (3-5).

We now turn to the planner's optimization problem. We will first assume that deviations in the concealment set are the only possible deviations, so that (3-5) and (3-6) are sufficient conditions for incentive compatibility. So we solve a sub-constrained optimization problem for the principal. We later show that the solution makes deviations outside the concealment set also unprofitable for the firm. Thus we are justified to consider the simpler optimization problem.

b) The planner's problem. We assumed that the planner has uniform beliefs on  $[\underline{\beta}, \bar{\beta}]$ . His optimization problem is then (using the definitions of  $U$  and  $C$ ):

$$(P) \quad (3-7) \quad \max E \int_{\underline{\beta}}^{\bar{\beta}} [S(q(\beta)) - (1+\lambda)(\psi(e(\beta)) + (\beta - e(\beta))q(\beta) + \epsilon) - \lambda U] d\beta$$

$$(3-6) \quad \text{s.t.} \quad \dot{U}(\beta) = -\psi'(e(\beta)) \quad \text{a.e.}$$

$$(3-5) \quad \dot{e}(\beta) < 1 \quad \text{a.e.}$$

$$(3-8) \quad U(\beta) > 0 \quad \forall \beta$$

(3-8), the individual rationality constraint, says that the firm is willing to participate. As mentioned earlier, (P) is a subconstrained problem. We make it even less constrained by ignoring the second-order condition (3-5). Naturally we will have to check that the two types of ignored constraints are indeed satisfied by the solution of the less constrained problem.

Note that from (3-6),  $U$  is a decreasing function of  $\beta$ . So (3-8) is satisfied if and only if  $U(\bar{\beta}) > 0$ . As social welfare decreases with  $U$ , we can as well replace (3-8) by  $U(\bar{\beta}) = 0$ . So we study the simplified program:

$$(P') \quad (3-7) \quad \max E \int_{\underline{\beta}}^{\bar{\beta}} (S(q(\beta)) - (1+\lambda)(\psi(e(\beta)) + (\beta - e(\beta))q(\beta) + \epsilon) - \lambda U(\beta)) d\beta$$

s.t.

endif

$$(3-6) \quad \dot{U}(\beta) = -\psi'(e(\beta)) \quad \text{a.e.}$$

and

$$(3-9) \quad U(\bar{\beta}) = 0.$$

We treat (P') as an optimal control problem with state variable U and control variables e and q. Appendix 3 studies this program, and can be summarized by:

Proposition 2: The necessary conditions for an optimum of (P') are:

$$(3-10) \quad \bar{U}(\beta) = 0$$

$$(3-6) \quad \dot{U}(\beta) = -\psi'(e(\beta))$$

$$(3-11) \quad S'(q) = (1+\lambda)(\beta - e)$$

$$(3-12) \quad \psi'(e) = q - \frac{\lambda}{1+\lambda} (\beta - \underline{\beta}) \psi''(e).$$

Appendix 3 also notices that the necessary conditions have a solution and shows that these conditions are also sufficient. For the necessary conditions to be sufficient, it suffices that the maximized Hamiltonian be concave in the state variable U, which is the case here.

c) Implementation: Under Assumption A1, (3-11) and (3-12) determine the levels of output and effort  $q^*(\beta)$  and  $e^*(\beta)$ ; (3-6) and (3-10) then determine the firm's utility  $U^*(\beta) = \int_{\beta}^{\bar{\beta}} \psi'(e^*(\delta)) d\delta$ . The expected transfer is then given by  $s^*(\beta) = U^*(\beta) + \psi(e^*(\beta))$ . As we mentioned earlier, we still must check that 1) the second-order condition (3-5) for the firm's maximization program is satisfied, so that the solution to (P') is also the solution to (P) and 2) we can find a transfer function  $t(\beta, C)$  which implements the optimum of (P) (in particular it should induce the right effort level and it should not induce the firm to deviate outside the concealment set either).

First, to see whether the second-order condition (3-5) is satisfied, one



must solve (3-11) and (3-12) and check that  $\dot{e}^*(\beta) < 1$ . This is in particular the case when  $\psi''' > 0$ , which implies that  $\dot{e}^*(\beta) < 0$ :

Indeed, from (3-11) (3-12) we have:

$$(3-13) \quad S'' \dot{q} = (1+\lambda) - (1+\lambda) \dot{e}$$

and

$$(3-14) \quad (\psi'' + \frac{\lambda}{1+\lambda} (\beta - \underline{\beta}) \psi''') \dot{e} - \dot{q} = - \frac{\lambda}{1+\lambda} \psi''$$

or

$$(3-15) \quad (\psi'' + \frac{1+\lambda}{S''} + \frac{\lambda}{1+\lambda} (\beta - \underline{\beta}) \psi''') \dot{e} = \frac{1+\lambda}{S''} - \frac{\lambda}{1+\lambda} \psi''.$$

Since  $S'' < 0$ ,  $\psi'' > 0$  and from A1iii),  $\psi'' + \frac{1+\lambda}{S''} > 0$ ; if  $\psi''' > 0$  then  $\dot{e} < 0$ .

Proposition 4: When  $\psi''' > 0$ , the firm's effort increases with its efficiency.

Therefore the firm's second-order condition is satisfied, and the solution to (P') is also the solution to the more constrained problem (P).

Second, let us study the implementation problem. As before, let  $\{e^*(\beta), q^*(\beta), U^*(\beta)\}$  denote the solution to (P'), and let  $s^*(\beta)$  and  $C^*(\beta)$  denote the corresponding expected transfer and expected cost.

The answer to the implementability question is trivial in the case of no disturbance ( $\varepsilon \equiv 0$ ). As we noticed earlier, only deviations within the concealment set can then go undetected. So the solution to (P) is also the solution to the global problem. To implement it, it suffices for the planner to i) ask the firm to announce its characteristic  $\beta$ ; ii) choose output  $q^*(\beta)$ ; iii) give transfer  $s^*(\beta)$  if  $C = C^*(\beta)$ , and  $(-\infty)$  otherwise. This simple "knife-edge" mechanism however is not robust to the introduction of any disturbance; if there is any noise, the

probability of incurring an extreme penalty becomes positive and makes the firm unwilling to participate.

Let us now turn to the general case of cost disturbance. To solve the problem completely, we must find a transfer function  $t(\beta, C)$  such that  $\{\hat{\beta}, e^*(\hat{\beta})\}$  is optimal for the firm:

$$\{\hat{\beta}, e^*(\hat{\beta})\} \text{ maximizes } E\{t(\beta, (\hat{\beta}-e)q^*(\beta)+\epsilon)-\psi(e)\}$$

and

$$Et(\hat{\beta}, (\hat{\beta}-e^*(\hat{\beta}))q^*(\beta)+\epsilon)=s^*(\hat{\beta}).$$

Imagine that the planner gives the firm the following transfer function (linear in observed cost):

$$(3-16) \quad t(\beta, C)=s^*(\beta)+K^*(\beta)(C^*(\beta)-C)$$

where

$$(3-17) \quad K^*(\beta)=\frac{\psi'(e^*(\beta))}{q^*(\beta)}$$

(remember that  $C^*(\beta)\equiv(\beta-e^*(\beta))q^*(\beta)$  and that  $s^*(\beta)\equiv\psi(e^*(\beta))+\int_{\beta}^{\bar{\beta}}\psi'(e^*(\delta))d\delta$ .)

Then firm  $\hat{\beta}$  solves:

$$(3-18) \quad \max_{\{\beta, e\}} E\{K^*(\beta)(C^*(\beta)-((\hat{\beta}-e)q^*(\beta)+\epsilon))+s^*(\beta)-\psi(e)\}$$

or

$$(3-19) \quad \max_{\{\beta, e\}} \{(s^*(\beta)-\psi'(e^*(\beta))\tilde{e}^*(\beta|\hat{\beta}))+(\psi'(e^*(\beta))e-\psi(e))\}$$

Optimization with respect to  $e$  clearly leads to:

$$(3-20) \quad e = e^*(\beta).$$

Using (3-20) and (3-3), the optimization with respect to  $\beta$  gives:

$$(3-21) \quad \beta = \hat{\beta}.$$

So this linear incentive scheme implements the optimal allocation if the firm's second order condition for (3-19) is satisfied. Straightforward computations show that this condition boils down to:

$$(3-22) \quad \dot{e}^*(\beta) < 0.$$

So we conclude:

Proposition 5: If (3-22) is satisfied (like, for instance, when  $\psi''' > 0$ ), the optimal allocation can be implemented by an incentive scheme that is linear in cost:  $t(\beta, C) = s^*(\beta) + K^*(\beta)(C^*(\beta) - C)$ .

The second-order condition (3-22) is more stringent than (3-5). This deserves some comment. (3-22) corresponds to one way of implementing the optimal solution. This way requires the transfer to be linear in cost. If (3-22) is satisfied (as is the case when  $\psi''' > 0$ ), then the linear scheme is a perfectly legitimate way of implementing the solution. If (3-22) is not satisfied, the linear scheme is not optimal, as it imposes too stringent a second-order condition. This point is best explained in the no uncertainty case ( $\varepsilon \equiv 0$ ). As we have seen, the knife-edge mechanism is an alternative way to implement the optimal allocation. This mechanism gives the most lenient second-order condition ((3-3)), as its extreme penalties for cost overruns restrict the set of possible deviations to the concealment set. To the contrary, the linear scheme defined by (3-16) allows more deviations; and its linearity in cost restricts the possible punishments for deviations out of the concealment set. Thus the second-order condition is unsurprisingly more stringent.

The linear scheme implements the optimal allocation as long as the optimal effort is decreasing. It furthermore has a very appealing property. Notice that the optimal allocation is independent of the distribution of cost uncertainty. Intuitively the linear scheme is the only scheme that implements the optimal allocation whatever the distribution of cost uncertainty:

Proposition 6: Assume that the optimal effort is non-increasing (as is the case if  $\psi'' > 0$ ). Then the linear scheme  $\{t(\beta, C) = s^*(\beta) + k^*(\beta)(C^*(\beta) - C)\}$  implements the optimal allocation for any cost uncertainty (with zero mean). It is the only scheme having this property.

The proof of the last part of proposition 6 can be found in Appendix 4.

d) Summary of the technical analysis: We studied the simple program (P') which maximizes expected social welfare under the individual rationality constraint for the least efficient firm and the first-order incentive constraint. We then looked at the firm's and the planner's second-order condition. And we showed how one can implement the optimal allocation. From now on, we will assume that

A2) For the solution to the necessary and sufficient conditions for (P'):  
 $e^*(\beta) < 0$  (the firm's second-order condition for the linear scheme is satisfied).

As we saw, A2) is satisfied if  $\psi'' > 0$ . Furthermore, if A2) is satisfied, the linear scheme is the optimal scheme (for any distribution of the cost disturbance). If A2) is not satisfied, the firm's second-order condition must be taken into account when constructing an incentive scheme that implements the optimal allocation. The analysis then becomes harder.

For the characterization of the optimal scheme, we will sometimes need a further assumption:

A3)  $\left\{ \frac{\psi''}{\psi} \right\}$  is non-increasing.



A3) puts a (positive) upper bound on the third derivative of the cost function. An example of a cost function satisfying both A2) and A3) is the quadratic cost function:  $\psi(e) = \frac{e^2}{2}$ . Contrary to A2), which will be assumed throughout, we will indicate when we make use of A3).

Remark: (3-5) (and a fortiori A2)) and (3-11) imply that the output is non-increasing in  $\beta$ . This remark will prove useful in the interpretation of the optimal scheme.

e) The optimal scheme under cost unobservability (Baron-Myerson).

In this subsection we want to compare the solution derived in section 3b) with the inferior solution which would obtain if the planner were unable to observe cost. The latter situation has been extensively studied in the literature (see, e.g., Baron-Myerson (1982) and Guesnerie-Laffont (1984)).

In this subsection only,  $s(\beta)$  will denote the gross transfer to the firm when it announces characteristic  $\beta$ . A net transfer does not make sense as  $C$  is not observed.

We derive the Baron-Myerson results for our model. To ease exposition, we will ignore second-order conditions in the presentation. The firm's program is:

$$(3-23) \quad U(\hat{\beta}) = \max_{\{\beta, e\}} \{s(\beta) - (\hat{\beta} - e)q(\beta) - \psi(e)\}$$

The firm's first-order conditions are

$$(3-24) \quad \dot{s}(\beta) = (\beta - e)\dot{q}(\beta)$$

$$(3-25) \quad \psi'(e(\beta)) = q(\beta).$$

(3-25) shows that effort is socially optimal conditional on output. This is intuitive as the cost, which is unobservable by the planner, is fully borne by the firm. Note also that the incentive constraints imply that:

$$(3-26) \quad \dot{U}(\beta) = -q(\beta).$$

So the planner's subconstrained program is:

$$(3-27) \quad \max_{\beta} \int_{\underline{\beta}}^{\bar{\beta}} (S(q(\beta)) - (1+\lambda)(\psi(e(\beta)) + (\beta - e(\beta))q(\beta)) - \lambda U(\beta)) d\beta$$

s.t.

$$(3-28) \quad \dot{U}(\beta) = -q(\beta)$$

$$(3-29) \quad U(\bar{\beta}) = 0.$$

It is easily shown that the necessary conditions for this program are:

$$(3-30) \quad S'(q) = (1+\lambda)(\beta - e) + \lambda(\beta - \underline{\beta})$$

$$(3-31) \quad \psi'(e) = q.$$

The role of cost observability will be studied in 4b).

#### 4. The optimal allocation and incentive scheme

This section draws the economic implications of the previous technical analysis (assuming A1) and A2)).

##### a) Comparison with the full information allocation

Proposition 7: The asymmetry in information implies for all  $\beta$  (except  $\underline{\beta}$ ) a lower output and a lower effort.

Proof: Compare {(2-4), (2-5)} and {(3-11), (3-12)} and apply A1. Q.E.D.

The intuition behind Proposition 7 is simple. Under moral hazard, the regulator cannot reimburse the totality of the firm's cost. It however does not want to adopt a fixed-price contract (which it would be forced to do under cost unobservability). Under such a contract no moral hazard problem arises. But the firm, bearing the full cost, has a tendency to understate its efficiency to be allocated a low output and thus incur a low cost (as shown by Baron-Myerson [1982]). This underproduction can be avoided if the firm is made the residual claimant for social welfare, i.e., if it is rewarded  $\{S(q)/(1+\lambda)\}$  (up to a con-

stant). But making the firm the residual claimant is too costly under redistributive considerations and incomplete information about the firm's productivity. Reimbursing part of the firm's cost helps alleviate this issue, by making the firm less concerned about cost and therefore less conservative in its output decision. Indeed if there were no moral hazard, the optimal contract would be cost-plus. Clearly the tradeoff between inducing revelation (cost-plus contract) and inducing effort (fixed-price contract) results in an "incentive contract" (partial sharing of cost), as shown by the optimal incentive scheme.

Given that the firm's cost is partially reimbursed, effort is suboptimal. Hence marginal cost is excessive, and output is therefore suboptimal, as shown by proposition 7.

b) The role of cost observability: The optimal scheme under cost unobservability has been studied by Baron-Myerson (1982). For our model, the comparison between the two cases is given by  $\{(3-11), (3-12)\}$  and  $\{(3-30), (3-31)\}$ . As explained above, Baron-Myerson's fixed-price contract implies no effort distortion for a given output contrary to the optimal "incentive contract" derived for cost observability. This effort distortion is more than offset from a welfare point of view by the lower price distortion  $\{S'(q)-(1+\lambda)(\beta-e)\}$  for the incentive contract. Indeed the fixed-price contract, which ignores the cost information, is also available to the planner under cost observability, but is not optimal because some cost reimbursement eases revelation of the technological information.

c) Efficiency and the choice of output and effort. In section 3 we saw that (under assumption A2) the optimal levels of effort, output and expected costs were all decreasing with the marginal cost parameter  $\beta$ . This is very natural. An increase in efficiency (decrease in  $\beta$ ) calls for a higher output. Therefore marginal cost reductions through effort become more valuable, as they apply to a higher number of units.

d) The optimal incentive scheme. Let us rephrase proposition 6 by assuming (more realistically) that the net transfer depends on output and observed cost (from the revelation principle the two approaches are equivalent). Since output is a monotonic function of  $\beta$  (see 3d), we have:

Proposition 8: For any distribution of the cost uncertainty, the optimal allocation is implemented through a linear scheme:

$$\bar{T}(q, C) = \bar{s}(q) + \bar{K}(q)(\bar{C}(q) - C)$$

where  $\bar{C}(q)$  is the optimal expected cost given  $q$  and  $0 < \bar{K}(q) < 1$ .

Furthermore

- i)  $\bar{s}(q)$  is an increasing function of  $q$ .
- ii)  $\bar{K}(q)$  is an increasing function of  $q$  if A3) is satisfied.
- iii)  $\bar{K}$  converges to 1 (fixed-price contract) when uncertainty becomes small ( $(\bar{\beta} - \beta) \rightarrow 0$ ).

Proof: The functions  $\{\bar{s}, \bar{K}, \bar{C}\}$  are derived from  $\{s^*, K^*, C^*\}$  by substituting  $q$  for  $\beta$  (as  $q$  is monotonic in  $\beta$ ). Monotonicity of  $\bar{s}$  results from its definition, and from the second-order condition. iii) results from the definition of  $\bar{K}$  and

(3-12). Lastly, differentiating (3-17) and using (3-12) gives

$$\frac{\dot{\bar{K}}}{\bar{K}} \alpha \frac{(q - \psi')}{\psi''} ((\psi'')^2 - \psi' \psi''') \dot{\beta} - \frac{\lambda}{1 + \lambda} \psi' \psi'' = 0 \quad \text{A2), A3) and (3-12) then imply that } \dot{\bar{K}} < 0.$$

Q.E.D.

Proposition 8 has several important implications for regulation. First, in the context of our model, the optimal allocation can be implemented by a particularly simple incentive scheme. Furthermore the knowledge of the distribution of the cost disturbance around zero is not required to build this scheme. The contract is an incentive contract. It can be decomposed into a fixed price contract  $\bar{s}(q)$  and a partial cost reimbursement. After agreeing on an output, the planner gives a first reward  $\bar{s}(q)$ , which increases with output. Then, after observing



the final cost, he gives a penalty or a bonus that is proportional to cost overruns.

Second, the coefficient of proportionality depends on the scale of the project. Indeed, under A3), the fraction  $(1-\bar{K})$  of costs that are reimbursed decreases with output. A rough intuition for this is as follows: low cost firms produce more. For those firms marginal cost reduction is more valuable; so effort should be particularly encouraged. To do so the planner can reimburse a lower fraction of costs in exchange of a higher fixed fee. Such a policy is consistent with screening, as low cost firms are more willing to be reimbursed a small fraction of cost. Furthermore, the firm's utility and, therefore, transfer are obtained by imposing the individual rationality constraint  $U(\bar{\beta})=0$  and integrating backwards the incentive compatibility constraint  $\dot{U}(\beta)=-\psi'(e(\beta))$ . A high level of effort for an inefficient firm ( $\beta$  close to  $\bar{\beta}$ ) is thus reflected into a higher utility for almost all types of firms. So effort again should be encouraged more for more efficient firms (indeed at the optimum, there is no effort distortion for  $\beta=\bar{\beta}$ ).

Third, when the uncertainty becomes small, reimbursing the cost to induce efficient revelation of information becomes valueless. Only the moral hazard problem remains relevant, and, under risk neutrality, the contract converges to a fixed-price contract. This phenomenon to some extent was observed by Ponsard and Pouvourville (1982, pp. 55) in the dynamic evolution of contracts in the French weapons industry. They observed that, as a project evolves over time, the contract resembles more and more a fixed price contract. This may be explained by the fact that the Government acquires information about the firm's cost function<sup>7</sup>.

An even more familiar way of interpreting proposition 6 uses the fact that the expected average cost  $c^*(\beta)$  is increasing in  $\beta$  (see section 3a). Imagine



that the firm, instead of announcing its efficiency parameter  $\beta$ , announces an expected average cost  $c^a$ ; it is then ordered  $\tilde{q}(c^a)$  units and is rewarded ex-post according to  $\tilde{t}(c^a, c)$  where  $c$  is the ex-post average cost. We have:

Proposition 9: The optimal allocation can be implemented by asking the firm to announce an expected average cost  $c^a$  and making the transfer depend on the expected and realized average costs:  $\tilde{t}(c^a, c) = \tilde{s}(c^a) + \tilde{K}(c^a)(c^a - c)$ ;  $\tilde{s}(c^a)$ ,  $\tilde{q}(c^a)$ , and (under A3))  $\tilde{K}(c^a)$  are decreasing functions, and  $0 < \tilde{K} < 1$ .

The ex-ante reward ( $\tilde{s}$ ) and the slope of the ex-post bonus scheme ( $\tilde{K}$ ) decrease with the announced cost. We can relate this result to evidence on actual incentive schemes. Contracts usually specify a higher transfer if the firm is willing to increase its share of cost overruns or underruns (see, e.g., Scherer (1964, p. 260)). This practice is given a normative justification by proposition 9: the latter shows that the transfer ( $\tilde{s}$ ) and the coefficient of cost sharing ( $\tilde{K}$ ) are positively correlated.

e) Influence of demand on the optimal contract. Let us briefly study how the sharing coefficient  $\tilde{K}$  varies with the demand function. Let us posit that the consumer's surplus depends on a parameter  $\theta$ :  $S(q, \theta)$ . A way of formalizing the idea that the output becomes (marginally) more valuable when  $\theta$  increases is to assume that  $S_{q\theta} > 0$ . In this case demand grows with the parameter  $\theta$  (for instance, for linear demand,  $\theta$  can represent the intercept or minus the slope of the demand curve).

Proposition 10: Under A3), the optimal contract resembles more a fixed-price contract when the demand for output increases.

Proof: Differentiate (3-11) and (3-12) and use assumptions A2) and A3) to obtain

$$\frac{\partial \tilde{K}}{\partial \theta} = S_{q\theta}$$

Q.E.D.

The intuition behind Proposition 10 is the following. A higher demand leads to higher output. So cost reduction through effort becomes more valuable. It then makes sense to have the firm bear a higher fraction of its cost overruns (not surprisingly, use is made of assumption A3), which also plays a role to show that  $\bar{K}$  must grow with  $q$ ).

f) Contracting on quality. Suppose that the scale variable to be determined is the quality of the output rather than its level (which we can take to be one). The model and its conclusions are unchanged if  $q$  denotes a quality parameter instead of a quantity as long as quality is observable ex-post. In particular, under asymmetric information:

- i) There is underprovision of effort and quality.
- ii) The sharing rate of the optimal linear scheme is positively correlated with quality.

Proposition 10, in the quality interpretation, tells us that the more concerned about quality the regulator is, the more the optimal contract resembles a fixed price contract. This is due to the fact that marginal cost reductions must be encouraged more when higher qualities are chosen. Alternative models may lead to the opposite conclusion. Imagine for instance that quality is observable, but not verifiable, so that the contract cannot be made contingent on quality. The firm must then trade off immediate cost savings (low quality) and reputation. A way to encourage the firm to choose a higher quality is then to share a higher fraction of cost. Similarly, even if no reputation is involved, the possibility of bankruptcy may also move the optimal contract towards a cost-plus contract. These models may fit better the casual observation (e.g., for

defense and building contracts) that contracts resembling cost-plus ones are often used when the level of quality matters much to the planner.

##### 5. Choice of technology and rate-of return regulations

The same approach can be applied to the case where the firm has, ex-ante, the possibility of choosing between various technologies which involve different splittings of cost between fixed costs and marginal costs.

Let

$$C = (\hat{\beta} + \beta_1 - e)q + \alpha(\beta_1) + \epsilon$$

where  $\alpha(\beta_1)$  is the firm's fixed cost and  $\epsilon$  is a cost disturbance with zero mean.  $\hat{\beta}$  is given. By increasing  $\beta_1$  the firm decreases its fixed cost ( $\alpha' < 0$ ,  $\alpha'' > 0$ ,  $\alpha'(0) = -\infty$ ) and increases its variable cost. In a first step neither the choice of  $\beta_1$ , nor the level of effort, nor the particular value of  $\hat{\beta}$  can be observed by the regulator.

Let us briefly argue that technological choices between fixed and variable costs and possibly their inobservability by the regulator may be relevant features of real world procurement situations. For example, a power company may choose between high fixed cost technologies (e.g., nuclear plants) and high variable cost ones (e.g., coal). Similarly increasing overhead within a plant (supervisors, foremen, engineers...) increases the fixed cost while reducing variable costs (associated with mistakes, delays, low effort, etc...). The latter example suggests that it is sometimes fairly hard for public accountants to split the total cost they observe into fixed and variable costs (for an example of how a firm can manipulate this accounting procedure, see Peck-Scherer (1962, p. 518)).

The analysis of section 3 is hardly modified by the introduction of the

extra choice variable  $\beta_1$ . It is easily seen that the optimal allocation must satisfy:

$$(5-1) \quad S'(q) = (1+\lambda)(\beta + \beta_1 - e)$$

$$(5-2) \quad \psi'(e) = q - \frac{\lambda}{1+\lambda} (\beta - \underline{\beta}) \psi''(e)$$

$$(5-3) \quad \alpha'(\beta_1) = -q.$$

From (5-1), (5-2) and assuming a unique full information allocation (analog of A1)), it is easy to see that  $q$  must be smaller than at the optimum under perfect information. Then from (5-3) we can conclude that there is a bias towards less fixed costs due to imperfect information<sup>8</sup>. The intuition behind this result is simple: Imperfect information about  $\hat{\beta}$  leads to suboptimal quantities. Thus marginal cost reductions (through  $\beta_1$ ) are effective on a lower number of units of output than in the perfect information case. Therefore there is an incentive to keep marginal cost high and fixed costs low. Let us however notice that the firm takes the right technological decision given its output. This is because, in the optimal contract, part of the cost is borne by the firm.

Let us now observe that this bias towards low fixed costs is also a local welfare property around the incomplete information optimum. To this purpose let us examine how the social welfare changes with  $\beta_1$  in the neighborhood of the second best solution when investment is not observable by the regulator

$$(5-4) \quad \frac{d}{d\beta_1} (S(q) - (1+\lambda)(EC + \psi(e)) - \lambda U) \\ = S(q) \dot{q} - (1+\lambda)(-q \dot{e} + (\beta - e) \dot{q} + \psi'(e) \dot{e}) \\ = \frac{\lambda (\beta - \underline{\beta}) \psi''(e)}{q} \dot{e},$$

where (5-4) uses (5-1), (5-2) and (5-3) in two ways: the partial derivatives of  $C$



and  $U$  with respect to  $\beta_1$  are zero.

So if the second-order condition for linear schemes is satisfied (A2),

$$(5-5) \quad \frac{d}{d\beta_1} (S(q) - (1+\lambda)(C+\phi(e)) - \lambda U) < 0.$$

This means that welfare would increase if the firm decreased  $\beta_1$  slightly below the cost minimization level. In other words, the regulator would like to force the firm to overinvest a bit in fixed costs. Let us now imagine that the level of investment  $\alpha(\beta_1)$  (and therefore  $\beta_1$ ) is observable by the regulator. One may wonder whether an Averch-Johnson type rule associated with a transfer function  $t(q, C)$  cannot implement the new second-best allocation (i.e., under investment observability). By Averch-Johnson type rule we mean a constraint on (for example) transfers to the firm ( $t$ ) per unit of capital invested ( $\alpha$ ) (where the regulated rate of return could depend on the firm's output). It is well-known that that rates of return regulation induce an upwards bias in capital accumulation. So a priori such a rule may improve the previous allocation while letting the firm choose its investment.

It turns out that, in our model, the optimal allocation under investment observability is the same as without observability. This result is contingent on the separable form we assumed for the cost function. Let us give some intuition for it. The firm, when free to choose its investment, has a common incentive with the regulator to minimize cost (see (5.3)). So incentives may differ only if the choice of  $\beta_1$  has an influence on the incentive compatibility constraint. But the latter ( $\hat{U} = -\phi'(e)$ ) is unaffected by the observability of  $\beta_1$ : knowing  $q(\beta)$  and knowing that the firm minimizes cost with respect to  $\beta_1$ , the planner infers the term  $(\beta_1 q(\beta) + \alpha(\beta_1))$  and therefore the "concealment set" (defined in section 3),  $\{\hat{\beta} - e = \beta - e(\beta)\}$  is not affected by investment observability. So for our specification there is nothing more that the regulator can do if he happens

to be able to observe investment<sup>9</sup>. In particular imposing a (binding) Averch-Johnson type regulatory constraint would be detrimental as it would destroy cost minimization<sup>10</sup>.

## 6. Risk aversion

Let us briefly explore the consequences of risk aversion on the firm's behavior and on the incentive scheme. Let us assume that the manager has the following expected utility function (which can be justified by an approximation argument):

$$(6-1) \quad U = Et - \gamma \text{var}t - \phi(e).$$

The cost function is  $C = (\beta - e)q + \varepsilon$ , where  $\varepsilon$  is a random variable with mean 0 and variance  $\sigma^2$ .

The analysis under risk aversion becomes complex. We will not derive the optimal mechanism but simply study how the coefficient  $K^*$  of the linear mechanism ( $t(\beta, C) = s^*(\beta) + K^*(\beta)(C^*(\beta) - C)$ ), (which is optimal under risk neutrality), must be changed because of risk aversion. Appendix 5 derives the optimal linear incentive scheme for small  $\gamma$  (since the linear scheme is nearly optimal when the firm is not very risk averse), and leads to the following (unsurprising) conclusion:

Proposition 11: Assume that A1) is satisfied; that  $\phi'' > 0$ ; and that the coefficient of risk aversion  $\gamma$  in (6-1) is "small". The fraction of cost that is reimbursed in the best linear scheme increases with the coefficient of risk aversion.

7. Marketed good

If the product is actually sold on a market at a price  $p(q)=S'(q)$ , the objective function of the planner is

$$(7-1) \quad \int_{\underline{\beta}}^{\bar{\beta}} ((S(q)-S'(q)q) - (1+\lambda)[E(s+C)-S'(q)q] + Es - \psi(e)) d\beta$$

$$= \int_{\underline{\beta}}^{\bar{\beta}} (S(q) - (1+\lambda)EC + \lambda S'(q)q - \lambda Es - \psi(e)) d\beta$$

The planner's optimization problem is thus the same as for a public good except that  $S(q)$  is replaced by  $(S(q) + \lambda S'(q)q)$ .

The equations defining the optimal  $q$  and  $e$  are:

$$(7-2) \quad S'(q) = (1+\lambda)(\beta - e) - \lambda MR(q)$$

$$(7-3) \quad \psi'(e) = q - \frac{\lambda}{1+\lambda} (\beta - \underline{\beta}) \psi''(e),$$

where  $MR(q) (\equiv \frac{d}{dq} (S'(q)q) = S'(q) + S''(q)q)$  is the firm's marginal revenue.

At the optimum under perfect information

$$(7-4) \quad S'(q) = (1+\lambda)(\beta - e) - \lambda MR(q)$$

and

$$(7-5) \quad \psi'(e) = q.$$

Here again the optimal production and effort levels under imperfect information is less than under perfect information, as is easily seen.

Comparing (7-2) (7-3) with (3-11) (3-12), the distortion from perfect information is decreased by the sale of the commodity. Both effort and production are higher than in the public good case. This is of course due to the

fact that the opportunity cost of money is larger than one for the planner. If  $\lambda=0$ , this divergence disappears.

One can also wonder what happens when the transfer cost ( $\lambda$ ) becomes large. It is easy to see that the pricing rule converges to the monopoly pricing rule if information is perfect (we should not forget however that second-order conditions become problematic for  $\lambda$  large<sup>11</sup>). This is natural as the emphasis is then on raising as much revenue as possible. Incentives, however costly, are still given to encourage effort as cost reduction remains very important (see (7-3)). Under imperfect information the quantity produced is lower than the monopoly output for  $\lambda$  sufficiently large.

Lastly we may ask whether the famous average cost pricing rule holds in an appropriate sense. At least two very informal arguments have been offered to justify average cost pricing:

- Average cost pricing may be a way to prevent the regulated firm from charging too high a price (for instance the monopoly one).
- For an increasing returns firm marginal cost pricing implies a deficit for the firm that must be subsidized. Subsidization is a risky policy when there is moral hazard. Average cost pricing avoids this problem.

Our model provides a first step to analyze these arguments. We do not find support for average cost pricing in our set up<sup>12</sup>. Maybe we should not be surprised by this. As cost and output are assumed to be observable, there are finer ways than the average cost rule to avoid monopoly pricing and to induce effort. The average pricing imposes rigid incentive and pricing structures. In particular it is not sensitive enough to the level of fixed cost, if any, and to the structure of information<sup>13</sup>.



## 8. Related work and conclusions

The desirability of an incentive contract is reminiscent of the moral hazard literature.<sup>14</sup> The main difference with this literature is that we added private information at the contracting date.<sup>15</sup> This accounts for the fact that, even under risk-neutrality, incentive contracts (of a linear form in our model) are desirable. More importantly, it allowed us to show how the sharing coefficient must vary with the fixed fee, or with the firm's intrinsic efficiency.

The paper most related to our work is Baron-Besanko's (1983). These authors consider a procurement situation analogous to ours. The planner does not know the marginal cost  $\hat{\beta}$ . Ex-post he observes a variable correlated with the firm's cost; so there is an observation error. The firm's only decision variable is the announcement  $\beta$ ; so there is no moral hazard. The authors assume that the planner is constrained to ex-post impose a penalty in some interval  $[0, \bar{N}]$ . They show that it is optimal to impose the penalty  $\bar{N}$  if the observed cost is "low" and 0 otherwise (what "low" means depends on the announcement  $\beta$ ). They also show that, under some conditions, the price (or quantity) policy  $q(\beta)$  is independent of the possibility of observing cost. In other words, ex-post auditing is only a way to reduce the transfer to the firm (separability property).

The idea behind these results is the following: if no cost observation is available, the model boils down to Baron-Myerson's, and the problem is simply to elicit the firm's marginal cost. As the cost is fully borne by the firm, a low marginal cost firm will tend to announce a high marginal cost to be allocated a low quantity to produce. Costly transfers are then required to prevent the firm from lying. Introducing cost observation does not affect the firm's real cost as there is no moral hazard; but it gives some information about the firm's marginal cost. To further prevent the firm from announcing high marginal costs, one puts penalties on low cost observation (if auditing is costly and therefore is not

done systematically, this statement must be qualified by the fact that high announced costs are more likely to be audited). The Baron-Besanko story might be that of an agency that applies for a yearly budget. If it has not used up its budget at the end of the year, it is punished for its excessive greed in the following year by being allocated a lower budget<sup>16</sup>.

Our conclusions differ strikingly from Baron-Besanko's. First, under moral hazard, the planner cannot reward high costs. Otherwise the firm could always manage to increase expenditures. Indeed we find that only a fraction of costs is reimbursed. Second our pricing policy relies heavily on the possibility of observing the firm's cost (see section 4). Cost observability reduces the distortion between price and (social) marginal cost.

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- i) We gave a complete characterization of the firm's and planner's problems.
- ii) Under more hazard and total cost observability, the firm's effort is suboptimal, and its price is too high compared with perfect information. We found no rationale in our model for average cost pricing.
- iii) As long as the second-order condition for the firm is satisfied, the planner can use a reward function that is linear in cost. The same linear function can be used for any distribution of the cost disturbance.
- iv) The fraction of realized cost that is reimbursed to the firm is not a constant, but decreases with the firm's output or increases with the firm's announced cost. This results from the fact that the different types of firm self-select when signing the contract with the regulator. The most efficient firm chooses a fixed-price contract. The less efficient firms opt for an incentive contract. The regulator agrees to reimburse a higher fraction of costs, the less efficient the firm is. Furthermore, the fixed transfer increases with the fraction of total cost that the firm is willing to share.

- v) The optimal contract moves towards a fixed-price contract when demand increases.
- vi) Cost observability improves welfare. It has a tendency to distort the effort decision, but it allows more control over the pricing policy.
- vii) The linear reward function deals with increasing risk aversion in the best way by increasing the fraction of reimbursed costs.
- viii) If the good is marketed, the pricing distortion due to incomplete information is alleviated.
- ix) If the firm makes an unobservable technological choice between fixed and variable costs, there is a bias towards low fixed costs and high variable costs. A rate-of-return regulation may not improve welfare in spite of insufficient capital accumulation.

Appendix 1: Differentiability of the effort, transfer and utility functions

Lemma 1:  $\beta < \hat{\beta} \rightarrow \tilde{e}(\beta | \hat{\beta}) > \tilde{e}(\hat{\beta} | \hat{\beta})$ .

Lemma 1 says that a firm with cost  $\hat{\beta}$  must make a higher effort when it announces a cost lower than the true one.

Proof: From the incentive compatibility constraints, we know that

$$(A-1) \quad s(\hat{\beta}) - \psi(\tilde{e}(\hat{\beta} | \hat{\beta})) > s(\beta) - \psi(\tilde{e}(\beta | \hat{\beta}))$$

$$s(\beta) - \psi(\tilde{e}(\beta | \beta)) > s(\hat{\beta}) - \psi(\tilde{e}(\hat{\beta} | \beta)).$$

Adding these two inequalities, we obtain

$$(A-2) \quad \psi(\tilde{e}(\beta | \hat{\beta})) - \psi(\tilde{e}(\beta | \beta)) > \psi(\tilde{e}(\hat{\beta} | \hat{\beta})) - \psi(\tilde{e}(\hat{\beta} | \beta)).$$

Imagine that

(A-3)  $\tilde{e}(\hat{\beta} | \hat{\beta}) > \tilde{e}(\beta | \hat{\beta})$  (which would contradict the lemma). We also know that, by definition of  $\tilde{e}$ ,

$$(A-4) \quad \tilde{e}(\beta | \hat{\beta}) - \tilde{e}(\beta | \beta) = \hat{\beta} - \beta = \tilde{e}(\hat{\beta} | \hat{\beta}) - \tilde{e}(\hat{\beta} | \beta) > 0.$$

Lastly, (A.3), (A.4) and the strict convexity of  $\psi$  contradict (A.2).

Q.E.D.

Lemma 2:  $\tilde{e}(\beta | \hat{\beta})$  is non-increasing in  $\beta$ .

Proof: Fix  $\beta > \beta'$  and define

$$\Delta(\hat{\beta}) \equiv \tilde{e}(\beta' | \hat{\beta}) - \tilde{e}(\beta | \hat{\beta}).$$



We have  $\Delta(\hat{\beta}) = (e(\beta') - \beta') - (e(\beta) - \beta)$ . Thus  $\Delta(\hat{\beta})$  does not depend on  $\hat{\beta}$ . But from the previous lemma  $\Delta(\hat{\beta}') < 0$ . Thus for all  $\hat{\beta}$ ,  $\Delta(\hat{\beta}) < 0$ .

Q.E.D.

Lemma 2 implies that  $\tilde{e}(\beta | \hat{\beta})$  is a.e. differentiable in  $\beta$ . So is  $e(\beta) = \tilde{e}(\beta | \hat{\beta}) + (\beta - \hat{\beta})$ .

Lemma 3:  $U(\beta | \hat{\beta})$ , as a function of  $\beta$ , is non-decreasing on  $[\underline{\beta}, \hat{\beta}]$  and non-increasing on  $[\hat{\beta}, \bar{\beta}]$ .

Proof: Let us first show monotonicity on  $[\underline{\beta}, \hat{\beta}]$ . Assume that  $\beta < \beta' < \hat{\beta}$  and  $U(\beta | \hat{\beta}) > U(\beta' | \hat{\beta})$ . Thus

$$(A-5) \quad s(\beta) - \psi(\tilde{e}(\beta | \hat{\beta})) > s(\beta') - \psi(\tilde{e}(\beta' | \hat{\beta})).$$

On the other hand we know that a firm with cost  $\beta'$  prefers to announce  $\beta'$  rather than announcing  $\beta$ . Thus

$$(A-6) \quad s(\beta') - \psi(\tilde{e}(\beta' | \beta')) > s(\beta) - \psi(\tilde{e}(\beta | \beta')).$$

Adding (A-5) and (A-6):

$$(A-7) \quad \psi(\tilde{e}(\beta | \beta')) - \psi(\tilde{e}(\beta' | \beta')) > \psi(\tilde{e}(\beta | \hat{\beta})) - \psi(\tilde{e}(\beta' | \hat{\beta})).$$

As in the proof of lemma 2, and using lemma 1:

$$(A-8) \quad \tilde{e}(\beta | \beta') - \tilde{e}(\beta' | \beta') = \tilde{e}(\beta | \hat{\beta}) - \tilde{e}(\beta' | \hat{\beta}) > 0$$

and from the definition of  $\tilde{e}$ :

$$(A-9) \quad \tilde{e}(\beta' | \beta') < \tilde{e}(\beta' | \hat{\beta}).$$

(A-8) and (A-9) and the convexity of  $\psi$  contradict (A-7).

Monotonicity on  $[\hat{\beta}, \bar{\beta}]$  is proved in the same way (using the incentive compatibility constraint for  $\beta$  this time).

Q.E.D.

Lemma 4:  $s(\beta)$  is non-increasing.

Proof: By definition

$$(A-10) \quad s(\beta) = U(\beta | \underline{\beta}) + \psi(\tilde{e}(\beta | \underline{\beta})).$$

From lemmas 2 and 3, the two functions on the right hand side are non-increasing.

So is  $s$ .

Q.E.D.

Lemmas 2 and 4 imply that the functions  $\tilde{e}$  (and therefore  $e$ ) and  $s$  are almost everywhere differentiable. Hence  $U(\beta) = s(\beta) - \psi(e(\beta))$  is also differentiable a.e. This completes the proof of the first part of proposition 1.

Appendix 2: The local second-order condition implies the global one

Lemma 5: If  $\frac{\partial U}{\partial \beta}$  is (strictly) monotonic in  $\hat{\beta}$ , then the local second-order condition implies the global one.

Proof of lemma 5: The local second-order condition implies that announcing the truth  $\hat{\beta}$  gives a local maximum for the firm. Is there another  $\beta \neq \hat{\beta}$  that satisfies the first-order condition? i.e., does there exist  $\beta \neq \hat{\beta}$  such that

$$\frac{\partial U}{\partial \beta}(\beta, \hat{\beta}) = \frac{\partial U}{\partial \beta}(\hat{\beta}, \hat{\beta}) = 0 ?$$

This would imply that

$$\frac{\partial U}{\partial \hat{\beta}}(\beta, \hat{\beta}) = \frac{\partial U}{\partial \beta}(\beta, \beta) = 0.$$

But this is inconsistent with the (strict) monotonicity of  $\frac{\partial U}{\partial \beta}$  with respect to its second argument. Q.E.D.

Lemma 6:  $\frac{\partial^2 U}{\partial \beta \partial \hat{\beta}}$  is (strictly) positive if the local second-order condition is (strictly) satisfied.

Proof of lemma 6: Differentiating (3-2) with respect to  $\hat{\beta}$  gives

$$(A-11) \quad \frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} = \psi''(\tilde{e}(\beta | \hat{\beta}))(1 - \dot{e}(\beta)).$$

Using (3-5) and the convexity of  $\psi$ , we obtain our conclusion. Q.E.D.

### Appendix 3: The planner's optimization problem

a) Necessary conditions.

Consider the subconstrained program (P'). The Hamiltonian is:

$$(A-12) \quad H = [S(q) - (1+\lambda)(\psi(e) + (\beta - e)q) - \lambda U] + \mu(-\psi'(e)),$$

where  $\mu$  is the multiplier associated with (3-6). The Pontryagin principle yields:

$$(A-13) \quad \frac{\partial H}{\partial q} = 0 = S'(q) - (1+\lambda)(\beta - e)$$

$$(A-14) \quad \frac{\partial H}{\partial e} = 0 = -(1+\lambda)(\psi'(e)-q) - \mu\psi'(e)$$

$$(A-15) \quad \dot{\mu} = -\frac{\partial H}{\partial U} = \lambda.$$

Furthermore  $\underline{\beta}$  is a free boundary, so that:

$$(A-16) \quad \mu(\underline{\beta})=0.$$

Integrating (A-15) and using (A-16), we obtain:

$$(A-17) \quad \mu(\beta)=\lambda(\beta - \underline{\beta}).$$

The necessary conditions given in Proposition 2 follow.

b) Sufficiency conditions and existence of a solution for the planner's problem.

Let us consider how to prove the existence of a solution and to characterize it. Two difficulties may exist. First, the program may be non-concave. Second, incentive compatibility imposes that the state variable  $U$  be almost everywhere differentiable while Pontryagin's principle assumes that the state variables are piecewise differentiable (with a finite number of pieces).

In step one we show that there exists a solution by restricting the analysis to the Pontryagin framework.

In step two we show that the solution to step one is indeed the solution.

Step 1: It is easy to show using assumption (A-1) that the Pontryagin necessary conditions have a solution. Next, if the Kamien-Schwartz (1971)'s sufficient condition holds, the solution to the first order condition is optimal. The sufficient condition is satisfied if the maximized Hamiltonian is concave in the state variable  $U$ . This is here always the case because the Hamiltonian is



linear in  $U$ , and the equations defining the control variables ((A-13) and (A-14)) are independent of  $U$ .

Step 2: The space of almost everywhere differentiable increasing functions on  $[\underline{\beta}, \bar{\beta}]$  is a closed convex subset of the Banach space  $L^\infty([\underline{\beta}, \bar{\beta}], \mathbb{R})$ . Let  $A$  be the subspace of piecewise-continuous functions. The objective function is continuous in  $e$  and  $s$ . Since any increasing function of  $L^\infty$  can be approximated as closely as desired in the supnorm topology by functions in  $A$  and since we have a solution to the maximization in  $A$ , it is a solution to the maximization.

#### Appendix 4: Nonlinearity and cost disturbances

Let us show that a scheme that is not linear in cost cannot implement the optimal solution for all distributions for the disturbance. We know that  $t(\beta, C)$  must satisfy:

$$(3-15) \quad s^*(\beta) = \mathbb{E}_t(\beta, (\beta - e^*(\beta))_q^*(\beta) + \epsilon).$$

If  $t$  is not linear in cost, there exist  $\beta, C_1, C_2, C_3$  such that

$$(A-18) \quad \frac{t(\beta, C_1) - t(\beta, C_2)}{C_1 - C_2} \neq \frac{t(\beta, C_1) - t(\beta, C_3)}{C_1 - C_3}.$$

Define  $\epsilon_i \equiv C_i - (\beta - e^*(\beta))_q^*(\beta)$ ; and consider the family of discrete distributions with three atoms at  $\epsilon_1, \epsilon_2$  and  $\epsilon_3$ , and no weight elsewhere (as these distributions can be approximated by continuous distributions, we could actually restrict ourselves to continuous distributions). It is clear that by varying the weights on the three disturbance levels, and given (A-18), (3-15) cannot always be satisfied.

Appendix 5: Risk aversion

Let us show that the coefficient  $K$  of risk sharing increases with  $\sigma^2$  and with  $\gamma$ . We know that

$$(A-19) \quad Et = s(\beta) + K(\beta)(\beta - e(\beta))q(\beta)$$

and

$$(A-20) \quad \text{var}t = K^2(\beta)\sigma^2,$$

so that

$$(A-21) \quad U = s(\beta) + K(\beta)(\beta - e(\beta))q(\beta) - \gamma K^2(\beta)\sigma^2.$$

The incentive constraint can be written:

$$(A-22) \quad \dot{U} = -\psi'(e) = -K(\beta)q(\beta).$$

The planner's optimization problem is (up to the second-order condition):

$$(A-23) \quad \max_{\beta} \int_{\beta}^{\bar{\beta}} (S(q) - (1+\lambda)((\beta - e)q + \psi(e)) - \lambda(U + \gamma\sigma^2 K^2)) d\beta$$

s.t. (A-22).

The Hamiltonian is:

$$(A-24) \quad H = S(q) - (1+\lambda)((\beta - e)q + \psi(e)) - \lambda(U + \gamma\sigma^2 K^2) - \mu\psi'(e) + v(Kq - \psi'(e)).$$

We treat  $U$  as a state variable and  $q$ ,  $e$  and  $K$  as control variables.

After writing the Pontryagin conditions, we obtain:

$$(A-25) \quad S'(q) = (1+\lambda)(\beta - e) - \frac{2\lambda\gamma\sigma^2\psi'(e)^2}{q^3}$$

$$(A-26) \quad \psi'(e) = q - \frac{\lambda}{1+\lambda} (\beta - \bar{\beta})\psi''(e) - \frac{2\lambda\gamma\sigma^2\psi'(e)\psi''(e)}{(1+\lambda)q^2}$$

Differentiating (A-25) and (A-26), one gets:

$$\text{sign} \left\{ \frac{dK}{d\gamma} \right\} = \text{sign} \left\{ q \left( \frac{s''\psi''}{1+\lambda} + 1 \right) - q \left( 1 - \frac{\psi'}{q} \right)^2 - \frac{\psi'^2 \psi'''}{(\psi'')^2} \right\}$$

A1) and the assumption that  $\psi''' > 0$  then leads to:

$$\frac{dK}{d\gamma} < 0 .$$

Q.E.D.

Footnotes

1. The literature on "Soviet bonus schemes" (see e.g., Ellman (1973), Fan (1975), Weitzman (1976), Bonin (1976)) and on the rate-of-return regulation (see, e.g., Crampes (1982) for a study with asymmetric information) considers the revelation of production possibilities. Another literature (Domar (1979), Tam (1979, 1981), Finsinger-Vogelsang (1982, 1983), Vogelsang (1983)) assumes that the firm has superior information about demand. Although the latter literature is relevant in some cases, we would expect informational asymmetries about production possibilities to be more important in general (for instance the regulator can run a consumer survey or use the firm's output and price data to estimate demand). Bergson (1978) has stressed the role of distributional aspects. Our use of a social cost of transfers can be viewed as a formalization of the concern for equity in the design of incentive schemes.

2. The analysis can be conceptually generalized to any number of parameters. In particular demand may also be parameterized in the same way as costs if the regulator does not know demand. But the optimal schemes then cannot be derived analytically. Also the assumption that these functions can be parameterized does not appear restrictive. Actually even the manager derives his information from a finite number of observations and can only have an approximation of the true cost and demand functions even in a stationary environment. See also Guesnerie and Laffont (1984) for an application to the control of labor managed firms and some additional theoretical developments.

3. Formally the planner can extract some information from aggregate cost observation. The point is that the high dimensionality of the characteristics space reduces the value of the information.

4. We exclude in this paper the solution proposed by Demsetz (1968) of designing an auction and giving the market to the best offer, by assuming that there is a single informed firm. One justification can be that huge increasing returns to scale do not make it worthwhile to set up several firms to benefit from their competition. A related reason, when the parties renegotiate the contract rather than set up a new relationship, comes from the advantages of sticking with the incumbent. For more details see Williamson (1976).

5. See section 7 for the case of a marketed good.

6. For a discussion of this formalism and of its (close) relation to a weighted social welfare function, see Caillaud et al (1985).

7. Of course, our model here is a static one. The Ponsard-Pouvoirville observations are vindicated by our model if both the planner and the firm take a myopic perspective in a dynamic context. When the parties take a dynamic perspective, the study should be completed by a dynamic analysis of the corresponding ratchet effect (see Laffont-Tirole (1985)).

8. This underinvestment property is closely related to that in Tirole (1984), who gives a general result for incomplete contracts (the result obtained here assumes a complete contract).



9. One may then wonder why a decrease in  $\beta_1$  can increase social welfare. The answer is that the decrease suggested by equation (5-5) is hypothetical in that it does not take into account the change in incentives required to bring it forth.
10. For more complex cost functions, we conjecture that a rate of return regulation may increase or decrease welfare relative to the now suboptimal scheme  $t(q,C)$ , depending on the effect of the investment on the concealment set.
11. This analysis in particular assumes that the planner does not want to shut the firm. Roughly this will not happen if a monopoly is viable.
12. One can think of several ways to formalize average cost pricing, depending on whether the managers' reward is included in the pricing rule. If  $S'(q)q=s+C$ , the firm behaves as a monopoly. If  $S'(q)q=C$ , then there is no incentive to expand effort:  $e=0$ .
13. See Freixas and Laffont (1985) for an analysis which compares only marginal cost pricing and average cost pricing in a framework with moral hazard.
14. See e.g. Mirrless (1974, 1975), Harris-Raviv (1979), Holmström (1979), Shavell (1979) and Grossman-Hart (1982). We should also mention the literature on the use of ex-post observations in insurance markets and optimal taxation (e.g., Polinsky-Shavell (1979), Mirrless (1974, 1975), Landsberger and Chazan (1983)). There it is shown that penalties based on, for instance, the occurrence of accident can help reduce moral hazard.
15. Baron (1982) studies a model of the demand for investment banking advising under adverse selection, moral hazard, uncertainty and risk neutrality. The complexity of the model does not allow for a derivation of the optimal incentive scheme.
16. Still it is very hard to avoid moral hazard. See the well-known stories of trucks driving around the barrack's yard at Christmas time to use up their gas endowment.

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