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> UNCERTAINTY AND TRADE THEORY: SOME COMPARATIVE-STATIC RESULTS

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# UNCERTAINTY AND TRADE THEORY: SOME COMPARATIVE-STATIC RESULTS\*

by

Pranab K. Bardhan

#### I. Introduction

Various kinds of uncertainty loom large in matters of international trade in the real world and yet trade theorists almost invariably tend to look away from them. $\frac{1}{1}$  In this paper we propose to illustrate an approach towards analysing the impact of uncertainty in simplified open-economy models. We are concerned with three (related) kinds of uncertainty: production uncertainty (say, due to weather fluctuations in agriculture), demand uncertainty (say, due to fluctuations of income or tastes in foreign markets) and price uncertainty (say, due to international price fluctuations). Our major emphasis will be on analysing the response to price uncertainty.

Since the introduction of uncertainty makes even otherwise very simple models quite complicated and since our approach in this paper

<sup>\*</sup> <sup>I</sup> am grateful to T. N. Srinivasan for many valuable comments at various stages of preparation of this paper. Errors remain mine.

<sup>1/</sup> The only article in the literature on trade theory dealing with uncertainty in any significant way that came to our attention are Brainard and Cooper (1968), Berry and Hymer (1969), Grubel (1964) and Snape and Yamey (1963). Of these the Brainard-Cooper paper is in the same genre as ours. It is an excellent paper, but it does not fully analyze the case where production decision is under uncertainty but the trading decision is taken when the world price is known; it does not discuss the income distribution and other aspects we have analyzed [footnote cont'd, on next page]

is more illustrative than comprehensive, we shall make drastic simplifications in the forms of utility and production functions that we use as well as in our characterisation of uncertainty, so that some suggestive but definite qualitative conclusions may be derived. In Section II we take a completely specialized small country which exports, say, a primary (consumable) product and imports an intermediate product; in this model we analyse how stabilization of prices or production (or reduction of price or production uncertainty) will affect the total foreign exchange earnings of the country. In Section III the economy is still completely specialized but it now has sufficient monopoly power in the international market to be able to vary the export price charged by the country (and, by implications on the level of the so-called 'optimum tariff). In Section IV, the economy is incompletely specialised allowing for general-equilibrium resource allocation effects and in this context we discuss the impact of price uncertainty on output and income distribution; in particular we find that under familiar assumptions about the pattern of risk-aversion, commodity price stabilization lowers the relative price of the factor less intensively used in the commodity the price of which is stabilized,

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in this paper; it takes the restrictive mean-variance approach and quadratic utility functions. The Berry-Hymer paper considers the question of how the impact of uncertainty depends on the "flexibility" of the economy. The Grubel and Snape-Yamey papers analyze the effect of buffer-stock and buffer-fund schemes of price stabilization on foreign exchange earnings.

i.e. in our model stabilization of the world price of a landintensive primary export good reduces the relative wage of labour. This last question is important because quite often we discuss the benefits of a stabilization policy without looking into its impact on income distribution; yet stabilization (like protection in the Stolper-Samuelson model) has significantly differential impact on different factor incomes and it is highly important that the policy-maker be aware of it.

> II. Price and Production Uncertainty in a Model with Complete Specialization

Suppose the economy is completely specialized in producing output of an exportable commodity, c, with the help of its fixed stocks of land and labour and an intermediate product (entirely imported from abroad in exchange of exports of c). With a Cobb-Douglas production function,

$$
f_c = Bk^{\alpha} v^{\beta} , \qquad 1 > \alpha + \beta
$$
 (1)

where  $f_{n}$  is output of c per unit of labour, k and  $v$  are land and intermediate input respectively per unit of labour and B is a given constant.

If P is the international price of the exportable good, the intermediate product is taken as a numeraire and if x is exports per unit of labour, the balance of trade equation is given by

$$
Px = v \tag{2}
$$

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We assume, for the time being, that the economy is too small to be able to affect the international price, P. But due to exogenous factors this price fluctuates in the international market, thus being a source of uncertainty to our economy.

How to characterize the degree of uncertainty in P without adopting the rather restrictive mean-variance approach? Following the approach of Sandmo (1970) we examine two kinds of shift in the probability distribution of P. One is an additive shift which is equivalent to an increase in the mean with all other moments constant. The other is a multiplicative shift by which the distribution is stretched around zero. A pure increase in dispersion can be defined as a stretching of the distribution around a constant mean. This is equivalent to a combination of additive and multiplicative parameter changes.

Let us write P as

$$
P = \lambda u + \rho \qquad (3)
$$

Where u is the random variate,  $\lambda$  is the multiplicative shift parameter and p the additive one.

With E as the expectation operator,

$$
EP = \lambda Eu + \rho \tag{4}
$$

A multiplicative shift around zero will increase the  $mean$ ; if the expected value is to be held constant, this should be matched by an

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additive shift in the negative direction. Taking the differential, this means that

$$
dEP = E [ud\lambda + d\rho] = 0 \quad , \tag{5}
$$

which implies that

$$
\frac{d\rho}{d\lambda} = - Eu \tag{6}
$$

Thus,

$$
\frac{d\rho}{d\lambda} = u + \frac{d\rho}{d\lambda} = u - Eu = \frac{1}{\lambda} (P - EP)
$$
 (7)

Per capita (assuming the number of labourers and people to be the same) national income in terms of value added in this economy is

$$
y = PBk^{\alpha}v^{\beta} - v \tag{8}
$$

Let us suppose that the economy maximizes  $EU(y)$ , expected utility of per capita income. The utility function is assumed to be strictly concave with a positive marginal utility of income for all y. Maximizing the expected utility function with respect to v, we get the first-order necessary condition for interior maximum as

$$
EU' [BPRk^{\alpha}v^{\beta-1} - 1] = 0
$$
 (9)

From (7) and (9),

$$
\frac{d\upsilon}{d\lambda} = \frac{[EU''(BPBk^{\alpha}\upsilon^{\beta-1}-1)B\upsilon^{\beta}k^{\alpha} + EU'BBk^{\alpha}\upsilon^{\beta-1}]\lambda^{-1}(P-EP)}{[EU'B(1-\beta)PBk^{\alpha}\upsilon^{\beta-2} - EU''(BPBk^{\alpha}\upsilon^{\beta-1}-1)^{2}]}
$$
(10)

In (10), with positive risk-aversion (i.e.  $U'' < 0$ ), the denominator on the R.H.S. is positive (this also happens to satisfy the secondorder condition for maximizing  $EU(y)$  with respect to v).

Let us denote the numerator on the R.H.S. of (10) as  $E\phi(P)(P-EP)$ . It is easy to show that  $\phi'(\mathbf{P}) < 0$ , under the following set of sufficient conditions: (i) if e =  $-U''(y)y$  is the measure of relative risk-aversion U'(y) in the Arrow-Pratt sense, de  $\geq$  , i.e. relative risk-aversion does not dy decrease as income increases (this is one of the assumptions Arrow (1965) uses in the context of portfolio decisions) and (ii) the measure of relative risk-aversion, e, is not above unity. Under these conditions, using Lemma <sup>1</sup> in the Appendix the numerator on the R.H.S. of (10) is negative, which implies that  $\nu$  is a decreasing function of  $\lambda$ . Given our characterization of uncertainty, this means that a pure increase in price uncertainty reduces v. Since from (2), v is equal to total export or foreign exchange earnings of the country, we may say that under our restrictions (i) and (ii) on risk-aversion an increase in export price uncertainty (conversely, stabilization of export price) will lower (raise) the foreign exchange earnings of this economy. So at least in terms of our model the contrary assertion of Nurkse {1958) does not follow.

Let us now for the time being forget about price uncertainty and deal briefly with production uncertainty. Let us suppose our exportable good, c, is an agricultural product subject to the vagaries of weather (the imported input v may be regarded as a fertilizer used in agricul-

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tural production). In equation (1) let us now take B as, instead of a given constant, a variable reflecting the influence of uncertain weather. The international price, P, is a given constant for this small economy. We are interested in finding out how in this model an increase in uncertainty in production of exportables affects foreign exchange earnings.

Analogous to our characterization of uncertainty before let us assume

$$
B = \lambda u + \rho \tag{11}
$$

where u now is the random variable representing some composite index of weather (rainfall, temperature, etc.),  $\lambda$  and  $\rho$  are, as before, the multiplicative and additive shift parameters respectively. Using exactly the same method of reasoning as before, we find that for a multiplicative shift around zero to keep the mean constant we must have

$$
\frac{d\rho}{d\lambda} = - Eu, \text{ and } \frac{dB}{d\lambda} = \frac{1}{\lambda} (B - EB).
$$

It can now be checked that if the economy maximizes EU(y), exactly the same conditions as (9) and (10) follow again; the only change is that, instead of P, B is now the random element. Following the same procedure of reasoning as before, we can say that an increase in uncertainty in production of exportables (conversely, stabilization of agricultural production through, say, irrigation, drainage, pesticides, etc.) will lower (raise) the foreign exchange earnings of this economy under our above-mentioned restrictions (i) and (ii) on the pattern of risk-aversion.

### III. Export Demand Uncertainty and the Optimum Export Price

In this Section we concern ourselves with export demand uncertainty due to, say, exogenous fluctuations in income or tastes in foreign markets. We shall continue with our assumption of complete specialization in the exportable good, c, but change the assumptions regarding the smallness of the economy in the international market and the imported intermediate product. Now the only good imported is a consumer good, m (no Intermediate input is used in the production of c), and in the export market the economy has sufficient monopoly power to affect the international price, P. We are interested in finding out the impact of an increase in export demand uncertainty on the optimum export price charged by the country.

The utility function has now two arguments in the consumption of the two goods. For simplification we shall assume that the utility function is additively separable  $\frac{2}{x}$ , so that our maximand

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<sup>2/</sup> The restrictions implied by additive von Neumann-Morgenstern utility functions are analyzed in Pollax (1967).

is E  $[U (C_c) + V (C_m)]$ , where  $C_c$  and  $C_m$  are the per capita amounts consumed of goods c and m respectively. Both U and V are strictly concave with positive marginal utilities.

Dispensing with the Cobb-Douglas assumption, the constant returns to scale production function for c is now given by

$$
f_c = f(k) \qquad (12)
$$

where  $f$  is, as before, the per capita amount of c produced and  $k$  is the given land-labour ratio in the economy.

While production of c is partly consumed and partly exported so that

$$
C_c = f_c - x \t\t(13)
$$

consumption of m is assumed to be entirely from imports, so that

$$
C_m = Px , \t\t(14)
$$

where x is the per capita amount of c exported and the imported consumer good is taken as the numeraire.

Again for simplification, we shall assume the following constantelasticity export demand function facing the economy:

$$
x = HP^{\eta} \t{15}
$$

where n is the constant (and negative) price elasticity of foreign demand for exports and H is the random variable representing demand uncertainty.

We shall assume the absolute value of the price elasticity to be larger than unity so that  $1 + \eta < 0$ .

Our characterization of the random element H follows the same lines as before, so that

$$
H = \lambda u + \rho \tag{16}
$$

where u now is the random variable representing the uncertain factors shifting the export demand curve and  $\lambda$  and  $\rho$  are the multiplicative and additive parameters respectively. Using exactly the same method of reasoning as before, we find that for a multiplicative shift around zero to keep the mean constant we must have  $\frac{d\rho}{d\lambda} = -$  Eu and  $\frac{dH}{d\lambda} = \frac{1}{(H-EH)}$ .

Maximizing the expected utility function with respect to P, subject to (12) - (15), we get the first-order condition for interior maximum as

$$
EHP^{\eta-1} [(1 + \eta) PV' - \eta U'] = 0
$$
 (17)

From (17),

$$
\frac{dP}{d\lambda} = \frac{E[{(1+n) PV' - nU'] + H ({(1+n) P2+nU'' + nPnU''] Pn-1\lambda-1(H-EH)}{D}
$$
 (18)

where D = -  $\frac{\partial^2 E[U + V]}{\partial}$  > 0 as the second-order condition for maxi- $\partial P$   $\overline{\phantom{1}}$ mizing the expected utility function.

Let us rewrite the numerator on the R.H.S. of (18) as  $E\emptyset(H)$  (H-EH). It is easy to show that  $\emptyset'$  (H) > 0 under the following sufficient condition:

if  $A_c = \frac{-U''}{U}$  and  $A_m = \frac{-V''}{V'}$ ,  $\frac{dA_i}{dC_s} \le 0$ , i = c,m (with our additive utility function this means that absolute risk-aversion $\frac{3}{ }$  in the Arrow-Pratt sense does not increase with increase in consumption). So under this condition it follows from Lemma 1 in the Appendix that the numerator on the R.H.S. of (18) is positive. This means that with a larger export demand uncertainty the country having monopoly power in the export market will charge a higher export price. This suggests that in this model with export demand uncertainty the optimum tariff is higher than in the conventional model with certainty .

# IV. Price Uncertainty with Incomplete Specialization and its Impact on Output and Factor Prices

In the preceding Sections by assuming complete specialization we have ignored general-equilibrium resource allocation effects of uncertainty. In order now to focus on this latter problem we assume in this Section that the economy has two production sectors with inter-sectorally mobile factors of production. We shall take the standard, strictly concave-to-origin production-possibility curve as a given technological datum so that the domestic output of one commodity is a given function of the other:

$$
Q_m = Q_m(Q_c)
$$
\n(19)

where  $Q_i$  is the per capita domestic output of the i-th good,  $Q_m^{\dagger}(Q_c) < 0$ and  $Q_{\mu}^{''}(Q_{\mu})$  < 0. Let us define  $\bar{P}$  as the slope of the production-possibility curve or the marginal rate of transformation between  $Q_c$  and  $Q_m$  so that

<sup>3/</sup> For a simple generalization of Arrow's results about absolute riskaversion with an additive utility function see Sandmo (1968).

 $\overline{P} = - Q_m^{\dagger}(Q_\circ).$ 

Given the strictly concave-to-the-origin production possibility curve and given competitive markets for the two factors, land and labour, it follows from the usual Ohlin-Lerner-Samuelson trade theory that a rise (fall) in  $\overline{P}$  will be associated with a rise (fall) in the relative price of the factor more intensively used in the production of c (of which  $\overline{P}$ is the relative price), or,

$$
\frac{dw}{dP} < 0 \tag{20}
$$

where w is the wage-rentals ratio and c is always the more land-intensive good. In other words price uncertainty in the export market that we are going to analyze will not affect the usual relationship between the marginal rate of transformation and the relative factor prices. So in order to analyze the impact of price uncertainty on factor prices our major task is to analyze its impact on  $\overline{P}$ . We shall take the price of the importable good, m, as unity, so that the world price, P, is the relative as well as the absolute price of the exportable good, c.

Our characterization of uncertainty in price P is exactly as in Section II, as summarized in equations  $(3)-(7)$ . As before we take a multiplicative shift around zero to keep the mean constant. We shall analyze the impact of a change in  $\lambda$ , the multiplicative shift parameter, to denote the pure change in uncertainty. We are thus interested in the sign of  $\frac{dP}{d\lambda}$ , dQ and since  $Q_m < 0$ , the latter has the same sign as  $\frac{1}{d\lambda}$ .

As in the preceding Section, we take an additively separable utility function  $[U(C_c) + V(C_m)]$ , where  $C_i$  is the per capita consumption of the i-th good.

$$
C_{\rm c} = Q_{\rm c} - x \tag{21}
$$
  

$$
C_{\rm m} = Q_{\rm m} + P x \tag{22}
$$

where, as before, x is the per capita amount (assumed positive) of c exported and trade is balanced.

We shall assume that production decisions are made when the world price is not known, but the country decides about how much to export after the world price is known.<sup>4</sup> In other words, we are not considering the case where contracts to export are entered into before the world price is known but exports fetch the price prevailing at the time of delivery.

Let us first take the trading decision. By the time it is taken production has already been decided upon and the world price is known. So we then maximize  $[U(C_{c}) + V(C_{m})]$  with respect to  $C_{c}$ , given  $Q_{c}$ ,  $Q_{m}$  and P, subject to the following relation equating national income and expenditure:

$$
PQ_c + Q_m = PC_c + C_m. \tag{23}
$$

This gives us as a necessary condition

$$
U' - V'P = 0.
$$
 (24)

But for the production decision which is taken under price uncertainty we maximize the expected utility function  $E[U(C_>) + V(C_])$  with respect to  $Q_$ . Using (19), (23), and (24), we get as necessary condition for this maximum

$$
EV^{\dagger}(P-\overline{P}) = 0. \tag{25}
$$

From  $(25)$  we can derive, with the help of  $(23)$  and  $(24)$ , that

<sup>4/</sup> Brainard and Cooper (1968) consider this case "analytically com plicated" and do not provide any clear-cut result.

$$
\frac{dQ_{c}}{d\lambda} = \frac{E \left[ V^{\dagger} + V^{\dagger} (P - \overline{P}) \frac{\left[ U^{\dagger} x - U^{\dagger} \right]}{(U^{\dagger} + P^{2} V^{\dagger})} \right] (P - EP) \lambda^{-1}}{-E \left[ V^{\dagger} Q_{m}^{\dagger \dagger} + \frac{V^{\dagger} U^{\dagger} (P - \overline{P})^{2}}{(U^{\dagger} + P^{2} V^{\dagger})} \right]}
$$
(26)

The denominator on the R.H.S. of (26) is obviously positive, since U", V" and Q" are negative and V' positive. So the sign of the numerator will determine the sign of  $\frac{dQ_c}{d\lambda}$ .

From Lemma 1 in the Appendix it is easy to check that  $EV'(P-EP) < 0$ , since  $\frac{dV'}{dP}$  < 0. Lemmas 2 and 3 prove that

$$
EV''(P-\overline{P})\frac{\left\{U''x-U'\right\}}{\left(U''+P^2V''\right)} > 0,
$$
\n
$$
(27)
$$

and

$$
EV''P(P-\overline{P})\frac{[U''x-U']}{(U''+P^2V'')} < 0, \qquad (28)
$$

under the following two conditions $\frac{5}{4}$  on the pattern of risk-aversion: (a) the absolute risk-aversion is non-increasing with increase in consumption and (b) the relative risk-aversion is non-decreasing with increase in consumption.

So if we now rewrite the numerator of (26) as

$$
EV^{\prime}(P-EP)\lambda^{-1} + EV^{\prime\prime}P(P-\overline{P})\frac{[U^{\prime\prime}x-U^{\prime}]\lambda}{(U^{\prime\prime}+P^2V^{\prime\prime})}^{-1} - EPEV^{\prime\prime}(P-\overline{P})\frac{[U^{\prime\prime}x-U^{\prime}]\lambda}{(U^{\prime\prime}+P^2V^{\prime\prime})}^{-1},
$$

we can see that it is negative. This means that under our conditions (a)

<sup>5/</sup> As we have mentioned before, both these conditions are familiar from Arrow's (1965) portfolio model. Condition (a) implies that the willingness to engage in small bets of a fixed size does not decrease as income (or consumption) increases; condition (b) implies that if both the size of the bet and income (or consumption) are increased in the same proportion, the willingness to accept the bet does not increase.

dQ<br>and (b) on risk-aversion,  $\frac{dQ}{dV}$  is negative, i.e. with increased price uncertainty for the export good its output decreases . This suggests the standard result that the degree of specialization in the export good will be smaller (or diversification larger) in the case with export price uncertainty than in the case with certainty.

Since we have already seen that  $\frac{dP}{dt}$  has the same sign as  $\frac{dQ}{dt}$ , this means that with increased price undertainty  $\overline{P}$ , the marginal rate of transformation of m into c along the production-possibility curve, goes down. From (20) this implies that with increased export price uncertainty the relative price of labour, the factor that is used less intensively in the export industry, goes up; conversely, stabilization of the price of landintensive exports (say, of primary products) lowers the relative price of labour. Thus in this model export price stabilization tends to turn the relative income distribution pattern against labour, or, more generally, against the owners of the factor less intensively used in the export industry .

In deriving (26) one can also check that, other things remaining the same, the larger is the absolute value of  $Q_{\text{m}}^{\text{U}}(Q_{\text{c}})$  the smaller is the dQ<br>absolute value of  $\frac{dQ}{dQ}$ . The absolute value of  $Q''$  may be regarded as an index of the "flexibility" of the economy; the larger it is the more does the transformation curve bulge outwards, and the smaller it is the less is the elasticity of substitution of the two goods along the transformation curve (in the extreme case of rigidity with fixed-coefficients production functions  $Q''_m$  is zero). If this index of flexibility is accepted, our result suggests that the output-reducing impact of increased price uncertainty will be smaller for the more flexible economy. In this respect our result is contrary to that obtained in the model of Berry and Hymer (1969).

In conclusion let us comment on some aspects of our basic model. First, in analyzing the response to uncertainty we have used a social utility function implicitly assuming the comfortable fiction that this is the utility function of the "representative" individual as well. As a matter of fact, social evaluation of risks may diverge significantly from private, and the social costs of variability in price, etc., may be different from the costs perceived by each individual. As Brainard and Cooper (1968) point out, in such circumstances an appropriate policy of taxes and subsidies, insurance and guarantees or more direct control may be called for.

Secondly, one should discuss the question of relating the impact of price fluctuations with the flexibility of the economy in terms of a fuller model in which certain costs of adjustment in moving along the productionpossibility curve should be introduced and inventory and other production smoothing methods should be taken into account.

Thirdly, in our discussion on increased price uncertainty we have kept the mean price constant while taking a pure increase in risk. But there are many interesting problems in which the effects of a simultaneous rise in the mean as well as in risk need analysis. To take one example, in discussing gains from trade in a model with uncertainty in the world market one may have to cope with a situation in which the opening of trade implies both a rise in the expected price for the exportable good and a larger variability in its price.

Fourth, we have ignored the possible effect of increased price uncertainty for the exportable good in a long-run adverse shift in its world demand curve because of the increased use of, say, a synthetic substitute that it induces.

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Fifth, one should discuss the impact of stabilization in a fuller model in which the differential effects of the specific stabilization policies are captured.

 $\mathcal{A}$ 

#### Appendix

Lemma 1

E $\oint (P)(P-EP) \geq 0$ , as  $\oint'(P) \geq 0$ 

Proof: We shall prove it only-for the case of  $\emptyset^{\dagger}(\mathbb{P}) \geq 0$ ; the other two cases may then be worked out easily.

If  $\phi'(\mathbf{P}) > 0$ ,  $\phi(\mathbf{P}) > \phi(\mathbf{E}\mathbf{P})$  for  $\mathbf{P} > \mathbf{E}\mathbf{P}$ and  $\phi(P) < \phi(EP)$  for  $P \leq EP$ ; hence  $E\phi(P)(P-EP) > \phi(EP)E(P-EP) = 0$ .

Lemma 2

$$
EV''(P-\overline{P})\frac{[U''x-U']}{[U''+P^{2}V'']}\geq 0,
$$
  
if the degree of absolute risk-aversion, 
$$
\left[\frac{-V''(C_{m})}{V'(C_{m})}\right], \text{ does not increase as } C_{m} \text{ increases.}
$$

Proof: Define 
$$
\Psi(P) = \frac{-V''(P)}{V'(P)} \frac{[U''x-U']}{[U''+P^2V''(P)]} = \frac{-V''(P)}{V'(P)} \frac{[U''x-U']}{[U''+U'\frac{V''(P)}{V'}}.
$$

Differentiating with respect to P, it is easily seen that under nonincreasing absolute risk-aversion  $\Psi'(P) < 0$ .

So  $\Psi(P) < \Psi(\overline{P})$  when  $P > \overline{P}$ and  $\Psi(P) > \Psi(\overline{P})$  when  $P < \overline{P}$ ;

hence  $\Psi(P)(P-\overline{P}) < \Psi(\overline{P})(P-\overline{P})$ 

or, 
$$
EV''(P-\overline{P})\frac{[U''x-U']}{[U''+P^2V'']}
$$
 > -  $\Psi(\overline{P})EV'(P)(P-\overline{P})$ , which is zero from (25).

Lemma 3

$$
EV''P(P-\overline{P})\frac{\left[U''x-U'\right]}{\left[U''+P^2V''\right]} < 0
$$

if the degree of relative risk-aversion,  $\left[\frac{-V''(C_m)C_m}{V'(C_m)}\right]$ , does not decrease  $\sim_{\text{m}}$ with increase in  $C_m$ .

<u>Proof</u>: Under non-decreasing relative risk-aversion  $\frac{ - \text{pV}^{\text{tr}}(\text{p})}{\text{V}^{\text{tr}}(\text{p})}$  is an increasing function of P. Hence, if we define V'(P)[U"+U' $\frac{PV''(P)}{V}$ ] it is easily seen that  $\phi'(\mathbb{P}) > 0$ .

So  $\phi(P) > \phi(\overline{P})$  when  $P > \overline{P}$ 

and  $\phi(P) < \phi(P)$  when  $P < \overline{P}$ ;

hence  $\phi(P)(P-\overline{P}) > \phi(\overline{P})(P-\overline{P})$ 

or EV"P(P- $\overline{P}$ ) $\frac{\left[U''x-U''\right]}{2}$  < -  $\phi(\overline{P})$ EV'(P)(P- $\overline{P}$ ), which is zero from (25) [U"+P V"]

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