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UNVERIFIABLE INFORMATION, INCOMPLETE  
CONTRACTS, AND RENEGOTIATION

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No. 92-6

Feb. 1992

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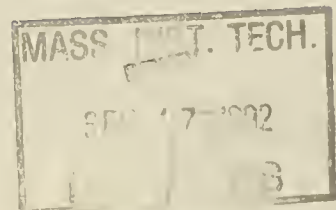


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# Unverifiable Information, Incomplete Contracts, and Renegotiation\*

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ABSTRACT: Hart and Moore (1988) argued that non-verifiability is a major cause for contract incompleteness and leads in general to underinvestment if the parties can not commit not to renegotiate their initial contract. We show that this result relies on the assumption that the courts cannot distinguish which party refused to trade. If this assumption is relaxed the first best can be achieved by giving control to one party which can decide unilaterally whether or not to enforce trade. Our result suggests that if relationship specific investments are non-verifiable, then vertical integration may perform better than separate ownership.

KEYWORDS: Incomplete Contracts, Renegotiation, Verifiability.

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# 1 Introduction

Long-term contracts are necessary as a safeguard against opportunistic behaviour if a joint project is undertaken by several parties who have to make relationship specific investments. However, long-term contracts are often incomplete. This observation is the starting point of a rapidly growing literature which interpretes institutions, such as ownership or the financial structure of the firm, as incomplete contracts and tries to explain their forms and functionings. This literature faces two basic theoretical questions: (i) Why are contracts incomplete? and (ii) When does contract incompleteness lead to inefficient investments?

While the early literature just assumed that contingent contracts are not feasible before the parties have to make their investment decisions<sup>1</sup>, Hart and Moore (1988) argued that non-verifiability is a major cause for contract incompleteness and may make it impossible to achieve first best investments. At first glance this result is surprising. It is well known from the literature on implementation in environments with symmetric information that quite generally the problem of non-verifiability can be overcome by conditioning the contract on the verifiable outcome of a revelation game or mechanism.<sup>2</sup> However, these mechanisms usually rely on the threat of inefficient punishments in case of a deviation from the equilibrium strategies. Hart and Moore argue that this threat is not credible in a contracting environment, because the parties can always tear up the old contract and renegotiate. To analyse this issue Hart and Moore provide a formal framework in which renegotiation is taken explicitly into account. Their path-breaking analysis shows that if contracts can be renegotiated then non-verifiability will, in general, lead to underinvestment.

Given that most of the incomplete contracts literature refers to this underinvestment result, we consider it important to point out that it relies not only on the renegotiation constraint, but also on one particular modelling assumption. Hart and Moore consider a situation in which the private investments of two parties, a seller and a buyer, affect the

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<sup>1</sup>Grossman and Hart (1986) is the most notable contribution using such an approach.

<sup>2</sup>See Moore (1990) for a recent survey of this literature.

that these additional degrees of freedom are not necessary to achieve the first best.<sup>5</sup>

The rest of the paper is organized as follows: In Section 2 we briefly summarize the model of Hart and Moore and show how the renegotiation outcome is affected if we allow for an option contract. Furthermore, we outline the intuition for why an option contract can achieve the first best while a Hart-Moore contract cannot. In Section 3 this argument is developed formally. Section 4 concludes.

## 2 The Basic Framework

Consider a buyer and a seller both of whom are risk neutral. At some initial date 0 they can write a contract specifying the terms of trade of one unit of an indivisible good which they may want to exchange at some future date 2. The buyer's valuation  $v$  and the seller's production costs  $c$  are random variables depending on the realization of the state of the world,  $\omega$ , which is determined at date 1. Let  $\omega$  be uniformly distributed on  $\Omega = [0, 1]$ <sup>6</sup>. After date 0, but before date 1, the buyer and the seller make relationship specific investments,  $\beta \in [0, \infty)$  and  $\sigma \in [0, \infty)$  respectively, which affect  $v$  and  $c$ . Investments are measured in terms of their costs, and these costs are sunk. We assume that  $v(\omega, \beta)$  and  $c(\omega, \sigma)$  are continuous in both arguments, uniformly bounded and non-negative; furthermore let us suppose that  $c(\omega, \sigma)$  is non-increasing in  $\sigma$  and  $v(\omega, \beta)$  is non-decreasing in  $\beta$  for all possible states of the world.

Let  $q \in \{0, 1\}$  be the level of trade and  $p$  the net payment of the buyer to the seller.

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<sup>5</sup>Also related to our work is a recent paper by Hermalin and Katz (1991), who use a somewhat different formal framework than Hart and Moore and show that in their model a class of contracts they call "fill-in-the-price mechanisms" may be used to implement efficient investment decisions. However, to derive this result Hermalin and Katz assume that the seller's cost of production and the buyer's benefit from consumption are independent random variables. The model we consider throughout the paper violates this requirement.

<sup>6</sup>If the set of relevant states of the world can be represented by a subset of the real numbers then the assumption of a uniform distribution on  $[0, 1]$  is without loss of generality. Suppose the "true" state of the world is  $\omega'$  which is drawn by nature out of some closed set  $\Omega' \subset \mathcal{R}$  according to the cumulative distribution function  $F(\omega')$ . Using the probability integral transformation we can define a new random variable  $\omega = F(\omega')$  which clearly is uniformly distributed on  $[0, 1]$ . Since  $F(\omega')$  is monotonic we can rescale all functions of  $\omega'$  such that they are functions of  $\omega$ .



Then the utilities of the buyer and the seller after date 2 are given by

$$u^B = q \cdot v(\omega, \beta) - p - \beta, \quad (1)$$

$$u^S = p - q \cdot c(\omega, \sigma) - \sigma. \quad (2)$$

Trade is efficient iff  $v(\omega, \beta) - c(\omega, \sigma) \geq 0$ .<sup>7</sup> At date 2 the buyer and the seller have to decide simultaneously whether they do or do not want to trade. Trade is assumed to be voluntary in the sense that it takes place if and only if both parties are willing to trade. The problem of the parties at date 0 is to design a contract which implements efficient investment and trade decisions, i.e. which maximizes expected social welfare

$$W(\beta, \sigma) = \int_0^1 [v(\omega, \beta) - c(\omega, \sigma)]^+ d\omega - \beta - \sigma, \quad (3)$$

where  $[\cdot]^+ = \max\{0, \cdot\}$ .

Note that, given our continuity assumptions on  $v(\cdot, \cdot)$  and  $c(\cdot, \cdot)$ ,  $W(\beta, \sigma)$  is always well-defined and continuous in  $\beta$  and  $\sigma$ . Furthermore, by the boundedness assumption the set of maximizers of  $W(\cdot, \cdot)$  is always non-empty. Denote by  $(\beta^*, \sigma^*)$  a pair of investment levels which maximizes (3).

The first best could easily be achieved if it were possible to contract upon the level of investment. However, we assume that although investments  $\beta$  and  $\sigma$  as well as the state of the world  $\omega$  (and so  $v$  and  $c$ ) are perfectly observable by both players, they cannot be verified to any third party, e.g. the courts. Thus the contract cannot enforce outcomes contingent on these variables. The courts can only observe payments and the trade decisions of the buyer and the seller at date 2. Thus the contract can specify four different prices,  $p_1$ ,  $p_0^B$ ,  $p_0^S$ , and  $p_0^{BS}$  depending on whether trade took place or whether the buyer, the seller or both parties refused to trade.

Hart and Moore make the following “simplification”: they assume that the courts can only observe whether  $q = 0$  or 1, but if  $q = 0$ , they cannot distinguish whether the seller or the buyer was unwilling to trade. Thus, in their setting only two different prices

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<sup>7</sup>Note that the specification of  $v(\cdot, \beta)$  and  $c(\cdot, \sigma)$  assumes that there are no direct externalities of the investments. However, there is of course an indirect externality because the investment affects the probability of trade. It is this indirect externality which is the focus of Williamson (1985) and Grossman and Hart (1986).

are feasible, namely  $\hat{p}_1 = p_1$  and  $\hat{p}_0 = p_0^B = p_0^S = p_0^{BS}$ . As noted in the Introduction, the main point of this paper is to show that it is this simplification which is crucial to the underinvestment result. If the courts can verify whether the seller failed to supply or the buyer failed to take delivery, then the first best can be achieved via a contract of the following form: Independent of whether or not the buyer wishes to trade, he has to pay  $\tilde{p}_1$  if the seller supplies the good, and  $\tilde{p}_0$  if the seller does not supply. Thus, there are again only two different prices, but this time  $\tilde{p}_0 = p_0^S = p_0^{BS}$  and  $\tilde{p}_1 = p_1 = p_0^B$ . This contract does not condition on  $q$  but on whether or not the seller supplied the good. Note, that given  $(\tilde{p}_0, \tilde{p}_1)$  and  $v(\cdot) \geq 0$  it is a weakly dominant strategy for the buyer always to trade at date 2, since the price to be paid is independent of the buyer's decision. Thus, it is *effectively* the seller who decides whether trade takes place. Such a contract can be interpreted as an "option contract", saying that the seller has the option to supply at price  $\hat{p}_1$ , but it is still left to the buyer to decide whether he wants to accept delivery of the good.

Given the fact that the buyer will never refuse to accept the good, we could have assumed right from the start that the buyer has no choice but to accept the decision of the seller. With this interpretation our contract may be viewed as an "ownership contract" as in Grossman and Hart (1986), giving the seller the "residual right to control" whether trade should take place, whereas the model of Hart and Moore corresponds to the case of separate ownership in which trade only takes place if both parties agree to it<sup>8</sup>. We will come back to this interpretation in Section 4.

The initial contract could also condition on messages exchanged between the parties or any other kind of revelation mechanism. Hart and Moore show that if the contract can be renegotiated, then (given the above assumption that only  $q$  can be verified) it is impossible to achieve the first best no matter how sophisticated the initial contract is. Since we want to show that a simple option contract does implement the first best, we do

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<sup>8</sup>This analogy is not quite precise, since there is no explicit mentioning of assets that might be owned either by the seller or by the buyer in our story. The existence of such assets however is the basis of the definition of ownership rights given in Grossman and Hart (1986). To justify our use of the term "ownership contract", it is necessary to re-interpret our model as one in which the trade decision corresponds to the decision of whether or not assets that are specific to the buyer, resp. the seller, should be used in some production process.

not consider these revelation games here.

However, renegotiation will be very important. We follow Hart and Moore in assuming that after date 1 the parties are free to send new contract offers to each other. If a new contract is signed by both parties and produced to the courts, then it replaces the old one. If there are conflicting new contracts, all signed by both parties, then the old contract remains valid. These offers are transmitted by a completely reliable mail service which takes one night to transmit the mail and delivers it once a day. There is a finite number of days between date 0 and 1.<sup>9</sup>

The following Proposition 1', which summarizes the outcome of the renegotiation game after an option contract  $(\tilde{p}_0, \tilde{p}_1)$  as defined above has been signed, is the counterpart to Proposition 1 of Hart and Moore.

**Proposition 1'** *Let  $(\tilde{p}_0, \tilde{p}_1)$  be the initial option contract signed at date 0. Then the traded quantity ( $q$ ) and the payment of the buyer to the seller ( $p$ ) are uniquely determined in all subgame perfect equilibria and given by:*

- (i) *if  $\tilde{p}_1 - \tilde{p}_0 < v < c$ , then  $q = 0$  and  $p = \tilde{p}_0$ ,*
- (ii) *if  $v \leq \tilde{p}_1 - \tilde{p}_0 < c$ , then  $q = 0$  and  $p = \tilde{p}_0$ ,*
- (iii) *if  $v < c \leq \tilde{p}_1 - \tilde{p}_0$ , then  $q = 0$  and  $p = \tilde{p}_1 - c$ ,*
- (iv) *if  $v \geq \tilde{p}_1 - \tilde{p}_0 \geq c$ , then  $q = 1$  and  $p = \tilde{p}_1$ ,*
- (v) *if  $v \geq c > \tilde{p}_1 - \tilde{p}_0$ , then  $q = 1$  and  $p = \tilde{p}_0 + c$ ,*
- (vi) *if  $\tilde{p}_1 - \tilde{p}_0 > v \geq c$ , then  $q = 1$  and  $p = \tilde{p}_1$ .*

A formal proof of Proposition 1' is easily constructed along the lines of the proof of Proposition 1 in Hart and Moore. So, let us just outline the basic intuition for this result here. Under an option contract the seller effectively decides whether or not trade takes place: In cases (i) and (ii) trade is inefficient and the seller does not want to trade because  $\tilde{p}_1 - \tilde{p}_0 < c$ . In cases (iv) and (vi) the seller wants to supply (because  $\tilde{p}_1 - \tilde{p}_0 > c$ ) and trade is also efficient. In all these cases there is no scope for renegotiation, since

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<sup>9</sup>See Hart and Moore (1988) for a discussion of this specification.



an efficient trade decision will already result from the initial contract and each player can guarantee himself at least his share of the total surplus specified through the initial contract, by simply refusing to sign any other agreement.

Now consider case (iii). Trade would be inefficient, but given the initial contract the seller wants to trade. He can also enforce trade which guarantees him at least  $\tilde{p}_1 - c > \tilde{p}_0$  and holds the buyer down to  $v - \tilde{p}_1 < -\tilde{p}_0$ . Consider the following strategy of the buyer. He sends a renegotiation offer to the seller on the very last day before date 2, raising the no-trade payment to  $p = \tilde{p}_1 - c + \epsilon$ . Given this offer, the seller can either supply the good and refer to the initial contract to get a payoff of  $\tilde{p}_1 - c$ , or he can give up production and take the new contract to enforce a payment of  $\tilde{p}_1 - c + \epsilon$ . If  $\epsilon > 0$  the seller prefers the new contract. Thus, in equilibrium the buyer will offer a no-trade payment of  $p = \tilde{p}_1 - c$ , the seller will not trade, and payoffs (net of investment costs) are  $u^S = \tilde{p}_1 - c$  and  $u^B = c - \tilde{p}_1 > v - \tilde{p}_1$ , respectively.

Finally, in case (v) trade would be efficient but the seller does not want to trade because refusing to supply gives him  $\tilde{p}_0 > \tilde{p}_1 - c$ . Again the buyer could wait until the last day before date 2 and then offer to raise the trade payment to  $p = \tilde{p}_0 + c$ . In equilibrium the seller will accept this offer and trade, so the payoffs are  $u^S = \tilde{p}_0$  and  $u^B = v - \tilde{p}_0 - c$ , respectively.

Comparing Proposition 1' with Proposition 1 in Hart and Moore two differences can be observed:

1. In case (iii), if  $v < c < \tilde{p}_1 - \tilde{p}_0$ , a Hart-Moore contract is not renegotiated and yields  $q = 0$  and  $p = \tilde{p}_0$ . Renegotiation is not necessary because the buyer, who doesn't want to trade can prevent inefficient trade unilaterally, guaranteeing himself  $u^B = -\tilde{p}_0 \geq c - \tilde{p}_1$ .
2. In case (vi), if  $\tilde{p}_1 - \tilde{p}_0 > v > c$ , a Hart-Moore contract is renegotiated while an option contract is not. In this case trade is efficient, but without renegotiation the buyer would veto trade. Thus, in equilibrium the seller will offer to lower the trade payment to  $p = v + \tilde{p}_0$  and payoffs are  $u^S = v + \tilde{p}_0 - c$  and  $u^B = -\tilde{p}_0$ .

What investment incentives are given by the option contract? Using Proposition 1' the date 2 payoffs of the buyer and the seller are given by:

$$u^B = -\beta + \begin{cases} -\tilde{p}_0 & \text{in (i), (ii)} \\ c - \tilde{p}_1 & \text{in (iii)} \\ v - \tilde{p}_1 & \text{in (iv), (vi)} \\ v - \tilde{p}_0 - c & \text{in (v)} \end{cases} \quad \text{and} \quad u^S = -\sigma + \begin{cases} \tilde{p}_0 & \text{in (i), (ii), (v)} \\ \tilde{p}_1 - c & \text{in (iii), (iv), (vi)} \end{cases}$$

whereas social welfare is

$$w = -\beta - \sigma + \begin{cases} 0 & \text{in (i) - (iii)} \\ v - c & \text{in (iv) - (vi)} \end{cases}.$$

Using these expressions it is possible to give a heuristic argument to explain how the first best can be achieved with an option contract. This argument glosses over a number of technical points, but is nevertheless instructive and captures the main intuition of the formal result to be presented in the following section. Briefly put, the idea is the following:

First, note that the private marginal returns of the buyer's investment coincide with the social marginal returns of his investment in all possible cases (for sake of exposition it is assumed that all the following derivatives are well-defined):

$$\frac{\partial u^B}{\partial \beta} = -1 + \begin{cases} 0 & \text{in (i)-(iii)} \\ \frac{\partial v}{\partial \beta} & \text{in (iv)-(vi)} \end{cases} = \frac{\partial w}{\partial \beta}. \quad (4)$$

Why? The seller has the right to decide whether or not trade takes place. Thus if it is necessary to renegotiate, then the renegotiated price will follow his costs. So the price is independent of the buyer's valuation and therefore independent of his investment. Hence, the buyer's payoff equals social welfare minus a constant (which depends only on the seller's cost and the price, but not on the buyer's investment). This gives him just the right incentives to invest no matter how  $\tilde{p}_0$  and  $\tilde{p}_1$  are chosen.

This is not true for the seller. Private and social marginal returns of his investment coincide only in cases (i), (ii), (iv), and (vi). In case (iii) the social return is 0 but the private return is  $-\frac{\partial c}{\partial \sigma} > 0$ , so there is an incentive to overinvest. On the other hand, in case (v) the social return is  $-\frac{\partial c}{\partial \sigma} > 0$  while the private return is 0, which gives an incentive to underinvest. To give the seller the right incentives to invest, it is thus necessary to

show, that by choosing  $\tilde{p}_1 - \tilde{p}_0$  appropriately we can fix the respective probabilities of cases (iii) and (v) such that *on average* the seller has just the right incentive to invest.

The idea why this should be possible is simple: Since  $v(\cdot)$  and  $c(\cdot)$  are bounded there exist real numbers  $\underline{k}$ ,  $\bar{k}$ , such that if  $\tilde{p}_1 - \tilde{p}_0 \leq \underline{k}$ , then  $\text{Prob}\{(i) \text{ or } (v)\}=1$ , and if  $\tilde{p}_1 - \tilde{p}_0 \geq \bar{k}$ , then  $\text{Prob}\{(iii) \text{ or } (vi)\}=1$ . In cases (i) and (v) the private marginal return of the seller's investment is 0, so if  $\tilde{p}_1 - \tilde{p}_0 = \underline{k}$  he will not invest at all. In cases (iii) and (vi) the private marginal return of his investment is  $-\frac{\partial c}{\partial \sigma}$  which is equal to the social marginal return if trade is efficient. However, the probability that trade is efficient is smaller or equal to 1, so if  $\tilde{p}_1 - \tilde{p}_0 = \bar{k}$  the seller is induced to overinvest. Hence, by varying the "option price" it is possible to induce the seller either to invest too much or to invest too little. Consequently, one may suspect that there exists a  $k^* \in [\underline{k}, \bar{k}]$  which gives the seller the desired incentive to choose his first best investment level.

Let us briefly explain the difference to Hart and Moore. Under a Hart-Moore contract private and social marginal returns of investments coincide in cases (i) - (iv). In case (v) the buyer's incentives are fine but the seller's marginal return is 0, so he will underinvest. In case (vi) it is just the other way round. The seller has the right incentives, but the buyer's investment does not pay off. Hart and Moore's underinvestment result stems from the fact that in general it is impossible to choose  $\tilde{p}_1 - \tilde{p}_0$  such that the probabilities of (v) and (vi) both vanish at the same time.

### 3 Options, Efficient Investments, and the Role of Reneogtiation

We are now going to develop a formal argument which makes the intuition given above precise and which highlights some further important properties of option contracts.

Consider the investment decision of the seller if an option contract  $(\tilde{p}_0, \tilde{p}_1)$  has been signed at date 0. By Proposition 1' his expected payoff is given by

$$U^S(\sigma, \tilde{p}_0, k) = -\sigma + \tilde{p}_0 + \int_0^1 [k - c(\omega, \sigma)]^+ d\omega, \quad (5)$$

where  $k = \tilde{p}_1 - \tilde{p}_0$  is the "option price". Clearly,  $U^S(\cdot)$  is well-defined and continuous in

all arguments. Given the option price the seller's problem is to find an investment level that maximizes  $U^S(\cdot)$ . The set of maximizers is always non-empty (by boundedness of the cost function) and since  $\tilde{p}_0$  enters the problem only as an additive constant will not depend on  $\tilde{p}_0$ . In general there is no presumption that there will be a unique maximizer for every possible  $k$ . We will discuss this issue in more detail below, for the time being we simply stipulate uniqueness as an additional assumption.

**Assumption 1** *There is a unique optimal investment level,  $\tilde{\sigma}(k)$ , for all  $k$ .*

Uniqueness of the seller's investment choice for all possible option prices is sufficient to ensure that the first best investment levels are implementable. More precisely our main result is the following.

**Proposition 2** *If Assumption 1 holds then there exists an option contract  $(\tilde{p}_0, \tilde{p}_1)$  which implements efficient investments  $\beta^*, \sigma^*$ . Furthermore, any division of the ex-ante surplus can be achieved via  $(\tilde{p}_0, \tilde{p}_1)$ .*

Proof: Since costs are always positive we have that for all  $\sigma, \tilde{p}_0$ ,  $U^S(\sigma, \tilde{p}_0, 0) = -\sigma + \tilde{p}_0$  which implies  $\tilde{\sigma}(0) = 0$ . Next, since costs are uniformly bounded there exists  $\bar{k}$  such that  $\forall k \geq \bar{k}$ :

$$U^S(\sigma, \tilde{p}_0, k) = -\sigma + \tilde{p}_0 + k - \int_0^1 c(\omega, \sigma) d\omega . \quad (6)$$

Let  $\bar{\sigma} = \tilde{\sigma}(\bar{k})$ . By the maximum theorem we know that Assumption 1 implies that  $\tilde{\sigma}(k)$  is continuous in  $k$ . Thus, it follows from the intermediate value theorem that any  $\sigma \in [0, \bar{\sigma}]$  can be implemented by choosing  $k \in [0, \bar{k}]$  appropriately. It remains to show that  $\sigma^* \in [0, \bar{\sigma}]$ . To see this, take any pair of first best investment levels  $(\beta^*, \sigma^*)$  and suppose  $\sigma^* > \bar{\sigma}$ . Now consider the functions

$$f(\sigma) = -\beta^* - \sigma + \int_0^1 [v(\omega, \beta^*) - c(\omega, \sigma)]^+ d\omega \quad (7)$$

and

$$h(\sigma) = -\beta^* - \sigma + \int_0^1 [v(\omega, \beta^*) - c(\omega, \sigma)] d\omega . \quad (8)$$



Note that  $\bar{\sigma} = \operatorname{argmax} h(\sigma)$  and that  $\sigma^* \in \operatorname{argmax} f(\sigma)$ . Thus we have  $h(\bar{\sigma}) > h(\sigma^*)$  and  $f(\sigma^*) \geq f(\bar{\sigma})$ . Combining these two inequalities yields:

$$f(\bar{\sigma}) - h(\bar{\sigma}) < f(\sigma^*) - h(\sigma^*) . \quad (9)$$

However, since production costs are non-increasing in  $\sigma$  for all  $\omega$  it follows that

$$f(\sigma) - h(\sigma) = \int_0^1 [c(\omega, \sigma) - v(\omega, \beta)]^+ d\omega \quad (10)$$

is non-increasing in  $\sigma$ . But this is a contradiction to (9) and  $\sigma^* > \bar{\sigma}$ .

Given the option price  $k^*$  that induces the seller to choose  $\sigma^*$  it follows immediately from the argument presented in Section 2 that the buyer's optimal choice is  $\beta^*$ . Finally, to share the expected surplus as desired,  $(\tilde{p}_0, \tilde{p}_1)$  can be chosen arbitrarily as long as  $\tilde{p}_1 - \tilde{p}_0 = k^*$ .

*Q.E.D.*

Assumption 1 is used in the proof of Proposition 2 only to guarantee that the seller's optimal investment level is continuous in the option price. Even without this requirement it is always possible to induce the seller to invest too little or too much by varying the option price. However, if Assumption 1 fails, there may be a discontinuity in  $\tilde{\sigma}(k)$ , i.e. it may happen that there exists no  $k$  such that  $\tilde{\sigma}(k) = \sigma^*$ . In this case the initial contract would have to use a randomization scheme (e.g. between  $\underline{k}$  and  $\bar{k}$ ) to induce efficient investment of the seller. Thus, Assumption 1 is a continuity requirement needed to ensure that by varying the option price it is possible to "finetune" the investment incentives of the seller without using randomization devices.

We have refrained from stating explicit assumptions which ensure that Assumption 1 holds. The reason for proceeding this way is simply that it is difficult to state meaningful, general assumptions which guarantee this property. To see the difficulty involved, note that there may be a non-concavity in the seller's utility function due to the following problem. Suppose that production costs are always strictly positive and consider an option price that is sufficiently low to imply that the seller never receives a return if he chooses  $\sigma$  close to zero. Then  $\sigma = 0$  is a local maximum of the seller's utility function. Let us suppose it is also a global maximum. Now, it may happen that for sufficiently large investments the option may become valuable. If this is the case the seller's utility is non-concave in  $\sigma$ . To rule out the possibility that there is a second global maximum at some

strictly positive investment level, it would be necessary to state conditions which ensure that the set of states of the world in which the seller receives a strictly positive payoff from the option is not increasing “too fast” in  $\sigma$ . Formulating such a requirement in any degree of generality, would have obscured the main point we wished to make, viz. that if the seller’s maximization problem is well-behaved a simple option contract is sufficient to achieve efficient investments.

However, the following example illustrates that there are natural cases in which the seller’s utility function is strictly concave in  $\sigma$ , which clearly implies that Assumption 1 holds. Suppose that the seller’s cost function is given by

$$c(\omega, \sigma) = \tilde{c}(\sigma) \cdot \omega, \quad (11)$$

where  $\tilde{c}(\sigma)$  is twice differentiable, and satisfies  $\tilde{c}(\cdot) > 0$ ,  $\tilde{c}'(\cdot) < 0$ ,  $\tilde{c}''(\cdot) > 0$ . Thus, the seller’s expected utility is given by

$$\begin{aligned} U^S &= -\sigma + \tilde{p}_0 + \int_0^{\min[1, \frac{k}{\tilde{c}(\sigma)}]} [k - \tilde{c}(\sigma) \cdot \omega] d\omega \\ &= -\sigma + \tilde{p}_0 + \min[1, \frac{k}{\tilde{c}(\sigma)}] \cdot k - \frac{1}{2} \min[1, \frac{k}{\tilde{c}(\sigma)}]^2 \cdot \tilde{c}(\sigma). \end{aligned} \quad (12)$$

Taking the derivative with respect to  $\sigma$  one obtains

$$\frac{\partial U^S}{\partial \sigma} = -1 - \frac{1}{2} \min[1, \frac{k}{\tilde{c}(\sigma)}]^2 \cdot \tilde{c}'(\sigma). \quad (13)$$

It is easy to see from this expression that for sufficiently small  $k$  the optimal investment is  $\tilde{\sigma}(k) = 0$  and for  $k \geq \tilde{c}(0)$  the derivative is independent of  $k$  and simply given by

$$\frac{\partial U^S}{\partial \sigma} = -1 - \frac{1}{2} \tilde{c}'(\sigma). \quad (14)$$

To obtain an interior solution let us assume that  $\tilde{c}'(0) < -2$  and  $\lim_{\sigma \rightarrow \infty} \tilde{c}'(\sigma) > -2$ . Then it follows from strict convexity of  $\tilde{c}(\cdot)$  that the optimal investment level is given by the unique solution to the FOC  $\tilde{c}'(\tilde{\sigma}(k)) = 2$  for all such  $k \geq \tilde{c}(0)$ . Consequently, as in Proposition 1, the optimal investment level will be independent of  $k$  if  $k$  is sufficiently high. Finally, since we allowed production cost to be identical equal to zero for  $\omega = 0$ , it is possible to state an assumption that guarantees that the seller’s problem is strictly

concave in  $\sigma$  for all  $k \geq 0$ . This condition is

$$\tilde{c}''(\cdot) > 2 \frac{[\tilde{c}'(\cdot)]^2}{\tilde{c}(\cdot)}. \quad (15)$$

If inequality (15) holds it is clearly the case that  $\tilde{\sigma}(k)$  is uniquely defined for all  $k$  and characterized by the first order condition for utility maximization whenever  $\tilde{\sigma}(k) > 0$ . Thus, by Proposition 1 it is possible to implement first best investments without any further assumption on the buyer's valuation function.

Returning now to the general setting, let us finally point out an interesting observation concerning the role of renegotiation. Renegotiation seems to be very important for the option contract to work. First of all, it is necessary to achieve efficient trade. Secondly, renegotiation of an option contract yields a price which is independent of the buyer's valuation, thus giving him the right incentives to invest. Finally, under our continuity assumption, by choosing  $\tilde{p}_1 - \tilde{p}_0$  appropriately it is possible to finetune the respective probabilities such that the renegotiated price gives the seller the correct incentive to invest. However, it is not clear whether renegotiations occur that frequently in reality<sup>10</sup>. Therefore, it may be worthwhile to note that there are interesting circumstances in which it is never necessary to renegotiate an option contract.

Suppose that the costs of the seller are increasing in  $\omega$  and assume that  $w(\omega, \cdot) = v(\omega, \cdot) - c(\omega, \cdot)$  is strictly decreasing in  $\omega$ . This means that the seller's utility and social welfare are always moving in the same direction: If the state of the world gets better from the seller's point of view, it is also getting better from a social perspective, even if it may become worse for the buyer. This case is illustrated in Figure 1. Note that there must be a unique  $\bar{\omega} \in [0, 1]$ , such that  $v(\omega, \cdot) \geq c(\omega, \cdot)$  if and only if  $\omega \leq \bar{\omega}$ . Suppose we fix the option price  $k^*$  such that  $k^* = c(\bar{\omega}(\beta^*, \sigma^*), \sigma^*)$ , i.e.  $k^*$  equals the costs of the seller in the marginal state  $\bar{\omega}$  given that both parties invested efficiently. The seller's investment yields private returns if and only if  $c(\omega, \cdot) \leq k^*$ , or, equivalently, iff  $\omega \leq \bar{\omega}(\beta^*, \sigma^*)$ . But this is the same set of states of the world in which the seller's investment yields a social return. Thus, the private and social marginal returns of his investment coincide. Furthermore, if  $k = k^*$ , cases (iii) and (v) of Proposition 1' (the only cases in which there is renegotiation)

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<sup>10</sup>In particular, if the contract is interpreted as an ownership contract we would not expect that the owner always has to renegotiate with his subordinate before he can take a decision.



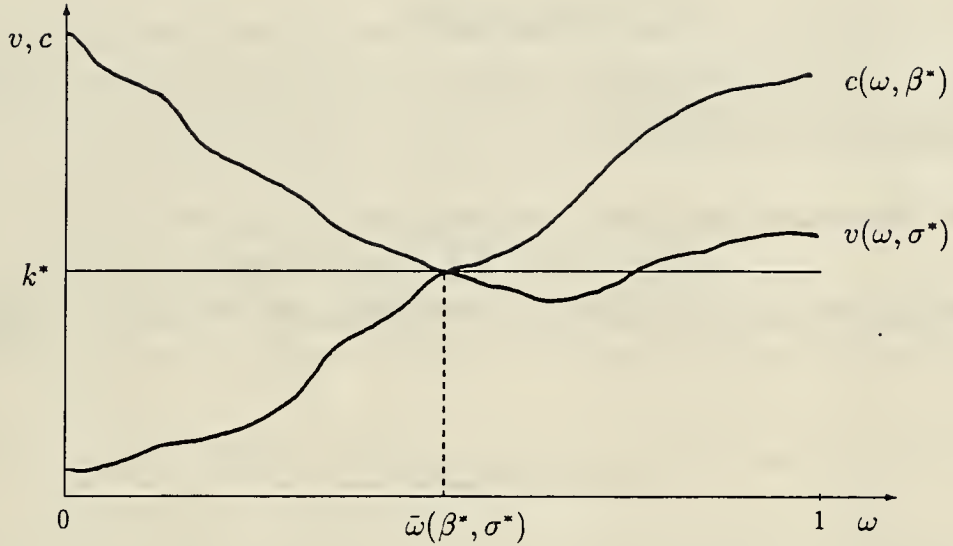


Figure 1: *Efficient Investments without Renegotiation*

must have zero probability, since  $v(\omega, \beta^*) \geq c(\omega, \sigma^*)$  iff  $c(\omega, \sigma^*) \leq k^*$ . Thus, there is no renegotiation in equilibrium. This is stated formally in the following proposition:

**Proposition 3** *Suppose that  $v(\omega, \beta)$  and  $c(\omega, \sigma)$  are continuously differentiable,  $c(\omega, \sigma)$  is increasing in  $\omega$  and  $w(\omega, \beta, \sigma) = v(\omega, \beta) - c(\omega, \sigma)$  is strictly decreasing in  $\omega$ . Furthermore, assume that the seller's maximization problem is strictly concave. Then there exists an option contract  $(\tilde{p}_1, \tilde{p}_0)$  that implements first best levels of investment and trade without renegotiation.<sup>11</sup>*

Proof: Without loss of generality we only consider the case in which  $\sigma^* \in (0, \bar{\sigma})$ . If  $w(\omega, \beta, \sigma)$  is strictly decreasing in  $\omega$  then there exists a  $\bar{\omega}(\beta, \sigma) \in [0, 1]$  such that  $v(\omega, \beta) - c(\omega, \sigma) \geq 0$  if and only if  $\omega \leq \bar{\omega}(\beta, \sigma)$ . The FOC for the social welfare maximizing  $\sigma^*$  is given by

$$-\int_0^{\bar{\omega}(\beta^*, \sigma^*)} \frac{\partial c(\omega, \sigma^*)}{\partial \sigma} = 1. \quad (16)$$

<sup>11</sup>It is easy to check that under the stated conditions a Hart-Moore contract with  $\hat{p}_1 - \hat{p}_0 = k^*$  will also implemented the first best without renegotiation if  $v(\omega, \beta^*)$  is *decreasing* in  $\omega$ . Without such an additional assumption a Hart-Moore contract will not achieve that goal; in particular in the situation shown in Figure 1 the Hart-Moore contract would be renegotiated for sufficiently high  $\omega$ .

Chose  $k^* = c(\bar{\omega}(\beta^*, \sigma^*), \sigma^*)$ . Then the FOC for the seller's maximization problem coincides with (16). Given that the seller's problem is strictly concave  $k^*$  thus implements first best investments. Furthermore,

$$v(\omega, \beta^*) - c(\omega, \sigma^*) \geq 0 \Leftrightarrow \omega \leq \bar{\omega}(\beta^*, \sigma^*) \Leftrightarrow c(\omega, \sigma^*) \leq k^* . \quad (17)$$

Thus, cases (iii) and (v) of Proposition 1' cannot occur, so there is no renegotiation.

*Q.E.D.*

## 4 Conclusions

We have shown that in the model of Hart and Moore first best investments can be implemented if the courts can observe whether or not the seller supplied the good. Thus, unverifiability of investments and of the state of the world alone does not yield an underinvestment effect. However, it does have an impact on the form of the contractual arrangement. The first best can be achieved if one of the parties, here the seller, gets control, i.e. if he can decide unilaterally whether or not trade takes place. On the other hand, Hart and Moore have demonstrated that the first best is not feasible if both parties have control. Thus our results suggest that if relationship specific investments are non-verifiable, then vertical integration may perform strictly better than separate ownership.

This is clearly a special case of the more general result in Grossman and Hart (1986) who analyzed the impact of different ownership structures on efficiency. However, they had to impose an "incomplete contracts assumption", saying that it is impossible to contract on the level of trade ( $q$ ) at date 0, but that it is possible to contract on it at date 1, after the state of the world has materialized. Although Grossman and Hart give convincing arguments why this assumption is sensible (see in particular their footnote 14), one would wish to replace it by a more basic assumption. For a special example, which has received considerable attention in both the economic and the legal literature) Hart and Moore's and our results show, that the superiority of one ownership structure to another can be established by just referring to unverifiability, without assuming any other source of contract incompleteness.

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