**Vibroacoustic launch analysis and alleviation of lightweight, active mirrors**

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Vibroacoustic launch analysis and alleviation of lightweight, active mirrors

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Abstract. Lightweight, active, silicon carbide mirrors can increase the capability of space-based optical systems. However, launch survival is a serious concern for such systems, with the vibrations and acoustics from launch threatening to damage the optics. Therefore, a dynamic, state-space launch model has been developed with which one can quickly analyze the survival probability of many designs and also directly analyze launch load alleviation techniques. This paper discusses the launch model from which launch stress and survival probability are obtained, as well as launch load alleviation techniques that may increase the probability of launch survival. Three launch load alleviation techniques are presented and analyzed: whole spacecraft isolation, passive shunt circuits using the existing embedded actuators, and active damping using the existing actuators. All of the techniques reduce the launch stress, but at the expense of mass and complexity. The launch model allows for early identification of lightweight, active mirror designs which will survive launch, and the analysis techniques expand the feasible design space by decreasing the launch stress and increasing the probability of launch survival.

1 Introduction

Better performing space-based optical systems can be accomplished through larger primary apertures. However, large apertures are difficult due to launch mass and size constraints, manufacturing difficulties, and lower flexible mode frequencies. One promising method of achieving large primary apertures is through a segmented aperture composed of lightweight, active mirror segments. These segments have a rib-stiffened silicon carbide substrate and many embedded surface parallel piezoelectric actuators to control the mirror surface figure. Using this technology, the achievable aperture size, and hence the potential imaging resolution, is very promising.

However, there are a number of challenges in the design of lightweight active mirrors. Launch survival is of particular concern, as the vibrations and acoustics from launch threaten to break the mirror, especially as the areal density decreases. Therefore, a dynamic, state-space model of the mirror in the vibroacoustic launch environment is developed. This model allows for the analysis of the launch stress and survival probability of the mirror segments. Additionally, it is constructed such that launch load alleviation techniques, including those making use of the existing embedded actuators to add damping to the mirror, can be analyzed.

The remainder of this paper will discuss the state-space mirror model and modeling methodology. Then, the launch load alleviation techniques, including both implementation and resulting performance, are discussed. Finally, a trade space of mirror designs is shown to illustrate which designs will survive launch, as well as demonstrate how the launch load alleviation can allow more designs to survive launch and expand the feasible design space to encompass designs with superior on-orbit performance.

2 Modeling Approach

A key aspect of the work is the use of an integrated mirror model. Parametric, integrated modeling has many benefits, including an ability to create and analyze many different designs over multiple disciplines, as described in Uebelhart and others. By keeping all relevant design parameters (geometry, structural, control, etc.) in a single input file and auto-generating models and analyses, the design space can be explored and optimization is possible. The integrated mirror model that is considered here combines finite element models, state-space models, control systems, and disturbance models to calculate performance outputs given a set of parametric design inputs.

This modeling approach is particularly useful for the mirror launch analysis, for which there is not a lot of design heritage (the best designs are unknown) and the mirrors are near launch survival limits. Therefore, the ability to analyze and understand the implications of various design choices on both launch survival and on-orbit performance during early design, afforded by the use of the integrated model, is valuable.

Furthermore, the mirrors that are most conducive to on-orbit correctability are the least likely to survive launch. Therefore, the dynamic, state-space model employed here enables one to add and analyze launch load alleviation techniques to determine their effects on the mirror, which would be difficult to determine with more traditional launch analyses. The method of modeling allows one to examine many different mirror designs, and include novel...
alleviation efforts, ultimately resulting in a better understanding of the mirror design early in the design process, and leading to better performing mirrors and fewer costly redesigns.

2.1 Mirror Model

The mirror model is created using finite element modeling (FEM) and state-space techniques. The structural model is made with FEM, and can be seen in Fig. 1. The model is of a single mirror segment that is rib-stiffened with silicon carbide material properties. There are surface-parallel piezoelectric actuators embedded in the ribs, allowing for actuation of the mirror, which are shown as bars in Fig. 1. The actuators expand or contract with an applied voltage, producing a moment on and therefore changing the shape of the mirror surface. The geometric properties of the mirror, such as curvature, areal density, number of ribs, rib aspect ratio, and percent of mass in the face sheet, are all parameters that can be varied in the model. The grid points, elements, and material properties are all defined automatically based on the input parameters within MATLAB, and the resulting normal modes solution (frequencies and mode shapes) is solved using Nastran. These results are brought back into MATLAB where they are manipulated into a state space model.

The frequencies (Ω) are used to define the dynamics of the system, or the A matrix in the state-space model. The mode shapes (Φ) are used in combination with the desired input and output grid points and types to determine the full state-space model, which is shown in Eq. (1).

\[
\begin{bmatrix}
\dot{q} \\
\dot{\dot{q}}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-\Omega^2 & -2\zeta\Omega
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} + B_u w + B_u u
\]

\[y = C_y \begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} + v\]

\[z = C_z \begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} + D_z u.\]  

In Eq. (1), q is the modal degrees of freedom, Ω is the modal frequency, ζ is the prescribed modal damping, w is the vibroacoustic disturbance input, u is the vector of control inputs for the piezoelectric actuators, y is the control sensor (when applicable for alleviation), and z is the stress in the mirror. The FEM defines the dynamics of the system (Ω, Φ), leaving the inputs and outputs to be defined through the B, C, and D matrices.

The stress performance outputs are defined using the Cz matrix, which contains interpolation functions taken from finite element theory. These interpolation functions transform nodal displacements, which are attainable from the FEM analysis, to elemental stresses. More details of the entire model can be found in Cohan and Miller.

2.2 Vibroacoustic Disturbances

The primary disturbance sources of concern for the mirrors during launch are random vibrations and acoustics. Acous-
tics are particularly problematic given the large surface area and low mass of the mirror. Both types of disturbances are included in the mirror model.

Random vibrations enter the spacecraft at the interface between the spacecraft and the launch vehicle. Since there is no spacecraft in this model, the vibrations enter through the rigid back structure on the mirror segment. The vibrations are described in terms of acceleration power spectral density (PSD) functions. To apply the acceleration power spectral density to the model, the “big M” method is used. In this method, a large concentrated mass is placed on a base structure of the finite element model. The acceleration spectral density is scaled by the square of the magnitude of the large mass, and the resulting scaled spectrum is applied as a force. Through Newton’s second law ($F = ma$), the scaled force PSD produces the desired acceleration in the mirror system.

The vibrations are applied in all three directions; one axis is considered translational, while the other two are considered transverse. Though the model allows for any axis to be the translational axis, the analysis herein assumes a stowed configuration in which the $x$-axis (surface parallel) is the translational axis. Both the translational and transverse acceleration spectral densities can be seen in Fig. 2.

The second disturbance source is acoustic loading within the launch vehicle fairing. The acoustic pressure is analyzed using the patch method. In this method, a pressure force is applied to the surface of the mirror in patches. The pressure is correlated over each patch and uncorrelated to other patches. The acoustic input spectrum is specified in terms of sound pressure levels (SPLs), which can be converted to a pressure spectral density and applied to the mirror. The pressure spectral density can be seen in Fig. 3. With this implementation, the vibroacoustic disturbance spectra can be applied to the mirror to determine the response to the harsh launch environment.

### Table 1 Parameters for baseline mirror.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment diameter (vertex to vertex)</td>
<td>1.2 m</td>
</tr>
<tr>
<td>SiC areal density</td>
<td>7.5 kg/m$^2$</td>
</tr>
<tr>
<td>Number of rib rings</td>
<td>4</td>
</tr>
<tr>
<td>Number of actuators</td>
<td>156</td>
</tr>
<tr>
<td>Face sheet mass fraction</td>
<td>0.63</td>
</tr>
<tr>
<td>Rib aspect ratio</td>
<td>25.4</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>1%</td>
</tr>
</tbody>
</table>

### Table 2 Baseline mirror peak stresses.

<table>
<thead>
<tr>
<th>Source</th>
<th>SiC Stress [MPa]</th>
<th>Actuator Stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration only</td>
<td>36</td>
<td>3</td>
</tr>
<tr>
<td>Acoustic only</td>
<td>76</td>
<td>6</td>
</tr>
<tr>
<td>Vibration and acoustic</td>
<td>83</td>
<td>12</td>
</tr>
</tbody>
</table>

### 2.3 Disturbance Analysis and Stress Outputs

Once the state-space model of the plant and the launch load power spectral densities are fully defined, a disturbance analysis can be performed to determine the stresses in the mirror when it is subjected to the prescribed disturbances. A frequency domain, steady state, dynamic disturbance analysis is performed to determine the rms stresses in the mirror. The resulting stress output is a 1-$\sigma$ value, meaning that the stress is expected to be below that value 68.2% of the time. In order to have greater confidence, 3- or 6-$\sigma$ values will be used to ensure greater than 99% launch survival confidence.

### 3 Baseline Mirror

The baseline mirror parameters can be seen in Table 1. This baseline is analyzed using the launch model and analysis described in the preceding sections, and is used to validate the model and as a reference point in trade studies.

The peak stresses in both the silicon carbide substrate and in the actuators for the baseline mirror can be seen in Table 2. The stress resulting from only vibration and only acoustic disturbances are shown in addition to the coupled case.
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Table 3 Stress limits for 3- and 6-σ confidence levels.

<table>
<thead>
<tr>
<th></th>
<th>Silicon Carbide</th>
<th>Actuators</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-σ (0.9973002)</td>
<td>100 MPa</td>
<td>23 MPa</td>
</tr>
<tr>
<td>6-σ (0.999999998)</td>
<td>50 MPa</td>
<td>11.5 MPa</td>
</tr>
</tbody>
</table>

with both disturbance sources. The acoustically induced stresses are significantly higher than the vibration induced stresses because it is a lightweight, large surface area system. Additionally, the stress distribution over the mirror due to both the vibration and acoustic disturbances can be seen in Fig. 4.

The stress outputs from this baseline model are validated to the extent possible. They are compared to available data from two similar systems, and match within 10% in all cases. The stresses are also compared to results from more traditional launch analyses, which match closely. The parametric nature of the model allows it to be validated against multiple sets of test data since the model can easily be made to represent the test setup.

As mentioned above, the model outputs a 1-σ value for stress (68.2% survival probability). In order to ensure either 3- or 6-σ survival certainty, the stress limits to which the model output is compared are adjusted, and Table 3 shows those adjusted stress limits. Therefore, in order to ensure the given probability level, the model output must be less than the desired limit level, so 6-σ survival certainty in the silicon carbide requires the model output to be less than 50 MPa.

Comparing the baseline mirror to the limit levels in Table 3 shows that the mirror will survive with 3-σ certainty, but not with 6-σ certainty. Also, though this particular mirror design may meet launch survival requirements, it is very near the limits, and many other mirror designs may not meet the survival limits. Therefore, launch load alleviation is introduced in order to increase the probability of launch survival.

4 Launch Load Alleviation

Though it is certainly possible to design mirrors that survive launch, the stresses due to launch loads are near the survival limits. Additionally, the mirrors that survive launch may not be optimally designed for on-orbit performance. This motivates using launch load alleviation to increase the number of designs that survive launch. Launch load alleviation techniques reduce the stresses in mirrors during launch, increasing the probability of launch survival, and therefore the feasible design space.

Three launch load alleviation techniques are considered: isolation, passive shunt circuits, and active damping. Isolation uses commercial, whole-spacecraft isolation technology to separate the fragile spacecraft from the noisy launch vehicle, effectively reducing the magnitude of the vibration seen by the mirror. The other two techniques use the existing embedded actuators in the mirror to add damping to the structure. Shunt circuits provide damping through a passive circuit, while active damping involves a full active control.

Fig. 4 Stress distribution in the baseline mirror.
The following sections elaborate on the implementation and effectiveness of each technique.

4.1 Isolation

The first launch load alleviation option is isolation. Whole spacecraft isolation has the potential to reduce the vibratory loads seen by the spacecraft, thus increasing the survivability of the system. CSA Engineering\textsuperscript{15} has developed a whole spacecraft isolation system called SoftRide, which is placed between the spacecraft and the launch vehicle to provide vibration and shock isolation. Details of the SoftRide isolation system can be found in Refs. 16–19.

A model of the isolation resulting from a system such as SoftRide is created in the mirror model. The isolator in the mirror model is based on a low-pass filter; the isolator is not modeled physically, but rather its effect on the system is modeled. The isolator filter has variable damping and corner frequencies and levels out at higher frequencies to represent the limitations of physical isolators. The baseline isolator parameters can be seen in Table 4. The corner frequency is calculated analytically, following the derivations in Hagood.\textsuperscript{20} The equation for the resistive shunt is:

\[
V + R\hat{Q} = 0,
\]

where \( R \) is the resistance. The optimal resistance can be calculated analytically, following the derivations in Hagood.\textsuperscript{20} The tuned resonant shunt circuit (RL-shunt) uses a resistor and an inductor in series to dissipate energy at a specific frequency. Additionally, the tuned shunt can be used with multiple modes, tuning different piezos to different modal frequencies.

4.2.1 Resistive shunt

The resistive shunt simply uses a resistor to dissipate energy. The equation for the resistive shunt is:

\[
\rho = \frac{1}{2}k_{33}^2,
\]

where \(\rho\) is the electrical damping ratio, \( k_{33} \) is the natural frequency of the system, \(\eta\) is the non-dimensional frequency corresponding to the maximum loss factor of the shunted piezo, \( k_{33} \) is the coupling coefficient, and \( C_f \) is the inherent capacitance at constant strain. The square of the coupling coefficient represents the percentage of mechanical strain energy that can be converted to electrical energy and vice versa. It is defined as:

\[
k_{33} = \frac{\epsilon}{\epsilon_0 T},
\]

where \(k_{33}\) is the piezoelectric constant, \(\epsilon\) is the compliance at short circuit, and \(\epsilon_0 T\) is the dielectric constant in a free condition. Therefore, the optimal resistance value, \( R \), can be calculated from the system natural frequency and the properties of the existing piezos.

4.2.2 Tuned resonant shunt

The tuned resonant shunt circuit (RL-shunt) uses a resistor and an inductor in series to dissipate energy at a specific frequency. The equation for the RL-shunt is:

\[
L\ddot{Q} + R\dot{Q} + V = 0,
\]
where $L$ is the inductance. In order to maximize the damping at a certain resonant frequency, the electrical resonant frequency is set equal to the natural frequency modified by a tuning parameter.

$$\omega_e = \frac{1}{\sqrt{LC_p}},$$

(7a)

$$\delta = \frac{\omega_e}{\omega_n},$$

(7b)

where $\omega_e$ is the electrical resonant frequency and $\delta$ is the nondimensional tuning parameter. Similar to the resistive shunt case, the resistance can be calculated using the electrical damping ratio, $r$.

$$r = RC_p\omega_n.$$  

(8)

However, in this case, the calculation of the electrical damping ratio, $r$, and tuning parameter, $\delta$ are not as simple. There are multiple ways to approximate the optimal parameters, two of which are outlined in Hagood.\(^{20}\) Using the transfer function optimization:

$$\delta_{opt} = \sqrt{1 + K_{ij}^2},$$

(9a)

$$r_{opt} \approx \sqrt{2K_{ij}} \frac{1}{1 + K_{ij}^2},$$

(9b)

where $K_{ij}$ is the generalized electromechanical coupling coefficient, and can be calculated with the open circuit and short circuit natural frequencies:

$$K_{ij} = \frac{(\omega_n^D)^2 - (\omega_e)^2}{(\omega_n)^2},$$

(10)

where $\omega_n$ is the natural frequency at short circuit and $\omega_n^D$ is the natural frequency at open circuit. These relationships can be used to calculate the optimal values of $L$ and $R$.

One potential issue with RL-shunt circuits is that they require high inductor values when trying to damp low frequencies. Traditional inductors with high inductance values are extremely massive. However, this issue can be counteracted through the use of a Riordan gyrator circuit\(^{21}\) to achieve the desired inductance. Additionally, Fleming et al.\(^{22}\) have demonstrated synthetic impedances for creating RL shunt circuits. Therefore, the issue of high inductances has been demonstrated synthetic impedances for creating RL shunt circuits. Therefore, the issue of high inductances has been addressed and can be overcome through these means, making the resonant shunt a viable option.

### 4.2.3 Multi-mode resonant shunt

The tuned resonant shunt formulation is derived for a single piezo and a single mode, and it is assumed that all of the piezos in the mirror are tuned to that mode. However, there may be multiple modes that significantly contribute to the mirror stress. Since the mirror system has many piezos, it is possible to tune some piezos to one mode, other piezos to a second mode, and so forth in order to minimize the stress in the mirror by targeting the set of modes that contribute most to the stress. This has the benefit of targeted damping of problematic modes offered by the resonant shunt, but with the ability to address multiple significant modes.

The multimode case uses the same resonant shunt derivation technique described in the previous section. However, the optimal parameters ($\delta$, $r$, $R$, $L$) are computed for each mode of interest. Each piezo uses the corresponding $R$ and $L$ values for the mode that it is assigned to damp. The difficulty involved with this case is in determining which piezos should damp which modes. Some piezos are in locations where a certain mode is unobservable or uncontrollable, making that shunt ineffective, and it is desirable to use the piezos in a capacity that maximizes the damping. Therefore, one would like each piezo to be tuned to a mode over which that piezo is effective, and that significantly contributes to the stress.

In order to determine which piezos should be tuned to which modes, a modal influence parameter is devised. First, the number of modes to be shunted and the corresponding modal frequencies must be determined. In the case of the mirror model, the first three modal frequencies typically have the most influence on the stress outputs. Next, the degrees of freedom corresponding to the piezos’ 3-direction are extracted from the mode shape matrix, and converted to a relative displacement, resulting in a single relative modal influence value for each piezo for each mode of interest. Then, the modes are weighted as to how important they are in the performance of the system. Finally, for each piezo, the mode with the highest weighted relative modal influence value is assigned to that piezo, and the shunt circuit associated with that piezo is tuned accordingly.

#### 4.2.4 Shunt circuit results

The shunted piezos are implemented in the mirror model, and all piezos are assumed to be shunted. The optimal values for the resistive and resonant shunt circuits are calculated, and PSDs of the shunted system, focused around the first three modal frequencies, can be seen in Fig. 5. The numbers in parentheses represent the rms stress in each case (for the single element being considered).

Notice that the resistive shunt (R-Shunt) damps all modes a small amount. There are two different resonant shunts; one corresponds to all of the piezos being tuned to the first modal frequency, and the second corresponds to all of the piezos being tuned to the second modal frequency. The resonant shunt tuned to the first frequency (RL-Shunt $\omega_1$) adds significant damping to the first mode, but the response for the second and third modes is high. Similarly, the resonant shunt tuned

### Table 5 Stress results with isolation.

<table>
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<tr>
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<th>Stress without isolation</th>
<th>Stress with isolation</th>
<th>Stress reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration only</td>
<td>36 MPa</td>
<td>13 MPa</td>
<td>65%</td>
</tr>
<tr>
<td>Vibration and acoustic</td>
<td>83 MPa</td>
<td>77 MPa</td>
<td>7%</td>
</tr>
</tbody>
</table>
to the second mode (RL-Shunt $\omega_2$) damps the second mode, but not the first or third modes. The multi-mode shunt tunes each of the piezos to either the first, second, or third modal frequency. This case balances the targeted damping at each of the three modes, and outperforms all other options.

The piezoelectric shunt circuit can provide significant damping and stress reduction under launch loads using the existing piezos and a passive circuit. Both a resistive and a resonant shunting circuit are considered, and the theoretically optimal values for the resistance and inductance that provide the maximum amount of additional damping are calculated. Furthermore, a multi-mode resonant shunt methodology has been created so that multiple modes can undergo targeted damping. The shunt circuit is a viable solution for launch load alleviation of lightweight, active mirrors.

### 4.3 Active Damping

The final launch load alleviation technique considered here is active damping. Active damping involves using an active control system to increase the amount of damping and decrease the system response to the disturbance. This necessitates a sensor, control algorithm, and control scheme in addition to the embedded actuators.

A single-input, single-output (SISO) control scheme is chosen. Though a multi-input, multi-output (MIMO) controller could theoretically provide better performance, it is not used due to the complexity, large controller size, and propensity for instability in structural control systems. In the SISO formulation, each piezo actuator has its own control loop and collocated sensor. Collocated control refers to a system where the sensor and actuator are placed at the same location, are aligned, and are energetically conjugated. This results in alternating poles and zeros, and can guarantee stability in many SISO control algorithms due to its phase-bounded behavior.

As mentioned above, active damping requires a sensor, and the collocated SISO control requires a sensor at each actuator location. Active damping with piezo actuators is best accomplished through using the piezos as sensors.\(^23\) The piezoelectric strain sensor provides a true collocated sensor/actuator pair with the piezo actuator, and the associated stability guarantees.

For this problem, a control method known as Positive Position Feedback (PPF) is used.\(^{24, 25}\) PPF uses generalized displacement measurements, and works well with strain-based sensing. Furthermore, it is particularly well suited for a structure with piezo sensors/actuators,\(^26\) making it appropriate for the mirror system. In addition, spillover dynamics into high frequency modes are stabilizing, and PPF is also stable in the presence of finite actuator dynamics, which threaten to destabilize other methods. Finally, though it is not unconditionally stable, a nondynamic stability condition can be derived for the system. Therefore, PPF is a good option for the damping of the mirror.

The PPF controller is a second order filter with the same form as a single degree-of-freedom (DOF) modal equation, but much higher damping. The position term from the structure is positively fed into the filter, hence the name PPF. This is best illustrated through a single degree of freedom system:\(^{25}\)

\[
\begin{align}
\text{Structure: } & \ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = g\omega^2 x_f, \\
\text{Compensator: } & \ddot{x}_f + 2\zeta_f\omega_f\dot{x}_f + \omega_f^2 x_f = \omega_f^2 x,
\end{align}
\]

where $x$ is the structural modal coordinate, $\omega$ is the modal frequency, $\zeta$ is the modal damping, $x_f$ is the filter coordinate, $\omega_f$ is the filter frequency, $\zeta_f$ is the filter damping, and $g$ is a scalar gain. It can be shown that the system is stable for $0 < g < 1$.\(^{24}\) Fanson and Caughey\(^{25}\) extend this stability criteria to multivariable systems.

The PPF control is implemented on the mirror using the collocated piezo sensor/actuator pairs. The stress reduction can be seen by looking at the PSD of the stress output. Figure 6 shows the PSD of the original system and with
active damping applied, zoomed in around the first three modes. The additional damping is clearly visible from the reduction in the peaks in the PSD. The active damping reduces the stress from 83 to 44 MPa, which is close to a 50% stress reduction, which, as expected, outperforms the shunted piezo techniques.

### 4.4 Mass and Complexity Implications

While it is clear that adding launch load alleviation can decrease the stress in the mirrors, it is important to also consider the mass and complexity implications of adding such techniques. Therefore, appropriate mass to implement each alleviation technique is added to the mirror mass. The isolation mass is added assuming SoftRide isolator flexures and a six segment primary aperture. Dividing the isolator mass among the six segments results in an additional 6.8 kg per mirror segment. The shunt circuit mass is made up of two parts: the mass of the circuit elements in the shunt and the mass of heat sinks to dissipate the energy extracted from the mirror. The heat dissipation mass is determined based on the power dissipated in the circuit. The power is calculated for each type of circuit using the voltage output from the state-space model. Then, an empirical relationship between dissipated power and mass is used to determine the approximate mass needed. Similarly, the active damping mass consists of component mass and heat dissipation mass, where the heat dissipation mass is determined using the power in the system.

In addition to the mass penalty, adding alleviation also results in a more complex system. Though complexity is not formally defined, one can infer that the complexity increases in the following order: No alleviation, isolation, resistive shunting, resonant shunting of the first flexible mode, resonant multi-mode shunting, and active damping. All of the parameters for the analysis are summarized in Table 6. The model is used to determine the peak stress and the mass of the mirror segment for each design, both of which one would like to minimize. The first set of results shows the trade space without any launch load alleviation (Sec. 5.1) and the second set of results shows the trade space with launch load alleviation included (Sec. 5.2).

### 5 Trade Space

As discussed in Sec. 2, the parametric nature of the model allows many mirror designs to be analyzed. Latin Hypercube sampling (LHS) is used to define 2500 designs, each defined by a set of input parameters. There are six parameters varied in the trade space: five describing the mirror geometry, and the launch load alleviation level. The launch load alleviation level parameter is defined as a discrete value with six possible levels corresponding to the numbers one through six: no alleviation, isolation, resistive shunting, resonant shunting of the first flexible mode, resonant multi-mode shunting, and active damping. All of the parameters for the analysis are summarized in Table 6. The model is used to determine the peak stress and the mass of the mirror segment for each design, both of which one would like to minimize. The first set of results shows the trade space without any launch load alleviation (Sec. 5.1) and the second set of results shows the trade space with launch load alleviation included (Sec. 5.2).

### 5.1 Trade Space without Alleviation

The trade space of designs, in terms of mirror mass and peak launch stress, can be seen in Fig. 7. Each point represents a distinct mirror design, specified by a set of parameters (Table 6). There are two lines marking the 3- and 6-$\sigma$ certainty levels; points below the lines have a launch survival probability of at least 3- and 6-$\sigma$, respectively. Also, the set of nondominated Pareto designs are circled. These designs represent the Pareto front, where one must sacrifice performance in one metric in order to improve performance in the
other. This results in a set of nondominated best designs, rather than a single best design.

Additionally, it is possible to distinguish the designs by each of the different parameters to visualize the trends. Figure 8 shows an example of the designs differentiated by the number of rib rings. This parameter defines the rib structure in the mirror, as well as the number of actuators. Designs with more rib rings have more actuators, and hence perform better on-orbit. However, it is clear from Fig. 8 that all of the nondominated designs have the minimum number of rib rings, and the designs with many rib rings are far from the Pareto front. Therefore, launch load alleviation is used to try to decrease the stress in those designs and bring them closer to the Pareto front.

5.2 Trade Space with Alleviation

Figure 9 shows the trade space when launch load alleviation is included, again showing the nondominated designs. Notice first that there are far more designs below the two stress limit lines, indicating that the addition of launch load alleviation did decrease the stress across the trade space. However, the mass does not decrease. As discussed in Sec. 5.4, adding alleviation necessitates the addition of mass. Figure 10 shows the trade space differentiated by the number of rib rings. Notice that there are many more designs with many ribs near the Pareto front, particularly at the mid-range and higher masses, indicating that the alleviation improves the performance of designs with many actuators, which perform better on-orbit.

Additionally, the designs can be visualized by the type of launch load alleviation used, as seen in Fig. 11. Here, notice that while the lowest mass designs tend to be without alleviation, the lowest stress designs all use shunt circuits or active damping, with active damping designs achieving the lowest stresses. Also, as discussed in Sec. 4.1, while isolation does reduce stress, the increase in mass and relatively small stress reduction make it ill-suited for this particular problem,
Figure 12 shows the two sets of nondominated designs: one and without launch load alleviation can be easily visualized by looking at the set of nondominated designs. Notice that the mirrors with alleviation dominate in the low stress portion of the design space. However, the very low mass designs with slightly higher stresses do not use alleviation.

Another way to compare the designs with and without alleviation is through examining the percentage of designs that meet the stress limits. The percentage of feasible designs that meet the 50 MPa stress limit are compared for designs with and without alleviation, and for each number or rib rings, as seen in the bar charts in Fig. 13. A higher percentage of designs meet the stress limit when launch load alleviation is included, and this trend becomes more apparent with more rib rings. Designs with more rib rings have more piezos which can be shunted or used with active damping, causing a more significant stress decrease and enlarging the feasible trade space.

Though it is clear that launch load alleviation, and particularly launch load alleviation making use of the existing embedded actuators can greatly decrease the peak launch stress and increase the feasible design space, it comes at the expense of both mass and complexity. However, it may be warranted to enhance the on-orbit performance of space telescopes, particularly when the areal density is low.

6 Conclusions

Lightweight, active, silicon carbide mirrors have the potential to increase capabilities in space-based optical systems. However, many mirror designs are extremely close to launch load limits. This paper presents a dynamic, state-space methodology for modeling and analyzing the disturbances encountered during launch. It computes the peak stress in the SiC substrate and in the actuators resulting from the vibroacoustic disturbances. While the baseline mirrors will likely survive launch, there are many designs which are better suited for on-orbit wavefront correction that are not likely to survive launch. The best mirror designs, with regard to launch, have very few ribs, and therefore few actuators. Additionally, the mirror response is dominated (2-to-1) by the acoustic disturbance, which is highly uncertain by nature. Therefore, it may be desirable to add launch load alleviation techniques to increase the probability of survival, as well as allow the use of mirrors that may be otherwise unable to survive launch.

Therefore, three launch load alleviation techniques are investigated in an attempt to expand the feasible design space. Whole spacecraft isolation can reduce vibrational loads on the mirror, though acoustic loading is the more problematic load case in this situation. Therefore, though isolation may be warranted for other fragile components in the spacecraft, it is not the best option when only considering the mirror. Passive shunt circuits are implemented using the existing embedded piezo actuators. Both resistive and resonant shunt circuits are analyzed, and can significantly reduce the stress in the mirror. Also, active damping using collocated SISO control and positive position feedback filters is implemented, and provides the greatest amount of stress reduction, but at the expense of having an active system during launch.

The parametric modeling implementation allows for a trade space analysis to examine the effect of different design parameters, as well as the alleviation techniques, on the mirror mass and peak launch stress. A trade space analysis shows that the launch load alleviation can reduce the stress in the mirrors, increasing the survival probability of many designs that have many ribs and actuators and are good for on-orbit performance. Systems that use active damping and shunting circuits achieve the lowest stresses. However, these techniques come at the expense of mass and complexity. This launch modeling methodology and alleviation analysis can allow one to discover the implications of the vibroacoustic launch disturbances early in the design process, thus pushing the boundaries of lightweight, active mirrors, and eventually resulting in better performing space-based optics.

References


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