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Optimal SISO and MIMO Spectral Efficiency to Minimize Hidden-Node Network Interference

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Optimal SISO and MIMO Spectral Efficiency to Minimize Hidden-Node Network Interference

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Abstract—In this letter, the optimal spectral efficiency for a given message size that minimizes the probability of causing disruptive interference for ad hoc wireless networks or cognitive radios is investigated. Implicitly, the trade being optimized is between longer transmit duration and wider bandwidth versus higher transmit power. Both single-input single-output (SISO) and multiple-input multiple-output (MIMO) links are considered. Here, a link optimizes its spectral efficiency to be a "good neighbor." The probability of interference is characterized by the probability that the signal power received by a hidden node in a wireless network exceeds some threshold. The optimization is a function of the transmitter-to-hidden-node channel exponent. It is shown that for typical channel exponents a spectral efficiency of slightly greater than 1 b/s/Hz per antenna is optimal.

Index Terms—Wireless LAN, local area networks, MIMO systems, cooperative systems.

I. INTRODUCTION

THE hidden-node problem is a significant issue for ad hoc wireless networks and cognitive radios. The problem is characterized by two links and four nodes: a transmitter of interest, a receiver of interest, a hidden transmitter, and hidden receiver [1]. It is assumed that the existence of the hidden link is not known by the link of interest and that the hidden link cannot adapt to interference. The hidden receiver (denoted the hidden node) may be near enough to the transmitter of interest that the interference causes the hidden link to fail. Being a good neighbor, the link of interest would like to minimize the adverse effects of transmitting a message of some finite number of bits. Both single-input single-output (SISO) and multiple-input multiple-output (MIMO) wireless communication links are considered.

As a specific example, imagine other links in an ad hoc wireless network (or legacy network for the cognitive radio problem) are communicating with random occupancy in frequency and time, as seen in Fig. 1(a). The geometry of a particular transmitter, receiver, and hidden node is depicted in Fig. 1(b). The distance from the transmitter to the hidden node is r, and the radius r_i contains the region within which the interference-to-noise ratio (INR), denoted η, is sufficient to disrupt the hidden link.
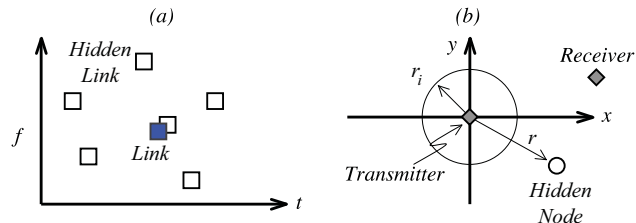
Fig. 1. (a) Displays a set of notional hidden links in time (t) and frequency (f) in the presence of a signal of interest transmission. (b) Depicts a notional geometry of transmitter, receiver, and hidden node. The distance from the transmitter to the hidden node is given by r. The region of disruptive interference is contained with radius r_i.

There are seven main assumptions used in this analysis. First, the probability of interfering is relatively small, which is equivalent to saying that the spatial, temporal, spectral occupancy of the network is not particularly high near the link of interest so that the probability of multiple collisions can be ignored. Second, the effects of interference on the hidden node can be factored into the probability of collision and the probability that the INR (η) at the hidden node exceeds some critical threshold η_i. Third, the hidden node location is sampled uniformly over some large area. Fourth, the average channel attenuation from the transmitter to the hidden node can be accurately modeled by using a power-law attenuation model. Fifth, the link performance can be characterized with reasonable accuracy by the channel capacity. Sixth, the hidden node does not have some interference mitigation capability that prefers a particular waveform structure. Finally, the desired data rate of the link of interest is sufficiently low that the link has the freedom to transmit in packets with relatively low spectral-temporal occupancy. Consequently, the optimization is developed for a single packet of a given number of information bits.

For the transmitter of interest to cause disruptive interference at the hidden node, the interfering signal must satisfy two requirements. First, it must overlap with a hidden link spectrally and temporally such that transmissions collide. The probability of collision (sufficient overlap) is denoted p_c. Second, the interfering signal must be of sufficient strength at the hidden node to cause disruptive interference, assuming sufficient overlap in time and frequency. The probability that the distance from the transmitter to hidden node is within sufficient range to cause disruptive interference is denoted p_r. The probability of disruptive interference is denoted p_i and is given by the product of p_c and p_r.

II. OPTIMAL SISO SPECTRAL EFFICIENCY

For some n_{info} information bits, the probability of interference p_i is a function of period of transmission T and bandwidth of transmission B, $p_i = p_i(T, B, n_{\text{info}})$. This functional

dependence is developed by noting that the probability of collisions p_c is linearly related to the transmitted duration and bandwidth

$$p_c \propto T B, \qquad (1)$$

because the fraction of the temporal and spectral space occupied the link [seen in Fig. 1(a)], and thus the probability of collision is proportional to the area subtended by the link in the temporal-spectral space, under the assumption that the distribution of packet occupancy over frequency and time is uniform.

If it is assumed that the hidden node is randomly located with respect to the transmitter in a two-dimensional physical space [Fig. 1(b)], then the probability that the hidden node is within sufficient range, p_r, to cause disruptive interference is proportional to the area, A, over which the signal has a sufficient INR, $\eta > \eta_i$, at the hidden node

$$p_r \propto A(\eta > \eta_i). \qquad (2)$$

Consequently, to a good approximation, the probability of interference exceeding some threshold is given by

$$p_i \propto T B A. \qquad (3)$$

The area is a function of the transmit energy and propagation loss to the hidden node.

For a SISO system, the information theoretic bound [2] on the number of bits that can be transmitted within time T and bandwidth B is given by

$$n_{\text{info}} \leq T B c \qquad (4)$$
$$c = \log_2(1 + \gamma) ,$$

where c is the information theoretic limit in bits/s/Hz on the SISO spectral efficiency (assuming a complex modulation), and γ is the signal-to-noise ratio (SNR) at the receiver. The bound is not achievable for finite n_{info}, but it is a reasonable approximation to the limiting performance.

By assuming the link of interest is operating at the information theoretic bound, the SNR at the receiver can be expressed in terms of the number of bits transmitted and the spectral efficiency

$$n_{\text{info}} \approx T B \log_2(1 + \gamma) ,$$
$$\gamma \approx (2^c - 1) . \qquad (5)$$

If the channel gain to the hidden node is denoted b^2 and the channel gain to the receiver of interest is denoted a^2, then the INR at the hidden node is

$$\eta = \frac{b^2}{a^2} \gamma . \qquad (6)$$

By using a simple power-law model for loss, with the channel gain to the hidden node proportional to $r^{-\alpha}$, the radius r_i at the critical interference level (at which $\eta = \eta_i$) is found by observing

$$\gamma = \frac{a^2}{b^2} \eta \propto \frac{a^2}{r^{-\alpha}} \eta \Rightarrow \frac{a^2}{r_i^{-\alpha}} \eta_i$$
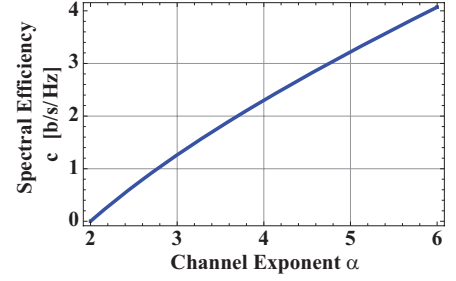$$r_i \propto \gamma^{1/\alpha} = (2^c - 1)^{1/\alpha} . \qquad (7)$$

Fig. 2. Optimal SISO spectral efficiency, c, for ideal coding in a static environment as a function of transmitter-to-hidden-node channel gain exponent, α.

Consequently, the probability of interference for the SISO system is given by

$$p_i \propto T B A$$
$$\approx \frac{n_{\text{info}}}{c} A \propto \frac{n_{\text{info}}}{c} r_i^2$$
$$\propto \frac{(2^c - 1)^{2/\alpha}}{c} . \qquad (8)$$

The optimal spectral efficiency for some α is given by

$$\frac{\partial p_i}{\partial c} \propto \frac{2^{c+1} (-1 + 2^c)^{\frac{2}{\alpha}-1} \log(2)}{c\alpha} - \frac{(-1 + 2^c)^{2/\alpha}}{c^2} = 0$$
$$c_{opt} = \frac{\alpha + 2 W_0 \left(-\frac{1}{2} e^{-\alpha/2} \alpha\right)}{2 \log(2)}$$
$$\approx 1.355(\alpha-2) - 0.118(\alpha-2)^2 + 0.008(\alpha-2)^3, \quad (9)$$

where $\log(\cdot)$ indicates the natural logarithm, and $W_0(x)$ is the product log or principal value of the Lambert W-function[1] [3]. It is remarkable that optimal spectral efficiency is dependent upon the channel exponent exclusively.

In Fig. 2, the optimal spectral efficiency for a given channel exponent, under the assumption of ideal coding in a static channel, is displayed. In the absence of multipath scattering, the line-of-sight exponent is $\alpha = 2$ (an anechoic chamber for example). For $\alpha = 2$, the optimal spectral efficiency approaches zero. For most scattering environments, $\alpha = 3$ to 4 [4] is a more reasonable characterization, suggesting a spectral efficiency around 2 b/s/Hz.

III. OPTIMAL MIMO SPECTRAL EFFICIENCY

The analysis for a MIMO link is similar to the SISO link. It is assumed that the MIMO link has an uninformed transmitter (without channel state information), and the number of transmitters by number of receivers, $n_t \times n_r$, MIMO channel is not frequency selective. The received signal is given by $\mathbf{Z} = \mathbf{H} \mathbf{S} + \mathbf{N}$, where $\mathbf{Z} \in \mathbb{C}^{n_r \times n_s}$ is the received signal, $\mathbf{S} \in \mathbb{C}^{n_t \times n_s}$ is the transmitted signal, $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix, and $\mathbf{N} \in \mathbb{C}^{n_r \times n_s}$ is the noise. The number of transmitted symbols is n_s.

For a MIMO system with an uninformed transmitter (a transmitter without channel state information), the information

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[1]The Lambert W-function is the inverse function of $f(W) = W e^W$. The solution of this function is multiply valued.

theoretic bound on the number of bits transmitted is given by

$$n_{\text{info}} \leq T B c$$
$$c = \log_2 \left| \mathbf{I} + \frac{P_0}{n_t} \mathbf{H} \mathbf{H}^\dagger \right| \tag{10}$$

where c is the information theoretic limit on the MIMO spectral efficiency (assuming a complex modulation), and P_0 is the total thermal-noise-normalized transmit power. The notation $|\cdot|$ indicates the determinant. Implicit in this formulation is the assumption that the interference-plus-noise covariance matrix is proportional to the identity matrix which is a reasonable model for most interference avoiding protocols.

Because the capacity is a function of a random SNR matrix, there is not a single solution as there is in the SISO analysis. However, by assuming that the channel matrix is proportional to a matrix sampled from an i.i.d. zero-mean element-unit-norm-variance complex Gaussian matrix, $\mathbf{H} = a\mathbf{G}$, an asymptotic analysis, in the limit of a large number of antennas, a solution can be found. Surprisingly, the asymptotic model is a reasonable approximation for even small numbers of antennas. [5] With this model, the term $a^2 P_0$ is the the average SNR per receive antenna at the receiver of interest. To simplify the analysis, it is assumed that $n_r = n_t \equiv n$. The optimal spectral efficiency under the assumption of other ratios of number transmitters to receivers can be found following a similar analysis. The asymptotic capacity [5] is given by

$$\frac{c}{n} \approx \frac{a^2 P_0}{\log 2} \,_3F_2([1,1,3/2],[2,3],-4\,a^2 P_0) \equiv f(a^2 P_0)$$
$$= \frac{4 \log\left(\sqrt{4a^2 P_0 + 1} + 1\right)}{\log(4)} + \frac{\sqrt{4a^2 P_0 + 1}}{a^2 P_0 \log(4)}$$
$$- \frac{1}{a^2 P_0 \log(4)} - 2 - \frac{2}{\log(4)}, \tag{11}$$

where $_pF_q$ is the generalized hypergeometric function [6], and the function $f(x)$ is used for notational convenience.

The SNR at the receiver can be expressed in terms of the number of bits transmitted and the spectral efficiency

$$n_{\text{info}} = T B n f(a^2 P_0),$$
$$a^2 P_0 = f^{-1}\left(\frac{c}{n}\right). \tag{12}$$

Unfortunately, a simple formulation of the functional inverse of $f(x)$ [denoted $f^{-1}(y)$] is not available; however, it is tractable numerically. By using a model similar to the link of interest, if the channel to the hidden node is given by $\mathbf{H}_{hn} = b^2 \mathbf{G}_{hn}$, then the average INR per receive antenna at the hidden node is given by

$$\eta = b^2 P_0. \tag{13}$$

By using a similar analysis to the SISO case and power-law model for the average channel gain b, the radius of disruptive interference is found by observing

$$a^2 P_0 = \frac{a^2}{b^2}\eta \propto \frac{a^2}{r^{-\alpha}}\eta \Rightarrow \frac{a^2}{r_i^{-\alpha}}\eta_i$$
$$r_i \propto (a^2 P_0)^{1/\alpha} = \left[f^{-1}\left(\frac{c}{n}\right)\right]^{1/\alpha}. \tag{14}$$
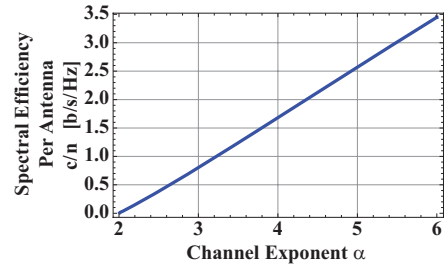

Fig. 3. Optimal MIMO spectral efficiency for ideal coding in a static environment as a function of the transmitter-to-hidden-node channel gain exponent.

Consequently, the probability of interference for the MIMO system is given by

$$p_i \propto T B A = \frac{n_{\text{info}}}{c} A \propto \frac{n_{\text{info}}}{c} r_i^2$$
$$\propto \frac{\left[f^{-1}(\beta)\right]^{2/\alpha}}{\beta}, \tag{15}$$

where the number-of-antenna-normalized spectral efficiency is given by $\beta \equiv c/n$. The optimal spectral efficiency for a given α is given by

$$\beta_{opt} = \mathrm{argmin}_\beta \frac{\left[f^{-1}(\beta)\right]^{2/\alpha}}{\beta}$$
$$\approx 0.795(\alpha-2) + 0.028(\alpha-2)^2 - 0.003(\alpha-2)^3. \tag{16}$$

In Fig. 3, the optimal spectral efficiency per antenna for a given channel exponent, under the assumption of ideal coding in a static channel, is displayed. For most scattering environments, $\alpha = 3$ to 4 [4] is a reasonable characterization, suggesting a spectral efficiency per antenna of a little more than 1 b/s/Hz.

IV. CONCLUSION

The optimal spectral efficiency was calculated to minimize the probability of causing disruptive interference in a wireless network. The optimization trades the time-bandwidth product versus the power of a transmission. Using this information, optimal per antenna spectral efficiency falls between 0.5 and 2 for typical channel exponents.

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