Performance of a Single Route

Outline

1. Wait time models
2. Service variation along route
3. Running time models

Wait Time Models

Simple deterministic model:

$E(w) = E(h)/2$

where $E(w)$ = expected waiting time

$E(h)$ = expected headway

Model assumptions:

- passenger arrival times are independent of vehicle departure times
- vehicles depart deterministically at equal intervals
- every passenger can board the first vehicle to arrive

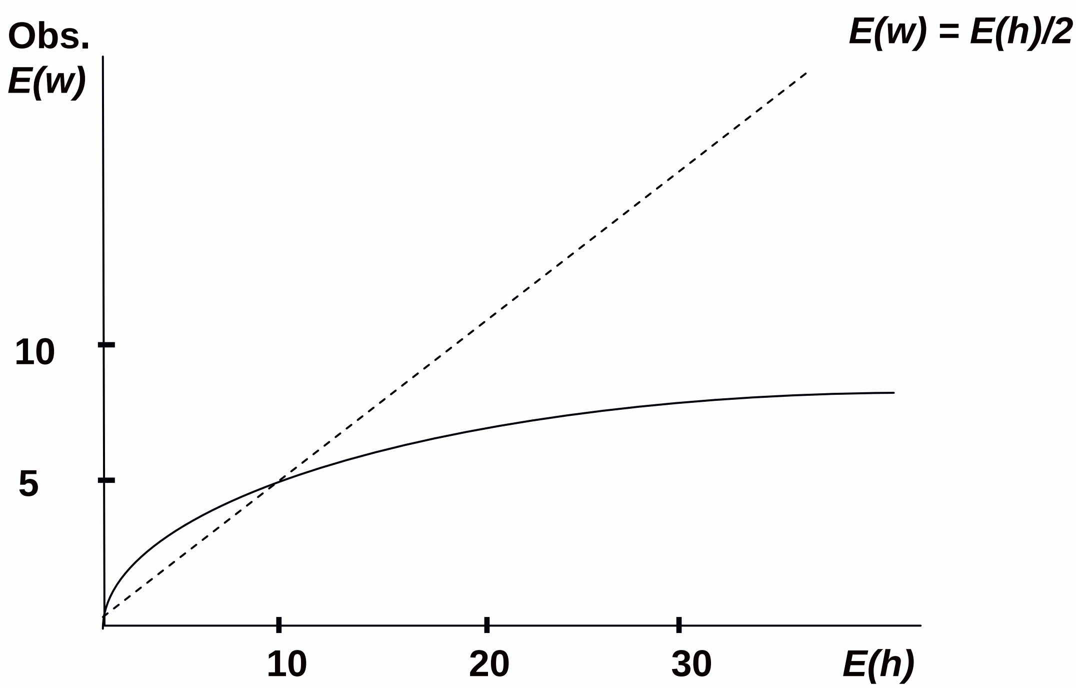
Passenger Arrival Process

- **Individual, group, and bulk passenger arrivals**

- **Passengers can be classified in terms of arrival process:**
  - **random arrivals**
  - **time arrival to minimize *E(w)***
  - **arrive with the vehicle, i.e. have *w* = 0**

Passenger Arrival Process (cont'd)

- For long headway service have "schedule delay" as well as wait time



Vehicle Departure Process

Vehicle departures typically not regular and deterministic

Wait Time Model refinement:

If: ***n(h)*** **= # of passengers arriving in a headway *h***

$\overline{w}(h)$ **= mean waiting time for passengers arriving in headway *h***

g(h) **= probability density function of headway**

Then:

E(w) **= *Expected Total Passenger Waiting Time per vehicle departure* / *Expected Passengers per vehicle departure***

$$= \frac{\int_0^\infty n(h)\overline{w}(h)g(h)dh}{\int_0^\infty n(h)g(h)dh}$$

Now if: $n(h) = \lambda \cdot h$ where λ is passenger arrival rate

$$\overline{w}(h) = \frac{h}{2}$$

Then: $$E(w) = \frac{E(h^2)}{2E(h)} = \frac{E(h)}{2}\left[1 + \frac{\text{var}(h)}{(E(h))^2}\right] = \frac{E(h)}{2}\left[1 + (\text{cov}(h))^2\right]$$

Vehicle Departure Process
Examples

A. If $var(h) = 0$: $E(w)=E(h)/2$

B. If vehicle departures are as in a Poisson process:

$var(h) = E(h))^2$ and $E(w)=E(h)$

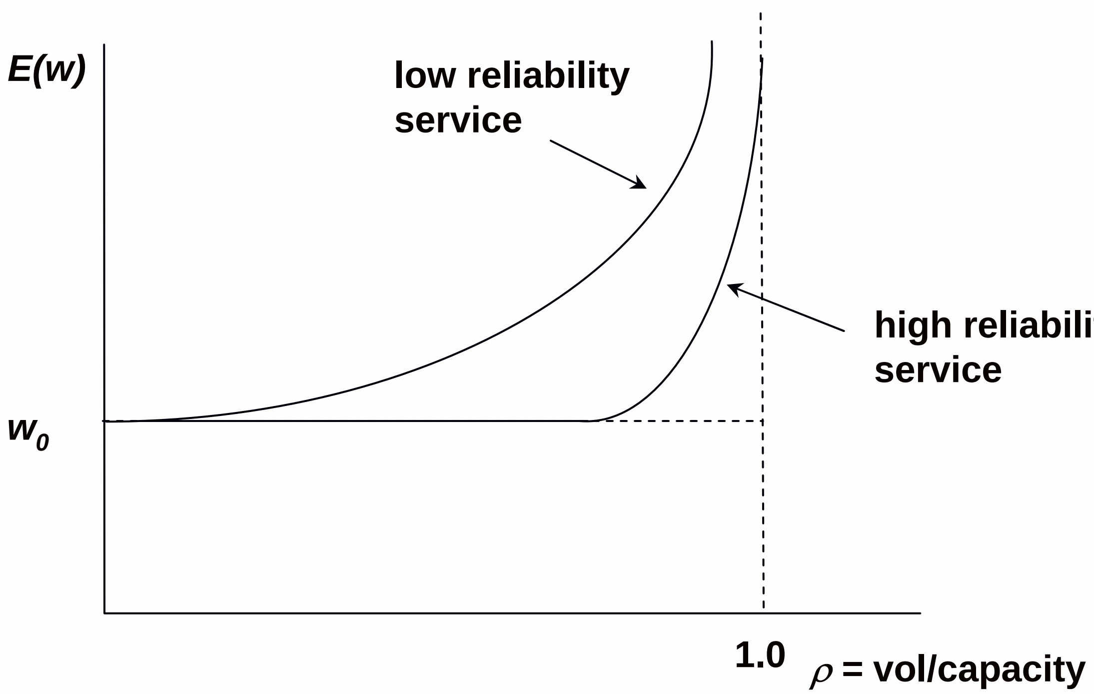
C. The headway sequence is 5, 15, 5, 15, ... then:

$E(h) = 10$

$E(w) = 2.5 * 0.25 + 7.5 * 0.75 = 6.25$ mins

Passenger Loads Approach Vehicle Capacity

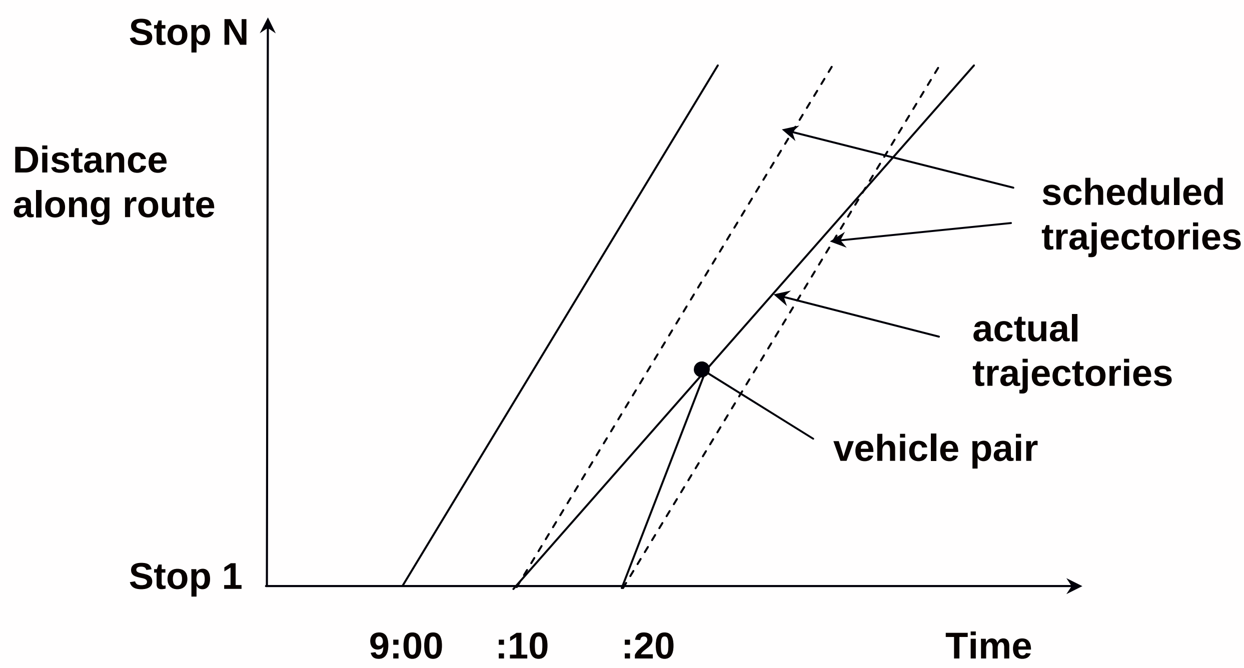
- Not all passengers can board the first vehicle to depart:



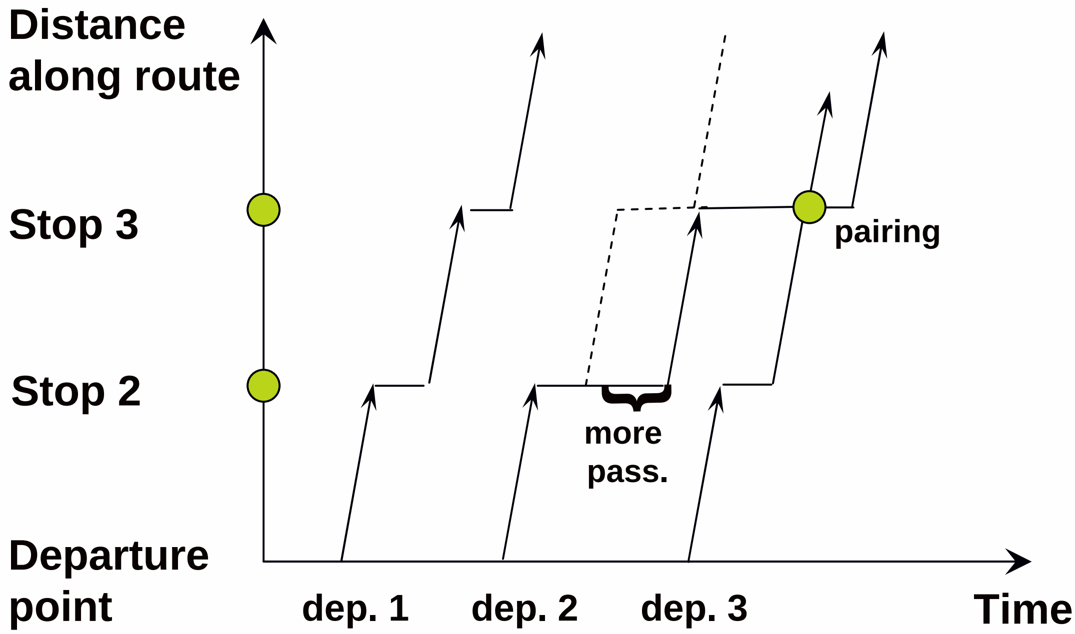
- General queuing relationship

Service Variation Along Route

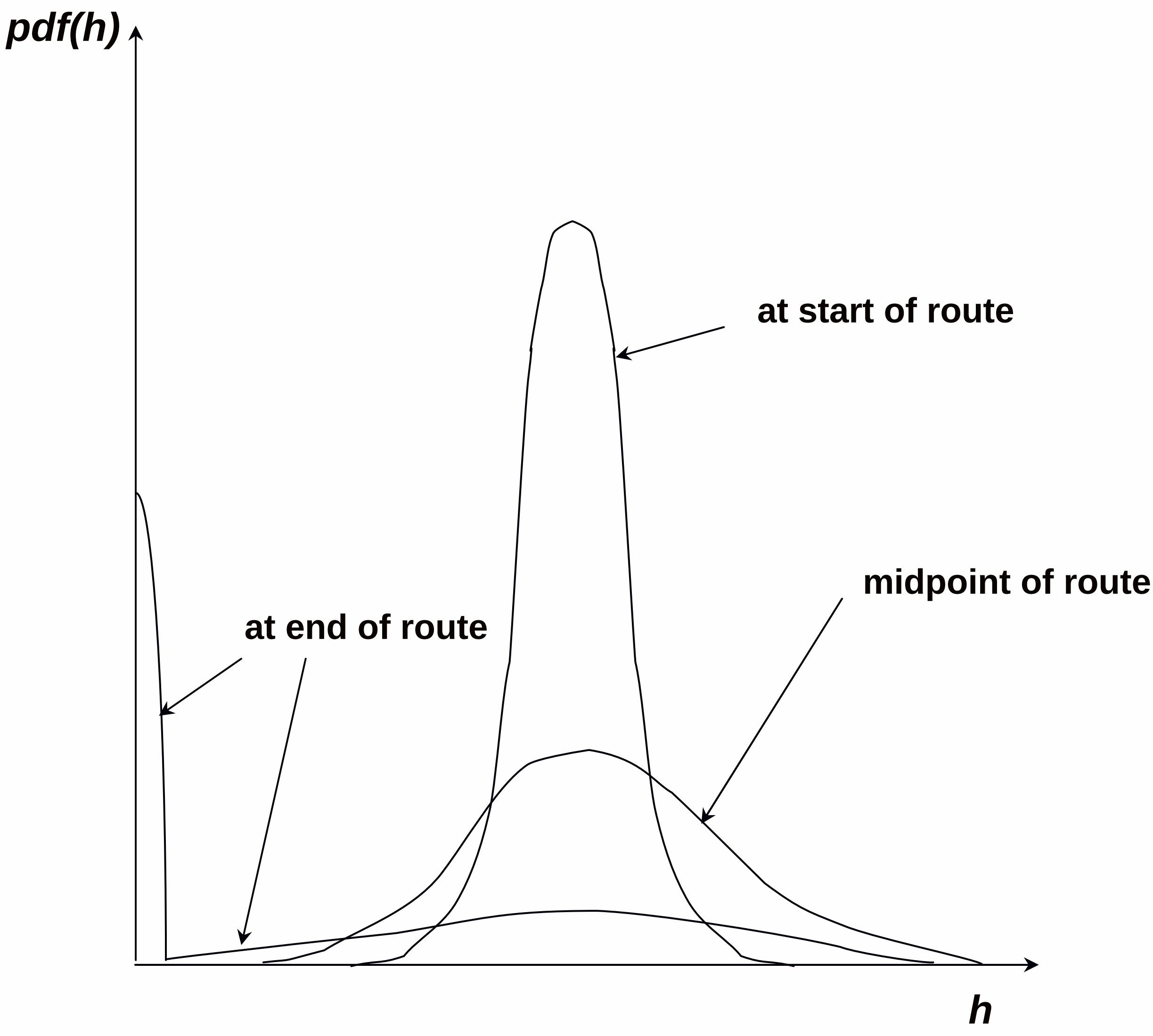
- **Service may vary along route even without capacity becoming binding:**
  - **the headway distribution can vary along the route, affecting E(w)**
  - **at the limit vehicles can be paired, or bunched**
  - **this can also result in passenger load variation between vehicles**



Service Variation Along Route (cont'd)



Service Variation Along Route (cont'd)



Factors Affecting Headway Deterioration

- **Length of route**
- **Marginal dwell time per passenger**
- **Stopping probability**
- **Scheduled headway**
- **Driver behavior**

<u>Simple model:</u>

$$e_i = (e_{i-1} + t_i)(1 + p_{i-1} \bullet b)$$

where e_i **= headway deviation (actual-scheduled) at stop** i

t_i **= travel time deviation (actual-scheduled) from stop** $i-1$ **to** i

p_i **= passenger arrival rate at stop** i

b **= boarding time per passenger**

Mathematical Model for Headway Variance*

$$\text{var}(h_i) = \text{var}(h_{i-1}) + \text{var}(\Delta t_{i-1}) + 2p_{i-1}(1-p_{i-1})(c\cdot\bar{q}_{i-1}+l)^2$$
$$+ 2c^2\,\text{var}(q_{i-1})[1-p_q + p_{i-1}p_q](1-p_{i+1})$$
$$+ c(1-p_{i-1})^2 \cdot \text{cov}(\Delta q_{i-1},h_{i-1})$$

Where:

$\text{var}(h_i)$	=	Headway variance at stop *i*
$\text{var}(\Delta t_i)$	=	Variance of the difference in running time between successive buses between stops *i*-1 and *i*
p_i	=	Probability bus will skip stop *i*
c	=	Marginal dwell time per passenger served at a stop
$\bar{q}_i$	=	Mean number of passengers per bus served at stop *i*
l	=	The constant term of the dwell time function
$\text{var}(q_i)$	=	Variance of the number of passengers served per bus at stop *i*
p_q	=	Correlation coefficient between the passengers served by successive buses at a stop
$\text{cov}(\Delta q_i, h_i)$	=	Covariance of the difference in number of passengers served by successive buses and the headway at stop i

Figure by MIT OCW.

*** Adebisi, O., "A Mathematical Model for Headway Variance of Fixed Bus Routes." Transportation Research B, Vol. 20B, No. 1, pp 59-70 (1986).**

Vehicle Running Time Models

Different levels of detail:

A. Very detailed, microscopic simulation:
   - represents vehicle motion and interaction with other vehicles, e.g. buses operating in mixed traffic, or train interaction through control system

B. Macroscopic:
   - identify factors which might affect running times
   - collect data and estimate model

Running Time includes dwell time, movement time, and delay time:

- dwell time is generally a function of number of passengers boarding and alighting as well as technology characteristics
- movement time and delay depend on other traffic and control system attributes

Typical bus running time breakdown in mixed traffic:

50-75% movement time

10-25% stop dwell time

10-25% traffic delays including traffic signals