MACRO DESIGN MODELS FOR A SINGLE ROUTE

Outline

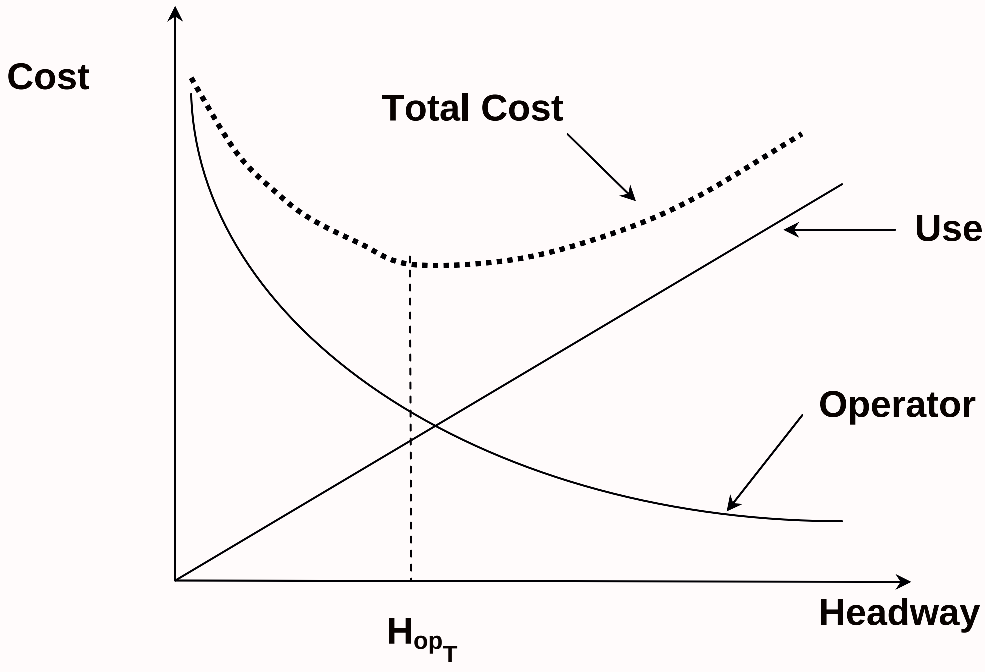
1. Introduction to analysis approach
2. Bus frequency model
3. Bus size model
4. Stop/station spacing model

Introduction to Analysis Approach

- Basic approach is to establish an aggregate total cost function including:
  - operator cost as *f*(design parameters)
  - user cost as *g*(design parameters)
- Minimize total cost function to determine optimal design parameter (s.t. constraints)

Variants include:

- Maximize service quality s.t. budget constraint
- Maximize consumer surplus s.t. budget constraint



Bus Frequency Model: the Square Root Model

Problem: define bus service frequency on a route as a function of ridership

Total Cost = operator cost + user cost

$$Z = c \bullet \frac{t}{h} + b \bullet r \bullet \frac{h}{2}$$

Where:

Z	=	Total (operator + user) cost per unit time
c	=	Operating cost per unit time
t	=	Round trip time
h	=	Headway – the decision variable to be determined
b	=	Value of unit passenger waiting time
r	=	Ridership per unit time

Minimizing *Z* w.r.t. *h* yields:

$$h = \sqrt{\frac{2ct}{br}} \quad or \quad \sqrt{2\left(\frac{c}{b}\right)\left(\frac{t}{r}\right)}$$

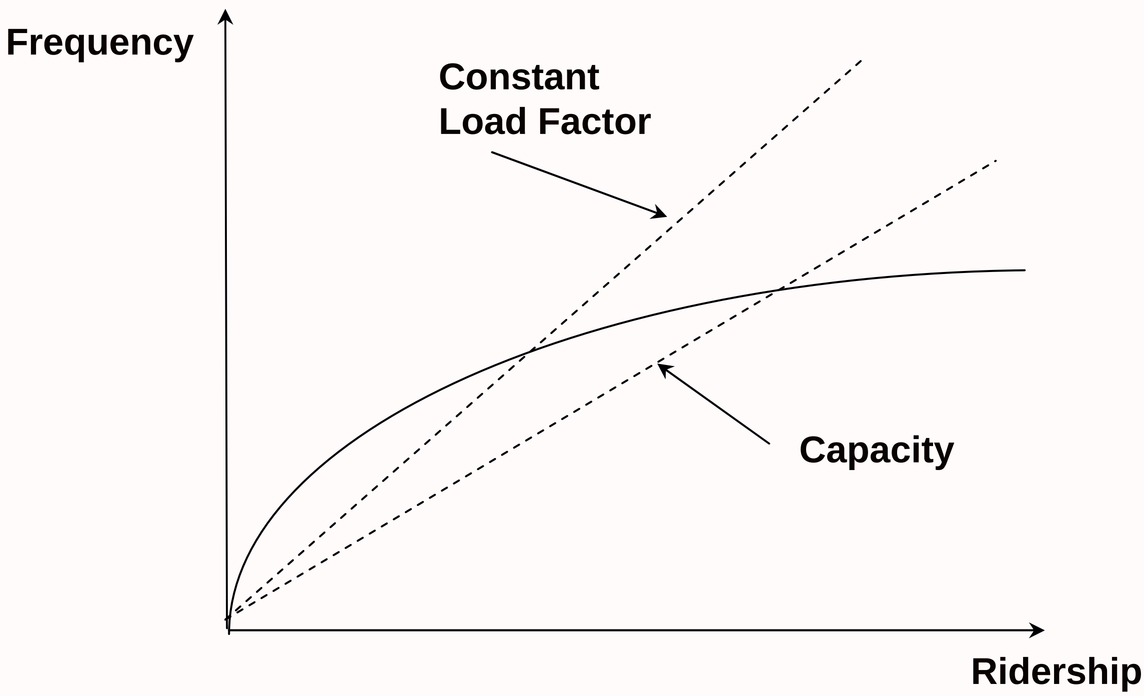
Figure by MIT OCW.

Square Root Model (cont'd)

This is the Square Rule with the following implications:

- **high frequency is appropriate where (cost of wait time/cost of operations time) is high**
- **frequency is proportional to the square root of ridership per unit time for routes of similar length**

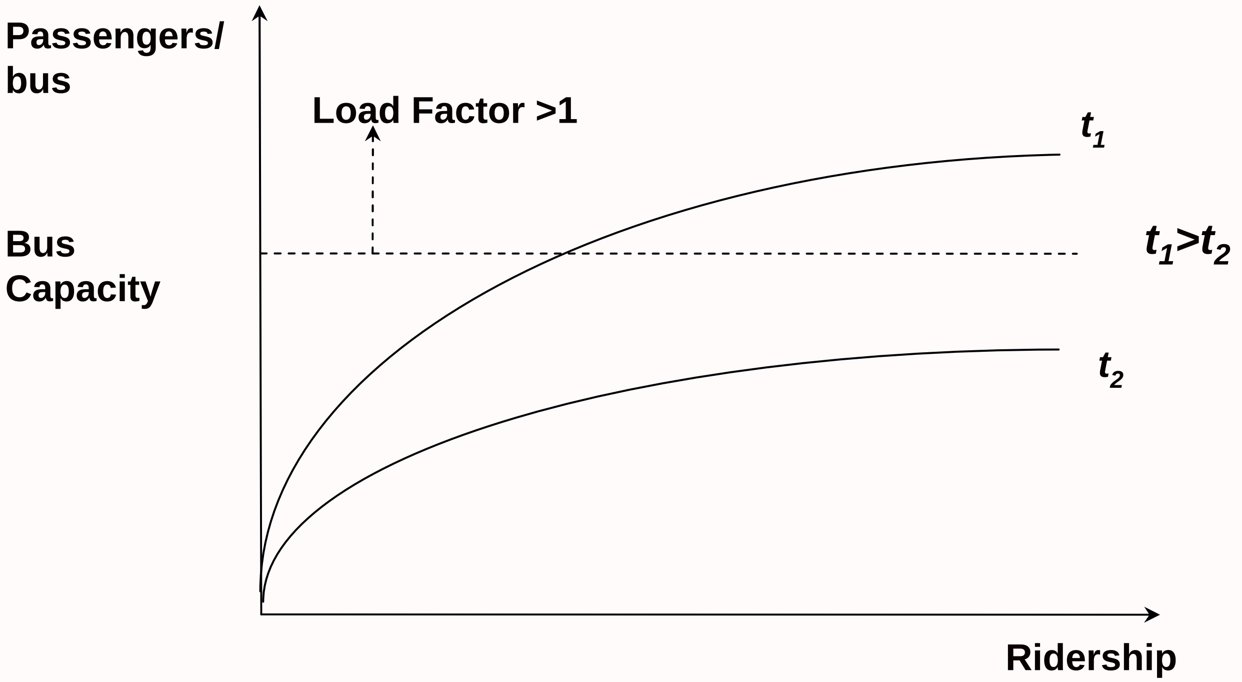
Frequency-Ridership Relationship



Square Root Model (cont'd)

- load factor is proportional to the square root of the product of ridership and route length.

<u>Bus Capacity-Ridership Relationship</u>



Square Root Model (cont'd)

Critical Assumptions:

- bus capacity is never binding
- only frequency benefits are wait time savings
- ridership $\neq f$ (frequency)
- simple wait time model
- budget constraint is not binding

Possible Remedies:

- introduce bus capacity constraint
- modify objective function
- introduce *r=f(h)* and re-define objective function
- modify objective function
- introduce budget constraint

Bus Frequency Example

If: c = \$90/bus hour,

b = \$10/passenger hour.

t = 90 mins,

r = 1000 passengers/hour,

Then: h_{OPT} = 11 mins

Bus Size Model

Problem: define optimal bus size on a route

Assumptions:

- Desired load factor is constant
- Labor cost/bus hour is independent of bus size
- Non-labor costs are proportional to bus size
- Bus dwell time costs per passenger are independent of bus size

Using same notation as before plus:

w = labor cost per bus hour

p = passenger flow past peak load point

k = desired bus load - the decision variable to be determined

Then $$Z = w \bullet \frac{t}{h} + b \bullet r \bullet \frac{h}{2}$$

Now $h = \frac{k}{p}$ by assumption above

Therefore $$Z = \frac{wtp}{k} + \frac{brk}{2p}$$

Minimizing Z w.r.t. k gives:

$$k_{OPT} = \sqrt{\frac{2p^2wt}{rb}}$$

Figure by MIT OCW.

Bus Size Model (cont'd)

Result is another square root model, implying that optimal bus size increases with:

- **round trip time**
- **ratio of labor cost to passenger wait time cost**
- **peak passenger flow**
- **concentration of passenger flows**

Previous example extended with:

p = 500 pass/hour,

w = \$40/bus hour;

all other parameters as before:

Then:

h_{OPT} = 55

Stop/Station Spacing Model

Problem: determine optimal stop or station spacing

Trade-off is between walk access time (which increases with station spacing), and in-vehicle time (which decreases as station spacing increases) for the user, and operating cost (which decreases as station spacing increases)

Define	Z	=	total cost per unit distance along route and per headway
and	T_{st}	=	time lost by vehicle making a stop
	c	=	vehicle operating cost per unit time
	s	=	station/stop spacing - the decision variable to be determined
	N	=	number of passengers on board vehicle
	v	=	value of passenger in-vehicle time
	D	=	demand density in passenger per unit route length per headway
	v_{acc}	=	value of passenger access time
	w	=	walk speed
	c_s	=	station/stop cost per headway

Stop/Station Spacing Model (cont'd)

$$Z = \frac{T_{st}}{s}(c + N \bullet v) + \frac{c_s}{s} + \frac{s}{4} \bullet D \bullet \frac{v_{acc}}{w}$$

Minimizing Z w.r.t. s gives:

$$s_{OPT} = \left[\frac{4w}{D v_{acc}} \left[c_s + T_{st}(c_v + Nv) \right] \right]^{1/2}$$

Figure by MIT OCW.

Yet another square root relationship, implying that station/stop spacing increases with:

- **walk speed**
- **station/stop cost**
- **time lost per stop**
- **vehicle operating cost**
- **number of passengers on board vehicle**
- **value of in-vehicle time**

and decreases with:

- **demand density**
- **value of access time**

Bus Stop Spacing

U.S. Practice

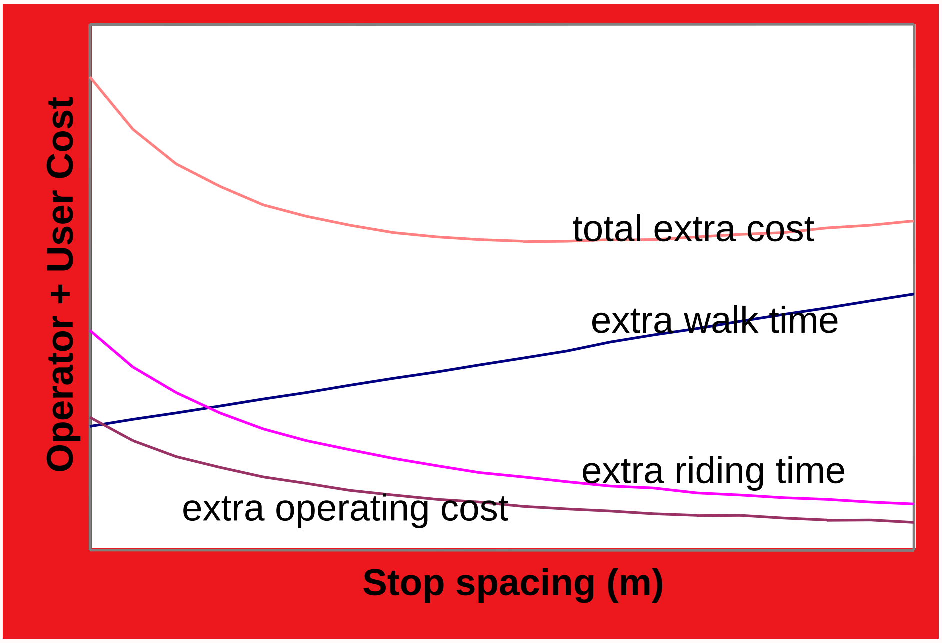
- **200 m between stops (8 per mile)**
- **shelters are rare**
- **little or no schedule information**

European Practice

- **320 m between stops (5 per mile)**
- **named & sheltered**
- **up to date schedule information**
- **scheduled time for every stop**

Stop Spacing Tradeoffs

- **Walking time**
- **Riding time**
- **Operating cost**
- **Ride quality**



Results: MBTA Route 39*

AM Peak Inbound results with Optimal Spacing

- Avg walking time up 40 s
- Avg riding time down 110 s
- Running time down 4.2 min
- Save 1, maybe 2 buses

Bus Stop Locations and Policies

- **Far-side** (vs. Near-side)
  - less queue interference
  - easier pull-in
  - fewer ped conflicts
  - snowbank problem demands priority in maintenance

- **Curb extensions** benefit transit, peds, and traffic (0.9 min/mi speed increase)
- **Pull-out priority** (it's the law in some states)
- **Reducing dwell time** (vehicle design, fare collection, fare policy)