Real-Time Control Strategies for Rail Transit

Outline:

- **Problem Description and Motivation**

- **Model Formulation**

- **Model Application and Results**

- **Implementation Issues**

- **Conclusions**

Problem Context

- **High frequency urban rail service (e.g. headways of 2-10 minutes)**

  - **passengers arrive randomly**
  - **service regularity is a key goal**
  - $E(WT) = \dfrac{E(H)}{2} [1 + cov(H)^2]$

- **Branching route structure**

- **Central real-time train location information and dispatch capability**

Three Levels of Control Problems

__Routine disturbances__ - several minutes' deviation from schedule

Control Strategies:

- speed adjustment
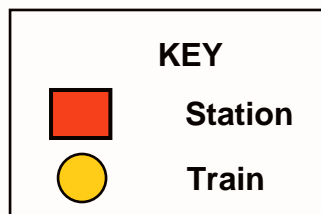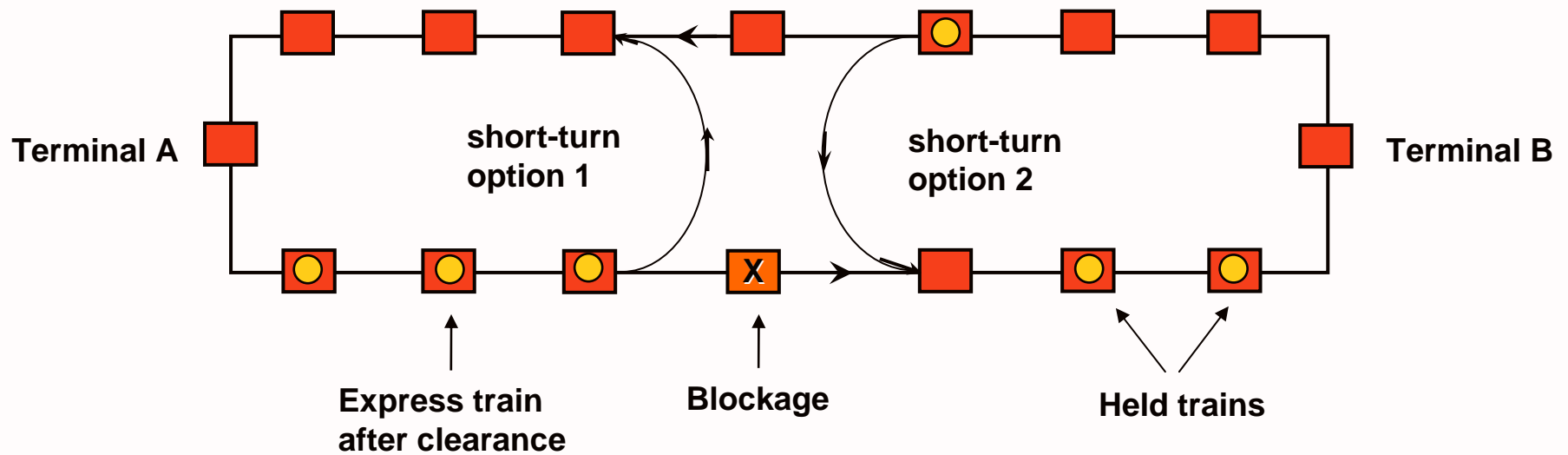- dwell time adjustment (selective holding) terminal recovery

__Short-term disruptions:__ 5-30 minute blockages on the line

__Longer-term disruptions__ - greater than 30 minute blockages

Control Strategies:

- single-track reverse direction operations
- replacement bus service around blockage

Disruption Response Strategies

Terminal A

short-turn option 1

short-turn option 2

Terminal B

X

Express train after clearance

Blockage

Held trains

KEY

Station

Train

Problem Description

- **Overall Objective:**
  -- Develop a real-time decision support system to determine control strategies to recover from disruptions

- **Specific Objective:**
  -- Minimize passenger waiting times (implies maintaining even headways)

- **Key Characteristics:**
  -- Instability of even headways
  -- Passenger sensitivity to long waiting time and crowding
  -- Cost insensitivity to different strategies

- **Possible Strategies:**
  -- Holding
  -- Short-turning
  -- Expressing

Example of Transit Control Strategies
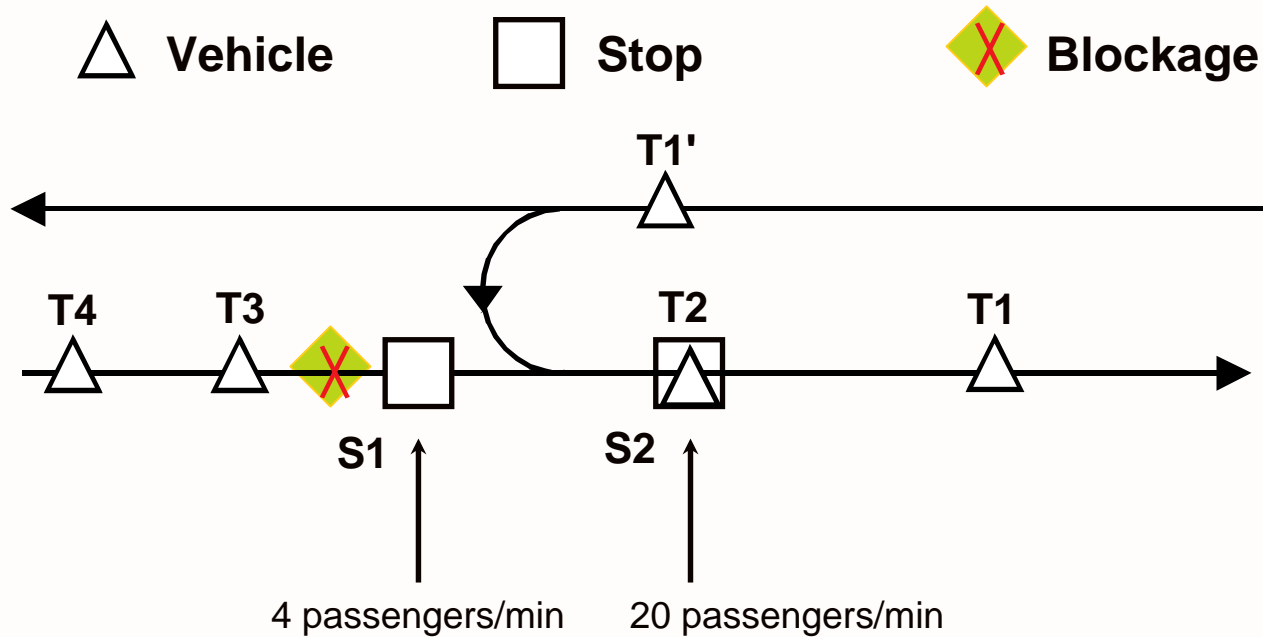
- **6-minute scheduled headways**
- **3-minute minimum safe headway**
- **10-minute disruption**
- **impact set includes trains T2, T3, and T4 and stations S1 and S2**

Example Results

1. Do nothing: h_{T2} = 6 mins.; h_{T3} = 16 mins.; h_{T4} = 3 mins.

$$Total\ Passenger\ Waiting\ Time\ =\ \frac{1}{2}\left[4(16^2+3^2)\right]\ +\ \frac{1}{2}\left[20(6^2+16^2+3^2)\right]\ =\ 3540\ \text{pass}-\text{mins.}$$

2. Holding: Hold T2 at S2 for 4 mins.
Then at S2: h_{T2} = 10 mins.; h_{T3} = 12 mins.; h_{T4} = 3 mins.

$$T\ P\ W\ T\ =\ \frac{1}{2}\left[\left(4(16^2+3^2)+20(10^2+12^2+3^2)\right)\right]\ =\ 3060\ \text{pass}-\text{mins.}$$

3. Expressing: Express T3 past S1 to save 1 minute in travel time.
Then at S2: h_{T2} = 6 mins.; h_{T3} = 15 mins.; h_{T4} = 4 mins.

$$T\ P\ W\ T\ =\ \frac{1}{2}\left[4*19^2+20(6^2+15^2+4^2)\right]\ =\ 3492\ \text{pass}-\text{mins.}$$

Model Formulation

Key Features:

- station specific parameters: passenger arrival rates, alighting fractions, minimum safe headways

- station dwell time a linear function of passengers boarding, alighting and crowding

- train order is variable
- train capacity constraint

Simplifications:

- predictable disruption length
- passenger flows estimated from historical data
- system is modelled as deterministic
- strategies selected to produce minimum inter-station travel times.

Model Formulation

Decision Variables: departure time of train *i* from station *k*

Objective function: minimization of passenger waiting time
- *quadratic function approximated by a piecewise linear function*

Impact Set: consider a finite set of trains and stations and approximate the effects beyond this set

Constraints: train running time and minimum safe headways
- *other relationships govern passenger loads, train dwell times*

Model Structure: mixed integer program except if passenger capacity is not binding when it is a linear program

Specific Models

Holding Strategy Models:

- **Hold all**

- **Hold once**

- **Hold at first station**

Combined Short-turning and Holding Models:

- **Predetermined train order**

- **Undetermined train order**

Model Application

MBTA Red Line Characteristics:

- **23 stations (including 3 terminals)**

- **27 six-car trains in A.M. peak**

- **3.4 minute trunk headways (6 and 8 minutes on branches)**

- **30,000 passengers in peak hour**

Red Line

Ashmont

North

Park Street

Harvard Square

Alewife

Kendall/MIT

Blockage Location, Incident 1

Blockage Location, Incident 2

Braintree

KEY:
- ▲ Ashmont Train
- ● Braintree Train
- ◻ Station
- ★ Blockage
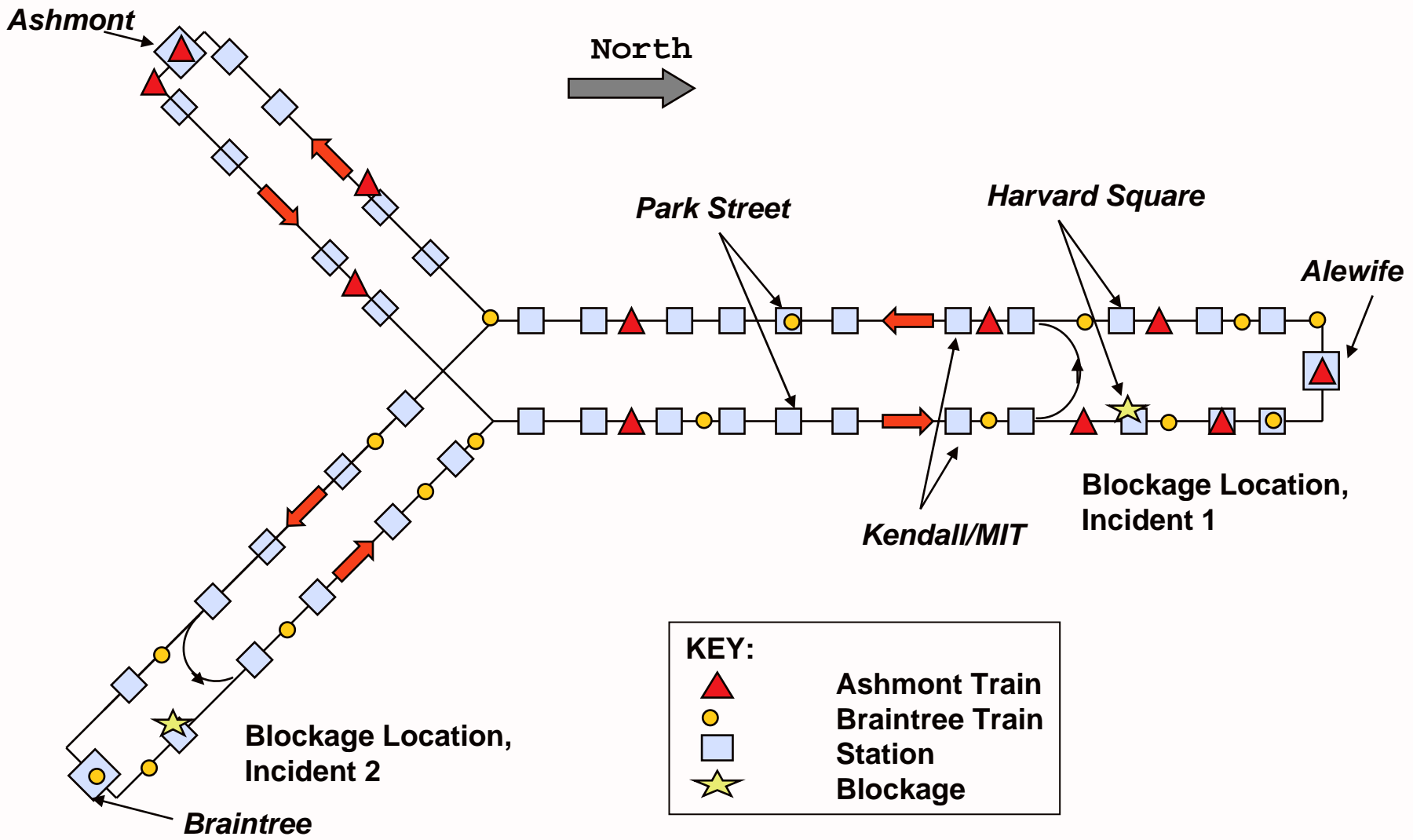
Incident 1, Ten Minute Delay

		Control Strategies			
		FOHPC			STPP
Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	11202	8863	8931	8961	9997
Savings (percent)		15%	14%	14%	8%
Behind Blockage	4791	4763			4753
Savings (percent)		0%			0%
Maximum Train Load	988	603	614	666	603
Problem Size		95	95	95	88
CPU Time (seconds)		22	37	21	16

Incident 1, Twenty Minute Delay

Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Control Strategies			
		FOHPC			STPP
		Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	36868	16934	17306	17385	16836
Savings (percent)		43%	42%	42%	43%
Behind Blockage	9218	7833			6842
Savings (percent)		3%			5%
Maximum Train Load	1646	666	759	805	651
Problem Size		95	95	95	88
CPU Time (seconds)		25	82	27	17

Incident 2, Ten Minute Delay

Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Control Strategies			
		FOHPC			STPP
		Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	32495	23101	24465	25327	23016
Savings (percent)		25%	21%	19%	25%
Behind Blockage	5593	5320			5404
Savings (percent)		<1%			<1%
Maximum Train Load	1336	1137	964	985	776
Problem Size		69	69	69	78
CPU Time (seconds)		17	274	23	12

Incident 2, Twenty Minute Delay

Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Control Strategies			
		FOHPC			STPP
		Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	32495	23101	24465	25327	23016
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Impact Set Size

Incident 1, Twenty Minute Delay
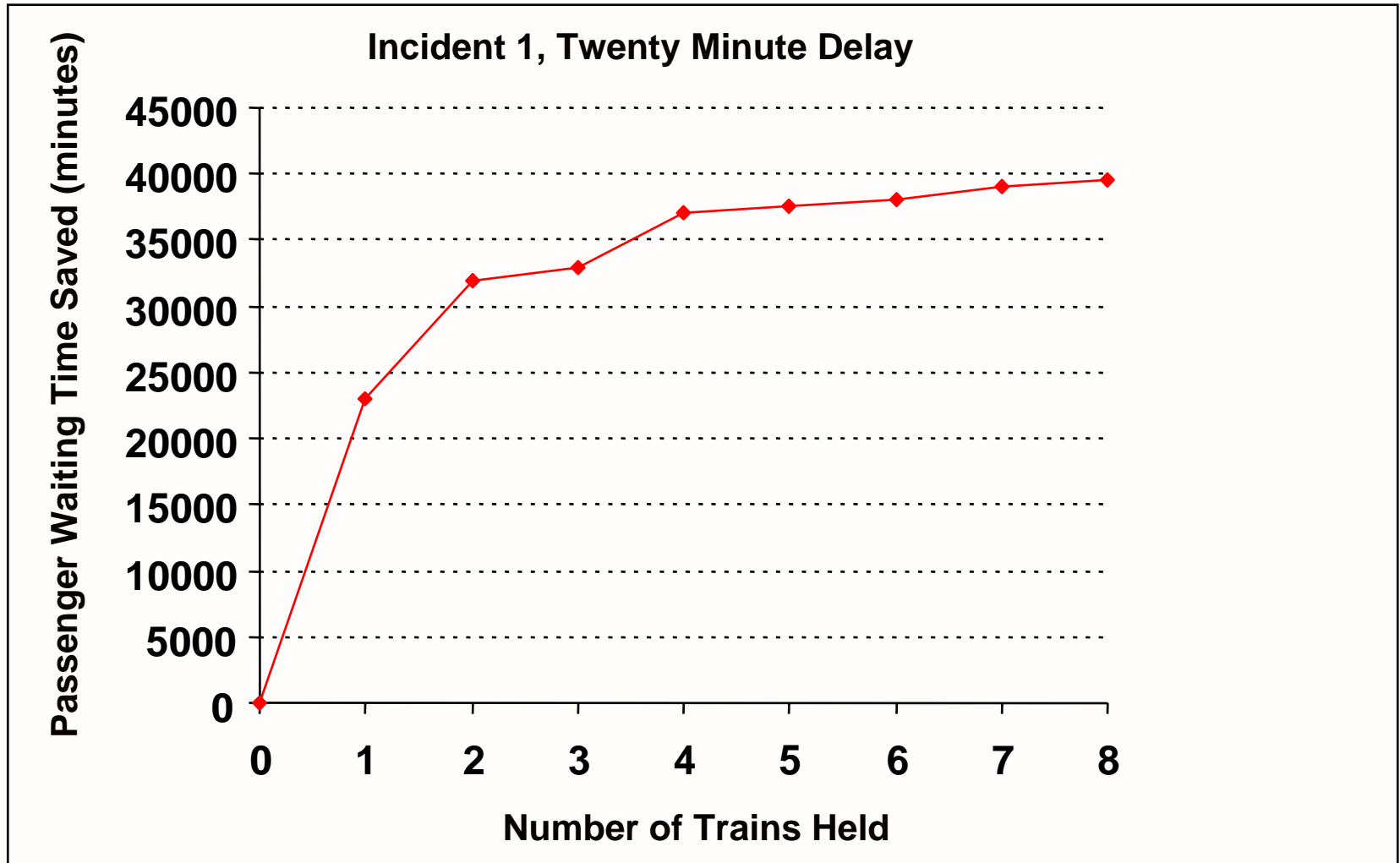
Passenger On-Board Time

Incident	Delay	Objective Function	Passenger Time		
			Waiting	**On-Board**	**Total (Weighted)**
1	10 Min.	PWT	8961	1543	9578
		TPT	9074	271	9182
1	20 Min.	PWT	17385	2372	18334
		TPT	17659	806	17982
2	10 Min.	PWT	23411	8666	26877
		TPT	23702	5920	26070
2	20 Min.	PWT	50018	17617	57065
		TPT	51201	10488	55396

Execution Times

- **Sun SPARC 20 workstation**
- **GAMS V. 2.25**
- **CPLEX V. 3.0**
- **Simple front-end heuristic to fix some binary variables**

Large Problems: 11-13 trains, 69-95 train/station decision var.
Execution Time: 10 out of 16 <30 sec.

Realistic Size: 7-8 trains, 40-50 train/station decision var.
Execution Time: 16 out of 16 <34 sec.

Conclusions

- **Holding and short-turning models formulated and solved to optimality**

- **Active control strategies result in significant passenger waiting time savings**

- **Train control set can be reduced to trains ahead of the blockage**

- **Train control set need not be large**

Conclusions

- **Hold at First or Hold Once strategies *can* be almost as effective as Hold All strategy**

- **Short-turning most effective where:**
  - **-- blockage is long relative to short-turn time**
  - **-- number of stations outside the short-turn loop is small**

- **Consideration of on-board time is desirable**

- **Execution time is 30 seconds or less but faster heuristics are probably achievable**