Abstract—In this paper, we propose a new statistical interference model for cognitive network based on the amplitude aggregate interference, which accounts for the parameters related to the sensing procedure, spatial reuse protocol employed by secondary users, and environment dependent conditions like channel fading and shadowing. We derive the characteristic function and the $\nu$th cumulant of the cognitive network interference on the primary user. By using the theory of truncated-stable distribution, we show how we can approximate the cognitive network interference analytically. We further show how to apply our model to derive system performance measure such as bit error probability in the presence of cognitive network interference. Moreover, this work can serve to bring additional understanding of cognitive network interference for successful deployment of cognitive networks in the future.

I. INTRODUCTION

Opportunistic spectrum access allows the opening of underutilized portions of the licensed spectrum for secondary reuse, provided that the transmissions of secondary radios do not cause harmful interference to primary network. To enable secondary users to accurately detect and to access the idle spectrum, cognitive radio has been proposed as the enabling technology. However, spectrum sensing is challenged by the uncertainty in the aggregate interference in the network. Such uncertainty can be resulted from the unknown number of interferers, the unknown location of the interferers, the effect of channel fading and shadowing, and other uncertain environment dependent conditions. Therefore, it is crucial to study how such uncertainty can be incorporated in the statistical aggregate interference model and the effect of the cognitive network interference on the primary network system performance.

Throughout this paper, we refer to the aggregate interference generated by secondary users sharing the same spectrum of the primary user as cognitive network interference [1]. In [2], the authors study the coexistence between wideband and narrowband communication systems, where they have applied stable distribution to model the aggregate interference. In [3], the authors assume the presence of a control channel to indicate the presence of primary receivers and the aggregate interference power is modeled as the sum of all interferers’ power. In [4], the authors compare bounded and unbounded path-loss models, showing that the unbounded path-loss model might results in significant deviations from more realistic performance. In [5], an expression for the moment of the aggregate interference distribution generated by nodes located in an arbitrary area is derived. Although the above models allow one to analyze cognitive network interference, they cannot be easily extended to perform error probability analysis of primary network in the presence of cognitive network interference. Moreover, these models usually assume unbounded path-loss model and are unable to capture the instantaneous cognitive interference conditions.

In this paper, we propose a new statistical interference model for cognitive network based on the amplitude aggregate interference, which accounts for the parameters related to the sensing procedure, spatial reuse protocol employed by secondary users, and environment dependent conditions like channel fading and shadowing. Moreover, our statistical model allows us to model the cognitive network interference generated by secondary users confined in a limited region. By using the theory of truncated-stable distribution, we show how we can approximate the cognitive network interference analytically. We further show how to derive system performance measure such as bit error probability (BEP) in the presence of cognitive network interference. The paper is organized as follows. Section II presents the system model. Section III derives the exact and approximate instantaneous interference distribution. Section IV demonstrates how these results can be applied in BEP analysis. Section V provides numerical results to illustrate the coexistence between primary and secondary networks depends on various system parameters. Section VI gives the conclusion.

II. SYSTEM MODEL

For cognitive networks, the secondary users need to sense channels before transmission in order not to cause harmful interference to a primary network. In this paper, we consider the primary network in frequency division duplex mode. Therefore, to detect the presence of active primary users, the secondary user senses the uplink channel while the transmission occupies the downlink channel of the primary system. Furthermore, we consider the secondary network as a simple ad-hoc network where secondary users join/exit the network and sense/access the channel independently without coordinating with other secondary users. As such, there exists the possibility that secondary users can transmit at the same
A. Cognitive Network Activity Model

The activity of each secondary user depends on its received signal strength of the uplink signal transmitted by the primary user. In the following, we consider two types of secondary spatial reuse protocols, namely: single-threshold and multiple-threshold protocols.

1) Single-Threshold Protocol: In this case, a secondary user is said to be active if
\[ \frac{KP_p Y}{r^{2\nu}} \leq \beta, \] 
(1)
or equivalently,
\[ r^{-2\nu} Y \leq \zeta, \] 
(2)
where \( \beta \) is the activating threshold; \( \zeta \triangleq \frac{\beta}{KP_p} \) is the normalized threshold; \( P_p \) is the transmitted power of the primary user; \( Y \) is the squared fading path strength from the primary user to the secondary user; \( K \) is the gain accounting for the loss in the near-field; \( r \) is the distance between the primary and secondary users; and \( \nu \) is the amplitude pass-loss exponent.\(^2\) Therefore, the activity of the secondary network users can be represented by the Bernoulli random variable:
\[ I_{[0,\zeta]} (r^{-2\nu} Y) \sim \text{Bern} \left( \frac{F_Y (r^{2\nu} \zeta)}{F_Y (r^{2\nu} \zeta)} \right), \] 
(3)
with the indicator function defined as
\[ I_{[a,b]} (x) = \begin{cases} 1, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}, \] 
(4)
where the value one of the Bernoulli variable denotes that the secondary user is active.

2) Multiple-Threshold Protocol: For this case, the transmission power of the secondary network users is set according to the detected power level of the primary network uplink signal [6]. We consider \( N - 1 \) normalized threshold values \( \zeta_1, \zeta_2, \ldots, \zeta_{N-1} \) in increasing order to identify \( N \) different zones (or sets) of active secondary users, denoted by \( A_k \), \( k = 1, 2, \ldots, N \). Let \( \zeta_0 = 0 \) and \( \zeta_N = \infty \). Then, the \( k \)th active zone \( A_k \) obeys the following activating rule:\(^3\)
\[ I_{[\zeta_{k-1},\zeta_k]} (r^{-2\nu} Y) \sim \text{Bern} \left( \frac{\mu_Y^{(pt)} (0, r^{2\nu} \zeta_{k-1}, r^{2\nu} \zeta_k)}{\mu_Y^{(pt)} (0, r^{2\nu} \zeta_{k-1}, r^{2\nu} \zeta_k)} \right). \] 
(5)
Note that the received primary network signal power at the active secondary user in the zone \( A_k \) is between \( KP_p \zeta_{k-1} \) and \( KP_p \zeta_k \).

B. Interference Model

The interference signal at the primary receiver generated by the \( i \)th cognitive interferer can be written as
\[ I_i = \sqrt{P_l r_i^{-\nu} X_i}, \] 
(6)
where \( P_l \) is the interference signal power at the limit of the near-far region;\(^4\) \( r_i \) is the distance between the \( i \)th interferer and the primary receiver; and \( X_i \) is the per-dimension fading channel gain, i.e., the real or imaginary part of the channel gain \( H_i \) from the \( i \)th cognitive interferer to the primary receiver. In the following, we assume that \( Y \) in (1) and \( X_i \) in (6) are statistically independent.

In our model, we consider that the secondary users are spatially scattered according to an homogeneous Poisson point process in a two-dimensional plane \( \mathbb{R}^2 \), where the victim primary user is assumed to be located at the center of the region. The probability that \( k \) secondary users lie inside a region \( R \subseteq \mathbb{R}^2 \) depends only on the total area \( A_R \) of the region, and is given by [7]
\[ P \{ k \ in \ R \} = \frac{(\lambda A_R)^k}{k!} e^{-\lambda A_R}, \quad k = 0, 1, 2, \ldots \] 
(7)
where \( \lambda \) is the spatial density (in nodes per unit area). Furthermore, we assume that the region \( R \) is limited by minimum and maximum distances from the primary receiver, denoted by \( R_s \) and \( R_t \), respectively.\(^5\) In this way, this model will enable us to consider a scenario where the secondary users are only located within a limited region.

III. Instantaneous Interference Distribution

To introduce our statistical model for the cognitive network interference, we first derive the cumulants of the cognitive network interference for random activity (the activity of each secondary user is regulated by a spatial reuse protocol) in Section III-A. Next, in Section III-B we develop the symmetric truncated-stable approximation for the cognitive network interference using its cumulant expressions.

A. Random Activity

1) Single-Threshold Protocol: In this spatial reuse protocol, the secondary users in the region \( R \) are activated by the single normalized threshold \( \zeta \) using (3). Therefore, the cognitive network interference for the single-threshold protocol can be written as
\[ I_{st} = \sqrt{P_l} \sum_{i \in A_{st}} r_i^{-\nu} X_i, \] 
(8)
where \( A_{st} \) defines the set of secondary users activated by the single normalized threshold \( \zeta \) in the region \( R \):
\[ A_{st} = \{ i \in R : I_{[0,\zeta]} (r_i^{-2\nu} Y) = 1 \}. \] 
(9)
\(^4\)We consider the near-far region limit at 1 meter.
\(^5\)Note that \( r_i \) in (6) can be smaller than 1. Therefore, the received interference power can be larger than \( P_l \) but it is finite since \( R_s > 0 \).
The characteristic function (CF) of $Z_{st} (\zeta; \mathcal{R})$ can be expressed as
\[
\psi_{Z_{st} (\zeta; \mathcal{R})} (js) = \exp \left( 2\pi \lambda \int_X \int_Y \int_{R_s} \int_{R_t} \left[ \exp (j s x r r^{-\nu}) - 1 \right] r^{-2\nu} f_X (x) f_Y (y) r dr dy dx \right).
\]
Using (10), we can then calculate the $n$th cumulant of the interference as
\[
Z_{nt} (\zeta; \mathcal{R}) = \frac{1}{n!} \frac{d^n \ln \psi_{Z_{st} (\zeta; \mathcal{R})} (js)}{ds^n} \bigg|_{s=0} = \frac{2\pi \lambda \mu_X (n)}{n\nu - 2} \left( R_s^{2-n\nu} - R_t^{2-n\nu} \right) F_Y (R_s^{2\nu} \zeta) + \zeta^{\frac{n\nu-2}{2\nu}} \left( \frac{2-n\nu}{2\nu}, R_s^{2\nu} \zeta, R_t^{2\nu} \zeta \right) - R_t^{2-n\nu} \left( 0, R_s^{2\nu} \zeta, R_t^{2\nu} \zeta \right) \right].
\]
Using the cumulant of $Z_{st} (\zeta; \mathcal{R})$, we obtain the $n$th cumulant of the cognitive network interference $l_{nt}$ for the single-threshold protocol as follows:
\[
\kappa_{l_{nt}} (n) = P_1^{n/2} \kappa_{Z_{st} (\zeta; \mathcal{R})} (n).
\]

2) Multiple-Threshold Protocol: Using (5), the per-zone cognitive interference generated by the secondary users in the $k$th active zone $A_k$ can be written as
\[
l_{mt, k} = \sqrt{P_{1, k}} \sum_{x \in A_k} r_i^{-\nu} X_i,
\]
where $P_{1, k}$ is the transmitted power of the secondary users in the $k$th active zone $A_k$ and
\[
A_k = \{ i \in \mathcal{R} : I_{[i_{k-1}, i_k]} (r_i^{2-\nu} Y_i) = 1 \}.
\]

Similar to (10), the CF of $Z_{k} (\mathcal{R})$ can be expressed as
\[
\psi_{Z_{k} (\mathcal{R})} (js) = \exp \left( 2\pi \lambda \int_X \int_Y \int_{R_s} \int_{R_t} \left[ \exp (j s x r r^{-\nu}) - 1 \right] r^{-2\nu} f_X (x) f_Y (y) r dr dy dx \right).
\]

The cognitive network interference generated by the secondary users in all the $N$ active zones is then given by
\[
l_{mt} = \sum_{k=1}^{N} \sqrt{P_{1, k}} Z_{k} (\mathcal{R}).
\]
Since all the $Z_k (\mathcal{R})$'s are statistically independent, we obtain the $n$th cumulant of the cognitive network interference $l_{mt}$ for the multiple-threshold protocol as
\[
\kappa_{l_{mt}} (n) = \sum_{k=1}^{N} P_{1, k}^{n/2} \kappa_{Z_k (\mathcal{R})} (n),
\]
where $\kappa_{Z_k (\mathcal{R})} (n)$ are given by (11), (18), and (20) for $k = 1, k = 2, 3, \ldots, N - 1$, and $k = N$, respectively.

B. Truncated-Stable Distribution Approximation

The truncated-stable distributions are a relatively new class of distributions that follow from the class of stable distributions [8]. The attractiveness of using stable distributions to model interference in wireless networks are: 1) the ability to capture the spatial distribution of the interfering nodes; and 2) the ability to accommodate heavy tails that exhibit the dominant contribution of a few interferers in the vicinity of the primary user [9]. However, as shown in [1], the aggregate interference converges to a stable distribution only if the number of interferers is unbounded and if $r_i \in [0, \infty]$. The main drawback of this model is that, with small but nonzero probability, the distance $r$ can be zero and hence, the interference power can be infinite. This singularity at $r = 0$ is taken account of in the stable distributions with unbounded (infinite) second or higher order moments. However, the truncated-stable distributions with smoothed tails and finite moments can offer an alternative statistical tool to model the aggregate interference in more realistic scenarios where this singularity can be avoided.

The CF of a symmetric truncated-stable random variable $T \sim S_t (\gamma', \alpha, g)$ is given by [11]
\[
\psi_T (js) = \exp \left( \gamma' \Gamma (-\alpha) \left[ \frac{g (g + js)^{\alpha}}{2} + \frac{(g + js)^{\alpha} - g^\alpha}{2} \right] \right),
\]
where $\Gamma (\cdot)$ is the Euler’s gamma function; and $\gamma', \alpha$ and $g$ are the parameters associated with the truncated-stable distribution. The parameters $\gamma'$ and $\alpha$ are akin to the dispersion and the characteristic exponent of the stable distribution, respectively. The parameter $g$ is the argument of the exponential function used to smooth the tail of the stable distribution. The $n$th cumulant of the truncated-stable distribution can be obtained using (21) as
\[
\frac{1}{j^n} \frac{d^n \ln \psi_T (js)}{ds^n} \bigg|_{s=0} = \gamma' \Gamma (-\alpha) g^{\alpha-n} \prod_{i=0}^{n-1} (\alpha - i).
\]
To approximate the cognitive network interference to the truncated-stable distribution, we first fix the characteristic exponent to $\alpha = 2/\nu$. This choice is motivated by the fact that as $R_s \to 0$ and $R_t \to \infty$, the cognitive network interference follows a stable distribution with the characteristic exponent $\alpha = 2/\nu$. Let $l_A$ be the cognitive network interference corresponding to the activity model $A \in \{sa, st, mt\}$, i.e., full activity, the single-threshold protocol, or the multiple-threshold protocol. Then, we can approximate the cognitive network interference $l_A$ to the symmetric truncated-stable random variable as $l_A \sim S_t (\gamma'_A, \alpha = 2/\nu, g_A)$, where the
\[
\kappa_{z_n}(\mathcal{R}) (n) = \frac{2\pi \lambda \mu_X (n)}{n \nu - 2} \left[ R_s^{2-n\nu} \mu_{\gamma}^{(pt)} \left(0, R_s^{2\nu} \zeta_{k-1}, \Delta_{\min}\right) - \frac{\nu-2}{\zeta_{k-1}} \mu_{\gamma}^{(pt)} \left(2 - \frac{n\nu}{2\nu}, R_s^{2\nu} \zeta_{k-1}, \Delta_{\min}\right) \right] \\
+ c_1 \mu_{\gamma}^{(pt)} (c_2, \Delta_{\min}, \Delta_{\max}) + \frac{\nu-2}{\zeta_{k}} \mu_{\gamma}^{(pt)} \left(2 - \frac{n\nu}{2\nu}, \Delta_{\max}, R_s^{2\nu} \zeta_{k}\right) \\
- R_s^{2-n\nu} \mu_{\gamma}^{(pt)} \left(0, \Delta_{\max}, R_s^{2\nu} \zeta_{k}\right),
\]

where \(\Delta_{\min} = \min \{ R_t^{2\nu} \zeta_{k-1}, R_s^{2\nu} \zeta_{k}\}, \Delta_{\max} = \max \{ R_t^{2\nu} \zeta_{k-1}, R_s^{2\nu} \zeta_{k}\}, and

\[
(c_1, c_2) = \begin{cases} 
\left( R_s^{2-n\nu} - R_t^{2-n\nu}, 0 \right), & \text{if } R_s^{2\nu} \zeta_{k} \geq R_t^{2\nu} \zeta_{k-1}, \\
\left( \frac{\nu-2}{\zeta_{k}} - \frac{2-n\nu}{2\nu}, \frac{2-n\nu}{2\nu} \right), & \text{if } R_s^{2\nu} \zeta_{k} < R_t^{2\nu} \zeta_{k-1}.
\end{cases}
\] 

\[
\kappa_{z_n}(\mathcal{R}) (n) = \frac{2\pi \lambda \mu_X (n)}{n \nu - 2} \left[ R_s^{2-n\nu} \mu_{\gamma}^{(pt)} \left(0, R_s^{2\nu} \zeta_{k-1}, R_t^{2\nu} \zeta_{k-1}\right) - \frac{\nu-2}{\zeta_{k-1}} \mu_{\gamma}^{(pt)} \left(2 - \frac{n\nu}{2\nu}, R_s^{2\nu} \zeta_{k-1}, R_t^{2\nu} \zeta_{k-1}\right) \right] \\
+ \left( R_s^{2-n\nu} - R_t^{2-n\nu} \right) \bar{F}_{\gamma} \left(R_t^{2\nu} \zeta_{k-1}\right),
\]

The CF of \(Z_{st}(\zeta; \mathcal{R}_{\ell})\) can be expressed as

\[
\begin{align*}
\psi_{Z_{st}(\zeta; \mathcal{R}_{\ell})}(js) & = \exp \left( \theta_\ell \lambda \int \int_{\mathcal{R}_{\ell}} \exp \left( jsx + \nu \gamma r \right) - 1 \right) \\
& \times \delta_{[0, \zeta]} \left( r^{2-\nu} \right) f_x(x) f_y(y) r dr dy dx.
\end{align*}
\]

where \(\gamma\) is the angle covered by \(\mathcal{R}_{\ell}\); and \(\theta_\ell\) is the angle covered by the obstacle. For a single obstacle placed at distance \(d\) from the origin, we have two thresholds (i.e., \(N = 3\)).
two subregions in front and behind the obstacle: \((a_1, b_1) = (R_s, d)\) and \((a_2, b_2) = (d, R_t)\), respectively. The \(n\)th cumulant of the cognitive network interference for the single-threshold protocol in the presence of shadowing can be written as

\[
\kappa_{l_{st}}(n) = \sum_{\ell=1}^{L} P_{n/2}^{\ell} \kappa_{Z_{st}}(\zeta \beta_{l}; R_t) (n) \tag{28}
\]

where the cumulant \(\kappa_{Z_{st}}(\zeta \beta_{l}; R_t) (n)\) is obtained from \(\kappa_{Z_{st}}(\zeta; R_t) (n)\) in (11) by setting \(\zeta, 2\pi, R_s,\) and \(R_t = \zeta \beta_{l}, \theta_{l}, a_{l},\) and \(b_{l}\), respectively. Fig. 2 shows realization snapshots of secondary users activated by the single-threshold protocol in the presence of shadowing.

### C. BEP Analysis

Consider a binary phase-shift keying (BPSK) narrowband system in the presence of interference generated by the cognitive network confined within the region \(R\). The nodes of the cognitive network are active according to (3). The decision variable of the primary received symbol after the correlation receiver can be written as

\[
V = GU \sqrt{E_b} + l_{st} + W, \tag{29}
\]

where \(G\) is the channel fading affecting the victim signal; \(U \in \{1, -1\}\) is the information data; \(E_b\) is the energy per bit; and \(W\) is the zero-mean additive white Gaussian noise with variance \(N_0/2\). Conditioned on \(G, l_{st},\) and \(U = +1\), the CF of the decision variable \(V\) can be written as

\[
\psi_V (js|G, l_{st}, U = +1) = \exp \left( js \left( G \sqrt{E_b} + l_{st} - \frac{N_0 s^2}{4} \right) \right). \tag{30}
\]

Assuming that \(G\) and \(l_{st}\) are statistically independent, the CF of the decision variable conditioned on \(U = +1\) is given by

\[
\psi_V (js|U = +1) = \psi_G (js \sqrt{E_b}) \psi_{l_{st}} (js) \exp \left( -\frac{N_0 s^2}{4} \right). \tag{31}
\]

For the cognitive network interference \(l_{st}\), we use the symmetric truncated-stable approximation \(l_{st} \sim S_t (\gamma_{st}, \alpha = 2/\nu, \gamma_{st})\), where the parameters \(\gamma_{st}\) and \(\gamma_{st}\) are determined by using (23) and (24), respectively. Since \(l_{st}\) is approximated as a symmetric random variable, the average BEP \(P_e\) is equal to the BEP conditioned on \(U = +1\), which can be expressed, using the inversion theorem [12], as

\[
P_e = \frac{1}{2} \int_0^{\infty} \frac{\psi_V (js|U = +1) + \psi_V (-js|U = +1)}{js} \, ds. \tag{32}
\]

### IV. Numerical Results

In this section, we illustrate how our interference model can provide insight into the coexistence of primary and secondary networks. In numerical examples, we consider \(R_s = 1\) meter, \(R_t = 60\) meters, \(\nu = 1\). We first investigate the effect of the cognitive network interference on the BEP performance of the primary user. In Fig. 3, the BEP of BPSK versus \(E_b/N_0\) in the presence of the cognitive network interference \(l_{st}\) for the single-threshold protocol when \(\text{SIR} = -16, -12,\) and \(-8\) dB. \(\lambda = 0.1\) and \(\zeta = -40\) dBm, and Rayleigh fading for primary and secondary links. For comparison, the BEP in the absence of interference is also plotted (black line).

Fig. 3. BEP of BPSK versus \(E_b/N_0\) in the presence of the cognitive network interference \(l_{st}\) for the single-threshold protocol when \(\text{SIR} = -16, -12,\) and \(-8\) dB. \(\lambda = 0.1\) and \(\zeta = -40\) dBm, and Rayleigh fading for primary and secondary links. For comparison, the BEP in the absence of interference is also plotted (black line).
of the maximum distance $R_t$ from the primary user. This example reveals that for fixed threshold $\zeta$, as the fading parameter $m$ increases (less severe fading), the cognitive interference vanishes in the vicinity of the primary user due to rare secondary activity. We can also see that less severe fading (i.e., larger $m$) reduces the cognitive interference power for all the values of $R_t$. This is due to the fact that more severe fading increases the secondary activity in the proximity of the primary user, leading consequently to a higher cognitive interference power. This finding is in contrast to common folklore that more severe fading on the interference signals would be beneficial. Moreover, we observe that the aggregate average interference power tends to saturate since secondary users located far from the primary user contribute marginally to cognitive interference.

An important insight is given in Fig. 5 where the interference power is plotted versus the amplitude path-loss coefficient. As shown in this figure, the cognitive network interference exhibits a non-monotonic behavior with an increasing path-loss coefficient. Similarly to the channel fading, a variation of the path-loss coefficient affects both the interference signal power and the primary signal detection. Under certain condition of $m$, $R_f$, and $\zeta$, an higher path-loss coefficient might lead to an higher activity of the nodes that is not compensated by a reduction of the interference signal power.

\[ V_m = \frac{P_m - P_s}{P_s} \]

\[ f_m = \frac{R_m}{R_s} \]

\[ \frac{\nu}{m} \sim \text{Nakagami}(m, 1) \]

\[ \sqrt{\nu} \sim \text{Nakagami}(2, 1) \]

\[ |H_i| \sim \text{Nakagami}(m, 1) \]

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