MIMO Networks: The Effects of Interference

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Marco Chiani, *Senior Member, IEEE*, Moe Z. Win, *Fellow, IEEE*, and Hyundong Shin, *Member, IEEE*

***Abstract*—Multiple-input multiple-output (MIMO) systems are being considered as one of the key enabling technologies for future wireless networks. However, the decrease in capacity due to the presence of interferers in MIMO networks is not well understood. In this paper, we develop an analytical framework to characterize the capacity of MIMO communication systems in the presence of multiple MIMO co-channel interferers and noise. We consider the situation in which transmitters have no channel state information, and all links undergo Rayleigh fading. We first generalize the determinant representation of hypergeometric functions with matrix arguments to the case when the argument matrices have eigenvalues of arbitrary multiplicity. This enables the derivation of the distribution of the eigenvalues of Gaussian quadratic forms and Wishart matrices with arbitrary correlation, with application to both single-user and multiuser MIMO systems. In particular, we derive the ergodic mutual information for MIMO systems in the presence of multiple MIMO interferers. Our analysis is valid for any number of interferers, each with arbitrary number of antennas having possibly unequal power levels. This framework, therefore, accommodates the study of distributed MIMO systems and accounts for different spatial positions of the MIMO interferers.**

***Index Terms*—Eigenvalues distribution, Gaussian quadratic forms, hypergeometric functions of matrix arguments, interference, multiple-input multiple-output (MIMO), Wishart matrices.**

I. Introduction

THE use of multiple transmitting and receiving antennas can provide high spectral efficiency and link reliability for point-to-point communication in fading environments [1], [2]. The analysis of capacity for multiple-input multiple-output (MIMO) channels in [3] suggested practical receiver structures to obtain such spectral efficiency. Since then, many studies have been devoted to the analysis of MIMO systems, starting from the ergodic [4] and outage [5] capacity for uncorrelated fading to the case where correlation is present at one of the two sides (either at the transmitter or at the receiver) or at both sides [6]–[8]. The effect of time correlation is studied in [9].

Only a few papers, by using simulation or approximations, have studied the capacity of MIMO systems in the presence of co-channel interference. In particular, a simulation study is presented in [10] for cellular systems, assuming up to three transmit and three receive antennas. The simulations showed that co-channel interference can seriously degrade the overall capacity when MIMO links are used in cellular networks. In [11] and [12] it is studied whether, in a MIMO multiuser scenario, it is always convenient to use all transmitting antennas. It was found that for some values of signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR), allocating all power into a single transmitting antenna, rather than dividing the power equally among independent streams from the different antennas, would lead to a higher overall system mutual information. The study in [11], [12] adopts simulation to evaluate the capacity of MIMO systems in the presence of co-channel interference, and the difficulties in the evaluations limited the results to a scenario with two MIMO users employing at most two antenna elements. In [13] the replica method is used to obtain approximate moments of the capacity for MIMO systems with large number of antenna elements including the presence of interference. The approximation requires iterative numerical methods to solve a system of non-linear equations, and its accuracy has to be verified by computer simulations. A multiuser MIMO system with specific receiver structures is analyzed for the interference-limited case in [14] and [15].

The MIMO capacity at high and low SNR for interference-limited scenarios is addressed in [16] and [17]. A worst-case analysis for MIMO capacity with channel state information (CSI) both at the transmitter and receiver, conditioned on the channel matrix, can be found in [18]. Asymptotic results for the Rician channel in the presence of interference can be found in [19].

In this paper, we develop a framework to analyze the ergodic capacity of MIMO systems in the presence of multiple MIMO co-channel interferers and additive white Gaussian noise (AWGN). We consider rich scattering environments in which transmitters have no CSI, the receiver has perfect CSI, and all links undergo frequency flat Rayleigh fading. The key contributions of the paper are as follows.

- Generalization of the determinant representation of hypergeometric functions with matrix arguments to the case where matrices in the arguments have eigenvalues with arbitrary multiplicity.
- Derivation, using the generalized representation, of the joint probability distribution function (pdf) of the eigenvalues of complex Gaussian quadratic forms and Wishart

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matrices, with arbitrary multiplicities for the eigenvalues of the associated covariance matrix.

- Derivation of the ergodic capacity of single-user MIMO systems that accounts for arbitrary power levels and arbitrary correlation across the transmitting antenna elements, or arbitrary correlation at the receiver side.
- Derivation of capacity expressions for MIMO systems in the presence of multiple MIMO interferers, valid for any number of interferers, each with arbitrary number of antennas having possibly unequal power levels.

The paper is organized as follows. In Section II, we introduce the system model for multiuser MIMO setting, relating the ergodic capacity of MIMO systems in the presence of multiple MIMO interferers to that of single-user MIMO systems with no interference. General results on hypergeometric functions of matrix arguments are given in Section III. The joint probability density function (pdf) of eigenvalues for Gaussian quadratic forms and Wishart matrices with arbitrary correlation is given in Section IV. In Section V, we give a unified expression for the capacity of single-user MIMO systems that accounts for arbitrary correlation at one side. Numerical results for MIMO relay networks and multiuser MIMO are presented in Section VI, and conclusions are given in Section VII.

Throughout the paper vectors and matrices are indicated by bold, $|\mathbf{A}|$ and $\det \mathbf{A}$ denote the determinant of matrix $\mathbf{A}$, and $a_{i,j}$ is the (i,j)th element of $\mathbf{A}$. Expectation operator is denoted by $\mathbb{E}\{\cdot\}$, and in particular $\mathbb{E}_X\{\cdot\}$ denotes expectation with respect to the random variable X. The superscript † denotes conjugation and transposition, $\mathbf{I}$ is the identity matrix (in particular $\mathbf{I}_n$ refers to the $(n \times n)$ identity matrix), $\mathrm{tr}\{\mathbf{A}\}$ is the trace of $\mathbf{A}$ and $\oplus$ is used for the direct sum of matrices defined as $\mathbf{A} \oplus \mathbf{B} = \mathrm{diag}(\mathbf{A}, \mathbf{B})$ [20].

II. System Models

We consider a network scenario as shown in Fig. 1, where a MIMO-$(N_{\mathrm{T}0}, N_{\mathrm{R}})$ link, with $N_{\mathrm{T}0}$ and N_{R} denoting the numbers of transmitting and receiving antennas, respectively, is subject to N_{I} MIMO co-channel interferers from other links, each with arbitrary number of antennas. The N_{R}-dimensional equivalent lowpass signal $\mathbf{y}$, after matched filtering and sampling, at the output of the receiving antennas can be written as

$$\mathbf{y} = \mathbf{H}_0 \mathbf{x}_0 + \sum_{k=1}^{N_{\mathrm{I}}} \mathbf{H}_k \mathbf{x}_k + \mathbf{n} \tag{1}$$

where $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{N_{\mathrm{I}}}$ denote the complex transmitted vectors with dimensions $N_{\mathrm{T}0}, N_{\mathrm{T}1}, \ldots, N_{\mathrm{T}N_{\mathrm{I}}}$, respectively. Subscript 0 is used for the desired signal, while subscripts $1, \ldots, N_{\mathrm{I}}$ are for the interferers. The additive noise $\mathbf{n}$ is an N_{R}-dimensional random vector with zero-mean independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian entries, each with independent real and imaginary parts having variance $\sigma^2/2$, so that $\mathbb{E}\{\mathbf{n}\mathbf{n}^\dagger\} = \sigma^2 \mathbf{I}$. The power transmitted from the kth user is $\mathbb{E}\{\mathbf{x}_k^\dagger \mathbf{x}_k\} = P_k$.

The matrices $\mathbf{H}_k$ in (1) denote the channel matrices of size $(N_{\mathrm{R}} \times N_{\mathrm{T}k})$ with complex elements $h_{i,j}^{(k)}$ describing the gain of the radio channel between the jth transmitting antenna of the kth MIMO interferers and the ith receiving antenna of the desired link. In particular, $\mathbf{H}_0$ is the matrix describing the channel of the desired link (see Fig. 1).

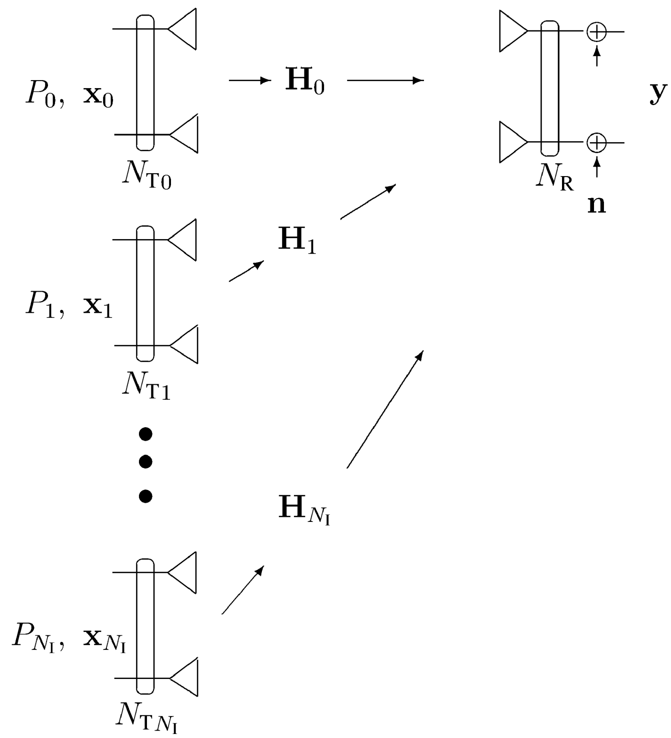

Fig. 1. MIMO network.

When considering statistical variations of the channel, the channel gains must be described as random variables (r.v.s). In particular, we assume uncorrelated MIMO Rayleigh fading channels for which the entries of $\mathbf{H}_k$ are i.i.d. circularly symmetric complex Gaussian r.v.s with zero-mean and variance one, i.e., $\mathbb{E}\{|h_{i,j}^{(k)}|^2\} = 1$. With this normalization, P_k represents the short-term average received power per antenna element from user k, which depends on the transmit power, path-loss, and shadowing between transmitter k and the (interfered) receiver. Thus, the P_k are in general different.

Conditioned to the channel matrices $\{\mathbf{H}_k\}_{k=0}^{N_{\mathrm{I}}}$, the mutual information between the received vector, $\mathbf{y}$, and the desired transmitted vector, $\mathbf{x}_0$, is

$$\begin{aligned}\mathcal{I}\left(\mathbf{x}_0; \mathbf{y} \,|\, \{\mathbf{H}_k\}_{k=0}^{N_{\mathrm{I}}}\right) &= \mathcal{H}\left(\mathbf{y} \,|\, \{\mathbf{H}_k\}_{k=0}^{N_{\mathrm{I}}}\right) \\ &\quad - \mathcal{H}\left(\mathbf{y} \,|\, \mathbf{x}_0, \{\mathbf{H}_k\}_{k=0}^{N_{\mathrm{I}}}\right)\end{aligned} \tag{2}$$

where $\mathcal{H}(\cdot)$ denotes differential entropy [21].

Here we consider the scenario in which the receiver has perfect CSI, and all the transmitters have no CSI. Note that the term CSI includes the information about the channels associated with all other MIMO interfering users. In this case, since the users do not know what is the interference seen at the receiver (if any), a reasonable strategy is that each user transmits circularly symmetric Gaussian vector signals with zero mean and i.i.d. elements. Thus, the transmit power per antenna element of the kth user is $P_k/N_{\mathrm{T}k}$. Note that this model includes the case in which the power levels of the individual antennas are different: it suffices to decompose a transmitter into virtual subtransmitters, each with the proper power level.

Hence, conditioned on all channel matrices $\{\mathbf{H}_k\}_{k=0}^{N_\mathrm{I}}$ in (1), both $\mathbf{y}$ and $\mathbf{y}\,|\,\mathbf{x}_0$ are circularly symmetric Gaussian. Since the differential entropy of a Gaussian vector is proportional to the logarithm of the determinant of its covariance matrix, we obtain the conditional mutual information

$$C_\mathrm{MU}\left(\{\mathbf{H}_k\}_{k=0}^{N_\mathrm{I}}\right) = \log\frac{\det\mathbf{K}_\mathbf{y}}{\det\mathbf{K}_{\mathbf{y}\,|\,\mathbf{x}_0}} \tag{3}$$

where $\mathbf{K}_\mathbf{y}$ and $\mathbf{K}_{\mathbf{y}\,|\,\mathbf{x}_0}$ respectively denote the covariance matrices of $\mathbf{y}$ and $\mathbf{y}\,|\,\mathbf{x}_0$, conditioned on the channel gains $\{\mathbf{H}_k\}_{k=0}^{N_\mathrm{I}}$. By expanding the covariance matrices using (1), the conditional mutual information of a MIMO link in the presence of multiple MIMO interferers, with CSI only at the receiver, is then given by

$$C_\mathrm{MU}\left(\{\mathbf{H}_k\}_{k=0}^{N_\mathrm{I}}\right) = \log\frac{\det\left(\mathbf{I}_{N_\mathrm{R}} + \tilde{\mathbf{H}}\tilde{\mathbf{\Psi}}\tilde{\mathbf{H}}^\dagger\right)}{\det\left(\mathbf{I}_{N_\mathrm{R}} + \mathbf{H}\mathbf{\Psi}\mathbf{H}^\dagger\right)} \tag{4}$$

where the $N_\mathrm{R}\times(\sum_{i=1}^{N_\mathrm{I}} N_{\mathrm{T}i})$ matrix $\mathbf{H}$ is

$$\mathbf{H} = [\mathbf{H}_1|\mathbf{H}_2|\cdots|\mathbf{H}_{N_\mathrm{I}}]$$

the $N_\mathrm{R}\times(\sum_{i=0}^{N_\mathrm{I}} N_{\mathrm{T}i})$ matrix $\tilde{\mathbf{H}}$ is

$$\tilde{\mathbf{H}} = [\mathbf{H}_0\,|\,\mathbf{H}]$$

and the covariance matrices $\mathbf{\Psi}$, $\tilde{\mathbf{\Psi}}$ are

$$\mathbf{\Psi} = \varrho_1\mathbf{I}_{N_{\mathrm{T}1}} \oplus \varrho_2\mathbf{I}_{N_{\mathrm{T}2}} \oplus\cdots\oplus \varrho_{N_\mathrm{I}}\mathbf{I}_{N_{\mathrm{T}N_\mathrm{I}}} \tag{5}$$

and

$$\tilde{\mathbf{\Psi}} = \varrho_0\mathbf{I}_{N_{\mathrm{T}0}} \oplus \mathbf{\Psi} \tag{6}$$

with

$$\varrho_i = \frac{P_i}{N_{\mathrm{T}i}\sigma^2}. \tag{7}$$

For random channel matrices the mutual information in (4) is the difference between random variables of the form $\log\det(\mathbf{I}+\mathbf{H}\mathbf{\Phi}\mathbf{H}^\dagger)$ where the elements of $\mathbf{H}$ are i.i.d. complex Gaussian and $\mathbf{\Phi}$ is a covariance matrix. The statistics of such random variables have been investigated in [6]–[8], assuming that the eigenvalues of $\mathbf{\Phi}$ were distinct. However, in the scenario under analysis these results cannot be used directly, since in (4) each eigenvalue ϱ_i of $\mathbf{\Psi}$ and $\tilde{\mathbf{\Psi}}$ has multiplicity $N_{\mathrm{T}i}$.

We consider the ergodic mutual information as a performance measure: taking the expectation of (4) with respect to the distribution of $\{\mathbf{H}_k\}_{k=0}^{N_\mathrm{I}}$, we get

$$\begin{aligned}\mathcal{C}_\mathrm{MU} &\triangleq \mathbb{E}\left\{C_\mathrm{MU}\left(\{\mathbf{H}_k\}_{k=0}^{N_\mathrm{I}}\right)\right\}\\ &= \mathcal{C}_\mathrm{SU}\left(\sum_{i=0}^{N_\mathrm{I}} N_{\mathrm{T}i}, N_\mathrm{R}, \tilde{\mathbf{\Psi}}\right) - \mathcal{C}_\mathrm{SU}\left(\sum_{i=1}^{N_\mathrm{I}} N_{\mathrm{T}i}, N_\mathrm{R}, \mathbf{\Psi}\right)\end{aligned} \tag{8}$$

where $\mathcal{C}_\mathrm{SU}(n_\mathrm{T}, n_\mathrm{R}, \mathbf{\Phi}) \triangleq \mathbb{E}_\mathbf{H}\{\log\det(\mathbf{I}_{n_\mathrm{R}} + \mathbf{H}\mathbf{\Phi}\mathbf{H}^\dagger)\}$ denotes the ergodic mutual information of a single-user MIMO-$(n_\mathrm{T}, n_\mathrm{R})$ Rayleigh fading channel with unit noise variance per receiving antenna and channel covariance matrix $\mathbf{\Phi}$ at the transmitter.

Note that the "building block" $\mathbb{E}_\mathbf{H}\{\log\det(\mathbf{I}+\mathbf{H}\mathbf{\Phi}\mathbf{H}^\dagger)\}$ is simple to evaluate when the covariance matrix $\mathbf{\Phi}$ is proportional to an identity matrix, which corresponds to a typical interference-free case with equal transmit power among all transmitting antennas (see, e.g., [4]). In contrast, in the presence of interference, the covariance matrix is of the type indicated in (5) and (6), where the power levels of the different users are in general different. Note that even when the power for the ith user is equally spread over the $N_{\mathrm{T}i}$ antennas, the matrices in (5) and (6) are generally not proportional to identity matrices and their eigenvalues have multiplicities greater than one. Therefore, studying MIMO systems in the presence of multiple MIMO co-channel interferers requires the characterization of $\mathcal{C}_\mathrm{SU}(n_\mathrm{T}, n_\mathrm{R}, \mathbf{\Phi})$ in a general setting in which the covariance matrix $\mathbf{\Phi}$ has eigenvalues of arbitrary multiplicities.

To this aim, we derive in the next sections simple expressions for the hypergeometric functions of matrix arguments with not necessarily distinct eigenvalues; then, we obtain the joint pdf of the eigenvalues of central Wishart matrices as well as that of Gaussian quadratic forms with arbitrary covariance matrix.

III. Hypergeometric Functions With Matrix Arguments Having Arbitrary Eigenvalues

Hypergeometric functions with matrix arguments have been used extensively in multivariate statistical analysis, especially in problems related to the distribution of random matrices [22], [23]. These functions are defined in terms of a series of zonal polynomials, and, as such, they are functions only of the eigenvalues (or latent roots) of the argument matrices [22], [23].

Definition 1: The hypergeometric functions of two Hermitian $(m\times m)$ matrices $\mathbf{\Lambda}$ and $\mathbf{W}$ are defined by [22]

$$\begin{aligned}&{}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;\mathbf{\Lambda},\mathbf{W})\\ &\qquad\triangleq \sum_{k=0}^{\infty}\sum_{\kappa}\frac{(a_1)_\kappa\cdots(a_p)_\kappa}{(b_1)_\kappa\cdots(b_q)_\kappa}\frac{C_\kappa(\mathbf{\Lambda})C_\kappa(\mathbf{W})}{k!C_\kappa(\mathbf{I}_m)}\end{aligned} \tag{9}$$

where $C_\kappa(\cdot)$ is a symmetric homogeneous polynomial of degree k in the eigenvalues of its argument, called *zonal polynomial*, the sum $\sum_\kappa$ is over all partitions of k, i.e., $\kappa = (k_1,\ldots,k_m)$ with $k_1\geq k_2\geq\cdots\geq k_m\geq 0$, $k_1+k_2+\cdots+k_m = k$, and the generalized hypergeometric coefficient $(a)_\kappa$ is given by $(a)_\kappa = \prod_{i=1}^m(a-\frac{1}{2}(i-1))_{k_i}$ with $(a)_k = a(a+1)\cdots(a+k-1)$, $(a)_0 = 1$.

We remark that zonal polynomials are symmetric polynomials in the eigenvalues of the matrix argument. Therefore, hypergeometric functions are only functions of the eigenvalues of their matrix arguments. In other words, without loss of generality we can replace $\mathbf{\Lambda}$ and $\mathbf{W}$ with the diagonal matrices $\mathrm{diag}(\lambda_1,\ldots\lambda_m)$ and $\mathrm{diag}(w_1,\ldots w_m)$, where λ_i and w_j are the eigenvalues of $\mathbf{\Lambda}$ and $\mathbf{W}$, respectively. Clearly the order of $\mathbf{\Lambda}$ and $\mathbf{W}$ is unimportant.

It is quite evident that these functions expressed as a series of zonal polynomials are in general very difficult to manage and the form of (9) is not tractable for further analysis. Fortunately, when the eigenvalues of $\mathbf{\Lambda}$ and $\mathbf{W}$ are all distinct, a simpler expression in terms of determinants of matrices whose elements are hypergeometric functions of scalar arguments can be obtained as follows [24, Lemma 3]:

Lemma 1: Let $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \dots \lambda_m)$ and $\mathbf{W} = \mathrm{diag}(w_1, \dots w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_m$. Then we have

$$ {}_p\tilde{F}_q(a_1, \dots, a_p; b_1, \dots, b_q; \mathbf{\Lambda}, \mathbf{W}) = \Gamma_{(m)}(m) \frac{\psi_q^{(m)}(\mathbf{b})}{\psi_p^{(m)}(\mathbf{a})} \frac{|\mathbf{G}|}{\prod_{i<j}(\lambda_i - \lambda_j) \prod_{i<j}(w_i - w_j)} \tag{10} $$

where $\Gamma_{(m)}(n) \triangleq \prod_{i=1}^{m}(n-i)!$, $\psi_q^{(m)}(\mathbf{b}) = \prod_{i=1}^{m}\prod_{j=1}^{q}(b_j - i + 1)^{i-1}$ and the ijth element of the $(m \times m)$ matrix $\mathbf{G}$ is defined in terms of hypergeometric functions of scalar arguments as follows

$$ g_{i,j} = {}_pF_q\left(\tilde{a}_1, \dots, \tilde{a}_p; \tilde{b}_1, \dots, \tilde{b}_q; \lambda_i w_j\right) \tag{11} $$

where $\tilde{a}_i = a_i - m + 1$, and $\tilde{b}_i = b_i - m + 1$.
Important particular cases are

$$ {}_0\tilde{F}_0(\mathbf{\Lambda}, \mathbf{W}) = \Gamma_{(m)}(m) \frac{|\mathbf{G}_0|}{\prod_{i<j}(\lambda_i - \lambda_j) \prod_{i<j}(w_i - w_j)} \tag{12} $$

and

$$ {}_1\tilde{F}_0(r; \mathbf{\Lambda}, \mathbf{W}) = \frac{\Gamma_{(m)}(m)}{\psi_1^{(m)}(r)} \frac{|\mathbf{G}_1|}{\prod_{i<j}(\lambda_i - \lambda_j) \prod_{i<j}(w_i - w_j)} \tag{13} $$

where the ijth elements of $\mathbf{G}_0$ and $\mathbf{G}_1$ are given by $e^{\lambda_i w_j}$ and $(1 - \lambda_i w_j)^{m-r-1}$, respectively.

These expressions have recently been used to study the distribution of Gaussian quadratic forms, to express the pdf of the eigenvalues of Wishart matrices, and to analyze the information-theoretic capacity and error rates of communication systems involving multiple antennas [5]–[8], [25]–[31]. However, it is important to underline that Lemma 1 requires the eigenvalues of the matrices to be all distinct.

Here, we generalize Lemma 1 to include the case where the eigenvalues are not necessarily distinct. To this aim we first need the following lemma.

Lemma 2: Let $P : \mathbf{A} \to \mathbb{R}$ be defined over $\mathbf{A} \subset \mathbb{R}^m$ as follows:

$$ P(w_1, \dots, w_m) \triangleq \frac{1}{\prod_{i<j}(w_i - w_j)} \times \begin{vmatrix} f_1(w_1) & f_1(w_2) & \cdots & f_1(w_m) \\ \vdots & \vdots & \cdots & \vdots \\ f_m(w_1) & f_m(w_2) & \cdots & f_m(w_m) \end{vmatrix} \tag{14} $$

where $w_1 > w_2 \cdots > w_m$, and the functions $f_i(w)$ have derivatives $f_i^{(n)}(w) = \frac{d^n f_i(w)}{dw^n}$ of orders at least $m-1$ throughout neighborhoods of the points $w_1, \dots, w_m$.

Then, the continuous extension $\breve{P}(w_1, w_2, \dots, w_m)$ of the function $P(w_1, w_2, \dots, w_m)$ to those points in $\mathbb{R}^m$ with L coincident arguments $w_K = w_{K+1} = \cdots w_{K+L-1}$ is obtained by removing the zero factors from the denominator in (14), replacing the columns of the matrix in (14) corresponding to the coincident arguments with the successive derivatives $f_i^{(L-l)}(w_K)$, $l = 1, \dots, L$, and then dividing by a scaling factor $\Gamma_{(L)}(L) = \prod_{i=1}^{L-1} i!$.

For example, for $w_1 = w_2 = \dots w_L$, this procedure gives (15) shown at the bottom of the page. More generally, a similar expression is valid if there are more groups of coinciding arguments: in this case, for each group of coincident arguments $w_K = \dots = w_{K+L-1}$ the correspondent columns of the matrix in (14) are to be replaced by $f_i^{(L-l)}(w_K)$, $l = 1, \dots, L$, with a scaling factor $\prod_{i=1}^{L-1} i!$.

Proof: See Appendix I. □

With Lemma 2 we can now generalize (10), (12), and (13).

Lemma 3: Let $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \dots \lambda_m)$ and $\mathbf{W} = \mathrm{diag}(w_1, \dots w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_k = w_{k+1} = \cdots = w_{k+L-1} > w_{k+L} > \cdots > w_m$. Then we have[1]

$$ {}_0\tilde{F}_0(\mathbf{\Lambda}, \mathbf{W}) = \frac{\Gamma_{(m)}(m)}{\Gamma_{(L)}(L)} \frac{|\mathbf{G}|}{\prod_{i<j}(\lambda_i - \lambda_j) \prod_{i<j, w_i \neq w_j}(w_i - w_j)} \tag{16} $$

[1]From here on, we will use the same symbols for the functions (10), (12), (13), and their continuous extension.

$$ \breve{P}(w_1, w_2, \dots, w_m) = \frac{1}{\prod_{i<j, w_i \neq w_j}(w_i - w_j) \prod_{i=1}^{L-1} i!} \times \begin{vmatrix} f_1^{(L-1)}(w_1) & f_1^{(L-2)}(w_1) & \cdots & f_1(w_1) & f_1(w_{L+1}) & \cdots & f_1(w_m) \\ \vdots & \vdots & & & \vdots & \cdots & \vdots \\ f_m^{(L-1)}(w_1) & f_m^{(L-2)}(w_1) & \cdots & f_m(w_1) & f_m(w_{L+1}) & \cdots & f_m(w_m) \end{vmatrix}. \tag{15} $$

where the elements of $\mathbf{G}$ are

$$g_{i,j} = \begin{cases} \lambda_i^{L-1+k-j} e^{\lambda_i w_k}, & j = k, \ldots, k+L-1 \\ e^{\lambda_i w_j}, & \text{elsewhere.} \end{cases} \quad (17)$$

That is, the matrix $\mathbf{G}$ is the same as that appearing in (12) except that the L columns corresponding to the coincident eigenvalues are $\lambda_i^{L-1} e^{\lambda_i w_k}, \lambda_i^{L-2} e^{\lambda_i w_k}, \ldots, \lambda_i^2 e^{\lambda_i w_k}, \lambda_i e^{\lambda_i w_k}, e^{\lambda_i w_k}$.

Proof: The proof is immediate by direct application of Lemma 2 with $f_i(w) = e^{\lambda_i w}$. □

Lemma 3 can be directly extended to more groups of coincident eigenvalues. In general, the rule is that each eigenvalue w of multiplicity $L > 1$ gives rise to L columns $\lambda_i^{L-1} e^{\lambda_i w}$, $\lambda_i^{L-2} e^{\lambda_i w}, \ldots, \lambda_i^2 e^{\lambda_i w}, \lambda_i e^{\lambda_i w}, e^{\lambda_i w}$ in the matrix $\mathbf{G}$ of (16), with the proper scaling factor $\Gamma_{(L)}(L)$.

Using Lemma 3 with $k = m-L+1$ and $w_k = 0$ results in the following corollary, valid for the case where some eigenvalues are equal to zero.

Corollary 1: Let $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots \lambda_m)$ and $\mathbf{W} = \text{diag}(w_1, \ldots w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_{m-L+1} = w_{m-L+2} = \cdots = w_m = 0$. Then we have

$$ {}_0\tilde{F}_0(\mathbf{\Lambda}, \mathbf{W}) = \frac{\Gamma_{(m)}(m)}{\Gamma_{(L)}(L)} \times \frac{|\mathbf{G}|}{\prod_{i<j}(\lambda_i - \lambda_j) \prod_{i<j\le m-L}(w_i - w_j) \prod_{i=1}^{m-L} w_i^L} \quad (18)$$

where the elements of $\mathbf{G}$ are as follows

$$g_{i,j} = \begin{cases} \lambda_i^{m-j}, & j = m-L+1, \ldots, m \\ e^{\lambda_i w_j}, & \text{elsewhere.} \end{cases} \quad (19)$$

We can apply a similar methodology to derive the general expression for ${}_1\tilde{F}_0(\cdot;\cdot,\cdot)$, as in the following Lemma.

Lemma 4: Let $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots \lambda_m)$ and $\mathbf{W} = \text{diag}(w_1, \ldots w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_k = w_{k+1} = \cdots = w_{k+L-1} > w_{k+L} > \cdots > w_m$. Then we have

$$ {}_1\tilde{F}_0(r; \mathbf{\Lambda}, \mathbf{W}) = \frac{\Gamma_{(m)}(m)}{\Gamma_{(L)}(L)} \frac{(-1)^{(L-1)L/2}}{\psi_1^{(m)}(r)} \times \frac{\gamma^{L-1}(\gamma-1)^{L-2}\cdots(\gamma-L+2)|\mathbf{A}|}{\prod_{i<j}(\lambda_i - \lambda_j)\prod_{i<j, w_i \ne w_j}(w_i - w_j)} \quad (20)$$

where $\gamma = m-r-1$ and the $(m \times m)$ matrix $\mathbf{A}$ has elements as shown in (21) at the bottom of the page. In other words, the matrix $\mathbf{A}$ is the same as that appearing in (13), except that the L columns corresponding to the L coincident eigenvalues are $\lambda_i^{L-1}(1-\lambda_i w_k)^{\gamma-(L-1)}, \ldots, \lambda_i(1-\lambda_i w_k)^{\gamma-1}, (1-\lambda_i w_k)^{\gamma}$.

Proof: For the proof we apply Lemma 2 with $f_i(w) = (1-\lambda_i w_k)^{\gamma}$, whose nth derivative is $f_i^{(n)}(w) = (-\lambda_i)^n \gamma(\gamma-1)\cdots(\gamma-n+1)(1-\lambda_i w_k)^{\gamma-n}$. □

Lemma 4 can be further generalized to more groups of coincident eigenvalues: each eigenvalue w of multiplicity $L > 1$ gives rise to L columns $\lambda_i^{L-1}(1-\lambda_i w)^{\gamma-(L-1)}, \ldots, \lambda_i^2(1-\lambda_i w)^{\gamma-2}, \lambda_i(1-\lambda_i w)^{\gamma-1}, (1-\lambda_i w)^{\gamma}$ in the matrix $\mathbf{A}$ of (20), and to a factor $(-1)^{(L-1)L/2}\gamma^{L-1}\cdots(\gamma-L+2)/\Gamma_{(L)}(L)$.

Using Lemma 4 with $k = m-L+1$ and $w_k = 0$ results in the following corollary.

Corollary 2: Let $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots \lambda_m)$ and $\mathbf{W} = \text{diag}(w_1, \ldots w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_{m-L+1} = w_{m-L+2} = \cdots = w_m = 0$. Then we have that (20) holds, with

$$a_{i,j} = \begin{cases} \lambda_i^{m-j}, & j = m-L+1, \ldots, m \\ (1-\lambda_i w_j)^{\gamma}, & \text{elsewhere.} \end{cases} \quad (22)$$

In other words, the matrix $\mathbf{A}$ has, in this case, the last L columns with elements $\lambda_i^{L-1}, \lambda_i^{L-2}, \ldots, \lambda_i, 1$.

Finally, we give the result for the ${}_p\tilde{F}_q(\cdot)$.

Lemma 5: Let $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots \lambda_m)$ and $\mathbf{W} = \text{diag}(w_1, \ldots w_m)$ with $\lambda_1 > \cdots > \lambda_m$ and $w_1 > \cdots > w_k = w_{k+1} = \cdots = w_{k+L-1} > w_{k+L} > \cdots > w_m$. Then we have

$$ {}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; \mathbf{\Lambda}, \mathbf{W}) = \Xi \frac{|\mathbf{C}|}{\prod_{i<j}(\lambda_i - \lambda_j)\prod_{i<j, w_i \ne w_j}(w_i - w_j)} \quad (23)$$

where the $(m \times m)$ matrix $\mathbf{C}$ has elements as follows

$$c_{i,j} = \lambda_i^{L-1+k-j} {}_p\tilde{F}_q(a_1 - m + L + k - j, \ldots, b_q - m + L + k - j; \lambda_i w_j) \quad (24)$$

for $j = k, \ldots, k+L-1$, and

$$c_{i,j} = {}_p\tilde{F}_q(\tilde{a}_1, \ldots, \tilde{a}_p; \tilde{b}_1, \ldots, \tilde{b}_q; \lambda_i w_j)$$

elsewhere. In (23) the constant Ξ is

$$\Xi = \frac{\Gamma_{(m)}(m)}{\Gamma_{(L)}(L)} \frac{\psi_q^{(m)}(\mathbf{b})}{\psi_q^{(m)}(\mathbf{a})} \prod_{i=1}^{L-1} \frac{(\tilde{a}_1)_i(\tilde{a}_2)_i \cdots (\tilde{a}_p)_i}{(\tilde{b}_1)_i(\tilde{b}_2)_i \cdots (\tilde{b}_q)_i}.$$

Proof: See Appendix I. □

IV. Gaussian Quadratic Forms With Covariance Matrix Having Eigenvalues of Arbitrary Multiplicity

We now derive the joint pdf of the eigenvalues for Gaussian quadratic forms and central Wishart matrices with arbitrary one-sided correlation matrix.

$$a_{i,j} = \begin{cases} \lambda_i^{L-1+k-j}(1-\lambda_i w_j)^{\gamma-(L-1+k-j)}, & j = k, \ldots, k+L-1 \\ (1-\lambda_i w_j)^{\gamma}, & \text{elsewhere.} \end{cases} \quad (21)$$

Lemma 6: Let $\mathbf{H}$ be a complex Gaussian $(p \times n)$ random matrix with circularly symmetric zero-mean, unit variance, i.i.d. entries and let $\mathbf{\Phi}$ be an $(n \times n)$ positive definite matrix. The joint pdf of the (real) non-zero ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{n_{\min}} \geq 0$ of the $(p \times p)$ quadratic form $\mathbf{W} = \mathbf{H\Phi H}^\dagger$ is given by

$$f_{\boldsymbol{\lambda}}(x_1, \ldots, x_{n_{\min}}) = K|\mathbf{V}(\mathbf{x})||\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})| \prod_{i=1}^{n_{\min}} x_i^{p-n_{\min}} \quad (25)$$

where $n_{\min} = \min(n, p)$, $\mathbf{V}(\mathbf{x})$ is the $(n_{\min} \times n_{\min})$ Vandermonde matrix with elements $v_{i,j} = x_j^{i-1}$,

$$K = \frac{(-1)^{p(n-n_{\min})}}{\Gamma_{(n_{\min})}(p)} \frac{\prod_{i=1}^{L} \mu_{(i)}^{m_i p}}{\prod_{i=1}^{L} \Gamma_{(m_i)}(m_i) \prod_{i<j} \left(\mu_{(i)} - \mu_{(j)}\right)^{m_i m_j}} \quad (26)$$

and $\mu_{(1)} > \mu_{(2)} \ldots > \mu_{(L)}$ are the L distinct eigenvalues of $\mathbf{\Phi}^{-1}$, with corresponding multiplicities $m_1, \ldots, m_L$ such that $\sum_{i=1}^{L} m_i = n$.

The $(n \times n)$ matrix $\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})$ has elements

$$g_{i,j} = \begin{cases} (-x_j)^{d_i} e^{-\mu_{(e_i)} x_j}, & j = 1, \ldots, n_{\min} \\ [n-j]_{d_i} \mu_{(e_i)}^{n-j-d_i}, & j = n_{\min}+1, \ldots, n \end{cases} \quad (27)$$

where $[a]_k = a(a-1)\cdots(a-k+1)$, $[a]_0 = 1$,, e_i denotes the unique integer such that

$$m_1 + \ldots + m_{e_i - 1} < i \leq m_1 + \ldots + m_{e_i}$$

and

$$d_i = \sum_{k=1}^{e_i} m_k - i.$$

Proof: See Appendix I.

Note that Lemma 6 gives, in a compact form, the general joint distribution for the eigenvalues of a central Wishart $(p \geq n)$, and central pseudo-Wishart or quadratic form $(n \geq p)$, with arbitrary one-sided correlation matrix with not-necessarily distinct eigenvalues.

In fact, Lemma 6 can be used for both $p \geq n$ and $n \geq p$; in particular, for $n \geq p$ we have $\prod_{i=1}^{n_{\min}} x_i^{p-n_{\min}} = 1$ in (25), while for $p \geq n$ the second row in (27) disappears and $(-1)^{p(n-n_{\min})} = 1$ in (26).

Moreover, using Lemma 6 and the results in [32] and [33] we can also derive the marginal distribution of individual eigenvalues or an arbitrary subset of the eigenvalues.

V. Ergodic Mutual Information of a Single-User MIMO System

In this section we provide a unified analysis of the ergodic mutual information of a single-user MIMO system with arbitrary power levels among the transmitting antenna elements or arbitrary correlation at the receiver, admitting covariance matrices with not-necessarily distinct eigenvalues.

Let us consider the function

$$\mathcal{C}_{\mathrm{SU}}(n, p, \mathbf{\Phi}) = \mathbb{E}_{\mathbf{H}}\{\log\det(\mathbf{I}_p + \mathbf{H\Phi H}^\dagger)\} \quad (28)$$

where $\mathbf{\Phi}$ is a generic $(n \times n)$ positive definite matrix and $\mathbf{H}$ is a $(p \times n)$ random matrix with circularly symmetric zero-mean, unit variance complex Gaussian i.i.d. entries.

Now, consider a single-user MIMO-$(n_{\mathrm{T}}, n_{\mathrm{R}})$ Rayleigh fading channel with $\mathbf{\Psi}_{\mathrm{T}}, \mathbf{\Psi}_{\mathrm{R}}$ denoting the $(n_{\mathrm{T}} \times n_{\mathrm{T}})$ transmit and $(n_{\mathrm{R}} \times n_{\mathrm{R}})$ receive correlation matrices, respectively, having diagonal elements equal to one. Assume the transmit vector $\mathbf{x}$ is zero-mean complex Gaussian, with arbitrary (but fixed) $(n_{\mathrm{T}} \times n_{\mathrm{T}})$ covariance matrix $\mathbf{Q} = \mathbb{E}\{\mathbf{xx}^\dagger\}$ so that $\mathrm{tr}\{\mathbf{Q}\} = P$. Then, the function (28) can be used to express the ergodic mutual information in the following cases [6]–[8].

1) The MIMO-$(n_{\mathrm{T}}, n_{\mathrm{R}})$ channel with no correlation at the receiver ($\mathbf{\Psi}_{\mathrm{R}} = \mathbf{I}$), covariance matrix at the transmitter side $\mathbf{\Psi}_{\mathrm{T}}$, and transmit covariance matrix $\mathbf{Q}$.
   In this case, the mutual information is $\mathcal{C}_{\mathrm{SU}}(n_{\mathrm{T}}, n_{\mathrm{R}}, \mathbf{\Phi})$ with $\mathbf{\Phi} = (1/\sigma^2)\mathbf{\Psi}_{\mathrm{T}}\mathbf{Q}$. If also $\mathbf{\Psi}_{\mathrm{T}} = \mathbf{I}$, we have $\mathbf{\Phi} = (1/\sigma^2)\mathbf{Q}$ and therefore $\mathrm{tr}\{\mathbf{\Phi}\} = P/\sigma^2$.
2) The MIMO-$(n_{\mathrm{T}}, n_{\mathrm{R}})$ channel with no correlation at the transmitter ($\mathbf{\Psi}_{\mathrm{T}} = \mathbf{I}$), covariance matrix at the receiver side $\mathbf{\Psi}_{\mathrm{R}}$, and equal power allocation $\mathbf{Q} = P/n_{\mathrm{T}}\mathbf{I}$.
   In this case the capacity is $\mathcal{C}_{\mathrm{SU}}(n_{\mathrm{R}}, n_{\mathrm{T}}, \mathbf{\Phi})$ with $\mathbf{\Phi} = P/(n_{\mathrm{T}}\sigma^2)\mathbf{\Psi}_{\mathrm{R}}$, giving $\mathrm{tr}\{\mathbf{\Phi}\} = (P/\sigma^2)(n_{\mathrm{R}}/n_{\mathrm{T}})$, in accordance to [6, Theorem 1].

In both cases, P/σ^2 represents the SNR per receiving antenna.

By indicating with $n_{\min} = \min(n, p)$ and with $f_{\boldsymbol{\lambda}}(\cdot, \ldots, \cdot)$ the joint pdf of the (real) ordered non-zero eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{n_{\min}} > 0$ of the $(p \times p)$ random matrix $\mathbf{W} = \mathbf{H\Phi H}^\dagger$, we can write:

$$\begin{aligned} \mathcal{C}_{\mathrm{SU}}(n, p, \mathbf{\Phi}) &= \mathbb{E}\left\{\sum_{i=1}^{n_{\min}} \log(1+\lambda_i)\right\} \\ &= \int \cdots \int_{\mathcal{D}_{\mathrm{ord}}} f_{\boldsymbol{\lambda}}(x_1, \ldots, x_{n_{\min}}) \\ &\quad \times \sum_{i=1}^{n_{\min}} \log(1+x_i) d\mathbf{x} \end{aligned} \quad (29)$$

where the multiple integral is over the domain $\mathcal{D}_{\mathrm{ord}} = \{\infty > x_1 \geq x_2 \geq \ldots \geq x_{n_{\min}} > 0\}$ and $d\mathbf{x} = dx_1\, dx_2 \cdots dx_{n_{\min}}$.

The nested integral in (29) can be evaluated using the results from previous sections and Appendix II, leading to the following theorem.

Theorem 1: The ergodic mutual information of a MIMO Rayleigh fading channel with CSI at the receiver only and one-sided correlation matrix $\mathbf{\Phi}$ having eigenvalues of arbitrary multiplicities described in (28) is given by

$$\mathcal{C}_{\mathrm{SU}}(n, p, \mathbf{\Phi}) = K \sum_{k=1}^{n_{\min}} \det\left(\mathbf{R}^{(k)}\right). \quad (30)$$

In the previous equation $n_{\min} = \min(n, p)$, the matrix $\mathbf{R}^{(k)}$ has elements shown in (31) at the bottom of the next page and $[a]_k$, e_i, d_i, K are defined as in Lemma 6, where $\mu_{(1)} > \mu_{(2)} \ldots >$

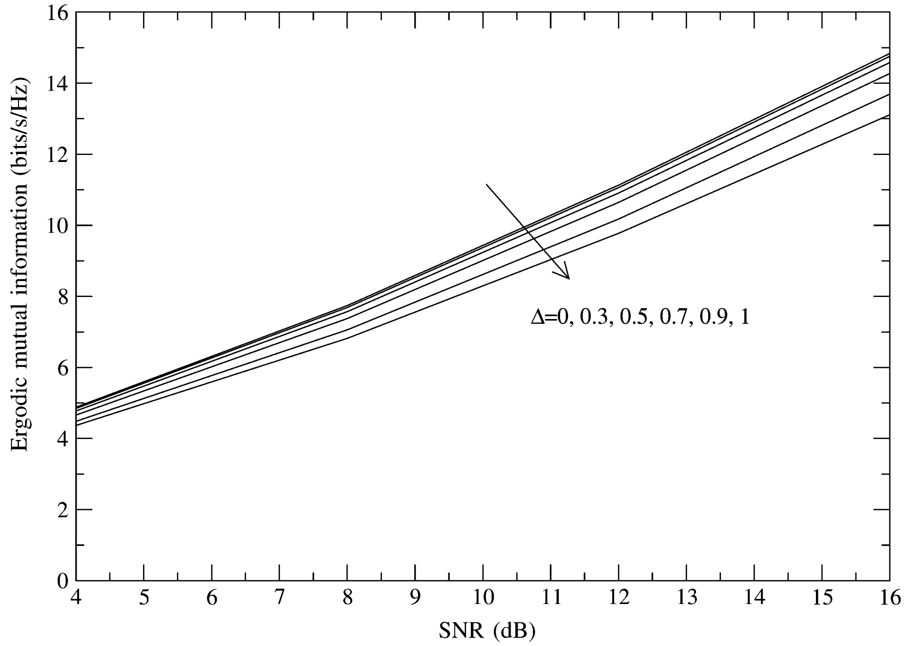

Fig. 2. Ergodic mutual information for single-user MIMO systems as a function of SNR over Rayleigh uncorrelated fading with $n_{\rm T} = 6$ and $n_{\rm R} = 3$. Half of the antennas with power (normalized) $1 + \Delta$, the others with $1 - \Delta$, i.e., with transmitted power levels (normalized) equal to $\{1 + \Delta, 1 + \Delta, 1 + \Delta, 1 - \Delta, 1 - \Delta, 1 - \Delta\}$.

$\mu_{(L)}$ are the L distinct eigenvalues of $\mathbf{\Phi}^{-1}$, with corresponding multiplicities $m_1, \dots, m_L$.

Proof: In Section IV it is shown that the joint pdf of the ordered eigenvalues of $\mathbf{W}$ can be written as (25), where the elements of $\mathbf{V}(\mathbf{x}), \mathbf{G}(\mathbf{x}, \boldsymbol{\mu})$ are real functions of $x_1, \dots, x_{n_{\min}}$. Thus, by using Appendix II, the multiple integral in (29) reduces to (30).

Note that the integral in (31) can be evaluated easily with standard numerical techniques; however, the integral can be further simplified, using the identities $\int_0^\infty x^m e^{-x\mu} dx = m!/\mu^{m+1}$, and $\int_0^\infty x^m e^{-x\mu} \ln(1+x) dx = m!\, e^{\mu} \sum_{i=0}^{m} \Gamma(i-m, \mu)/\mu^{i+1}$, where $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function.

Theorem 1 gives, in a unified way, the exact mutual information for MIMO systems, encompassing the cases of $n_{\rm R} \geq n_{\rm T}$ and $n_{\rm T} \geq n_{\rm R}$ with arbitrary correlation at the transmitter or the receiver, avoiding the need for Monte Carlo evaluation. The application of the results in Sections III–V enables a unified analysis for MIMO systems, which allow the generalization of ergodic and outage capacity [6]–[8], [29], for optimum combining multiple antenna systems [26], [27], for MIMO-MMSE systems [28], for MIMO relay networks [34], [35], as well as for multiuser MIMO systems and for distributed MIMO systems, accounting arbitrary covariance matrices. For example, after the first derivation of the hypergeometric functions of matrices with nondistinct eigenvalues in [36], other applications to multiple antenna systems have appeared in [32], [37]–[40].

VI. Numerical Results

Let us first apply Theorem 1 to the analysis of a single-user MIMO system with unequal power levels among the transmitting antennas. Fig. 2 shows the ergodic mutual information[2] of a MIMO-$(6, 3)$ Rayleigh channel, where the relative transmitted power levels are $\{1+\Delta, 1+\Delta, 1+\Delta, 1-\Delta, 1-\Delta, 1-\Delta\}$. The particular cases $\Delta = 0$ and $\Delta = 1$ are equivalent to the equal power levels over 6 and 3 transmitting antennas, respectively. This figure shows how the capacity decreases as Δ increases from 0 to 1, with a behavior in accordance to analysis based on majorization theory [41].

As another example of application, we evaluate the performance of MIMO relay networks in Rayleigh fading [34], [35]. For such networks the network capacity is upper bounded by [35, (5)], which can be easily put in the form $C_u = \frac{1}{2}\mathbb{E}_{\mathbf{H}}\{\log\det(\mathbf{I} + \mathbf{H}\mathbf{\Phi}\mathbf{H}^{\dagger})\}$, and evaluated in closed form by Theorem 1. In Fig. 3 we report the exact C_u as obtained from Theorem 1, compared to the Jensen's inequality [35, Theorem 1]. The figure has been obtained for a source node with four antennas, five relays each equipped with two antennas, as a function of the total equivalent SNR here defined as $\mathsf{SNR} = \mathrm{tr}\{\mathbf{\Phi}\}$. We assume, for the 5 relays, that the received power is distributed proportionally to the weights

[2]For the numerical results we use the base-2 of logarithm in all formulas, giving a mutual information in bits/s/Hz.

$$r_{i,j}^{(k)} = \begin{cases} (-1)^{d_i} \int_0^\infty x^{p-n_{\min}+j-1+d_i} e^{-x\,\mu_{(e_i)}} dx, & j = 1, \dots, n_{\min},\, j \neq k \\ (-1)^{d_i} \int_0^\infty x^{p-n_{\min}+j-1+d_i} e^{-x\,\mu_{(e_i)}} \log(1+x) dx, & j = 1, \dots, n_{\min},\, j = k \\ [n-j]_{d_i}\, \mu_{(e_i)}^{n-d_i-j}, & j = n_{\min}+1, \dots, n \end{cases} \quad (31)$$

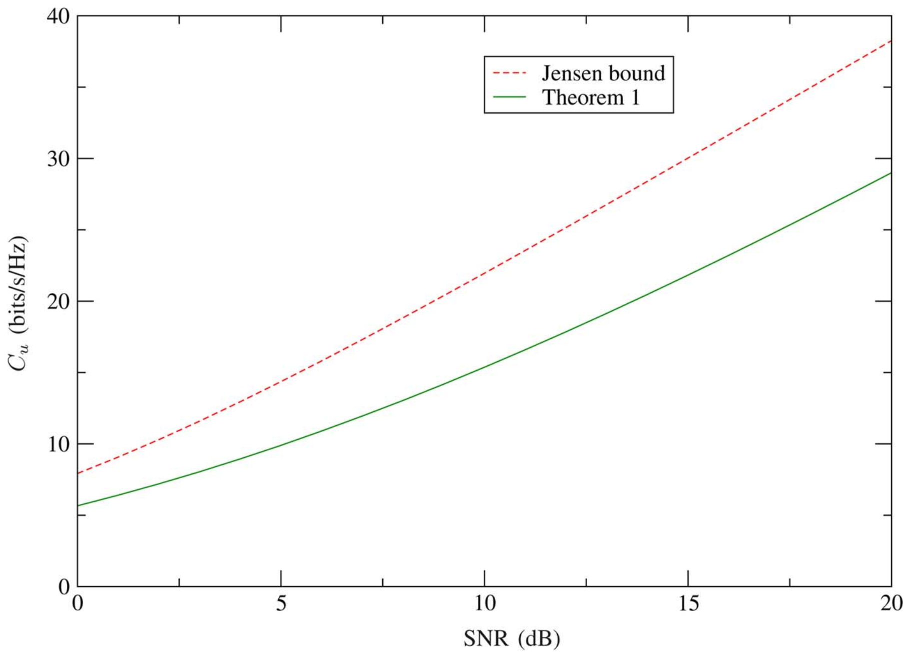

Fig. 3. Upper bounds on the network capacity for MIMO relay networks. Source with four antennas, five relays with two antennas each, power levels per relay proportional to $\{1, 2, 5, 10, 20\}$.

$\{1, 2, 5, 10, 20\}$. It can observed that the results based on the Jensen's inequality can be overly optimistic.

As a third example of application we evaluate, using (8) together with Theorem 1, the exact expression of the ergodic mutual information of MIMO systems in the presence of multiple MIMO interferers in Rayleigh fading. In particular, the eigenvalues to be used in Theorem 1 are given by $\mu_{(i)} = 1/\varrho_i = \sigma^2 N_{\mathrm{T}i}/P_i$, allowing an easy analysis for several scenarios. We define the average SNR per receiving antenna as $\mathsf{SNR} = P_0/\sigma^2$ giving $\varrho_0 = \mathsf{SNR}/N_{\mathrm{T}0}$, and the SIR as $\mathsf{SIR} = P_0/\sum_{i\geq 1} P_i$.[3] Fig. 4 shows the ergodic mutual information for a MIMO-(6,6) system as a function of the SIR, in the presence of one MIMO co-channel interferer having $N_{\mathrm{T}1}$ equal power transmitting antennas. It can be noted that the capacity decreases with the increase in the number of interfering antenna elements, tending to the curve obtained by using the Gaussian approximation.[4] Despite the fact that the received vector $\mathbf{y}$ in (1) is Gaussian conditioned on the channel matrices, and that the elements of $\mathbf{H}_k$ are Gaussian, approximating the cumulative interference as a spatially white complex Gaussian vector is pessimistic for analyzing MIMO systems in the presence of interference, unless the number of transmitting antenna of the interferer is large compared with that of the desired user. This is because the Gaussian approximation implicitly assumes that the receiver does not exploit the CSI of the interferers (single-user receiver), whereas the exact capacity accounts for the knowledge of all CSI at the receiver. In the same figure we also report, using circles, the capacity of a single-user MIMO-$(N_{\mathrm{T}0}, N_{\mathrm{R}} - N_{\mathrm{T}1})$ for $N_{\mathrm{R}} > N_{\mathrm{T}1}$. It can be observed that the capacity of the MIMO-$(N_{\mathrm{T}0}, N_{\mathrm{R}})$ in the presence of $N_{\mathrm{T}1}$ interfering antenna elements approaches, asymptotically for large interference power, to a floor given by the capacity of a single-user MIMO-$(N_{\mathrm{T}0}, N_{\mathrm{R}} - N_{\mathrm{T}1})$ system. This behavior can be thought of as using $N_{\mathrm{T}1}$ degrees of freedom (DoF) at the receiver to null the interference in a small SIR regime. On the other hand, when $N_{\mathrm{R}} \leq N_{\mathrm{T}1}$ the capacity approaches to zero for small SIR. This is due to the limited DoF at the receiver (related to the number N_{R} of receiving antenna elements) that prevents mitigating all interfering signals (one from each antenna elements) while, at the same time, processing the $N_{\mathrm{T}0}$ useful parallel streams, as previously observed for multiple antenna systems with optimum combining [2], [26], [27].

Finally, in Fig. 5 we consider a MIMO-$(N_{\mathrm{T}0}, 6)$ system in the presence of one and two MIMO interferers in the network, each equipped with the same number of antennas as for the desired user. We clearly see here two different regions: for small SIR the interference effect is dominant, and it is better for all users to employ the minimum number of transmitting antennas (i.e., MIMO-$(3, 6)$ for all users), so as to allow the receiver to mitigate the interfering signals. On the contrary, for large SIR the channel tends to that of a single-user MIMO system and it is better to employ the maximum number of transmitting antennas. In the same figure we also report the capacity for interference-free channels, which represents the asymptotes of the four curves, as well as the Gaussian approximation, which incorrectly indicates that it is always better to use the largest possible number of transmitting antennas.

It can be also verified that, in a network where all nodes are using the same MIMO-(n, n) systems, larger values of n achieve higher mutual information, for all values of SIR and

[3]We recall that, with our normalization on the channel gains, the mean received power from user i is P_i, and our definition of SIR account for the *total* interference power.

[4]With Gaussian approximation the performance is evaluated as if interference were absent, except the overall noise power is set to $\sigma^2 + \sum_{i\geq 1} P_i$, giving a signal-to-interference-plus-noise ratio $\mathsf{SINR} = (\frac{1}{\mathsf{SNR}} + \frac{1}{\mathsf{SIR}})^{-1}$.

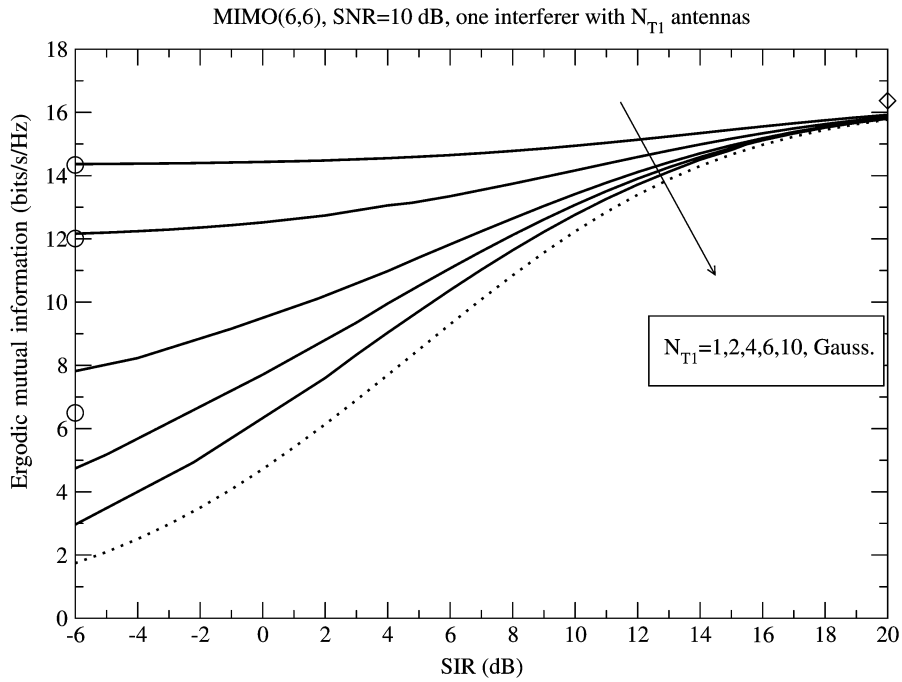

Fig. 4. Ergodic mutual information for MIMO-$(6,6)$ as a function of SIR in the presence of one MIMO co-channel interferer with $N_{\mathrm{T}1} = 1, 2, 4, 6, 10$. The SNR is set to 10 dB. The Gaussian approximation of the interference is also shown. Diamond: capacity of a single-user MIMO-$(6,6)$. Circles: capacity of a single-user MIMO-$(6, 6 - N_{\mathrm{T}1})$ (only for $N_{\mathrm{T}1} = 1, 2, 4$).

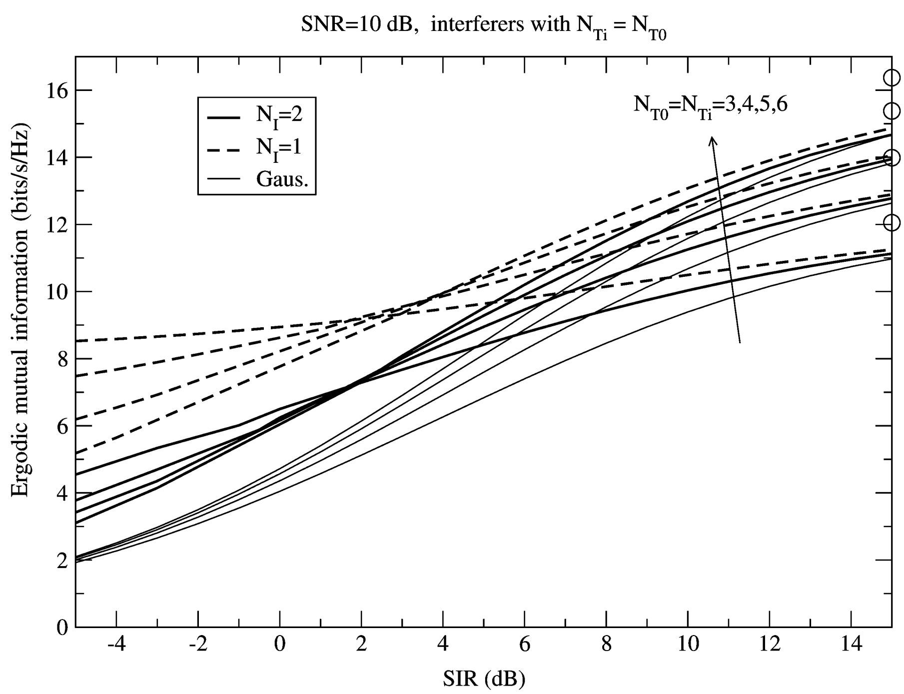

Fig. 5. Ergodic mutual information as a function of the signal-to-total interference ratio. MIMO system with $N_{\mathrm{R}} = 6$ receiving antenna, $\mathsf{SNR} = 10$ dB. The Gaussian approximation of the interference is also shown. Scenario with one and two interferers, each with the same number of transmitting antennas as the desired user. Cases of 3, 4, 5, and 6 transmitting antennas. Circles: capacity of single-user MIMO-$(N_{\mathrm{T}0}, N_{\mathrm{R}})$.

SNR. Note, however, that increasing the number of antennas and users, correlation may arise in the channel matrices.

VII. Conclusion

We have studied MIMO communication systems in the presence of multiple MIMO interferers and noise. To this aim, we first generalized the determinant representations for hypergeometric functions with matrix arguments to the case where the eigenvalues of the argument matrices have arbitrary multiplicities. Then, we derived a unified formula for the joint pdf of the eigenvalues for central Wishart matrices and Gaussian quadratic forms, allowing arbitrary multiplicities for the covariance matrix eigenvalues. These new results enable the analysis of many scenarios involving MIMO systems. For example, we derived a unified expression for the ergodic mutual information of MIMO Rayleigh fading channels, which applies to transmit or receive correlation matrices with eigenvalues of arbitrary multiplicities. We have shown how to apply the new expressions to MIMO networks, deriving in closed form the ergodic mutual information of MIMO systems in the presence of multiple MIMO interferers.

APPENDIX I
PROOFS

A. Proof of Lemma 2

For ease of notation and without loss of generality, we consider the case of $K = 1$, where the application of the lemma leads to (15). For the proof we proceed by induction. First, the result in (15) is obvious for $L = 1$, since in this case (15) coincides with (14). Then, we must show that if (15) is true for any L then it is also true for $L + 1$. So, assuming that (15) holds for L, we must find

$$\lim_{w_{L+1}\to w_L} \breve{P}(w_1, \ldots, w_m).$$

In this regard, note that, with $w_1 = w_2 = \cdots = w_L$ the product $\prod_{i<j, w_i \neq w_j}(w_i - w_j)$ in (15) contains exactly L factors with value $\epsilon \triangleq w_L - w_{L+1}$. Thus, by rewriting $w_{L+1} = w_L - \epsilon$ we have (32) shown at the bottom of the page. We can now apply the Taylor expansion to the functions

$$f_i(w-\epsilon) = \sum_{n=0}^{L} f_i^{(n)}(w)\frac{(-\epsilon)^n}{n!} + O(\epsilon^{L+1}) \tag{33}$$

where $O(\epsilon)$ denotes the omitted terms of order ϵ. We also know from basic algebra that, seen as a function of a column with the others fixed, the determinant is a linear function of the entries in the given column, as is clear for example from the Laplace expansion. Therefore, we have (34) shown at the bottom of the page.

In (33), the determinants for $n = 0, \ldots, L-1$ are zero since there are coincident columns. Hence, in the limit for $\epsilon \to 0$ only the term of grade L remains.

By simplifying and reordering the first $L+1$ columns of the matrix in (34), with a cyclic permutation having sign equal to $(-1)^L$, we finally have (35) shown at the bottom of the page which is again in the form of (15). This concludes the proof by induction of Lemma 2 for $w_1 = \cdots = w_L$.

The extension to different K and more groups of coincident arguments is straightforward.

B. Proof of Lemma 5

The derivatives of the hypergeometric function of scalar arguments can be expressed as

$$\begin{aligned}\frac{d^n}{dz^n}{}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z) &= \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \\ &\quad \times {}_p\tilde{F}_q(a_1+n, \ldots, a_p+n; b_1+n, \ldots, b_q+n; z).\end{aligned}$$

$$\lim_{w_{L+1}\to w_L} \breve{P}(w_1, \ldots, w_m) = \frac{1}{\prod_{i<j, w_i\neq w_j, j\neq L+1}(w_i - w_j)\prod_{i=1}^{L-1} i!} \times \lim_{\epsilon\to 0}\frac{1}{\epsilon^L}\begin{vmatrix} f_1^{(L-1)}(w_L) & \cdots & f_1(w_L) & f_1(w_L-\epsilon) & \cdots & f_1(w_m) \\ \vdots & & \vdots & & \cdots & \vdots \\ f_m^{(L-1)}(w_L) & \cdots & f_m(w_L) & f_m(w_L-\epsilon) & \cdots & f_m(w_m)\end{vmatrix}. \tag{32}$$

$$\lim_{w_{L+1}\to w_L} \breve{P}(w_1, \ldots, w_m) = \frac{1}{\prod_{i<j, w_i\neq w_j, j\neq L+1}(w_i - w_j)\prod_{i=1}^{L-1} i!} \times \lim_{\epsilon\to 0}\left(O(\epsilon) + \frac{1}{\epsilon^L}\sum_{n=0}^{L}\frac{(-\epsilon)^n}{n!}\cdot\begin{vmatrix} f_1^{(L-1)}(w_L) & \cdots & f_1(w_L) & f_1^{(n)}(w_L) & \cdots & f_1(w_m) \\ \vdots & & \vdots & & \cdots & \vdots \\ f_m^{(L-1)}(w_L) & \cdots & f_m(w_L) & f_m^{(n)}(w_L) & \cdots & f_m(w_m)\end{vmatrix}\right). \tag{34}$$

$$\lim_{w_L\to w_{L+1}} \breve{P}(w_1, \ldots, w_m) = \frac{1}{\prod_{i<j, w_i\neq w_j}(w_i - w_j)\prod_{i=1}^{L} i!}\cdot\begin{vmatrix} f_1^{(L)}(w_{L+1}) & \cdots & f_1(w_{L+1}) & f_1(w_{L+2}) & \cdots & f_1(w_m) \\ \vdots & \vdots & & & \cdots & \vdots \\ f_m^{(L)}(w_{L+1}) & \cdots & f_m(w_{L+1}) & f_m(w_{L+2}) & \cdots & f_m(w_m)\end{vmatrix} \tag{35}$$

Using this result in Lemma 2 and (10) with

$$f_i(w) = {}_p\tilde{F}_q(\tilde{a}_1, \ldots, \tilde{a}_p; \tilde{b}_1, \ldots, \tilde{b}_q; \lambda_i w)$$

gives Lemma 5.

C. Proof of Lemma 6

Here, based on Section III, we prove Lemma 6 concerning the eigenvalues distribution of Gaussian quadratic forms. The problem is related to the distribution of random matrices of the form $\mathbf{W} = \mathbf{H\Phi H}^\dagger$, where $\mathbf{H}$ is a Gaussian $(p \times n)$ matrix with uncorrelated entries and $\mathbf{\Phi}$ is a $(n \times n)$ positive definite matrix that represents the covariance matrix of the channel. The eigenvalues distribution has been studied for the two possible cases $n \geq p$ and $p \geq n$ in [6] and [7], assuming a covariance matrix $\mathbf{\Phi}$ with distinct eigenvalues (i.e., unit multiplicity). We here generalize the results to matrices $\mathbf{\Phi}$ with arbitrary eigenvalue multiplicities.

Let us first recall the distributions for the case of covariance matrix with distinct eigenvalues.

1) Correlation on the Shortest Side—Distinct Eigenvalues: The case $p \geq n$ has been analyzed in [6], where it is shown that the joint pdf of the (real) ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ of $\mathbf{W}$ is

$$f_{\boldsymbol{\lambda}}(x_1, \ldots, x_n) = \frac{1}{\Gamma_{(n)}(p)} \frac{\prod_{i=1}^n \mu_i^p}{\prod_{i<j}(\mu_i - \mu_j)} \times |\mathbf{V}(\mathbf{x})||\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})| \prod_{j=1}^n x_j^{p-n} \quad (36)$$

where μ_i are the n distinct eigenvalues of $\mathbf{\Phi}^{-1}$, $\mathbf{V}(\mathbf{x})$ is the $(n \times n)$ Vandermonde matrix with elements $v_{i,j} = x_j^{i-1}$ and where $\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})$ is a $(n \times n)$ matrix with elements $g_{i,j} = e^{-\mu_i x_j}$.

2) Correlation on the Largest Side—Distinct Eigenvalues: We briefly derive the joint pdf for the eigenvalues of $\mathbf{W}$ when $\mathbf{\Phi}$ has all distinct eigenvalues and $n \geq p$, based on the results in Section III. Note that this case has been analyzed also in [7] by following a different approach.

First we recall that, given a $(p \times n)$ random matrix $\mathbf{H}$ with $n \geq p$ and pdf

$$\pi^{-pn} e^{-\mathrm{tr}\,\mathbf{HH}^\dagger} \quad (37)$$

the pdf of the $(p \times p)$ quadratic form

$$\mathbf{W} = \mathbf{H\Phi H}^\dagger \quad (38)$$

where the $(n \times n)$ matrix $\mathbf{\Phi}$ is positive definite, is given by [42], [43]

$$f(\mathbf{W}) = \frac{|\mathbf{W}|^{n-p}}{\pi^{(p-1)p/2}\Gamma_{(p)}(n)|\mathbf{\Phi}|^p} {}_0\tilde{F}_0(\mathbf{\Phi}^{-1}, -\mathbf{W}). \quad (39)$$

Then, the joint pdf of the (real) ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$ of $\mathbf{W}$ is given by using the results in [22, (93)] as

$$f_{\boldsymbol{\lambda}}(x_1, \ldots, x_p) = K_1 |\mathbf{\Phi}|^{-p} {}_0\tilde{F}_0(\mathbf{\Phi}^{-1}, -\mathbf{W}) |\mathbf{W}|^{n-p} \cdot \prod_{i<j}^p (x_i - x_j)^2 \quad (40)$$

where

$$K_1 = \frac{1}{\Gamma_p(n)\Gamma_p(p)}. \quad (41)$$

Note that in (40) the two matrices $\mathbf{\Phi}^{-1}$ and $\mathbf{W}$ are of dimensions $(n \times n)$ and $(p \times p)$, respectively. Hence, in (40) we evaluate ${}_0\tilde{F}_0(\mathbf{\Phi}^{-1}, \mathbf{B})$ where $\mathbf{B} = -\mathbf{W} \oplus 0 \cdot \mathbf{I}_p$ is obtained by adding $n - p$ zero eigenvalues to $-\mathbf{W}$ [7].

Differently from the previous literature, we can now directly use Corollary 1 and get immediately the joint pdf of the ordered eigenvalues of the $(p \times p)$ matrix $\mathbf{W}$ when $n \geq p$ as:

$$f_{\boldsymbol{\lambda}}(x_1, \ldots, x_p) = \frac{(-1)^{p(n-p)}}{\Gamma_{(p)}(p)} \times \frac{\prod_{i=1}^n \mu_i^p}{\prod_{i<j}(\mu_i - \mu_j)} |\mathbf{V}(\mathbf{x})||\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})| \quad (42)$$

where μ_i are the eigenvalues of $\mathbf{\Phi}^{-1}$, and are of multiplicity one. $\mathbf{V}(\mathbf{x})$ is the $(p \times p)$ Vandermonde matrix, and the $(m \times m)$ matrix $\mathbf{G}(x, \boldsymbol{\mu})$ has elements as follows:

$$g_{i,j} = \begin{cases} e^{-\mu_i x_j} & j = 1, \ldots, p \\ \mu_i^{n-j} & j = p+1, \ldots, n. \end{cases} \quad (43)$$

That is, the matrix $\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})$ is

$$\mathbf{G}(\mathbf{x}, \boldsymbol{\mu}) \triangleq \begin{bmatrix} e^{-\mu_1 x_1} & \cdots & e^{-\mu_1 x_p} & \mu_1^{n-p-1} & \mu_1^{n-p-2} & \cdots & \mu_1 & 1 \\ e^{-\mu_2 x_1} & \cdots & e^{-\mu_2 x_p} & \mu_2^{n-p-1} & \mu_2^{n-p-2} & \cdots & \mu_2 & 1 \\ \vdots & & \vdots & \vdots & \vdots & \cdots & \vdots & \\ e^{-\mu_n x_1} & \cdots & e^{-\mu_n x_p} & \mu_n^{n-p-1} & \mu_n^{n-p-2} & \cdots & \mu_n & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{g}(\mathbf{x}, \mu_1) \\ \mathbf{g}(\mathbf{x}, \mu_2) \\ \vdots \\ \mathbf{g}(\mathbf{x}, \mu_n) \end{bmatrix}. \quad (44)$$

3) Generalization to Covariance Matrix With Arbitrary Eigenvalues: Note that (36) and (42) are only valid for covariance matrices with all distinct eigenvalues (multiplicity one). Hence, we must now generalize these expressions to the case of interest, i.e., eigenvalues μ_i with arbitrary multiplicities. This step is possible by using Lemma 2.

In fact, we note that in both (36) and (42) we have a ratio of the form

$$\frac{|\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})|}{\prod_{i<j}(\mu_i - \mu_j)}. \quad (45)$$

By using Lemma 2, for each eigenvalue with multiplicity m_i we must replace the rows of $\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})$ with their successive derivatives with respect to the eigenvalue, and divide by $\Gamma_{(m_i)}(m_i)$, obtaining

$$\frac{|\mathbf{G}(\mathbf{x}, \boldsymbol{\mu})|}{\prod_{i<j}(\mu_i - \mu_j)} \to \frac{1}{\prod_i \Gamma_{(m_i)}(m_i) \prod_{i<j}(\mu_{(i)} - \mu_{(j)})^{m_i m_j}}$$

$$\times \det \begin{bmatrix} \mathbf{g}^{(m_1-1)}(\mathbf{x}, \mu_{(1)}) \\ \vdots \\ \mathbf{g}^{(1)}(\mathbf{x}, \mu_{(1)}) \\ \mathbf{g}^{(0)}(\mathbf{x}, \mu_{(1)}) \\ \vdots \\ \mathbf{g}^{(m_L-1)}(\mathbf{x}, \mu_{(L)}) \\ \vdots \\ \mathbf{g}^{(1)}(\mathbf{x}, \mu_{(L)}) \\ \mathbf{g}^{(0)}(\mathbf{x}, \mu_{(L)}) \end{bmatrix} \tag{46}$$

where the row vector $\mathbf{g}^{(l)}(\mathbf{x}, \mu)$ is the lth derivative of the row $\mathbf{g}(\mathbf{x}, \mu)$ in (36) or (44). The jth element of $\mathbf{g}^{(l)}(\mathbf{x}, \mu)$ is so derived to be

$$\begin{aligned} g_j^{(l)} &= g_j^{(l)}(\mu) \\ &= \begin{cases} (-x_j)^l e^{-\mu x_j}, & j = 1, \dots, p \\ [n-j]_l \mu^{n-j-l}, & j = p+1, \dots, n. \end{cases} \end{aligned} \tag{47}$$

The relation between the row index, i, and the derivative order, l, can be established by introducing the function e_i indicating the eigenvalue $\mu_{(e_i)} \in \{\mu_{(1)}, \dots, \mu_{(L)}\}$ to be used in row i of the matrix in the RHS of (46). It is easy to verify that e_i is the unique integer such that

$$m_1 + \ldots + m_{e_i - 1} < i \leq m_1 + \ldots + m_{e_i}.$$

Then, the derivative order for the row i is $l = d_i$, where

$$d_i = \sum_{k=1}^{e_i} m_k - i.$$

Thus, the generic element of the matrix in the RHS of (46) is $g_j^{(d_i)}(\mu_{(e_i)})$. By combining (36), (42), and (46) we have Lemma 6.

APPENDIX II
AN IDENTITY ON MULTIPLE INTEGRALS INVOLVING DETERMINANTS

Theorem 2: Given an arbitrary $p \times p$ matrix $(\mathbf{\Phi}(\mathbf{x}))$ with ijth elements $\Phi_i(x_j)$, an arbitrary $(n \times n)$ matrix $\mathbf{\Psi}(\mathbf{x}), n \geq p$, with elements

$$\begin{cases} \Psi_i(x_j) & j = 1, \dots, p \\ \Psi_{i,j} & j = p+1, \dots, n \end{cases}$$

and two arbitrary functions $\xi(\cdot)$ and $\tilde{\xi}(\cdot)$ the following identity holds:

$$\int \cdots \int_{\mathcal{D}_{\text{ord}}} |\mathbf{\Phi}(\mathbf{x})| \cdot |\mathbf{\Psi}(\mathbf{x})| \prod_{m=1}^{p} \xi(x_m) \sum_{i=1}^{p} \tilde{\xi}(x_i) d\mathbf{x} = \sum_{k=1}^{p} \det \left(\left\{ c_{i,j}^{(k)} \right\}_{i,j=1\dots,n} \right) \tag{48}$$

where the multiple integral is over the domain $\mathcal{D}_{\text{ord}} = \{b \geq x_1 \geq x_2 \geq \ldots \geq x_p \geq a\}$

$$c_{i,j}^{(k)} = \begin{cases} \int_a^b \Phi_i(x) \Psi_j(x) \xi(x), U_{k,j}(\tilde{\xi}(x)) dx, & j = 1, \dots, p \\ \Psi_{i,j}, & j = p+1, \dots, n \end{cases}$$

and the function $U_{k,j}(x)$ is defined by

$$U_{k,j}(x) \triangleq \begin{cases} x, & \text{if } k = j \\ 1, & \text{if } k \neq j. \end{cases} \tag{49}$$

Proof: As this theorem is an extension of [6, Theorem 3], it is sufficient for the proof to follow the same steps reported there.

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