## THE DESIGN OF EVOLUTION

by

## STEPHEN NATHANIEL FLANDERS

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#### DESIGNING EVOLUTION

Stephen N. Flanders Submitted to the Department of Architecture on May 14, 1971 in partial fulfillment of the requirements for the degree of Bachelor of Architecture.

The complexity of problems facing mankind is increasing beyond our ability to deal with them unaided. We seek to comprehend meaningful complexity and to diminish inimicable complexity. An artificial intelligence can aid us only if it can understand us. To do so, it must possess ways of sensing and effecting the same world we do, as we seek to understand all that is about us and within us.

Artificial intelligence is itself probably too complex a problem to design in its entirety. If so, we require some process that will design itself into the desired artifact. The design of such an evolutionary process should require a less-than-total understanding of the entire problem. Even less-thanintelligent evolution embodied in problem environments would enable them to reconfigure themselves on the spur of the moment to limited changes in user context. This thesis seeks to explore evolution as a medium of self-design. The study of evolution presents the problems of feedback of form, self-organization and self-reproduction. The work of this thesis is to develop cellular automata as a medium for experimentation.

Thesis Supervisor: Nicholas P. Negroponte Title: Assistant Professor of Architecture

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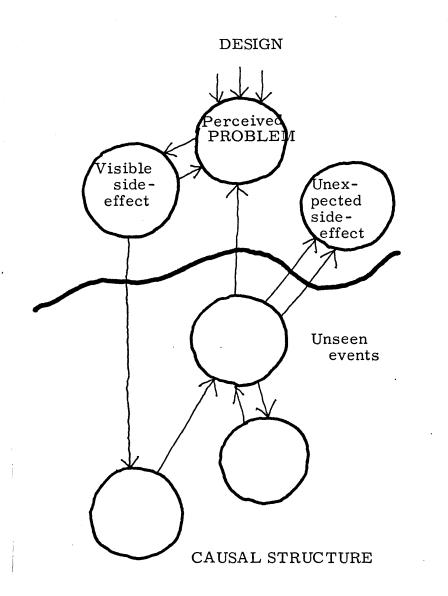
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### INTRODUCTION

The growth of civilization worries me. It seems to create ever more complex problems as it attempts to solve others. Working to house the millions more people of tomorrow ignores and perpetuates the cause of their arrival. Yet even if a city is a place where people compete in furious disarray for limited space and services, it is also a place where the most organized human interactions can flourish. J.W. Forrester demonstrates<sup>1</sup> that the prospects for a world that does not grow in population or in its depletion rate of sources of energy and material

and yet continues to improve to the satisfaction of all its inhabitants are slim. Yet if such a world is possible, it requires an understanding of how to respond to our environment more than it requires a response with our present ability to understand.

The process of design takes place in a world of interwoven causal connections. Beneath the surface there are mechanisms causing the events we perceive. When we manipulate one set of phenomena, we set off a chain of events that extends beyond our direct influence, but often returns to affect conversely our intended region of



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manipulation. Such a mistake is a result of misunderstanding causal structure. Given some knowledge of a structure, we still require information about the quanta that participate in it. A designer cannot hope to have complete knowledge of either structure or quanta, consequently <u>he cannot hope to know</u> <u>how to solve completely a problem as</u> <u>he perceives it or to perceive a problem</u> <u>he creates as he solves another.</u>

Clearly this is a very deterministic view of how all things work. Nevertheless, it does not preclude or contradict aiding the process of design. In one

sense everything is deterministic. At any given time only one set of events is possible and that consists of those events that actually happen at that time. Every event that takes place emits information about its occurence in the form of energy. Depending on how and when it is received, the information determines an event in one set of ways only, be the event a collision between gas molecules or a turn of affairs in international diplomacy. No event takes place under the influence of what will occur in the future because energy does not arrive from such events before they occur. On the other hand some events are very difficult to predict based on past information. This depends largely on how we are receiving information, how much is going on and whether we are altering events by our presence. At one level of perception the behavior of air in a balloon is very simple: its pressure changes uniformly with pressure and volume. At a vantage-point where we can discern many gas molecules individually, all is in chaos; Brownian motion is famous for its randomness. But in observing a single collision between two gas molecules, the events become more comprehensible. Randomness as unpredictability in one frame of reference becomes uniformity or total predict-

ability in another, yet both are determined by information about past events. In terms of this description, <u>design is deciding how to affect wisely</u> <u>that which affects oneself and others;</u> design is (self-) control.

The difference in meaning between deterministic and probabilistic becomes useful in describing our ability (or the ability of any other entity) for predicting future events based on information about past events. The need for prediction arises out of the need to have a response at each point in time. Design forms the response to such a prediction. The prediction represents an implicit model of the structure and behavior of the environment it takes place in. The designer bases his model of an environment on information he receives from it. That information can only be less than and different from the environment emitting it. Therefore the model can never be complete. In order to make the model as useful as possible, the designer must solicit information about the environment by testing it with responses which work in his current model. Only by discriminating between model and test results can he reformulate his model. Somehow, he can now choose a response which will probably represent an improved model.

Unaided, human beings are suited for modeling only a narrow range of their environment. We receive information and understand structure of causal events in a limited way. Although we organize our individual environments in a way comprehensible to each of us, we contribute to an over-all organization beyond the comprehension of any one of Only through association with an us. intelligent entity with powers of information gathering and understanding both greater than and different from ours, can we be aided in modeling the complexity that organizes our lives.

To many the proposition of living in

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consort with a machine seems shallow and demeaning at best. How can a machine understand and benefit us? How can it understand how we feel in terms meaningful to us? The answer lies in the rhetorical question, how can anybody do all these things? No-one can, completely. One must settle for a model of other people as well as of ourselves. The hardware necessary to construct a model consists of some black box (we call it intelligence) which organizes the model, based on information it solicits by controlling extensions of the body (effectors) in experiments that result in information about the environment which is detected

by energy receivers (sensors) and transmitted back to the black box for comparison with the current model. The quality of the model in man or machine depends on its black box <u>and</u> its sensors and effectors.

One response we make to our environment is art. Art is our model of human experience. Art is not a model of the reality that human experience is itself a model of. Because art, both already made and in the making, is itself a portion of human experience, this description of art applies recursively to itself. A sufficiently intelligent machine to meet our requirements would not only model machine experience as a form of art, <u>but would also model</u> <u>human experience</u>.

The M.I.T. Architecture Machine is an effort in the direction of a responsive environment. Along the way there are many sub-goals that contribute to the process of design. These have centered on developing an ability for the machine to enhance the designer's and user's understanding of the environment they deal with and their relationships to it. To do this, the machine must solicit information both from the environment under discussion and the people who use

it and design it. The machine can then represent the problem in a manner that its human proteges would find difficult to construct, but find easy to understand.

Much work on the Architecture Machine has gone into interfacing it with the real world. Eyes have been constructed; an arm is under construction; a mechanism called Seek builds and alters an environment for gerbils and a sketch recognition program accepts a sketch and translates through Seek into a physical configuration of blocks.' The sensors and effectors are receiving a lot of attention, <u>but our black box</u> <u>is not</u>, beyond there being an expressed desire for routines that write themselves and heuristics that continually modify themselves.

Artificial intelligence is an awesome problem, so awesome that approaching it as an all-encompassing design problem may well be more difficult than any problem we might first apply the intelligence to. The most sensible approach may be to design something that somehow organizes itself into intelligence, in short, which evolves. Finding an elementary component capable of evolution is difficult by itself.

Using the real world as a continuous

and complete environment for evolution has obvious advantages over attempting to stash away a data point inside a computer memory to represent every possible piece of energy that might influence evolution. On the other hand, if one can construct a probabilistic model maker for generating responses to an environment, one should be able to make a deterministic model to represent the environment where needed only. If one can not conveniently interface with the real world, an internal environment generator might be a substitute for, but not a simulation of, reality. Of course, one would have to

be able to monitor internal events from the outside.

Studying evolution as an approach to artificial intelligence will not assure that any given evolutionary mechanism will achieve our goal in a reasonable amount of time, but it will impute at each stage of progress how to improve the elementary entity one uses to precipitate the process in each experiment.

Having watched the progress of electrical engineering from tubes to integrated circuits and from crystalline semiconductors to amorphous semi-conductors, I am tempted to make a science fiction

speculation about the technology that might accomplish evolution of environments as well as intelligence. A self-designing, self-building and self-tending set of components, whose total complexity exceeds our or its own total understanding, may chemically resemble the components of life. They may evolve from initial specifications only as complicated as the human sperm and egg relative to their fully developed state. We might not live in machines after all, but in organisms.

In proceeding with this paper, I will:

 Contrast the power of designing evolutionary artifacts with the drawbacks of what I call "total design" that we now pursue;

- Demonstrate the role of feedback in evolutionary design by analogy to one of the few responsive feedback control systems used in built environments; (To do this, I will clarify the principles of positive and negative feedback).
- Model several evolutionary phenomena as embodying feedback of form -- feedback in a multiplicity of changing dimensions;
- Demonstrate, in a survey of the interest in evolving environments and components of evolution, the roles of self-organization and self-reproduction;
- Describe the possibilities of using cellular automata as a medium for experimentation of the properties mentioned above;
- 6) Finally, discuss my work on the M.I.T.
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Architecture Machine providing a vehicle for further investigation of evolution.

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## 1.0 THE DESIGN OF EVOLUTION

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### 1.1 Total Design versus Evolution

God created the universe in six days. In this bootstrap operation He made order out of chaos. Where "the earth was without form, and void" (Gen. 1, 2), He made a firmament in the midst of the waters. He caused the sun, moon and stars to move in an orderly fashion in the heavens. He created plants and animals which interact with their surroundings in a complex, but orderly manner. Finally, out of the dust He made his most complex creation, man. This is one story.

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Other lore maintains that a process of

"natural selection" enabled certain creatures and plants to endure the vagaries of a changing environment by the "survival of the fittest" of successive generations. This process led from a relatively primeval state to the complex degree of organization we are aware of today.

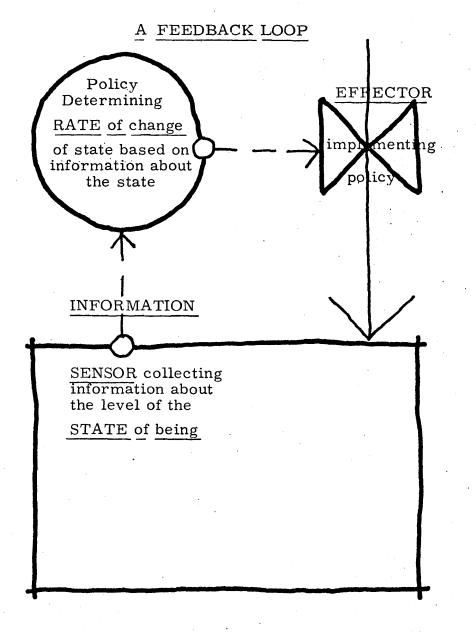
However it happened, regard the Creation as a design event. The process described in the Bible required complete knowledge of the future if the grand design was to be adequate to meet the challenge of its own existence. <u>Evolution</u>, <u>however</u>, <u>describes a process that responds solely</u> to events in the (usually immediate) past. No matter what happens to alter a state of affairs, there is a mechanism capable of restoring an equilibrium to the now changed circumstances. The human design process resembles the Biblical creation in that it draws on a great deal of information (as complete as possible) to result in a fully developed artifact, not a developing one. In compensation for our lack of omniscience, we make the artifact somewhat open-ended and call it a design for "flexibility," "growth" and the "future." Very little is known about how to create the selfdesigning process of evolution.

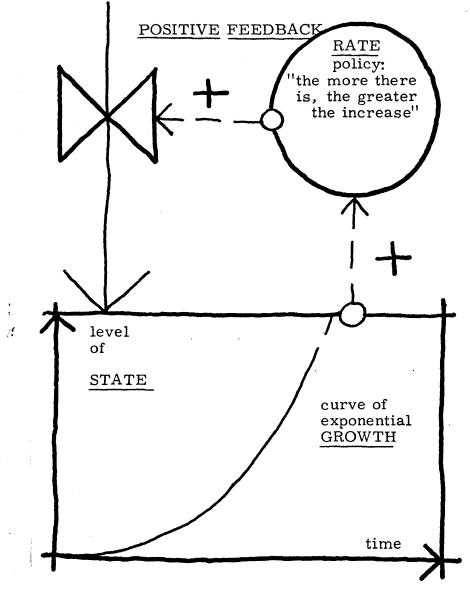
Thermostatic control of temperature

provides the closest analogy in our everyday experience to self-designing environment. We take this regulating system for granted. But consider the total design alternative. Probably one would define a schedule for heat or cooling output based on average seasonal and diurnal changes of outside temperature. Lacking prior knowledge of exactly what goes on at any given moment, like someone leaving the outside door open in order to bring in the groceries, the designer must rely on very general likelihoods which result in a building that attains the desired temperature only by coincidence.

### 1.2 Growth and Control

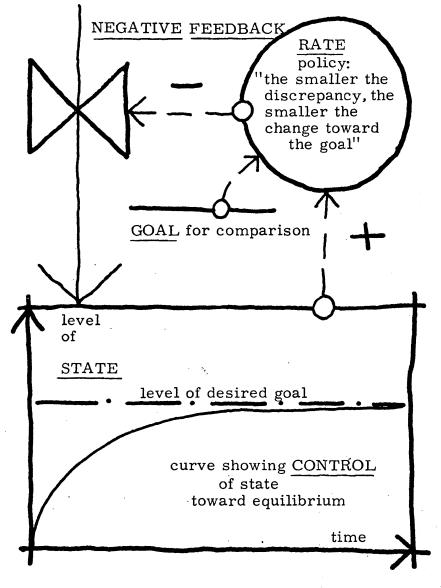
The thermostat's advantages are manifest; furthermore, it provides the opportunity to extract the principles governing its usefulness. It is part of a system that includes an information receiver (sensor) and a responding mechanism (effector), both dealing with the same medium, temperature. The sensor guides a policy determining what the effector does. When the results of the effector's response reach the sensor, they complete a feedback loop. The feedback loop governs the change (rate) of temperature in the environment (state of being). A feedback





loop would not exist if the sensor monitored the temperature in the Boston Edison smokestacks or the water level of the Charles River, since the effector would not alter the sensed state of being.

Consider some feedback policies the effector might pursue. One possibility is to change temperature by the following rule: "the more there is (state of being), the greater the increase (rate of change)." Such a policy, governing what one calls a <u>positive feedback system</u>, would have exponentially increasing <u>growth of</u> <u>both rate and state</u>. Another possibility is to set a goal and strive for it. In



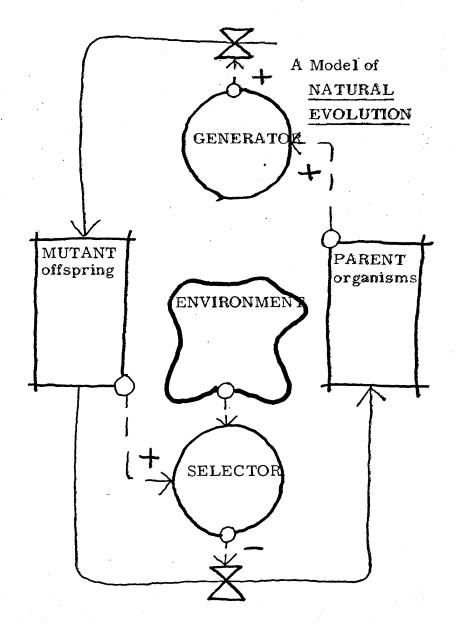
this case the policy would follow the rule: "the greater the discrepancy between goal and state, the greater the rate of change in the direction of the goal." This policy is called <u>negative</u> <u>feedback</u> and characteristically <u>approaches</u> <u>its goal ever more slowly</u>. Every control system, notably the thermostat, is an attempt to emulate this behavior.

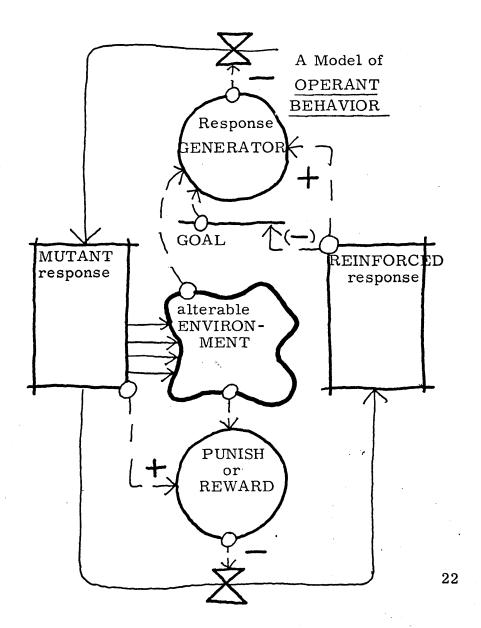
The control system determining temperature in a house is larger than a set of rooms, a furnace and a thermostat. It includes a person who adjusts the thermostat (determines its goal) at a setting comfortable for what he is doing. His comparison of the surrounding temperature with his own goals (a function of the amount of his activity, clothing, weariness, etc.) determines how he adjusts the thermostat.

## 1.3 Evolution as Feedback of Form

Evolution is not a thermostat. As a controller of an environment, a thermostat deals only with the dimension of temperature. An evolutionary controller must be able to alter the shape as well as the size of its response to an environmental situation. It might be adaptable enough only to provide lighting, shapes and textures for enjoyable living. Or it might be intelligent enough to take care of and educate a child.

Evolution is a feedback process in two parts: one is reproduction, a generator of mutant versions of previous generations; the other is environment, a selector of fit entities that provides the basis for a new generation. Consonance of mutants with the environment provides teleology insofar as the mutants do not alter significantly in their favor the environment that is the basis for selection. With a teleological environment the generator of mutants must pursue a policy of positive feedback of form; "the greater the differences in form among parent entities, the greater the increase of differences among offspring." The environment then provides negative feedback of form; "the greater the dis-





parity between the qualities of the environment and those of the mutants, the greater the rate of rejection." If a generator stubbornly pursues a goal disparate from the environment without being able to alter it, there is no tendency for mutants to move toward either goal; the surviving mutants will be scattered in quality between the requirements of the two goals. The generator may change its goal as a result of its inability to change the environment. If the mutants can converge in quality with that of the environment or the environment with that of the mutants, then an equilibrium occurs. If the environment does not

represent a constant goal because the mutants are capable of changing it in their favor and the generating process pursues no converging goal, the total system will grow exponentially in mutation of succeeding generations.

The coelacanth lives unchanged for millions of years in one of the most constant of environments, the bottom of the sea. Other fish, accepting the coelacanth as a part of their environment, as it must accept them, find sustenance in other niches of the same world. The cooling of the earth's crust and its motion in relation to the sun

drive the change of the surface environment; the evolution of species follows suit. One creature, man, as a mutant is able to alter significantly the environment which affects him. The power of adaptation depends on the speed and variety of mutation. Single-cell creatures can reproduce rapidly, but their range of mutations per generation is small. Where such creatures specialize within the organization of a sexually reproducing animal, like a dinosaur, their rate of regeneration as a whole is much slower, but the combinations of possible genetic instructions are greatly increased. The ability of a given body to evolve its own responses to its environment offers a crude description of intelligent behavior.

Any feedback response to an environment is an implicit prediction about the state of the environment, since the response is in the future relative to the sensing that prompted it. If the sensors and effectors involved deal with only one stimulus dimension (like temperature), then there is no basis for improving the prediction beyond fielding responses to stimuli as rapidly as the time lag in feedback permits. The ability to sense time improves the prediction of sequential patterns of stimuli, such as the cyclical numbers

in Fogel's experiments described below. The greater the number of stimulus dimensions an organism can respond to in correlation to one another, the greater its power of prediction (adaptation). The ability to respond to correlated stimuli requires abstraction, classifying stimuli so as to detach them from the specific objects that possess them, and a means of recalling events from before the most recent impulse of feedback information. Armed with these additional observations, let us refine our understanding of intelligence as an evolutionary phenomenon.

One can model intelligence at many

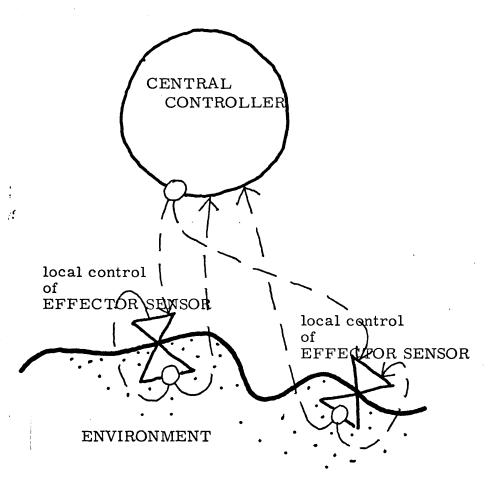
levels. Following is a very elementary description: operant behavior is the ability to acquire new responses to stimuli without any prior propensity to shape them. In the course of natural evolution a mass of one-celled creatures has no greater propensity to organize into a rhinoceros than into an elm tree. A person has no more propensity to evolve the responses of his body to play the piano than to speak Japanese. Ability for these things to occur depends on the mechanisms involved; propensity depends on the environment they operate in. Operant behavior is evolutionary. This is in contrast to reflexive behavior, such as breathing

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or eye-blinking, which is a hard-wired response that can come under the control of stimuli other than the original, as happened to Pavlov's dogs when a belltone controlled their salivation in place of the stimulus of food.

Unlike natural evolution, operant behavior relies on a direct information link between the environment and the response generator. The information provides a reinforcing stimulus to the response that caused the environment to emit it and when compared with a goal, it provides a discriminating stimulus determining the generator's response. This system represents feedback at the highest level. In addition, there are many feedback loops controlling

## HIERARCHY OF CONTROL



the effector-like movements of sensors seeking information from their environment, and the local sensing that controls the effectors' response as directed from the brain. Local control of sensors and effectors provides the information necessary for central control, but <u>the speed of the</u> <u>convergence of responses to an environmental context hinge on the ability of</u> <u>the response generator to form mutants</u> <u>of previously successful responses to</u> a similar context.

### 1.4 The Response Generator, an Iceberg

"Response generator" is a term that hides a formidable problem. The information that determines the shape of a mutant response cannot be as complete as the response itself without being identical with it. Surely the sperm and the egg do not contain information equilavent to every portion of a human being such that for every atom in the complete entity, there is a corresponding atom in the information representing the entity. Each cell in a human being is about as complex as the original two. The development of a human being from two cells embodies positive feedback of form;

the information transmitted from the parent entities to the generator is a set of instructions on how to make another more complicated set of instructions on how to make the next set, etc. One speculates that a sparse amount of information initiates a self-organizing process with complex results. In an absurd analogy one could select two books from a library shelf, choose a number between the Library of Congress numbers of the books selected, and insert it into a book generator; the book received, would be not just a random distribution of paper and ink, but an original work with real ideas in it, be they of high or low quality.

The purpose of such a generator would be to propagate widely varied responses when the mismatch between mutant and environment is great, and not so varied responses as the disparity diminishes. As such, it embodies a probabilistic model of an environment. If one can construct such a generator of form for mutant responses, one can use a similar method to simulate an environment within a machine without real-world contact. It would require a set of data as sparse as that which generates mutants. A deterministic generating process would simulate environment where only sensors and effectors were in operation. The French used a ruse similar in principle

for deceiving the Germans in the movie, "The Train." The Germans loaded a train with art work stolen from the Louvre to take to Germany in case Paris was to be burned. The French partisans diverted the train away from Germany. Each station along the way had the name changed to correspond to the appropriate one along the route intended by the Germans. This obviated the need to simulate all of France and Germany to save the paintings.

The advantage of using the real world as an environment for the simulation of evolution is that it provides a continuous multiformity of energy to be sensed

and altered. Wherever a response occurs in whatever form, the real world offers something to parry it. No matter what the inclination or wavelength of a sensor, the real world is emitting energy ready for reception. Internal representation of an environment might be useful only if it were more convenient than interfacing with the real world; it would be foolish to simulate the real world instead of some convenient imaginary one. For the machine it would be a logically constrained dream. The problem with machine dreams as with human ones is to witness the internal events.

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1.5 Relevant Research and Interest

The understanding and especially the simulation of evolution are steps less than, but probably necessary to, understanding and creating artificial intelligence. This is evident from the progress in nature of organizing intelligence, not because intelligence must evolve from less organized phenomena, but because it is a special and particularly complex manifestation of evolutionary processes.

Warren Brody expects evolutionary environments to be intelligent in order to be sufficiently discriminating, rapid

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in response, and able to learn when confronted with the changing contexts of their users. "Human Enhancement through Evolutionary Technology" argues that we are enslaved by the necessity to adapt to the viscissitudes of "stupid" unresponsive machines and (by extension) environments. "Our entire machine environment needs to be given a selforganizing capability that is similar to the self-organizing capability of men, so that both kinds of systems can evolve and survive over the long run."<sup>2</sup>

"Human Enhancement: Beyond the Machine Age" is a survey of efforts in pursuit of the aims espoused in Brody's above-

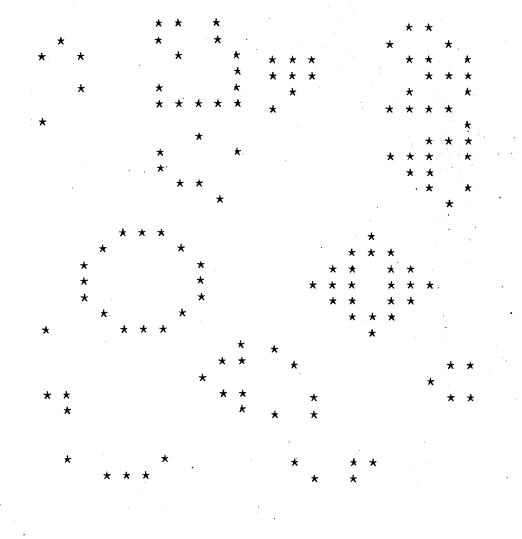
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mentioned article. Of importance to this paper are the concepts of self-organizing control systems and artificial intelligence through evolutionary programming. The self-organizing controller was developed by Gilstrap, Barron, et al. to relieve pilots of the burden of controlling aircraft with unstable handling properties due to the delay between a pilot's response and the aircraft's reaction. "There is (1) a goal circuit (performance assessment logic), which is a means for evaluating current performance; (2) a conditioning logic for computing and effecting suitable changes of the controller parameters and/or output signals; and (3) a memory for

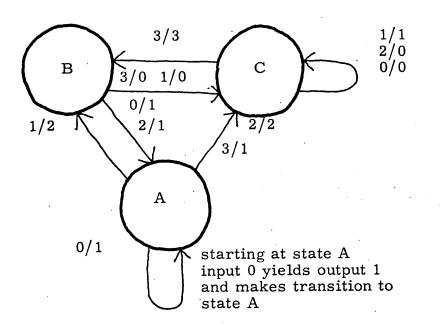
storing information concerning past parameter states. The memory exhibits an 'exponential forgetting', important in control applications because experiences in the remote past usually have less pertinence to present actions than do relatively recent experiences."<sup>3</sup>

The importance of a self-organizing controller is that it uses a certain amount of randomness in its responses to search out a suitable response to its environment at each point in time. Its responses are along a finite set of stimulus dimensions, defined in terms of pitch, roll and yaw. The controller's goals are to cause the unstable aircraft's motion to conform to its model of the pilot's intentions. These are communicated through his controls in response to the plane's motion. This system is not truly evolutionary because it generates vectors only for those fixed dimensions, rather than qualitatively changing its behavior when the rules of its environment change.

In his paper, "Environments of Self-Organizing Systems," Heinz von Foerster demonstrates that a closed, bounded self-organizing system cannot exist because it would be in contravention to the Second Law of Thermodynamics. Instead, it must exist in close contact with an environment from which it can draw energy and order. Hence the earth, which is becoming increasingly more organized, relies on the sun for the energy necessary to evolution, and the rules of interaction among the elements represent the source of order. "Cheap, undirected" energy enables rule-following entities to increase the orderliness of their association by encouraging random contact with other entities. In terms of the Bible, these conditions create order from chaos; in terms of the engineer, they create a signal out of noise.



SELF-ORGANIZATION



A FINITE-STATE AUTOMATION AND												
ITS PREDICTIONS <sup>4</sup>												
State	В	Α	С	С	С	С	в	С	С	в	Α	В
Input	2	2	1	0	1	3	3	0	3	0	1	2
Output		1	2	1	0	1	3	0	0	3	1	2
Error Cost		1	1	1	1	1	0	0	1	1	0	

Fogel, Owens and Walsh's Artificial Intelligence through Simulated Evolution demonstrates the approximate state of explicit interest in artificial evolution. Fogel uses finite-state automata to provide responses to an environment of cyclical number series. A finite-state automation exists in only one of a number of states at a given time. Each responds to any one of a finite number of inputs by emitting an output and changing its state (perhaps back to the same one) in a way that differentiates it from any other possible state. In this way a finitestate automaton determines a sequence of outputs depending on the inputs it receives. In order to successfully respond

to the environment Fogel has defined, it must correctly predict each subsequent signal emitted by the environment. In terms of the operation of the machine, each input signal from the environment should cause the machine in its particular state to emit the signal identical to the one that the environment will emit next. Input signals that elicit the incorrect response from the machine are noise until a machine is chosen that can correctly predict that response.

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Fogel's automata are sensitive to two stimulus dimensions: integers and cyclicality. He does not discuss time, but his machines demonstrate what is necessary to respond to it. Time is the abstraction of any sequence of events apart from the specific events themselves. In order to respond to time a machine must change states thereby distinguishing the events of one time from those of another.

Because there are really only two fixed dimensions of response these experiments do not demonstrate evolution. Mutant variations of parent machines are evaluated in comparison with their forerunners and improved versions are selected on the basis of their performance. However Fogel points out that "as long as evolutionary programming is restricted to the use of finite-state machines as the representation for the evolving organism, it is possible to encounter plants or environments that can never be completely expressed within the logic of a single organism. For example, the binary sequence 101100111000... and the characteristic function of the prime numbers are sequences that cannot be perfectly described by a single finitestate machine:"<sup>b</sup> The process for generating mutant machines itself does not improve.

The noise in the generating process permits it to organize favorable mutant response predictions about the environment. Since the regeneration of an entity is itself a response to the environment, the process of regeneration should mutate itself in order for the responses to improve.

Self-reproduction requires much more than the original response-machine. There must be some kind of machine sufficient to construct the things we require. In order for the constructor to make the original machine, there must be a set of instructions for it. Upon following the instructions, there exists a copy of the response-machine without

the means for reproduction. The constructor must execute a set of instructions on how to construct itself, how to construct a controller to decide the order of construction and activate newly created machines that might otherwise interfere with the ongoing process, and how to construct instructions for the controller as well. Now it can construct everything except that it has not reconstructed all the instructions on how to repeat the process. With a machine to be reproduced, a constructor, a controller and a set of instructions on how to reconstruct everything including themselves, self-reproduction can take place.

instructions and mechanisms to implement them, the problem is how to keep the information necessary for self-reproduction small relative to the complexity of the eventual organism. Positive feedback of form must implement successively more complex stages of growth. If the germinating information is small relative to the eventual entity, one must still be able to effect a small change in the simple beginning stages without causing a major change in the final complex stages, if one is to have a reasonable range of variation in mutability.

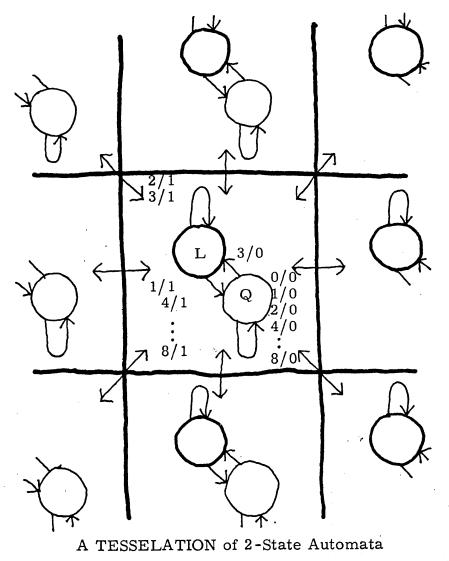
Once equipped with the necessary

#### 1.6 Cellular Automata, a Medium

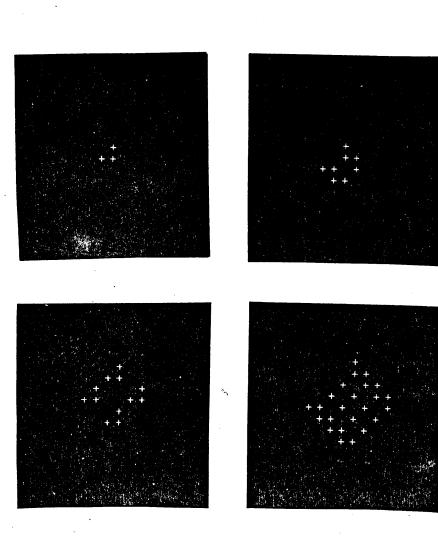
states.

The medium of cellular automata provides an avenue for probing positive feedback of form, self-reproduction and possibly evolution. Cellular automata are similar to the finite-state machines used by Fogel, except that there are many of them arranged in a grid of consistent geometry and each represents a part of its neighbors' environments. Each cell changes state and emits a signal depending on the pattern of signals it receives from its neighbors. Most interest is focused on very simple automata with a small number of states and a small number of rules for changing

Edward Moore in "Machine Models of Self-Reproduction" gives a more complete description. Consider "a universe which is a two-dimensional Euclidean space subdivided into square cells of equal size, like squares of graph paper [Call] such a space a tesselation Located in each of the cells of this tesselation there is to be one copy of a finite-state machine. Each cell-machine is to be deterministic and synchronous; that is, at each integervalued time T g.t. O, the state of each cell-machine is to depend only on its own state at time T-1 and on the states



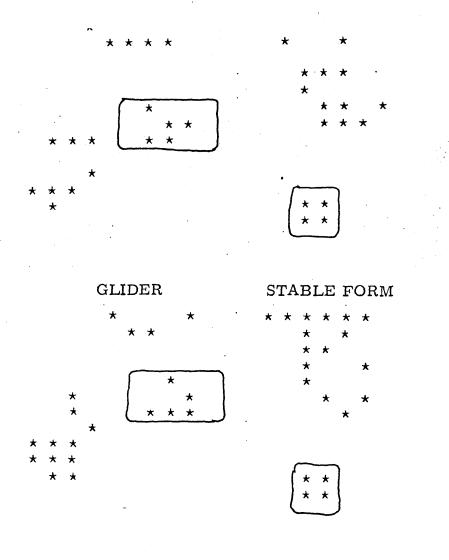
of its neighboring cell-machines at time T-1. All of the cell-machines are to be exactly alike as to their list of states and the rule determining their transitions, but different cell-machines are permitted to be in different states. The list of the possible states of the cell-machines must include a special state called the quiescent state, and all except a finite number of cellmachines will be in the quiescent state. The quiescent state is to have the property that if any cell-machine and all of its neighbors are in the quiescent state at time T-1, then the cellmachine will be in the quiescent state at time T".



FREDKIN'S SELF-REPRODUCING TROMINO It is easy to make rules capable of trivial self-reproduction. Moore points out that a rule stating, "there are only two states, X and O, and having the transition function f such that each cell will be in state X at time T if at least one of its neighbors was in state X at time T-1, then the configuration consisting of one cell in state X will be a self-reproducing configuration."<sup>7</sup> Fredkin's tromino<sup>8</sup> in orthogonal four-neighbor cellular space is a slightly less trivial example of trivial self-reproduction.

In the October 1970 issue of <u>Scientific</u> American there appears a tesselation

devised by John Conway, called Life. His rules are of particular interest because they involve only two-state automata and a simple set of rules governing the change and survival of states, but at the same time are sufficiently tenuous to make a configuration that continuously increases the active population of the tesselation, not obvious even after initial experimentation. Patterns that do not decay are rare, although there do exist patterns that are stable, some that oscillate and some that move across the space (so-called gliders and rocket ships). More interesting was the discovery of a glider gun



which did indeed increase the population by creating a new glider every fifteenth time period. Each glider moves out of the way for the next one. Furthermore, a certain fortuitous collision of gliders causes the formation of a glider gun.

This is not sufficient to demonstrate self-reproduction in Conway's space. If one considers the gun to be the object of reproduction, then the parent guns have to be positioned and phased in such a way as to line up the trajectories of their gliders on the appropriate collision courses. This process requires thirteen guns to generate the thirteen necessary gliders to create, not thirteen, but only one gun that is being bombarded by the original sources of gliders. Reproducing gliders also falls into a trap if it requires the same immobile guns.

It is difficult to determine how simple an elementary machine can be to participate in self-reproduction. Von Neumann demonstrates<sup>9</sup> a 29-state transition function capable of reproduction of its own 200,000 cells. Edgar Codd requires a much smaller 8-state tesselation.<sup>10</sup> Distinguishing trivial from non-trivial self-reproduction in the absence of a satisfactory definition of

either, further complicates the problem. Certainly simulation of evolution using self-reproduction will qualify as non-trivial, and not merely complex. The elementary machine need not be evolutionary in itself. It must have sufficiently complex rules to permit an increase in overall order in association with others like itself. Witness physics as a basis for chemistry, chemistry as a basis for biology, biology as a basis for physiology, physiology as a basis for psychology, psychology as a basis for sociology, and sociology as a basis for political science. All these studies are looking at hierarchies of order, whose boundaries in the universe are neither distinct nor finite-dimensional, but represent an increase in self-organization from a relatively constant set of principles to a set of evolutionary phenomena.

If they do not yet demonstrate selfreproduction, Conway's rules for Life do permit positive feedback of formation. The <u>r-pentomino</u> is a configuration of five live counters which after more than a thousand time-increments stabilizes at a formation of several blinkers, gliders and other stable forms. There are many other less impressive examples.

Although a mathematical environment can

test cellular configurations for stability, growth, or even self-reproduction, it cannot generate such forms. No one has yet discovered a heuristic for generating interesting formations. Nature has a large advantage. There is a sufficiently complex set of elementary rules to permit self-organization and plenty of "cheap, undirected energy" permitting many concurrent events to stumble across evolutionary configurations after billions of years.

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Finding a self-reproducing formation and especially an evolutionary one requires patience. If one starts out with the right rules for evolution

(without yet knowing it), one has only to permit random events to take place long enough before one has an evolving system. That probably takes a long time, but not so long as the now-famous monkeys with their typewriters who will probably randomly type the works of Shakespeare making many mistakes with commas, when other things are correct, and vice versa, before they type a correct version. When the evolutionary state approaches "Shakespeareness," it will work on that quality in the presence of an environment encouraging it until its "Shakespeareness" becomes quite good. After waiting a long time in vain, one might never know that the rules were not

sufficient for evolution.

1.7 Recapitulation or Capitulation?

Evolution is self-organizing; it draws on an environment for energy and order. Evolution must be self-reproducing, otherwise the process of improvement will not improve itself.

## Evolution includes:

Responses embodying a model of an environment that determines which responses a selection process will permit to participate in the generation of new mutant responses. It is easier to create an evolutionary process with evolving components than with non-evolving components.

## 2.0 THE EXPERIMENTAL MEDIUMS

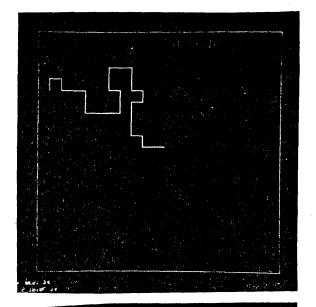
#### 2.1 A Probability Walk

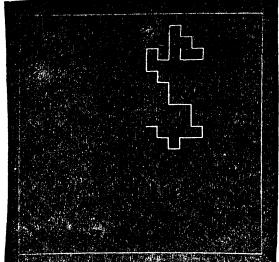
My work started with an experiment to explore how well a hierarchy of probabilities could improve a mechanism's ability to predict. A simulated machine wandered in a bounded region. Capable of turning only  $90^{\circ}$  right or left, or going forward, its task was to avoid the boundaries. At a hierarchy level of one, having no memory, it would accumulate a probability for going in each of the three possible directions. Unable to sense its environment, its longevity of movement would determine the value of its current set of probabilities; these would change in the direction

indicated by subsequent, more successful trials.

An obvious secret to longevity in this environment is to make nothing but right turns. The machine had no initial propensity to do this and had only a small likelihood of discovering this strategy from the many other possible combination of responses. Although the machine's ability to perform could improve, its ability to improve could not.

Beyond the first level of hierarchy was the possiblity of controlling each of the movements to follow each of the other movements. Beyond the second level





one could control (for example) the likelihood for right to follow left where left had just followed forward movement. <u>The hierarchical structure</u> <u>burgeons as it handles all the possible</u> <u>combinations of responses specifically.</u> That experiment was a straw man, and was not worth pursuing as a route toward evolution.

## LEFT: PROBABILITY WALKS

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## 2.2 The Construction of a Tesselation

The February 1971 issue of Scientific American describing discoveries about Conway's game of Life suggested that cellular automata would be an interesting medium for exploring the problems of evolution, not by representing a population of creatures as Conway's metaphorical name suggests, but as a self-organizing system. My task became one of building a tesselation that would accept cellular automata, varying in numbers of states and rules of behavior.

I started using the Interdata Fortran programming language with I/O (input/

output) software developed to enable Fortran to communicate with ARDS storage tubes and Sylvania tablet. My initial set of rules operated with a neighborhood of the four orthogonally located cells determining quiescence and activation. Quiescence remained in effect for a cell unless it acquired exactly three neighbors; activation continued as long as there were two or three neighbors.

Because Fortran placed severe limitations on the size of the tesselation, I soon started to write a similar program in assembly language, the learning of which was crucial to this thesis. Assembly language permits detailed control of events and economy of usage since it is the closest thing to the "machine" instructions that govern the behavior of the computer. Its inconvenience stems from the same fact; it compels one to pay great attention to minute detail which a higher-order language, like fortran, takes care of automatically. The effort was worthwhile, since it changed the limits of the tesselation size in the Interdata Model 3 from 10 x 10 in Fortran to 55 x 55 in assembly language, an increase by a factor of more than 30. The speed of operation for the respective languages increased from about 3 elements per second

to about 600 elements per second. Several different rule structures were tried including Conway's with an eight cell neighborhood and several rules that represented "bugs" in the program.

From one's transition function, one hopes to see self-organization through positive feedback of form, where a small population germinates a more complicated one. Necessary to selfreproduction is a configuration that moves out of the way of other similar figures during the process. Evolution requires both positive feedback and selfreproduction with the possiblity for mutation. Inspection of interesting configurations already discovered by others revealed that there is no apparent heuristic that would be likely to generate interesting forms. Although symmetry was a common phenomenon, the most interesting formations usually had at least one asymmetric property. The glider gun was not even a contiguous pattern. Failing a heuristic, the choice was to generate random initial conditions and watch their progress in self-organization. Although not as desirable as trying out many "almost good" configurations, it is a lot more satisfactory than generating random static solutions. If one seeks to find a stable form that is resilient to a

hostile environment, it is interesting to bombard the tesselation with stray live counters occasionally; this might provide mutations that have more interesting behavior than the stable forms they mutate.

The problem remains to find a selfreproducing entity that is mutatable enough to provide a large number of alternative configurations at any stage in a self-improving process. Von Neumann's universal constructor with a neighborhood of four orthogonal elements and 29 states would be capable of self-reproduction, but it would still require mutation. After mutation it might not be able to reconstruct itself.

Von Neumann's demonstration does not address itself to the positive feedback of form from a simple to a complex entity.

The appendices contain portions of sample output representing an initial state of "primordial ooze" that organizes itself into stable formations. The assembly language program is also included with comments on how to modify the number of states, transition functions, etc. The end of the usefulness of this medium for studying evolution will provide the conclusion for this thesis.

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# APPENDICES

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	008ER			BAL	0, BLANK		
	0092R	007ER		BXLE	4, POOL2		
	00921	005ER		DAGE			
	0096R	4100		BAL	0,LFCR		
	009AR	COAAR		BXLE	1,P00L1		
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	009ER			SSR	9,8		
	00A0R			BTC	9,*-2		
	00A4R	009ER		W D	9,FF		
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	GOA8R			BR	10		
	OOAAR		LFCR	SSR	9,8		
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	00C2R			LHI	2,1		
	00C6R	0001		LHR	3,3		
	OOC8R		LOOPl	AHR	7,14		•
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	OOCCR	C850 0001		LHI	5,1		
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	00E2R	0A9F		AHR	9,15		
	00E4R	D389		LB	8,COUNT(9	)	
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		0E22R	
00E8R CA80 0001		AHI	8,1
00ECR D289		STB	8,COUNT(9)
0E22R 00F0R CA70 0001	SAMEL	AHI	7,1
00F4R C140		BXLE	4,L00P2
00D2R 00F8R 0A7D 00FAR C110 00C8R		AHR BXLE	7,13 1,LOOP1
00FER 030A 0100R 0830 0102R C870	TESTS RIGHT	BR SHR LHI	10 0,0 7,0
0000 0106R 4830		LH	3 <b>,</b> UP
022AR 010AR CAEO		AHI	14,1
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0124R 0B77			7,7
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012AR 41A0 00COR		BAL	10,TALLY
012ER 4830 022CR	BELON	LH	3,UPM
0132R 4860 022AR	·	LH	6,UP
0136R 4870		LH	7,UP1
O228R O13AR OBDD		SHR	13,13
013CR 48F0 0228R		LH	15,UP1
0140R CBFO		SHI	15,1
COO1 0144R C7F0		XHI	15,X*FFFF*
FFFF 0148R 41A0 00COR		BAL	10, TALLY
014CR 0877	ABOVE	SHR	7,7
014ER C8E0 0000		LHI	14,0
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022CR 0166R C8E0		LHI	14.1
0001 016AR 48F0		LH	15,UP1
0228R 016ER C7F0		XHI	15,X*FFFF*
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017CR CAF0 0002		AHI	15,2
0180R C8D0 0001		LHI	13,1
0184R 41A0		BAL	10,TALLY
00COR 0188R 0B77 018AR 48F0 0228R	LORITE	SHR LH	7,7 15,UP1
018ER CAFO		AHI	15,1
0001 0192R 41A0 00COR		BAL	10, TALLY
0196R C8E0 0001	LOLEFT	LHI	14,1
019AR 0B77 019CR 48F0 022AR		SHR LH	7,7 15,UP
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01A6R C500 0000		CLHI	0,0
01AAR 4330 0000R		BZ ·	LOAD
OlAER OB77 Olbor Obll Olb2R C820	RBRIH	SHR SHR LHI	7,7 1,1 2,1
0001 01B6R 4830		LH	3,UP
022AR 01BAR 0B44 01BCR C850	LOPI	SHR LHI	4,4 5,1
0001 01COR 4860		ΓH	6,UP
022AR 01C4R D387 0238R	LOP2	LB	8,FIELD(7)
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## THE BEGINNING OF SELF-ORGANIZATION

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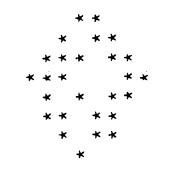
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