6.061 / 6.690 Introduction to Electric Power Systems
Spring 2007

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

Problem Set 10 Solutions May 6, 2007

The bulk of this problem set solution is contained in the single (fairly long) MATLAB script that is appended.

Problem 1: The torque-speed curve is generated in a straightforward way by first finding the input impedance of the machine, finding terminal current, doing the current divider thing to find rotor current and using that to find torque.The resulting curve is shown in Figure 1

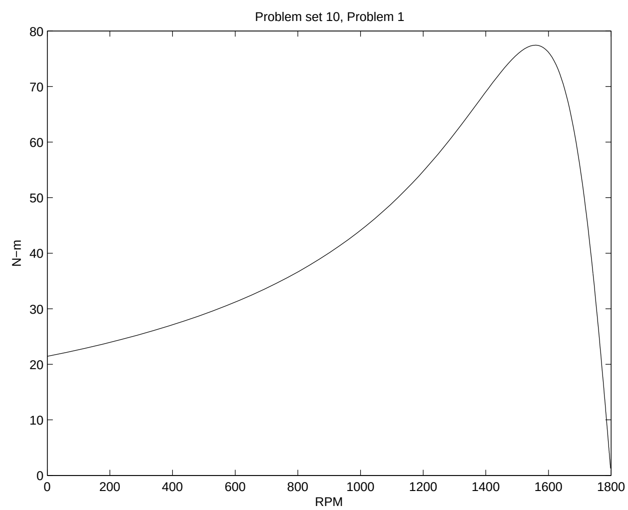

Figure 1: Torque-Speed Curve

Problem 2: Terminal current comes essentially for free and is shown in Figure 2. Note that this is not a very good machine because its magnetizing inductance is too low, leading to substantial current while running light. As we will see below, it has very poor power factor.

Problem 3: To find breakdown torque, recognize that is produced when the rotor equivalent circuit is dissipating maximum power, and that condition is when rotor resistance $\frac{R_2}{s}$ is equal to its source impedance. So we first calculate source impedance, set $\frac{R_s}{s}$ equal, which means setting s, and finding what the machine does. Here is a snippet of output from the script:

```
6.061 Problem Set 10, Problem 3
Breakdown Torque = 77.4476
Slip at Breakdown  = 0.134227
Speed at Breakdown = 1558.39
Current at Breakdown Torque = 45.6225
Power Factor at Breakdown Torque = 0.46847
```

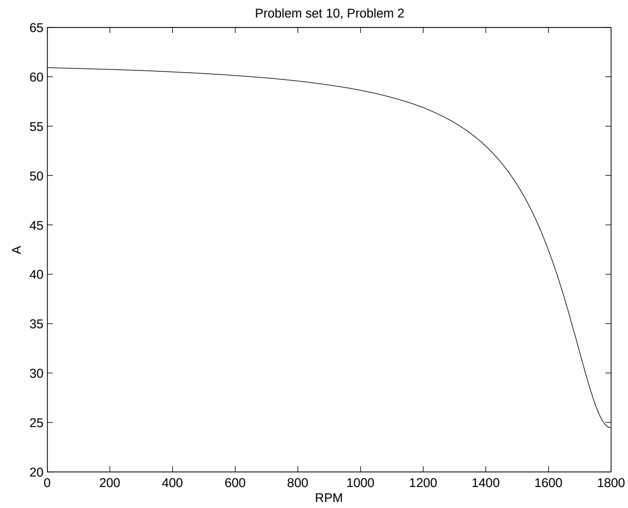

Figure 2: Current-Speed Curve

Problem 4: Running 'light', the only power output is windage, and we have to do a little searching to find the correct slip. Fortunately, in the low slip region, torque and power are both very nearly proportional to slip so the search procedure is simple. The resulting input power is very little more than core plus windage loss, but the power factor is, as expected, quite low.

```
Problem 4: Runninng Light
Rotational Speed = 1799.81 RPM
Real Power = 1003.8
Reactive Power = 20342.7
```

Problem 5 Blocked rotor is easier as current is defined. So we find voltage along with torque.

```
Problem 5: Blocked Rotor at 9 A
Terminal Voltage = 40.9353
Real Power = 210.186
Reactive Power = 1085.08
Blocked Rotor Torque = 0.467588
```

Problem 6: For 6.690 The trick here is to find the values of slip corresponding with five and one hundred percent of rated torque. This is done using a technique very similar to that in finding the 'running light' condition, but in this case we must add rated and five percent torque. Just to check, I calculated output power for the rated case. Once the two values of slip are calculated we can do a sweep between them to get efficiency and power factor, which are in Figures 3 and 4 respectively.

Problem 7: For 6.690 This is straightforward: all that we change is frequency and voltage between curves. The results are plotted in Figure 5

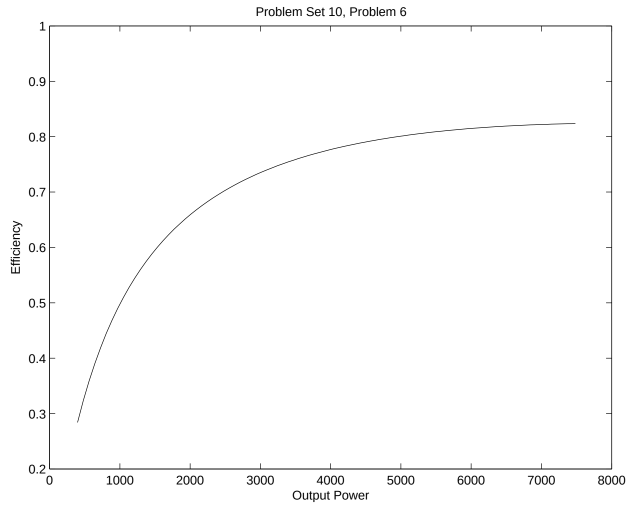

Figure 3: Rated Frequency Efficiency

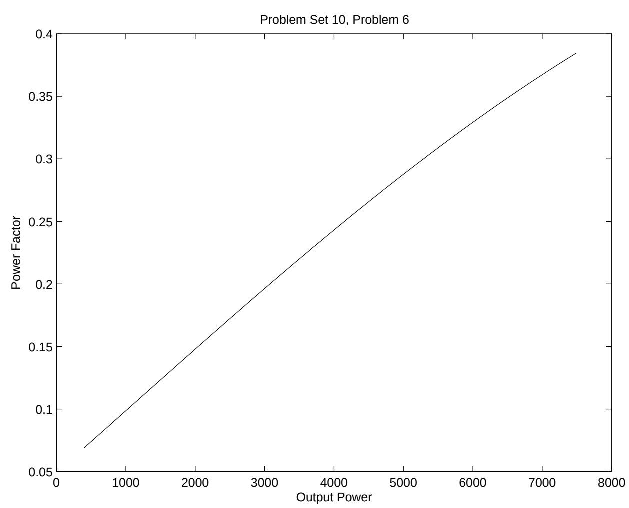

Figure 4: Rated Frequency Power Factor

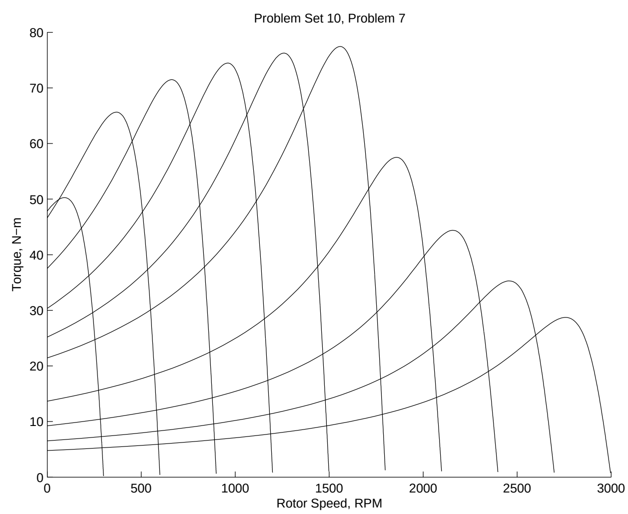

Figure 5: Torque Curves: Multiple Frequencies

```
% 6.061/6.690 Spring 2007 Problem Set 10
% Parameters:
Vll = 480;                   % line-line voltage
p = 2;                       % four pole machine
f = 60;                      % line frequency, Hz
R1 = .5;
R2 = .6;
X1 = 2.5;
X2 = 2.5;
Xm = 8.8;
Pc = 130;
Pw = 25;
Prat = 7460;                % this is rated power
tol =1e-14;                   % tolerance for iterative stuff
% find core loss element
Vlg = Vll/sqrt(3);
Rc = Vlg^2/(Pc/3);            % only 1/3 per phase!
% Problem 1
s = logspace(-3, 0, 200);     % trick to spacing points
ome = 2*pi*f;                 % electrical frequency
omm = (ome/p) .* (1 - s);     % mechanical frequency
N = (f/(2*pi)) .* omm;       % speed in RPM
Zr = j*X2 + R2 ./ s;          % rotor circuit branch impedance
Zg = j*Xm*Rc / (j*Xm + Rc);   % gap and core elements
Zag = Zg .* Zr ./ (Zg + Zr);  % air-gap impedance
Zt = Zag + j*X1 + R1;         % terminal impedance
It = Vlg ./ Zt;               % terminal current
I2 = It .* Zg ./ (Zg + Zr);   % current divider ratio
T = (3*p/ome) .* abs(I2) .^2 .* R2 ./ s;

figure(1)
plot(N, T)
title('Problem set 10, Problem 1')
ylabel('N-m')
xlabel('RPM')

% problem 2
figure(2)
plot(N, abs(It))
title('Problem set 10, Problem 2')
ylabel('A')
xlabel('RPM')

% Problem 3: To find breakdown torque
Za = R1 + j*X1;                 % armature branch
```

```
Zs = Za*Zg/(Za+Zg)+j*X2;          % source impedance
sM = R2/(abs(Zs));                % slip at max torque
Zrm = j*X2 + R2 / sM;              % rotor circuit branch impedance
Zagm = Zg * Zrm / (Zg + Zrm);     % air-gap impedance
Ztm = Za + Zagm;                  % terminal impedance
Itm = Vlg/Ztm;                    % terminal current
I2m = Itm *Zg/(Zg+Zrm);           % current divider
Tm = (3*p/ome) * abs(I2m) ^2 * R2 / sM;
Nb = (60/(2*pi))*(ome/p)*(1-sM);
pfm = real(Itm)/abs(Itm);         % power factor
fprintf('6.061 Problem Set 10, Problem 3\n');
fprintf('Breakdown Torque = %g\n',Tm);
fprintf('Slip at Breakdown  = %g\n',sM);
fprintf('Speed at Breakdown = %g\n',Nb);
fprintf('Current at Breakdown Torque = %g\n',abs(Itm));
fprintf('Power Factor at Breakdown Torque = %g\n',pfm);

%problem 4: running light
not_done=1;                       % we need to iterate here
sl = .001;                        % starting guess
while not_done==1,
  Zr = j*X2 + R2 / sl;            % rotor circuit branch impedance
  Zg = j*Xm*Rc / (j*Xm + Rc);     % gap and core elements
  Zag = Zg * Zr / (Zg + Zr);      % air-gap impedance
  Zt = Zag + j*X1 + R1;           % terminal impedance
  It = Vlg / Zt;                  % terminal current
  I2 = It .* Zg ./ (Zg + Zr);     % current divider ratio
  Pm = 3*I2^2*(R2/sl)*(1-sl);
  if abs(1-Pm/Pw)^2 <tol
      not_done=0;                 % slip is about right
  else
      sl = sl*(Pw/Pm);            % new guess for slip
      fprintf('.')
  end
end
Pl = real(3*Vlg*conj(It));
Ql = imag(3*Vlg*conj(It));
Nl = 60*(f/p)*(1-sl);
fprintf('\nProblem 4: Runninng Light\n');
fprintf('Rotational Speed = %g RPM\n',Nl);
fprintf('Real Power = %g\n',Pl)
fprintf('Reactive Power = %g\n',Ql);

% Problem 5: Blocked Rotor
Zr = j*X2 + R2;                 % rotor circuit branch impedance (s=1)
```

```
Zg = j*Xm*Rc / (j*Xm + Rc);     % gap and core elements
Zag = Zg * Zr / (Zg + Zr);      % air-gap impedance
Zt = Zag + j*X1 + R1;           % terminal impedance
It = 9;                        % terminal current is fixed
Vb = It*Zt;                     % required terminal voltage
I2 = It .* Zg ./ (Zg + Zr);     % current divider ratio
Pb = real(3*Vb*conj(It));
Qb = imag(3*Vb*conj(It));
Tb = 3*abs(I2)^2*R2*p/ome;      % torque

fprintf('\nProblem 5: Blocked Rotor at %g A\n', It);
fprintf('Terminal Voltage = %g\n',abs(Vb));
fprintf('Real Power = %g\n',Pb)
fprintf('Reactive Power = %g\n',Qb);
fprintf('Blocked Rotor Torque = %g\n',Tb);
% Problem 6
% Now we are going to get power factor and efficiency: first
% we need to find slip for the full power point
Ptarget = Prat+Pw;                 % gonna adjust slip to get this out
notdone = 1;
s0 = .01;
while notdone == 1,
  Zr = j*X2 + R2 / s0;             % rotor circuit branch impedance
  Zag = Zg * Zr / (Zg + Zr);       % air-gap impedance
  Zt = Zag + j*X1 + R1;            % terminal impedance
  It = Vlg / Zt;                   % terminal current
  I2 = It * Zg / (Zg + Zr);        % current divider ratio
  Pag = 3*abs(I2)^2*R2/s0;         % air-gap power
  Pm = Pag*(1-s0)-Pw;
  if abs(Pm/Ptarget -1)^2 < tol,       % close enough
    notdone = 0;
  else


  s0 = s0*Ptarget/Pm;                % try new value for slip
  fprintf('.');
  end                                % end of if then else
end                                % end of slip loop
Sr = s0;                           % max value of slip
% just to check...
Tm = (p/ome)*(Pm-Pw)/(1-s0);            % output torque
Nm = 60*ome*(1-s0)/(2*pi*p);       % running speed
Pout = Tm*2*pi*Nm/60;              % check output power
fprintf('Check: Full Power Running\n')
fprintf('Speed = %g RPM (slip = %g)\n',Nm, s0);
```

```
fprintf('Torque = %g  Power = %g\n',Tm, Pout);

% now to find the value for five percent of power
Ptarget = .05*Prat+Pw;
notdone = 1;
s0 = .01;
while notdone == 1,
  Zr = j*X2 + R2 / s0;               % rotor circuit branch impedance
  Zag = Zg * Zr / (Zg + Zr);         % air-gap impedance
  Zt = Zag + j*X1 + R1;              % terminal impedance
  It = Vlg / Zt;                     % terminal current
  I2 = It * Zg / (Zg + Zr);          % current divider ratio
  Pag = 3*abs(I2)^2*R2/s0;           % air-gap power
  Pm = Pag*(1-s0)-Pw;
  if abs(Pm/Ptarget -1) < tol,         % close enough
    notdone = 0;
  else
  s0 = s0*Ptarget/Pm;                  % try new value for slip
  fprintf('.');
  end                                  % end of if then else
end                                    % end of slip loop
Sm = s0;                               % min value of slip

% now we can do this against slip

s = Sm:(Sr-Sm)/100:Sr;                 % slips to use
Zr = j*X2 + R2 ./ s;                   % rotor circuit branch impedance
Zg = j*Xm*Rc / (j*Xm + Rc);            % gap and core elements
Zag = Zg .* Zr ./ (Zg + Zr);           % air-gap impedance
Zt = Zag + j*X1 + R1;                  % terminal impedance
It = Vlg ./ Zt;                        % terminal current
I2 = It .* Zg ./ (Zg + Zr);            % current divider ratio
Pag = 3 .* abs(I2) .^2 .* R2 ./ s;
Pt = 3 .* real(Vlg .* conj(It));
Pm = Pag .* (1 - s) - Pw;
eff = Pm ./ Pt;
pf = Pt ./ (3*Vlg .* abs(It));

figure(3)
     plot(Pm, eff)
     title('Problem Set 10, Problem 6')
     xlabel('Output Power')
     ylabel('Efficiency')

 figure(4)
```

```
     plot(Pm, pf)
     title('Problem Set 10, Problem 6')
     xlabel('Output Power')
     ylabel('Power Factor')

% Problem 7: voltgs per Hz control
F_r = [1/6 2/6 3/6 4/6 5/6 1 7/6 8/6 9/6 10/6];  % for these relative frequencies
V_r = [1/6 2/6 3/6 4/6 5/6 1 1 1 1 1];           % and voltages

figure(5)
clf
hold on

% now it is just like the very first part of the problem

for i=1:length(F_r);
  fa = f*F_r(i);                  % here is our absolute frequency
  vlga = Vlg*V_r(i);              % need to adjust reactances for frequency
  X2a = X2*F_r(i);
  Xma = Xm*F_r(i);
  X1a = X1*F_r(i);

  s = logspace(-3, 0, 200);       % trick to spacing points
  ome = 2*pi*fa;                  % electrical frequency
  omm = (ome/p) .* (1 - s);       % mechanical frequency
  N = (60/(2*pi)) .* omm;         % speed in RPM
  Zr = j*X2a + R2 ./ s;            % rotor circuit branch impedance
  Zg = j*Xma*Rc / (j*Xma + Rc);     % gap and core elements
  Zag = Zg .* Zr ./ (Zg + Zr);    % air-gap impedance
  Zt = Zag + j*X1a + R1;           % terminal impedance
  It = vlga ./ Zt;                 % terminal current
  I2 = It .* Zg ./ (Zg + Zr);     % current divider ratio
  T = (3*p/ome) .* abs(I2) .^2 .* R2 ./ s;

  plot (N, T)
end
hold off
title('Problem Set 10, Problem 7')
ylabel('Torque, N-m')
xlabel('Rotor Speed, RPM')
```