6.061 / 6.690 Introduction to Electric Power Systems
Spring 2007

Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.690 Introduction to Power Systems

Problem Set 11 Solutions May 6, 2007

Problem 1: 1. If the motor is producing 500 kW at 125 radians/second, torque is $T = \frac{50000}{125} =$ 4000N-m. The motor constant is then:

$$G = \frac{T}{I^2} = \frac{5000\text{N-m}}{10^6 A^2} = .004\Omega\text{sec}$$

Now, back voltage would be $E_b = G\Omega I = .004 \times 125 \times 1000 = 500\text{V}$, and if terminal voltage is 600 V, the motor resistance must be $RI = 600 - 500 = 100\text{V}$ or $R = .1\Omega$. The torque/speed relationship is:

$$T = \frac{GV^2}{R + G\Omega}$$

This is evaluated in the attached script and is shown in Figure 1.

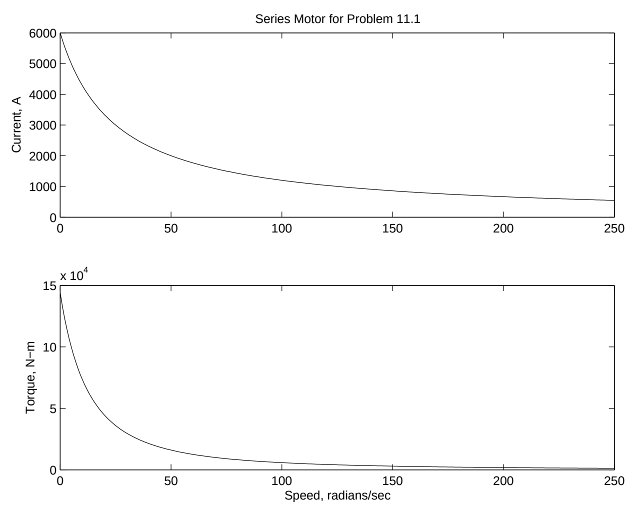

Figure 1: Torque vs. Speed Characteristic: Series Connected Motor

2. For the vehicle application,
   (a) The equivalent wheel radius is

$$R_{eq} = \frac{10\text{m/sec}}{125\text{Rad/sec}} = 0.08m$$

   (b) If the car weighs 20,000 kg and is accelerating at $1m/s^2$, the accelerating force is $F_A = 20000$ N.

(c) Drag force is

$$F_D = 5000 \left(\frac{u}{10\text{m/s}} \right)^2$$

(d) Then the drive force required is:

$$F_A + F_D = \left(\frac{G}{R_{eq}} \right) I^2 \qquad \text{Or} \qquad I = \sqrt{\frac{F_A + F_D}{\frac{G}{R_{eq}}}}$$

Velocity and required force are shown in Figure 2. Speed is simply $u = At$, so then we can find voltage at the terminals by:

$$V = RI + \frac{G}{R_{eq}} uI$$

Figure 3 shows required motor current, which for the resistive controller is also drive current. The middle chart in this figure is voltage drop across the controller. Power required from the source is just $P_s = V_s I$.

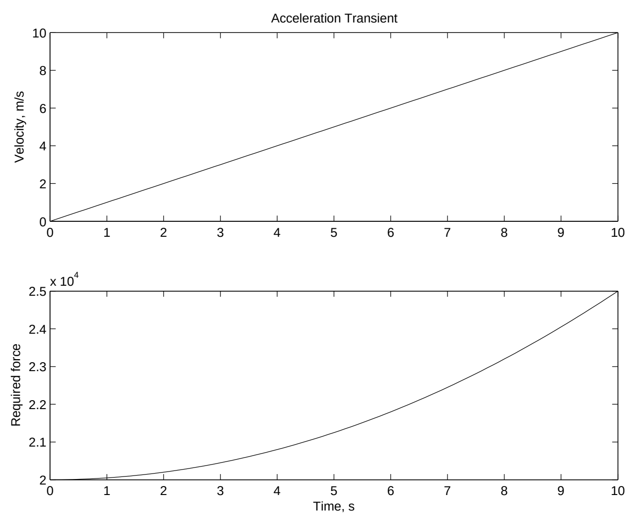

Figure 2: Velocity and drive force

In the next part we replace the resistive controller by an ideal buck converter (chopper). The duty cycle of the buck converter is

$$d = \frac{V}{V_s}$$

and of course, once that is found source current is:

$$I_s = dI$$

Current in the motor and required power are shown in Figure 4. Of course if the buck converter (chopper) is ideal, power into the motor and power from the source are the same.

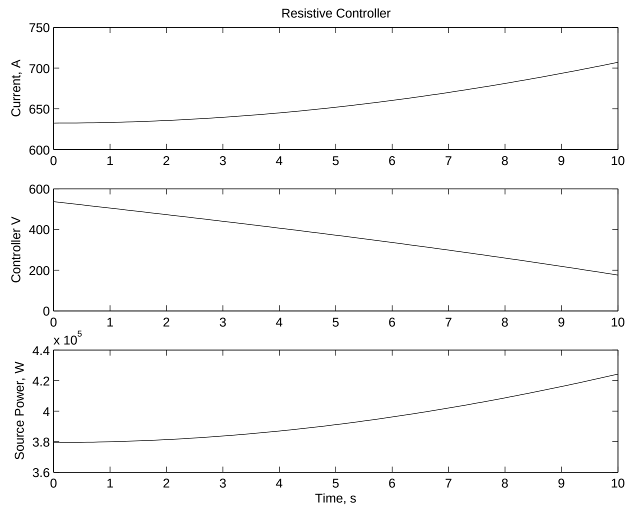

Figure 3: Motor Current, Resistance Voltage Drop and Source Power: Resistive Controller

3. **For 6.979**. Simulation of this situation is a fairly straightfoward exercise. Perhaps the most ambitious assumption here is the the car does not blow a circuit breaker. The scripts required to do this simulation are appended and the result is shown in Figure 5.

Problem 2 Since the two motors achieve base power at different speeds they have different low speed torque limits. For Motor A that low speed torque limit is $500,000/125 = 4000$ N, for Motor B it is $500000/62.5 = 8000$ N. Then the motor coefficients are $G_A I_f = 4000/920 \approx 4.35$ and $G_B I_f = 8000/92 \approx 8.69$. The same force requirements as were calculated for the series connected motor hold and current is just

$$I_a = \frac{F_A + F_D}{\frac{GI_f}{R_{eq}}}$$

Field current is constant: $I_f = 12,000/600 = 20$ A. With a resistive controller source power is simply

$$P_s = V_s \times (I_a + I_f)$$

The comparison is shown in Figure 6.

The limited jerk rate acceleration example assumes a jerk rate of $0.5 m/s^3$. This is about 1/20 g per second and is actually pretty gentle. During this limited period velocity is $u = \frac{1}{2} \times \frac{1}{2} t^2$ After an initial transient of 2 seconds we reach the stated acceleration rate of $a = 1 m/s^2$ (about 1/10 g) and $u = 1 m/s$. The end of the acceleration is a mirror image. The resulting acceleration and velocity profiles are shown in Figure 7.

All of the other details of this calculation are simple extensions of those done earlier. As this is a separately excited machine, armature current is directly proportional to required force. Acceleration and drag force are shown in Figure 8. Voltage components: back voltage $E_b = G\Omega I_f$, resistive drop and total terminal voltage and curents (armature current is proportional to force and source current at the input to the armature chopper) are shown in Figure 9.

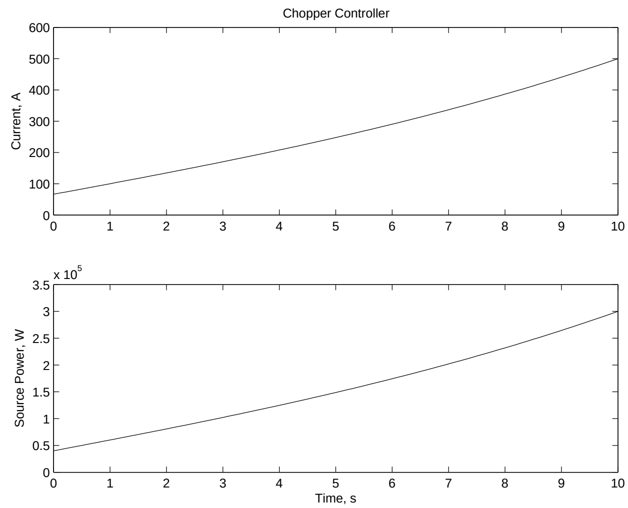

Figure 4: Source Current and Source Power: Ideal Chopper Controller

Finally, power is shown in Figure 10: mechanical power is force times velocity (dotted) and electrical power is source voltage times current (solid).

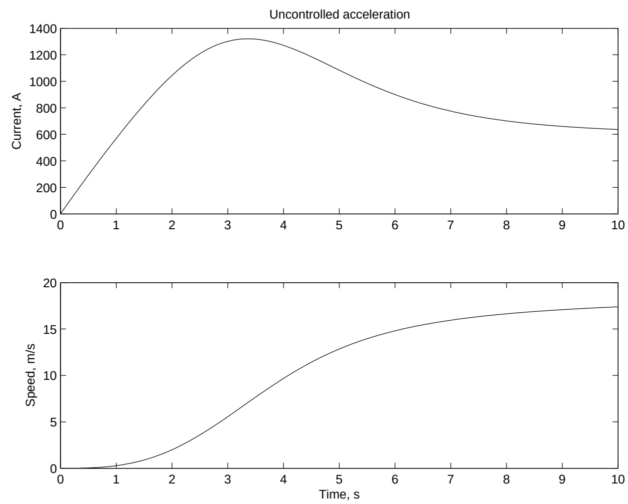

Figure 5: Simulation of uncontrolled acceleration

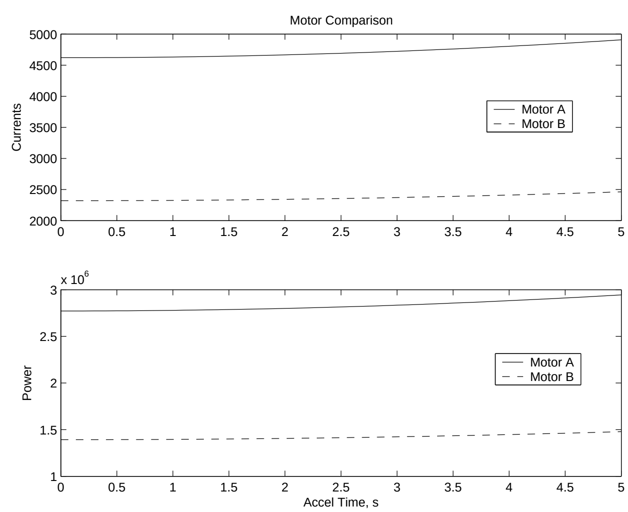

Figure 6: Motor Comparison

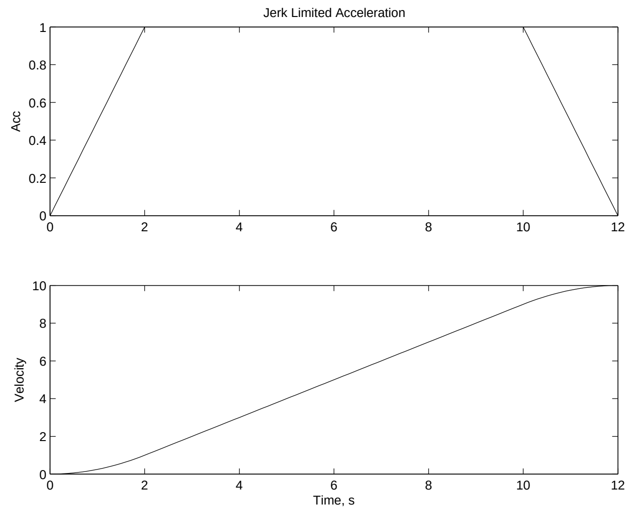

Figure 7: Limited Jerk Rate Acceleration

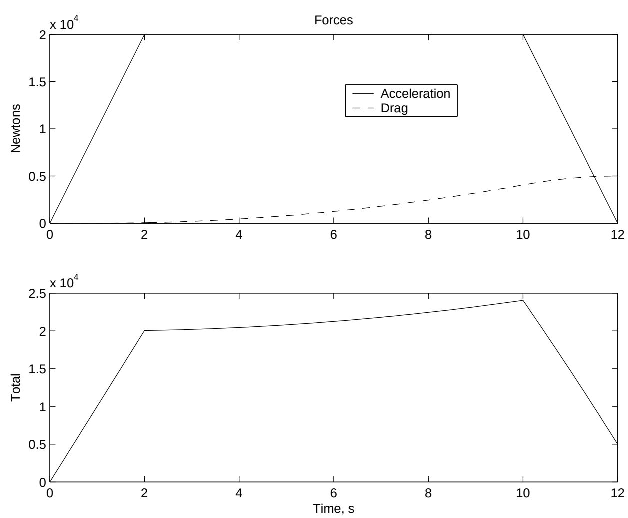

Figure 8: Force addition

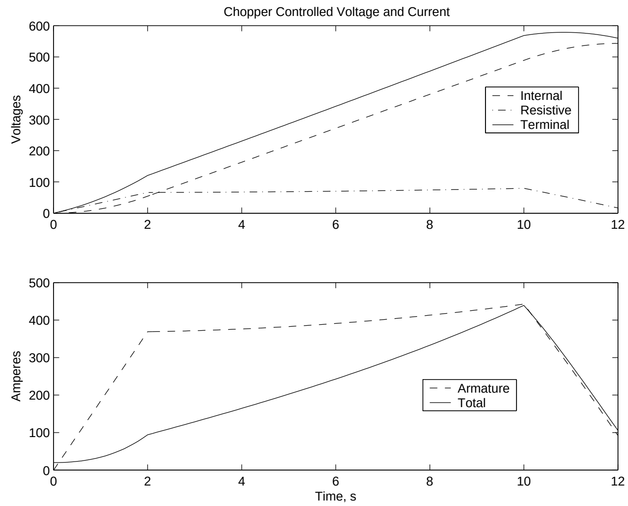

Figure 9: Voltages and Currents

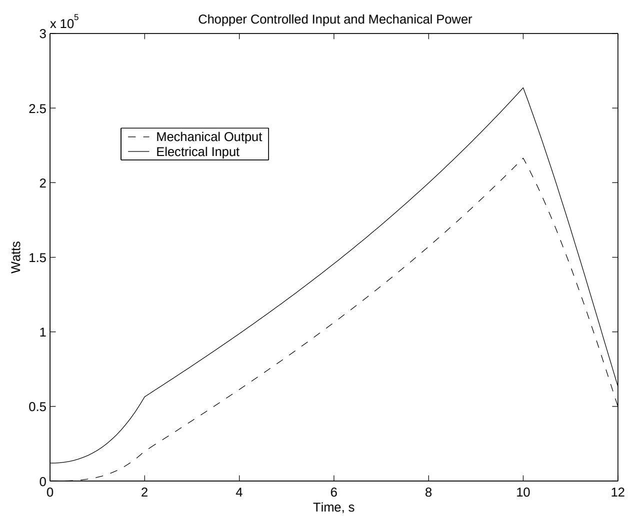

Figure 10: Mechanical and Electrical Power

```
% 6.061/6.960 Spring 07 Problem Set 11

% Problem 1
R = .1;                         % motor series resistance, ohms
G = .004;                       % motor constant, ohm-sec
V = 600;                        % terminal voltage
om = 0:1:250;                   % speed range

I = V ./ (R + G .* om);         % current drawn by series motor
T = G .* I .^2;                 % and resulting torque

figure(1)
subplot(211)
plot(om, I)
title('Series Motor for Problem 11.1')
ylabel('Current, A')
subplot(212)
plot(om, T)
ylabel('Torque, N-m')
xlabel('Speed, radians/sec')

% Part 2
M = 20000;                      % we are gonna accelerate this thing
D = 5000;                       % drag force (50 kW at 10 m/s) at 10 m/s
Req = .08;                      % equivalent wheel radius
a = 1;                          % required acceleration
t = 0:.1:10;                    % time for acceleration
u = a .* t;                     % so this is speed

Fa = M*a;                       % acceleration force
Fd = D .* (u ./ 10) .^2;         % drag force
Ft = Fa+Fd;
Ia = sqrt(Ft ./ (G/Req));       % required current
Psr = V .* Ia;                  % power from the source: Resistive controller

Eb = (G/Req) .* Ia .* u;        % back voltage
Er = R .* Ia;                   % resistive drop in motor
Erc = V - Eb - Er;              % drop in resistive controller
Vt = Eb + Er;                   % chopper output voltage
vr = Vt ./ V;                   % chopper duty cycle
Is = Ia .* vr;                  % current from source
Psc = V .* Is;                  % power from source: chopper controller

figure(2)
subplot 211
```

```
plot(t, u)
title('Acceleration Transient')
ylabel('Velocity, m/s')
subplot 212
plot(t, Ft)
ylabel('Required force')
xlabel('Time, s')

figure(3)
subplot 311
plot(t, Ia)
title('Resistive Controller')
ylabel('Current, A')
subplot 312
plot(t, Erc)
ylabel('Controller V')
subplot 313
plot(t, Psr)
ylabel('Source Power, W')
xlabel('Time, s')

figure(4)
subplot 211
plot(t, Is)
title('Chopper Controller')
ylabel('Current, A')
subplot 212
plot(t, Psc)
ylabel('Source Power, W')
xlabel('Time, s')

% now to simulate the uncontrolled acceleration example
ts = 0:.01:10;
S0 = [0 0]';
[tsim, S] = ode23('tc', ts, S0);
Isim = S(:,1);
Usim = S(:,2);

figure(5)
subplot 211
plot(tsim, Isim)
title('Uncontrolled acceleration')
ylabel('Current, A')
subplot 212
plot(tsim, Usim)
```

```
ylabel('Speed, m/s')
xlabel('Time, s')

% Problem 2
GA = 500000/(125*920);
GB = 500000/(62.5*920);
If = 12000/600;
t2 = 0:.1:5;
u2 = a .* t2;
Fa = M*a;                     % acceleration force
Fd = D .* (u2 ./ 10) .^2;      % drag force
Ft = Fa+Fd;
R_A = (V -GA*100)/920;
fprintf('Motor_A R = %g\n', R_A);

IA = Ft ./ GA;                % required armature current
IB = Ft ./ GB;

I_a = IA + If;                % total current: add back field
I_b = IB + If;

P_a = V .* I_a;               % power required
P_b = V .* I_b;

figure(6)
subplot(211)
plot(t2, I_a, '-', t2, I_b, '--')
legend('Motor A', 'Motor B')
ylabel('Currents')
title('Motor Comparison')
subplot(212)
plot(t2, P_a, '-', t2, P_b, '--')
ylabel('Power')
xlabel('Accel Time, s')
legend('Motor A', 'Motor B')


% Jerk rate limit
J = .5;
a0 = 1;
dt = .1;
T1 = 2;
T2 = 10;
T3 = 12;
t1 = 0:dt:T1;
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```
a1 = J .* t1;
u1 = .5*J .* t1 .^2;
U1 = u1(length(t1));
t2 = T1+dt:dt:T2;
a2 = a0 .* ones(size(t2));
u2 = U1 + a0 .* (t2-T1);
U2 = u2(length(u2));
t3 = T2+dt:dt:T3;
a3 = a0 - J .* (t3 - T2);
u3 = U2 + a0 .* (t3-T2) - .5*J .* (t3 - T2) .^2;
t = [t1 t2 t3];
a = [a1 a2 a3];
u = [u1 u2 u3];

figure(7)
subplot 211
plot(t, a)
title('Jerk Limited Acceleration')
ylabel('Acc')
subplot 212
plot(t, u)
ylabel('Velocity')
xlabel('Time, s')

Fa = M .* a;
Fd = D .* (u ./ 10) .^2;
Ft = Fa + Fd;
Ia = Ft ./ (GA/Req);                  % required current
V_r = R_A .* Ia;                % internal resistance drop
E_a = (GA/Req) .* u;            % back voltage
Vt = E_a + V_r;
vr = Vt/V;                      % chopper ratio
Is = vr .* Ia + If;             % source current
Ps = V .* Is;
Pm = u .* Ft;

figure(9)
subplot 211
plot(t, Fa, t, Fd, '--')
title('Forces')
ylabel('Newtons')
legend('Acceleration', 'Drag')
subplot 212
plot(t, Ft)
ylabel('Total')
```

```
xlabel('Time, s')

figure(10)
subplot 211
plot(t, E_a, '--', t, V_r, '-.', t, Vt)
title('Chopper Controlled Voltage and Current')
ylabel('Voltages')
legend('Internal', 'Resistive', 'Terminal')
subplot 212
plot(t, Ia, '--', t, Is)
legend('Armature', 'Total')
ylabel('Amperes')
xlabel('Time, s')

figure(8)
plot(t, Pm, '--', t, Ps)
title('Chopper Controlled Input and Mechanical Power')
ylabel('Watts');
xlabel('Time, s')
legend('Mechanical Output', 'Electrical Input')
```