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6.061 / 6.690 Introduction to Electric Power Systems
Spring 2007

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Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
 6.061/6.690 Introduction to Power Systems

Problem Set 3 Solutions

February 25, 2007

Problem 1: Assuming that sending-end and receiving end voltages are of equal magnitude and differ in phase angle by δ , *real* power flow is

$$P = \frac{|V|^2}{X_L} \sin \delta$$

or, in terms of the values given in the problem:

$$10 \times 10^6 = \frac{(10 \times 10^3)^2}{5} \times \sin \delta$$

Which implies that $\sin \delta = \frac{1}{2}$, or $\delta = 0.5236 \text{ Radians} = 30^\circ$.

Reactive power drawn by the line at the load end, which must be supplied by the capacitor that is supporting voltage is:

$$Q = \frac{V^2}{X_L} (1 - \cos \delta) = \frac{V^2}{X_L} \left(1 - \frac{\sqrt{3}}{2}\right) \approx 0.134 \times \frac{100 \times 10^6}{5} \approx 2.68 \text{ MVAR}$$

The capacitor is then sized by:

$$Q = V^2 \omega C$$

or

$$C = \frac{2.68 \times 10^6}{277 \times 100 \times 10^6} \approx 71.1 \mu F$$

If the voltages are equal, the problem is actually symmetric, and the same value of capacitance results in unity power factor at the source.

Figure 1 shows, approximately, the phasor diagram.

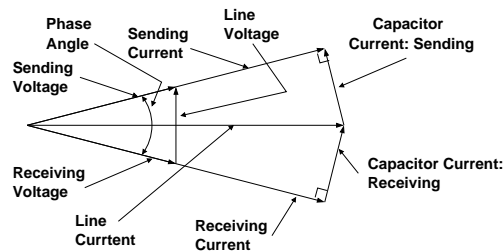


Figure 1: Transmission Line Voltages and Currents

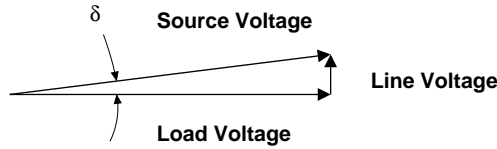


Figure 2: Phasor Diagram for Problem 2

Problem 2: As seen from the source, line plus load impedance are

$$Z = j\omega L + R$$

Since $\omega L = 377 \times .0265 \approx 10\Omega$, the total impedance has magnitude of 64.78Ω and a phase angle of very nearly 9°

Load current is

$$|I_L| = \frac{8000}{64.78} \approx 123.5A$$

and power in the load is

$$P_L = |I_L|^2 R \approx 976.2kW$$

For the second part, including the parallel capacitance, we may write the expression for load voltage, a simple voltage divider:

$$V_L = V_S \frac{R \parallel \frac{1}{j\omega C}}{R \parallel \frac{1}{j\omega C} + j\omega L} = \frac{V_S}{(1 - \omega^2 LC) + j\omega \frac{L}{R}}$$

To make $V_L = V_S$ we need to make

$$\left(1 - \omega^2 LC\right)^2 + \left(\omega \frac{L}{R}\right)^2 = 1$$

Which, after a little algebra, boils to

$$\omega C = \frac{1}{\omega L} \pm \sqrt{\left(\frac{1}{\omega L}\right)^2 - \left(\frac{1}{R}\right)^2}$$

The smaller capacitance results from taking the minus sign, for which the value of $C \approx 3.25\mu F$.

It is possible to solve, using MATLAB, the magnitude of receiving end voltage vs. capacitance (shown in Figure 3). The script that does this simply solves for the magnitude of receiving voltage. It appears in the appendix.

Problem 3: If the capacitor is really holding voltage constant, maximum and minimum currents in the inductor are:

$$\begin{aligned} I_m &= I_\ell + \frac{V_s - V_o}{L} t_{on} \\ I_\ell &= I_m - \frac{V_o}{L} t_{off} \end{aligned}$$

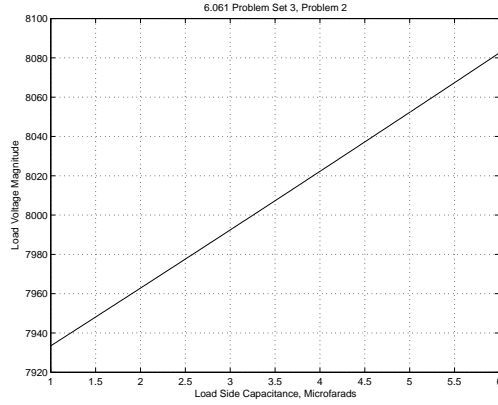


Figure 3: Receiving End Voltage vs. Compensating Cap

which may be handily solved for, as expected:

$$V_o = V_s \frac{t_{on}}{t_{on} + t_{off}}$$

Now, if output voltage is indeed constant, we may also conclude that average inductor current is the same as output current:

$$\frac{I_\ell + I_m}{2} = \frac{V_o}{R}$$

Combining these relationships we find:

$$I_m = \frac{V_o}{R} + \frac{1}{2} \frac{V_o}{L} t_{off} = 6.25A$$

$$I_\ell = \frac{V_o}{R} - \frac{1}{2} \frac{V_o}{L} t_{off} = 3.75A$$

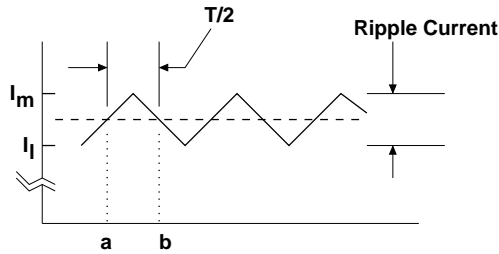


Figure 4: Cartoon of ripple current

Now, to get ripple voltage, note that the area of current ripple as shown in Figure 4 (the area above the average current line) represents charge added to the capacitor during the positive ripple excursion. This is:

$$\Delta Q = \frac{1}{2} \Delta I \frac{T}{2} = \frac{1}{2} \times \frac{1}{2} \times 10^{-4} \times .625A \approx 1.56 \times 10^{-5} \text{ coulombs}$$

With $C = 10\mu F$, this implies a voltage excursion of about 312 mV.

The script for simulating this circuit is appended. What it does is shown in Figure 5, which shows voltage buildup and Figure 6 which shows a couple of cycles near steady state to show ripple.

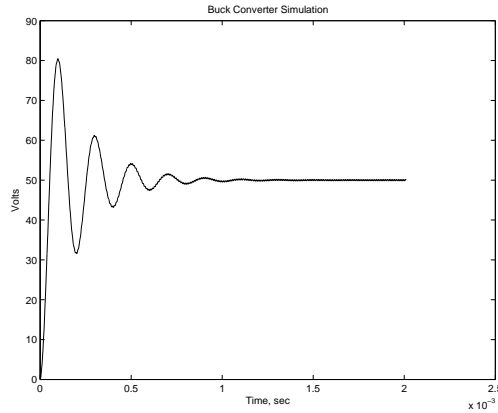


Figure 5: Voltage Buildup in Capacitor Filtered Buck Converter

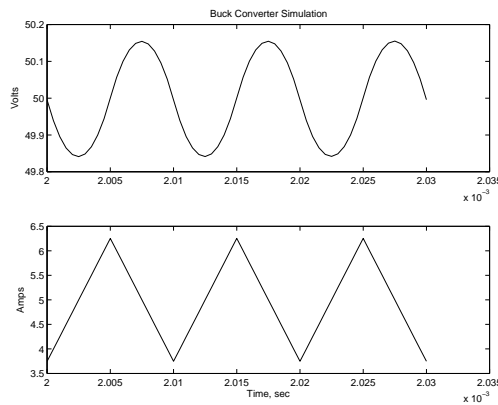


Figure 6: Ripple in Capacitor Filtered Buck Converter

Problem 4: In the Boost Converter, with $t_{on} = t_{off}$, output voltage should be just twice input voltage, or 100 V. With a load of 10Ω , this would draw 10 A in the load resistor, or an average of about 20 A out of the source.

Voltage change in the filter capacitor is then:

$$\Delta V = \frac{I t_{off}}{C}$$

which, for 10 A, $t_{off} = \frac{1}{2} \times 10^{-5}$ seconds and $C = 10^{-5}$ F, is just about 5 V.

This circuit, too, has been simulated, and the scripts are appended. The results are shown in Figure 7 (voltage buildup) and Figure 8 (steady state ripple).

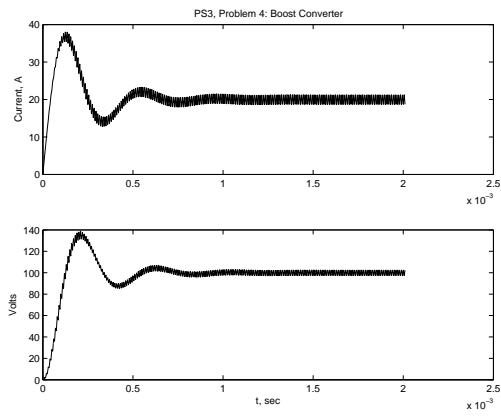


Figure 7: Boost Converter Voltage Buildup

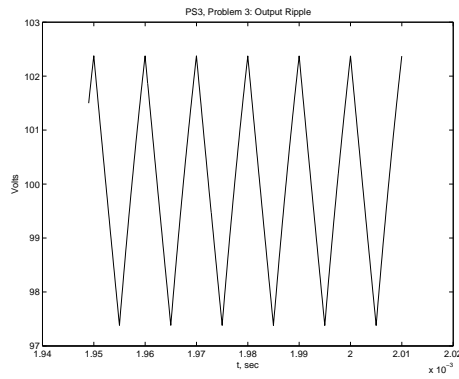


Figure 8: Boost Converter Voltage Ripple

Script for Problem 2

```
% Problem set 3, Problem 2
```

```
R = 64;  
L = .0265;  
Vs = 8000;  
om = 377;  
Cmf = 1:.01:6;           % in microfarads  
C = 1e-6 .* Cmf;
```

```
Vl = Vs .* 1 ./((1- om^2*L .* C) + j*om*L/R);  
Vm = abs(Vl);
```

```
plot(Cmf, Vm);  
title('6.061 Problem Set 3, Problem 2')  
ylabel('Load Voltage Magnitude');  
xlabel('Load Side Capacitance, Microfarads')  
grid on
```

```
% power dissipated with no capacitance  
P_nc = Vs^2*R/((om*L)^2 + R^2);  
fprintf('Problem Set 3, Problem 2\n')  
fprintf('Resistive Dissipation with no cap = %8.0f watts\n', P_nc)  
% calculation of required C:
```

```
Xl = om*L;  
Yc = 1/Xl - sqrt(1/Xl^2 - 1/R^2);  
Creq = Yc/om;  
fprintf('Required C = %g\n',Creq);
```

```
% now equal ended compensation  
delta = asin(Xl/R);  
V_l = Vs*2*sin(delta/2);  
Ql = V_l^2/Xl;  
Xc = 2*Vs^2/Ql;  
Ceq = 1/(om*Xc);
```

```
fprintf('Part 2:\n');  
fprintf('Angle = %6.3f radians = %5.1f degrees\n',delta, 180*delta/pi);  
fprintf('Voltage across line = %g\n',V_l);  
fprintf('Line Reactive Power = %g\n', Ql);  
fprintf('Required Capacitive Impedance = %g\n',Xc);  
fprintf('Capacitance = %g\n',Ceq);
```

Here are the buck converter scripts:

```

% Problem set 2, Problem 3 simulation
% buck converter with output capacitor
global Vs L R C
Vs=100;
L=.0001;
R=10;
C=10e-6;
X=[];
t = [];
T = 1e-5;
d = .5;
ton = T*d;
toff = T*(1-d);
S0=[0; 0];
for n = 0:200
    [tt, S] = ode23('bon', [n*T n*T+ton], S0);
    t = [t' tt']';
    X = [X S'];
    S0 = S(length(tt),:);
    [tt, S] = ode23('boff', [n*T+toff (n+1)*T], S0);
    t = [t' tt']';
    X = [X S'];
    S0 = S(length(tt),:);
end
I1 = X(1,:);
vo = X(2,:);
figure(1)
clf
plot(t, vo)
title('Buck Converter Simulation')
ylabel('Volts');
xlabel('Time, sec');
% now just a couple of cycles to get ripple
tf = [];
Xf = [];
for n = 200:202
    [tt, S] = ode23('bon', [n*T n*T+ton], S0);
    tf = [tf' tt']';
    Xf = [Xf S'];
    S0 = S(length(tt),:);
    [tt, S] = ode23('boff', [n*T+toff (n+1)*T], S0);
    tf = [tf' tt']';
    Xf = [Xf S'];
    S0 = S(length(tt),:);
end

```



```

Ilf = Xf(1,:);
vof = Xf(2,:);
figure(2)
clf
subplot 211
plot(tf, vof)
title('Buck Converter Simulation')
ylabel('Volts');
subplot 212
plot(tf, Ilf)
ylabel('Amps')
xlabel('Time, sec');

```

```

-----
function DS = bon(t, X)
global Vs L R C
il = X(1);
vc = X(2);
didt = (Vs-vc)/L;
dvdt = (il-vc/R)/C;
DS = [didt dvdt]';

```

```

-----
function DS = boff(t, X)
global Vs L R C
il = X(1);
vc = X(2);
didt = -vc/L;
dvdt = (il-vc/R)/C;
DS = [didt dvdt]';
-----

```

```

-----
% trivial boost converter model
global vs L C R
vs = 50;
f = 100000;
alf = .5;
L = 1e-4;
C = 10e-6;
R = 10;
T=1/f;
Dton = alf/f;
Dtoff = (1-alf)/f;
dton = Dton/10;
dtoff = Dtoff/10;
v0 = 0;
i0 = 0;
t = [];
i = [];
v = [];
for n = 0:200;
[tc, S] = ode45('upon',[n*T n*T+Dton] , [i0 v0]');
t = [t tc'];
ic = S(:,1);
vc = S(:,2);
i = [i ic'];
v = [v vc'];
i0 = ic(length(tc));
v0 = vc(length(tc));
[tc, S] = ode45('upoff',[n*T+Dton (n+1)*T] , [i0 v0]);
ic = S(:,1);
vc = S(:,2);
t = [t tc'];
i = [i ic'];
v = [v vc'];
i0 = ic(length(tc));
v0 = vc(length(tc));
end
figure(1)
clf
subplot 211
plot(t, i)
title('PS3, Problem 4: Boost Converter')
ylabel('Current, A');
subplot 212
plot(t, v)

```

```

ylabel('Volts');
xlabel('t, sec');
N = length(t);
figure(2)
clf
plot(t(N-500:N), v(N-500:N))
title('PS3, Problem 3: Output Ripple')
ylabel('Volts');
xlabel('t, sec');

```

```

-----
function Sdot = up(t, S)
global vs L C R
    il = S(1);
    vc = S(2);
    vdot = (1/C) * ( - vc/R);
    idot = (1/L) * (vs);
    Sdot = [idot vdot]';

```

```

-----
function Sdot = up(t, S)
global vs L C R
    il = S(1);
    vc = S(2);
    vdot = (1/C) * (il - vc/R);
    idot = (1/L) * (vs - vc);
    Sdot = [idot vdot]';

```