6.061 / 6.690 Introduction to Electric Power Systems
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Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

Problem Set 7 Solutions April 7, 2007

Problem 1: 1. Area at maximum alignment (as shown) is $A = RL\theta_p = .05 \times .1 \times \frac{\pi}{4} \approx .0039m^2$ Maximum inductance is then:

$$L_{\max} = \frac{\mu_0 A N^2}{2g} = \frac{\mu_0 \times .05 \times .1 \times \frac{\pi}{4} \times 200^2}{2 \times .0005} \approx 0.197\text{Hy}$$

2. Since the engagement area falls linearly with rotational angle, reaching zero at $\theta = \pm\theta_p$, the inductance as a function of rotor angle is as shown in Figure 1.

3. Torque is

$$T^e = \frac{i^2}{2}\frac{\partial L}{\partial \theta} = \frac{5^2}{2} \times .197 \times \frac{4}{\pi} \approx 3.13\text{N-m}$$

Energy that can be delivered over the range of engagement is simply torque times angle or

$$W = \theta_p T^e = \frac{\pi}{4} \times 3.13 \approx 2.46J$$

One more thing you *should have* thought of is saturation. With 5 amperes in the coil the flux density in the air gap is $B_g = \frac{\mu_0 N i}{2g} = \frac{\mu_0 \times 100 \times 5}{2 \times .0002} \approx 1.57T$, Which is within the saturation limits of the kinds of materials we might think of making such an actuator from.

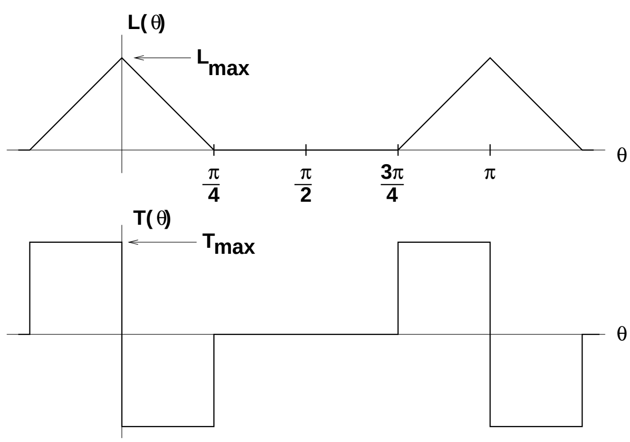

Figure 1: Answer to Problem 1

Problem 2: 1. Core area is $A_c = .2 \times .4 = .08 m^2$. So flux per turn is $\phi = 1.5 \times .08 = .12 Wb$. In turn this implies volts per turn is: $\frac{V}{N} = \frac{377}{\sqrt{2}} \times .12 \approx 32$ volts/turn. This implies that:

$$\begin{aligned} N_p &= \tfrac{4200}{32} = & 131 \\ N_s &= \tfrac{277}{32} = & 9 \end{aligned}$$

Note that the ratios are not exact: the actual turns ratio should be $\frac{4200}{277} = 15.16$. We will get a lot closer to the actual required voltage ratio if the turns ratio is 136:9.

2. Current is $I = \frac{VA}{V}$, or:

$$\begin{aligned} I_p &= \tfrac{300000}{4200} = & 71.4A \\ I_s &= \tfrac{300000}{277} = & 1083A \end{aligned}$$

If we assume half the winding area per winding (this is suspect because of the wide range of voltages: we would need more insulation in the high voltage side), we have $A_w = \frac{1}{2} \times .2 \times .1 = .01 m^2$, or

$$J_w = \frac{N \times I}{A_w} = \frac{136 \times 71.4}{.01} = .971 \times 10^6 A/m^2$$

(Note since the secondary has the same number of amp-turns the answer there would be the same.

3. Peak flux density of 1.5 T, and looking at the data sheet for M-15, we see that core loss density is about 1.4 W/lb or about 3 W/kg. Core mass is

$$M_c = 7650 \times .4 \times (.6 \times .4 - 2 \times .2 \times .1) \approx 612 kg$$

Then core loss evaluates to $612 \times 3 \approx 1836W$.
The equivalent core loss resistance element is:

$$R_c = \frac{4200^2}{1836} \approx 9.6 k\Omega$$

4. To get winding resistance we need both the length and area of each winding. Eyeballing the figures, we see that the end turns of the winding are circular with an average radius which is a bit larger for the secondary. We estimate the average radius of the primary to be about .125 m and for the secondary it is about .175 m. Length per turn is:

$$L_{\text{turn}} = 2 \times L_c + 2\pi R_e$$

This evaluates to:

$$\begin{aligned} L_p &= 136 \times (2 \times .4 + 2\pi \times .125) & \approx 136 \times 1.585m \approx 215.6m \\ L_s &= 9 \times (2 \times .4 + 2\pi \times .175) & \approx 9 \times 1.19m \approx 17.1m \end{aligned}$$

Area of the winding is space factor times winding area per turn, or

$$\begin{aligned} A_p &= \tfrac{1}{2} \times \tfrac{.01}{136} \approx & 3.7 \times 10^{-5} m^2 \\ A_s &= \tfrac{1}{2} \times \tfrac{.01}{9} \approx & 5.6 \times 10^{-4} m^2 \end{aligned}$$

Resistances are, assuming $\sigma = 5.8 \times 10^7 S/m$,

$$\begin{aligned} R_p &= \frac{215.6}{3.7\times10^{-5}\times5.8\times10^7} \approx 0.1\Omega \\ R_s &= \frac{17.1}{5.6\times10^{-4}\times5.8\times10^7} \approx 0.53m\Omega \end{aligned}$$

Note that translated across the turns ratio

$$R_{sR} = \left(\frac{136}{9}\right)^2 \times .00053 \approx .12\Omega$$

which stands to reason as the secondary is a bit longer than the primary.
Dissipation at full load is:

$$P_w = I^2R = 71.4^2 \times .1 + 1083^2 \times .00053 \approx 516W + 622W = 1138W$$

Problem 3: The total area of the rails is $2 \times 8 \times .1 = 1.6m^2$. The weight of the car is 20,000 kg times 9.8 N/kg = 196,000 N. Force per unit area is 196,000 N/1.6 m^2=122,500 Pa. (This is about 17.8 PSI). Since the attractive force is $P = \frac{1}{2}\frac{B^2}{\mu_0}$, required air-gap flux density is $B = \sqrt{2\mu_0 P} \approx .555T$.
To produce this flux density we need $NI = \frac{g}{\mu_0}B$ ampere-turns per pole,
or $NI = \frac{.555\times.01}{4\pi\times10^{-7}} \approx 4,417$ ampere-turns per pole.
Force is inversely proportional to gap, as shown in Figure 2.

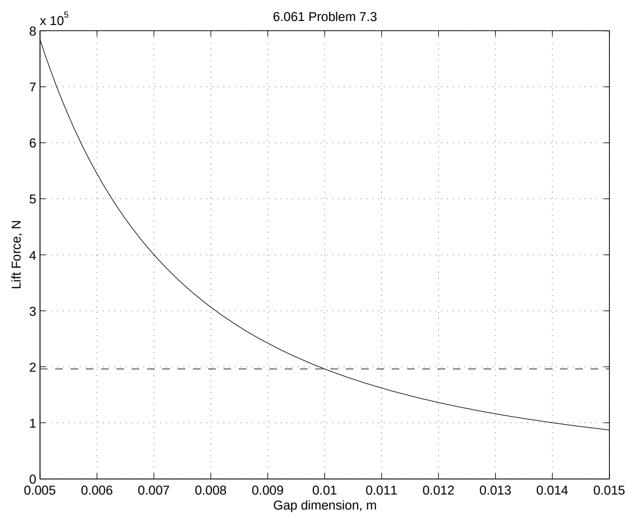

Figure 2: Transverse Force

To get transverse force, look at Figure 3. On the left hand side of the figure we have noted what the situation would look like were the car to be displaced by some distance x. Assuming a current-source drive and constant air-gap, energy stored in the overlapping region is:

$$W_m = \frac{1}{2}\frac{B^2}{\mu_0} \times \text{vol} = \frac{1}{2}\frac{B^2}{\mu_0} \times L \times g \times (w - x)$$

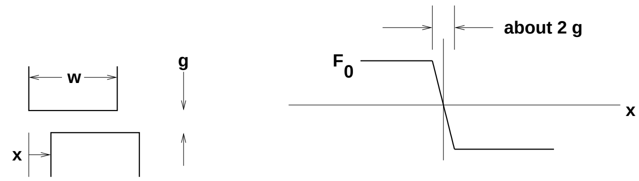

Figure 3: Transverse Force

Force is

$$F^m = -\frac{\partial W_m}{\partial x} = -\frac{1}{2}\frac{B^2}{\mu_0} \times L \times g$$

(Of course this is force per rail and we must multiply by two to get the whole solution). Noting that $\frac{1}{2}\frac{B^2}{\mu_0} = 122,500 Pa$,

$$F^e = \mp 2 \times 8 \times .01 \times 122,500 \approx 19,614 N$$

This indicates that the force is positive for negative displacement and negative for positive displacement with a sharp change at the center. Of course this won't be right, but will look more like the right-hand side of Figure 3. This is stable but probably not well damped at all.