6.061 / 6.690 Introduction to Electric Power Systems
Spring 2007

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.979 Introduction to Power Systems

Problem Set 8 Solutions April 10, 2003

The first few parts of this problem set are concerned with the same synchronous machine which is characterized by the following parameters:

Number of Poles	p	4
Field to Armature Mutual Inductance (Peak)	M	37.5 mHy
Armature Phase Self Inductance	L_a	1.9 mHy
Armature Phase-to-Phase Mutual Inductance	L_{ab}	-0.86 mHy
Field Winding Resistance	R_f	165 mΩ
Armature Winding Resistance (per phase)	R_a	1 mΩ
Rotational Speed		1800 RPM
Terminal Voltage (RMS, Line-Line)	V_a	26 kv
Rated Power		1300 MVA
Frequency		60 Hz

Problem 1: This machine is operated synchronously with a balanced *current* source of RMS amplitude 120 A and a field current of 10 A. The torque-angle curve is plotted in Figure 1. The peak of this

$$T_p = \frac{3}{2} \times 2 \times .0375 \times 3,500 \times \sqrt{2} \times 28,000 \approx 1.56 \times 10^7 \text{N-m}$$

Rated torque is

$$T_r = \frac{2}{377} \times .9 \times 1.3 \times 10^9 \approx 6.2 \times 10^6 \text{N-m}$$

so torque angle to get rated torque is about

$$\delta_i = \sin^{-1} \frac{6.2}{15.6} \approx 0.409 \quad \text{R}adians \approx 23.4^\circ$$

The voltage at the terminals is shown in Figure 2. A script which produced these figures is appended at the end of this solution set.

Problem 2: Now the machine is operating as a synchronous generator, connected to a 26 kV, (RMS, Line-Line) 60 Hz voltage source.Initially, the field current is still 3,500 A. The torque/angle curve for this operation is plotted in Figure 3

Internal voltage is

$$E_{af} = \omega M I_f = 2 \times \pi \times 60 \times 3500 \times .0375 \approx 49,481\text{V}$$

Terminal voltage is

$$V = \sqrt{\frac{2}{3}} \times 26,000 \approx 21,229\text{V}$$

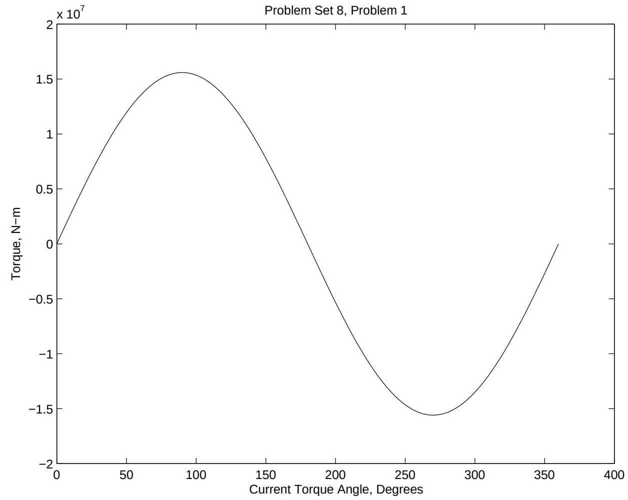

Figure 1: Torque vs. angle: Current Excitation

And reactance is:

$$X_d = 2 \times \pi \times 60 \times .00276 \approx 1.04\Omega$$

So peak power is:

$$P_p = \frac{3}{2}\frac{VE_{af}}{X_d} \approx 1.515 \times 10^9\text{W}$$

Then required torque angle is

$$|\delta| = \sin^{-1}\frac{1.3}{1.515} \approx 1.029 \quad \text{Radians} \approx 59^\circ$$

The relevant phasor diagram is shown in Figure 4.

To find efficiency we simply find losses and add them to required input power. They are: In the Field:

$$P_{df} = .165\Omega \times 3500^2 \approx 2\text{MW}$$

In the Stator:

$$P_{ds} = 3 \times 28868^2 \times .001\Omega \approx 2.5\text{MW}$$

Field loss is about 0.15rating. This would imply an efficiency of about 99.65indeed unreasonably high. Once core loss is included and more realistic numbers for armature and field are used, efficiency would be closer to 99.2

Problem 3: for 6.690 Before we can do the compounding curve we need to know the highest current we can carry before exceeding the stability limit in the under-excited range.

Examining figure 5, note that the angle $\delta = \frac{\pi}{2}$ will be zero if:

$$V + X_d I_a \sin\psi = 0$$

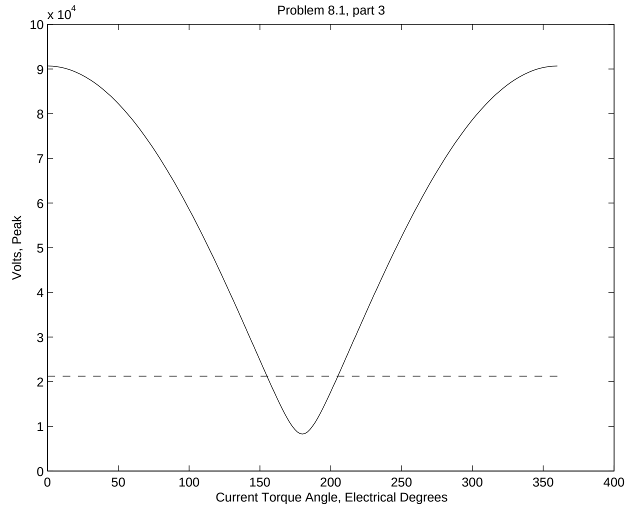

Figure 2: Terminal voltage (Peak, not RMS) vs. angle: Current Excitation

or

$$I_a\text{max} = -\frac{V}{X_d \sin\psi}$$

Stability is only a limitation for underexcited operation.

Then it is straightforward to get internal voltage:

$$E_{af}^2 = (V + X_d I_a \sin\psi)^2 + (X_d I_a \cos\psi)^2$$

and of course

$$I_f = \frac{E_{af}}{\omega M}$$

Remember we are still working in *peak* amplitudes.

The resulting compounding curves are shown in Figure 6.

Vee curves are characterization of armature current for a range of field current and fixed values of real power. It is most convenient to parameterize these curves using the torque angle δ as the parameter. Then, since

$$P = -\frac{3}{2}\frac{V E_{af}}{X_d}\sin\delta$$

it We may find E_{af} (and so I_f) and then

$$Q = \frac{3}{2}\left(\frac{V^2}{X_d} - \frac{V E_{af}}{X_d}\cos\delta\right)$$

And, then

$$I_a = \frac{\sqrt{P^2 + Q^2}}{\frac{3}{2}V}$$

The trick is to find the limiting values of δ.

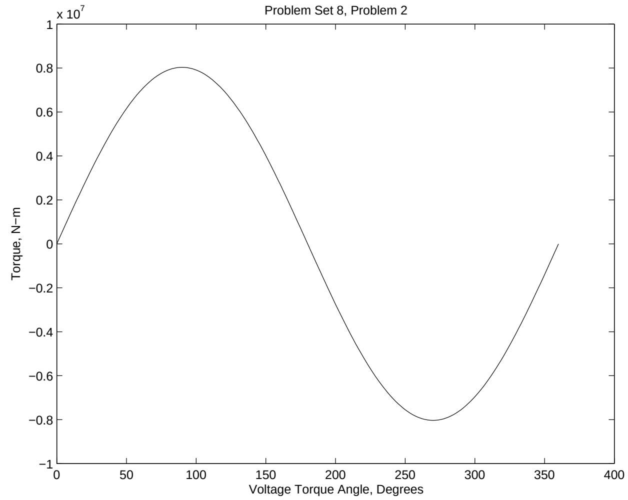

Figure 3: Torque vs. angle: Voltage Excitation

1. At the *under*excited limit, δ is the larger (smaller magnitude of either $\frac{\pi}{2}$ (stability) or the value which results in armature current being at its limit. This would be found by:

$$|Q| = \sqrt{VA^2 - P^2}$$

Working in complex variables, we find the stator current for this condition:

$$\underline{I}_{au} = \frac{P - j|Q|}{\frac{3}{2}V}$$

and internal voltage is

$$\underline{E}_{af} = V - jX_d \underline{I}_{au}$$

If the angle of this voltage is less than $-90°$, the stability limit is the lower limit of δ.

2. At the *over*excited limit, δ is limited by either reaching the stator limit or the field limit. These are found by testing for the required internal voltage to reach the overexcited stator limit:

$$\underline{E}_{af} = V - jX_d \underline{I}_{ao}$$

where:

$$\underline{I}_{ao} = \frac{P + j|Q|}{\frac{3}{2}V}$$

if the magnitude of this is less than the limiting value of E_{af} then its angle is the upper limit. If its magnitude is greater, then we use

$$\delta_u = -\sin^{-1}\frac{PX_d}{\frac{3}{2}VE_{a\ell}}$$

where $E_{a\ell}$ is the limiting value, easily determined using the same techniques as in finding the compounding curve.

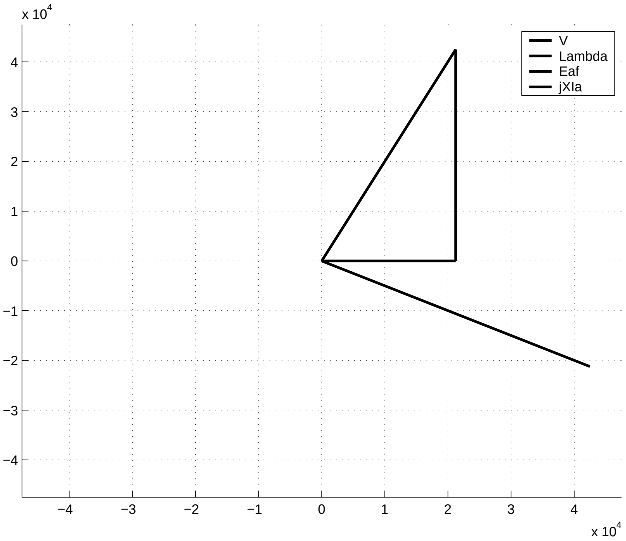

Figure 4: Rated Torque, Unity Power Factor Operation

Details are shown in the attached script. The resulting vee curve is in Figure 7

Problem 4 If you could somehow operate over the whole of that figure, you could circle an area of:

$$A = \frac{1}{2} \times 1\text{A} \times 0.4\text{Wb} + 9\text{A} \times 0.4\text{Wb} = 3.8\text{J}$$

There are $3 \times 4 = 12$ such interactions per rotational cycle of the machine, so average torque is

$$< T >= \frac{12 \times 3.8}{2\pi} \approx 7.26\text{N-m}$$

Appendix: Scripts

```
% 6.061/6.690 Problem set 8, Problems 1, 2 and 3

M = .0375;          % mutual inductance
If = 3500;          % field current
om = 2*pi*60;       % electrical frequency
Ia = 28000*sqrt(2);  % peak armature current
La = .0019;         % phase self inductance
Lab = -.00086;      % phase to phase mutual inductance
Ld = La-Lab;        % synchronous inductance
p = 2;              % four pole machine
Preq = 1.3e9;       % rated power
Treq = p*Preq/om;   % corresponding torque

Ea = om*M*If;       % internal voltage
Vx = om*Ld*Ia;      % reactance drop
```

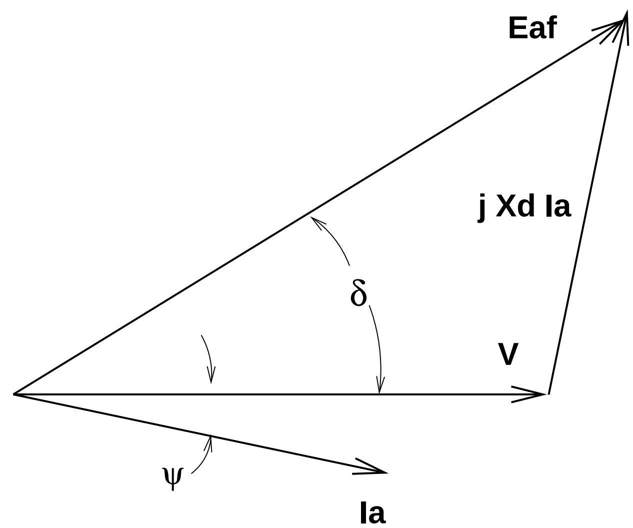

Figure 5: Phasor Diagram for Compounding Curve

```
Vterm = sqrt(2/3)*26000;% rated terminal voltage (phase, peak)

delt = 0:pi/100:2*pi;
dd = (180/pi) .* delt;

Te = p*(3/2)*M*If*Ia*sin(delt);
delt_i = asin(Treq/(1.5*p*M*If*Ia));

Vt = Vx + Ea .* exp(j .* delt);
C1 = [0 360];
C2 = [Vterm Vterm];

figure(1)
plot(dd, Te)
title('Problem Set 8, Problem 1')
ylabel('Torque, N-m')
xlabel('Current Torque Angle, Degrees')

figure(2)
plot(dd, abs(Vt), C1, C2,'--');
title('Problem 8.1, part 3')
ylabel('Volts, Peak')
xlabel('Current Torque Angle, Electrical Degrees')

% now do problem 2
V = Vterm;   % peak, phase voltage
```

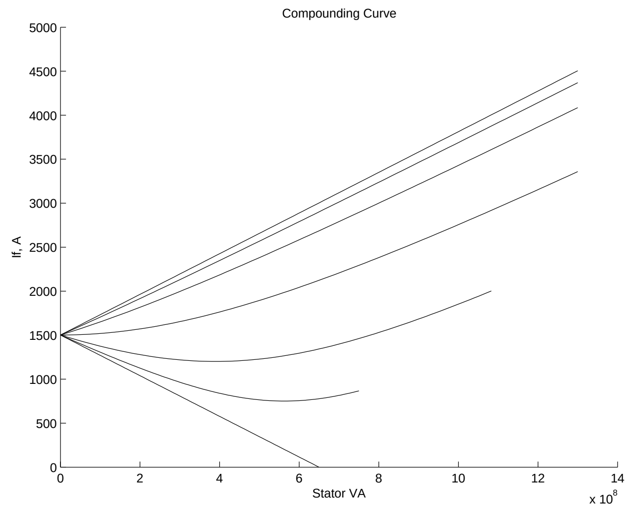

Figure 6: Compounding Curves

```
Xd = om*Ld;
P = (3/2)*(V*Ea/Xd) .* sin(delt);
T = (p/om) .* P;

figure(3)
plot(dd, T);
title('Problem Set 8, Problem 2');
ylabel('Torque, N-m');
xlabel('Voltage Torque Angle, Degrees')

I = Preq/(1.5*V);                                  % peak current at rated point (V is peak phase
Eaf = V + j*Xd*I;                               % this is internal voltage (peak)

dr = angle(Eaf);                                % this is rated angle
Ifr = abs(Eaf)/(om*M);                          % and rated field current
Ia = (Eaf - V)/(j*Xd);
Pcheck = (3/2)*real(V*Ia);
pf = Pcheck/(1.5*V*abs(Ia));

fprintf('Field Current If = %g\n',Ifr)
fprintf('Internal Voltage Ea = %g Angle %g\n',abs(Eaf), angle(Eaf));
fprintf('Current Ia = %g Angle %g\n',abs(Ia), angle(Ia));
fprintf('Power Check = %g  Power Factor = %g\n',Pcheck, pf);

% now let's get MATLAB to plot the vector diagram

VVr = [0 Vterm];
```

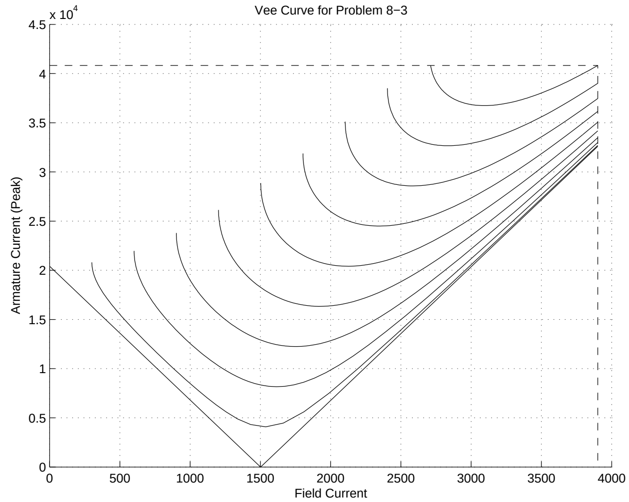

Figure 7: Vee Curves

```
VVi = [0 0];
LVr = [0 real(-j*Eaf)];
LVi = [0 imag(-j*Eaf)];
EVr = [0 real(Eaf)];
EVi = [0 imag(Eaf)];
VXr = [Vterm real(Eaf)];
VXi = [0 imag(Eaf)];

figure(4)
clf
hold on
plot(VVr, VVi, LVr, LVi, EVr, EVi, VXr, VXi)
S = abs(Eaf);
axis([-S S -S S])
legend('V', 'Lambda', 'Eaf', 'jXIa')
hold off

% Now we do the compounding curve

Psi = [pi/2 acos(.8) acos(.5) 0 -acos(.5) -acos(.8) -pi/2];
Xd = om*Ld;
Imax = Preq/(1.5*V);

figure(5)
clf
hold on
```

```
for i = 1:length(Psi);
    psi = Psi(i)
    if (psi<0)           % stability check
        Ism = -V/(Xd*sin(psi))     % stability limit
        Im = min(Imax, Ism);
    else
        Im = Imax;
    end
    I = 0:Im/100:Im;
    VA = 1.5*V .* I;
    Eaf = sqrt((V + Xd*sin(psi) .* I) .^2 + (Xd*cos(psi) .* I) .^2);
    If = Eaf ./ (om *M);
    plot(VA, If);
end
hold off
title('Compounding Curve')
ylabel('If, A');
xlabel('Stator VA')

% and now the vee curve

VA = Preq;                  % armature rating
pfr = .9;                   % rating point
Pv = VA .* (.1:.1:.9);      % this is the real power for each curve
% find armature current capability
Iar = VA/(1.5 * V);           % remember we are working in peak

% find maximum field capability:

psir = acos(pfr);           % power factor angle
% Ia is the complex current at rating point
Iac = Iar*cos(psir) - j*Iar*sin(psir);
Eafr = V + j*Xd*Iac;
Eafm = abs(Eafr);           % this is max magnitude of field voltage
Ifm = Eafm/(om*M);          % limiting field current

fprintf('Rated Ia = %g  Max If = %g\n',Iar, Ifm)
figure(6)
clf
hold on

% now we generate the vee curve

for i = 1:length(Pv)
     P = Pv(i);
```

```
    % minimum I_f of the vee curve is determined by stability
    % or by maximum armature current
    Q = sqrt(VA^2-P^2);
    % underexcited limit is on torque angle
    Ia_min = (P+j*Q)/(1.5*V);
    Eaf_min = V + j*Xd*Ia_min;
    if angle(Eaf_min) > pi/2,
    dmax =  pi/2;
  else
            dmax = angle(Eaf_min);
    end
    Iamax = (P-j*Q) / (1.5*V);
    Eaf_max = V + j*Xd*Iamax;
    if abs(Eaf_max) < Eafm,
   dmin = angle(Eaf_max);
    else
   dmin = asin(P*Xd/(1.5*V*Eafm));
    end
    fprintf('P= %g  dmin = %g  dmax = %g\n', P, dmin, dmax);
    fprintf('Eaf_min = %g, Eaf_max=%g, Eafm=%g\n',Eaf_min, Eaf_max, Eafm);
% now that we have limits we go parameterize by delta

    delt = dmin:(dmax-dmin)/100:dmax;
    Eaf = P*Xd ./ (1.5*V .* sin(delt));
    Q = 1.5*(V^2/Xd - (V/Xd) .* Eaf .* cos(delt));
    Ia = sqrt(P^2 + Q .^2)/(1.5*V);
    If = Eaf ./ (om*M);

    plot(If, Ia)
    %pause
end

% now for the zero power curve
    Iaz = [V/Xd 0 (Eafm-V)/Xd];
    Ifz = [0 V/(om*M) Eafm/(om*M)];
plot(Ifz, Iaz)

% and now we plot the limit lines
    Ial = [Iar Iar 0];
    Ifl = [0 Ifm Ifm]
plot(Ifl, Ial, '--')

    title('Vee Curve for Problem 8-3')
    ylabel('Armature Current (Peak)')
    xlabel('Field Current')
```

```
grid on
hold off
```