6.061 / 6.690 Introduction to Electric Power Systems
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Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.690 Introduction to Power Systems

Problem Set 9 Solutions — April 29, 2007

Problem 1: This problem deals with a salient pole machine with the following characteristics:

Number of Poles	p	14
Frequency	f	60 Hz
Peak Field to Armature Mutual Inductance	M	300 mHy
Direct Axis Stator Inductance	L_d	5 mHy
Quadrature Axis Stator Inductance	L_q	3 mHy
Rated (Line-Line, RMS) Terminal Voltage	V_B	4,200 V
Machine Rating	P_B	15 MVA

1. (AFNL) is simply found by, first, estimating the peak voltage:

$$|V_a| = \sqrt{\frac{2}{3}} \times 4,200 \approx 3,429\text{V}$$

Then field current to reach this voltage at no load is:

$$I_{fnl} = \frac{V_a}{\omega M} = \frac{3,429}{377 \times .3} \approx 30.32\text{A}$$

2. In operating conditions, it is appropriate to, first, find the components of operating current This is cone with the aid of the phasor diagram shown in Figure 1. The details are shown on the appended script, with the numbers generated repeated here. (I have edited out a number of blank lines)

```
6.061/6.690 Homework Set 9, Problem 1
Phase Voltage = 2424.87 RMS
I_fnl = 30.3215
I_a = 2061.97
V_x = 3886.71
I_ffl = 62.0682
I_f for full VA, Zero PF = 78.9225
I_f for underexcited Stability Limit = -20.2144
Absorbed VARs at underexcited Stability Limit = 1.55972e+07
```

3. To operate in the under-excited region it is necessary to remember that power and torque are proportional and that generator power is:

$$P = \frac{3}{2}\left(\frac{VE_{af}}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\right)$$

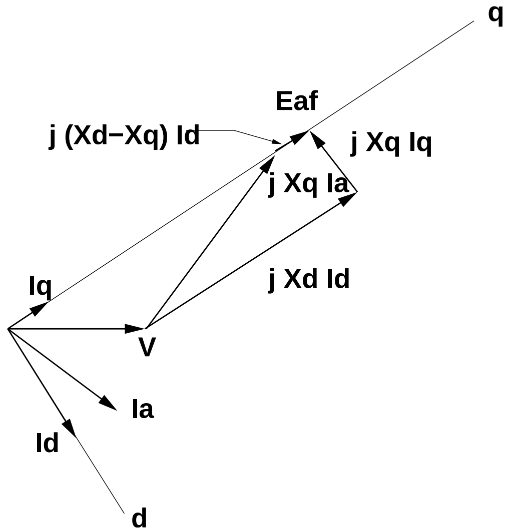

Figure 1: Salient Pole Phasor Diagram

Stability requires that the derivative of torque with respect to angle be negative, so we can find the point of stability by doing the derivative assuming $\delta = 0$:

$$\frac{\partial P}{\partial \delta} = -\frac{3}{2}\left(\frac{VE_{af}}{X_d} + V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\right)$$

This is zero (the edge of the stable region) when

$$E_{af} = -V\left(\frac{X_d}{X_q} - 1\right)$$

Substituting this into the expression for reactive power:

$$Q = \frac{3}{2}\frac{V^2}{X_q}$$

The rest of this is in the script. the results indicate that the machine can supply reactive power from about -5.6 to +15 MVAR.

A summary and approximate vee curve for zero power operation is sketched in Figure 2.

Problem 2: The slip-ring machine can be represented as shown in Figure 3. This looks just like an induction machine equivalent circuit (at least the flux linkage parts). In this case the magnetizing branch reactance is $L_m = \frac{3}{2}M = 12$mHy. The leakage inductance is therefore $L_1 = L_d - L_m = 0.2$mHy.

The voltage at the slip ring (left-hand) terminals is proportional to rotor frequency:

$$\underline{V}_r = s\underline{V}_x = \frac{\omega_e - \omega_m}{\omega_e}\underline{V}_x$$

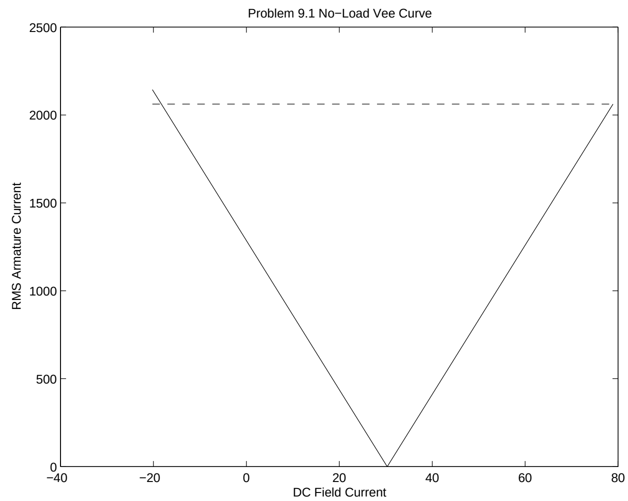

Figure 2: Zero Real Power Vee Curve

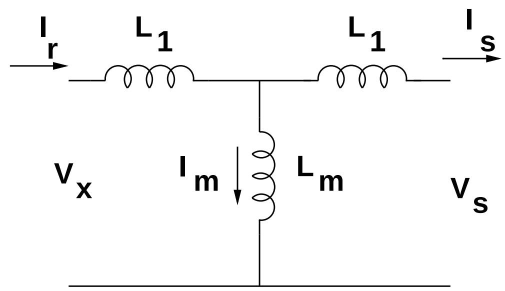

Figure 3: Slip Ring Machine Equivalent Circuit

And, quite conveniently, V_x is the voltage that would appear in the stator frame.

For operation as a generator at overexcited conditions, the relationship between voltage and current is as shown in Figure 4. Components of current are:

$$\underline{I}_s = \frac{V - jQ}{\frac{3}{2}V}$$

Assuming that generation voltage is known, we can easily estimate the other voltages and currents in the circuit:

$$\begin{aligned}
\underline{V}_m &= V_s + j\omega_e L_1 \underline{I}_s \\
\underline{I}_m &= \frac{\underline{V}_m}{j\omega L_m} \\
\underline{I}_r &= \underline{I}_s + \underline{I}_m \\
\underline{V}_x &= \underline{V}_m + j\omega_e L_1 \underline{I}_r
\end{aligned}$$

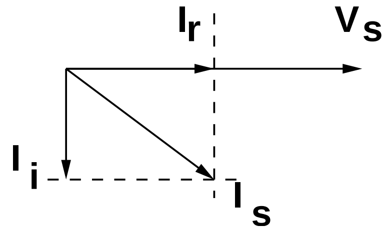

Figure 4: Generation Voltage and Current

Then, finally, real and reactive power are found as:

$$P_r + jQ_r = \frac{3}{2}\underline{V}_r\underline{I}_r^*$$

Of course at 75% speed, slip $s = .25$ and at 125% speed, slip is $s = -.25$. A script which carries out these calculations is appended. Here is an edited (to eliminate white space) transcript of the running of that script. The last two lines show rotor input real and reactive power for positive and negative slips.

Note that when slip is negative, the relative phase relationships reverse, and this reverses the sign of reactive power.

```
6.061/6.960 Problem Set 9, Problem 2
Stator Output P + j Q = 80000 + j 60000
Rotor Input at Slip = 0.25  P + j Q = 20000 + j 30630.3
Rotor Input at Slip = -0.25  P + j Q = -20000 + j 30630.3
```

Problem 3: for 6.690 Essentially all of the development of this solution is the same as for Problem 2. The only difference is to note that the ratio between rotor and stator real power is, taking the sign convention for a generator:

$$P_r = sP_s$$

and, since $P_m = P_{\text{out}} = P_s - P_r$ (this assumes the power electronics is lossless, but that is another story), we have:

$$P_s = \frac{P_{\text{out}}}{1 - s}$$

Since the power electronics is assumed to operate at unity power factor at the stator side of the system:

$$Q_s = Q_{\text{out}}$$

The rest is automated in the script which is appended. The resulting real and reactive power curves vs. machine speed are shown in Figures 5 and 6.

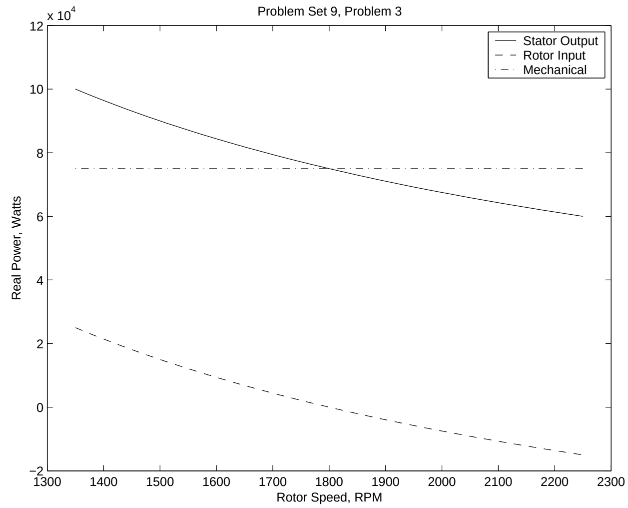

Figure 5: Real Power: Stator, Rotor and Mechanical

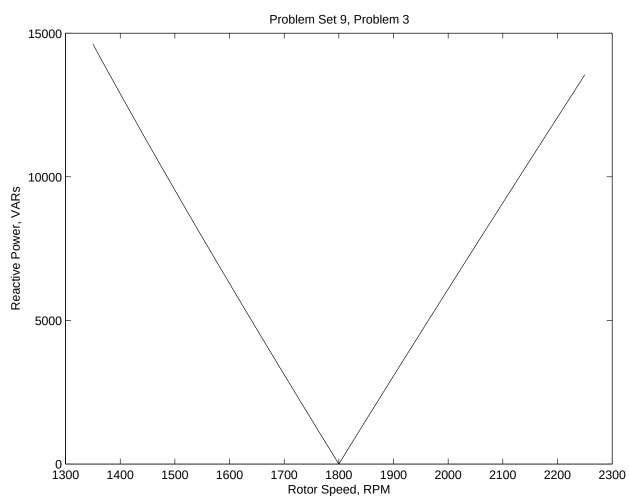

Figure 6: Rotor Input Reactive Power

Appendix: Scripts

```
% 6.061 Problem Set 9, Problem 1

om = 2*pi*60;          % real power
M=.3;                  % mutual inductance
Ld=.011;               % d-axis inductance
Lq=.008;               % q-axis inductance
Xd=om*Ld;
Xq=om*Lq;
V=sqrt(2/3)*4200;     % peak phase voltage
VA=15e6;              % machine rating
pf=.8;                % operating power factor
psi=acos(pf);          % power factor angle
Ia= VA/(1.5*V);        % this is peak armature phase current
Ir = Ia*cos(psi);      % this is real current
Ii = Ia*sin(psi);      % this is reactive current
E1 = V+Ii*Xq+j*Ir*Xq;  % this establishes the d-axis
delt = angle(E1);      % and this is the torque angle
angi = delt+psi;       % this is the angle between current and d-axis
Id = Ia*cos(angi);     % this is d-axis current
Ef = abs(E1)+(Xd-Xq)*Id;    % and this is internal voltage
Iffl = Ef/(om*M);      % field current required to achieve same
Ifnl = V/(om*M);       % field current required to achieve no-load voltage
Vx = Xd*Ia;
Ifsc = (V+Vx)/(om*M);  % field current for overexcited sync condenser operation
Efue = -V*(Xd/Xq-1);    % stability limit if negative field voltage
Ifue = Efue/(om*M);    % stability limiting negative field  current
Que = (3/2)*V^2/Xq;    % max absorbed reactive power

fprintf('6.061/6.690 Homework Set 9, Problem 1\n')
fprintf('I_fnl = %g\n', Ifnl)
fprintf('I_ffl = %g\n', Iffl)
fprintf('I_f for full VA, Zero PF = %g\n', Ifsc)
fprintf('I_f for underexcited Stability Limit = %g\n', Ifue)
fprintf('Absorbed VARs at underexcited Stability Limit = %g\n', Que)
```

```
% Problem set 9, Problem2

om = 2*pi*60;
Lm = .012;                % magnetizing inductance
Ll = .0002;               % leakage inductance
Xm = om*Lm;
Xl = om*Ll;
V = 480*sqrt(2/3)        % working in peak amplitudes
P = 80000;                % real part of 100 kVA, 80% power factor
Q = 60000;                % reactive part
Ir = P/(1.5*V);           % real part of current
Ii = Q/(1.5*V);           % reactive part of current
Is = Ir-j*Ii;             % complex stator current
Vm = V+j*Xl*Is;           % voltage at magnetizing branch
Im = Vm/(j*Xm);           % magnetizing branch current
Ir = Is+Im;               % current into the rotor
Vx = Vm+j*Xl*Ir;          % rotor voltage in stator frame
ss = .25;                 % slip at 75%
sf = -.25;                % slip at 125%
Vrs = ss*Vx;              % rotor voltage at low speed
Pcs = 1.5*Vrs*conj(Ir); % complex power into rotor at low speed
Vrf = sf*Vx;              % rotor voltage at high speed
Pcf = conj(1.5*Vrf*conj(Ir));          % complex power into rotor at high speed
                          % note rotor phase sequence is reversed (conj(conj()) = ..;
fprintf('6.061/6.960 Problem Set 9, Problem 2\n')
fprintf('Stator Output P + j Q = %g + j %g\n', P, Q)
fprintf('Rotor Input at Slip = %g  P + j Q = %g + j %g\n', ss, real(Pcs), imag(Pcs))
fprintf('Rotor Input at Slip = %g  P + j Q = %g + j %g\n', sf, real(Pcf), imag(Pcf))
```

```
% 6.960 Problem set 9, Problem 3

om = 2*pi*60;
Lm = .012;                  % magnetizing inductance
Ll = .0002;                 % leakage inductance
Xm = om*Lm;
Xl = om*Ll;
V = 480*sqrt(2/3);          % working in peak amplitudes
omm = om .* (.75:.01:1.25); % range of working speeds
N = 30/(2*pi) .* omm;
s = 1 - omm ./ om;          % and resulting slips
Pout = 75000;               % at system terminals
Qout = 0;
P = Pout ./ (1 - s);        % real part at machine terminals
Q = Qout;                   % reactive part
Ir = P ./ (1.5*V);          % real part of current
Ii = Q ./ (1.5*V);          % reactive part of current
Is = Ir-j .* Ii;            % complex stator current
Vm = V + j*Xl .* Is;          % voltage at magnetizing branch
Im = Vm ./ (j*Xm);          % magnetizing branch current
Ir = Is+Im;                 % current into the rotor
Vx = Vm+j*Xl .* Ir;         % rotor voltage in stator frame
Vrs = s .* Vx;              % rotor voltage at low speed
Pcr = 1.5 .*Vrs .* conj(Ir);     % complex power into rotor
Pr = real(Pcr);
Qr = imag(Pcr) .* sign(s);
Pw = P-Pr;
figure(1)
plot(N, P, N, Pr, '--', N, Pw, '-.')
title('Problem Set 9, Problem 3')
ylabel('Real Power, Watts')
xlabel('Rotor Speed, RPM')
legend('Stator Output', 'Rotor Input', 'Mechanical')
figure(2)
plot(N, Qr)
title('Problem Set 9, Problem 3')
ylabel('Reactive Power, VARs')
xlabel('Rotor Speed, RPM')
```