6.061 / 6.690 Introduction to Electric Power Systems
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Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.690 Introduction to Power Systems

Quiz 1 Solutions

Problem 1: To help with this, consider the two resistor currents defined in Figure 1. The currents are:

$$I_{AB} = \frac{1}{R}(V_A - V_B) = \frac{V_{ll}}{R}e^{j\frac{\pi}{6}}$$
$$I_{AC} = \frac{1}{R}(V_A - V_C) = \frac{V_{ll}}{R}e^{-j\frac{\pi}{6}}$$

then the phase currents are:

$$\begin{aligned} I_a &= I_{AB} + I_{AC} = \frac{\sqrt{3}V_{ll}}{R} = \frac{360}{10} = 36A \\ I_b &= -I_{AB} = -\frac{V_{ll}}{R}e^{j\frac{\pi}{6}} \approx -20.8e^{j\frac{\pi}{6}} \\ I_c &= -I_{AC} = -\frac{V_{ll}}{R}e^{j\frac{-\pi}{6}} \approx -20.8e^{-j\frac{\pi}{6}} \end{aligned}$$

Real and reactive power are then:

$$\begin{aligned} (P+jQ)_A &= 120 \times 36 \approx 4320W \\ (P+jQ)_B &= 120 \times 20.8 \times \left(\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\right) \approx 2162 + j1246 \\ (P+jQ)_C &= 120 \times 20.8 \times \left(\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right) \approx 2162 - j1246 \end{aligned}$$

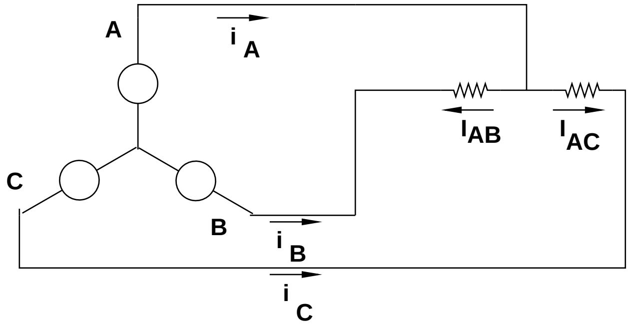

Figure 1: Source connected to load

1. The currents I_A, I_B and I_C and are drawn on the template shown in Figure 1.

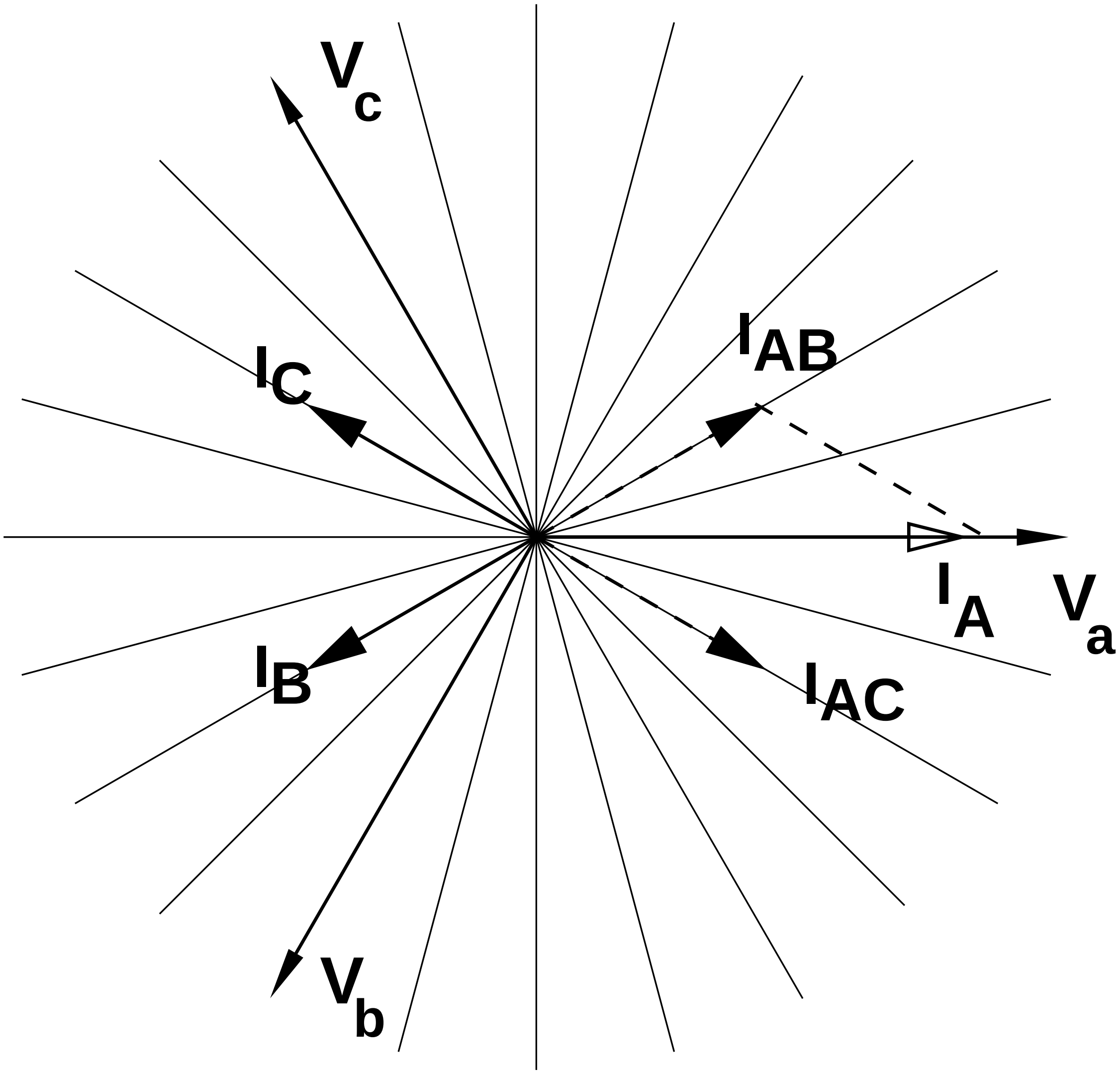

Figure 2: Template for your answer to Problem 1

Problem 2: The ratio between input and output voltage is given by the divider ratio:

$$\frac{V_R}{V_S} = \frac{\frac{-RjX_c}{R-jX_C}}{jX_L - \frac{RjX_c}{/R-jX_C}} = \frac{1}{\left(1 - \frac{X_C}{X_L} + j\frac{X_L}{R}\right)}$$

Voltage magnitudes are equal when the denominator of the second term is equal to one. With a bit of manipulation we find the proper value of capacitive reactance is:

$$X_C = \frac{X_L}{1 \pm \sqrt{1 - \left(\frac{X_L}{R}\right)^2}}$$

Taking the larger value (smaller capacitance), we find:

$$X_c = \frac{6}{1 - \sqrt{1 - .6^2}} = 30\Omega$$

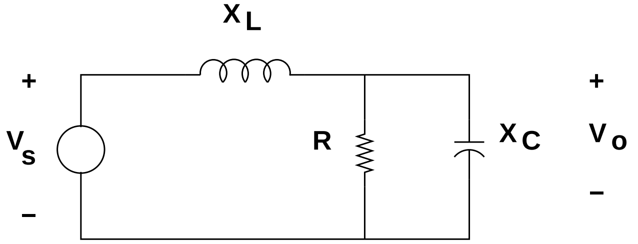

Figure 3: L-R-C Circuit

Real power is:

$$P = \frac{120^2}{10} = 1440\text{watts}$$

Reactive power provided by the capacitor is:

$$Q_c = -\frac{120^2}{30} \approx -480\text{VAR}$$

Reactive power absorbed by the line inductance is:

$$Q_L = |I|^2 X_L$$

and current is, depending on your time origin:

$$I = \frac{120}{10} - j\frac{120}{30} = 12 - j4$$

So

$$Q_L = \left(12^2 + 4^2\right) \times 6 = 930VAR$$

Then the power supply provides:

$$P + jQ = 1440 + j450$$

Problem 3 What is important to remember about this one is that the starting point is one in which the forward going wave is, initially, of the same magnitude as the reverse going wave. $I_+ = I_- = \frac{I_0}{2}$. The voltages add to zero: $V_+ = \frac{I_0}{2} Z_0$ and $V_- = -\frac{I_0}{2} Z_0$ so that $V_+ + V_- = 0$. After the current source shutd down, the positive going voltage V_+ goes to zero, leaving only the negative going wave. After two transit times all goes away.

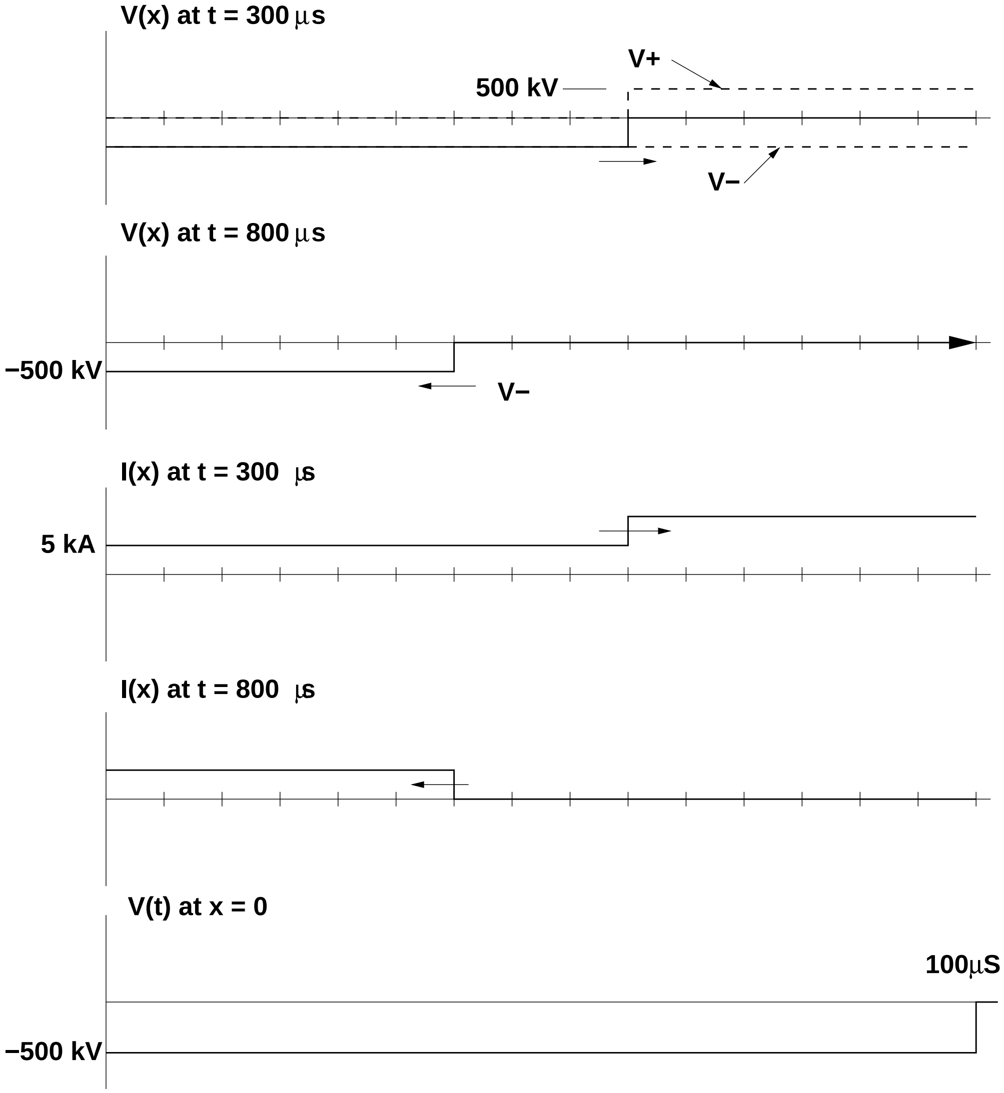

Figure 4: Transmission Line