6.061 / 6.690 Introduction to Electric Power Systems
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Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6,979 Introduction to Power Systems

Quiz 2

Closed Book: Two Handwritten Crib Sheets Allowed
Please put your answers in the spaces provided on the quiz. You may, if you wish, turn in your work on additional sheets of paper. Hopefully you will get all the answers correct so I don't have to look at those sheets.

Problem 1: Two resistors are connected, line-line, to an otherwise balanced three-phase voltage source as shown in Figure 1. Assume both resistors are 10Ω and that the voltage sources all have **RMS** amplitude of 100 V.

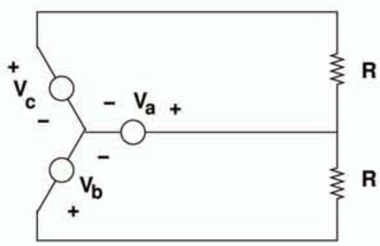

Figure 1: Source and Load

1. Find real and reactive power **out** of each of the three voltage sources:

$$
\begin{aligned}
I_a &= \frac{100\sqrt{3}}{10}\left(e^{j\frac{\pi}{6}} + e^{-j\frac{\pi}{6}}\right) = 10\sqrt{3} \times 2 \times \cos\frac{\pi}{6} = 10\sqrt{3} \times 2 \times \frac{\sqrt{3}}{2} = 30 \\
I_b &= -\frac{100\sqrt{3}}{10} e^{j\frac{\pi}{6}} = 10\sqrt{3} e^{-j\frac{5\pi}{6}} = -15 - j5\sqrt{3} \\
I_c &= -\frac{100\sqrt{3}}{10} e^{-j\frac{\pi}{6}} = 10\sqrt{3} e^{j\frac{5\pi}{6}} = -15 + j5\sqrt{3}
\end{aligned}
$$

To get real and reactive power, use $P + jQ = VI^*$ if amplitudes are RMS:

$$
\begin{aligned}
P_a &= 100 \times 30 = 3000W \qquad Q_a = 0 \qquad \text{5 points} \\
P_b + jQ_b &= 100e^{-j\frac{2\pi}{3}} \times 10\sqrt{3} e^{j\frac{5\pi}{6}} = 1000\sqrt{3} e^{j\frac{\pi}{6}} = 1500 + j500\sqrt{3} \qquad \text{5 points} \\
P_c + jQ_c &= 100e^{j\frac{2\pi}{3}} \times 10\sqrt{3} e^{-j\frac{5\pi}{6}} = 1000\sqrt{3} e^{-j\frac{\pi}{6}} = 1500 - j500\sqrt{3} \qquad \text{5 points}
\end{aligned}
$$

2. Find positive, negative and zero sequence currents, **out** of the three-phase source.

$$\begin{aligned}
I_1 &= \frac{1}{3}10\sqrt{3}\left(e^{j\frac{\pi}{6}} + e^{-j\frac{\pi}{6}} - e^{j\frac{2\pi}{3}}e^{j\frac{\pi}{6}} - e^{-j\frac{2\pi}{3}}e^{-j\frac{\pi}{6}}\right) \\
&= \frac{1}{3}10\sqrt{3} \times 4 \times \cos\frac{\pi}{6} = \frac{1}{3}10\sqrt{3} \times 4 \times \frac{\sqrt{3}}{2} = 20 \qquad \text{5 points} \\
I_2 &= \frac{1}{3}10\sqrt{3}\left(e^{j\frac{\pi}{6}} + e^{-j\frac{\pi}{6}} - e^{-j\frac{2\pi}{3}}e^{j\frac{\pi}{6}} - e^{j\frac{2\pi}{3}}e^{-j\frac{\pi}{6}}\right) \qquad \text{5 points} \\
&= \frac{1}{3}10\sqrt{3} \times \left(2 \times \cos\frac{\pi}{6} - j + j\right) = \frac{1}{3}10\sqrt{3} \times 2 \times \frac{\sqrt{3}}{2} = 10 \\
I_0 &= 01 \qquad \text{5 points}
\end{aligned}$$

Here we have used

$$\begin{aligned}
e^{j\frac{2\pi}{3}}e^{j\frac{\pi}{6}} &= e^{j\frac{5\pi}{6}} = -e^{-j\frac{\pi}{6}} \\
e^{-j\frac{2\pi}{3}}e^{j\frac{\pi}{6}} &= e^{-j\frac{\pi}{2}} = -j
\end{aligned}$$

And, of course, zero sequence current is zero because there is no neutral current.

Problem 2: A DC motor is known to have armature (plus brush) resistance of 1Ω and, when running 'light' (no mechanical load) with a terminal voltage of 100 V turns at 100 Radians per second.

1. How fast does it turn with a torque load of 10 N-m?
   If $GI_f\Omega_0 = 100V$, and $\Omega = 100$, then $GI_f = 1$, and so current to make 10 N-m is 10 A. Voltage drop across the armature resistance is 10 V and $E_a = GI_f\Omega_L = 90V$ or

$$\Omega_L = 90\text{Radians/sec} \qquad \text{10 points}$$

2. Ignoring all losses besides armature resistance, what is the indicated efficiency of the motor with this load? Power out is

$$P_{\text{out}} = 90 \times 10 = 900W$$

Power in is:

$$P_{\text{in}} = 10 \times 100 = 1000W$$

So efficiency is

$$\eta = \frac{900}{1000} = 0.9 \qquad \text{10 points}$$

Problem 3 A three-phase, two-pole, round-rotor synchronous machine has a field to phase mutual inductance of 200 mHy and a synchronous inductance of 20 mHy. Assume a rotational speed of 500 radians/second and a field current of 10 A.

1. If the machine is driven at its armature terminals by a *balanced* current source with *peak* amplitude of 100 A, what is the maximum torque it can produce?

$$T_{\text{max}} = \frac{3}{2} \times .200 \times 10 \times 100 = 300\text{N-m} \qquad \text{10 points}$$

2. Now assume the machine is driven by a *balanced* voltage source with *peak* phase voltage of 600 V, (line-neutral if the machine is connected line-neutral). What is the peak torque the machine can produce?

$$T_{\max} = \frac{3}{2}\times\frac{1}{500}\times\frac{600\times 500\times .2\times 10}{500\times .02} = \frac{3}{2}\times\frac{600}{500}\times\frac{2}{.02} = \frac{3}{2}\times\frac{600}{500}\times 100 = 180\text{N-m} \qquad 10 \text{ points}$$

3. If the machine's terminals are shorted, and ignoring any armature resistance, what is the armature current when field current is 10 A?

$$I_{\text{peak}} = 10\times\frac{.2}{.02} = 100A \qquad 10 \text{ points}$$

Problem 4 Figure 2 shows the single-phase equivalent circuit for an induction motor. Assume the parameters of the motor are, at its operating frequency:

$$\begin{aligned} X_\phi &= 10\Omega \\ X_1 &= 2.5\Omega \\ X_2 &= 2.0\Omega \\ R_2 &= 1.0\Omega \end{aligned}$$

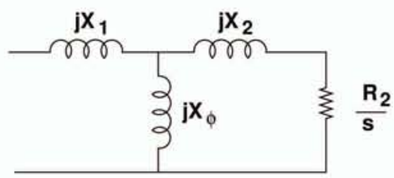

Figure 2: Induction Motor Equivalent Circuit

For the purposes of this problem we will ignore armature resistance. Assume this is a two-pole machine and that it is driven by a voltage source of 1,250 V, *Peak*, phase-neutral. Also, perhaps improbably, the excitation is at a frequency of 500 Radians/second.

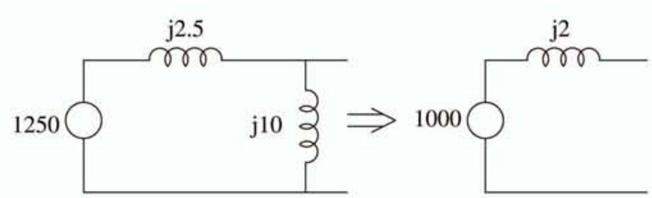

Figure 3: Source End

1. What is the *breakdown* torque for this machine?
   To find breakdown torque, look at Figure 3. Then the rotor current becomes:

$$I_2 = \frac{1000}{j4 + \frac{R_2}{s}}$$

Maximum torque will be when rotor resistance is equal to source resistance:

$$\frac{R_2}{s} = 4$$

Then

$$I_2 = \frac{1000}{4+j4} \qquad \text{or} \qquad |I_2| = \frac{1000}{\sqrt{2}\times 4}$$

Torque is

$$T_e = \frac{3}{2}\frac{1}{\omega}|I_2|^2\frac{R_2}{2} = \frac{3}{2}\frac{1}{500}\frac{1000^2}{2\times 16} = \frac{3}{2}\times\frac{2\times 1000\times 4}{32} = 375\text{N-m} \qquad \text{5 points}$$

2. At what slip does the machine reach breakdown torque?

   If $R_2 = 1$ and $\frac{R_2}{s} = 4$, $s = .25$ (10 points), or $\Omega = .75 \times 500 = 375$Radians/sec

3. Running light (zero torque), what current does the motor draw from the source? The rotor branch is open so

$$I = \frac{1250}{j2.5 + j10} = -j100 \qquad 5 points$$