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Transactions Letters

Outage Behavior of Selective Relaying Schemes

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Abstract—Cooperative diversity techniques can improve the transmission rate and reliability of wireless networks. For systems employing such diversity techniques in slow-fading channels, outage probability and outage capacity are important performance measures. Existing studies have derived approximate expressions for these performance measures in different scenarios. In this paper, we derive the exact expressions for outage probabilities and outage capacities of three proactive cooperative diversity schemes that select a best relay from a set of relays to forward the information. The derived expressions are valid for arbitrary network topology and operating signal-to-noise ratio, and serve as a useful tool for network design.

Index Terms—Selective relaying, cooperative communication, outage probability, outage capacity, decode-and-forward.

I. INTRODUCTION

VARIOUS cooperative diversity schemes have been explored in the literature. These schemes use one or more relay nodes to forward signals transmitted from the source node to the destination node. The relay nodes can forward signals with a decode-and-forward (DF) or amplify-and-forward (AF) strategy. Here, we focus on DF, where each relay node fully decodes, re-encodes, and retransmits the source messages. For DF relay networks, [1]–[3] consider single relay transmission, while [4]–[6] exploit the possibility of using multiple relay nodes. To reduce the required overhead from using multiple relay nodes, a single *best* relay can be selected among multiple relays [7]–[13]. Such relay selection can be either proactive (i.e., before source data transmission)

[8], [9], [11] or reactive (i.e., after source data transmission) [12], [13].

For DF relaying in slow-fading channels, outage probability and outage capacity are important performance measures. However, it is often difficult to derive closed-form expressions of these measures, and, as a result, many approximations have been made in the literature [1]–[6]. For the single-relay case, upper bounds of the outage capacities were examined for different protocols in [1]. For the single-relay case with collisions, approximations of the outage probabilities were provided in [2] for a high signal-to-noise ratio (SNR) and in [3] for a low SNR. For the multiple-relay case without relay selection, outage probabilities for schemes with maximum ratio combining (MRC) were derived in [4], [5], and for a simplified scheme in [6]. Three important schemes for proactive relay selection from multiple relays were considered in [8], [9], [11]. First, in [8], the outage probability is derived for the opportunistic cooperative diversity scheme without MRC at the destination, referred to as *fixed selective decode-and-forward (FSDF) without direct link combining*.¹ Second, a similar scheme with MRC at the destination to combine signals from the selected relay and the source, referred to as *FSDF with direct link combining*, was evaluated in terms of outage probability in the high SNR regime [9], and later in the low and medium SNR regimes [10]. Finally, a scheme that uses a relay only when it is beneficial, was proposed and evaluated numerically in terms of outage capacity in [11]. This scheme is referred to as *smart selective decode-and-forward (SSDF)*.

In this paper, we will evaluate outage capacity and outage probability for FSDF with and without direct link combining, as well as SSDF. Instead of relying on an asymptotic analysis or an approximation, we derive the exact outage probabilities and outage capacities. The obtained results are simple, exact, and applicable to arbitrary network topology and operational SNR under Rayleigh fading. From the derived expressions, we obtain insight into the design of practical relay networks.

The paper is organized as follows. In Section II, we describe the network and channel model. We then detail the three relaying schemes, and discuss implementation trade-offs in Section III. In Section IV, we present the outage analysis of the selective relaying schemes, and present numerical results in Section V. Finally, we draw our conclusions in Section VI.

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¹For consistency, we will discard the aggregate power constraint from [8].

II. SYSTEM MODEL

We consider a wireless relay network consisting of $K + 2$ single-antenna nodes: a source node S, a destination node D, and K relay nodes. The topology of the relay network is arbitrary but deterministic and static. Without loss of generality, the distance between the source and the destination nodes is normalized to one unit. The source node can transmit information to the destination node directly, or transmit information to the destination node via a relay. The relays operate in DF mode, whereby a single relay is selected proactively to forward the information. The use of relays results in a division of the transmission time into two slots: the first slot for the transmission from the source and the second slot for the transmission from the relay.

We consider Rayleigh frequency-flat fading with the coherence time that is long enough for the system to complete transmitting a block of data. The model for the received signal and the channel for a link between any pair of nodes i and j is given by

$$y_j = h_{ij}x_i + n_j, \quad (1)$$

where² x_i is the signal transmitted by node i , $h_{ij} \sim \mathcal{CN}(0, \Omega_{ij})$ is the complex channel gain over the link $i \rightarrow j$, $n_j \sim \mathcal{CN}(0, N_0)$ is additive white Gaussian noise at node j . The channel gains, noise, and transmitted signals are independent. The channel gain h_{ij} captures the effects of fading as well as path loss by setting $\Omega_{ij} = d_{ij}^{-\alpha}$, where d_{ij} denotes the distance between node i and node j , and α is the path loss exponent. We denote the channel from S to the k th relay by $h_{B,k}$, and the channel from the k th relay to D by $h_{F,k}$.³ Every node i transmits with the same average transmit signal power $P \triangleq \mathbb{E}\{|x_i|^2\}$. Finally, we define SNR as the average signal-to-noise ratio from the source node to the destination node, given by

$$\text{SNR} \triangleq \frac{P}{N_0W}, \quad (2)$$

where W is the transmission bandwidth. Hence, the average signal-to-noise ratio from node i to node j can be written as SNR/d_{ij}^α .

III. RELAYING SCHEMES: DESCRIPTION AND IMPLEMENTATION

In this section, we will describe how proactive relay selection occurs, and how it can be implemented for three selective relaying schemes: FSDF with direct link combining; FSDF without direct link combining; and SSDF. Relays are selected so as to maximize the instantaneous mutual information, as is detailed below. As a benchmark, we note that when the source node transmits the signals directly to the destination node, the maximum mutual information (between y_D and x_S), conditioned on the instantaneous channel, is given by

$$R_{DC} = \log_2(1 + |h_{SD}|^2 \text{SNR}). \quad (3)$$

We refer to $|h_{SD}|^2 \text{SNR}$ as the instantaneous SNR between the source and destination.

²We will denote by $\mathcal{CN}(\mu, \sigma^2)$ a complex circularly symmetric Gaussian distribution with mean μ and variance $\sigma^2/2$ for the real and imaginary components.

³The subscripts ‘‘B’’ and ‘‘F’’ refer to the backward and forward channels, respectively.

A. FSDF with Direct Link Combining

This scheme optimally selects one relay to decode. If the decoding fails, the transmission is declared unsuccessful. If the decoding succeeds, the relay forwards the information to the destination node. The destination combines the signals from both the source and the destination nodes using MRC. Thus, the maximum instantaneous end-to-end mutual information is given by

$$R_{\text{FSDF-direct}} = \max_{k \in \mathcal{K}} \min\{R_{B,k}, R_{\text{MRC},k}\}, \quad (4)$$

where $\mathcal{K} = \{1, 2, \dots, K\}$ denoting the set of relay nodes, and

$$R_{B,k} = \frac{1}{2} \log_2(1 + |h_{B,k}|^2 \text{SNR}),$$

$$R_{\text{MRC},k} = \frac{1}{2} \log_2(1 + (|h_{SD}|^2 + |h_{F,k}|^2) \text{SNR}). \quad (5)$$

Note that the factor of $1/2$ in (5) accounts for the fact that the transmission occurs over two time slots. The relay that gives rise to the maximum in (4) is the selected relay.

B. FSDF without Direct Link Combining

This scheme is similar to the previous scheme, except there is no direct link. Hence, the maximum instantaneous end-to-end mutual information is given by

$$R_{\text{FSDF-nodirect}} = \max_{k \in \mathcal{K}} \min\{R_{B,k}, R_{F,k}\}, \quad (6)$$

where $R_{B,k}$ was defined in (5) and

$$R_{F,k} = \frac{1}{2} \log_2(1 + (|h_{F,k}|^2) \text{SNR}). \quad (7)$$

C. SSDF

Dividing the transmission time into two slots results in a rate loss (i.e., the factor $1/2$ in (5) and (7)), which may offset the benefit of using a relay. The SSDF scheme is similar to the FSDF scheme with direct link combining, but will not use a relay when direct communication gives a larger instantaneous mutual information. Thus, the maximum instantaneous end-to-end mutual information is given by

$$R_{\text{SSDF}} = \max\{R_{\text{FSDF-direct}}, R_{DC}\}. \quad (8)$$

D. Implementation

The relay selection procedure in the three relaying schemes relies on the instantaneous channel state information (CSI). In related works, the implementation of FSDF without direct link combining is described in [14]; FSDF with direct link combining and SSDF are described conceptually in [9], [11]. Here, we will describe how CSI may be obtained, and how relay selection can be performed in a distributed manner. Recall that the channel remains static during the relay selection procedure and the data communication. Also, recall that the data communication occurs at a certain rate R , after the relay selection procedure. We will focus on FSDF with direct link combining and SSDF. The initial five steps are the same for both schemes.

- 1) The source transmits a training sequence. All relays and the destination estimate the CSI and the instantaneous SNR from the source to themselves.

- 2) The destination broadcasts a training sequence. All relays estimate the CSI and the instantaneous SNR between themselves and the destination.
- 3) The destination broadcasts the instantaneous SNR between the source and the destination, $|h_{SD}|^2 \text{SNR}$. All relays receive this information.
- 4) Every relay k can now compute $\min\{R_{B,k}, R_{MRC,k}\}$, based on the available CSI and instantaneous SNR information. Every relay k now sets a timer and remains silent for the duration inversely proportional to $\min\{R_{B,k}, R_{MRC,k}\}$.
- 5) The relay whose timer expires first will broadcast a signal to the other relays, indicating that they can go to a sleep mode for the rest of the current transmission period. The source also receives this broadcast.

FSDF with direct link combining :

- 6) The source transmits the data message in the first time slot at rate R . Both the destination and the selected relay, say relay k , listen. If $R > R_{B,k}$, an outage occurs, and the data transmission is unsuccessful. If $R < R_{B,k}$, the relay can decode and forward the message in the second time slot at rate R .
- 7) The destination performs MRC of the information from the first and second time slot. If $R > R_{MRC,k}$, an outage occurs, and the data transmission is unsuccessful. Otherwise, the destination can decode the message and the data transmission is successful.

SSDF :

- 6) The selected relay compares $R_{\text{FSDF-direct}}$ with R_{DC} . The relay then notifies the source whether or not relaying should occur. If $R_{\text{DC}} > R_{\text{FSDF-direct}}$, the relay then goes to the sleep mode.
- 7) The source transmits the data message at rate R using either one slot (when relaying is used) or two slots (when direct communication is used). In the latter case, an outage occurs when $R > R_{\text{DC}}$.

IV. OUTAGE ANALYSIS

In this section, we determine the outage probability (the probability that an outage occurs for a given rate R) and the outage capacity (the rate that can be supported if outages are allowed to occur with probability ε) of the three relaying schemes. We first note that for direct communication at a rate R and an average signal-to-noise ratio SNR , the outage probability is given by [15, pp. 220, eq. (5.57)]

$$\mathbb{P}_{\text{out}}^{\text{DC}}(R, \text{SNR}) = 1 - \exp\left(-d_{\text{SD}}^{\alpha} \left(\frac{2^R - 1}{\text{SNR}}\right)\right), \quad (9)$$

and the outage capacity at an outage probability of ε is

$$C_{\varepsilon}^{\text{DC}} = \log_2\left(1 + \text{SNR} \ln\left(\frac{1}{1 - \varepsilon}\right)\right). \quad (10)$$

A. FSDF with Direct Link Combining

The outage probability for the FSDF scheme with direct link combining is

$$\begin{aligned} & \mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(R, \text{SNR}) \\ &= 1 + \sum_{\ell=1}^K \sum_{\substack{S \subseteq \mathcal{K} \\ |S|=\ell}} \left\{ (-1)^{\ell} \frac{c_2(\mathcal{S}) w^{1+c_1(\mathcal{S})} - w^{c_1(\mathcal{S})+c_2(\mathcal{S})}}{-1 - c_2(\mathcal{S})} \right\}, \end{aligned} \quad (11)$$

where w , c_1 , and c_2 are given by

$$w \triangleq \exp\left(-\left(\frac{2^{2R} - 1}{\text{SNR}}\right)\right), \quad (12)$$

$$c_1(\mathcal{S}) \triangleq \sum_{k \in \mathcal{S}} d_{\text{Sk}}^{\alpha}, \quad (13)$$

$$c_2(\mathcal{S}) \triangleq \sum_{k \in \mathcal{S}} d_{\text{kD}}^{\alpha} \quad (14)$$

(see [16] for the proof). It can be seen that R and SNR appear only through w , and hence the right-hand side of (11) can be treated as a function $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(w)$ of w . We prove in Appendix A that $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(w)$ is a continuous and strictly decreasing function of w , for $0 < w \leq 1$. Hence, for a fixed outage probability ε , we can determine a unique $w_{\varepsilon}^{\text{FSDF-direct}}$ such that $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(w_{\varepsilon}^{\text{FSDF-direct}}) = \varepsilon$, using any suitable numerical method. Then, the obtained $w_{\varepsilon}^{\text{FSDF-direct}}$ together with (12) gives the outage capacity

$$C_{\varepsilon}^{\text{FSDF-direct}} = \frac{1}{2} \log_2\left(1 + \text{SNR} \ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right). \quad (15)$$

B. FSDF without Direct Link Combining

The outage probability for this scheme is given by (see Appendix B):⁴

$$\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(R, \text{SNR}) = \prod_{k=1}^K \left[1 - w^{d_{\text{Sk}}^{\alpha} + d_{\text{kD}}^{\alpha}}\right], \quad (16)$$

where w was introduced in (12). Similar to the previous scheme, for a fixed outage probability ε , we can determine a unique $w_{\varepsilon}^{\text{FSDF-nodirect}}$ such that $\mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}}(w_{\varepsilon}^{\text{FSDF-nodirect}}) = \varepsilon$, and have the outage capacity

$$C_{\varepsilon}^{\text{FSDF-nodirect}} = \frac{1}{2} \log_2\left(1 + \text{SNR} \ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-nodirect}}}\right)\right). \quad (17)$$

C. SSDF

The outage probability for the SSDF scheme is

$$\begin{aligned} & \mathbb{P}_{\text{out}}^{\text{SSDF}}(R, \text{SNR}) = 1 - v \\ & + \sum_{\ell=1}^K \sum_{\substack{S \subseteq \mathcal{K} \\ |S|=\ell}} \left\{ \frac{(-1)^{\ell} w^{c_1(\mathcal{S})+c_2(\mathcal{S})} (v^{1-c_2(\mathcal{S})} - 1)}{-1 - c_2(\mathcal{S})} \right\}, \end{aligned} \quad (18)$$

⁴Another way to derive the outage probability is to follow an approach of [8], setting the power in both the backward hop and the forward hop to be 1. This approach gives the expression that is more complicated than (16) but they are equivalent after some manipulation.

where w , c_1 and c_2 were introduced in (12)–(14), and v is defined by

$$v \triangleq \exp\left(-\left(\frac{2^R - 1}{\text{SNR}}\right)\right) \quad (19)$$

(see [16] for the proof). Note that w can be expressed in terms of v and SNR as

$$w = \exp(2 \ln v - (\ln v)^2 \text{SNR}), \quad (20)$$

and hence the right-hand side of (18) can be treated as a function $\mathbb{P}_{\text{out}}^{\text{SSDF}}(v; \text{SNR})$ of v and SNR . For each SNR , a similar proof to Appendix A shows that $\mathbb{P}_{\text{out}}^{\text{SSDF}}(v; \text{SNR})$ is a continuous and strictly decreasing function of v , for $0 < v \leq 1$. Hence, for a fixed outage probability ε and a fixed SNR , we can determine a unique $v_{\varepsilon, \text{SNR}}$ such that $\mathbb{P}_{\text{out}}^{\text{SSDF}}(v_{\varepsilon, \text{SNR}}; \text{SNR}) = \varepsilon$. The obtained $v_{\varepsilon, \text{SNR}}$ provides us with the outage capacity

$$C_{\varepsilon}^{\text{SSDF}} = \log_2\left(1 + \text{SNR} \ln\left(\frac{1}{v_{\varepsilon, \text{SNR}}}\right)\right). \quad (21)$$

D. Performance Comparison

It can be verified that

$$R_{\text{FSDF-nodirect}} \leq R_{\text{FSDF-direct}} \leq R_{\text{SSDF}}.$$

Therefore, the outage capacities of the three relaying schemes satisfy

$$C_{\varepsilon}^{\text{FSDF-nodirect}} \leq C_{\varepsilon}^{\text{FSDF-direct}} \leq C_{\varepsilon}^{\text{SSDF}}$$

for any ε and SNR . In the high SNR regime, we have

$$\lim_{\text{SNR} \rightarrow \infty} \frac{C_{\varepsilon}^{\text{FSDF-direct}}}{C_{\varepsilon}^{\text{DC}}} = \frac{1}{2} \quad (22)$$

by l'Hôpital's rule. Thus, we expect direct communication to outperform FSDF with direct link combining when SNR is above a certain threshold. The SNR threshold can be found by equating (10) and (15). In addition, we observe that

$$\lim_{\text{SNR} \rightarrow \infty} \frac{C_{\varepsilon}^{\text{FSDF-nodirect}}}{C_{\varepsilon}^{\text{DC}}} = \frac{1}{2}. \quad (23)$$

Thus, the outage capacity of FSDF without direct link combining converges to that of FSDF with direct link combining scheme at high SNR . Finally, we can verify that⁵

$$\lim_{\text{SNR} \rightarrow \infty} \frac{C_{\varepsilon}^{\text{SSDF}}}{C_{\varepsilon}^{\text{DC}}} = 1, \quad (24)$$

which means that SSDF reverts to direct communication at high SNR , regardless of the relay network topology. Equation (24) also implies that in the high SNR regime, a loss in the effective transmission rate from dividing the transmission time into two slots for relaying offsets the benefit of using the relay.

⁵The limit can be obtained by observing that $R_{\text{DC}} \leq R_{\text{SSDF}} \leq \max_{k \in \mathcal{K}} 2R_{\text{MRC}, k}$, which implies that $C_{\varepsilon}^{\text{DC}} \leq C_{\varepsilon}^{\text{SSDF}} \leq C_{\varepsilon}$, where $C_{\varepsilon} = \log_2(1 + \text{SNR} F^{-1}(\varepsilon))$ and F is the cumulative distribution function of random variable $|h_{\text{SD}}|^2 + \max_{k \in \mathcal{K}} |h_{\text{F}, k}|^2$. Dividing the inequalities by $C_{\varepsilon}^{\text{DC}}$ and taking the limits as SNR approaches infinity give the desired result.

V. NUMERICAL RESULTS

For illustration purposes, we will evaluate the outage capacities of different schemes at a fixed outage probability of $\varepsilon = 10^{-2}$. We consider two types of networks: (i) grid topologies with $\sqrt{K} \times \sqrt{K}$ relay nodes for an integer \sqrt{K} , where each of the horizontal and vertical distances between adjacent relay nodes is $1/(\sqrt{K} + 1)$ units, and the source and destination nodes are located outside the grid, at the horizontal distance of $1/(\sqrt{K} + 1)$ units from the middle of the left side and the middle of the right side of the grid, respectively; (ii) random topologies, where the nodes are distributed uniformly within a unit square, and the source and the destination nodes are located at the middle of the left side and the middle of the right side of the square, respectively.

Fig. 1 shows the outage capacity for the 3×3 grid topology. When SNR becomes large, both FSDF schemes, which always use a relay, are worse than direct communication. This is because both FSDF schemes cannot switch to direct communication when the benefit of relaying diminishes. The SNR thresholds at which both FSDF schemes become worse than direct communication are obtained by equating (10) to (15) and to (17), and are marked by two vertical lines in the figure.

In Fig. 2 we consider a random topology at a fixed SNR of 20 dB for $K = 1, 4, 9$. The outage capacity of the network is now a random variable, depending on the specific network topology, and the figure shows the cumulative distribution function (CDF) of the outage capacity.⁶ The cross on each CDF gives the outage capacity (on the x-axis) of deterministic grid topology using the same scheme and number of relay nodes. From the figure, the performance differences among the schemes are reduced when the number of relays increases from 4 to 9. This indicates a saturation in the improvement of outage capacity as the number of relay nodes increases. We also observe that when $K = 9$, the crosses give the probabilities of approximately 1 on the y-axis, implying that almost none of random network realizations provide better outage capacities than the grid topology provides. Hence, for $K = 9$, the grid topology is a good topology to use in deploying the relays. On the other hand, when the number of relays is reduced to $K = 4$, about 35% of random network realizations provide better performance than the grid topology for all relaying schemes.

We now focus our discussion on the grid topology in Figs. 3 and 4, which show the effect of K on the performance of FSDF with direct link combining and SSDF, respectively. In both figures, increasing the number of relays from 1×1 to 2×2 provides a larger gain than does the increment of K from 3×3 to 4×4 . In Fig. 3 we see that the differences in outage capacities among curves are constant when SNR is large. This behavior can be explained by considering (15) for large SNR :

$$\begin{aligned} C_{\varepsilon}^{\text{FSDF-direct}} &\approx \frac{1}{2} \log_2\left(\text{SNR} \ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right) \\ &= \frac{1}{2} \log_2\left(\ln\left(\frac{1}{w_{\varepsilon}^{\text{FSDF-direct}}}\right)\right) + \frac{1}{2} \log_2(\text{SNR}), \end{aligned} \quad (25)$$

⁶Based on 10,000 network realizations.

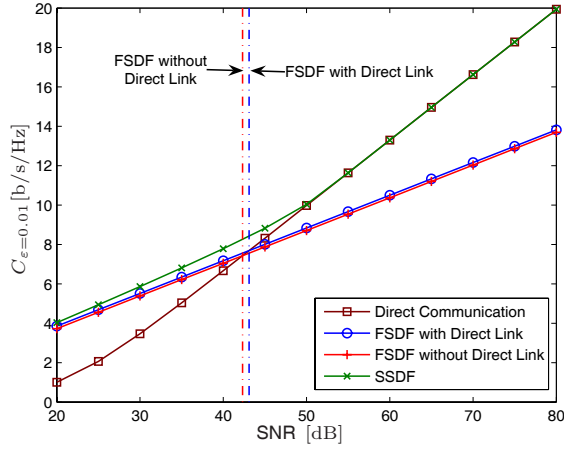


Fig. 1. Outage capacities of various schemes as functions of SNR at an outage probability of 0.01 for the 3×3 grid topology. The vertical lines show the SNR thresholds.

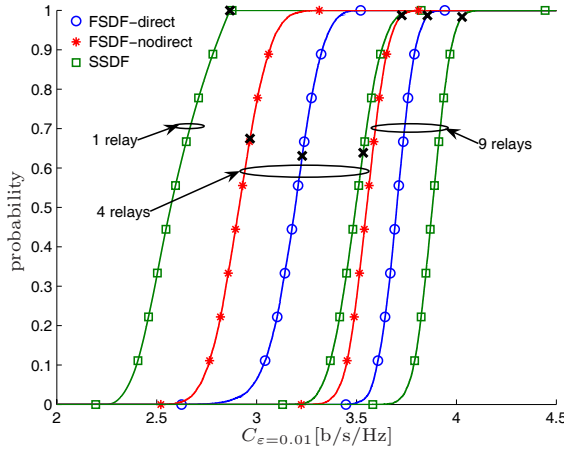


Fig. 2. Cumulative distribution functions of outage capacities at SNR of 20 dB of all schemes at an outage probability of 0.01 for random topologies. Each cross shows the outage capacity when a grid topology is used.

where the first term results in a vertical offset of the capacity curve, and the second term makes the outage capacity a linear function of SNR, when expressed in decibels. In Fig. 4 we observe that the outage capacity of the SSDF scheme in the high SNR regime does not increase by adding more relay nodes. This can be attributed to the fact that SSDF converges to direct communication when the SNR increases. On the other hand, adding more relays increases the outage capacity in the medium SNR regime.

VI. CONCLUSION

We derived exact expressions for outage probabilities and outage capacities for several decode-and-forward cooperative diversity schemes with proactive relay selection. The derived expressions are simple, and applicable for arbitrary network topologies and SNR values. We found that FSDF with or without direct link combining improves the outage capacity compared to direct communication only when the SNR is below a certain threshold. This allows us to characterize the

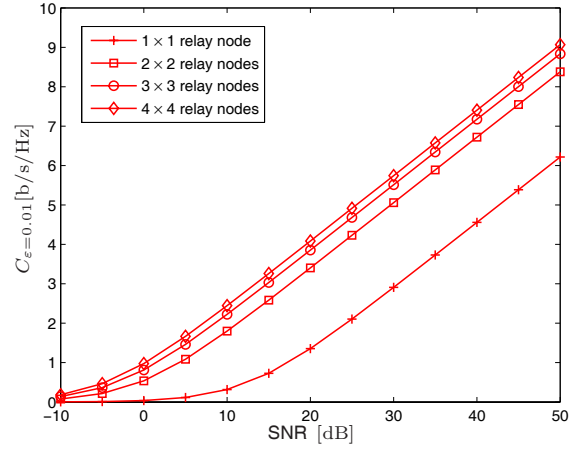


Fig. 3. Outage capacities of FSDF with direct link combining at an outage probability of 0.01 for grid relay networks with various sizes.

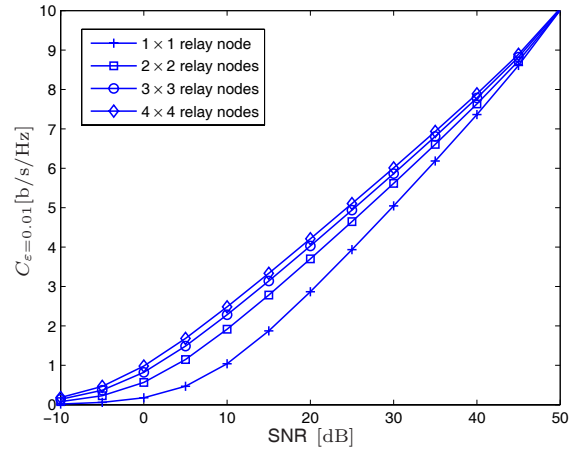


Fig. 4. Outage capacities of SSDF at an outage probability of 0.01 for grid relay networks with various sizes.

SNR regions for which relaying is beneficial. In the high SNR regime, the outage capacity of FSDF is approximately half of that provided by direct communication. We also found that the outage capacity of SSDF (which is the best relaying scheme under consideration, but requires every relay to know the channel between the source and destination) converges to that of direct communication in the high SNR regime.

APPENDIX A CONTINUITY AND MONOTONICITY

Let $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(w)$ denote the right-hand side of (11), which is a polynomial in w , and hence a continuous function. Monotonicity of $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(\cdot)$ can be established after we prove the following proposition.

Proposition 1: Function $p : [0, \infty) \rightarrow [0, 1]$ below is strictly increasing:

$$p(x) \triangleq \mathbb{P} \left\{ \max_{k \in \mathcal{K}} \min \left\{ |h_{B,k}|^2, |h_{F,k}|^2 + |h_{SD}|^2 \right\} \leq x \right\}.$$

Proof: Let $x_1 < x_2$ be given. Then,

$$\begin{aligned}
 & p(x_2) - p(x_1) \\
 &= \mathbb{P} \left\{ x_1 < \max_{k \in \mathcal{K}} \min \left\{ |h_{B,k}|^2, |h_{F,k}|^2 + |h_{SD}|^2 \right\} \leq x_2 \right\} \\
 &\geq \mathbb{P} \left\{ \bigcap_{k=1}^K \left\{ x_1 < |h_{B,k}|^2 \leq x_2, \frac{1}{2}x_1 < |h_{F,k}|^2 \leq \frac{1}{2}x_2 \right\} \right. \\
 &\quad \left. \bigcap \left\{ \frac{1}{2}x_1 < |h_{SD}|^2 \leq \frac{1}{2}x_2 \right\} \right\} \\
 &\quad \text{(one event is a subset of the other)} \\
 &= \mathbb{P} \left\{ \frac{1}{2}x_1 < |h_{SD}|^2 \leq \frac{1}{2}x_2 \right\} \\
 &\quad \prod_{k=1}^K \left[\mathbb{P} \left\{ x_1 < |h_{B,k}|^2 \leq x_2 \right\} \mathbb{P} \left\{ \frac{1}{2}x_1 < |h_{F,k}|^2 \leq \frac{1}{2}x_2 \right\} \right] \\
 &> 0.
 \end{aligned}$$

□

From [16, Appendix A], it can be verified that

$$\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(w) = p \left(\ln \left(\frac{1}{w} \right) \right), \quad 0 < w \leq 1, \quad (26)$$

and hence $\mathbb{P}_{\text{out}}^{\text{FSDF-direct}}(\cdot)$ is strictly decreasing on $(0, 1]$.

APPENDIX B OUTAGE PROBABILITY OF FSDF WITHOUT DIRECT LINK COMBINING

We write

$$\begin{aligned}
 & \mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}} \\
 &= \mathbb{P} \left\{ \max_{k \in \mathcal{K}} \min \{ R_{B,k}, R_{F,k} \} < R \right\} \\
 &= \prod_{k=1}^K \mathbb{P} \left\{ \min \{ R_{B,k}, R_{F,k} \} < R \right\} \quad \text{(independence)} \\
 &= \prod_{k=1}^K \left[1 - (1 - \mathbb{P}\{R_{B,k} < R\})(1 - \mathbb{P}\{R_{F,k} < R\}) \right]. \quad (28)
 \end{aligned}$$

Substituting

$$\begin{aligned}
 \mathbb{P} \{ R_{B,k} < R \} &= \mathbb{P} \left\{ |h_{B,k}|^2 < \frac{2^{2R} - 1}{\text{SNR}} \right\} \\
 &= 1 - \exp \left(-d_{S_k}^\alpha \left(\frac{2^{2R} - 1}{\text{SNR}} \right) \right), \quad (29)
 \end{aligned}$$

and substituting

$$\begin{aligned}
 \mathbb{P} \{ R_{F,k} < R \} &= \mathbb{P} \left\{ |h_{F,k}|^2 < \frac{2^{2R} - 1}{\text{SNR}} \right\} \\
 &= 1 - \exp \left(-d_{kD}^\alpha \left(\frac{2^{2R} - 1}{\text{SNR}} \right) \right) \quad (30)
 \end{aligned}$$

into (28) give

$$\begin{aligned}
 \mathbb{P}_{\text{out}}^{\text{FSDF-nodirect}} &= \prod_{k=1}^K \left[1 - \exp \left(-d_{S_k}^\alpha \left(\frac{2^{2R} - 1}{\text{SNR}} \right) \right) \right. \\
 &\quad \left. \exp \left(-d_{kD}^\alpha \left(\frac{2^{2R} - 1}{\text{SNR}} \right) \right) \right],
 \end{aligned}$$

which simplifies to (16).

REFERENCES

- [1] R. U. Nabar, H. Bölcskei, and F. W. Kneubühler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099-1109, Aug. 2004.
- [2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [3] A. S. Avestimehr and D. N. C. Tse, "Outage capacity of the fading relay channel in the low-SNR regime," *IEEE Trans. Inform. Theory*, vol. 53, no. 4, pp. 1401-1415, Apr. 2007.
- [4] J. Hu and N. C. Beaulieu, "Closed-form expressions for the outage and error probabilities of decode-and-forward relaying in dissimilar Rayleigh fading channels," in *Proc. IEEE Int. Conf. Commun.*, Glasgow, Scotland, June 2007, pp. 5553-5557.
- [5] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700-702, Aug. 2005.
- [6] J. Hu and N. C. Beaulieu, "Performance analysis of decode-and-forward relaying with selection combining," *IEEE Commun. Lett.*, vol. 11, no. 6, pp. 489-491, June 2007.
- [7] A. Bletsas, H. Shin, M. Z. Win, and A. Lippman, "Cooperative diversity with opportunistic relaying," in *Proc. IEEE Wireless Commun. Networking Conf.*, vol. 2, Las Vegas, NV, Mar. 2006, pp. 1034-1039.
- [8] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450-3460, Sept. 2007.
- [9] E. Beres and R. Adve, "On selection cooperation in distributed networks," in *Proc. Conf. Inform. Sci. Sys.*, Princeton, NJ, USA, Mar. 2006, pp. 1056-1061.
- [10] —, "Outage probability of selection cooperation in the low to medium SNR regime," *IEEE Commun. Lett.*, vol. 11, no. 7, pp. 589-597, July 2007.
- [11] —, "Cooperation and routing in multi-hop networks," in *Proc. IEEE Int. Conf. Commun.*, Glasgow, Scotland, June 2007, pp. 4767-4772.
- [12] J. Luo, R. S. Blum, L. Cimini, L. Greenstein, and A. Haimovich, "New approaches for cooperative use of multiple antennas in ad hoc wireless networks," in *Proc. IEEE Semiannual Veh. Technol. Conf.*, vol. 4, Los Angeles, CA, Sept. 2004, pp. 2769-2773.
- [13] L. Zhang and L. Cimini, "Cooperative network coding in selective decode-and-forward networks with multiple source-destination pairs," in *Proc. Conf. on Inform. Sci. and Sys.*, Princeton, NJ, Mar. 2008, pp. 521-526.
- [14] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 659-672, Mar. 2006.
- [15] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. New York: Cambridge University Press, 2005.
- [16] K. Woradit, T. Q. S. Quek, W. Suwansantisuk, H. Wymeersch, L. Wuttisittikulij, and M. Z. Win, "Outage behavior of cooperative diversity with relay selection," in *Proc. IEEE Global Telecomm. Conf.*, New Orleans, LA, Dec. 2008.