



# MIT Sloan School of Management

MIT Sloan School Working Paper 4765-09

Class Pricing

Birger Wernerfelt

© Birger Wernerfelt

All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission, provided that full credit including © notice is given to the source.

This paper also can be downloaded without charge from the  
Social Science Research Network Electronic Paper Collection:  
<http://ssrn.com/abstract=1084113>

Electronic copy available at: <http://ssrn.com/abstract=1084113>

## **CLASS PRICING**

by

Birger Wernerfelt\*

January 15, 2008

\* MIT Sloan School of Management, Cambridge, MA 02142, 617 253 7192,  
[bwerner@mit.edu](mailto:bwerner@mit.edu). I am indebted to seminar audiences for several important comments.  
Of course, the usual disclaimer applies.

## CLASS PRICING

### Abstract

A contract with  $K$ -class pricing divides a large set of goods or services into  $K$  classes and assigns a single price to any element of a class. Class pricing can be efficient when several different versions may be traded and it is costly to assign individual prices to all of them. It is more likely to be used when the number of buyers is smaller, the number of versions is larger, the variance in costs is smaller, and demand ex ante differs less between versions. Under simple conditions classes should be designed to minimize the sum of squared within-class cost deviations. In bilateral trades, the most efficient game form is that in which classes are designed by the player with less varied gains from trade, while the traded version is chosen by the other player. Decisions are thus made by the player who cares most about them, while the opponent prescribes a set of limits.

## I. INTRODUCTION

Jelly Beans are sold by weight and the buyer can decide on the mix of flavors, college tuition does not depend on the courses taken by the student, a haircut can be as the customer “likes it”, a fast-food hamburger can include any number of napkins and condiments picked up by the guest, a stay at an “all inclusive” resort may take an infinite number of forms. Along similar lines, a food-service contractor is allowed several substitutions by the “seasonal vegetables” language of the contract, and employees will perform any of a large set of tasks as requested by their bosses. The above are examples of what we will call “one-class pricing”. It is absurdly expensive to determine a price for each of the possible versions of the goods (or services) mentioned, so one player (often the seller) groups the versions into a class, a single price applies to all elements in the class, and other player decides which is traded. We can more generally think about “ $K$ -class” pricing in which the versions are grouped into  $K$  classes with  $K$  different prices. Examples of this include different prices for different types of candies, different fees to cut longer or shorter hair, and higher wages for overtime or hardship. The canonical example is, of course, the “5 and 10” stores.

In the present paper we attempt to rationalize the existence and properties of class pricing. We offer results on the determinants of class prices, the number of classes used, how versions are grouped into classes, and who should design the classes. First, and not surprisingly, class prices are increasing in average class costs and demands. Second, fewer classes are used when the number of buyers is smaller, the number of versions is larger, the variance in costs is smaller, and demand ex ante differs less between versions. Third, sellers will define classes by cost/demand-intervals: if one high and one low

cost/demand version are in the same class, then all versions with intermediate cost/demand will be included. In a simple but natural case, we find that the profit maximizing class intervals are those that minimize the sum of squared within-class cost deviations (holding the number of classes constant). Fourth, in bilateral trades, it is most efficient if the classes are designed by the player with less varied gains from trade, while the traded version are chosen by the other player. The ultimate decision right is given to the player who cares the most, but the opponent can constrain the amount of discretion yielded.

Throughout the paper, we take as a premise that pricing costs are subject to economies of scale, such that the cost of assigning a single price to a class of versions is lower than the cost of pricing all the versions one-by-one. Since the vast majority of economic models are based on the assumption that pricing is free in the first place, our premise rests on controversial grounds. At issue is not whether pricing actually is completely costless, but whether these costs are large enough to have important implications. The paper addresses this underlying controversy by assuming, *arguendo*, that pricing is costly, and deriving some implications thereof.

Once we provisionally accept that pricing is costly, it is quite natural to assume that these costs are subject to economies of scale, at least in markets where prices are posted. Depending on the source of bargaining costs, economies of scale may be less clear if prices are arrived at through bargaining, particularly since one could imagine that players would tolerate costs in proportion to stakes. However, most people will find it plausible that a single \$30,000 deal could be negotiated in less time than thirty \$1,000 deals.

## *Literature*

In the seventy years since Coase argued that “there is a cost of using the price mechanism” (1937, p. 390), the literature has identified several such costs. However, instead of focusing on costs that are “common”, there has been a preference for costs that are “large” (such as the threat of hold-up). As suggested by the examples in the Introduction, class pricing is a widely observed phenomenon, but one of Cents rather than Dollars. Any convincing explanation must rely on what has been called the “mundane” (Williamson, 1985, p. 105) costs of pricing; the common but small costs associated with any price determination process, such as bargaining or take-it-or-leave-it offers.

In the broader economics literature, we find the most important use of “common but small” pricing costs in macro-economic works on the dynamic implications of menu-costs (Mankiw, 1985). The general idea in this literature is that pricing costs lead to sticky prices and that the resulting incomplete equilibration can provide a micro-foundation for Keynesian macroeconomics (Blinder et al., 1998). One can look at this literature as aiming to explain why prices are coarse over time, rather than across products (as is our focus here).

The optimality of cross-sectional class pricing has received significant attention in the context of screening models where it often is optimal to “bunch” several types into the same contract (Mussa and Rosen, 1978). There has been much less work justifying the phenomenon by the costs of detailed pricing. To the best of our knowledge, Seim and Viard (2006) and Wernerfelt (1997) are the only two papers to do so, and then only in rather narrow contexts.

Outside the academic literature, the U. S. Supreme court has used pricing costs in a ruling about license fees. The American Society of Composers, Authors and Publishers, which license the work of individual artists in the music industry, charges a blanket fee to bars, radio stations etc.. When the Columbia Broadcast System challenged this practice, the court found in favor of the defendant, arguing that “A middle man with a blanket license was an obvious necessity if the thousands of individual negotiations, a virtual impossibility, were to be avoided” (Broadcast Music Inc. vs. Columbia Broadcast System, p. 20).

At a more abstract level, our characterization of the optimal composition of pricing classes leads us to minimize the same function as that minimized in a very commonly used computer science algorithm for optimal quantization (Gersho and Gray, 1991) as well as in the statistical technique used in “k-means” clustering algorithms (MacQueen, 1967).

### *Plan of the Paper*

Since the cost of pricing is the controversial premise of the argument, we will devote the Section II to a discussion of the nature of these costs. The analysis will be presented in Section III, followed by a brief discussion in Section IV.

## **II. THE COSTS OF PRICING**

Arguments about pricing costs are often objected to with proposals to use alternative trading mechanisms. Since no single class of pricing costs, such as those

incurred in connection with bargaining or signage, apply convincingly to all ways of determining prices, the players can avoid any specific problem by using some other mechanism.

We will here offer a four-pronged counter-argument. First, there are many types of pricing costs and the argument does not depend on any one type being important. Second, while the existence of mechanisms without pricing costs is a theoretical possibility, widely used practices, such as bargaining and take-it-or-leave-it (“posted”) offers, arguably entail several important types of pricing costs. Whatever the reasons for their use, the prevalence of these mechanisms makes it important to study the implications of the associated pricing costs. Third, and perhaps more speculatively, the existence of a cost-free mechanism is immaterial unless the players can use a cost-free mechanism to agree to use it. Fourth, even if pricing costs are unimportant in many applications, they could still be critical in others.

Aiming to make the first two prongs of the argument, we now offer a brief literature survey to document the existence and importance of several pricing costs associated with bargaining and posted price mechanisms.

### *Bargaining Costs*

Some version of alternating offer bargaining is commonly used to determine prices under conditions with a flavor of bilateral monopoly. Examples include big ticket consumer goods, industrial products, employment contracts, and other services. Because the idea is to make a list of the costs associated with this price determination process, we will group them in three categories.



1. Costs associated with the bargaining process itself. Any explicit model of alternating offer bargaining must posit some costs of refusing an offer and making a counter, since the process otherwise would go on ad infinitum. Delays are strictly out-of-equilibrium outcomes in the most simple models (Rubinstein, 1982), but not in richer settings (Watson, 1998).

Perhaps more importantly, it is obvious that bargaining often does take quite a bit of time in the real world. The costs of this time include the salary of bargainers, the loss from delays in trade, and the disutility many people feel from participating in the back-and-forth process (think of asking for a raise or buying a car). At a more aggregate level, the Bureau of Labor Statistics (May, 2005) estimates that there are 69,300 Purchasing Managers in the US each making an average of \$81,440 per year. Since a survey by *Purchasing Magazine* suggests that these managers spend 15% of their time on price negotiations, we can estimate that the firms employing them incur close to one billion dollars in direct negotiation costs.

2. Costs associated with the outcomes: It has recently been argued that any not-ex-ante-agreed-upon outcome produces ill-will towards the trading partner and a reduction in gains from trade (Hart and Moore, 2008). More generally, players may experience lingering negative (and counter-productive) sentiments towards past bargaining opponents.

3. Costs incurred in anticipation of bargaining. It is well-documented that better-informed bargainers get better results (Busse, Silva-Risso, and Zettelmeyer, 2006). While this result does not figure prominently in the theoretical literature, it is not hard to understand. The idea is that players, prior to bargaining, can invest to get information that

will help them in the bargaining process. Unless equilibrium investments equal collusive investments, there will be a distortion, i.e. a bargaining cost. In most cases it seems natural to assume that the cost comes in the form of both players over-investing in jointly wasteful information.

Consistent with the importance of anticipatory bargaining costs, the above-mentioned survey of purchasing managers found that they spent about 25% of their time “Preparing Bids” and “Researching Prices”. At a more strategic level, players may refrain from suggesting improved trades in order to avoid bargaining or withhold information about such opportunities in order to protect their own future bargaining power (Simester and Knez, 2002).

### *Costs of Posting Prices*

In situations where a single seller faces several buyers, it is common for the former to post a price which is understood to be a take-it-or-leave-it offer. I once again list several costs of the process.

1. Direct costs of posting a price. Levy et al (1997) estimate the direct costs of changing a supermarket price to be \$.52. Depending on the setting, additional costs may be incurred in order to communicate the prices to buyers. In the case of a large industrial supplier, Zbaracki et al (2004) find that the firm’s total expense on pricing amounts to 1.22% of revenues.

2. Seller costs of managing several different prices. It is not unheard of for retailers to put all products in a small number of price classes in order to keep operations as simple as possible (“5 and 10”, “Dollar Stores”). In the study of Zbaracki et al. (2004),

the costs of managerial information gathering, decision-making, and communication were much larger than the conventional menu-costs.

3. Buyers' reactions to facing several different prices. Several laboratory studies have suggested that buyers may purchase more when faced with fewer price classes (Chernev, 2006). A recent field experiment by Bertini, Frederick and Simester (2006), yielded similar results and the authors attribute this to buyers' aversion to making complex tradeoffs.

### III. ANALYSIS

As noted in the previous Section, the costs of determining a price depend on the mechanism through which this is done e. g. by alternating offer bargaining or unilateral price posting. Since these costs contribute to the overall efficiencies of alternative price-determination mechanisms, attempts to economize on them should ideally take the endogeneity of the mechanism into account. However, to the extent that the nature of pricing costs varies between mechanisms, it would be very hard to perform a comparative analysis. We will therefore sidestep the issue and focus the analysis on unilateral price-posting only.

In our reduced form model, the cost of pricing is represented only by the total cost  $r$  that must be incurred on a per price basis. We endow the model with economies of scale by assuming that this cost is independent of the number of versions to which the price applies.

## 0. Preliminaries.

The product comes in  $N$  possible versions, and if these are divided into  $K \leq N$  classes, the parties incur total pricing costs  $Kr$ . There is one seller (he) and  $B$  potential buyers (she), each of whom buys one or no units of the product. Versions are indexed by  $n$  or  $q = 1, 2, \dots, N$ , classes by  $k$  or  $j = 1, 2, \dots, K$ , and buyers by  $b = 1, 2, \dots, B$ . The set of versions in class  $k$  is  $S^k$  and we use  $p_n$  to denote the price of version  $n$ , while  $p^k$  is the price for any version in class  $k$ . We will use the notation  $n \in k$  as shorthand for  $n \in S_k$ , such that  $n \in k$  implies that  $p_n = p^k$ .

To keep the effects of costly pricing separate from those of screening, we will assume that each buyer has very specific needs, such that only one version is “right” – has positive value - for any specific buyer (cf. Aghion and Tirole, 1997). In this context, we start with the case in which versions have identical prior demands, but different costs. The second Sub-Section is focused on the opposite case with different prior demands, but identical costs; while we look at the general case, with differing costs and demands, in the third Sub-Section. In the fourth and final Sub-Section, we compare two game forms to ask whether buyers or sellers would design the classes most efficiently.

### 1. Identical prior demands, different costs

The seller can produce the  $n$ 'th version of the product for  $c_n \in [0, 1]$  and buyer  $b$  values this version at  $v_{nb} \in [0, 1]$ . The seller knows these costs at the outset and we label the versions such that  $c_1 < c_2 < \dots < c_N$ . All  $BN$  values are ex ante unknown to the seller, and because pricing takes time and it is important to trade quickly, he has to set prices for all versions before hearing from the buyers. For buyer  $b$ , only one version, indicated by

$rb$ , is “right.” Ex ante, all versions are equally likely to be right for  $b$ , and thus have probability  $1/N$  of being so. The value of  $rb$ ,  $v_{rb}$ , is drawn from a commonly known distribution  $F: [0, 1] \rightarrow [0, 1]$ , independent of costs and I.I.D. across buyers and versions. The buyer values all other versions at 0. We indicate “no trade” by the version label 0, and define  $v_{0b} = c_0 = p_0 = 0$ . The addition of the no trade possibility means that the set of possible outcomes for a specific buyer can be described as an element of the set  $\{0, 1, 2, \dots, N\}$ .

Except in Subsection 4, we assume that the seller designs the classes while the buyer selects the version to be traded. The sequence of events is:

1. The seller learns his costs  $(c_1, c_2, \dots, c_N)$  and each buyer learns her valuations  $(v_{b1}, v_{b2}, \dots, v_{bN}) = (0, 0, \dots, v_{rb}, \dots, 0, 0)$ .<sup>1</sup>
2. The seller groups the  $N$  versions into  $K$  classes and sets a price for each class.
3. Each buyer picks the version she wants to trade, if any.
4. Trades and payoffs.

Pricing costs are incurred even if no trades take place and since the seller sets the prices unilaterally, we here charge all pricing costs to him. Analyzing the game backwards, buyer  $b$  selects version  $rb$  if  $v_{rb} - p_{rb} > 0$  and otherwise makes no trade. To indicate the decisions made by buyers, we define  $B(N + 1)$  indicator variables such that  $t_{bq} = 1$  if buyer  $b$  makes trade  $q \in \{0, 1, 2, \dots, N\}$  and  $t_{bq} = 0$  otherwise. So

$$\begin{aligned}
 (t_{b0}, t_{brb}) &= (0, 1) \text{ if } v_{rb} - p_{rb} > 0, \\
 (t_{b0}, t_{brb}) &= (1, 0) \text{ if } v_{rb} - p_{rb} \leq 0, \text{ and} \\
 t_{bq} &= 0 \text{ if } q \neq 0, rb.
 \end{aligned} \tag{1}$$

---

<sup>1</sup> It will often be more natural to assume that the buyer learns his valuation ex interim, between 2 and 3, but the present formulation will give the same results and preserves symmetry between the players.

In stage 2, the seller wants to find a number of classes  $K$ , a way to partition the  $N$  versions into these classes,  $S = (S^1, S^2, \dots, S^K)$ , and a set of  $K$  prices to maximize the expected profits. With some abuse of notation, we use the  $\text{Max}_S$  operator as shorthand for the first two steps, such that we can write the seller's problem as

$$\begin{aligned} & \text{Max}_S \text{Max}_p E \sum_B \sum_K \sum_{q \in k} (p^k - c_q) t_{bq} - Kr, \\ & \text{s. t. (1), } \bigcup_k S^k = N, \bigcap_{j \neq k} S^k S^j = \emptyset, \text{ and } S^k \neq \emptyset \text{ for all } k. \end{aligned} \quad (2)$$

While (2) in general is a very difficult problem, we have endowed it with sufficient structure to allow us to characterize the optimal solution in some detail.

First, since all buyers are ex ante identical, such that we can solve (2) at the level of a representative buyer, thus replacing the summation over  $B$  in favor of multiplication. Second, since all versions have identical prior valuations, the ex ante choice probabilities depend only on  $p$ . All versions in a class will therefore have equal choice probabilities and the seller's expected per-buyer profits are  $\sum_K (p^k - c^k) [1 - F(p^k)] |S^k| / N - Kr/B$ , where  $|S^k|$  is the cardinality of  $S^k$  and  $c^k$  is the average cost of the versions in it. The optimal prices are then given by

$$p^{k*} = c^k + [1 - F(p^{k*})] / f(p^{k*}), \quad (3)$$

and the partitioning problem is

$$\begin{aligned} & \text{Max}_S \sum_K [1 - F(p^{k*})]^2 |S^k| / [N f(p^{k*})] - Kr/B. \\ & \text{s. t. (3), } \bigcup_k S^k = N, \bigcap_{j \neq k} S^k S^j = \emptyset, \text{ and } S^k \neq \emptyset \text{ for all } k. \end{aligned} \quad (4)$$

While we are unable to solve this problem analytically, we can characterize the solution.

**Proposition 1:** *If prior demands are identical, it is never profit maximizing to have classes with interlacing costs.*

**Proof:** If versions  $n$  and  $n + 2$  are in class  $k$ , then version  $n + 1$  should be in class  $k$  as well. To see this, note that if version  $n + 1$  is in a higher priced class, it is traded with lower probability than version  $n + 2$  implying that the seller could do better by switching the two versions. Similarly, if version  $n + 1$  is in a lower priced class, it is traded with higher probability than version  $n$ .

Q.E.D.

This immediately gives

**Corollary 1:** *If prior demands are identical, the optimal classes can be defined by cost-intervals and class prices increase as average class costs go up.*

Recalling that versions are labeled in order of increasing costs, we can label the classes such that the (average class) costs and profit maximizing prices  $p^{k*}$  are increasing in  $k$ .

With these labels, if  $c_q < c_n$ ,  $n \in k$  and  $q \in j$ , then  $j \leq k$ .

We can offer a much stronger characterization of the solution if  $F$  is uniform.

**Finding 1:** *If  $F$  is uniform, the profit maximizing partition for a given  $K$  is that which minimizes the sum of squared within-class deviations in costs.<sup>2</sup>*

---

<sup>2</sup> For example, if  $(c_1, c_2, \dots, c_N) = (.01, .02, \dots, [.01]N)$ , where  $N < 100$ , and we ignore integer problems, we can write (10) as  $(N-K)r/B - (.0001)(N-2K)(N-K)/(12K^2)$ , and the optimal  $K$  is a root of  $120000rK^3 - BK^2 + 3NBK - 2BN^2 = 0$ .

**Proof:** By substituting in the optimal class prices,  $p^{k*} = 1/2 + c^k/2$ , we find the ex ante expected per-buyer profits as

$$[1 + \sum_K (c^k)^2 |S^k| / N - 2\sum_N c_n / N] / 4 - Kr/B. \quad (5)$$

If  $K = N$ , (5) becomes  $[1 + \sum_N (c_n)^2 / N - 2\sum_N c_n / N] / 4 - Nr/B$ , and we can express the net advantage of class pricing as

$$(N - K)r/B - [\sum_N (c_n)^2 - \sum_K |S^k| (c^k)^2] / N. \quad (6)$$

The first term in (6) reflects the saved pricing costs and the second the loss from less than optimal pricing of individual versions. The finding follows from rewriting (6) as

$$(N - K)r/B - [\sum_K \sum_{n \in k} (c_n - c^k)^2] / N. \quad (7)$$

*Q.E.D.*

The criterion (7) appears in other fields as well. For example, both the “k-means” clustering algorithm (MacQueen, 1967) from statistics and a widely used “quantization” technique from computer science (Gersho and Gray, 1991) minimize the same criterion.<sup>3</sup> In our case, we have an analytical proof that the procedure is optimal, although only in the knife-edge condition considered above.

**Remark 1A.** It is tempting to conclude that the profit maximizing  $K$  is smaller when the variance in costs is smaller, but one can easily construct examples in which this conjecture is false. For example, if  $c_1 = c_2 = .1$  and  $c_3 = c_4 = .9$ ,  $K$  will be at most 2, while  $c_1 = .1$ ,  $c_2 = .4$ ,  $c_3 = .6$ , and  $c_4 = .9$  may lead to a larger  $K = 3$  or 4, in spite of

---

<sup>3</sup> The object of quantization is to compress a finer scale into a smaller set of discrete categories, while retaining as much relevant information as possible and the object of clustering is to organize a set of elements into groups that are in some way similar.



having lower variance. However, we can conclude that the seller is more likely to set  $K = I$  when the variance in costs is lower.

**Remark 1B.** As can be seen from (7), the variance in values plays no role as long as all the values all are drawn from the same distribution. Suppose, however, that some versions, if “right”, have values drawn from a uniform distribution on  $[\rho, I]$ , where  $\rho \in [0, I]$ , while others have values drawn from a uniform distribution on  $[0, I - \rho]$ . In this case, the seller could reap larger benefits from putting the two types of versions in separate classes for larger values of  $\rho$ . So in this example, the profit maximizing  $K$  will be weakly larger if the ex ante variance in values is larger in the sense that the distributions differ “more” between versions. (Formulation and demonstration of a more general result of this type is a topic for future research.)

The results obtained so far apply to the case in which all versions have identical prior demand, while ex post, each buyer assigns positive value to only one version. Continuing to maintain the latter feature (and thus ruling out screening); we now consider the case in which versions have different prior demands, but identical costs.

## 2. Different prior demands, identical costs

Formally, the seller can produce any of the  $N$  versions of the product for  $c \in [0, I]$  and buyer  $b$  values version  $n$  at  $v_{nb} \in [0, I]$ . For buyer  $b$  only one version, indicated by  $rb$ , is “right.” As in Subsection 1, a buyer values all versions other than the one that is right for her at 0. Ex ante, all versions are equally likely to be right for  $b$ , and thus have

probability  $1/N$  of being so. If version  $n$  is right for  $b$  (such that  $rb = n$ ), its value  $v_{rb}$  is drawn from a commonly known distribution  $F_n: [0, 1] \rightarrow [0, 1]$  I.I.D. across buyers. The family of distributions  $F_1, F_2, \dots, F_N$  satisfy a monotone likelihood ratio property such that there exists a labeling for which

$$f_{n+1}(P)/f_n(P) \geq f_{n+1}(p)/f_n(p) \quad (MLR)$$

for all  $(p, P) \in ([0, 1], (p, 1])$  and any  $n \in \{1, 2, \dots, N-1\}$ . We will adopt those labels.

The sequence of events is as in Subsection 1 and we can again represent the seller's problem by (2). However, it is no longer true that all versions in a class have identical ex ante choice probabilities. Since the probability of  $v_{rb} - p_{rb} > 0$  is  $1 - F_{rb}(p^{rb})$ , the seller's expected per-buyer profits are  $\sum_K \{ (p^k - c) \sum_{n \in k} [1 - F_n(p^k)] / N \} - Kr/B$ .

The optimal prices are therefore given by

$$p^{k*} = c + \sum_{n \in k} [1 - F_n(p^{k*})] / \sum_{n \in k} f_n(p^{k*}) \quad (8)$$

(such that  $p^{k*}$  is increasing in  $k$  if  $K = N$ ). The partitioning problem is

$$\text{Max}_S \sum_K \{ \sum_{n \in k} [1 - F_n(p^{k*})] \}^2 / \{ \sum_{n \in k} f_n(p^{k*}) N \} - Kr/B. \quad (9)$$

$$s. t. (8), \bigcup_k S^k = N, \bigcap_{j \neq k} S^k \cap S^j = \emptyset, \text{ and } S^k \neq \emptyset \text{ for all } k.$$

The solution to this problem has characteristics analog to those of (4).

**Proposition 2:** *If costs are identical, it is never profit maximizing to have classes with interlacing prior demands.*

**Proof:** Suppose that version  $n + 1$  is priced below version  $n$ , such that  $p_{n+1} = p$  and  $p_n = P$ , where  $p < P$ . The seller's expected profit from these two versions is then

$[P - c][1 - F_n(P)] + [p - c][1 - F_{n+1}(p)]$ . This is higher than the profit from charging  $p$  for both versions if

$$[P - c][1 - F_n(P)] > [p - c][1 - F(p)], \quad (10)$$

and it is higher than the profits from charging  $P$  for both versions if

$$[p - c][1 - F_{n+1}(p)] > [P - c][1 - F_{n+1}(P)]. \quad (11)$$

Taken together (10) and (11) require that

$$[1 - F_n(p)]/[1 - F_n(P)] < [P - c]/[p - c] < [1 - F_{n+1}(p)]/[1 - F_{n+1}(P)] \quad (12)$$

and that

$$[1 - F_{n+1}(p)]/[1 - F_n(p)] > [1 - F_{n+1}(P)]/[1 - F_n(P)] \quad (13)$$

This again requires that the ratio  $[1 - F_{n+1}(x)]/[1 - F_n(x)]$  be decreasing in  $x \in [0, 1]$ , such that

$$d\{[1 - F_{n+1}(x)]/[1 - F_n(x)]\}/dx < 0 \quad (14)$$

or

$$f_n(x)[1 - F_{n+1}(x)] < f_{n+1}(x)[1 - F_n(x)]. \quad (15)$$

But this is inconsistent with (MLR), since we can rewrite (MLR) as

$$f_{n+1}(P)f_n(p) \geq f_{n+1}(p)f_n(P) \quad (16)$$

and integrate both sides over  $P$  from  $p$  to 1 to get

$$f_n(p)[1 - F_{n+1}(p)] \geq f_{n+1}(p)[1 - F_n(p)]. \quad (17)$$

QED.

This immediately gives

**Corollary 2:** *If costs are identical, the optimal classes can be defined by demand-intervals and class prices increase as average class demand goes up.*

Combining the effects considered above, we next consider the case in which both prior demands and costs differ between versions.

### 3. Different prior demands, different costs

The assumptions in this subsection combine those in subsections 1 and 2. In particular, the seller can produce the  $n$ 'th version for  $c_n \in [0, 1]$  and buyer  $b$  values this version at  $v_{nb} \in [0, 1]$ . For buyer  $b$  only one version, indicated by  $rb$ , is "right." A buyer values all versions other than the one that is right for her at 0. Ex ante, all versions are equally likely to be right for  $b$ , and thus have probability  $1/N$  of being so. If version  $n$  is right for  $b$  (such that  $rb = n$ ), its value  $v_{rb}$  is drawn from a commonly known distribution  $F_n: [0, 1] \rightarrow [0, 1]$  I.I.D. across buyers. We assume that the vector of costs  $c_1, c_2, \dots, c_N$  and the family of distributions  $F_1, F_2, \dots, F_N$  satisfy a monotone "scaled costs plus likelihood ratio" property in the sense that there exists a labeling for which:

$$\begin{aligned} & [c_n/p][F_n(P) - F_n(p)]/[1 - F_n(P)] - [1 - F_n(p)]/[1 - F_n(P)] < \\ & [c_{n+1}/p][F_{n+1}(P) - F_{n+1}(p)]/[1 - F_{n+1}(P)] - [1 - F_{n+1}(p)]/[1 - F_{n+1}(P)] \quad (MSC+LR) \end{aligned}$$

for all  $p \in [0, 1]$ , all  $P \in [p, 1]$ , and any  $n \in \{1, 2, \dots, N-1\}$ . We will refer to these labels as "indices" of the corresponding versions. We adopt these labels.

The sequence of events is as in subsection 1 and we can again represent the seller's problem by (2). Since the probability of  $v_{rb} - p_{rb} > 0$  is  $1 - F_{rb}(p^{rb})$ , the seller's expected per-buyer profits are  $\sum_{K \setminus \{n\}} \{ \sum_{n \in K} (p^k - c_n)[1 - F_n(p^k)]/N \} - Kr/B$ .

The optimal prices are therefore given by

$$p^{k*} = \{\sum_{n \in k} c_n f_n(p^{k*}) + \sum_{n \in k} [1 - F_n(p^{k*})]\} / \sum_{n \in k} f_n(p^{k*}), \quad (18)$$

and the partitioning problem is

$$\begin{aligned} & \text{Max}_S \sum_k \{(\sum_{n \in k} [1 - F_n(p^{k*})])^2 / \{\sum_{n \in k} f_n(p^{k*}) N\} + \\ & \{\sum_{n \in k} c_n f_n(p^{k*}) \sum_{n \in k} [1 - F_n(p^{k*})]\} / \{\sum_{n \in k} f_n(p^{k*}) N\} - \sum_{n \in k} c_n [1 - F_n(p^{k*})] / N - Kr/B. \end{aligned} \quad (19)$$

$$s. t. (18), \bigcup_k S^k = N, \bigcap_{j \neq k} S^k \cap S^j = \emptyset, \text{ and } S^k \neq \emptyset \text{ for all } k.$$

**Proposition 3:** *It is never profit maximizing to have classes with interlacing index values.*

**Proof:** Suppose that version  $n + 1$  is priced below version  $n$ , such that  $p_{n+1} = p$  and  $p_n = P$ , where  $p < P$ . The seller's expected profit from these two versions is then

$[P - c_n][1 - F_n(P)] + [p - c_{n+1}][1 - F_{n+1}(p)]$ . This is higher than the profit from charging  $p$  for both versions if

$$[P - c_n][1 - F_n(P)] > [p - c_n][1 - F_n(p)], \quad (20)$$

and it is higher than the profits from charging  $P$  for both versions if

$$[p - c_{n+1}][1 - F_{n+1}(p)] > [P - c_{n+1}][1 - F_{n+1}(P)]. \quad (21)$$

Taken together (20) and (21) require that

$$\begin{aligned} & \{1 - F_n(p) - [F_n(P) - F_n(p)] c_n/p\} / \{1 - F_n(P)\} < P/p \\ & < \{1 - F_{n+1}(p) - [F_{n+1}(P) - F_{n+1}(p)] c_{n+1}/p\} / \{1 - F_{n+1}(P)\} \end{aligned} \quad (22)$$

But this contradicts (MSC+LR).

Q.E.D.

**Corollary 3:** *The optimal classes can be defined by index-intervals and class prices increase as average class indices goes up.*

We will now look in some detail at a simple example in which we can identify the most efficient allocation of roles between buyer and seller.

4. Finding the most efficient game form (division of roles between buyer and seller).

We have so far looked exclusively at the *Seller-Design-Buyer-Choice* game form, in which the seller designs the classes and sets the prices, after which the buyer chooses the version to be traded. We now want to investigate the optimality of this arrangement when the two sides of the market are symmetric in the sense that there is a single buyer. The most appealing candidate is the symmetric alternative in which the buyer designs the classes and sets the prices after which the seller chooses the version to be traded.

There are, however, other possibilities as well. The two alternatives in which the same player designs and chooses are most easily thought of in a sequential sense in which (i) one player first designs the classes, (ii) the opponent then sets prices, and (iii) the first player finally chooses. In both of these cases, the design of the classes reflect private information (costs or values), which is not available to the player setting prices. The resulting jockeying for information-rents will burden both game forms with additional inefficiencies, and we will therefore ignore them in the following. The last two logical possibilities, in which the same player sets prices and chooses, are clearly very inefficient unless the opponent is given the right to refuse. Since this introduces factors not present in the simpler game forms, we will also ignore this last pair of alternatives.

Aiming to keep the analysis simple, we look at a case with two versions and assume that costs that are binomial and I.I.D. across versions. In particular, values are  $\gamma \in$

$[0, \frac{1}{2}]$  or  $1 - \gamma$ , each with probability  $\frac{1}{2}$ , and costs are  $1/3$  or  $2/3$ , also each with probability  $\frac{1}{2}$ . Noting that the buyer's valuations are more (less) varied than the seller's when  $\gamma < 1/3$  ( $\gamma > 1/3$ ), we will evaluate the relative efficiency of alternative game forms for varying values of  $\gamma$  relative to  $1/3$ . The cost of determining a price is  $r \geq 0$ .

Recall that the sequence of events in *Seller-Design-Buyer-Choice* is as follows

1. The seller learns his costs and the buyer learns her valuations.
2. The seller groups the 2 versions into 1 or 2 classes and sets prices.
3. The buyer picks the version she wants to trade, if any.
4. Trades and payoffs.

We will compare the ex ante efficiency of this to the logical alternative, *Buyer-Design-Seller-Choice*, in which

1. The seller learns his costs and the buyer learns her valuations.
2. The buyer groups the 2 versions into 1 or 2 classes and sets prices.
3. The seller picks the version she wants to trade, if any.
4. Trades and payoffs.

By proceeding mechanically and working through a lot of rather trite algebra, we can derive an appealing result about the relative efficiency of the two game forms. In particular, we can show

**Finding 4:** *For any  $r \geq 0$ , the Buyer-Design-Seller-Choice is the more efficient game form when  $\gamma < 1/3$ , while Seller-Design-Buyer-Choice is more efficient when  $\gamma > 1/3$ .*

**Proof:** See Appendix

Since the seller's (the buyer's) valuation is relatively less variable exactly when  $\gamma > 1/3$  ( $\gamma > 1/3$ ), this implies:

**Corollary 4:** *Classes in bilateral trades should be designed by the player with the less variable valuations, while the version traded should be chosen by the player with the more variable valuations.*

Although classes in most examples are designed by the seller, the above result helps us make sense of this, and the exceptions. In particular, many incomplete contracts leave the give the seller latitude in determining the attributes about which the buyer cares little. For example, food-service contracts with the term “seasonal vegetables” give the seller some latitude to take advantage of market prices. Another example is home renovation contracts, which leave the contractor with the flexibility to make many minor decisions in light of local conditions. However, in most cases it is the buyer who cares more about which version is traded, and thus given the right to choose.

While this result has strong intuitive appeal and considerable face-validity, other factors may play a role in more general settings. Specifically, if trade is not bilateral, it might make sense if the players on the thin side of the market are allowed to design the classes.

#### IV. DISCUSSION

We have introduced the concept of class pricing as a response to the costs of assigning prices to large numbers of products. Class pricing is more likely to be used



when the number of buyers is smaller, the number of versions is larger, the variance in costs is smaller, and demand ex ante differs less between versions. Sellers will define classes by cost-intervals and in a simple but natural case, the profit maximizing class design is that which minimizes the sum of squared within-class cost deviations. In bilateral trades, the most efficient game form is that in which classes are designed by the player with less varied gains from trade, while the traded version is chosen by the other player.

Once you start looking for it, class pricing is, as suggested by the examples in the opening paragraph of the paper, a very widely observed phenomenon. In many cases, (Jellybeans, haircuts, fast food,...) it is possible to argue that the aggregate implications of exactly identical prices are very similar to those of slightly differing prices. In other cases, (university educations, all-inclusive resorts,..) one could question the practice and probe its justification. Finally, there are some cases in which the implications are much less trivial.

It is possible to think of class pricing as an endogenously incomplete contract. We can define a very large set of versions by considering all levels of all attributes left out of the contract. Each of these attributes is often de facto left for a specific player to decide. For example, in a home-renovation contract, it is understood that the buyer can select colors, while the seller decides on almost all hidden aspects of construction. According to the analysis presented here, these are attributes about which the deciding player cares the most. In contrast, the opponent is so indifferent that it simply is not worth it to negotiate different prices for all versions.<sup>4</sup>

---

<sup>4</sup> Price negotiation may result if the opponent is not indifferent, but this is relatively rare.

Let us now briefly discuss some important questions left for future research. First, since the models and examples analyzed here are quite special, it will be important to develop a more general version of the theory. The computational difficulties associated with the partitioning problem will likely prove a significant hurdle. However, if the computer science literature is a good analogy, one can hope to show that the solution from Finding 1: “Minimize the sum of squared within-class cost variations”, is approximately optimal in a larger class of problems. If so, it would be much easier to do further work in the area. Secondly, while we have assumed a single seller throughout, it would be interesting to look at competitive class design. Suppose that there are many sellers with privately known costs drawn from the same commonly known distribution. In this case, prices should go down, thus depressing the advantages of having more classes. If the per-seller costs of pricing stay the same, one would therefore expect fewer classes with more competition. Alternatively, a small number of sellers may be able to play an equilibrium in which they use different classes, allowing each to cater to its own market segment, much like in a model with spatial differentiation. Third, while we have focused on classes defined by identical unit prices, one could imagine a more general theory of classes defined by identical contracts. Employees work on hourly pay and most multi-product sales forces receive the same commission rate regardless of the product sold. This line may eventually lead to a theory of contractual simplicity. Fourth, one can not help but notice that the versions, in many of the examples given, are quite close substitutes. Since the current theory does not explain this, it may be possible to sharpen the argument. This would most likely require us to allow for ex post substitutability and thus introduce screening considerations. Finally, and perhaps most importantly, it would

be interesting to test some of the predictions made in the paper, perhaps by exploiting different pricing practices between countries or across different periods in history.

## APPENDIX

### Proof of Finding 5

In the following,  $E\pi$  denotes the seller's expected profits,  $EU$  is the buyer's expected utility, and  $EL$  is the amount by which the sum of these falls short of the first best. We proceed mechanically and compare the performance of the two game forms for  $\gamma < 1/3$  and  $\gamma > 1/3$ .

*Case 1:  $\gamma < 1/3$ , Seller-Design-Buyer-Choice.*

For  $K = 2$ , the profit maximizing prices, associated efficiency losses, and profits prior to pricing costs, are:

$$(p_1, p_2 \mid c_1 = c_2 = 1/3) = (1 - \gamma, 1 - \gamma), EL = 0, E\pi = 1/2 - 3\gamma/4,$$

$$(p_1, p_2 \mid c_1 = 1/3, c_2 = 2/3) = (1 - \gamma - \varepsilon, 1 - \gamma), EL = 0, E\pi = 5/12 - 3\gamma/4,$$

$$(p_1, p_2 \mid c_1 = c_2 = 2/3) = (1 - \gamma, 1 - \gamma), EL = 0, E\pi = 1/4 - 3\gamma/4.$$

For  $K = 1$ , the profit maximizing price, associated efficiency losses, and profits prior to pricing costs, are:

$$p^1 = 1 - \gamma, EL = 0, E\pi \text{ are as if } K = 2, \text{ except that } E\pi = 3/8 - 3\gamma/4 \text{ if } c_1 \neq c_2.$$

So the seller will ex ante prefer  $K = 2$  if  $r < 1/48$ . However, the game form implements the first best for either value of  $K$  and thus for all values of  $r \geq 0$ .

*Case 2:  $\gamma < 1/3$  Buyer-Design-Seller-Choice.*

For  $K=2$ , the utility maximizing prices, associated efficiency losses, and utilities prior to pricing costs, are:

$$(p_1, p_2 \mid v_1 = v_2 = \gamma) = (0, 0), EL = 0, EU = 0,$$

$$(p_1, p_2 \mid v_1 = \gamma, v_2 = 1 - \gamma) = (0, 1/3), EL = 1/6 - \gamma/2, EU = 1/3 - \gamma/2,$$

$$(p_1, p_2 \mid v_1 = v_2 = 1 - \gamma) = (1/3, 1/3), EL = 1/12 - \gamma/4, EU = 1/2 - 3\gamma/4,$$

If  $K = 1$ , the utility maximizing price, associated efficiency losses, and utilities prior to pricing costs, are

$$(p^1 \mid v_1 = v_2 = \gamma) = 0, EL = 0, EU = 0.$$

$$(p^1 \mid v_1 = \gamma, v_2 = 1 - \gamma) = 1/3, EL = 1/8 - 3\gamma/8, EU = 1/8.$$

$$(p^1 \mid v_1 = v_2 = 1 - \gamma) = 1/3, EL = 1/12 - \gamma/4, EU = 1/2 - 3\gamma/4.$$

So the seller will ex ante prefer  $K = 2$  if  $r < (5 - 12\gamma)/48$ . However, the game form implements the first best for neither value of  $K$  and thus for no values of  $r \geq 0$ .

*Case 3:  $\gamma > 1/3$ , Seller-Design-Buyer-Choice.*

For  $K=2$ , the profit maximizing prices, associated efficiency losses, and profits prior to pricing costs, are

$$(p_1, p_2 \mid c_1 = c_2 = 1/3) = (1 - \gamma, 1 - \gamma), EL = \gamma/4 - 1/12, E\pi = 1/2 - 3\gamma/4$$

$$(p_1, p_2 \mid c_1 = 1/3, c_2 = 2/3) = (1 - \gamma, 1) \text{ if } \gamma \in (1/3, 4/9], EL = \gamma/2 - 1/6, E\pi = 1/3 - \gamma/2$$

$$= (\gamma, 1) \text{ if } \gamma \in (4/9, 1/2], EL = 0, E\pi = \gamma - 1/3$$

$$(p_1, p_2 \mid c_1 = c_2 = 2/3) = (1, 1), EL = 0, E\pi = 0.$$

If  $K = 1$ , the profit maximizing price, associated efficiency losses, and profits prior to pricing costs, are

$$(p^1 \mid c_1 = c_2 = 1/3) = 1 - \gamma \text{ if } \gamma \in (1/3, 10/21], EL = \gamma/4 - 1/12, E\pi = 1/2 - 3\gamma/4$$

$$= \gamma \text{ if } \gamma \in (10/21, 1/2], EL = 0, E\pi = \gamma - 1/3$$

$$(p^1 \mid c_1 = 1/3, c_2 = 2/3) = 1 - \gamma, EL = 1/24 - \gamma/8, E\pi = 1/2 - \gamma$$

$$(p^1 \mid c_1 = c_2 = 2/3) = 1, EL = 0, E\pi = 0$$

So the seller will ex ante prefer  $K=2$  if  $r < (3\gamma - 1)/12$  and  $\gamma \in (1/3, 4/9]$ ,  $r < \gamma - 1/3$  and  $\gamma \in (4/9, 10/21]$ , and if  $r < (27\gamma - 10)/48$  and  $\gamma \in (10/21, 1/2]$ . However, the game form implements the first best for neither value of  $K$  and thus for no values of  $r \geq 0$ .

*Case 4:  $\gamma > 1/3$ , Buyer-Design-Seller-Choice*

For  $K=2$ , the utility maximizing prices, associated efficiency losses, and utilities prior to pricing costs, are:

$$(p_1, p_2 \mid v_1 = v_2 = \gamma) = (1/3, 1/3), EL = 0, EU = 3\gamma/4 - 1/4$$

$$(p_1, p_2 \mid v_1 = \gamma, v_2 = 1 - \gamma) = (1/3, 1/3 + \epsilon), EL = 0, EU = 1/4 - \gamma/4$$

$$(p_1, p_2 \mid v_1 = v_2 = 1 - \gamma) = (1/3, 1/3), EL = 0, EU = 1/2 - 3\gamma/4.$$

If  $K = 1$ , the utility maximizing price, associated efficiency losses, and utilities prior to pricing costs, are

$$(p^1 \mid v_1 = v_2 = \gamma) = 1/3, EL = 0, EU = 3\gamma/4 - 1/4$$

$$(p^1 \mid v_1 = \gamma, v_2 = 1 - \gamma) = 1/3, EL = 0, EU = 1/8$$

$$(p^1 \mid v_1 = v_2 = 1 - \gamma) = 1/3, EL = 0, EU = 1/2 - 3\gamma/4$$

So the buyer will ex ante prefer  $K = 2$  if  $r < (1 - 2\gamma)/8$ . However, the game form implements the first best for either value of  $K$  and thus for all values of  $r \geq 0$ .

The desired result then follows by combining the results from the four cases.

Q.E.D.

## REFERENCES

Aghion, Philippe, and Jean Tirole, "Formal and Real Authority in Organizations", *Journal of Political Economy*, 107, February, pp. 1-29, 1997.

Bertini, Marco, Frederick, Shane, and Duncan Simester "The Psychological and Economic Costs of Fourth Degree Price Discrimination: Why Chocolate Lovers Hate Vanilla Discounts", Working Paper, M.I.T., 2006.

Blinder, Alan S., Elie R. D. Canetti, David E. Lebow, and Jeremy B. Rudd, Asking about Prices: A new Approach to Understanding Price Stickiness, New York, NY: The Russell Sage Foundation, 1998.

Bradley, Peter, "Juggling Tasks: It is just Another Buying Day", *Purchasing*, Available from the author on request.

*Broadcast Music Inc. vs. Columbia Broadcast System*. 441 U.S. Report, Washington DC: US Government Printing Office, 1978.

Busse, Meghan, Jorge Silva-Risso, and Florian Zettelmeyer, "\$1000 Cash Back: The Pass-Through of Auto Manufacturer Promotions", *American Economic Review*, 96, no. 4, September, pp. 1253-70, 2006.

Chernev, Alexander, "Differentiation and Parity in Assortment Pricing", *Journal of Consumer Research*, 33, September, pp. 199-210, 2006.

Coase, Ronald H., "The Nature of the Firm", *Economica*, New Series, 4, no. 16, pp. 386-405, 1937.

Gersho, Allen, and Robert M. Gray, Vector Quantization and Signal Compression, Boston, MA: Kluwer Academic Publishers, 1991.

Hart, Oliver D., and John Moore, "Contracts as Reference Points", forthcoming, *Quarterly Journal of Economics*, 123, 2008.

Levy, Daniel; Mark Bergen, Shantanu Dutta, and Robert Venable, "The Magnitude of Menu Costs: Direct Evidence from Large U.S. Supermarket Chains", *Quarterly Journal of Economics*, 112, August, pp. 791-825, 1997.

MacQueen, James B., "Some Methods for classification and Analysis of Multivariate Observations", *Proceedings of 5-th Berkeley Symposium on Mathematical Statistics and Probability*, L. M. Le Cam & J. Neyman (eds.), Berkeley, CA: University of California Press, 1:281-297, 1967.



Mankiw, Gregory N., “Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly”, *Quarterly Journal of Economics*, 100, May, pp. 529-39, 1985.

Mussa, Michael, and Sherwin Rosen, “Monopoly and Product Quality”, *Journal of Economic Theory*, 18, August, pp. 301-17, 1978.

Rubinstein, Ariel, “Perfect Equilibrium in a Bargaining Model”, *Econometrica*, 50, No. 1, January, pp. 97-109, 1982.

Seim, Katja, and V. Brian Viard, “The Effect of Entry and Market Structure on Cellular Pricing Tactics”, Working Paper, University of Pennsylvania, 2006.

Simester, Duncan, and Marc Knez, “Direct and Indirect Bargaining Costs and the Scope of the Firm”, *Journal of Business*, 75, April, pp. 283-304, 2002.

Watson, Joel, “Alternating-Offer Bargaining with Two-Sided Incomplete Information”, *Review of Economic Studies*, 65, July, pp. 573-94, 1998.

Wernerfelt, Birger, “On the Nature and Scope of the Firm: An Adjustment-Cost Theory”, *Journal of Business*, 70, October, pp. 489-514, 1997.

Williamson, Oliver E. *The Economic Institutions of Capitalism*, New York, NY: The Free Press, 1985.

Zbaracki, Mark; Mark Ritson, Daniel Levy, Shantanu Dutta, and Mark Bergen

"Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets", *The Review of Economics and Statistics*; 86, May, pp. 514-533, 2004.