Friction-based Control System for Seismic Energy Dissipation with Isolated Stories

By

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Abstract:

The implementation of various structural control systems, such as passive, semi-active or active is not a new concept. They are incorporated in structures to increase the performance under seismic and/or wind loading either by adding stiffness or inducing counteracting forces which dissipate energy in various ways. In order to efficiently dissipate the seismic energy with existing schemes, large structural displacements are required. However, structures that are most vulnerable to earthquakes such as low-rise relatively stiff buildings, cannot experience significant displacements. Therein lies the challenge the author attempts to address by proposing a structural scheme which can be applied to low-rise concrete buildings to efficiently dissipate seismic energy and at the same time to considerably decrease the forces in the structural members for a given seismic excitation. In this thesis the design of this new structural scheme is described and a case study is performed in order to demonstrate its effectiveness and applicability.

Thesis Supervisor: Jerome Connor
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1 Introduction

In recent years there has been a considerable amount of research on the use of structural control systems to optimize the response of buildings to various excitations. Serious efforts focused on enabling this new technology to be implemented in existing buildings and bridges and today, there are many control devices installed in civil engineering structures throughout the world. There are three categories of control: passive, semi-active and active control that all make use of various properties to reduce damage sustained from various environmental factors such as wind and earthquake loads. Control methods have been slow in their acceptance in the structural design community because the systems are often prohibitively complicated, large and expensive. Over time, however, their utility is becoming more recognized and improvements in the technology are making them more viable options in new construction and retrofits.

Three methods currently in use for structural control include friction damping, base isolation. This paper ties the two together to form a possible solution for reducing the response of a design building to an earthquake in the form of a friction pendulum system. Rather than isolating the building base, the proposed scheme isolates sections of the design structure, each four stories high, from the rest of the building. Each block is allowed to displace independently of the rest of building under some seismic load. The behavior is modeled using MATLAB in conjunction with Simulink.
2 Overview of Control in Civil Structures

Structural control systems can be divided into two major categories: passive and active control systems. Passive control systems consist of a wide variety of methods and function by increasing the damping of the structure as well as stiffness and strength.

2.1 Passive Control Systems

Passive control systems can be best described as passive energy dissipation systems and can be used both in the design of new structures and retrofits. Passive energy dissipation systems have been installed in approximately 105 buildings in North America. Some of the most common devices implemented in passive energy dissipating systems include:

- **Friction dampers** dissipate energy from the applied load by utilizing friction that develops between solid bodies sliding against each other. Examples include cross-braced friction dampers, Pall friction devices and slotted bolted connections.

- **Metallic yield dampers** provide energy dissipation through the inelastic deformation of the metals used. Typically, X-shaped plate dampers, slit dampers, shear panels and bending type of honeycombs are implemented and, apart from steel, other material are also used including shape-memory alloys and lead. The advantages of such devices include their good hysteretic behavior, low-cycle fatigue properties and their resistance to environmental temperature changes.

- There are **two viscous fluid devices** used: wall and fluid dampers. The first consists of a plate enclosed in a steel case filled with viscous fluid in which the plate can move. The latter is usually a piston inside a damper filled with highly viscous fluid, and the piston has several small holes through which the fluid can move from the one part of the damper to the other as the piston moves to dissipate energy.

- **Viscoelastic dampers** make use of materials such as glassy substances or copolymers that dissipate energy via shear deformations. Usually, viscoelastic plates are plaed in
between steel flanges and energy dissipation happens when the outer flanges are displaced relatively to the plates in the middle.

- **Tuned mass dampers** were originally applied to limit excitations caused by the wind but recently, their capability of optimizing the seismic response for a building is being examined. A major disadvantage is that the mass damper is tuned to the first mode of the structure and thus it does not sufficiently reduce the response of the structure when the dominant frequency of the excitation excites a mode other than the fundamental.
2.2 Active, Hybrid and Semi-Active Control Systems

Active, hybrid and semi-active systems evolved from traditional passive control systems and have required expertise from fields including sensing technology, data processing, computer science, material engineering, stochastic processes and many more.

- **Active mass dampers** are equipped with active control technology. **Actuators** are installed in the perimeter of the mass and apply forces on it in order to enhance seismic and wind response to ensure comfort of the occupants.

- A **hybrid mass damper** is a combination of a tuned mass damper and an active control actuator. It reduces the structural response mainly by the natural motion of the mass but also the forces employed by the actuator increase its efficiency. The energy needed for the operation of a hybrid mass damper is much less than that needed for the operation of a fully active mass damper system.

- Control methods that are based on semi-active devices consist of a combination of the best attributes of both passive and active control systems. Apart from the optimum performance they can utilize for a structure, they consume little energy compared to fully active control dampers. This means that they can operate on battery power which is critical during an event that could cause power failures.

- **Semi-active controllable fluid dampers** are another group of semi-active systems using fluids that can be controlled. Compared to the semi-active systems described above, the controllable fluid systems have the advantage of containing no moving parts apart from the piston, which makes them more reliable. Mainly two kinds of fluids are used in the development of semi-active controllable fluid dampers: electrorheological and magnetorheological fluids. These fluids have the advantage of changing from a linear viscous fluid to a semi-solid with a yield-strength that can be controlled when exposed to an electric or a magnetic field for the electrorheological and magnetorheological fluids respectively.
2.2.1 Active Control Theory

A controlling system is generally made up of three components: the monitoring system, which uses sensors to gather information on a structure, a controller which uses this data to determine the state of a structure, and actuators which respond in accordance with a control system protocol to produce the desired effect on the structure (Sangati et al., 2004). Such a control system requires an understanding of the interactions within the structure in order to create an algorithm by which the controller can determine the reaction of actuators. These control algorithms may either be static (invariant) or may be adaptive and change over time in order to optimize the control actuation in the structure. Adaptive control systems therefore have a better ability to deal with complex variable or unanticipated loads (Connor, 2002).

Figure 1 Diagram of a generic control algorithm.

Figure 1 shows a generic diagram representation of a control algorithm. The structure experiences some deformations that are represented as the excitation (for example a human being walking on a beam), and sensors record the response of the structure and send the data to controllers that activate (or not) the actuators. The actuators are made to change the structure and thus minimize the bending moment that occurs in its core.
2.3 Friction

The model proposed in this paper isolates various levels of the structure from each other, allowing for fairly large displacements. In this scheme, a large portion of the energy from the earthquake is dissipated through friction between the sliding surfaces of the control system. The friction at the interfaces needs to be designed such that desired amount of energy dissipation is achieved. The energy dissipated will be given by the area defined in the friction force-displacement hysteresis cycle.

2.3.1 Various Friction Models

(The following information is taken from the Proceedings of the 10\textsuperscript{th} Mediterranean conference on control and automation, Lisbon, Portugal, July 9-12, Experimental comparison of different friction models for accurate low-velocity tracking\textbackslash V. Lampaert, J. Swevers, F. Al-bender et. al.)

Friction is divided into two different stages, the pre-sliding stage and the sliding stage. For small displacements which correspond to the pre-sliding stage, the force generated by friction is a hysteresis function of the displacement. While for large displacements which correspond to the sliding stage, the force generated by friction is a nonlinear function of velocity. The following outlines theory for modeling friction.
2.3.1.1 Static Model

The static model is velocity $v$ dependend:

$$F_f = \sigma_2 v + \text{sign}(v) \left(F_c + (F_s + F_c) \exp \left(-\frac{|v|}{V_s}\right)\right)$$

The term "$\sigma_2 v$" represents the viscous friction force while the second term represents the Stribeck effect.

Where

- $F_s = \text{Static Force}$
- $F_c = \text{Coulomb Force}$
- $V_s = \text{Stribeck Force}$
- $\delta = \text{Shape Factor}$
- $\sigma_2 = \text{Viscous Friction Coefficient}$

Dahl Model

This model introduced by Dahl describes the pre-sliding stage of friction. The sliding stage is modeled by $F_s$ (static friction).

$$\frac{dF_f}{dx} = \sigma_0 \text{sign} \left(1 - \frac{F_f}{F_s}\right) \left|1 - \frac{F_f}{F_s}\right|^n$$

Where

- $\sigma_0 = \text{Micro-stiffness}$
- $F_s = \text{Static force}$
- $n = \text{Shape factor}$

LuGre Model

This model was developed by Canudas de Wit and combines the previous two models. Both friction stages are modeled after the same equations without using a switching function. This results to the smooth transition between both friction stages.

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{s(v)} z$$
\[ s(v) = F_c + (F_s + F_c) \exp\left(-\left|\frac{v}{V_s}\right|^\delta\right) \]

\[ F_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \]

Using a first order nonlinear differential equation and by introducing an internal state variable \( z \) the friction lag in the sliding stage, the hysteresis-like behavior in the pre-sliding stage and the varying break-away force, can be described. By introducing \( \frac{dz}{dt} = 0 \), the LuGre model reduces to the static model. By introducing \( \sigma_0 = \sigma_1 = 0, \ F_c = F_s \), the Lugre model reduces to the Dahl model with \( n = 1 \).

**Leuven Model**

This model is developed by Swevers and is a modified LuGre model. The difference lies within the pre-sliding stage, the behavior being a hysteretic function \( F_h(z) \) with nonlocal behavior. For the sliding stage both the Leuven and Lugre model share the same properties.

\[ \frac{dz}{dt} = v \left(1 - \text{sign}\left(\frac{F_h(z)}{s(v)}\right) \left|\frac{F_h(z)}{s(v)}\right|^n\right) \]

\[ s(v) = (F_s - F_c) \exp\left(-\left|\frac{v}{V_s}\right|\delta\right) \]

\[ F_f = F_h(z) + \sigma_1 \frac{dz}{dt} + \sigma_2 v \]
2.3.1.2 Friction Damping

Coulomb damping is generated by a damping force which is in phase with the deformation.

\[ F = \overline{F} \text{sgn}(\dot{u}) \]

\[ u = \dot{u} \sin \Omega t \]

\[ \overline{u} = \dot{u} \]

The above figure corresponds to periodic excitation. The work per cycle is calculated with:

\[ W_{\text{coulomb}} = 4\overline{F} \overline{u} \]

In structural damping we consider the magnitude of the damping force to be proportional to the displacement amplitude. The model is described by:

\[ F = k_s |u| \text{sgn}(\dot{u}) \]

Where \( k_s \) is a pseudo-stiffness factor. The following figure shows the cyclic response.
The energy dissipated per cycle is given by:

\[ W_{\text{structural}} = 4 \left( \frac{k_s \ddot{u}^2}{2} \right) = 2k_s \ddot{u}^2 \]
2.3.2 Implementation of Friction in Structural Control

The following figure shows one of the most widely used friction devices in structural engineering. The devices are generally implemented within X-bracing systems in framed structures and the friction pads are placed at the bolt-placed connections. The energy dissipated is equal to the work being done by the frictional moments and the relative rotation at the connections which are created by the inter-story displacement of the structure. This friction device for the case of X-bracing is placed where the two legs of the bracing connect.

![Figure 4. Friction brace damper (J. J. Connor, 2002)](image)

The following friction damper device, which can be easily installed on existing structures, has was innovated by Mualla and Belev. An analytical description of its behavior that follows an idealized hysteretic loop is shown in Figure 1.
Figure 5. Friction damper innovated by Mualla and Belev, 2002
The simplified model of the damping scheme is shown in the following figure.

![Figure 6](image)

**Figure 6. Force-deformation diagram for friction damper system (Mualla and Belev, 2002)**

The friction devices described above are widely implemented and can be easily applied in all different forms of bracing used by the industry such as X-bracing, K- or chevron bracing, diagonal bracing, knee bracing etc. as shown in Figure 7.

![Figure 7](image)

**Figure 7. Various bracing schemes.**

From the above we can see that the great majority of friction based devices that are installed in framed structures are activated by the inter-story drift of the structure. There, is can be surmised that inter-storey drift is a requirement for such friction devices to function properly. In rigid reinforced concrete buildings inter-storey drift is small.
2.4 Base Isolation Systems for Earthquake Resistance

One form of structural control is base isolation. This works by reducing the amount of interaction between two objects, for example, between machinery and its support or a building and the ground. Isolation techniques have been used for mounting mechanical systems for over seventy years, but have only recently been considered a viable solution for civil structures (Connor, 2003). The idea was first proposed in the early nineteenth century, but development of the concept for its application to civil structures began in earnest in the mid-1980’s (Connor, 2003). During an earthquake, the shaking ground interacts with the structure, causing it to move. This can cause considerable damage to the structure and lead to severe safety issues for the users. The principle of isolation involves isolating the base of a structure from the shaking ground such that the building does not feel the movements.

There are some issues concerning base isolation systems. For example, isolated structures have very low stiffnesses at their base, which makes them vulnerable to large displacements under low level loadings such as wind (Connor, 2003). If the displacements are too large, the base of the building could potentially interact with surrounding objects. The main systems that have been proposed are rubber bearings, sliding on a curved surface (as described in this paper) and sleeve piles.
2.4.1 Spherical Sliding Isolation Systems

Another method of isolation is placing columns on a spherical sliding bearing (Figure 8). During a seismic excitation, the building can slide on the curved surface, sliding both vertically and horizontally. The force needed to move the building upwards dissipates energy and the radius of the bearing can be adjusted to control the building’s vibration period.

Figure 8. Spherical sliding isolation system.
2.4.2 Friction Pendulum System (FPS)

The extension of spherical sliding systems is the friction pendulum system (FPS) which has been researched since the 1980s (Woo et al., 2009). Previous study results showed that the use of such devices for seismic isolation allowed a notable increase in the structure’s capacity to withstand earthquake forces and spherical bearings were the simplest way of achieving long periods of a structure under low gravity loads. The paper by Woo et al. cites a study by Al-Husseini et al. (1994) which focuses on a seven-storey building with an FPS. The shear forces and inter-storey drifts were reduced by factors of four to six and allowed the structural system to remain elastic during severe seismic loading (Woo et al., 2009).

Woo et al. employs an FPS for the isolation of a main control room of a nuclear plant. Their FPS uses a concave surface with low friction to support the weight of the structure which provides little resistance to sliding in the lateral direction. The structure is supported on a spherical bearing (Figure 9).

Figure 9. Schematic diagram of a friction pendulum system (Woo et al., 2009)

The horizontal force \( F \) at the point of horizontal displacement is as follows:

\[
F = \frac{W}{R} u + \mu W \text{sign}(\dot{u})
\]
Where

\( W = \text{vertical load of structure} \)

\( \mu = \text{is the friction coefficient of the sliding surface} \)

\( \dot{u} = \text{velocity at the sliding surface} \)

The equation for the natural period of the structure with the friction pendulum bearing is compared to that of a pendulum as follows:

\[
T = 2\pi \sqrt{\frac{R}{g}}
\]

Equation 2

Where

\( g = \text{ground acceleration} \)

\( R = \text{radius (see Figure 9)} \)

A similar set-up is used in the analysis that follows, utilizing the same principles of the pendulum as well as the governing equations.
3 Analysis Methodology

The system being analyzed comprises a building with inter-storey isolation. The system that isolates the stories is a pendulum friction system, inserted every third floor. In reality, this means that every third floor has a concave ceiling and the floor directly above has a concave floor. The floors with continuous columns have a particular stiffness \((k_c)\) while the FPS systems have their own, much lower stiffness \((k_r)\) that depend on the radius of curvature and the mass of structure bearing down on that particular sliding surface (as in Equation 2). Unlike the examples given in the literature review, the system employed in this paper models the entire floor or ceiling as the sliding surface, rather than just the surface between the columns.

The sliding in the curved surface aims to isolate several stories from one another, thereby reducing their absolute and relative motions, as well as reducing forces in the structural systems during a seismic event.
3.1 Formulation of Analysis

A low-rise building with the properties shown in Figure 10 was used for analysis.

For the purposes of the computer analysis, the building was modeled as a lumped mass, spring-damper system as shown in Figure 12, (where \(m_i, c_i\) and \(k_i\) represent the masses, dampers and springs, respectively).

![Diagram of the system modeled in MATLAB.](image)

Establishing the equations of motion:

\[
m \ddot{u}_i + k_i (u_i - u_{i-1}) + c_i (\dot{u}_i - \dot{u}_{i-1}) - k_{ci+1} (u_{i+1} - u_i) - c_{i+1} (\dot{u}_{i+1} - \dot{u}_i) = -m \ddot{u}_g
\]

Written in matrix form as:

\[
M \ddot{U} + C \dot{U} + KU = -M \ddot{U}_g
\]

The damping matrix \(C\) corresponds to the damping of the structure of the building and is taken as
proportional to the stiffness \( (C = \alpha K) \). The stiffnesses of the floors were calculated as follows:

\[
Sk' = M\Phi \\
K = \omega^2 k'
\]

These equations were plugged into Simulink using the state-space vector function (see Appendices A and B). An unscaled earthquake from the PEER Berkeley database was downloaded, scaled so that its maximum reaches 0.12g and plugged into the Simulink model. The model was subjected to this earthquake and the plot obtained was used as a reference to which the controlled structure would later be compared. The goal was then to decrease the acceleration and inter-storey displacement and observe the behavior. This was achieved using the theory of the friction pendulum method (described above) and simulated by including a stiffness \( k_r \) every three floors as shown in Figure 12.

Figure 12. Model of the building with the new stiffness added every three floors.

This change made the model more complicated as it added three degrees of freedom, making the total fifteen degrees of freedom. To cope with such a problem, two Simulink models were used.
(see Appendix B), one controlled and one uncontrolled. The stiffness of the FPS was taken as:

$$ k_r = \frac{nmg}{R} $$

Equation 6

Where

- \( n \) = the number of stories above the given FPS
- \( m \) = the mass of one storey
- \( R \) = the radius of the curved floor

For a given FPS, the mass in Equation 6 is equal to the total mass of the floors above that point. \( R \) was taken to be equal to 100m, which corresponds to a deflection of 0.3m at mid-span of the floor. The slabs of each FPS slide against each other during excitation, giving rise to significant amounts of friction, which is included in the MATLAB model as:

$$ F_r = \mu mg $$

Equation 7

Where

- \( g \) = gravity (9.81m/s^2)
- \( \mu \) = coefficient of friction

Comparing to the literature (Equation 1), the force is written as

$$ F = k_r u + (\mu mg) \text{sign}(\dot{u}) $$

Equation 8

As a result, the equation of motion for the 4th degree of freedom (which is the top half of the 3rd floor) becomes:

$$ m \ddot{u}_i + k_{ri} (u_i - u_{i-1}) - F_r - k_{ci+1} (u_{i+1} - u_i) - c_{i+1} (\dot{u}_{i+1} - \dot{u}_i) = -m\ddot{u}_g $$

Equation 9

This equation is extended to the three cuts in the building; modeling the buildings as such effectively isolates blocks of three stories from one another, while the friction caused by sliding dissipates energy thereby damping oscillations caused by the earthquake. The MATLAB code and Simulink models were used to plot inter-storey displacements and accelerations and are shown in Appendices A and B.
4 Results

![Graph showing acceleration of top floor versus time.](image)

Figure 13. Acceleration of the top floor versus time.

This plot shows the acceleration of the top floor of the uncontrolled system (magenta) and the controlled with and without the inclusion of friction on the sliding surface (black and blue, respectively). It is clear that having the FPS significantly reduces the acceleration of the top floor. However, the acceleration of the top floor increases slightly when friction is included in the model.
Figure 14. Inter-story displacement of column above the first FPS with $\mu=0.05$ (black).

Figure 15. Inter-story displacement of column above the first FPS with $\mu=0.01$ (black).
Figure 14 plots the inter-storey displacement for the first continuous column directly above the lowest FPS with and without friction (black and blue, respectively). The coefficient of friction is equal to 0.05. Figure 15 shows the same plot but with the coefficient of friction equal to 0.01. Comparing these two plots shows that decreasing the coefficient of friction at the sliding surface decreases the inter-storey displacement.

Figure 16 shows the inter-slab displacement at the lowest FPS with friction (black) and without friction (blue). As expected, there is much less displacement between the two halves of the FPS i.e. between the top of the third floor and the bottom of the fourth floor when friction is included.

![Inter-slab Displacement of Lowest FPS](image_url)

**Figure 16.** Inter-storey displacement of the first FPS with $\mu = 0.05$ (black).
Figure 17 Displacement of First Floor vs. Time

Figure 17 shows that the displacement of the first floor is significantly reduced with the presence of the FPS.

Figure 18. Displacement of all floors relative to the ground displacement.

Figure 18 shows the displacements of every floor relative to ground motion. The colors indicate the degrees of freedom.
Figure 19. Hysteretic behavior of the friction forces versus displacement.

Figure 19 shows the cyclic behavior of friction force of the FPS, which can be compared to the theoretical behavior shown in Figure 2 which shows the behavior of Coulomb damping.
5 Discussion

Figure 13 shows that the friction pendulum is effective in mitigating accelerations imposed by ground seismic excitation for the given input earthquake. When friction is included, the acceleration at the top floor is increased. This is due to the fact that the high friction coefficient drastically reduces the inter-storey displacement, effectively resulting in a system which is closer to the uncontrolled structure. Hence, accelerations at the top floor are higher as they absorb the seismic forces and act as the energy-dissipating mechanism.

This idea is further clarified in Figure 14, which shows the displacement of the continuous column directly above the first FPS with a coefficient of friction equal to 0.05. When friction is included, the relative column displacement is far higher than when the base of that block of floors is isolated and allowed to displace freely (shown in the same figure in blue). In the case where friction is included, forces are transferred up through the column, causing the columns to displace to dissipate energy.

Clearly, the friction coefficient between the sliding surfaces is critical to the effectiveness of the system: if the friction is too high, the seismic forces and corresponding displacements are transferred to upper parts of the building, which defeats the purpose of installing the system. However, by carefully selecting the amount of friction between the sliding surfaces, energy could be dissipated while transferring less to the upper floors. Figure 15 shows the inter-storey displacement of the column above the FPS with $\mu = 0.01$ (compared to $\mu = 0.05$ in Figure 14). It can be seen that the displacement of the column is less with the lower friction coefficient, due to the fact that the entire block of floors is allowed to displace more freely. This is a more desirable result as less forces are transferred into the upper parts of the building and hence, less shear force.

Figure 18 shows the displacements of every degree of freedom relative to ground motion. The four isolated blocks are indicated. Floors 1-3 barely move relative to the ground, meaning that they move together with the ground. Two blocks that are next to each other can be seen to move out of phase.
The hysteretic behavior demonstrated in Figure 19 can be compared to the shape Figure 2 and demonstrates that the friction works as expected.
6 Conclusions

From the inception of this structural scheme it became clear that a specific amount of stiffness would be needed between the levels of discontinuity to avoid performing analysis for an unstable system. This can be achieved in various ways, but for this case study we chose the implementation of pedulum systems. The simulation carried out in this paper shows that friction pendulum damper systems implemented as inter-storey isolation devices could be an effective way of mitigating high forces in a building subjected to seismic excitation. However, the success of such a system is strongly dependent on the amount of friction produced in the sliding system, as higher friction forces cause the columns above to absorb a higher amount of seismic forces. The radius of curvature, which directly affects the stiffness of the system, also needs to be carefully controlled.

By increasing the friction forces, the relative displacements at the levels of discontinuity are limited but at the same time the forces in the structural members due to seismic excitation are increased. That means that for a given excitation, the maximum displacements at the levels of discontinuity our structure can withstand should be defined before we can proceed towards the optimization of the friction factors. In addition, this structural scheme limits the possibility of floor mechanism creation under a seismic event with higher maximum acceleration than the one it was designed against and our attention is focused more on P-Delta effects caused by larger displacements.

There are also several practical implications of such a system that may prevent its implementation. Specially designed flexible mechanical systems would need to be installed that could move with the building without being damaged. Furthermore, the movement presents problems for elevator shafts, which are (at present) required to be rigid. In addition, the economic implications must be assessed. On one hand, there could be savings in materials as the structural system has smaller forces to resist to. On the other hand, base isolation systems are expensive and this is exacerbated by needing several throughout the building’s elevations. The system would also have to be carefully tuned so that displacements at the FPS levels are not too large and potentially cause the blocks to topple off one another. In addition, the curved floors and ceiling could present issues of serviceability for the specific case of friction pendulums systems.

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Further studies should include tests of larger earthquakes as well as harmonic excitations to assess the behavior of the individual blocks. A more in-depth analysis should be made of the optimal number of FPS installed within a structure of a given number of stories high, as well as the most advantageous stiffness values between the levels of discontinuity.

Further research should focus on the development of more efficient systems that can provide stiffness between the levels of discontinuity, the development of friction surfaces that will be able to be mass produced and at the same time provide the friction forces demand each design dictates and the development of mechanical systems that will remain operable under the significant displacements induced at the levels of isolation.
7 References

Connor, Jerome J., “Introduction to Structural Motion Control”, 2002


Constantinou, M. C., “Friction Pendulum Double Concave Bearing”, University of Buffalo, New York, 2004
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Wang, Y., “Introduction to Passive, Active, and Semi-Active Structural Control”, Georgia Institute of Technology.
http://people.ce.gatech.edu/~ywang/courses/adv_dyn_smt_struct/Smart_Structures_L16.pdf

Woo, B. K. et al., “Application of Friction Pendulum System to the Main Control Room of a Nuclear Power Plant”, Canadian Journal of Civil Engineering, 2009, 36:63-72, 10.1139/L08-111

Earthquake Protection Systems
http://www.earthquakeprotection.com/triple_pendulum_bearing.html

Advanced Earthquake Resistant Design
http://mceer.buffalo.edu/infoservice/reference_services/advEQdesign.asp
Appendix A
The following is the MATLAB code used for the simulation.

```matlab
clear all
clc

H = 36; %m
m = 5*10^4; %kg
umax = H/350; %m
g=9.81;
R=100;

M = m*eye(12);
phi = umax/12*[1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12];
S1=[1 -1 0 0 0 0 0 0 0 0 0 0;
    0 1 -1 0 0 0 0 0 0 0 0 0;
    0 0 1 -1 0 0 0 0 0 0 0 0;
    0 0 0 1 -1 0 0 0 0 0 0 0;
    0 0 0 0 1 -1 0 0 0 0 0 0;
    0 0 0 0 0 1 -1 0 0 0 0 0;
    0 0 0 0 0 0 1 -1 0 0 0 0;
    0 0 0 0 0 0 0 1 -1 0 0 0;
    0 0 0 0 0 0 0 0 1 -1 0 0;
    0 0 0 0 0 0 0 0 0 1 -1 0;
    0 0 0 0 0 0 0 0 0 0 1 -1;];
S = umax/12*S1;

Kval = S\M*phi*4*pi^2; %Kval = K'w^2, where w = 2*pi
K=zeros(12);

% assembles big K matrix
for i = 1:12
    if i == 1
        K(i,i)=Kval(1)+Kval(2);
        K(i,i+1)=-Kval(2);
    elseif i == 12
        K(i,i-1)=-Kval(12);
        K(i,i)=Kval(12);
    else
        K(i,i-1)=-Kval(i);
        K(i,i)=Kval(i)+Kval(i+1);
        K(i,i+1)=-Kval(i+1);
    end
end

% values for the classic building
alpha = 2*0.02/(2*pi);
C = alpha*K;

Bg = [zeros(12,1); ones(12,1)]*1.2;

Ap = [zeros(12) eye(12); -inv(M)*K -inv(M)*C];
Bp = eye(24);

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\[ C_p = \text{eye}(24); \]
\[ D_p = \text{zeros}(24); \]
\[ \text{INTER1} = [S1' \ \text{zeros}(12) ; \ \text{zeros}(12, 24)]; \]
\[ \% \text{values for the new building} \]
\[ M_1 = [m \ 0 \ 0 \ 0; \ 0 \ m \ 0 \ 0; \ 0 \ 0 \ m/2 \ 0; \ 0 \ 0 \ 0 \ m/2]; \]
\[ \text{Mnew} = [M_1 \ \text{zeros}(4, 11); \ \text{zeros}(4) \ M_1 \ \text{zeros}(4, 7); \ \text{zeros}(4, 8) \ M_1 \ \text{zeros}(4, 3); \ \text{zeros}(3, 12) \ m*\text{eye}(3)]; \]
\[ \% \text{assemble big K matrix} \]
\[ \text{Knew} = [K_{\text{val}}(1)+K_{\text{val}}(2) -K_{\text{val}}(2) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ -K_{\text{val}}(2) \ K_{\text{val}}(2)+K_{\text{val}}(3) -K_{\text{val}}(3) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ -K_{\text{val}}(3) \ K_{\text{val}}(3)+10*m^2/g/R -10*m^2/g/R 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ -10*m^2/g/R 10*m^2/g/R+K_{\text{val}}(4) -K_{\text{val}}(4) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ -K_{\text{val}}(4) \ K_{\text{val}}(4)+K_{\text{val}}(5) -K_{\text{val}}(5) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(5) \ K_{\text{val}}(5)+K_{\text{val}}(6) -K_{\text{val}}(6) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(6) \ K_{\text{val}}(6)+7*m^2/g/R -7*m^2/g/R 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(7) \ K_{\text{val}}(7)+K_{\text{val}}(8) -K_{\text{val}}(8) \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(8) \ K_{\text{val}}(8)+K_{\text{val}}(9) -K_{\text{val}}(9) \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(9) \ K_{\text{val}}(9)+K_{\text{val}}(10) -K_{\text{val}}(10) \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(10) \ K_{\text{val}}(10)+K_{\text{val}}(11) -K_{\text{val}}(11) \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(11) \ K_{\text{val}}(11)+K_{\text{val}}(12) -K_{\text{val}}(12); \]
\[ \text{Cnew} = \text{alpha}*[K_{\text{val}}(1)+K_{\text{val}}(2) -K_{\text{val}}(2) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ -K_{\text{val}}(2) \ K_{\text{val}}(2)+K_{\text{val}}(3) -K_{\text{val}}(3) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ -K_{\text{val}}(3) \ K_{\text{val}}(3)+10*m^2/g/R -10*m^2/g/R 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ -10*m^2/g/R 10*m^2/g/R+K_{\text{val}}(4) -K_{\text{val}}(4) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ -K_{\text{val}}(4) \ K_{\text{val}}(4)+K_{\text{val}}(5) -K_{\text{val}}(5) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(5) \ K_{\text{val}}(5)+K_{\text{val}}(6) -K_{\text{val}}(6) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(6) \ K_{\text{val}}(6)+7*m^2/g/R -7*m^2/g/R 0 \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(7) \ K_{\text{val}}(7)+K_{\text{val}}(8) -K_{\text{val}}(8) \ 0 \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(8) \ K_{\text{val}}(8)+K_{\text{val}}(9) -K_{\text{val}}(9) \ 0 \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(9) \ K_{\text{val}}(9)+K_{\text{val}}(10) -K_{\text{val}}(10) \ 0 \ 0 \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(10) \ K_{\text{val}}(10)+K_{\text{val}}(11) -K_{\text{val}}(11) \ 0; \]
\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -K_{\text{val}}(11) \ K_{\text{val}}(11)+K_{\text{val}}(12) -K_{\text{val}}(12); \]
\[ \text{Bgl} = [\text{zeros}(15,1); \ \text{ones}(15,1)]*1.2; \]
\[ \text{Ap1} = [\text{zeros}(15) \ \text{eye}(15); -\text{inv}(\text{Mnew})*\text{Knew} -\text{inv}(\text{Mnew})*\text{Cnew}]; \]
\[ \text{Bpl} = \text{eye}(30); \]
\[ \text{Cpl} = \text{eye}(30); \]
\[ \text{Dpl} = \text{zeros}(30); \]
\[ \text{S2} = [1 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \]

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\[ Bf = [\text{zeros}(15); -S2]*1/m; \]

\[ \text{INTER2} = [S2' \text{zeros}(15); \text{zeros}(15,30)]; \]

\[ \text{load earthquake.AT2}; \]
\[ X1 = \text{earthquake}(:,1); \]
\[ X2 = \text{earthquake}(:,2); \]
\[ X3 = \text{earthquake}(:,3); \]
\[ X4 = \text{earthquake}(:,4); \]
\[ X5 = \text{earthquake}(:,5); \]

\[ \text{for } i=1:822 \]
\[ \quad Y(5*i-4) = X1(i); \]
\[ \quad Y(5*i+1-4) = X2(i); \]
\[ \quad Y(5*i+2-4) = X3(i); \]
\[ \quad Y(5*i+3-4) = X4(i); \]
\[ \quad Y(5*i+4-4) = X5(i); \]
\[ \text{end} \]

\[ X = 0:.01:41.09; \]

\% uncontrolled response
\[ [t\_unc, x\_unc, y\_unc] = \text{sim('MBDHW6project1.mdl',50)}; \]

\% controlled response no friction
\[ \text{friction on}=0; \]
\[ [t\_nf, x\_nf, y\_nf] = \text{sim('MBDHW6project2.mdl',50)}; \]

\% controlled response
\[ \text{friction on}=1; \]
\[ [t, x, y] = \text{sim('MBDHW6project2.mdl',50)}; \]

\text{figure;}
\%\text{plot(t\_unc,y\_unc(:,24),'m'); hold all}
\%\text{plot(t\_nf,y\_nf(:,30),'b'); hold all}
\%\text{plot(t,y(:,30),'k');}
\%\text{plot(t\_unc,y\_unc(:,25),'m'); hold all}
\text{plot(t\_nf,y\_nf(:,16),'m'); hold all}
\text{plot(t\_nf,y\_nf(:,23),'b'); hold all}
\text{plot(t\_nf,y\_nf(:,30),'k'); hold all}
\%\text{plot(t,y(:,31),'k');}
\%\text{plot(y(:,4),y(:,64),'b');}
Embedded MATLAB function (used in controlled Simulink model):

```matlab
function friction = fcn(u)

Fr=10^5;
forx=zeros(15,1);

for i=1:3
    forx(4*i) = sgn(u(15+4*i)-u(15+4*i-1))*Fr;
end

friction=forx
```

Appendix B

The following show the Simulink models used for the simulation.

Uncontrolled building:

Controlled building: