18.701 Algebra I
Fall 2007

September 29, 2007

18.701 Quiz 1

This is a modified version of last year's quiz. I changed one problem because it was an example done in class.

As a general rule, you are expected to prove your assertions. You may state and use without proof any results from lectures or from the assigned reading, unless asked to prove it here.

1. (15 *points*) Let H be a subgroup of a group G, and let a relation be defined by $a \sim b$ if $b^2a^{-2} \in H$. Decide whether or not this is an equivalence relation.

2. (15 *points*) What orders occur among the elements of the symmetric group S_5?

3. (20 *points*) Let $F = \mathbb{F}_3$ be the field of integers modulo 3. Determine the number of (ordered) bases of F^2.

4. (20 *points*) Let a be an element of a group G, and assume that $a \neq 1_G$.

(i) Under what circumstances is the set of two elements $S = \{1, a\}$ a subgroup of G?

(ii) Suppose that S is a normal subgroup of G. Prove that $ga = ag$ for every element $g \in G$.

5. (20 *points*) Let G denote the multiplicative group of nonzero congruence classes modulo 17.

(i) Prove directly that G is a cyclic group.

(ii) Let $\phi : G \to G'$ be a homomorphism from G to a group G' of order 6, not the trivial homomorphism (which sends every element of G to 1). Say as much as you can about the kernel and image of ϕ.