18.701 Algebra I
Fall 2007

Practice Quiz 3

(This is last year's quiz.)

1. *(15 points)* Let A, B be positive definite real symmetric matrices. Which of the following matrices are positive definite symmetric: $A^2, A^{-1}, AB, A + B$?

2. *(20 points)* Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $(1, 1, 0)^t$ and $(0, 1, 1)^t$. Determine the orthogonal projection of the vector $e_1 = (1, 0, 0,)^t$ to W.

3. *(20 points)* Let $A = R + Si$ be a hermitian matrix, with R, S real.

(i) Show that R is symmetric and that S is skew-symmetric.

(ii) Show that if A is a positive definite hermitian matrix , then R is a real positive definite symmetrix matrix.

4. *(15 points)* What does the spectral theorem for normal operators say about the conjugacy classes in the unitary group U_n?

5. *(15 points)* Determine the type of the conic $x^2 - 4xy + 4y^2 + 3x - 2y - 2 = 0$.

6. *(15 points)* Let G be the group of upper triangular real $n \times n$ matrices with diagonal entries 1. Determine the 1-parameter groups in G. Prove your assertions.