A COMPUTER-BASED OPTIMIZATION METHOD FOR PLASTIC DESIGN OF BRACED MULTI-STORY STEEL FRAMES

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ABSTRACT

A COMPUTER-BASED OPTIMIZATION METHOD FOR PLASTIC DESIGN OF BRACED MULTI-STORY STEEL FRAMES

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Submitted to the Department of Civil Engineering on October 3, 1969 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

A method is developed for plastic design of both braced and unbraced multi-story steel frames including a consideration of elastic stress and elastic deflection constraints. It is applicable to rectangular multi-story planar frames of steel with coplanar loading.

The method consists of three parts which are the plastic analysis and design part, the elastic analysis and elastic stress design part, and the elastic stiffness design part. The plastic analysis and design part for factored loads follows a story by story optimization procedure in order to determine the most favorable force distribution in the frame. The optimization procedure utilizes a gradient search technique intended to minimize material cost. In addition, by means of an iterative procedure, the so-called P-Δ effect due to gravity loads acting in the laterally displaced position of the structure is accounted for. All member proportioning is in accordance with the 1969 AISC Manual of Steel Construction. The elastic analysis and elastic stress design part for service loadings performs an 'exact' matrix stiffness analysis of the structure and redesigns members in order to satisfy imposed elastic stress constraints. Finally, the elastic stiffness design part for service loadings is executed in order to satisfy imposed elastic lateral deflection constraints. This part also utilizes a story by story gradient search optimization technique in order to minimize the material cost increase needed to satisfy the elastic deflection constraints.

A computer design system, written in the Fortran IV language, is also developed to execute the proposed design method.

The practicality and efficiency of the design method is illustrated by several example problems. The results indicate that satisfactory and economical designs may be obtained by the proposed design method.

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DEFINITIONS OF SYMBOLS

$A_B(i,j)$	= area of beam (i,j).
$A_{BR}(i,j)$	= area of panel (i,j) tension brace.
$A_{BR}(i,j,k)$	= $A_{BR}(i,j)$ for wind from left (k=2) or wind from right (k=1)
$A_C(i,j)$	= area of column (i,j).
$\overline{AK}(p_k)$	= one-level array representation of $\underset{\sim}{K}_S$.
B(i,j)	= beam (i,j).
C	= plastic design flag indicating mode of panel resistance (0.0 = moment resistance, -1.0 = truss resistance).
C(i,j)	= Column (i,j).
$C\Delta H_J$	= horizontal component of tension brace force.
$C_v(j)$	= ratio of P_2' to P_1'.
$d_b(i,j)$	= depth of beam (i,j).
$d_b'(i,j)$	= average beam depth in panel (i,j).
$d_c(i,j)$	= depth of column (i,j).
$d_c'(i,j)$	= average column depth in panel (i,j).
$\Delta(i)$	= relative story i deflection.
$\Delta(i,j)$	= panel (i,j) relative story deflection.
Δ_a	= total approximate elastic relative story deflection.

Δ_b	= elastic relative story deflection due to beam elongation and shortening effects.
Δ_{c0}	= $\Delta_c(i)$ calculated at the beginning of the elastic stiffness design.
$\Delta_c(i)$	= approximate elastic relative story i deflection due to column elongation and shortening effects.
Δ_e	= 'exact' relative story deflection.
Δ_g	= 'exact' relative story gravity sway deflection.
Δ_{s0}	= $\Delta_s(i)$ calculated at the beginning of the elastic stiffness design.
$\Delta_s(i)$	= approximate elastic relative story i deflection due to beam and column bending and tension bracing elongation.
$\bar{\Delta}$	= approximate elastic relative story deflection computed at the end of each execution of the elastic stiffness design.
ΔH_J	= total incremental shear applied to panel j.
ΔH_1	= incremental shear applied to top left joint of panel j.
ΔH_2	= incremental shear applied to top right joint of panel j.
ΔM	= change in panel top beam right joint moment.
$\Delta M'$	= change in panel beam end moment (at face of column).
ΔM_1	= ΔM due to ΔH_1.
ΔM_2	= ΔM due to ΔH_2.

$\Delta M'_B$ = change in panel beam end moment (at face of column).

ΔM_{BC} = increment of mid-span beam moment.

ΔM_{BL} = increment of left joint beam moment.

ΔM_{BR} = increment of right joint beam moment.

$\Delta M'_C$ = change in panel column end moment (at face of beam).

ΔM_T = total change in top right joint beam moment.

$(\frac{\partial f}{\partial H_J})_{CBP}$ = sub-sensitivity coefficient of columns below panel j.

$(\frac{\partial f}{\partial H_J})_{BLP}$ = sub-sensitivity coefficient of story i beams to left of panel j.

$(\frac{\partial f}{\partial H_J})_{BRP}$ = sub-sensitivity coefficient of story i beams to right of panel j.

$(\frac{\partial f}{\partial H_J})_{\text{panel member}}$ = sub-sensitivity coefficient of a panel member.

$(\frac{\partial f}{\partial M_P})_{\text{Beam}}$ = change in beam cost with respect to changes in beam plastic moment capacity.

$(\frac{\partial f}{\partial F})_{\text{Beam}}$ = change in beam cost with respect to changes in beam axial force capacity.

$(\frac{\partial f}{\partial M_P})_{\text{Column}}$ = change in column cost with respect to changes in column plastic moment capacity.

$(\frac{\partial f}{\partial F})_{\text{Column}}$ = change in column cost with respect to changes in column axial force capacity.

$(\frac{\partial f}{\partial F})_{\text{Tension Brace}}$ = change in tension brace cost with respect to changes in brace axial force capacity.

E = modulus of elasticity.

$\bar{E}$ = total error in the Δ_a calculation.

e = error in the latest approximate deflection calculation not including gravity sway effects.

F = required axial force capacity.

f = cost of all members affecting relative story deflection.

$F_B(i,j)$ = beam (i,j) axial force.

F_{BL} = beam axial force to left of panel j.

f_{BL} = cost of a single beam to left of panel j.

F_{BR} = beam axial force to right of panel j.

$F_{BR}(i,j)$ = panel (i,j) tension brace axial force.

$F_{BR}(i,j,k)$ = axial force in panel (i,j) tension brace for wind from the left (k=2) or wind from the right (k=1).

F_{CB} = column axial force below panel j.

f_{CB} = cost of a single column below panel j.

$F_c(i,j)$ = column (i,j) axial force.

f_T = cost of all member which experience force changes due to the application of ΔH_J to panel j.

$(f)_{\text{beams adjacent to panel}}$ = cost of top beams adjacent to panel j.

$(f)_{\substack{\text{columns}\\\text{below panel}}}$ = cost of columns below panel j.

$(f)_{\substack{\text{panel}\\\text{beams}}}$ = cost of beams in panel j.

$(f)_{\substack{\text{panel}\\\text{columns}}}$ = cost of columns in panel j.

$(f)_{\substack{\text{panel}\\\text{tension brace}}}$ = cost of tension brace in panel j.

$H(i)$ = lateral unfactored concentrated load applied at exterior joint of story level i from the left or right.

$H_T(i)$ = sum of factored lateral loads from story 1 down to and including story i.

$h(i)$ = height of story i.

$I_B(i,j)$ = moment of inertia of beam (i,j).

$I_c(i,j)$ = moment of inertia of column (i,j).

K = ratio of ΔH_2 to ΔH_1.

$\underset{\sim}{K}_B$ = beam stiffness matrix.

$K_B(i,j)$ = beam (i,j) stiffness $(I_B(i,j)/L(j))$.

$\underset{\sim}{K}_{BR1}$ = tension brace type 1 stiffness matrix.

$\underset{\sim}{K}_{BR2}$ = tension brace type 2 stiffness matrix.

$\underset{\sim}{K}_C$ = column stiffness matrix.

$K_c(i,j)$ = column (i,j) stiffness $(I_c(i,j)/h(i))$.

$\underset{\sim}{K}_S$ = structure stiffness matrix.

$k_{B,ij}$ = element (i,j) of beam stiffness matrix.

$k_{c,ij}$ = element (i,j) of column stiffness matrix.

$k_{s,ij}$ = element (i,j) of beam stiffness matrix.

L(j) = length of bay j.

L'(j) = clear span length of beam j.

$L_B(i,j)$ = diagonal brace length in panel (i,j).

λ = arbitrary load factor.

λ_1 = load factor for the gravity load condition.

λ_2 = load factor for the combination gravity plus wind load condition.

M = number of stories.

$\overline{M}(j)$ = sum of column joint moments at joint j in story level under consideration.

$M_{BC}(i,j)$ = beam (i,j) mid-span moment.

$M_{BC}(k,j)$ = panel j mid-span moment in top beam (k=1) or bottom beam (k=2).

$M_{BL}(i,j)$ = beam (i,j) left joint moment.

$M'_{BL}(i,j)$ = beam (i,j) left end moment (at face of column).

$M_{BL}(k,j)$ = panel j left joint moment in top beam (k=1) or bottom beam (k=2).

$M_{BP}(i,j)$ = beam (i,j) required plastic moment capacity.

$M_{BP}(k,j)$ = panel j required plastic moment capacity for top beam (k=1) or bottom beam (k=2).

$M_{BR}(i,j)$ = beam (i,j) right joint moment.

$M'_{BR}(i,j)$ = beam (i,j) right end moment (at face of column).

$M_{BR}(k,j)$ = panel j right joint moment in top beam (k=1) or bottom beam (k=2).

$M'_{BR}(k,j)$ = panel j right end moment (at face of column) in top beam (k=1) or bottom beam (k=2).

$M_{BR,max}$ = terminal value of upper right joint beam moment.

$M'_{BR,max}$ = terminal value of upper right end beam moment (at face of column).

$M_{CB}(i,j)$ = column (i,j) bottom joint moment.

$M'_{CB}(i,j)$ = column (i,j) bottom end moment (at face of beam).

$M_{CB}(j)$ = column j bottom joint moment.

$M_{CP}(i,j)$ = column (i,j) required plastic moment capacity.

$M_{CT}(i,j)$ = column (i,j) top joint moment.

$M'_{CT}(i,j)$ = column (i,j) top end moment (at face of beam).

$M_{CT}(j)$ = column j top joint moment.

M_P = required plastic moment capacity.

N = number of stories.

NIC = number of columns in a story.

NJ = total number of joints in frame not including joints at the foundation.

$P_j(i,j)$ = applied concentrated unfactored gravity joint loads.

P_y = axial member force equal to the product of member area and steel yield stress.

$P_W(i,j)$ = in plastic design: equivalent concentrated load applied at mid-span of beam (i,j).

$P_W(i,j)$ = in elastic design: applied unfactored uniform gravity beam load.

P_1' = equivalent concentrated load applied at mid-span of top beam in panel model.

P_2' = equivalent concentrated load applied at mid-span of bottom beam in panel model.

$\phi(i,j)$ = column (i,j) rigid body rotation.

$\psi(i)$ = average of story i rigid body column rotations.

$Q_B(i,j)$ = beam (i,j) deflection sensitivity coefficient.

$Q_{BR}(i,j)$ = panel (i,j) tension brace deflection sensitivity coefficient.

$Q_c(i,j)$ = column (i,j) deflection sensitivity coefficient.

$R(i)$ = sum of horizontal components of tension brace force in story i.

$\overline{R}(i,j)$ = horizontal component of tension brace force in panel (i,j).

r = non-dimensional distance from bottom column joint to column inflection point.

r_x = radius of gyration about major axis.

r_y = radius of gyration about minor axis.

ρ = mass density of steel.

$S(j)$ = in plastic design, lateral shear load applied to joint j in story level under consideration.

$S(i)$ = in elastic stiffness design, sum of unfactored wind loads from top story level down to and including story level i.

S_E = elastic member stress.

$S_T(i)$ = equivalent required shear capacity of story i.

σ_{max} = maximum elastic member stress.

σ_Y = steel yield stress.

σ_{YB} = beam steel yield stress.

σ_{YBR} = brace steel yield stress.

σ_{YC} = column steel yield stress.

θ = in plastic design, beam rotation at a plastic hinge.

$\theta(i)$ = in elastic stiffness design, story level i joint rotations or story level i rigid body beam rotations.

θ_L = left joint beam rotation.

θ_R = right joint beam rotation.

$U_B(i,j)$ = unit material cost of beam (i,j).

$U_{BR}(i,j)$ = unit material cost of panel (i,j) tension brace.

$U_C(i,j)$ = unit material cost of column (i,j).

$V_C(i,j)$ = column (i,j) joint shear.

$w(i,j)$ = uniformly applied unfactored gravity beam load.

$W(i,j)$ = uniformly applied unfactored gravity beam load.

W_D = uniformly applied dead floor load.

W_L = uniformly applied live floor load.

Z_B = beam plastic section modulus.

Z_C = column plastic section modulus.

CHAPTER 1

INTRODUCTION

1.1 Objective of this Dissertation.

As a result of many years of concentrated research both in this country and abroad plastic design of steel framed structures has become an accepted design method. For several years the American Institute of Steel Construction's Specification for the Design, Fabrication and Erection of Structural Steel for Buildings has permitted plastic design for simple one and two story frames as well as continuous beams. For such simple structures plastic design is easily amenable to hand computation and, in many respects, is more easily accomplished than the alternative elastic design. The February 1969 revision of the AISC Specification now permits plastic design of braced multistory frames and it seems likely that a subsequent revisionn will permit plastic design of unbraced multistory frames as well. Unlike the situation for continuous beams and one story frames the plastic design procedures of multistory frames are not simple. In many ways they are more complex and demanding than the alternative elastic design procedures which have been in use for some years.

The objective of this dissertation was to develop a practical and efficient computer system for the plastic design of both braced and unbraced multistory steel frames. For unbraced frames the system simply proportions individual beams and columns according to a certain optimization procedure. For braced frames the system also examines the question of where braces should be located, given that there is some flexibility in this regard. The work is computer oriented not because plastic design of such structures by hand computation is not possible, but rather because the enormous amount of data which must be processed makes a computer solution the most rational approach.

This dissertation follows and builds upon an earlier doctoral dissertation of Y. Nakamura[6]. First of all, Nakamura's dissertation contains an excellent historical review of the research efforts related to plastic design. The review will not be repeated or expanded here. More importantly, his work, while limited to unbraced frames, contains a basic organizational focus which appears sound and was followed.

1.2 Description of the Design Methodology

The computer system described herein consists of the following five parts:

1. Input of the design problem.
2. A strength design for factored ultimate loads.
3. An elastic analysis for working loads. Modification of member sizes when elastic stress limits are exceeded.
4. An elastic stiffness design if, at working loads, lateral story deflection limits are exceeded.
5. Output of design results.

Figure 1.1 shows a macro flow-chart of the overall computer system. In somewhat more detail, Figure 1.2 shows a macro flow-chart for the plastic design for factored combined load condition, Figure 1.3 shows a similar chart for the 'exact' elastic analyses and elastic stress design, and Figure 1.4 shows a chart for the elastic stiffness design.

1.2.1 Input of the Design

The units of input are kips and inches unless specifically mentioned. Details of the form of the input are given in Appendix D.

1. Geometrical Conditions.

The proposed design method considers multistory plane frames with coplanar loads as illustrated in Fig. 1.5. The story levels of the frame are numbered from top to bottom. Geometrical data to be given are as follows.

M : Number of stories, $(2 \leq M \leq 30)$.

N : Number of bays, $(2 \leq N \leq 5)$.

L(j) : Span length of bay j.

h(i) : Height of story i.

2. Loading Conditions.

The working loads applied to the frame are also shown in Fig. 1.5. Loading conditions are as follows.

i. D.L. + L.L.

ii. D.L. + L.L. + W.L.

As input data, D.L. + L.L. are uniformly distributed vertical loads on beams and concentrated vertical loads on joints. W.L. are concentrated horizontal loads at the external joints of the frame. Ultimate loads applied to the frame are calculated by the computer programs by multiplying loading conditions i and ii by the appropriate load factors.

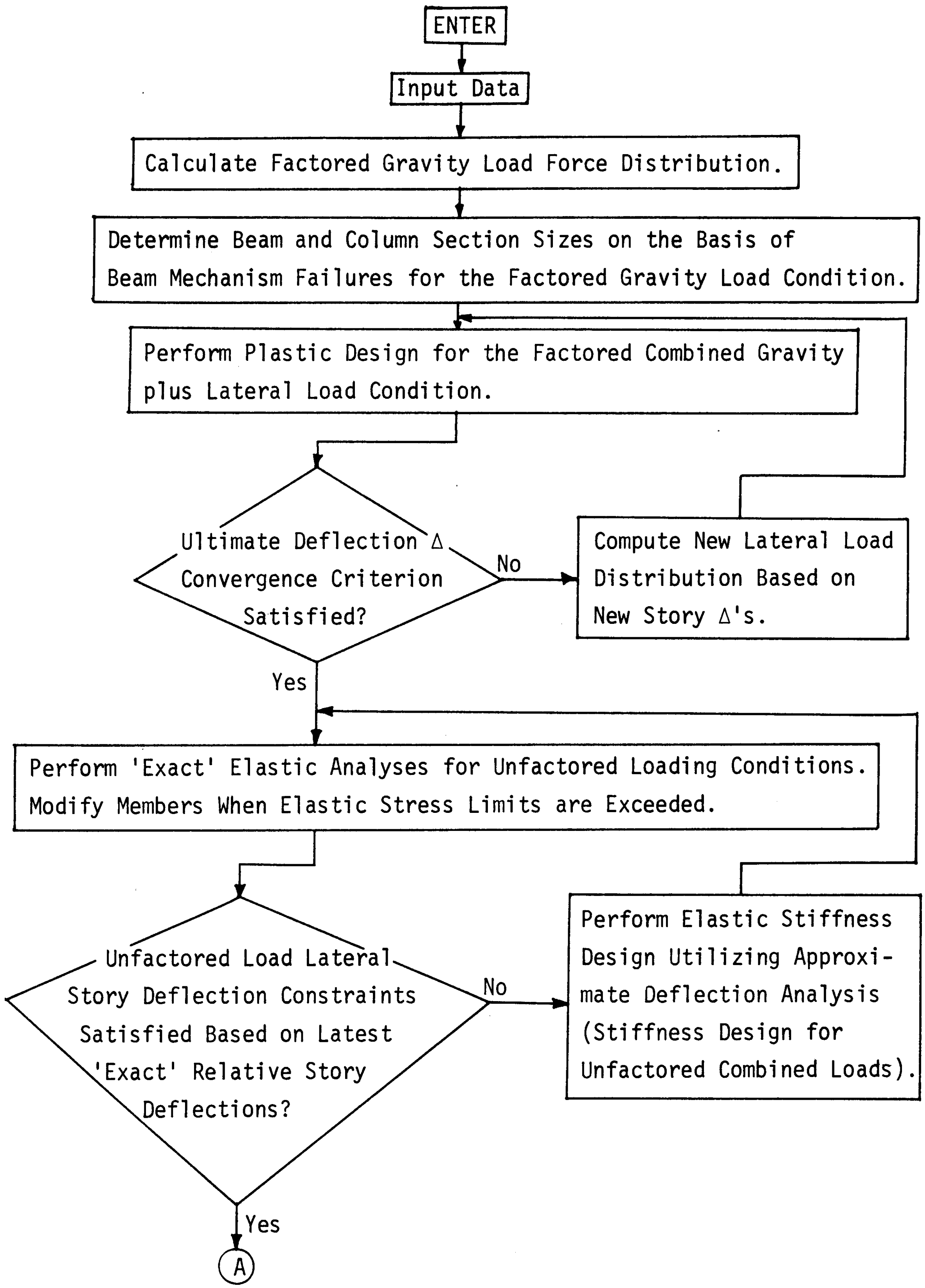


Figure 1.1 Macro Flow Chart of Total Design System

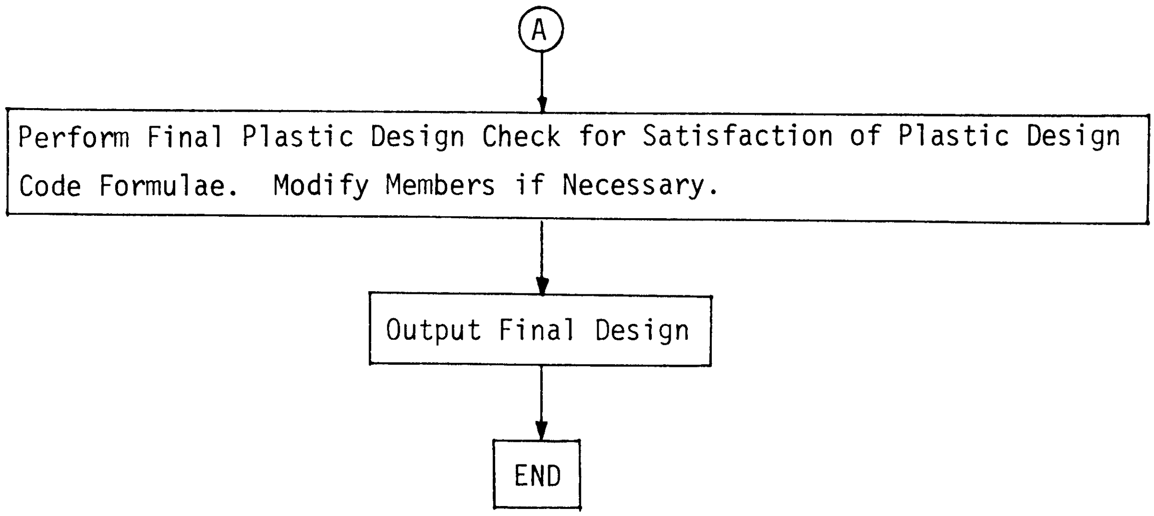


Figure 1.1 Continued

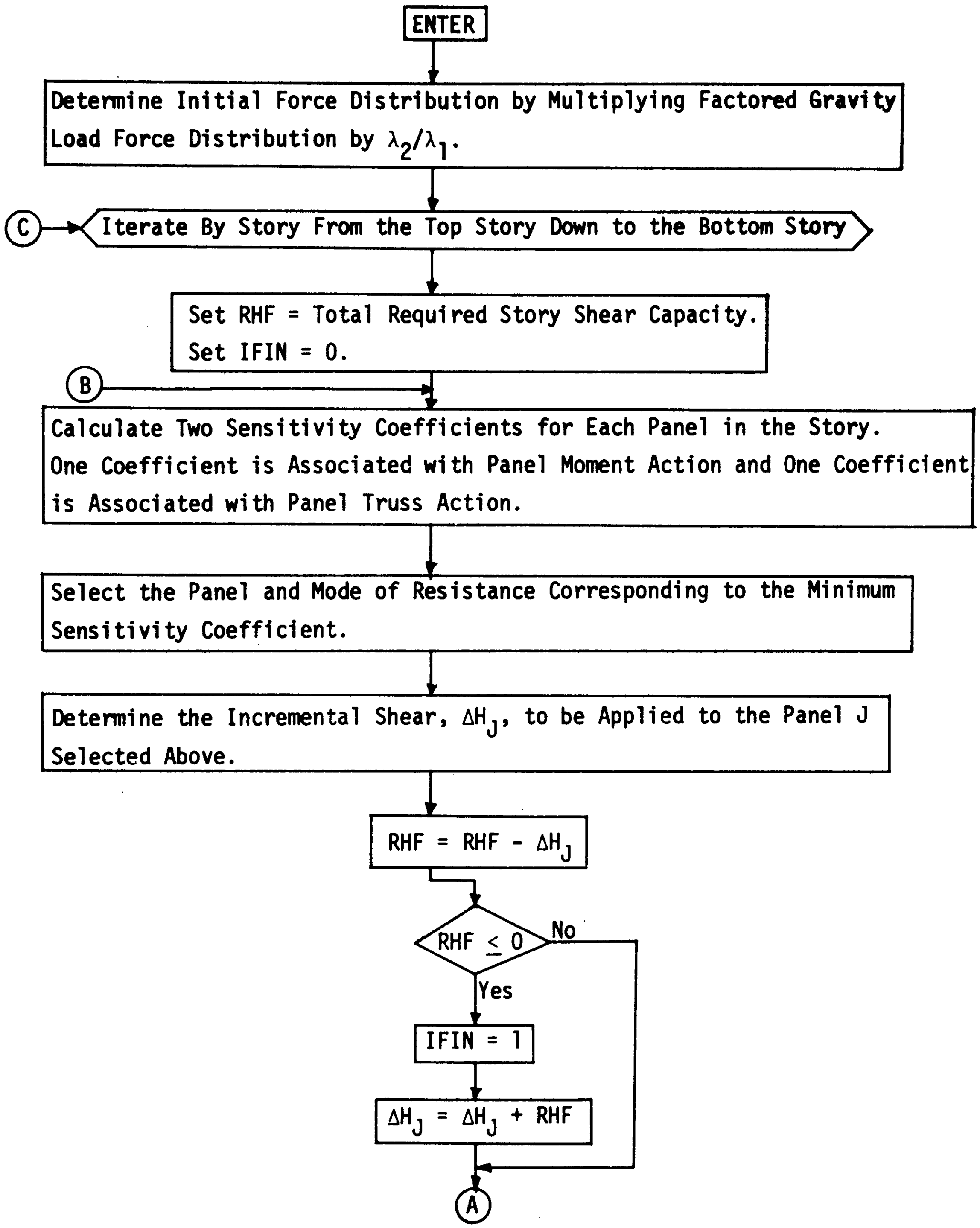


Figure 1.2 Macro Flow Chart of Plastic Design Method for Combined Load Condition

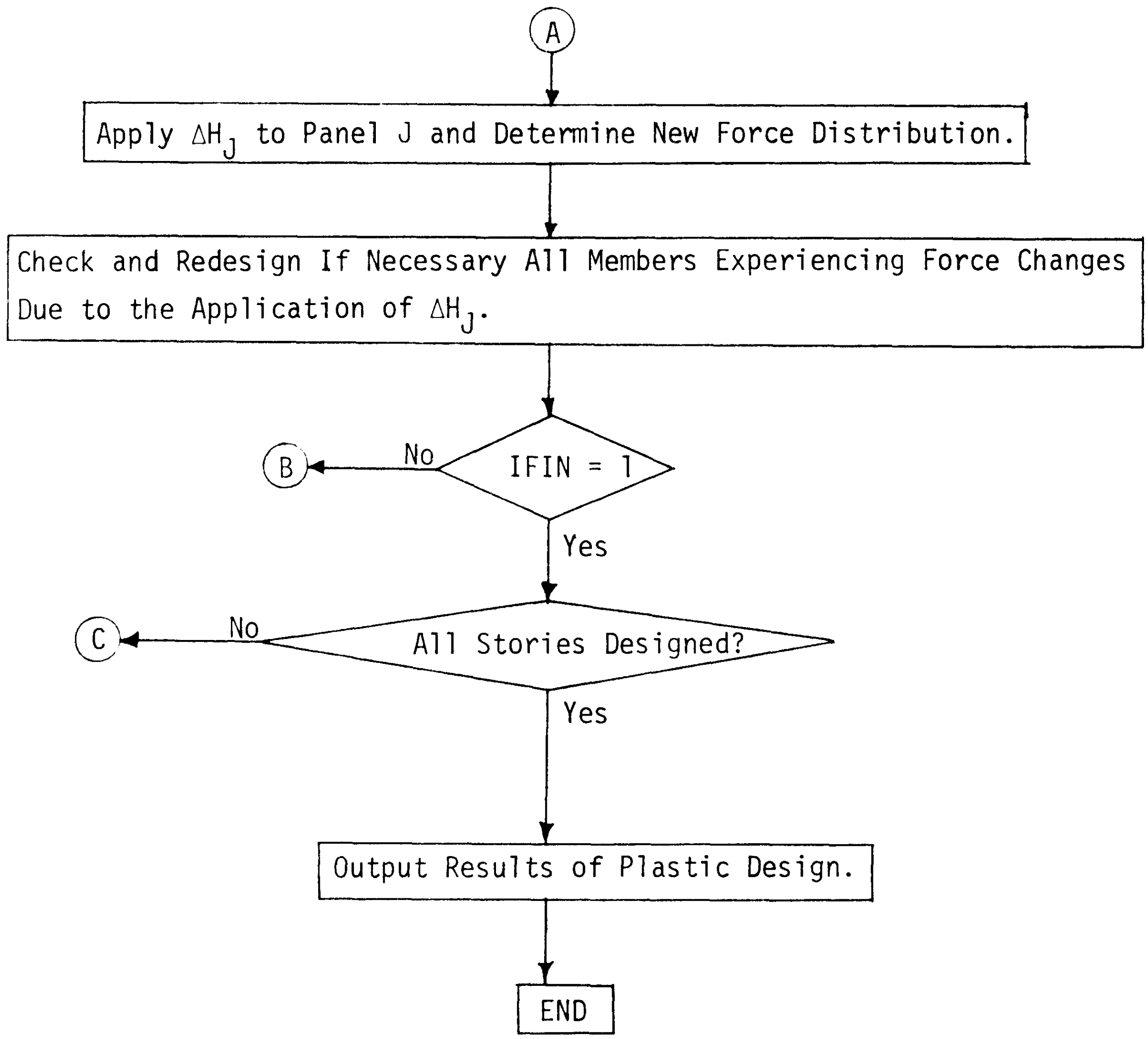


Figure 1.2 Continued

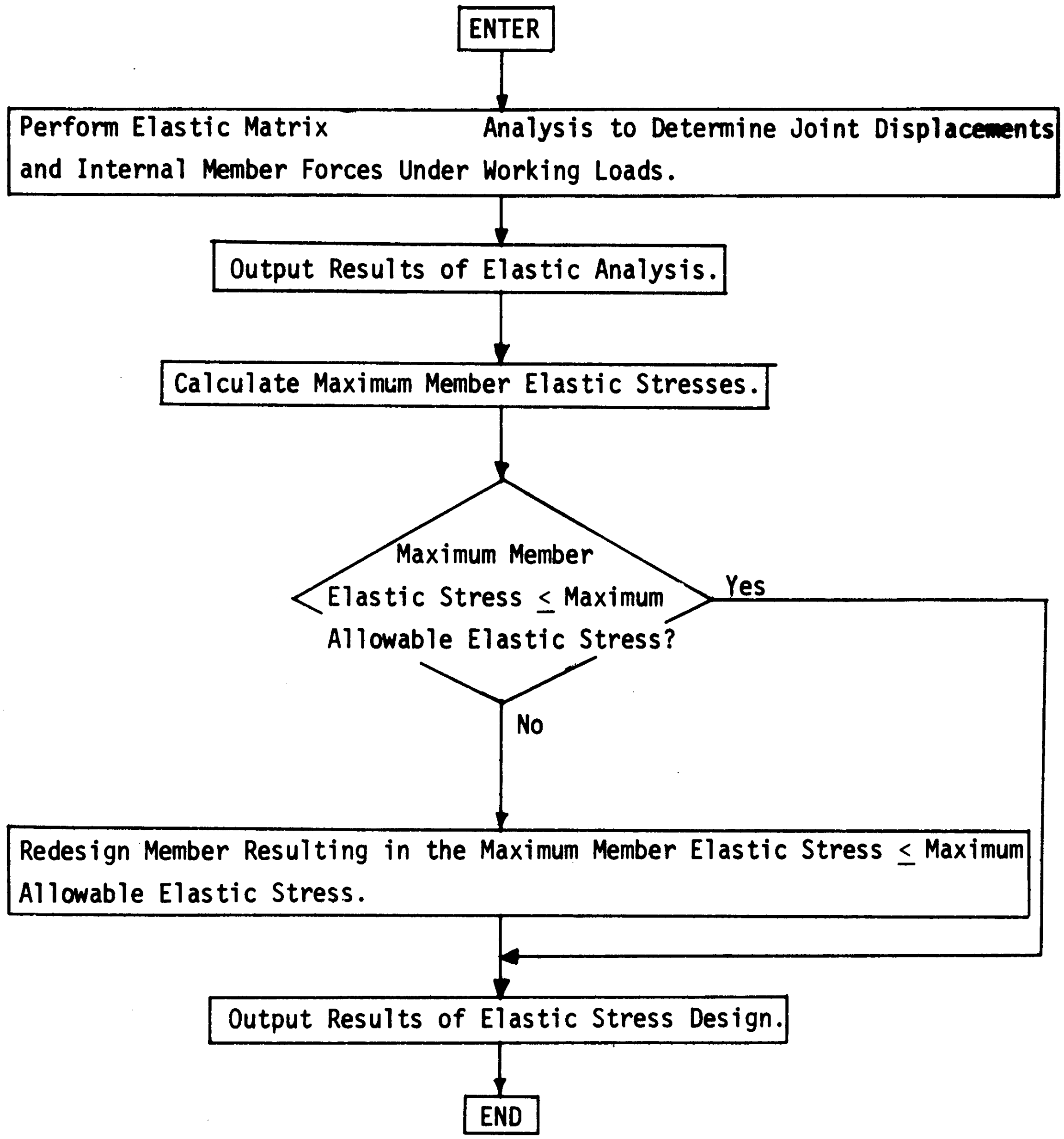


Figure 1.3 Macro Flow Chart of Elastic Stress Design Method

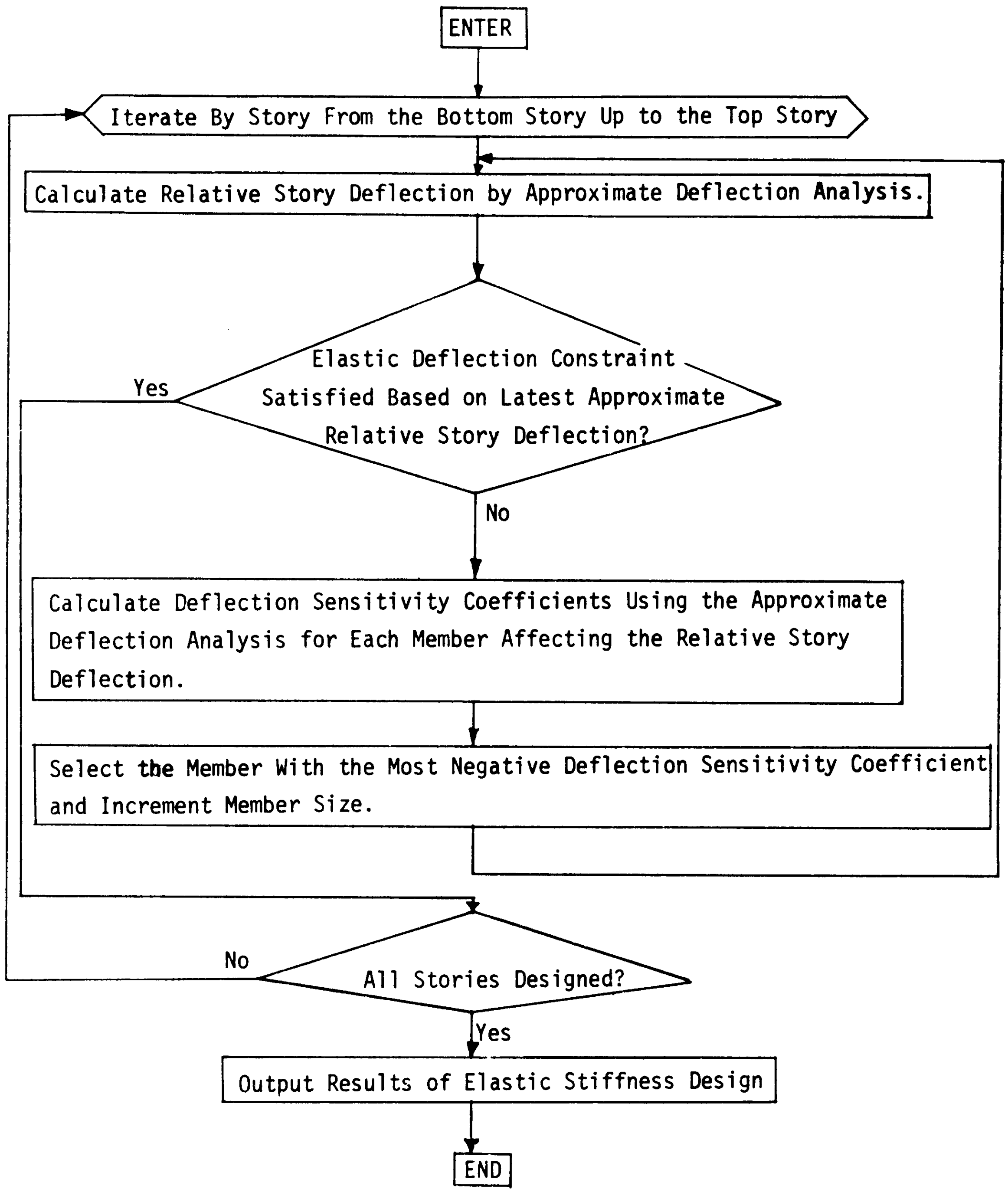


Figure 1.4 Macro Flow Chart of Elastic Stiffness Design Method

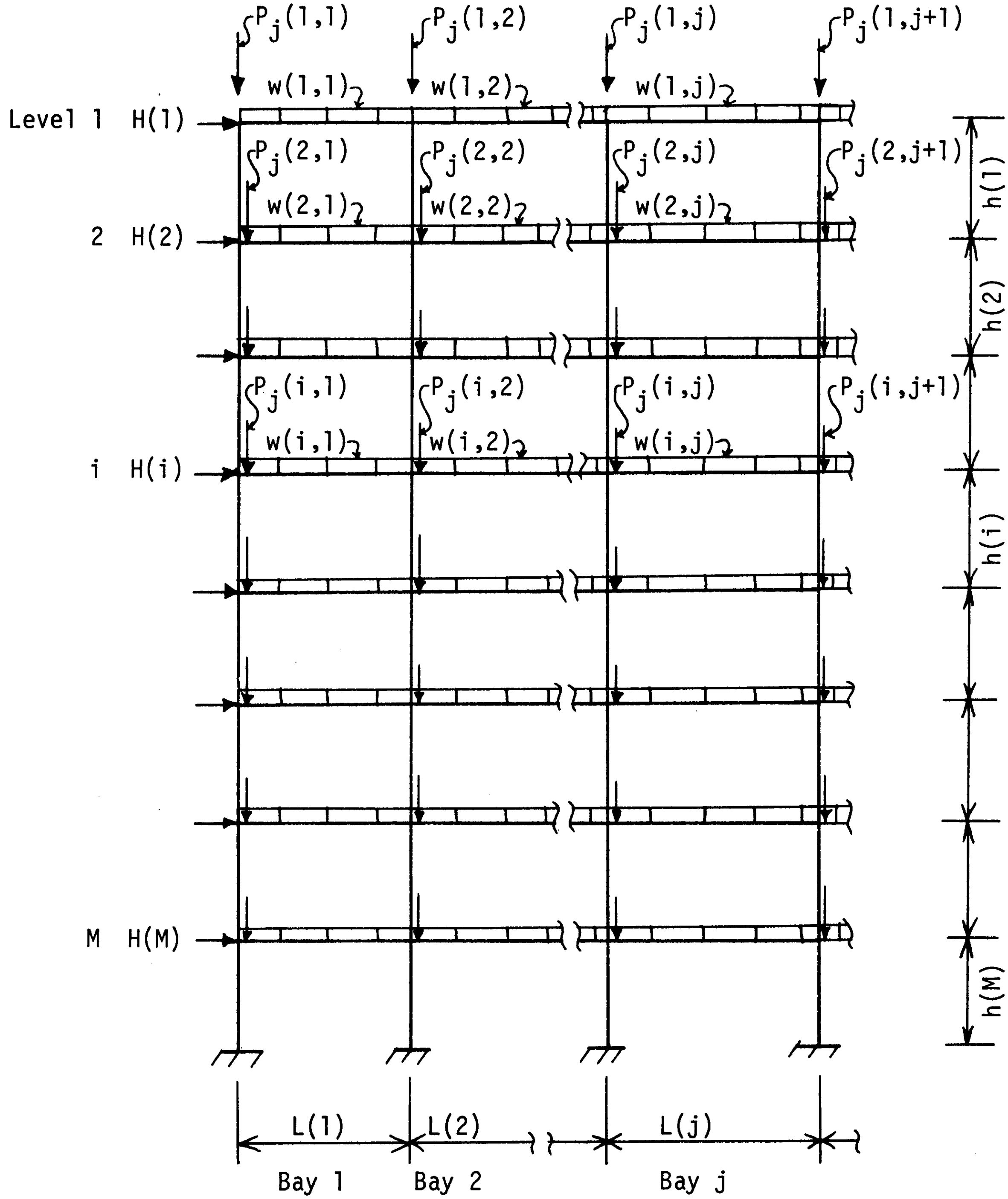


Figure 1.5 Geometrical and Loading Conditions

3. Material Properties.

Any grade of steel may be used. However, the grade of steel (i.e. A36, A441, etc.) must be input for each member in the frame including bracing elements even if bracing is not used.

Data to be given are

σ_{YB} : Yield stress of a beam. One value for each beam.

σ_{YC} : Yield stress of a column. One value for each column.

σ_{YBR}: Yield stress of a brace. One value for each set of diagonal braces in each bay.

U_B, U_C, U_{BR}: Unit material costs (cents/pound) corresponding to each of the above specified yield stresses.

4. Load Factors.

λ_1 : Load factor under D.L. + L.L.

λ_2 : Load factor under D.L. + L.L. + W.L.

5. Design Constraints.

i. Maximum permissible relative story deflections under working loads.

ii. Maximum permissible elastic member stresses under working loads.

iii. Maximum permissible beam and column depths.

iv. Actual maximum unsupported beam and column lengths with respect to out-of-plane deformation.

v. Panel codes indicating allowable modes of panel resistance.

6. Assumed Story Deflections Δ at the Collapse Mechanism.

The P-Δ effects are accounted for by an iterative procedure. Zero displacements may be input as the initial values of Δ. If this is the case the computer programs will assume initial values to be 0.0005h. However, for unbraced frames, $\Delta/h = 0.02$ may give faster convergence. For braced frames, where bracing may exist in any story, a maximum of two iterations will lead to convergence. For the braced frame case, it is advised that zero initial values of Δ be input so that two iterations are executed. The reason for this is that advantage of the joint size effect can only be taken with iterations greater than or equal to two.

7. Available Sections.

Any series of rolled sections for beams, columns, and braces may be used, although the illustrative examples use wide flange sections listed in the AISC Manual. The section tables used in the illustrative examples are given in Appendix A.

i. The beam section table consists of two parts. The first part consists of economy beam sections ordered on increasing section area without regard to beam depth constraints. This part is required. The second part, which is optional, consists of non-economy beam sections ordered on increasing plastic section modulus. This part is used in the design when beam depth constraints are critical.

ii. The column section table also consists of two parts. The first part consists of a commonly used column series ordered on increasing section area without regard to column depth constraints. This part is required. The second part, which is optional, consists of additional column sections ordered on increasing area but with depths that satisfy specific column depth constraints. This part is used in the design when column depth constraints are critical.

iii. The brace section table consists of only one part. It is simply a series of available brace section sizes ordered on increasing area.

8. Side Constraint.

The following side constraint is not input to the design system. Instead it is assumed in the design method. It is that the same column section be used in two successive stories.

1.2.2 Plastic Design.

1. Gravity Load Condition (D.L.+L.L.)

i. The gravity loads (D.L.+L.L.) are multiplied by the load factor λ_1.

ii. The moment distribution in the beams due to the factored gravity loads are calculated on the basis of beam mechanism failures.

iii. Moment and axial force distributions in the columns are calculated from force equilibrium equations. Moments for the adjoining columns at a joint are assumed to be equal.

iv. Based on the resulting force distribution, beams and columns are designed according to the 1969 AISC Manual(1). These beam and column sections represent the minimum section sizes for the design.

v. Empirical relationships between beam and column section properties are calculated by a least squares technique. These relations are used in the calculation of the sensitivity coefficients.

2. Combination Load Condition (D.L.+L.L.+W.L.).

i. Initial force distributions are calculated by multiplying the factored gravity load force distribution by the factor λ_2/λ_1.

ii. The required story shear capacity, S_T, is equal to the sum of the factored lateral loads from the top story down to and including the story under consideration plus the equivalent shear due to the P-Δ effect in the story under consideration. The equivalent P-Δ shear is calculated as the total factored gravity load column axial forces times the relative story deflection at the collapse mechanism divided by the story height.

iii. The design proceeds on a story-by-story basis beginning with the topmost story and proceeding down the frame to the bottom story.

iv. Sensitivity factors are defined as the increase in cost of a panel due to an increase in lateral shear capacity of the panel. Two sensitivity coefficients are calculated for each panel in the story. One coefficient is associated with the panel providing the next increment of shear capacity through moment changes in the beams and columns (panel moment action). The second coefficient is associated with the panel providing the next increment of shear capacity through axial force changes in the beams, columns and diagonal tension brace (panel truss action). The values of the sensitivity coefficients are functions of the geometrical condition, the current member properties, and the current state of the force distribution.

v. The panel and mode of resistance (moment action or truss action) corresponding to the minimum sensitivity coefficient is selected. The selected panel and mode of resistance will lead to the least increase in cost for an increase in lateral shear capacity.

vi. The value of the incremental shear, ΔH_J, to be applied to the panel J selected in (v) above is calculated. ΔH_J is a function of the current member properties and the current state of the force distribution.

vii. The value of ΔH_J is subtracted from the required total story shear remaining to be distributed into the story. If the difference is less than zero, ΔH_J is modified so that the difference is exactly zero.

viii. ΔH_J is applied to panel J and a new force distribution is calculated.

ix. All members experiencing force changes are checked for adequacy against the appropriate 1969 AISC code formulae and are redesigned if necessary.

x. If the remaining story shear to be resisted is greater than zero, go back to (iv) and continue in the current story. Otherwise, the design process proceeds to the next story and begins with (iv). After all stories have been designed by the above process, continue with (xi).

xi. Deflections at the collapse mechanism are calculated for each story. If these deflections satisfy the convergnece criterion such that the change in deflection is less than five per cent of its absolute value, the design process goes to (xii). Otherwise, the calculated deflections become those of the P-Δ effect in the next cycle of design which begins at (ii). The design process will continue until the deflections at the collapse mechanism satisfy the convergence condition, or the number of cycles of iteration equals the maximum number input to the computer programs. Note that the benefit of the joint size effect is realized only with the second or greater cycle number.

xii. Output results of the plastic design method. Output includes required member sizes for both the factored gravity load condition and the factored combination load condition, the force distributions under the two loading conditions, the panel shear capacity distributions, and the final total weight and material cost of the plastically designed frame.

1.2.3 Elastic Stress Design

i. Perform an elastic matrix analysis of the frame subjected to working loads in order to determine the joint displacements.

ii. From the joint displacements determined above, calculate the internal member forces.

iii. Maximum elastic member stresses are calculated on the basis of the internal member forces for all members.

iv. For each member in the frame, check if the maximum elastic member stress is less than or equal to the specified maximum allowable elastic stress. If this is the case, proceed to the next member and repeat the check. If this is not the case, the member in question is redesigned so that the elastic stress constraint is satisfied.

v. After all members are checked against the elastic stress constraint and redesigned if necessary, the results of the elastic stress design are output. Output includes the joint displacements from the stiffness analysis, internal member forces, elastic member stresses, member sizes before and after the elastic stress design, and the final total material weight and cost.

1.2.4 Elastic Stiffness Design.

i. This design part proceeds on a story-by-story basis beginning with the bottom story and proceeding up the frame to the top story.

ii. For each story, check if the 'exact' relative story deflection calculated from the matrix analysis in Section 1.2.3 is less than or equal to the specified maximum allowable relative story deflection.

iii. If the elastic relative story deflection constraint is satisfied, proceed to the next story and return to (ii). If the constraints in all stories are satisfied proceed to a final plastic design check and output of final results. If this constraint is not satisfied at any story level, member sizes must be modified in order to reduce the relative story deflection. The redesign begins with the calculation of deflection sensitivity coefficients for each member affecting the relative story deflection using an 'approximate' deflection analysis. The deflection sensitivity coefficient reflects the decrease in relative story deflection due to a unit increase in the cost of a member.

iv. Select the member with the most negative deflection sensitivity coefficient. This member will cause the maximum decrease in relative story deflection for a unit increase in cost. Increment the selected member by one section in the economy section table or to the least weight beam or column section in the non-economy section table with a moment of inertia greater than the moment of inertia of the current section.

v. Calculate the new relative story deflection by the 'approximate' deflection analysis and check the elastic deflection constraint. Return to (iii). After all stories satisfy the elastic relative story deflection constraint, the results of the elastic stiffness design are output. Output includes the final relative story deflections as calculated by the 'approximate' deflection analysis, the member sizes before and after the elastic stiffness design, and the final total material weight and cost.

vi. While the deflection constraints are now satisfied according to the 'approximate' deflection analysis, a new 'exact' elastic analysis is made to insure that the constraints are in fact satisfied. That is, the computer program at this point returns to Section 1.2.3.

This concludes the description of the general philosophy of the design system. More detailed descriptions are contained in Chapters 3, 4, and 5.

1.2.5 Output of the Design Results

As described in the preceding sections results are output not only for the final design but for several intermediate stages. By examination of all results the user can identify how the various design conditions and constraints affect the final results.

1.3 General Design Conditions and Limitations.

The type of frames considered in this design method are braced or unbraced rectangular multistory steel plane frames. All beams and columns are prismatic and rigidly connected at the joints of the frame. When bracing is used in the design, only diagonal bracing elements are considered. The bracing elements are prismatic and assumed to be pin-connected to the joints of the frame. In addition, bracing elements only span between diagonal joints of a bay in a story. Furthermore, the bottom story columns are assumed to be completely fixed to the foundation.

The proposed design method does not attempt to perform an optimization of the geometrical configuration of the beams and columns of the frame. On the contrary, it is assumed that the geometrical and topological conditions of the frame, such as the number of stories and bays, the story heights and the bay lengths, are determined from functional considerations for the frame. Consequently, these geometrical conditions are considered fixed and are input to the design system.

The loading configuration considered in the proposed design method conforms to the recommendations in the AISC Manual "Commentary on Plastic Design in Steel."[2] The first loading condition is the combination of dead loads plus live loads while the second loading condition is the combination of dead loads plus live loads plus wind or earthquake forces. Dead loads and live loads are taken as uniformly distributed gravity loads applied to the beams of the frame and concentrated gravity loads applied to the joints of the frame. Wind or earthquake loads are taken as concentrated horizontal loads applied to the external joints from either side of the frame. All loads are taken as static loads.

Two important design constraints that are considered in the design system contribute significantly to the practicality of the proposed method. The first constraint is a maximum depth constraint for beams and columns. The user may specify maximum beam and column depths which may not be exceeded in the design process. If unspecified, the design system will assume no limitation on the corresponding depths. Allowing for this constraint is necessary due to numerous functional requirements stemming from architectural, mechanical and other considerations. The second constraint is the two story column constraint. The design system designs column sections in two story lengths. For an even number of stories there are an even number of two-story column lengths. For an odd number of stories, the bottom story columns are taken as one story column lengths. This constraint is followed due to a consideration of the economics of the construction of the frame.

Finally, the following basic assumptions are made in the proposed design method:

1. The stress-strain curve of steel is represented as an ideal

elastic-plastic, bilinear line where strain hardening effects are neglected.

2. The spread of yielding in a member is not considered. Instead, the concept of plastic hinge formation is adopted.

3. The frame and loading are coplanar. Consequently, biaxial bending moments are not considered.

4. For plastic design under gravity loads only all diagonal bracing is neglected and the resulting unbraced frame alone is considered to provide the strength of the frame.

5. Under the application of the combination gravity plus wind load condition, only diagonal tension bracing and beams and columns are assumed to contribute to the strength and stiffness of the frame. Diagonal compression bracing is assumed to take on a buckled configuration under the application of gravity and lateral loads.

1.4 Computer Requirements.

The computer system has been coded largely in the Fortran IV language. The current size of the system requires the use of a computer with a minimum working core size of 100,000 words. All of the programs have been tested on an IBM 360/65.

CHAPTER 2

SUMMARY OF RESULTS

This Chapter will present example problems illustrating the application of the author's design system. Two frame geometries and loadings have been selected. They are Frames B and C contained in the Lehigh University lecture notes[10] of their 1965 summer conference on plastic design of multistory frames. The design data for these frames which will be presented in this section can also be found in these lecture notes. Before presenting the design data, the following important points should be kept in mind.

i. Unit costs used in the examples are different from those identified in the Lehigh lecture notes. The values used herein attempt to reflect fabrication and erection costs as well as material costs.

ii. Direct comparisons with Lehigh design solutions are made only on the basis of the results of the plastic design. In addition, although Lehigh design solutions do not include either elastic stress constraints nor elastic deflection constraints, several example solutions by the proposed design method will be presented which include these additional constraints.

iii. The 1969 AISC[1] plastic design code requirement which states that axial member forces are not permitted to exceed 0.85 P_y has been neglected in the example problems in order to be consistant with the design constraints of Lehigh University. Instead, the alternate constraint imposed is that axial member forces are not permitted to exceed 1.0 P_y.

iv. Material weights presented for the author's design solutions are based on nominal member lengths (i.e. joint center to joint center).

2.1 Frame B General Design Data.

General design data applicable to all Frame B example problems are presented in this section. Additional design data are presented in Section 2.3 and Appendix B.

i. Geometrical Conditions.

The geometrical conditions for Frame B are illustrated in Fig. 2.1.

ii. Material.

a. Modulus of Elasticity: $E = 2.9 \times 10^4$ k/in.^{2}

b. Yield Strength.

ASTM A36 Steel: $\sigma_y = 36.0$ k/in.^{2}

ASTM A441 Steel: $\sigma_y = 50.0$ k/in.^{2}

c. Unit Cost.

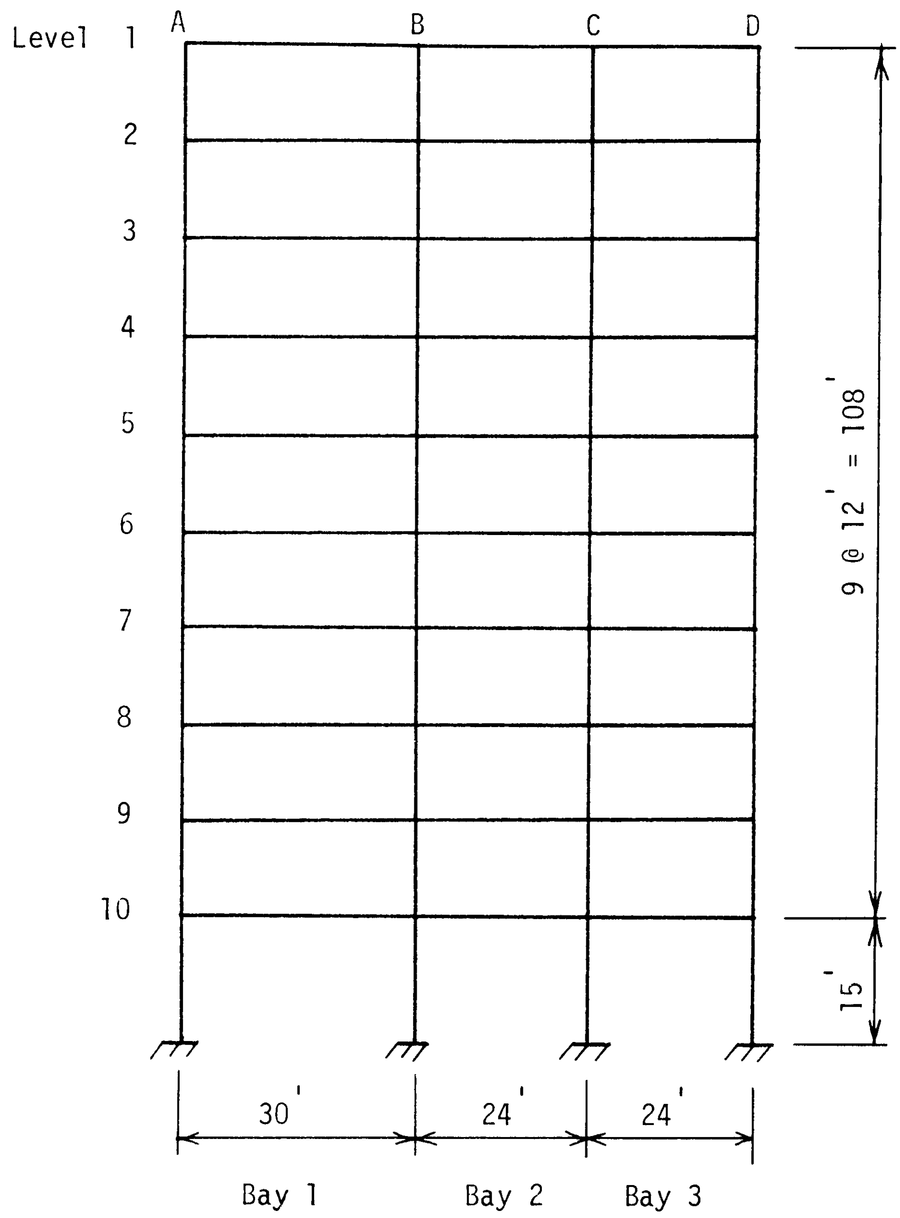
ASTM A36 Steel: U = 20 cents/lb.

ASTM A441 Steel: U = 24 cents/lb.

iii. Load Factors .

λ_1 = 1.70 (for D.L.+L.L.)

λ_2 = 1.30 (for D.L.+L.L.+W.L.)



Bent Spacing = 24 ft.

Maximum Lateral Unbraced Length: Beams = 5 ft.

Columns = 5 ft.

Figure 2.1 Frame B Geometry

iv. Loading Conditions, Working Loads.

Roof: W_L = 30 psf.

W_D = 60 psf.

Floors: W_L = 80 psf.

W_D = 80 psf.

Exterior Wall: W_D = 45 psf.

Wind: 20 psf.

2.2 Frame C General Design Data.

General design data applicable to all Frame C example problems are presented in this section. Additional design data are presented in Section 2.3 and Appendix C.

i. Geometrical Conditions.

The geometrical conditions for Frame C are illustrated in Fig. 2.2.

ii. Material.

a. Modulus of Elasticity: 2.9×10^4 k/in.^{2}

b. Yield Strength.

ASTM A36 Steel: σ_y = 36.0 k/in.^{2}

ASTM A441 Steel: σ_y = 50.0 k/in.^{2}

c. Unit Cost.

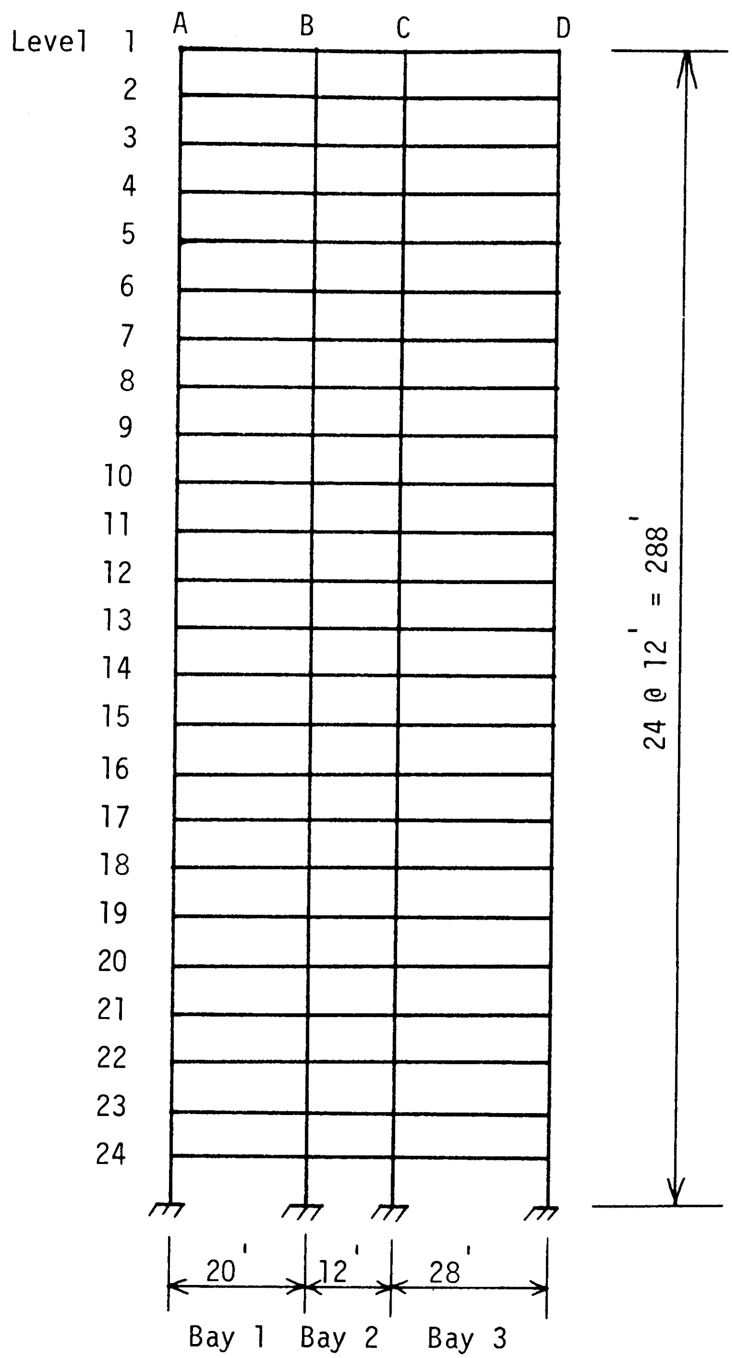
ASTM A36 Steel: U = 20 cents/lb.

ASTM A441 Steel: U = 24 cents/lb.

iii. Load Factors .

λ_1 = 1.70 (for D.L.+L.L.)

λ_2 = 1.30 (for D.L.+L.L.+W.L.)



Bent Spacing = 24 ft.

Maximum Lateral Unbraced Length: Beams = 3 ft.

Columns = 6 ft.

Figure 2.2 Frame C Geometry

iv. Loading Conditions, Working Loads.

Roof: W_L = 30 psf.

W_D = 95 psf.

Floors: W_L = 100 psf.

W_D = 120 psf.

Exterior Walls: W_D = 85 psf.

Wind: 20 psf.

2.3 List of Example Problems with Additional Design Data.

Twenty-eight example design solutions are presented. Section sizes, material weights, and material costs for each example are illustrated in Figs. 2.3 through 2.30.

The following points of clarification should be noted:

(a) Plastic design implies a consideration of only the plastic design constraints and no consideration of either elastic stress constraints nor elastic deflection constraints.

(b) Total design implies a consideration of plastic design constraints, elastic stress constraints, and elastic deflection constraints.

(c) When elastic stress constraints are considered:

$\sigma_{max} \leq \sigma_y$.

(d) When elastic deflection constraints are considered:

$\frac{\Delta}{h} \leq \frac{1}{400}$. For Frame B examples, $\Delta \leq 0.36$ in. for stories 1 to 9 and $\Delta \leq 0.45$ in. for story 10. For Frame C examples, $\Delta \leq 0.36$ in. for all stories.

(e) Unless otherwise stated, panel moment action is permitted.

Recall that a panel's resistance to lateral shear by either moment or truss action can only be controlled in the plastic design part. In the elastic design parts, the distribution of force is based on the actual elastic stiffness characteristics of the frame.

(f) Unless otherwise stated, no beam or column depth constraints are imposed.

(g) The author's material weights presented are rounded to the nearest tenth of a ton. However, the author's material costs presented are based on actual material weights. In addition, material costs presented for Lehigh designs are based on the author's unit material costs applied to the rounded weights presented in the Lehigh lecture notes.[10]

The twenty-eight example problems are as follows:

i. Frame B, All A36 Steel.

1. Example Problem B1.1A: author's design solution; unbraced frame; plastic design.
2. Example Problem B1.1L: Lehigh's design solution; unbraced frame; plastic design.
3. Example Problem B2.1A: author's design solution; bracing permitted in bay 3 only; plastic design.
4. Example Problem B2.1L: Lehigh's design solution; bracing permitted in bay 3 only; plastic design.
5. Example Problem B3.1A: author's design solution; bracing permitted in any bay; plastic design.

ii. Frame C, All A36 Steel, Braced Frames Only.

6. Example Problem C1.1A: author's design solution; bracing permitted in bay 3 only; plastic design.
7. Example Problem C1.1L: Lehigh's design solution; bracing permitted in bay 3 only; plastic design.
8. Example Problem C1.2A: author's design solution; bracing permitted in bay 3 only; total design.
9. Example Problem C2.1A: author's design solution; bracing permitted in any bay; plastic design.
10. Example Problem C2.2A: author's design solution; bracing permitted in any bay; total design.
11. Example Problem C3.1A: author's design solution; bracing permitted in bay 1 only; plastic design.
12. Example Problem C3.2A: author's design solution; bracing permitted in bay 1 only; total design.
13. Example Problem C4.1A: author's design solution; bracing permitted in bay 2 only; plastic design.
14. Example Problem C4.2A: author's design solution; bracing permitted in bay 2 only; total design.
15. Example Problem C5.1A: author's design solution; bracing permitted in bays 1 and 3 only; plastic design.
16. Example Problem C5.2A: author's design solution; bracing permitted in bays 1 and 3 only; total design.
17. Example Problem C6.1A: author's design solution; bracing permitted in bay 3 only; no panel moment action

permitted (i.e. all shear in plastic design part resisted by bay 3 vertical cantilever truss); plastic design.

18. Example Problem C6.2A: author's design solution; bracing permitted in bay 3 only (i.e. all shear in plastic design part resisted by bay 3 vertical cantilever truss); total design.

iii. Frame C, A441 Steel for All Columns in Stories 13 to 24, A36 Steel Elsewhere, Braced Frames Only.

19. Example Problem C7.1A: author's design solution; bracing permitted in bay 3 only: plastic design.

20. Example Problem C7.1L: Lehigh's design solution; bracing permitted in bay 3 only; plastic design.

21. Example Problem C7.2A: author's design solution; bracing permitted in bay 3 only; total design.

iv. Frame C, All A36 Steel, Braced Frames, Beam Depth Constraint.

22. Example Problem C8.1A: author's design solution: bracing permitted in bay 3 only: maximum allowable beam depth = 17 in.; plastic design.

23. Example Problem C8.2A: author's design solution; bracing permitted in bay 3 only: maximum allowable beam depth = 17 in.; total design.

v. Frame C, A441 Steel for All Columns in Stories 11 to 24, A36 Steel Elsewhere, Unbraced Frames Only.

24. Example Problem C9.1A: author's design solution; unbraced; plastic design.

25. Example Problem C9.1L: Lehigh's design solution; unbraced; plastic design.

26. Example Problem C9.2A: author's design solution; unbraced; total design.

vi. Frame C, All A36 Steel, Unbraced Frames Only.

27. Example Problem C10.1A: author's design solution; unbraced; plastic design.

28. Example Problem C10.2A: author's design solution; unbraced; total design.

2.4 General Discussion of Results.

This section presents general results illustrating the practicality and efficiency of the proposed design system. Included in the presentation are direct comparisons between the author's design solutions and Lehigh's design solutions, the effects of the beam depth constraint on a selected author's design solution, and a comparison between a braced frame with no restriction on brace location and several other braced frames with restricted bracing patterns. In addition, Tables 2.1 through 2.5 summarize the results for all example problems.

2.4.1 Comparisons Between the Author's and Lehigh University's Design Solutions.

Five comparisons are made between the author's and Lehigh's design solutions.

1. Comparison with Lehigh University Frame B, Unbraced, All A36 Steel.

Lehigh's design solution, B1.1L, and the author's design solution, B1.1A, are illustrated in Figs. 2.4 and 2.3, respectively. In both of these examples only the plastic design constraints are considered. Note that the total material weight according to the Lehigh design is 0.5 per cent lighter than the author's design. This is due to the fact that Lehigh's total girder weight is based on clear span lengths while the author's total girder weight is based on nominal joint center to joint center girder lengths. In fact, when nominal girder lengths are used to calculate the girder weights of Lehigh's design, the author's design solution is 1.5 per cent lighter.

The author's design required six cycles to satisfy the P-Δ convergence criterion. The execution time was 48.1 seconds.

2. Comparison with Lehigh University Frame B, Braced, All A36 Steel.

Lehigh's design solution, B2.1L, and the author's design solution, B2.1A, are illustrated in Figs. 2.6 and 2.5, respectively. In both of these examples only the plastic design constraints are considered. Note that in the author's design, no braces have been placed in the top story. This occurs since the author's design permits shear to be resisted by unbraced panels as described in Chapter 3. The design indicates that it is more economical to resist the total top story shear purely by panel moment action.

Furthermore, whereas Lehigh's brace design is based on resisting the total story shear, the author's brace design allows a reduction in brace force capacity depending on the total shear resisted by the unbraced story panels according to the method described in Chapter 3.

The author's design is two per cent lighter than Lehigh's design. Only two cycles are required for the P-Δ convergence criterion to be satisfied. The execution time was 25.3 seconds.

Attention is now called to the author's design solution, B3.1A, where the location of bracing elements is unspecified and permitted to be placed in any panel. The result, shown in Fig. 2.7, is identical to the author's design B2.1A except that all braces are located in bay 2. This illustrates the fact that the proposed design method is at most capable of only determining a local optimum solution and that no guarantee is given that the solution found is a global optimum solution. In fact, no guarantee can be given that the resulting solution is even a local optimum. However, the comparative study being presented is intended to show that economical designs can be realized by the proposed design method. The execution time for example B3.1A was 27.3 seconds.

3. Comparison with Lehigh University Frame C, Braced. All A36 Steel.

Lehigh's design solution, C1.1L, and the author's design solution, C1.1A, are illustrated in Figs. 2.9 and 2.8, respectively. In both of these examples only the plastic design constraints are considered. For the same reasons as explained for example B2.1A, no braces are necessary in the top story of the author's design. Otherwise, the member distribution is very similar between the two designs.

The author's design is 1.7 per cent lighter than Lehigh's design. The exeuction time was 1.8 minutes.

Figure 2.10 illustrates the results of example problem C1.2A which is the same as example C1.1A except that the elastic stress and elastic deflection constraints are considered in addition to the plastic design constraints. Only beam size changes were necessary to satisfy the elastic stress constraints while only brace size changes were necessary to satisfy the elastic deflection constraints. The result is that the author's design experienced a five per cent weight increase. The execution time for example C1.2A was 3.7 minutes.

Another interesting comparison is between the above discussed author's design C1.1A where panel moment action is permitted in the plastic design and the author's design C6.1A where no panel moment action is permitted (see Fig. 2.19). In design C6.1A, only a vertical cantilever truss in bay 3 is used to resist the total required story shears. In this case, example C6.1A is 3.4 per cent heavier than example C1.1A where the increased weight is due to larger beams, columns, and bracing elements in the bay 3 truss system. The execution time for example C6.1A was 1.5 minutes.

4. Comparison with Lehigh University Frame C, Braced, A36 and A441 Steel.

Lehigh's design solution, C7.1L, and the author's design solution, C7.1A, are illustrated in Figs. 2.22 and 2.21, respectively. In both of these examples only the plastic design constraints are considered. The author's design is eight per cent lighter than Lehigh's design. This large decrease in weight is due to the very much lighter A441 column sections. The difference in column section size may be due to the fact that the proposed design method allows a reduction in required column plastic moment capacity due to column plastic hinge formation at the intersection of a column centerline with a beam flange. In addition, the complete details of the design data by Lehigh University are not available. With this in mind, another reason for the large difference in A441 column weight may be due to a difference in maximum laterally unsupported column lengths (assumed to be six feet in the author's design). This effect would cause larger differences between the more slender A441 columns of examples C7.1A and C7.1L than in the less slender A36 columns of examples C1.1A and C1.1L.

The execution time for the author's design C7.1A was 1.6 minutes.

The effects of considering elastic stress and elastic stiffness constraints in addition to the plastic design constraints are demonstrated by example C7.2A as shown in Fig. 2.23. Only beam size increases were necessary to satisfy the elastic stress constraints while only brace size increases were necessary to satisfy the elastic deflection constraints. The result is that the author's design experienced a seven per cent weight increase in A36 steel. The execution time was 3.6 minutes.

5. Comparison with Lehigh University Frame C, Unbraced, A36 and A441 Steel.

Lehigh's design solution, C9.1L, and the author's design solution, C9.1A, are illustrated in Figs. 2.27 and 2.26, respectively. In both of these examples only the plastic design constraints are considered. The author's design is 11 per cent lighter than Lehigh's design. Most of this weight difference is due to lighter A441 column sections. The reasons for this are the same as the ones presented for example C7.1A above.

The execution time for the author's design C9.1A was 3.4 minutes based on five cycles of plastic design in order to satisfy the P-Δ convergence criterion.

The effects of considering elastic stress and elastic stiffness constraints in addition to the plastic design constraints are demonstrated by example C9.2A as shown in Fig. 2.28. A detailed description of the elastic stiffness design for this example is presented in Section 2.5.4.

2.4.2 Effects of Depth Constraints.

A beam depth constraint which limits maximum beam depths to 17 in. is imposed on the author's design solutions C1.1A and C1.2A. The results are given by examples C8.1A and C8.2A as illustrated in

Figs. 2.24 and 2.25. When only the plastic design constraints are considered (C1.1A and C8.1A), the beam depth constraint affects the beams in bay 3 only. In addition, several columns in column lines 3 and 4 (C and D) have increased in size. This is due to larger required column plastic moment capacities resulting from smaller beam depths. Furthermore, referring to the list of sections in the AISC Manual[1], it is apparent that there are several non-economy beam sections which satisfy the beam depth constraint as well as the plastic design constraints and which are of lesser weight than the non-economy beam sections selected in example C8.1A. This occurs since the design system can only select the least weight available section input to the program. Referring to the non-economy beam section table (Table A2 in Appendix A), the non economy sections selected in example C8.1A represent the least weight sections that are available. Obviously, if the entire AISC section table had been input, lighter beam sections would have been selected.

When elastic stress and elastic deflection constraints are considered in addition to the plastic design constraints (C1.2A and C8.2A), the beam depth constraint controls beam sizes in bays 1 and 3. However, columns are still only effected in column lines 3 and 4. Also note that the bracing weight is higher for example C8.2A than for C1.2A. This occurs since the smaller depth beams in C8.2A provide less stiffness and thus lead to larger elastic deflections. Since the braces are more economical for deflection control, only the braces change size in the elastic stiffness design. Thus, more brace weight was required in C8.2A.

A 4.9 per cent weight increase is necessary to satisfy the beam depth constraint when only plastic design constraints are considered and a 7.7 per cent weight increase is necessary when elastic stress and elastic deflection constraints are also considered. The execution times for examples C8.1A and C8.2A was 1.8 minutes and 3.9 minutes, respectively.

2.4.3 Comparison of the Free Bracing Case with Several Selected Bracing Patterns.

A study is made in order to determine the validity of the proposed design method with respect to determining a bracing pattern that leads to a least weight structure. As discussed in Chapter 3, the optimization procedure is heuristic in nature and, thus, no guarantee can be given that resulting designs are optimal (either local or global). Consequently, only through comparative studies such as presented here can one develop a feeling for the validity of the optimization procedure.

The comparison is made using Frame C with all A 36 steel. Table 2.2 summarizes the comparative study except that examples C8.1A and C8.2A should not be considered since these examples include a beam depth constraint. Examples C2.1A and C2.2A are the cases where bracing is permitted in any panel (free bracing cases). The results are also illustrated in Figs. 2.8 to 2.20. When only the plastic design constraints are considered, example C2.1A is 0.48 per cent lighter than the next heavier comparative example (C1.1A) and 3.7 per cent lighter than the heaviest comparative example (C6.1A). When the elastic stress and elastic deflection constraints are considered in addition to the plastic design constraints, example C2.2A is 0.39 per cent lighter than the next heavier comparative example (C1.2A) and 10.3 per cent lighter than the heaviest comparative example (C4.2A).

The bracing pattern selected by the design method for the free bracing case is illustrated in Fig. 2.11. Admittedly, although the bracing pattern selected led to the least weight structure of those compared, it is not a practical pattern. However, this free bracing pattern may be used to suggest an alternate, but practical, bracing pattern which might be best. One alternate suggested is to allow bracing in bays 1 and 2 only.

The following points summarize the general results presented in Section 2.4:

(i) As indicated by the comparisons with the Lehigh University designs, the proposed system leads to reasonable solutions. In four of the five comparisons, the author's designs were of lesser weight. The one exception, B1.1A, is explained by the fact that the author computes beam weight on the basis of nominal beam lengths, rather than on the basis of clear span lengths, whereas Lehigh computes their beam weights on the basis of the clear span lengths.

(ii) Of all braced Frame C, A36 steel, examples considered, the lightest structure occurred when the brace location was completely unconstrained. However, of more importance is the fact that all of the different bracing arrangements lead to frames having approximately the same total weight.

(iii) The consideration of elastic stress and elastic deflection constraints in all cases led to heavier structures.

(iv) Beam depth constraints not only result in heavier beam sections, but may also lead to heavier column sections to satisfy the plastic design constraints and heavier bracing sections to satisfy the elastic deflection constraints.

2.5 A Detailed Consideration of Selected Results.

This section will present a more detailed consideration of selected results from several example problems.

2.5.1 Examples of the Approximate Deflection Analysis.

Tables 2.6 and 2.7 compare the results of the approximate and exact relative story deflection analyses for the combination gravity and lateral service loads for example problems C2.2A and C1.2A, respectively. The elastic deflections presented are based on the member property distribution determined in the plastic design part (before the execution of the elastic stress or elastic stiffness designs).

The table headings are defined as follows.

Δ_s = approximate relative story deflections due to beam and column bending and tension brace elongation under the application of service lateral wind loads.

Δ_c = approximate relative story deflections due to column elongation and shortening under the application of service lateral wind loads.

Δ_g = 'exact' relative story deflections due to service gravity loads calculated by the matrix stiffness method of analysis.

Δ_a = $\Delta_s + \Delta_c + \Delta_g$

= total approximate relative story deflection neglecting the effects of beam elongation and shortening due to the combination gravity and lateral wind load.

Δ_e = 'exact' relative story deflections calculated by the stiffness method of analysis.

Error = $\Delta_e - \Delta_a$

= error in the approximation including the effects of beam elongation and shortening which are not taken into consideration in the approximation.

Values of Δ_g are calculated for presentation purposes only. In addition, note that values of Δ_g which are added to values of Δ_s and Δ_c for wind from the left represent relative story deflections due to gravity loads and associated with a frame geometry that includes tension bracing for wind from the left (brace type 2). When wind from the right is considered, values of Δ_g which are added to Δ_s and Δ_c are associated with a frame geometry that includes tension bracing for wind from the right (brace type 1).

Note that the error terms in Table 2.7 are larger than those in Table 2.6. This may be due to larger beam shortening effects in example C1.2A than in example C2.2A.

2.5.2 Effects of the Elastic Stress Design.

In all examples presented that consider the elastic stress constraint, it was found that most beams designed on the basis of the plastic design constraints violated the elastic stress constraint (i.e. these beam elastic stresses were greater than the corresponding steel yield stress), and were modified in the elastic stress design part. In addition, for all braced frame examples, it was found that after each execution of the elastic stiffness design part a few beams again violated the elastic stress constraint and were again modified on the basis of elastic stress. In no case were columns or braces found to violate the imposed elastic stress constraint.

2.5.3 Effects of the Elastic Stiffness Design.

In all but two examples presented that consider the elastic deflection constraint, only one execution of the elastic stiffness design was necessary in order to reduce the elastic relative story deflections to a value less than or equal to $\frac{1}{400}$ h or 0.36 in. for the Frame C stories. One of the exceptions, example C6.2A, was found to satisfy the elastic deflection constraint following the first elastic stress design. Consequently, the elastic stiffness design was not executed. The second exception, example C9.2A, required six cycles of the elastic stiffness design. This result will be discussed in more detail in Section 2.5.4 since it uncovered a difficulty in the elastic stiffness design procedure.

In all but one of the braced Frame C examples in which the elastic stiffness design was executed, only increases in brace section size was necessary in order to satisfy the elastic deflection constraint. The exception, example C4.2A where braces were only permitted in bay 2, required increases in both brace and beam section sizes in order to satisfy the elastic deflection constraints. In no braced frame example were columns modified during the elastic stiffness design.

In the unbraced example C10.2A, only beams were modified in order to satisfy the elastic deflection constraints.

The effects of the elastic stiffness design on elastic relative story deflections are presented for examples C1.2A, C2.2A, C4.2A, and C10.2A in Figs. 2.31, 2.32, 2.33, and 2.34, respectively. In these figures:

$(\Delta_e)_{initial}$ = 'exact' elastic relative story deflections based on member sizes from the plastic design part alone.

$(\Delta_e)_{final}$ = 'exact' elastic relative story deflections based on the final design member sizes.

2.5.4 A Difficulty in the Elastic Stiffness Design Method.

A difficulty in the elastic stiffness design method was uncovered in its application to example problem C9.2A. The difficulty stems from the fact that the formulation of the elastic stiffness design does not attempt to minimize gravity sway deflections when modifying members in order to satisfy the elastic deflection constraints. In example problem C9.2A, during the elastic stiffness design, the gravity sway deflections become sufficiently large so as to cause an unreasonably large weight increase before the elastic deflection constraints were satisfied. This difficulty is described in some detail in what follows.

Figures 2.35 to 2.41 show the elastic deflection characteristics of example C9.2A after the plastic design part and after each execution of the elastic stiffness design where the 0.36 in. horizontal line represents the maximum allowable elastic relative story deflection. The following definitions apply to each figure.

Δ_e = 'exact' relative story deflection computed by the stiffness method of analysis for the combined service gravity and lateral wind loads.

Δ_s = approximate relative story deflection due to beam and column bending under the application of service lateral wind loads.

Δ_c = approximate relative story deflection due to column elongation and shortening under the application of service lateral wind loads.

Δ_g = 'exact' relative sway deflection due to service gravity loads.

Δ_a = $\Delta_s + \Delta_c + \Delta_g$

= total approximate relative story deflection for the combined service gravity and lateral wind loads and neglecting beam elongation and shortening effects.

$e \;=\; \Delta_e - \Delta_a$

= error in the latest approximation based on the 'exact' relative story deflection computed after each execution of the elastic stiffness design.

$\bar{\Delta}$ = the final value of relative story deflection computed at the end of each execution of the elastic stiffness design and based on the error term computed at the beginning of each execution of the elastic stiffness design.

Note that the error term includes the effects of beam elongation and shortening since the approximate calculation does not compute a value for this effect.

The convergence characteristics of the elastic stiffness design will be described as follows:

(i) Immediately following the plastic design, 'exact' elastic relative story deflections (Δ_e) are computed. As shown in Fig. 2.35, numerous values of Δ_e exceed the maximum permitted of 0.36 in. Consequently, values of Δ_a are calculated so that the initial error terms, e, may be calculated. The total weight of the structure following the plastic design is 132.4 tons.

(ii) The first execution of the elastic stiffness design is now executed. After all member size changes are made, the predicted approximate relative story deflections after this first elastic stiffness design, $\bar{\Delta}$, based on the error term in Fig. 2.35, are plotted in Fig. 2.36. In addition, the current values of Δ_a as well as new exact values of Δ_e, Δ_g, and e are plotted. Note that the plotted values of Δ_a plus the error terms of Fig. 2.35 equal the plotted values of $\bar{\Delta}$. Also, the new exact values of Δ_e do not all equal the values of $\bar{\Delta}$ since the error term has changed slightly for several stories. In fact, the error term has changed sufficiently to just cause a few values of Δ_e to again exceed the maximum permissible. The total weight of the structure following the first execution of the elastic stiffness design is 138.9 tons, where all of the weight increase is due to beam size changes. Note however that 3.5 tons of the 6.5 ton increase is due to numerous beam size changes necessary to satisfy the elastic stress constraint which was checked immediately prior to the first elastic stiffness design execution. Also note that all succeeding weight increases due to elastic stress design modifications are extremely small relative to the weight increases due to elastic stiffness design modifications and thus will no longer be mentioned.

(iii) Since several values of Δ_e plotted in Fig. 2.36 exceed the maximum permissible deflection for wind from the left and right, a second execution of the elastic stiffness design is performed. Again, resulting values of $\bar{\Delta}$, based on the error term in Fig. 2.36, are plotted in Fig. 2.37. In addition, the current values of Δ_a as well as new exact values of Δ_e, Δ_g and e are plotted. Again, the error term has changed sufficiently to just cause a few values of Δ_e to again exceed the maximum permissible. The total weight of the structure following the second execution of the elastic stiffness design was 141.3 tons (2.4 ton increase). All member size changes but one were due to beam size increases in bay 2. One column in story 14, column line 3, increased in size. Note that the effects of gravity sway plotted in Fig. 2.37 due to the current member sizes are still quite small.

Up to this point only one column has changed size while all other elastic stiffness design member size changes occurred in the beams in bay 2.

(iv) Since several values of Δ_e plotted in Fig. 2.37 exceed the maximum permissible deflection, a third execution of the elastic stiffness design is performed. The same types of deflection measures that were plotted in Fig. 2.37 are again plotted in Fig. 2.38. This is a particularly significant plot. Note the extremely large changes in the gravity sway deflections, Δ_g, as compared to the much smaller changes in the error term e. In fact, if the values of Δ_g were subtracted from the exact deflections, Δ_e, the elastic deflection constraints would be satisfied. Instead, the increased sway deflections have caused several values of Δ_e to exceed the maximum permissible deflection by larger amounts than in the previous cycle for both wind from the left and wind from the right.

The reason for these large gravity sway deflections is that the third execution of the elastic stiffness design resulted in large column size increases in column line 3 of stories 14 and 16 as well as beam size increases in several stories of bay 2. These changes, in addition to those made in previous cycles, lead to a highly unsymmetrical stiffness configuration which in turn leads to large gravity sway deflections. It is at this point where the gravity sway deflections begin to control the elastic stiffness design since, as indicated above, if values of Δ_g were subtracted from values of Δ_e, the design would terminate.

The total weight of the structure following the third execution of the elastic stiffness design was 145.3 tons (4.0 ton increase).

(v) The next two executions of the elastic stiffness design again caused large variations in the gravity sway deflections as illustrated in Figs. 2.39 and 2.40. In both of these stiffness designs, beams and columns within stories as well as columns below stories under consideration were increased in size. Finally, the sixth execution of the elastic stiffness design (Fig. 2.41) caused negligable changes in the gravity sway deflections and the design terminated. The final structure weight was 192.4 tons. The execution time was 10.0 minutes.

In summary, although changes in the error term caused two additional executions of the elastic stiffness design (the second and third), the total increase in structure weight was at least reasonable. In fact, if the gravity sway deflections caused by unsymmetrical member size changes in the third execution of the elastic stiffness design were neglected, the design would have terminated with a total structure weight of 145.3 tons which is considerably less than the 160.2 tons necessary to satisfy the elastic deflection constraints of the all A36 steel example C10.2A. However, gravity sway deflections cannot be neglected in the deflection calculation, and consequently, the sudden variations of Δ_g after the third execution of the elastic stiffness design resulted in additional executions of the elastic stiffness design leading to an excessively high final structure weight.

One possible simple solution to this difficulty might be to require the elastic stiffness design, based on the approximate deflection calculation, to satisfy a deflection constraint somewhat less than the constraint the exact deflections must satisfy (say 85 per cent). However, even this procedure would not guarantee the avoidance of this problem in all situations. On the other hand, the best solution would be to somehow account for gravity sway deflections in the member selection procedure during the elastic stiffness design. It is not immediately obvious how this could be done, and thus is recommended as a future area of study.

One last point is to be made. In the doctoral dissertation of Y. Nakamura(6), an elastic stiffness design is also performed which leads to similar member size changes as in the first execution of the author's elastic stiffness design method. However, Nakamura only performs one execution of the elastic stiffness design and makes no attempt to check the results by an exact analysis. Consequently, Nakamura cannot guarantee with his design method that exact relative story deflections satisfy the imposed elastic deflection constraints.

Example	Material Weight (TONS)				Materal Cost (Dollars)
	Girders	Columns	Bracing	Total	
B1.1A	16.9	20.4	—	37.3	14,904
B1.1L	16.5	20.6	—	37.1	14,840
B2.1A	15.75	19.14	1.55	36.44	14,575
B2.1L	14.5	19.6	3.2	37.3	14,920
B3.1A	15.75	19.14	1.55	36.44	14,575

Table 2.1 Materal Cost and Weight for Frame B, All A36 Steel

Example	Material Weight (TONS)				Material Cost
	Girders	Columns	Bracing	Total	(Dollars)
(a) Plastic Design:					
C1.1A	29.5	108.5	7.7	145.7	58,289
C1.1L	28.1	110.7	9.4	148.2	59,280
C2.1A	29.9	109.2	5.9	145.0	57,982
C3.1A	29.9	110.8	6.4	147.1	58,833
C4.1A	29.8	114.4	5.3	149.3	59,812
C5.1A	29.9	109.4	6.7	145.9	58,374
C6.1A	30.0	109.8	10.8	150.6	60,258
C8.1A	35.8	109.3	7.7	152.8	61,121
(b) Total Design:					
C1.2A	35.2	108.5	8.5	152.2	60,888
C2.2A	35.7	109.2	6.7	151.6	60,640
C3.2A	35.7	110.8	9.1	155.6	62,225
C4.2A	41.3	115.4	12.3	169.0	67,618
C5.2A	35.7	109.4	8.2	153.3	61,300
C6.2A	34.4	109.8	10.8	155.0	61,995
C8.2A	45.3	109.3	9.3	163.9	65,554

Table 2.2 Material Cost and Weight for Braced Frame C, All A36 Steel

Example	Material Weight (TONS)					Material Cost (Dollars)
	Girders (A36)	Columns (A36)	Columns (A441)	Bracing (A36)	Total A36 / A441	
(a) Plastic Design:						
C7.1A	29.5	31.1	56.0	8.0	68.6 / 56.0	54,329
C7.1L	28.1	31.1	66.5	9.4	68.6 / 66.5	59,300
(b) Total Design:						
C7.2A	35.0	31.1	56.0	11.4	77.5 / 56.0	57,867

Table 2.3 Material Cost and Weight for Braced Frame C, A36 and A441 Steel

Example	Material Weight (TONS)				Material Cost (Dollars)
	Girders (A36)	Columns (A36)	Columns (A441)	Total A36/A441	
(a) Plastic Design:					
C9.1A	40.6	22.9	68.9	63.5 / 68.9	58,463
C9.1L	42.8	23.0	83.4	65.8 / 83.4	66,352
(b) Total Design:					
C9.2A	57.2	22.9	112.3	80.1 / 112.3	85,942

Table 2.4 Material Cost and Weight for Unbraced Frame C, A36 and A441 Steel

Example	Material Weight (TONS)			Material Cost (Dollars)
	Girders	Columns	Total	
(a) Plastic Design:				
C10.1A	39.4	116.4	155.8	62,312
(b) Total Design:				
C10.2A	43.8	116.4	160.2	64,077

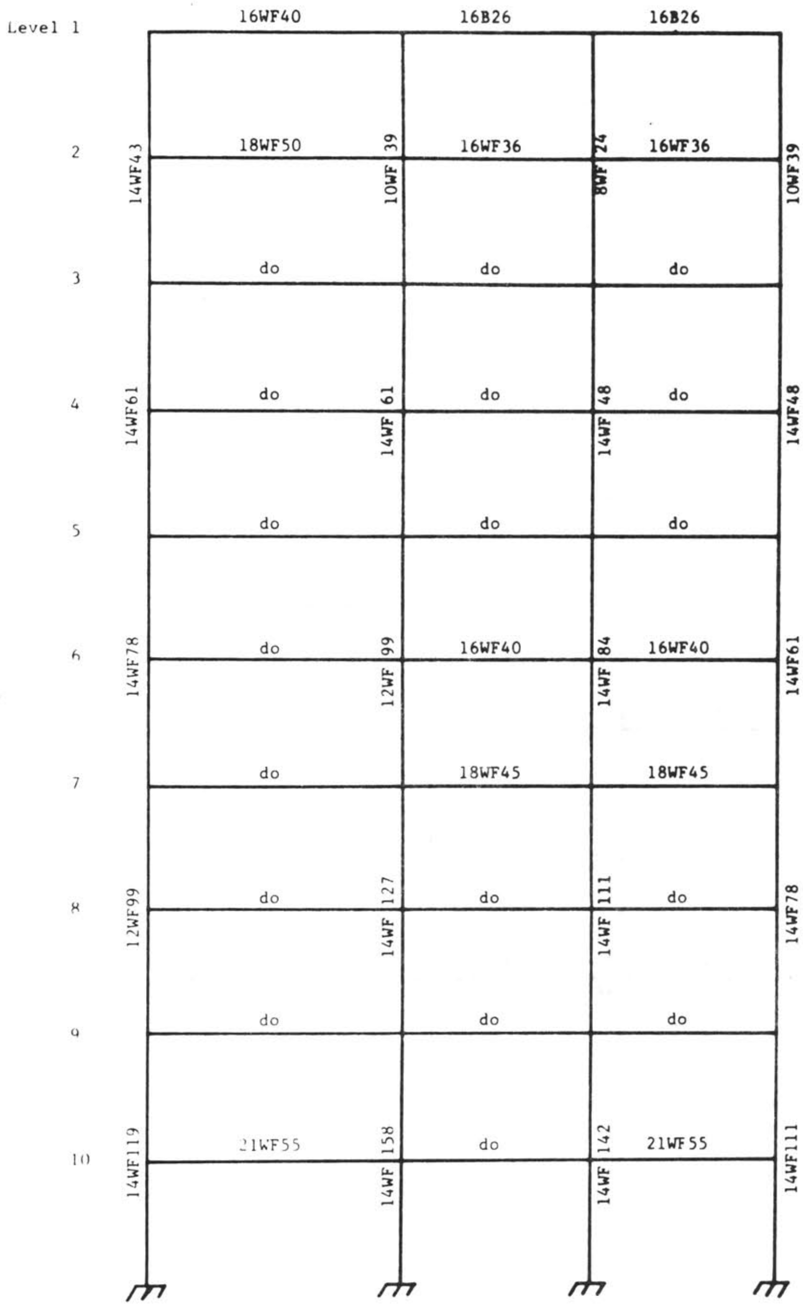
Table 2.5 Material Cost and Weight for Unbraced Frame C, All A36 Steel

Story	Δ_s	Δ_c	Δ_g	$\Delta_a = \Delta_s + \Delta_c + \Delta_g$	Δ_e	Error = $\Delta_e - \Delta_a$ (includes beam elongation and shortening effects)
1	.06	.18	.00	.24	.14	-.10
2	.06	.18	-.01	.23	.21	-.02
3	.08	.18	.00	.26	.24	-.02
4	.11	.18	.01	.30	.29	-.01
5	.13	.18	.01	.32	.31	-.01
6	.16	.18	.02	.36	.35	-.01
7	.18	.18	.02	.38	.37	-.01
8	.20	.17	.02	.39	.40	.01
9	.19	.17	.03	.39	.41	.02
10	.20	.17	.03	.40	.42	.02
11	.20	.16	.03	.39	.41	.02
12	.18	.16	.04	.38	.37	.01
13	.20	.15	.04	.39	.41	.02
14	.20	.14	.05	.39	.38	-.01
15	.24	.13	.04	.41	.39	-.02
16	.23	.10	.05	.38	.40	.02
17	.18	.09	.05	.32	.36	.04
18	.21	.07	.05	.33	.35	.02
19	.22	.07	.05	.34	.34	.00
20	.21	.05	.05	.31	.33	.02
21	.22	.05	.05	.32	.32	.00
22	.22	.04	.05	.31	.33	.02
23	.24	.01	.05	.30	.31	.01
24	.18	.00	.03	.21	.17	-.04

Table 2.6 Example C2.2A, Elastic Relative Story Deflections Prior to First Execution of Elastic Stress and Elastic Stiffness Design, Wind from Left.

Story	Δ_s	Δ_c	Δ_g	$\Delta_a = \Delta_s + \Delta_c + \Delta_g$	Δ_e	Error = $\Delta_e - \Delta_a$ (Includes beam elongation and shortening effects)
1	.06	.09	.00	.15	.17	.02
2	.06	.09	-.01	.14	.19	.05
3	.08	.09	.00	.17	.22	.05
4	.11	.09	.00	.20	.25	.05
5	.13	.09	.00	.22	.27	.05
6	.16	.09	.00	.27	.31	.04
7	.19	.09	.00	.28	.33	.05
8	.21	.09	.01	.31	.36	.05
9	.24	.08	.01	.33	.38	.05
10	.26	.08	.01	.35	.40	.05
11	.24	.08	.01	.33	.39	.06
12	.25	.08	.02	.35	.41	.06
13	.27	.07	.02	.36	.41	.05
14	.25	.07	.02	.34	.41	.07
15	.26	.06	.02	.34	.42	.08
16	.28	.06	.02	.36	.42	.06
17	.28	.05	.02	.35	.42	.07
18	.27	.05	.02	.34	.41	.07
19	.27	.04	.02	.33	.41	.08
20	.27	.03	.03	.33	.40	.07
21	.28	.03	.03	.34	.40	.06
22	.27	.02	.03	.32	.38	.06
23	.29	.01	.03	.33	.33	.00
24	.20	.00	.01	.21	.18	-.03

Table 2.7 Example C1.2A, Elastic Relative Story Deflections Prior to First Execution of Elastic Stress and Elastic Stiffness Design, Wind from Left.



DESIGN CONDITIONS

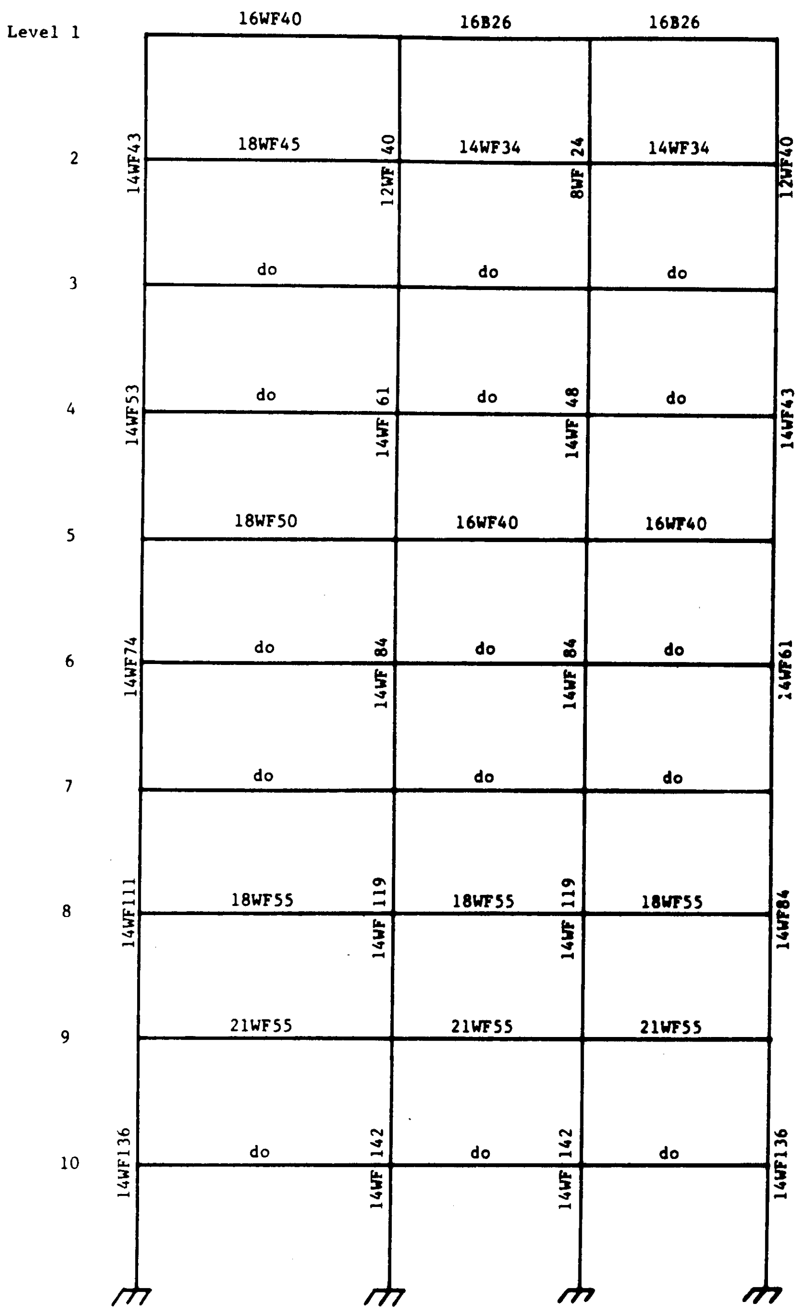
1. Unbraced Frame.
2. Plastic Design.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 16.9 Tons
Column Wt.	= 20.4 Tons
Total Wt.	= 37.3 Tons
Material Cost	= $14,904

Figure 2.3 Example Problem B1.1A



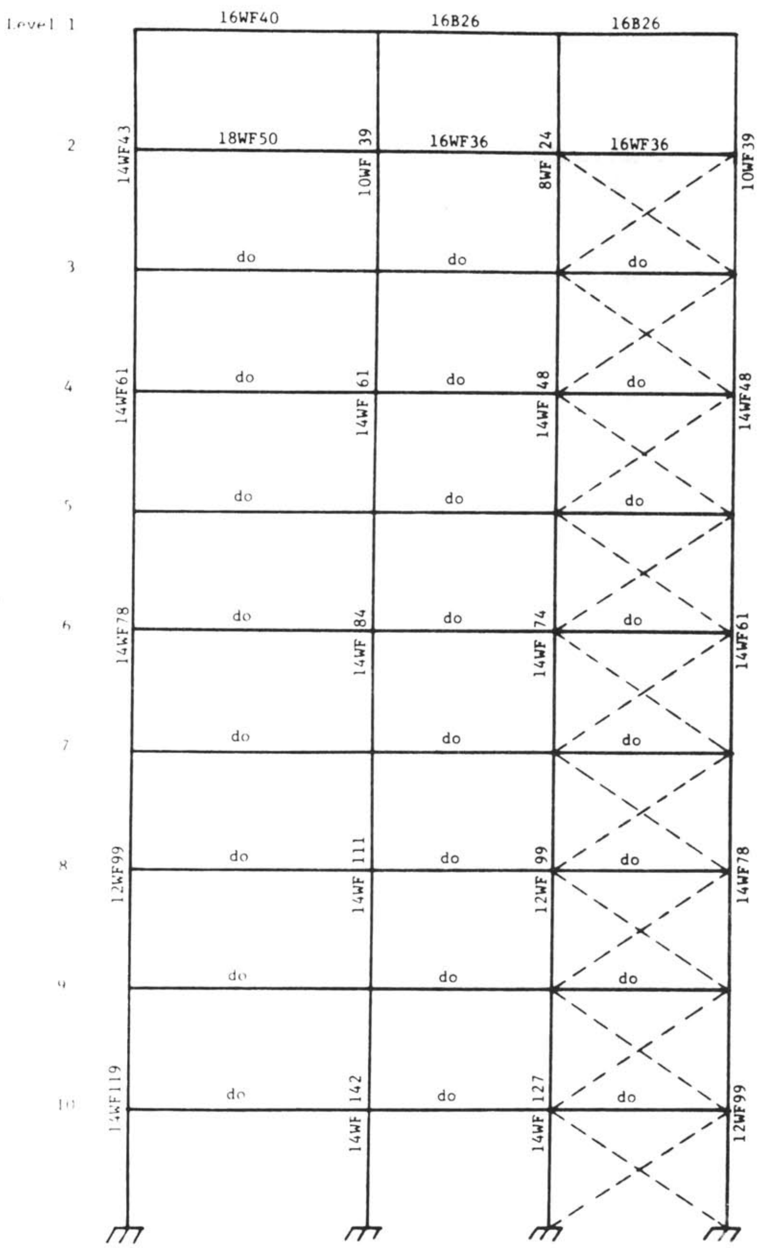
DESIGN CONDITIONS

1. Unbraced Frame.
2. Plastic Design.

MATERIAL WEIGHT AND COST
A36 Steel

Girder Wt.	= 16.5 Tons
Column Wt.	= 20.6 Tons
Total Wt.	= 37.1 Tons
Material Cost	= $14,840

Figure 2.4 Example Problem B1.1L



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, ╥)

Size	Level	Type*
2x2x1/4	2 to 10	1,2

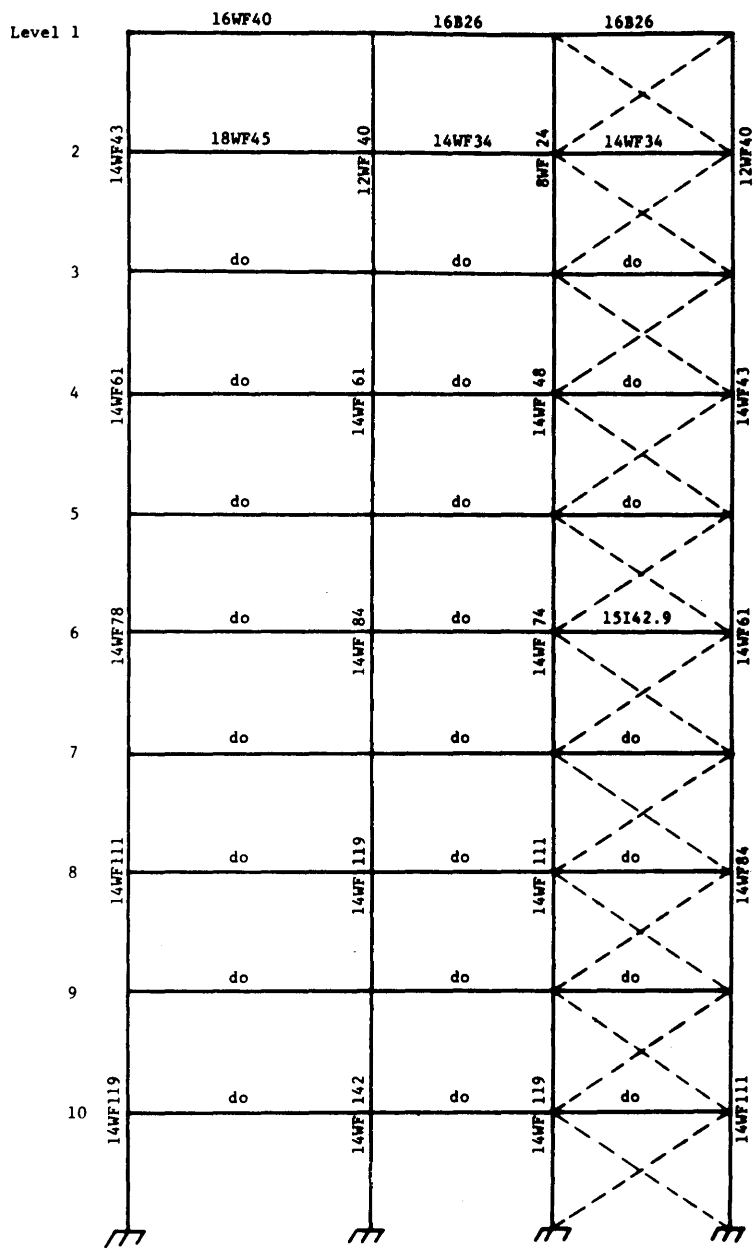
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 15.75 Tons
Column Wt.	= 19.14 Tons
Bracing Wt.	= 1.55 Tons
Total Wt.	= 36.44 Tons
Material Cost	= $14,575

Figure 2.5 Example Problem B2.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Single Angles, ⅂)

Size	Level	Type*
6x6x5/16	1 to 10	1,2

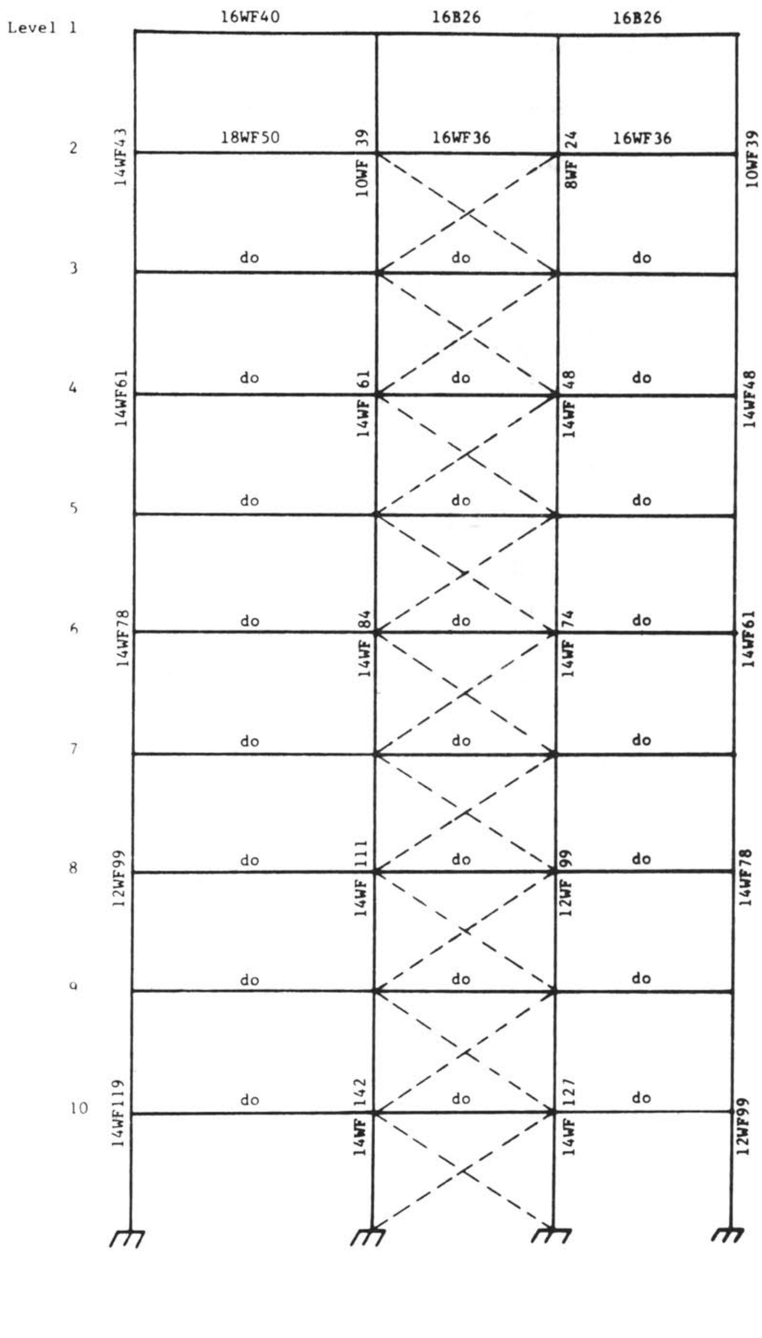
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 14.5 Tons
Column Wt. = 19.6 Tons
Bracing Wt. = 3.2 Tons
Total Wt. = 37.3 Tons
Material Cost = $14,920

Figure 2.6 Example Problem B2.1L



DESIGN CONDITIONS

1. Bracing Permitted in Any Panel.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles,]⌈)

Size	Level	Bay	Type*
2x2x1/4	2 to 10	2	1,2

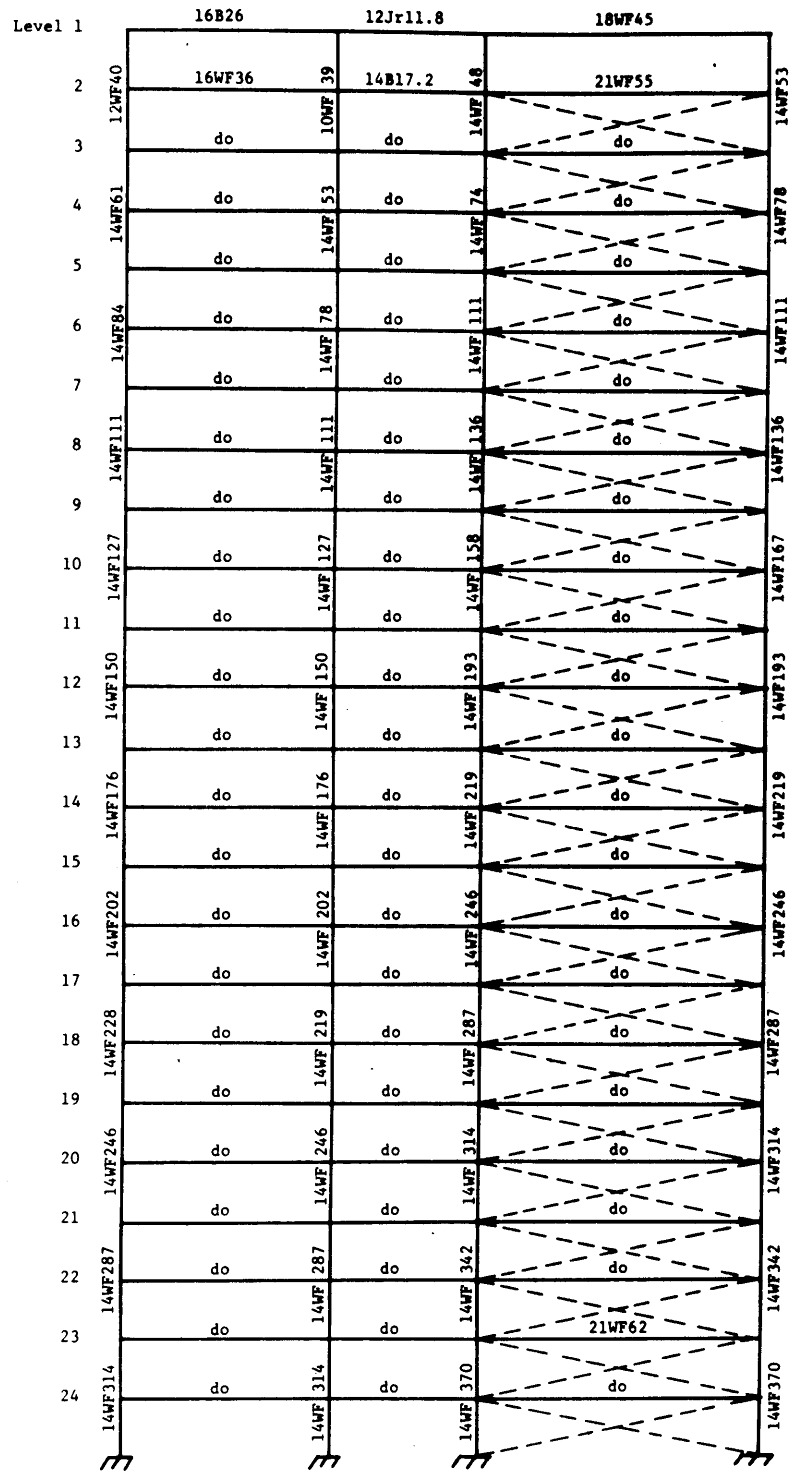
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 15.75 Tons
Column Wt.	= 19.14 Tons
Bracing Wt.	= 1.55 Tons
Total Wt.	= 36.44 Tons
Material Cost	= $14,575

Figure 2.7 Example Problem B3.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, T)

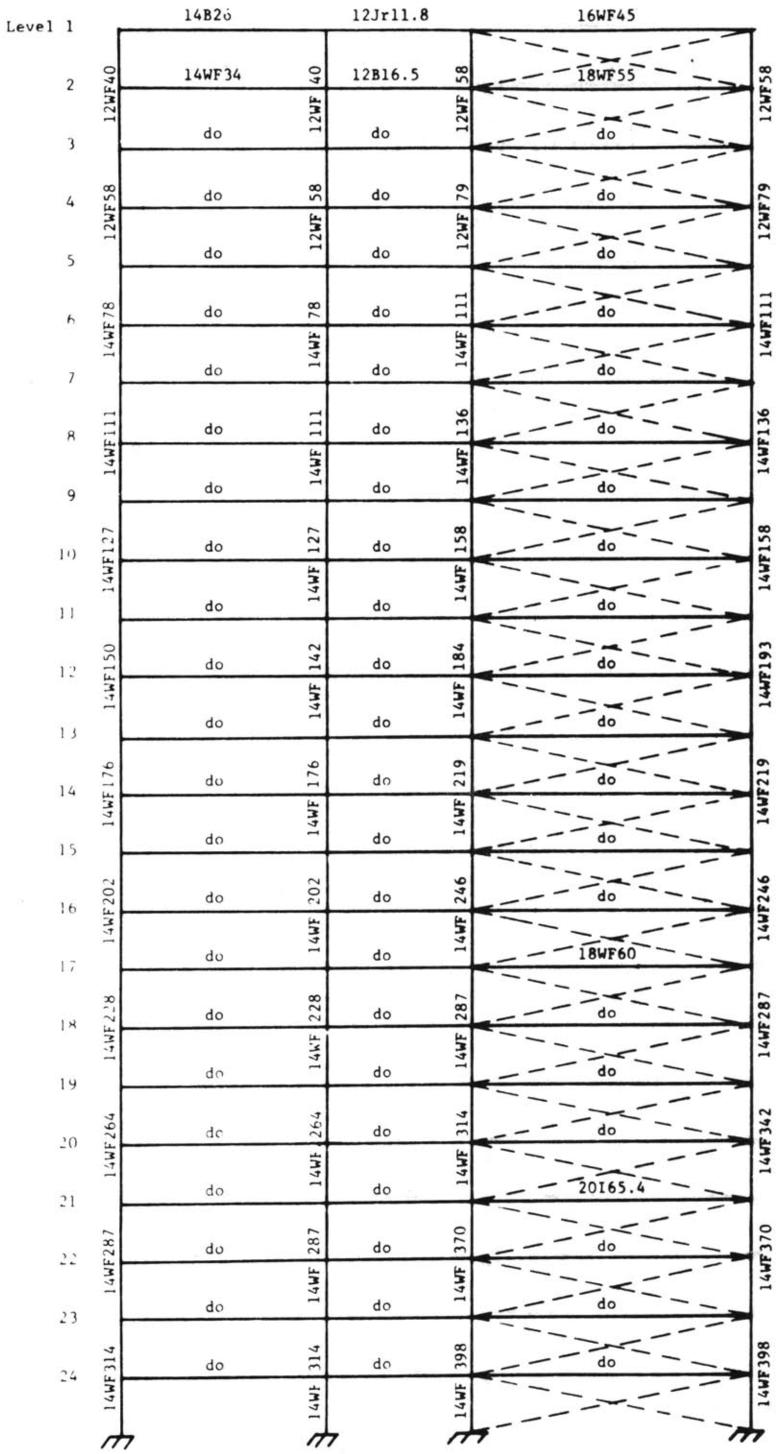
Size	Level	Type*
3x2x3/16	2 to 10	1,2
$3x2\frac{1}{2}x1/4$	11 to 13	1,2
4x3x1/4	14 to 16	1,2
$3x2\frac{1}{2}x3/8$	17	1,2
4x3x5/16	18	1,2
$4x3\frac{1}{2}x5/16$	19	1,2
4x3x3/8	20 to 21	1,2
$4x3\frac{1}{2}x3/8$	22	1,2
4x3x7/16	23	1,2
$4x3\frac{1}{2}x7/16$	24	1,2

MATERIAL WEIGHT AND COST
A36 Steel

Girder Wt. = 29.5 Tons
Column Wt. = 108.5 Tons
Bracing Wt. = 7.7 Tons
Total Wt. = 145.7 Tons
Material Cost = $58,289

* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

Figure 2.8 Example Problem C1.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, ⊤)

Size	Level	Type*
$3x2\frac{1}{2}x1/4$	1 to 9	1,2
$3\frac{1}{2}x3x1/4$	10 to 11	1,2
4x3x1/4	12	1,2
$4x3\frac{1}{2}x1/4$	13 to 16	1,2
5x3x5/16	17 to 18	1,2
$5x3\frac{1}{2}x3/8$	19 to 22	1,2
5x3x7/16	23 to 24	1,2

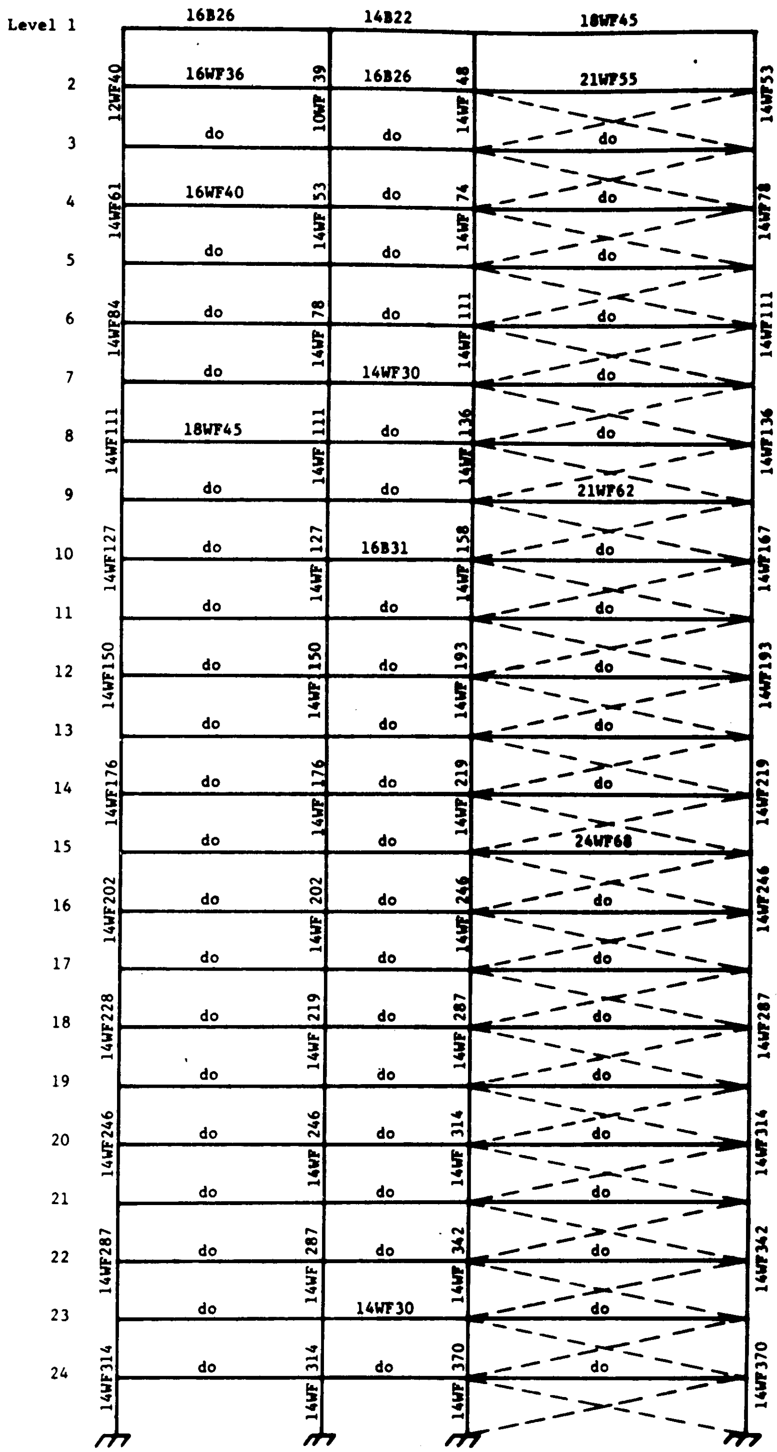
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 28.1 Tons
Column Wt. = 110.7 Tons
Bracing Wt. = 9.4 Tons
Total Wt. = 148.2 Tons
Material Cost = $59,280

Figure 2.9 Example Problem C1.1L



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.
4. Elastic Stress Design($\sigma_{max} \leq \sigma_y$).
5. Elastic Stiffness Design($\frac{\Delta}{h} \leq \frac{1}{400}$).

BRACING SIZES (Double Angles, ⊤)

Size	Level	Type*
3x2x3/16	2 to 9	1,2
	10	2
$3x2\frac{1}{2}x1/4$	10	1
4x3x1/4	11 to 13	1,2
$4x3\frac{1}{2}x5/16$	14	1
	17	2
4x3x5/16	14,16	2
	15,16	1
$3x2\frac{1}{2}x3/8$	15	2
4x3x3/8	17	1
	18,19,21	1,2
4x3x7/16	20	1
	23	1,2
$4x3\frac{1}{2}x3/8$	20	2
	22	1,2
$4x3\frac{1}{2}x7/16$	24	1,2

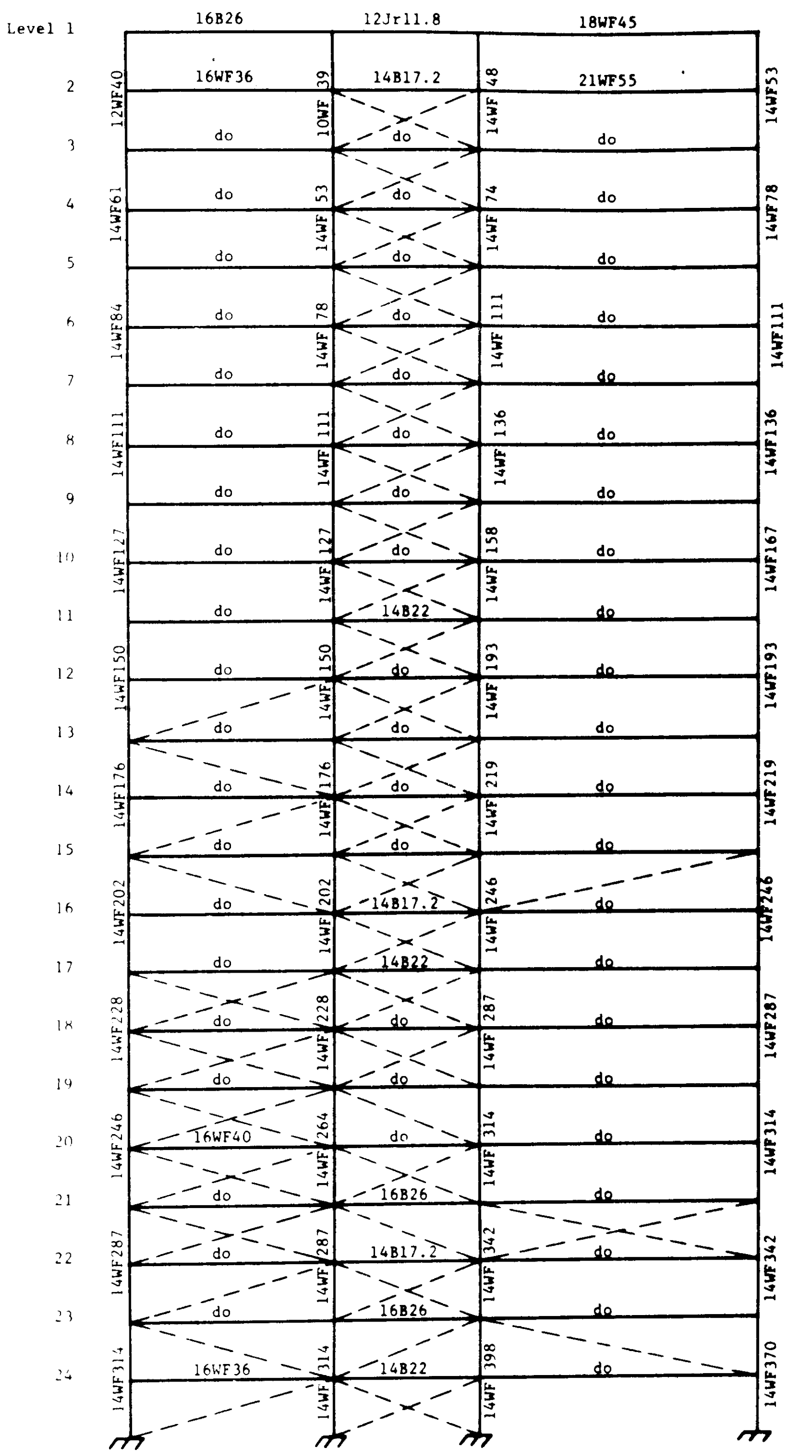
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 35.2 Tons
Column Wt. = 108.5 Tons
Bracing Wt. = 8.5 Tons
Total Wt. = 152.2 Tons
Material Cost = $60,888

Figure 2.10 Example Problem C1.2A



DESIGN CONDITIONS

1. Bracing Permitted in Any Panel.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, ⊤)

Size	Level	Bay	Type*
3x2x3/16	2 to 8	2	1,2
	12,14,17,18	1	2
	13,15,18,20	1	1
	13,15,17	2	1
	15,24	2	2
	15,21	3	2
	21	3	1
$3x2\frac{1}{2}x1/4$	9 to 11	2	1,2
	12,14	2	2
	17,19	1	1
	20,22	1	2
	21	2	1
	23	3	1
4x3x1/4	12,19	2	1
	13,18,20	2	2
$3x2\frac{1}{2}x3/8$	14,18	2	1
	21,23	1	1
	22	2	2
4x3x5/16	16	2	1,2
	21	1	2
$4x3\frac{1}{2}x5/16$	17	2	2
$4x3\frac{1}{2}x3/8$	19	1	2
4x3x3/8	20	2	1
	24	1	2
4x3x1/2	22	2	1
$6x3\frac{1}{2}x3/8$	23	2	2
5x3x1/2	24	2	1

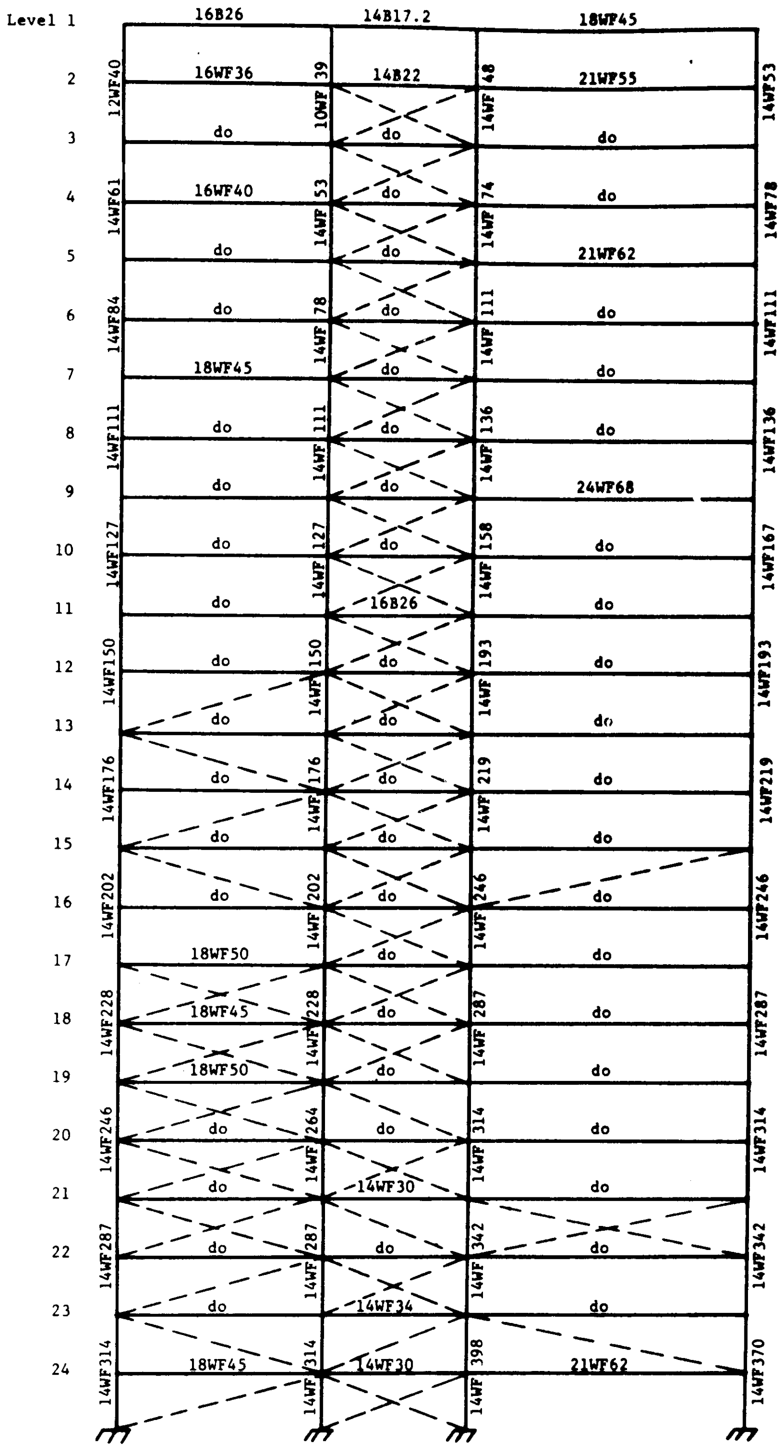
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 29.9 Tons
Column Wt. = 109.2 Tons
Bracing Wt. = 5.9 Tons
Total Wt. = 145.0 Tons
Material Cost = $57,982

Figure 2.11 Example Problem C2.1A



DESIGN CONDITIONS

1. Bracing Permitted in Any Panel.
2. Panel Moment Action Permitted.
3. Plastic Design.
4. Elastic Stress Design ($\sigma_{max} \le \sigma_y$).
5. Elastic Stiffness Design ($\frac{\Delta}{h} \le \frac{1}{400}$).

BRACING SIZES (Double Angles, ┬)

Size	Level	Bay	Type*
3x2x3/16	2 to 6	2	1,2
	7,15,24	2	2
	12,14,17,18	1	2
	13,15,18,20	1	1
	15,21	3	2
	21	3	1
$3x2\frac{1}{2}x1/4$	7,13,15	2	1
	17,21	2	1
	8,12	2	2
	17,19	1	1
	20,22	1	2
	23	3	1
4x3x1/4	8,19	2	1
	9,14,18,20	2	2
$4x3\frac{1}{2}x5/16$	9	2	1
	13,17	2	2
4x3x7/16	10	2	1
$3x2\frac{1}{2}x3/8$	10,11,22	2	2
	18	2	1
	21,23	1	1
$4x3\frac{1}{2}x3/8$	11	2	1
	19	1	2
$6x3\frac{1}{2}x3/8$	12	2	1
	23	2	2
$4x3\frac{1}{2}x7/16$	14,16	2	1
4x3x3/8	16	2	2
	20	2	1
	24	1	2
4x3x5/16	21	1	2
4x3x1/2	22	2	1
5x3x1/2	24	2	1

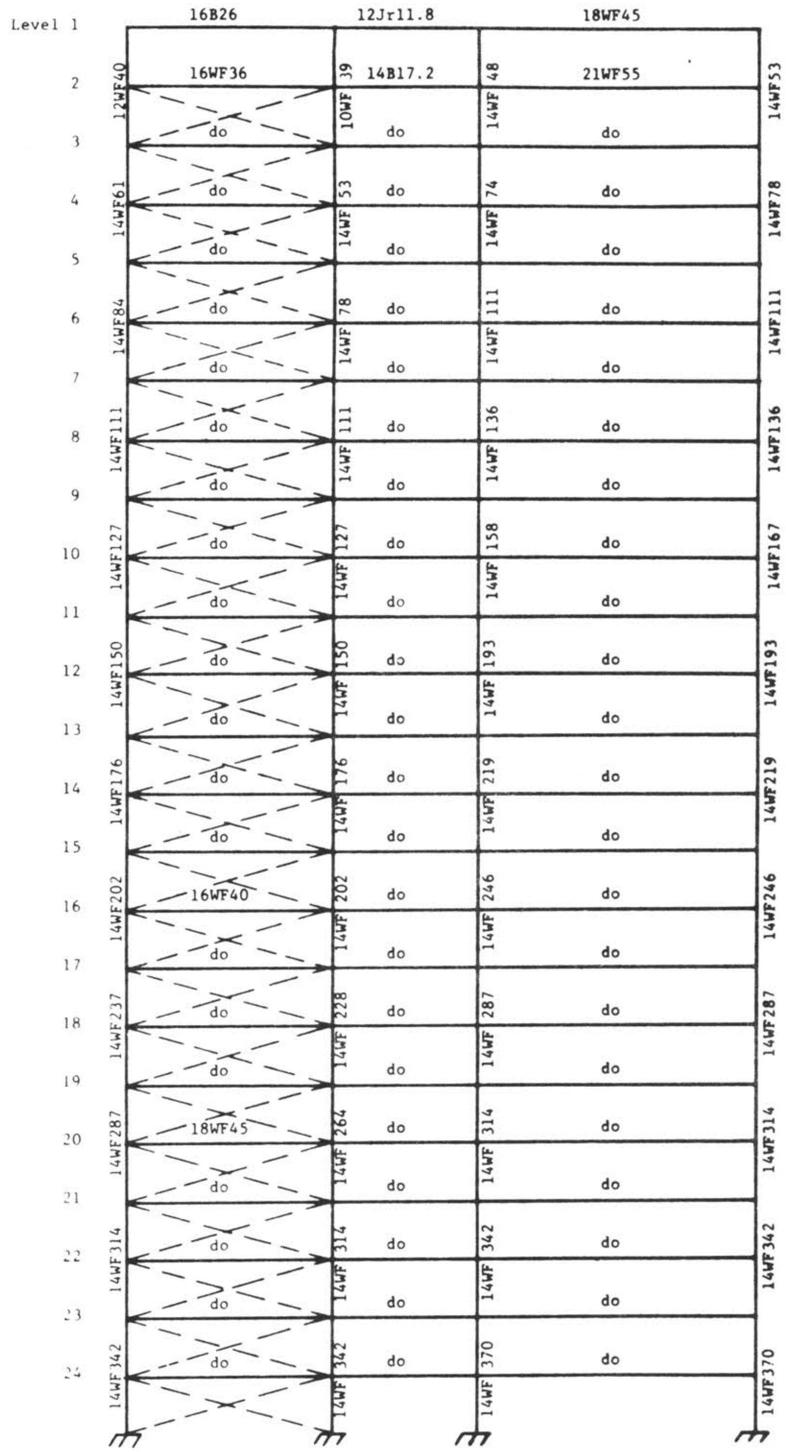
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 35.7 Tons
Column Wt. = 109.2 Tons
Bracing Wt. = 6.7 Tons
Total Wt. = 151.6 Tons
Material Cost = $60,640

Figure 2.12 Example Problem C2.2A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 1 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, T)

Size	Level	Type*
3x2x3/16	2 to 9	1,2
$3\text{x}2\frac{1}{2}\text{x}1/4$	10 to 13	1,2
4x3x1/4	14 to 15	1,2
$3\text{x}2\frac{1}{2}\text{x}3/8$	16 to 17	1,2
4x3x5/16	18	1,2
$4\text{x}3\frac{1}{2}\text{x}3/8$	19	1,2
4x3x7/16	20	1,2
$4\text{x}3\frac{1}{2}\text{x}7/16$	21	1,2
4x3x1/2	22 to 24	1,2

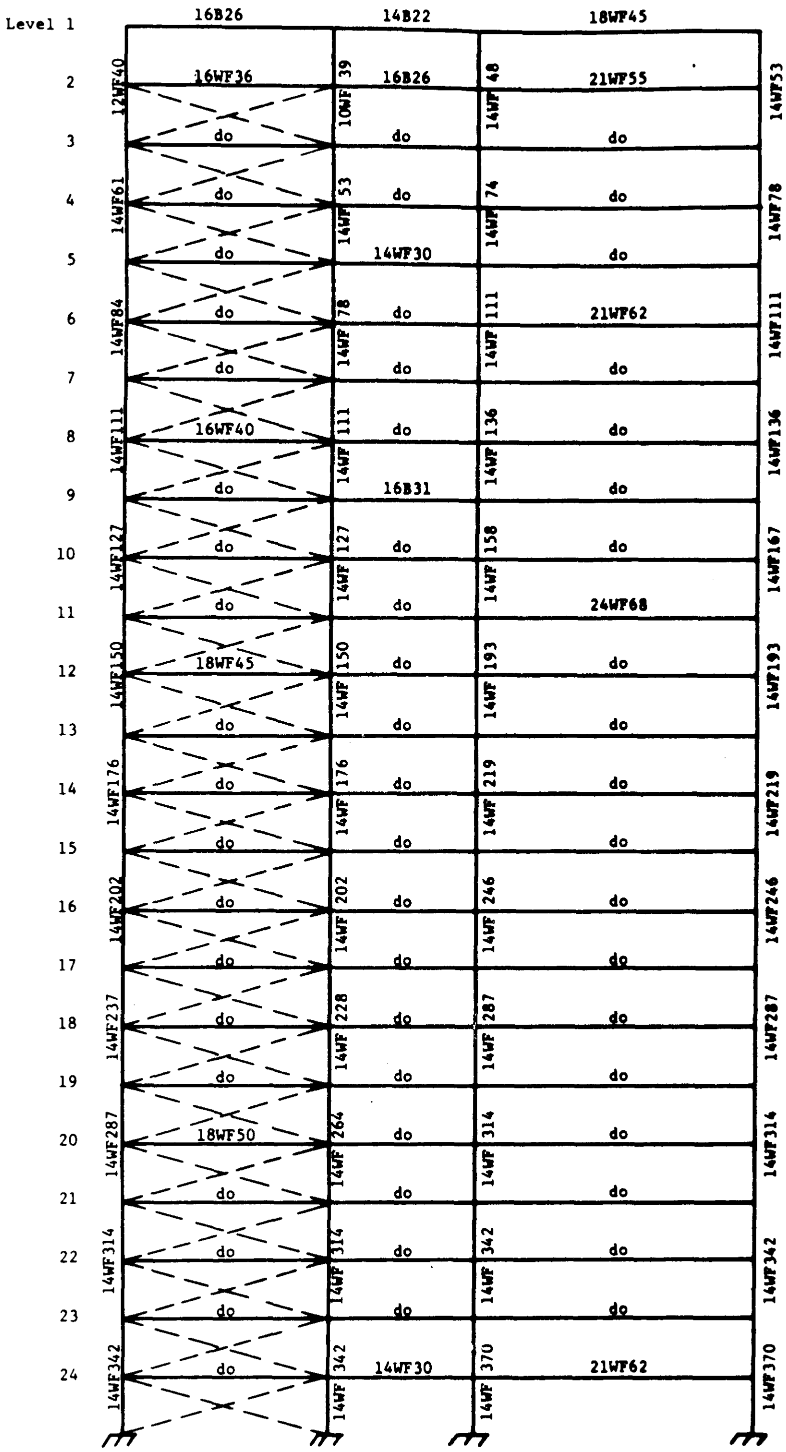
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	=	29.9 Tons
Column Wt.	=	110.7 Tons
Bracing Wt.	=	6.4 Tons
Total Wt.	=	147.1 Tons
Material Cost	=	$58,833

Figure 2.13 Example Problem C3.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 1 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.
4. Elastic Stress Design ($\sigma_{max} \leq \sigma_y$).
5. Elastic Stiffness Design ($\frac{\Delta}{h} \leq \frac{1}{400}$).

BRACING SIZES (Double Angles, ⊤)

Size	Level	Type*
3x2x3/16	2 to 5	1,2
	6	1
$3x2\frac{1}{2}x1/4$	6	2
	7	1,2
	8	1
4x3x1/4	8	2
	9	1,2
4x3x3/8	10,12,13	1
4x3x7/16	10,11	2
$4x3\frac{1}{2}x5/16$	11	1
$4x3\frac{1}{2}x7/16$	12,13	2
	15,17	1
	21	1,2
4x3x1/2	14,18,19	1
	20,22,23,24	1,2
$5x3\frac{1}{2}x1/2$	14	2
5x3x1/2	15	2
$6x3\frac{1}{2}x3/8$	16	1
6x4x7/16	16	2
6x4x3/8	17,18,19	2

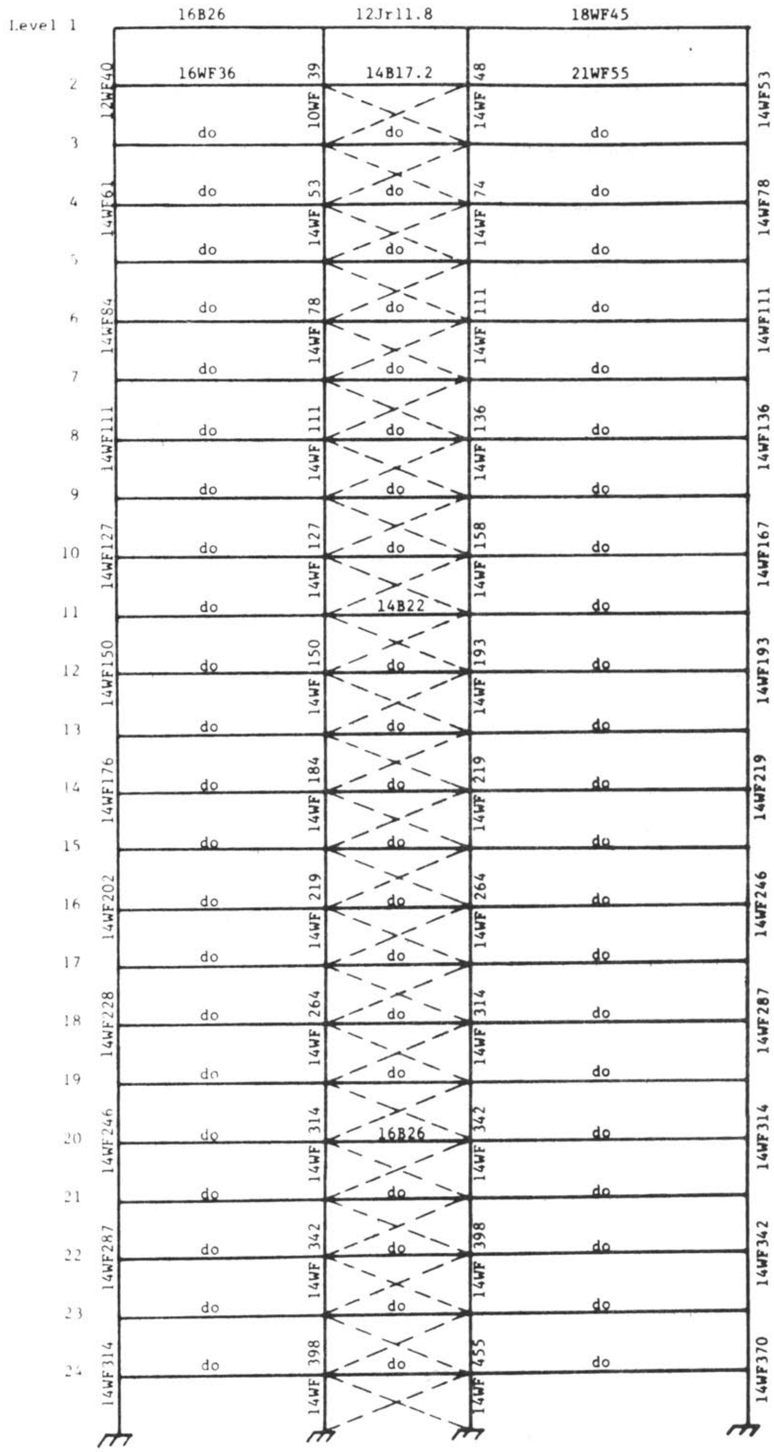
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 35.7 Tons
Column Wt. = 110.7 Tons
Bracing Wt. = 9.1 Tons
Total Wt. = 155.6 Tons
Material Cost = $62,225

Figure 2.14 Example Problem C3.2A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 2 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, ⊤)

Size	Level	Type*
3x2x3/16	2 to 8	1,2
3x2$\frac{1}{2}$x1/4	9 to 11	1,2
4x3x1/4	12 to 13	1,2
3x2$\frac{1}{2}$x3/8	14 to 15	1,2
4x3x5/16	16	1,2
4x3x3/8	17	1,2
4x3$\frac{1}{2}$x3/8	18	1,2
4x3x7/16	19	1,2
4x3x1/2	20	1,2
6x3$\frac{1}{2}$x3/8	21	1,2
6x4x3/8	22	1,2
5x3x1/2	23	1,2
5x3$\frac{1}{2}$x1/2	24	1,2

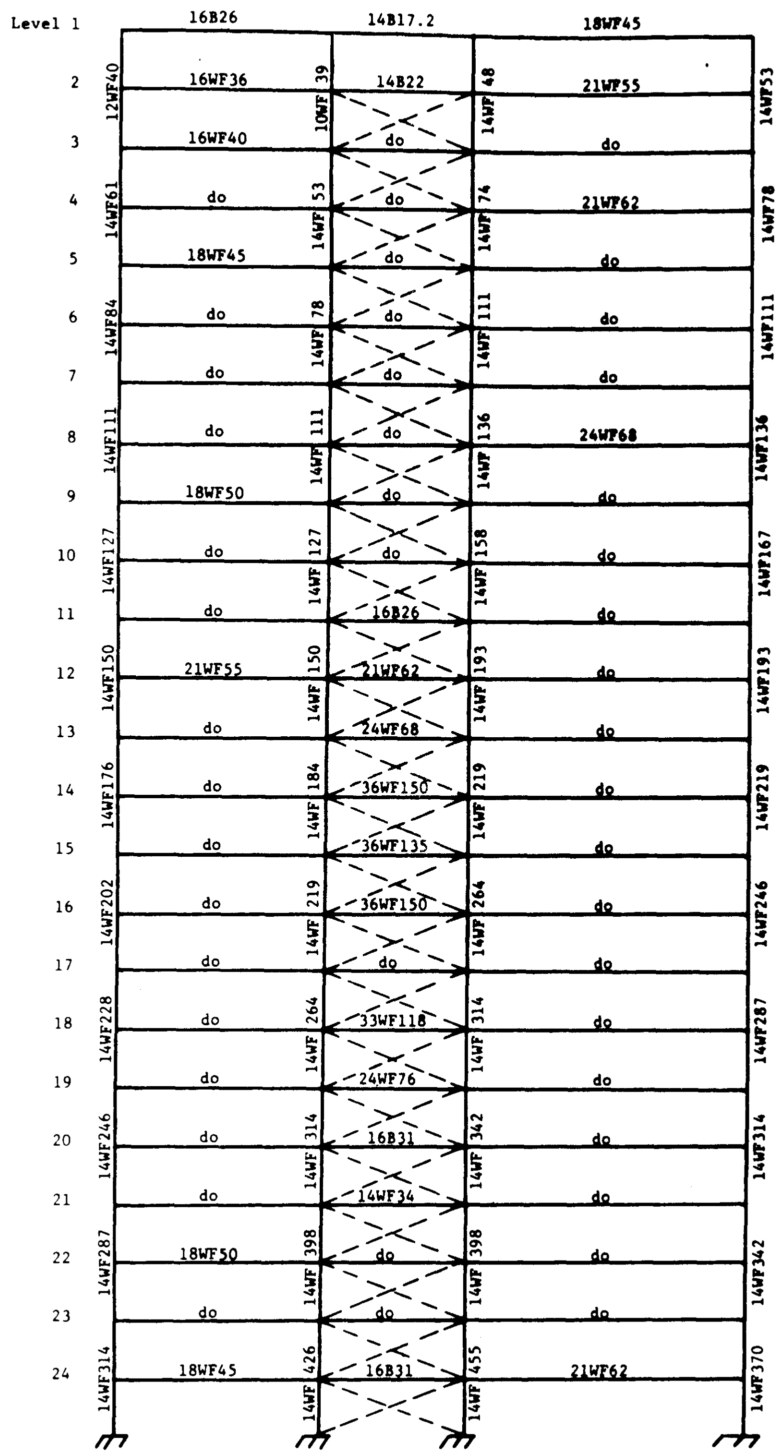
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 29.8 Tons
Column Wt.	= 114.4 Tons
Bracing Wt.	= 5.3 Tons
Total Wt.	= 149.5 Tons
Material Cost	= $59,812

Figure 2.15 Example Problem C4.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 2 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.
4. Elastic Stress Design ($\sigma_{max} \leq \sigma_y$).
5. Elastic Stiffness Design ($\frac{\Delta}{h} \leq \frac{1}{400}$).

BRACING SIZES (Double Angles, ⊥⊥)

Size	Level	Type*
3x2x3/16	2 to 5	1,2
	6	1
$3x2\frac{1}{2}x1/4$	6	2
	7	1,2
4x3x1/4	8	1
$3x2\frac{1}{2}x3/8$	8	2
$4x3\frac{1}{2}x3/8$	9	1
$6x3\frac{1}{2}x3/8$	9	2
	10	1
8x4x7/16	10	2
	12,21	1
4x3x1/2	11	1
8x4x1/2	11,21	2
	15,19	1
8x4x3/4	12,13,15,19	2
	14,16,17	1,2
	18,20	1,2
7x4x7/16	13	1
	22	2
6x4x7/16	22	1
5x3x1/2	23	1,2
$5x3\frac{1}{2}x1/2$	24	1,2

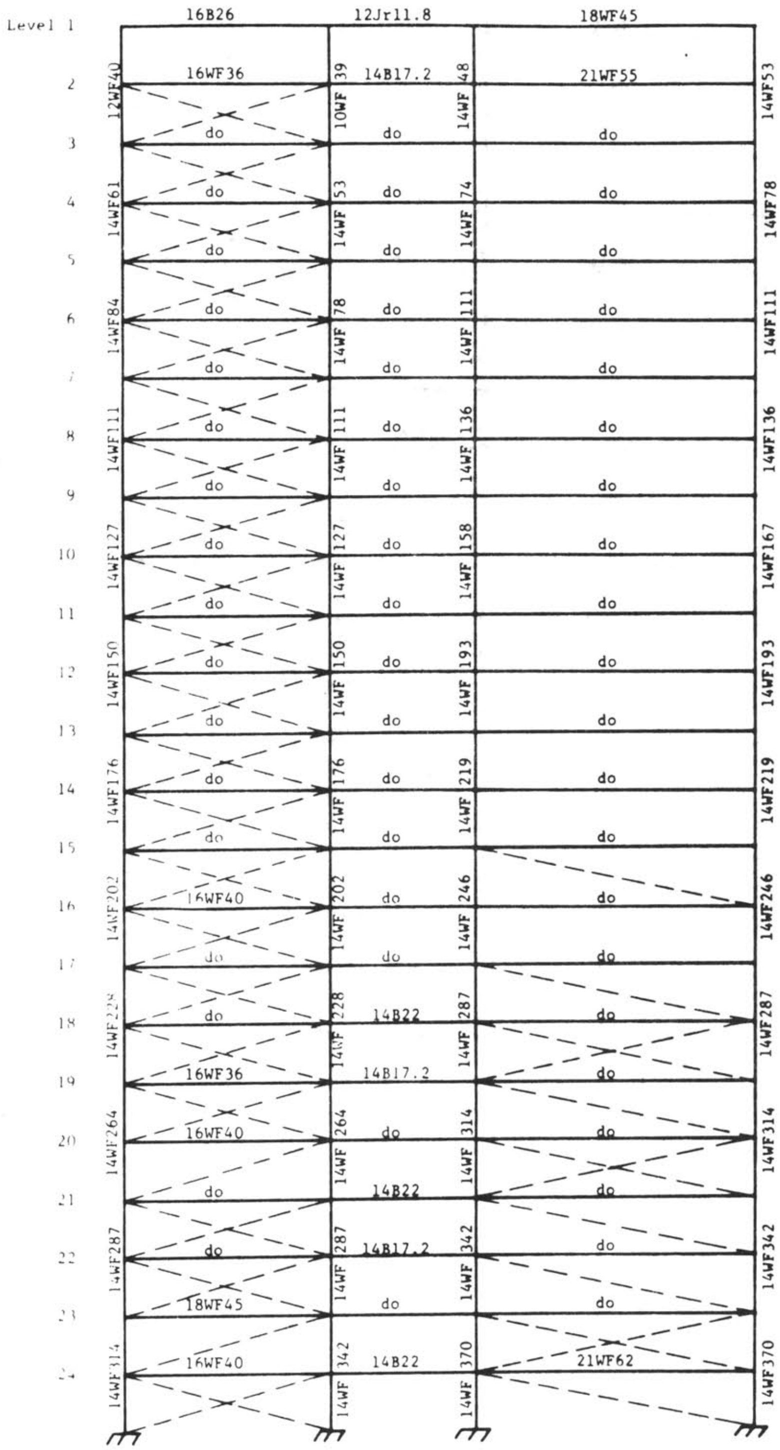
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 41.3 Tons
Column Wt.	= 115.4 Tons
Bracing Wt.	= 12.3 Tons
Total Wt.	= 169.0 Tons
Material Cost	= $67,618

Figure 2.16 Example Problem C4.2A



DESIGN CONDITIONS

1. Bracing Permitted in Bays 1 and 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles,][ )

Size	Level	Bay	Type*
3x2x3/16	2 to 9	1	1,2
	15,18,21	3	1
	17,19	1	1
	18,20,23	3	2
$3\times2\frac{1}{2}\times1/4$	10 to 13	1	1,2
	15	1	1
	17,19,22,24	3	1
	18	1	2
4x3x1/4	14,22	1	1
	14,15	1	2
$3\times2\frac{1}{2}\times3/8$	16,18,21,24	1	1
	16,17,20	1	2
$4\times3\frac{1}{2}\times5/16$	19,23	1	2
$4\times3\frac{1}{2}\times3/8$	20	3	1
	21	1	2
4x3x7/16	22	1	2
	23	3	1
$4\times3\frac{1}{2}\times7/16$	24	1	2

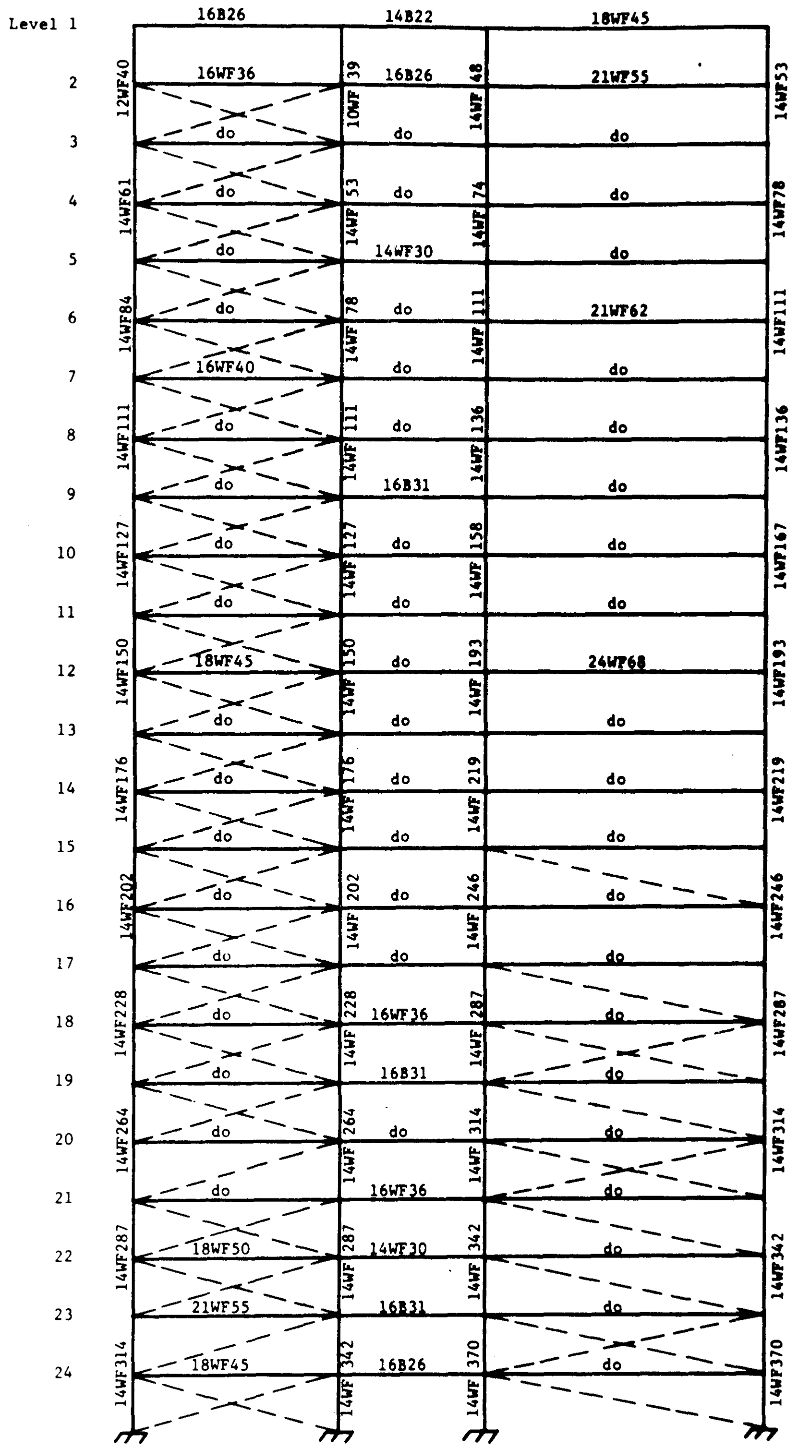
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	=	29.9 Tons
Column Wt.	=	109.4 Tons
Bracing Wt.	=	6.7 Tons
Total Wt.	=	145.9 Tons
Material Cost	=	$58,374

Figure 2.17 Example Problem C5.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bays 1 and 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.
4. Elastic Stress Design ($\sigma_{max} \leq \sigma_y$).
5. Elastic Stiffness Design ($\frac{\Delta}{h} \leq \frac{1}{400}$).

BRACING SIZES (Double Angles, ⊤)

Size	Level	Bay	Type*
3x2x3/16	2 to 7	1	1,2
	15,18,21	3	1
	17,19	1	1
	18,20,23	3	2
$3x2\frac{1}{2}x1/4$	8	1	1,2
	17,19,22,24	3	1
4x3x1/4	9,15,22	1	1
	9,18	1	2
$4x3\frac{1}{2}x5/16$	10 to 13,16	1	1
	23	1	2
4x3x3/8	10 to 12	1	2
$4x3\frac{1}{2}x3/8$	13,19,21	1	2
	14	1	1
	20	3	1
4x3x1/2	14,15	1	2
$6x3\frac{1}{2}x3/8$	16	1	2
4x3x7/16	17,22	1	2
	23	3	1
$3x2\frac{1}{2}x3/8$	18,21,24	1	1
	20	1	2
$4x3\frac{1}{2}x7/16$	24	1	2

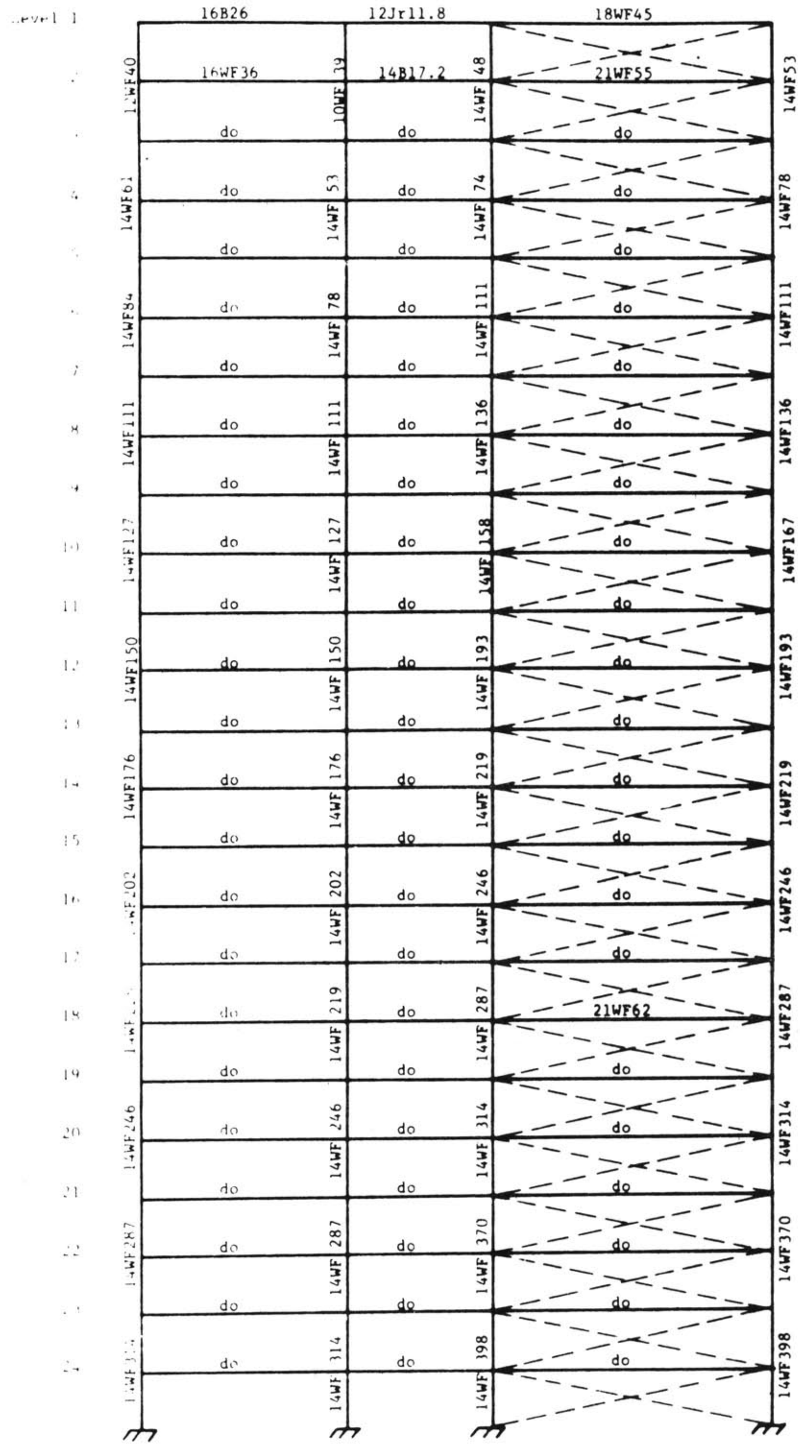
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 35.7 Tons
Column Wt.	= 109.4 Tons
Bracing Wt.	= 8.2 Tons
Total Wt.	= 153.2 Tons
Material Cost	= $61,300

Figure 2.18 Example Problem C5.2A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. No Panel Moment Action Permitted (Lateral shear resisted by bracing system in bay 3).
3. Plastic Design.

BRACING SIZES (Double Angles, ╥)

Size	Level	Type*
3x2x3/16	1 to 5	1,2
$3x2\frac{1}{2}x1/4$	6 to 8	1,2
4x3x1/4	9 to 10	1,2
$3x2\frac{1}{2}x3/8$	11 to 12	1,2
4x3x5/16	13	1,2
$4x3\frac{1}{2}x5/16$	14	1,2
4x3x3/8	15	1,2
$4x3\frac{1}{2}x3/8$	16 to 17	1,2
4x3x7/16	18	1,2
$4x3\frac{1}{2}x7/16$	19	1,2
4x3x1/2	20	1,2
$6x3\frac{1}{2}x3/8$	21	1,2
6x4x3/8	22	1,2
5x3x1/2	23	1,2
$5x3\frac{1}{2}x1/2$	24	1,2

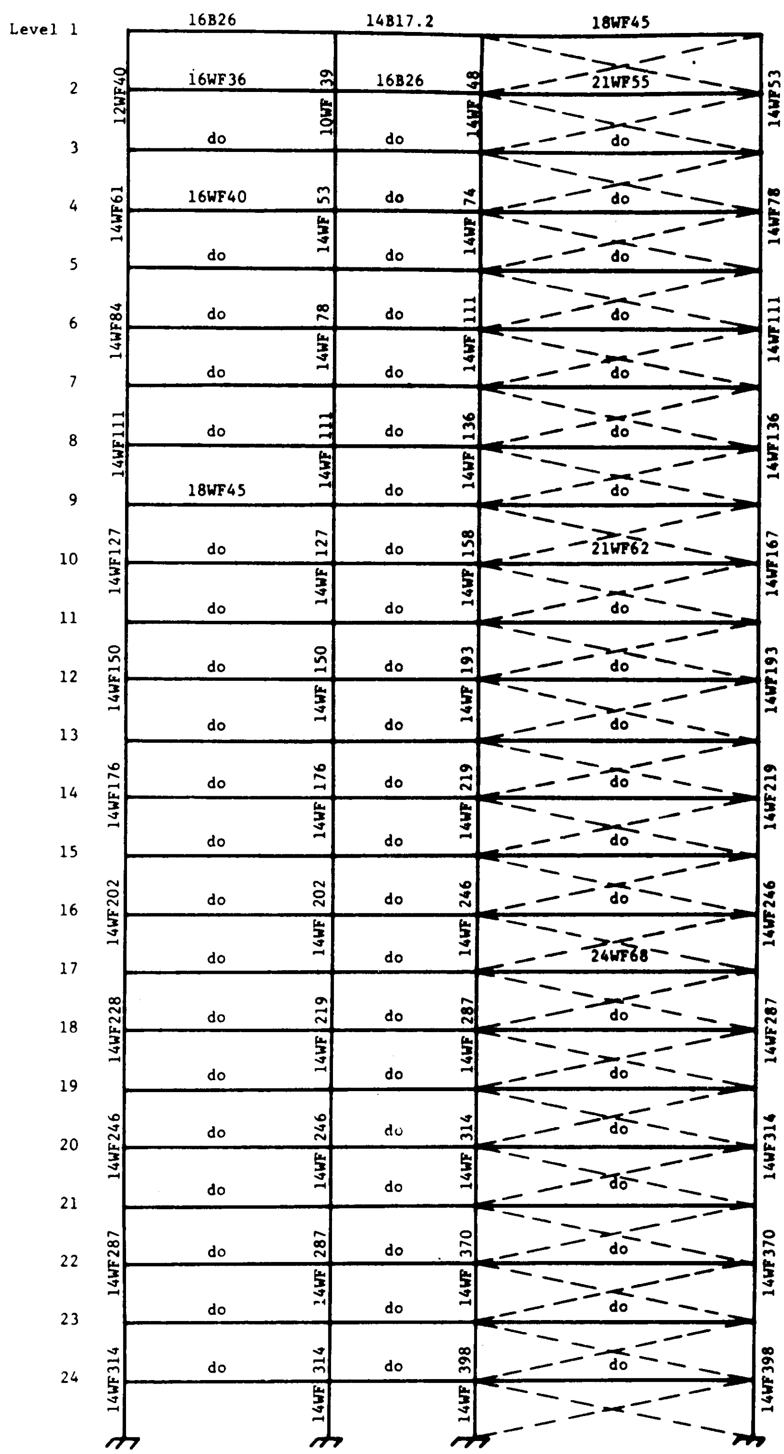
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	=	30.0 Tons
Column Wt.	=	109.8 Tons
Bracing Wt.	=	10.8 Tons
Total Wt.	=	150.6 Tons
Material Cost	=	$60,258

Figure 2.19 Example Problem C6.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. No Panel Moment Action Permitted (Lateral shear resisted by bracing system in bay 3).
3. Plastic Design.
4. Elastic Stress Design ($\sigma_{max} \leq \sigma_y$).
5. Elastic Stiffness Design ($\frac{\Delta}{h} \leq \frac{1}{400}$).

BRACING SIZES (Double Angles, T)

Size	Level	Type*
3x2x3/16	1 to 5	1,2
3x2$\frac{1}{2}$x1/4	6 to 8	1,2
4x3x1/4	9 to 10	1,2
3x2$\frac{1}{2}$x3/8	11 to 12	1,2
4x3x5/16	13	1,2
4x3$\frac{1}{2}$x5/16	14	1,2
4x3x3/8	15	1,2
4x3$\frac{1}{2}$x3/8	16 to 17	1,2
4x3x7/16	18	1,2
4x3$\frac{1}{2}$x7/16	19	1,2
4x3x1/2	20	1,2
6x3$\frac{1}{2}$x3/8	21	1,2
6x4x3/8	22	1,2
5x3x1/2	23	1,2
5x3$\frac{1}{2}$x1/2	24	1,2

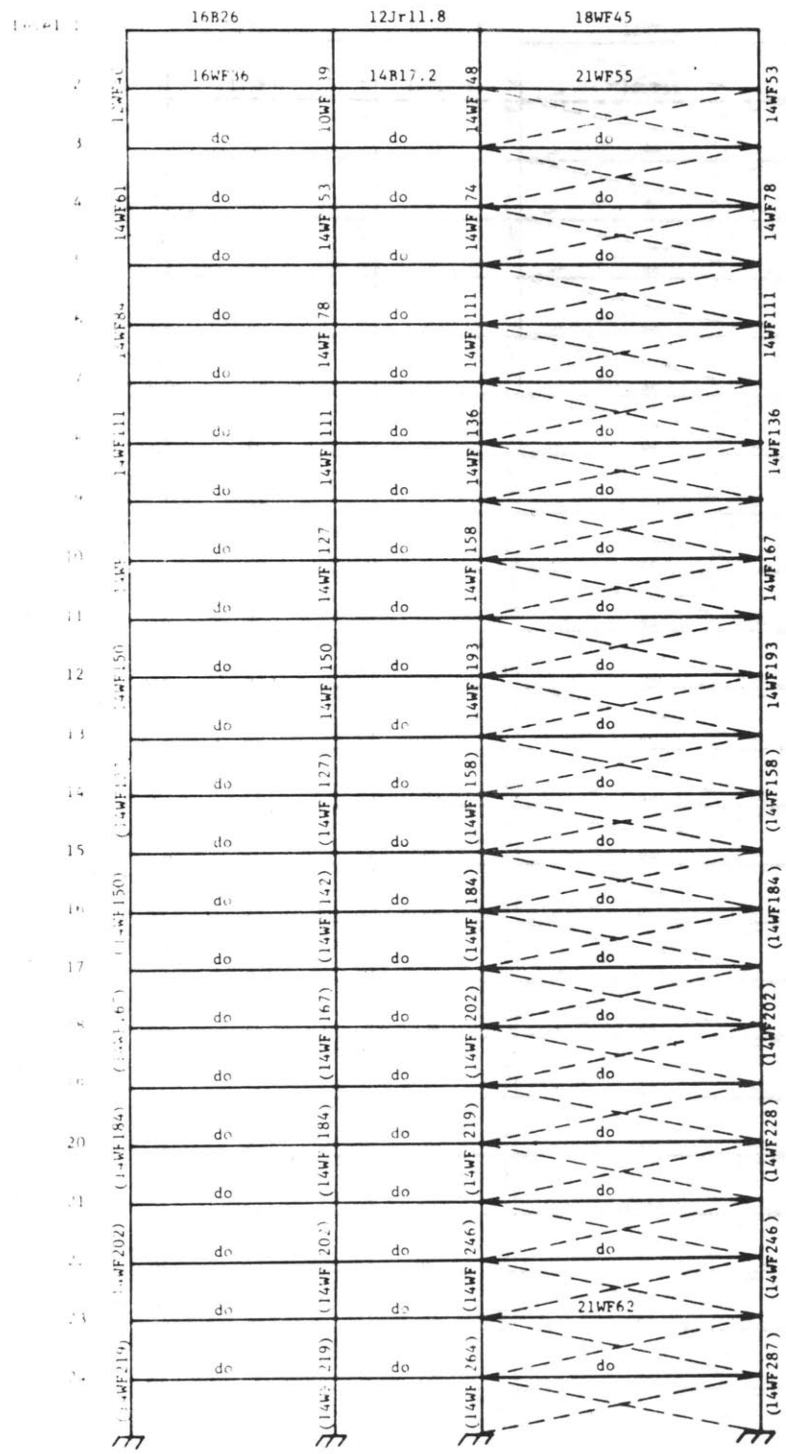
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 34.4 Tons
Column Wt. = 109.8 Tons
Bracing Wt. = 10.8 Tons
Total Wt. = 155.0 Tons
Material Cost = $61,995

Figure 2.20 Example Problem C6.2A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles, ⊤)

Size	Level	Type*
2x2x1/4	2 to 10	1,2
2x2x5/16	11 to 12	1,2
$2\frac{1}{2}$x$2\frac{1}{2}$x5/16	13 to 14	1,2
3x3x3/8	15 to 18	1,2
$3\frac{1}{2}$x$3\frac{1}{2}$x3/8	19 to 21	1,2
4x4x3/8	22 to 23	1,2
$3\frac{1}{2}$x$3\frac{1}{2}$x1/2	24	1,2

* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

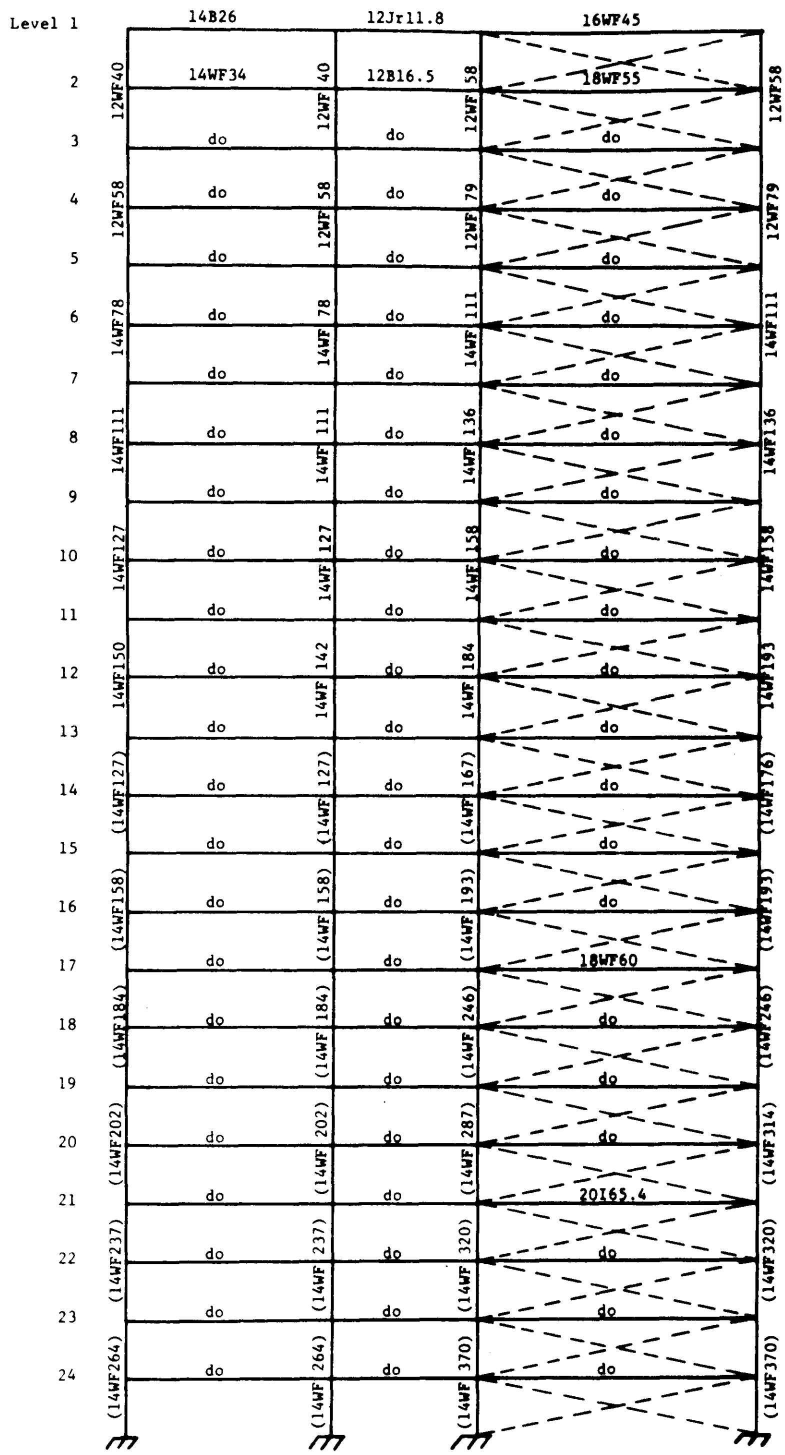
MATERIAL WEIGHT AND COST

A36 and A441 Steel

	A36	A441**
Girder Wt.	= 29.5 Tons	—
Column Wt.	= 31.1 Tons	56.0 Tons
Bracing Wt.	= 8.0 Tons	—
Total Wt.	= 68.6 Tons	56.0 Tons
Material Cost	= $54,329	

**A441 column names given in parentheses.

Figure 2.21 Example Problem C7.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.

BRACING SIZES (Double Angles,][ )

Size	Level	Type*
$3x2\frac{1}{2}x1/4$	1 to 9	1,2
$3\frac{1}{2}x3x1/4$	10 to 11	1,2
4x3x1/4	12	1,2
$4x3\frac{1}{2}x1/4$	13 to 16	1,2
5x3x5/16	17 to 18	1,2
$5x3\frac{1}{2}x3/8$	19 to 22	1,2
5x3x7/16	23 to 24	1,2

* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

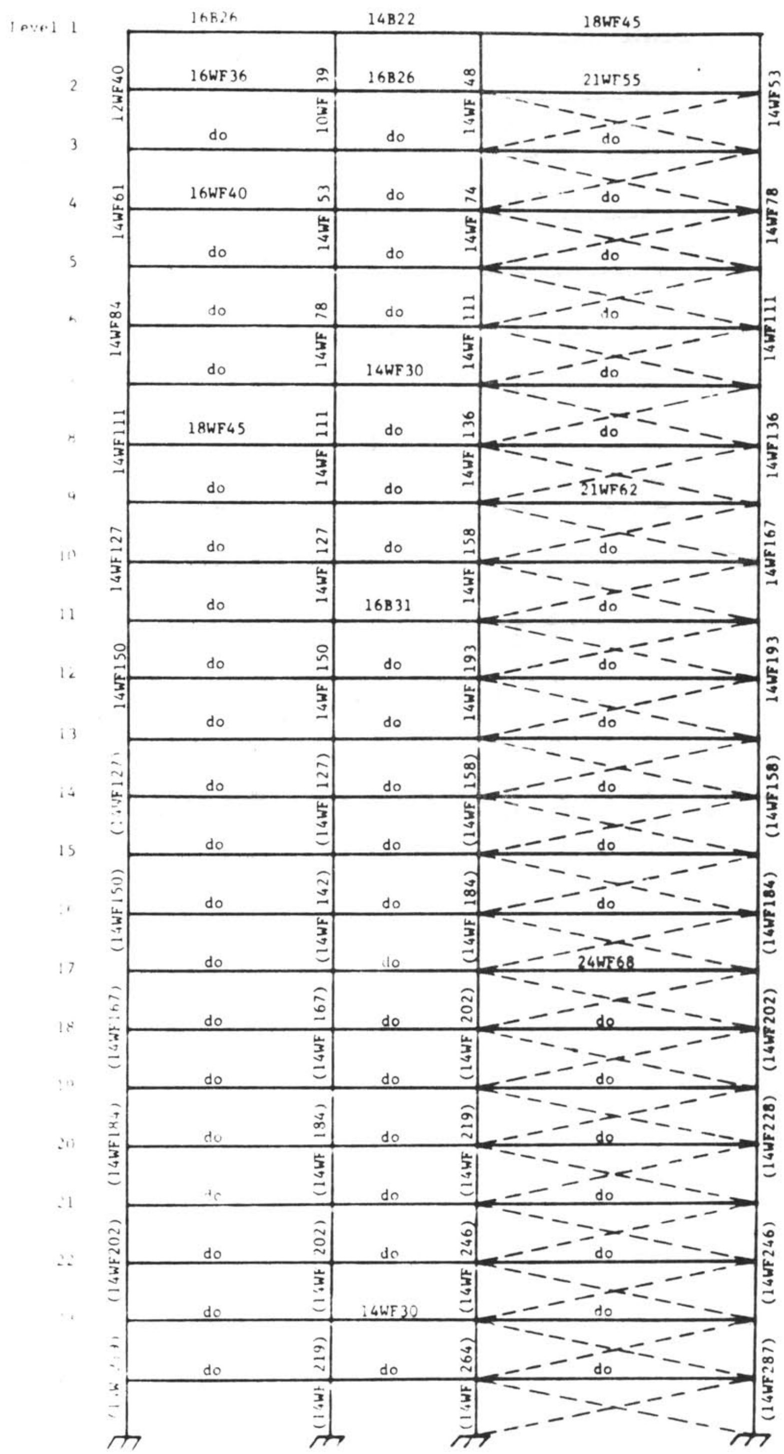
MATERIAL WEIGHT AND COST

A36 and A441 Steel

	A36	A441**
Girder Wt.	= 28.1 Tons	—
Column Wt.	= 31.1 Tons	66.5 Tons
Bracing Wt.	= 9.4 Tons	—
Total Wt.	= 68.6 Tons	66.5 Tons
Material Cost	= $59,300	

**A441 column names given in parenthese

Figure 2.22 Example Problem C7.1L



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Plastic Design.
4. Elastic Stress Design ($\sigma_{max} \le \sigma_y$).
5. Elastic Stiffness Design ($\frac{\Delta}{h} \le \frac{1}{400}$).

BRACING SIZES (Double Angles,][ )

Size	Level	Type*
2x2x1/4	2 to 7	1,2
	8	2
$2\frac{1}{2}$x$2\frac{1}{2}$x5/16	8,9	1
	10	2
2x2x5/16	9	2
3x3x3/8	10,11	1
	11,12	2
$3\frac{1}{2}$x$3\frac{1}{2}$x3/8	12	1
	13	2
$3\frac{1}{2}$x$3\frac{1}{2}$x1/2	13,14,21,24	1
	15 to 22,24	2
4x4x3/8	14,23	2
	23	1
5x5x7/16	15,16	1
4x4x1/2	17 to 20,22	1

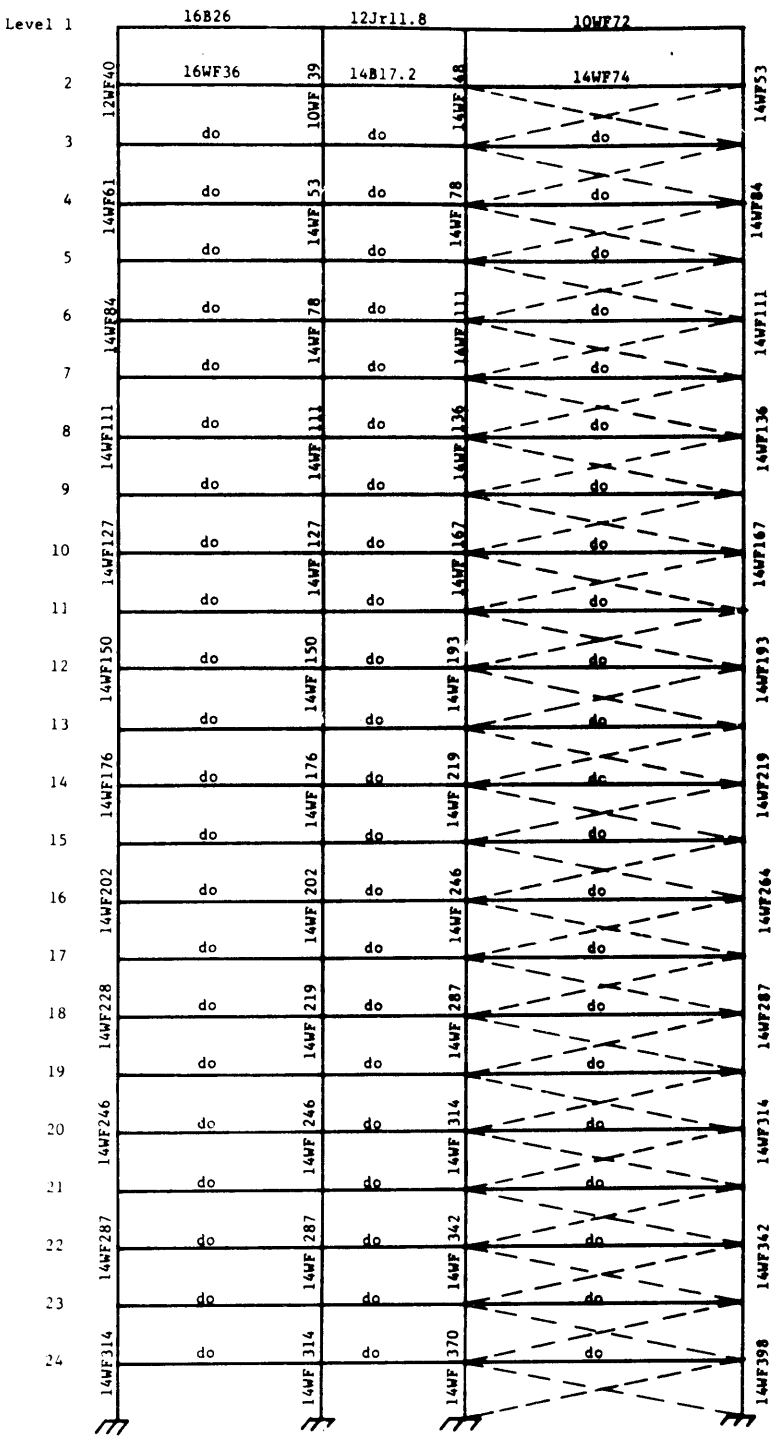
* Type = 1 = Tension, wind from right.
        = 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST
A36 and A441 Steel

	A36	A441**
Girder Wt.	= 35.0 Tons	—
Column Wt.	= 31.1 Tons	56.0 Tons
Bracing Wt.	= 11.4 Tons	—
Total Wt	= 77.5 Tons	56.0 Tons
Material Cost	= $57,867	

**A441 column names given in parentheses

Figure 2.23 Example Problem C7.2A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Maximum Allowable Beam Depth = 17".
4. Plastic Design.

BRACING SIZES (Double Angles,][ )

Size	Level	Type*
3x2x3/16	2 to 10	1,2
$3x2\frac{1}{2}x1/4$	11 to 13	1,2
4x3x1/4	14 to 16	1,2
$3x2\frac{1}{2}x3/8$	17	1,2
4x3x5/16	18	1,2
$4x3\frac{1}{2}x5/16$	19	1,2
4x3x3/8	20 to 21	1,2
$4x3\frac{1}{2}x3/8$	22	1,2
4x3x7/16	23	1,2
$4x3\frac{1}{2}x7/16$	24	1,2

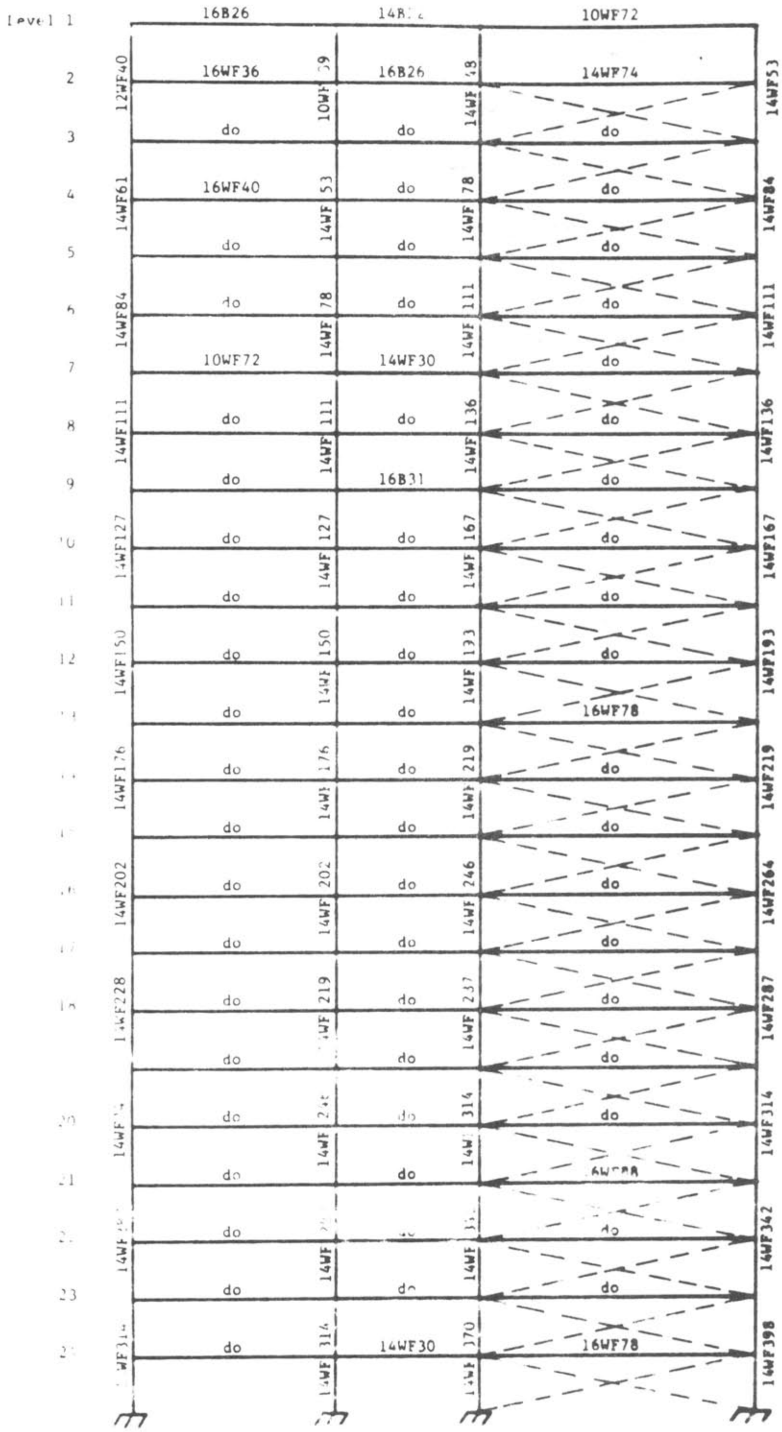
* Type = 1=Tension, wind from right.
= 2=Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 35.8 Tons
Column Wt. = 109.3 Tons
Bracing Wt. = 7.7 Tons
Total Wt. = 152.8 Tons
Material Cost = $61,120

Figure 2.24 Example Problem C8.1A



DESIGN CONDITIONS

1. Bracing Permitted in Bay 3 Only.
2. Panel Moment Action Permitted.
3. Maximum Allowable Beam Depth = 17".
4. Plastic Design.
5. Elastic Stress Design ($\sigma_{max} \le \sigma_y$).
6. Elastic Stiffness Design ($\frac{\Delta}{h} \le \frac{1}{400}$).

BRACING SIZES (Double Angles,][)

Size	Level	Type*
3x2x3/16	2 to 7	1,2
	8,9	2
$3x2\frac{1}{2}x1/4$	8,9	1
	10	2
4x3x1/4	11,12,13	2
	10	1
$3x2\frac{1}{2}x3/8$	11	1
4x3x5/16	12,13	1
	14	2
4x3x3/8	14,15	1
	17	2
$4x3\frac{1}{2}x5/16$	15,16	2
$4x3\frac{1}{2}x3/8$	16,17	1
	18,19,21	2
4x3x7/16	18,19,21	1
	22,23	1,2
$4x3\frac{1}{2}x7/16$	20,24	1,2

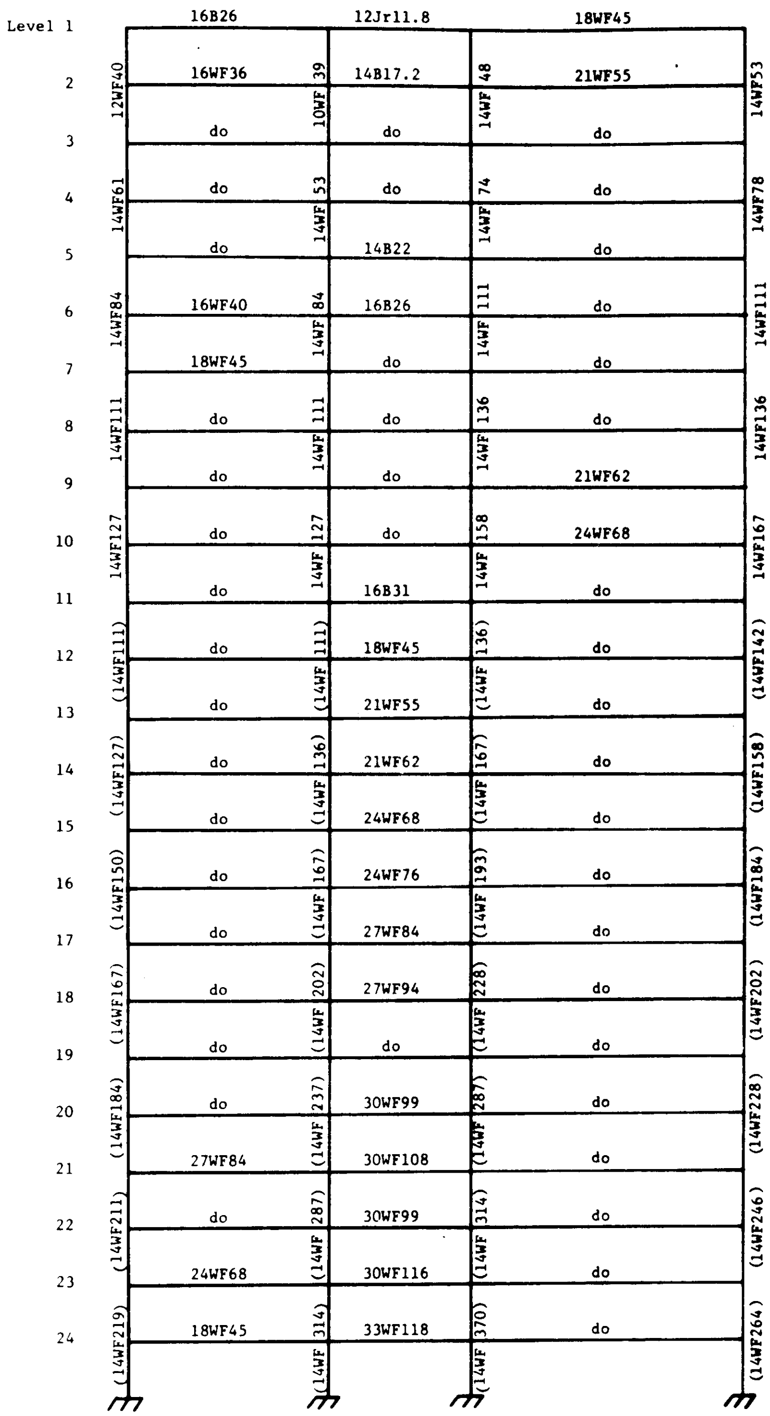
* Type = 1 = Tension, wind from right.
= 2 = Tension, wind from left.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 45.3 Tons
Column Wt.	= 109.3 Tons
Bracing Wt.	= 9.3 Tons
Total Wt.	= 163.9 Tons
Material Cost	= $65,554

Figure 2.25 Example Problem C8.2A



DESIGN CONDITIONS

1. Unbraced Frame.
2. Plastic Design.

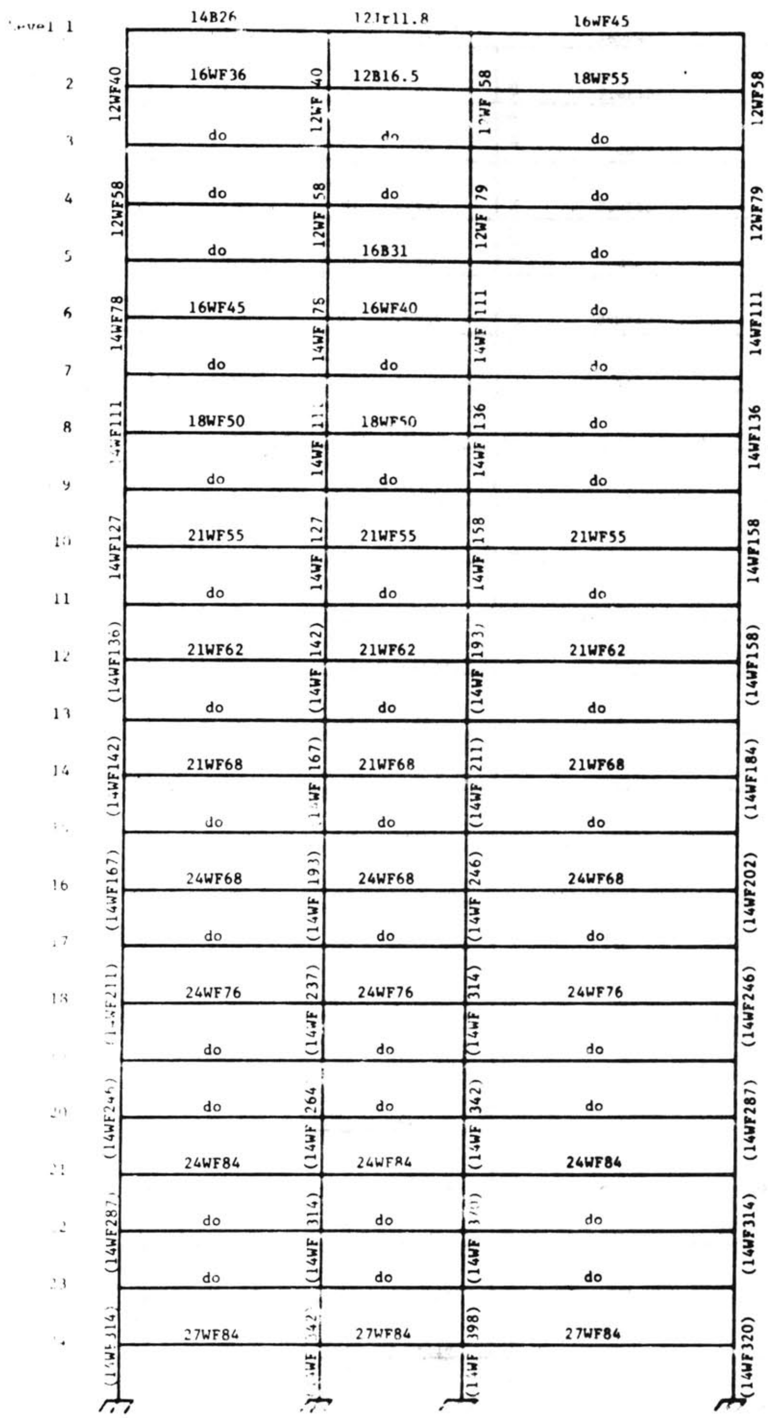
MATERIAL WEIGHT AND COST

A36 and A441 Steel

		A36	A441**
Girder Wt.	=	40.6 Tons	—
Column Wt.	=	22.9 Tons	68.9 Tons
Total Wt.	=	63.5 Tons	68.9 Tons
Material Cost	=	$58,463	

**A441 column names given in parentheses.

Figure 2.26 Example Problem C9.1A



DESIGN CONDITIONS

1. Unbraced Frame.
2. Plastic Design.

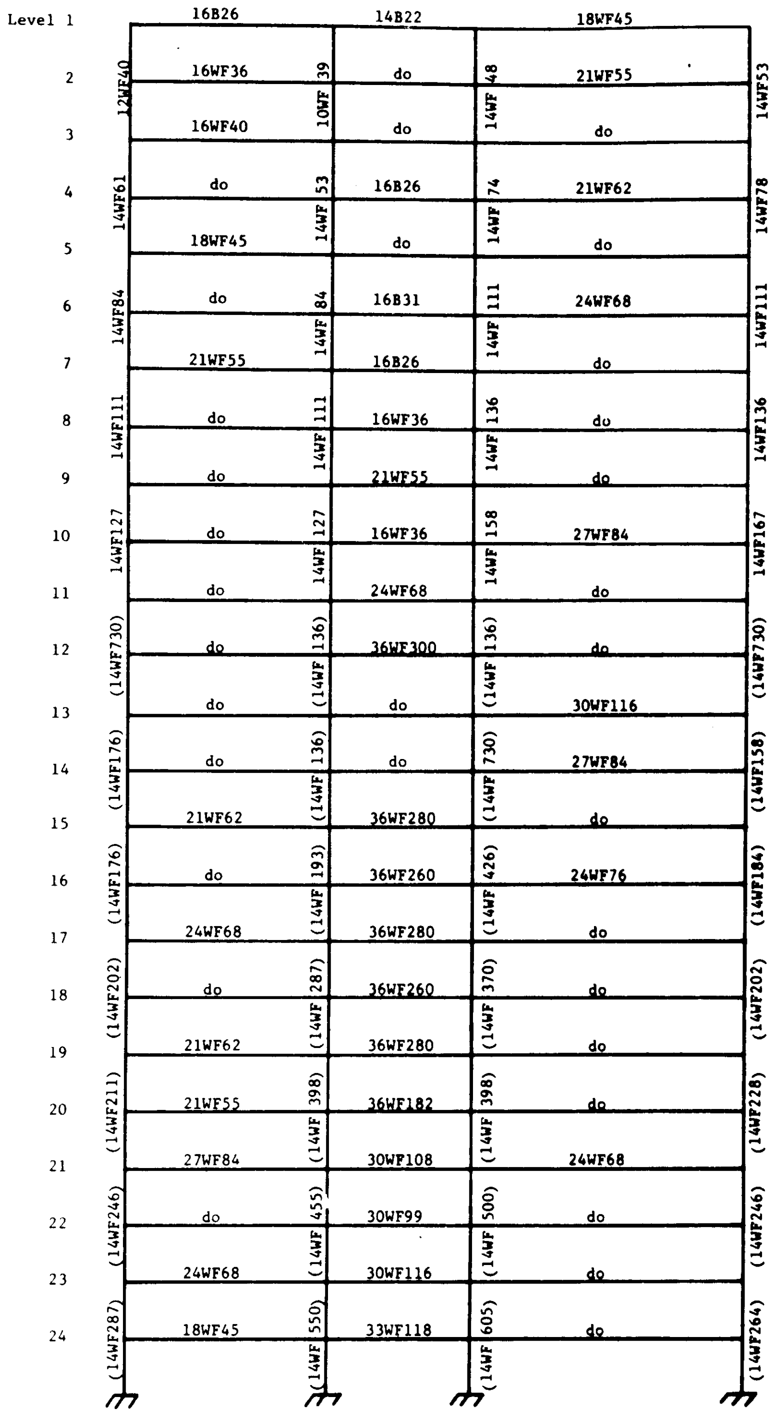
MATERIAL WEIGHT AND COST

A36 and A441 Steel

	A36	A441**
Girder Wt.	= 42.8 Tons	—
Column Wt.	= 23.0 Tons	83.4 Tons
Total Wt.	= 65.8 Tons	83.4 Tons
Material Cost	= $66,352	

**A441 column names given in parentheses.

Figure 2.27 Example Problem C9.1L



DESIGN CONDITIONS

1. Unbraced Frame.
2. Plastic Design.
3. Elastic Stress Design ($\sigma_{max} \le \sigma_y$).
4. Elastic Stiffness Design ($\frac{\Delta}{h} \le \frac{1}{400}$).

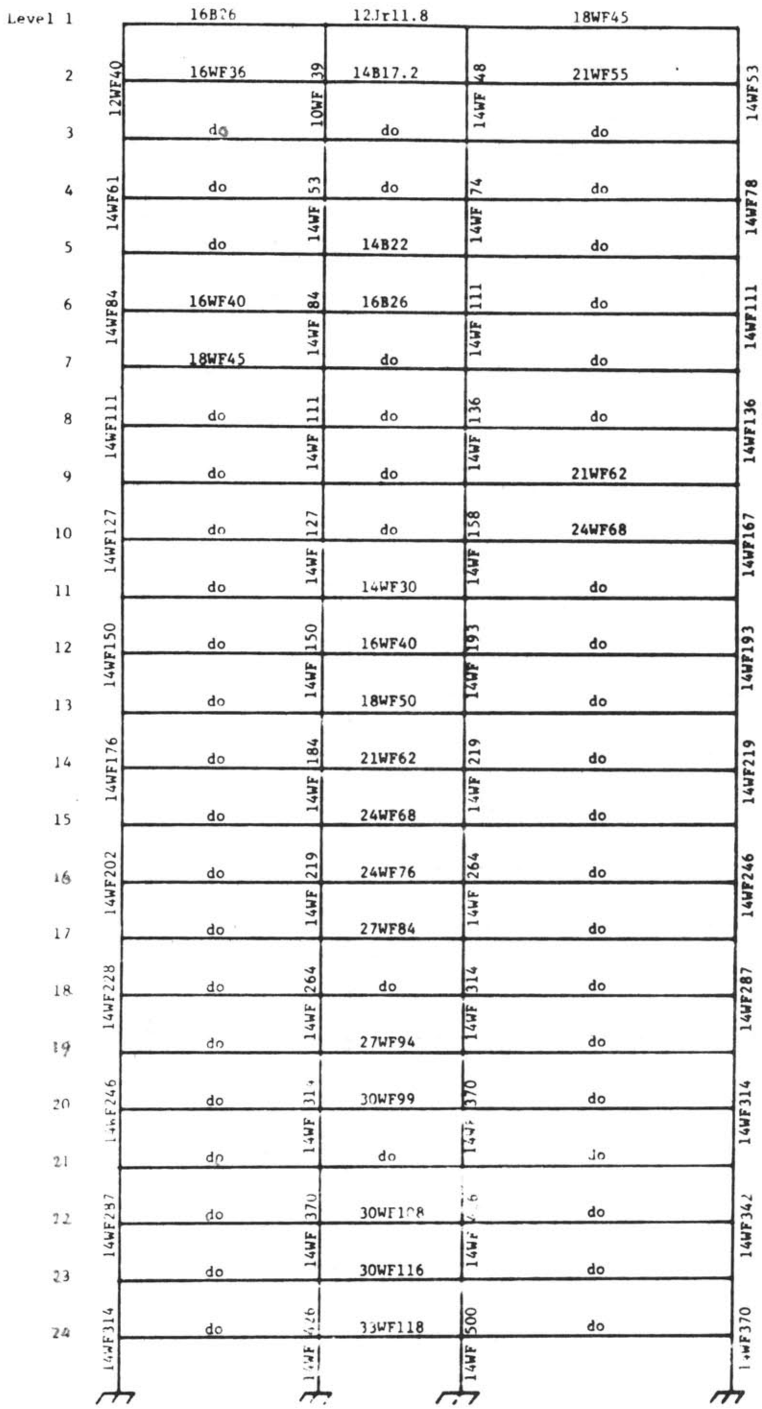
MATERIAL WEIGHT AND COST

A36 and A441 Steel

		A36	A441**
Girder Wt.	=	57.2 Tons	—
Column Wt.	=	22.9 Tons	112.3 Tons
Total Wt.	=	80.1 Tons	112.3 Tons
Material Cost	=	$85,942	

**A441 columns given in parentheses.

Figure 2.28 Example Problem C9.2A



DESIGN CONDITIONS

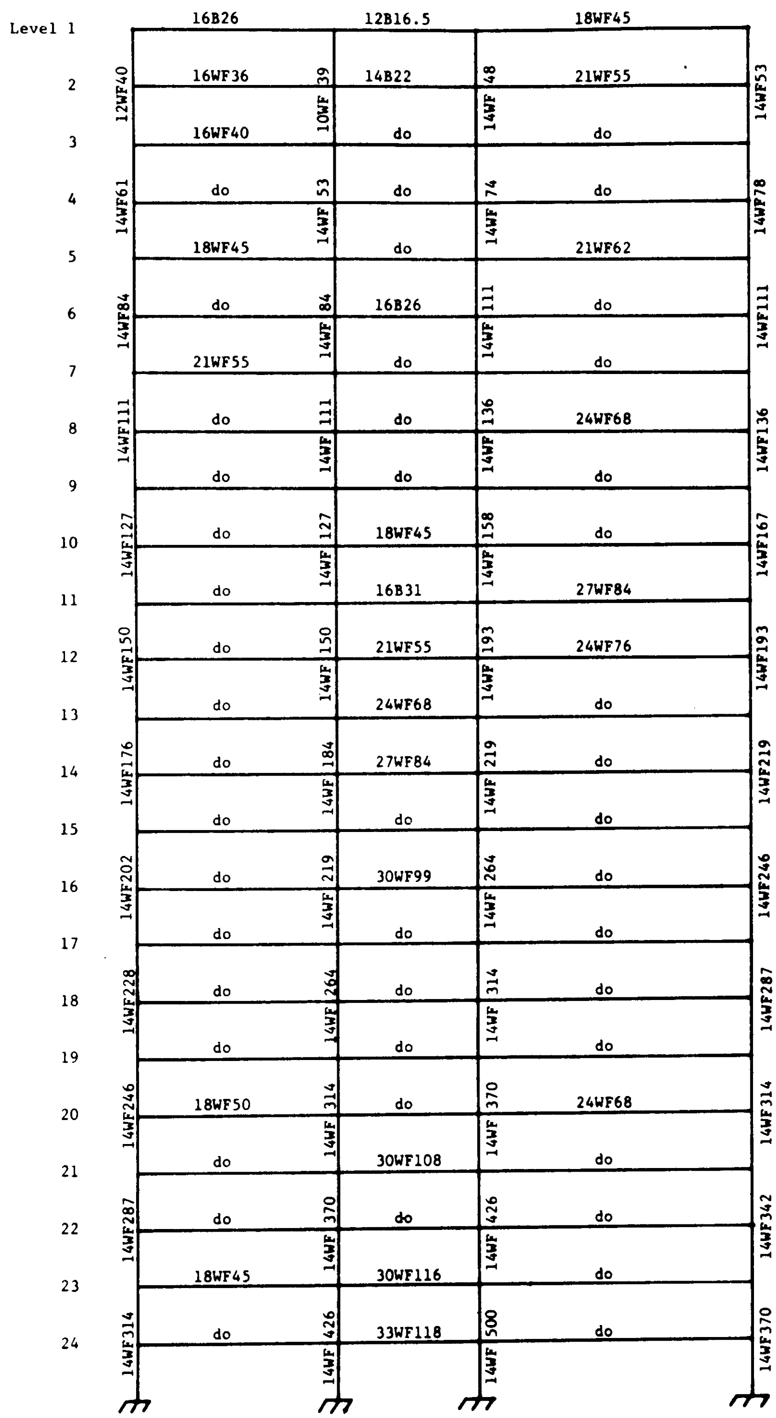
1. Unbraced Frame.
2. Plastic Design.

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt. = 39.4 Tons
Column Wt. = 116.4 Tons
Total Wt. = 155.8 Tons
Material Cost = $62,312

Figure 2.29 Example Problem C10.1A



DESIGN CONDITIONS

1. Unbraced Frame.
2. Plastic Design.
3. Elastic Stress Design ($\sigma_{max} \le \sigma_y$).
4. Elastic Stiffness Design ($\frac{\Delta}{h} \le \frac{1}{400}$).

MATERIAL WEIGHT AND COST

A36 Steel

Girder Wt.	= 43.8 Tons
Column Wt.	= 116.4 Tons
Total Wt.	= 160.2 Tons
Material Cost	= $64,077

Figure 2.30 Example Problem C10.2A

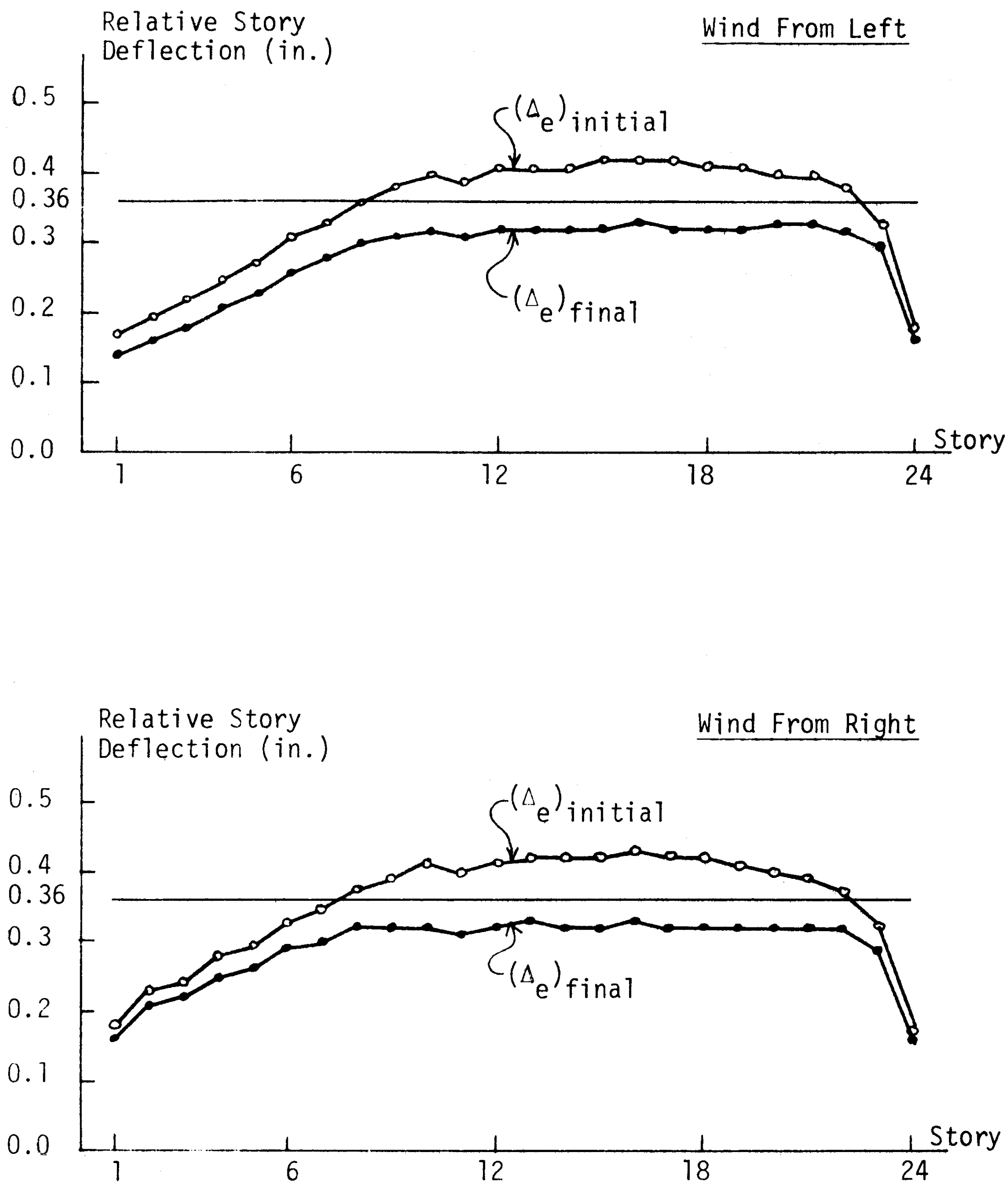
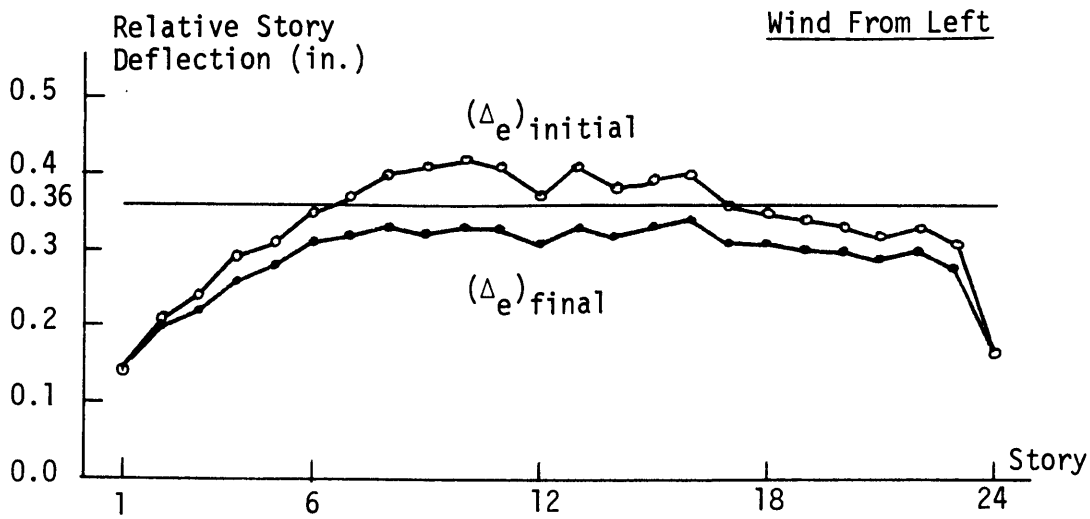


Figure 2.31 Example C1.2A, Initial and Final Elastic Relative Story Deflections.



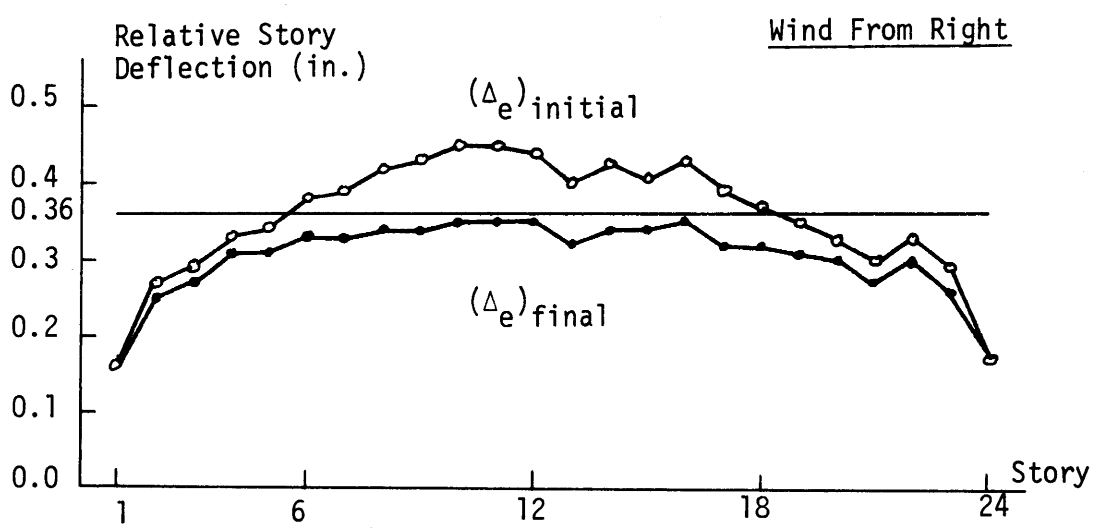
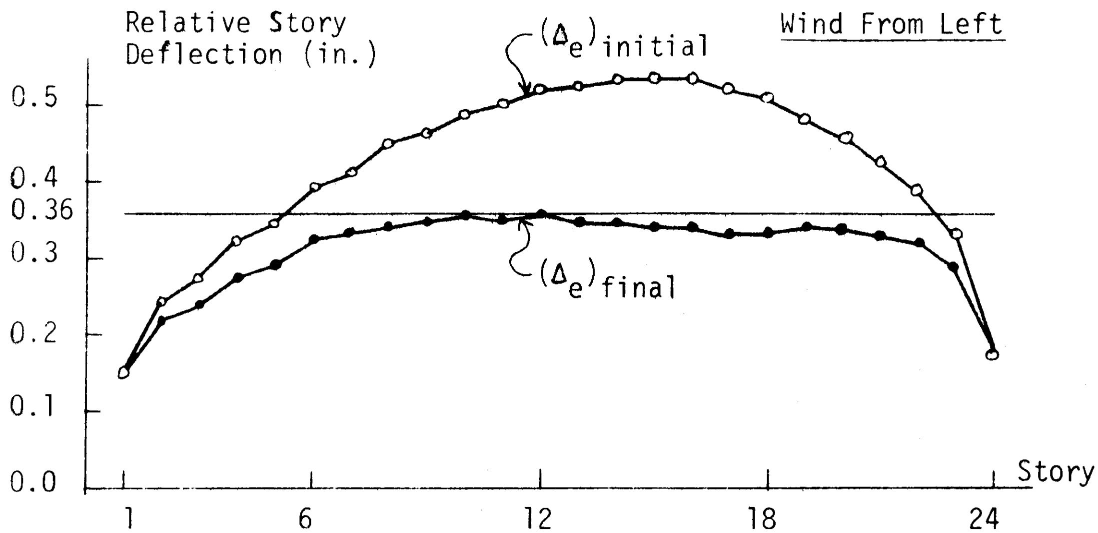


Figure 2.32 Example C2.2A, Initial and Final Elastic Relative Story Deflections.



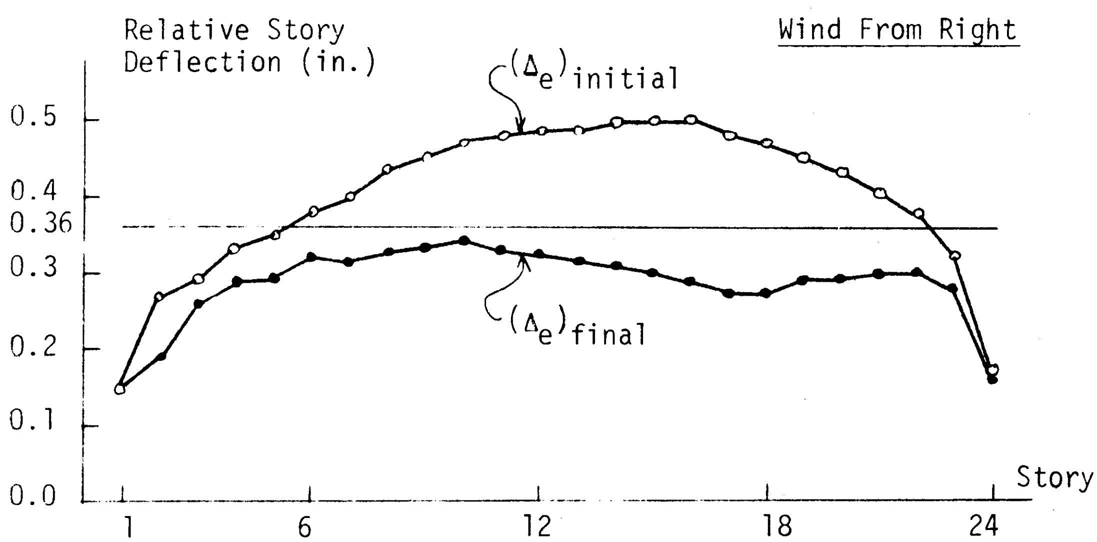
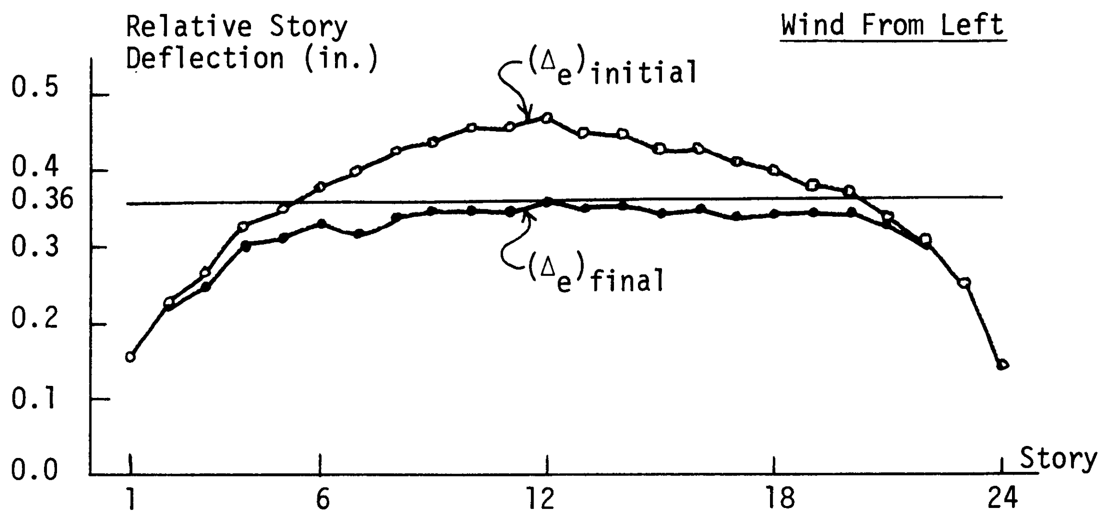


Figure 2.33 Example C4.2A, Initial and Final Elastic Relative Story Deflections.



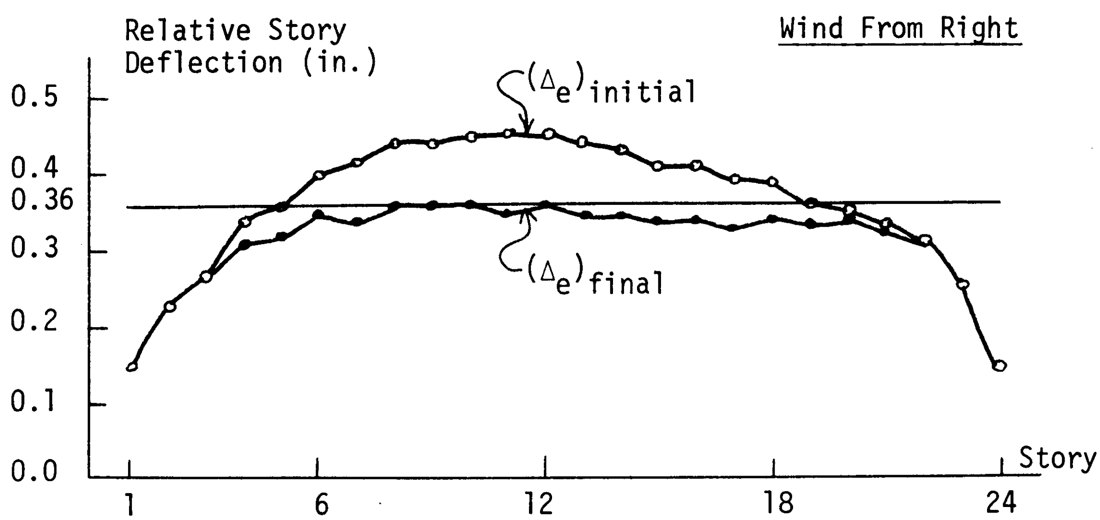
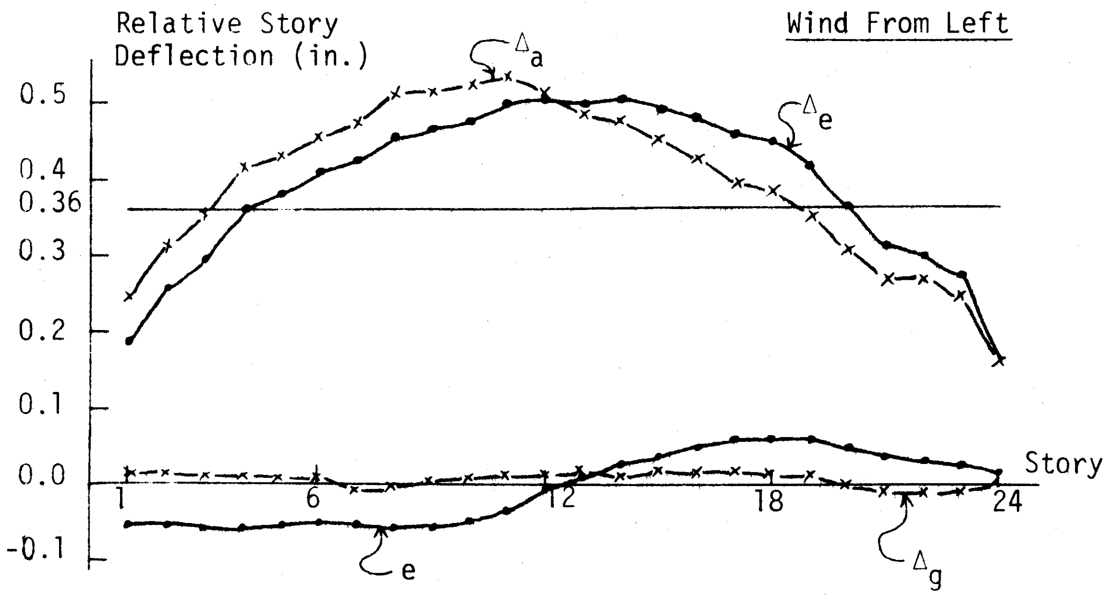


Figure 2.34 Example C10.2A, Initial and Final Elastic Relative Story Deflections.



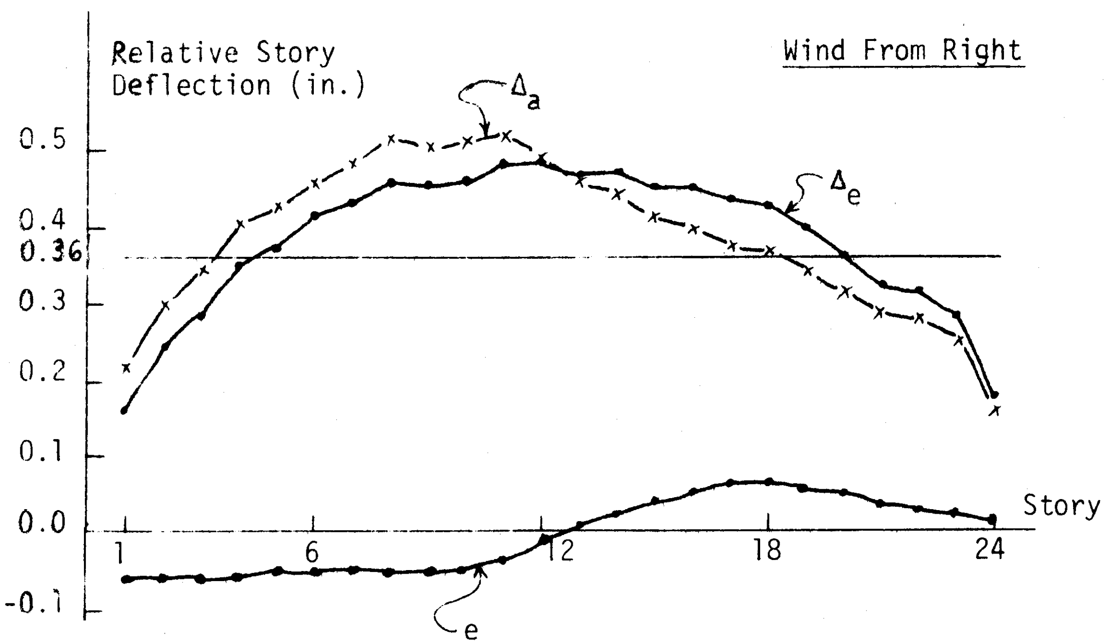
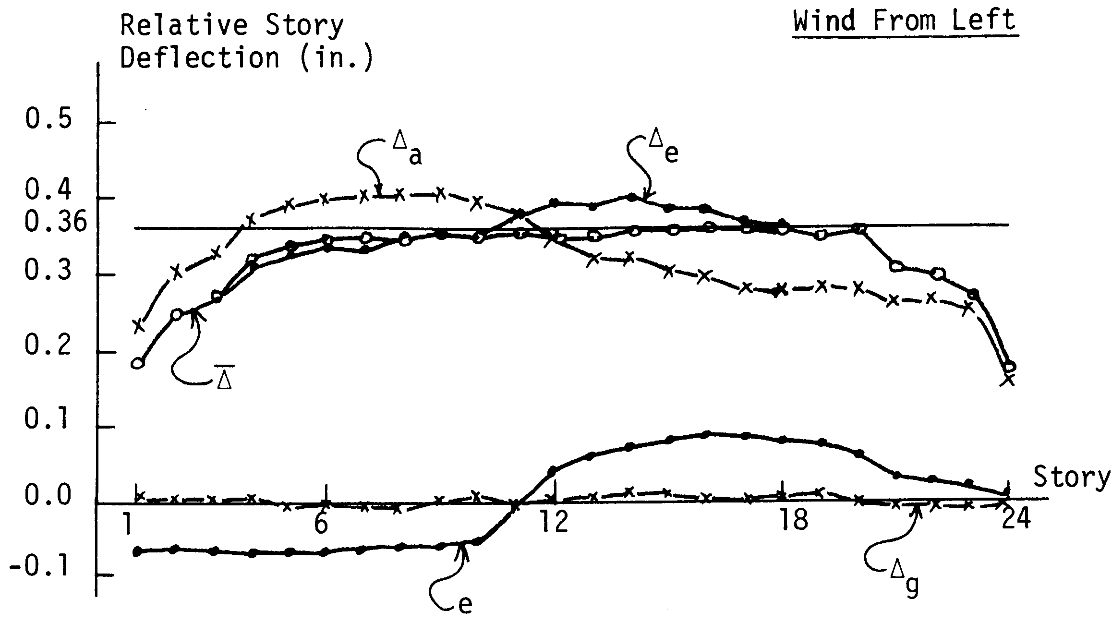


Figure 2.35 Example C9.2A, Initial Elastic Relative Story Deflections.



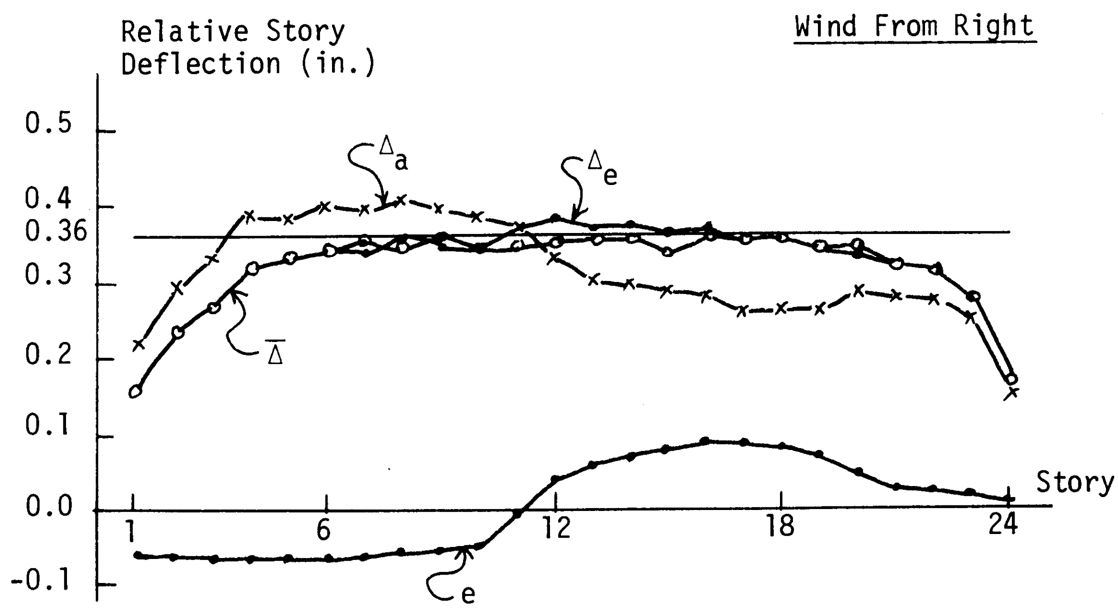


Figure 2.36 Example C9.2A, Elastic Relative Story Deflections After First Execution of the Elastic Stiffness Design.

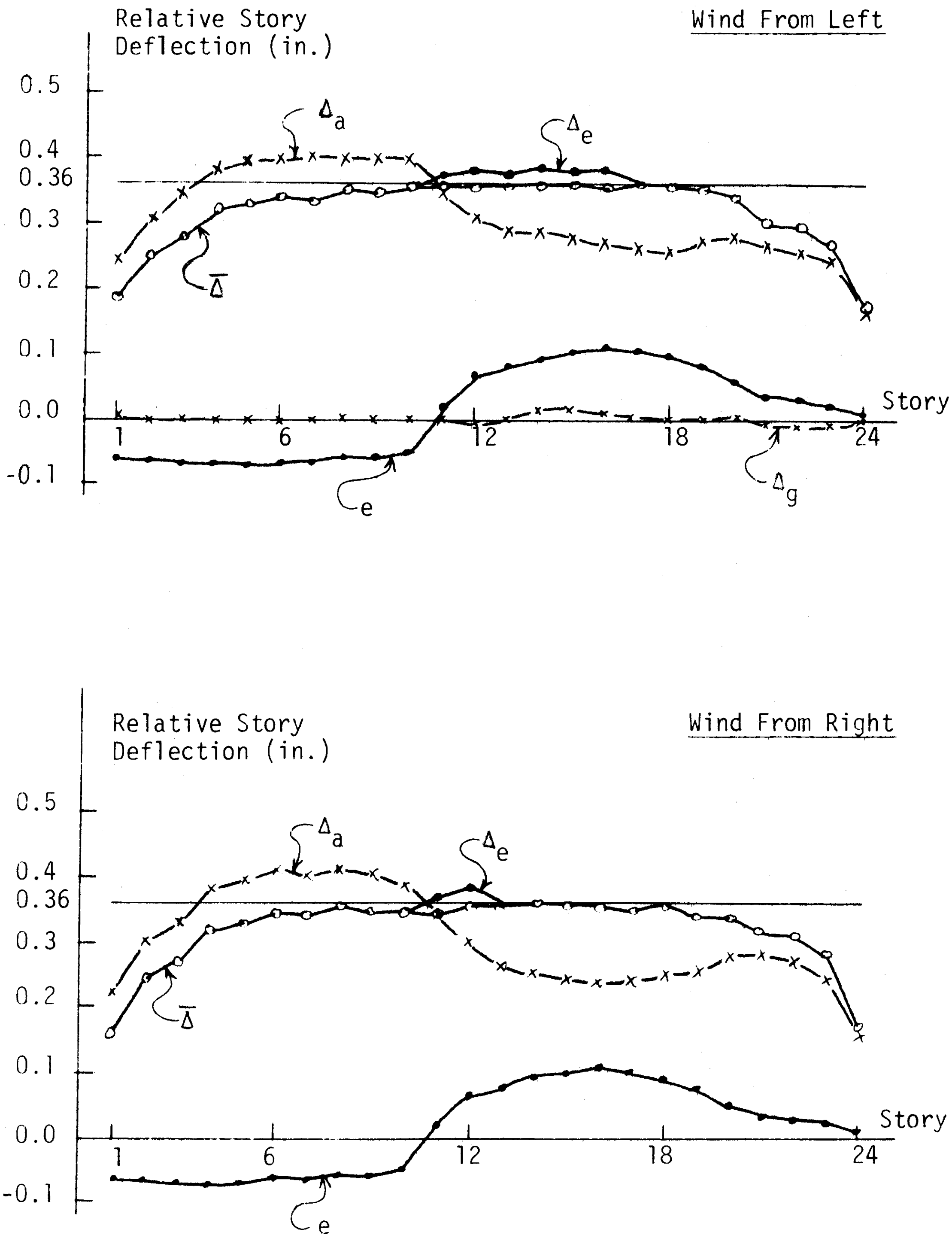


Figure 2.37 Example C9.2A, Elastic Relative Story Deflections After Second Execution of the Elastic Stiffness Design.

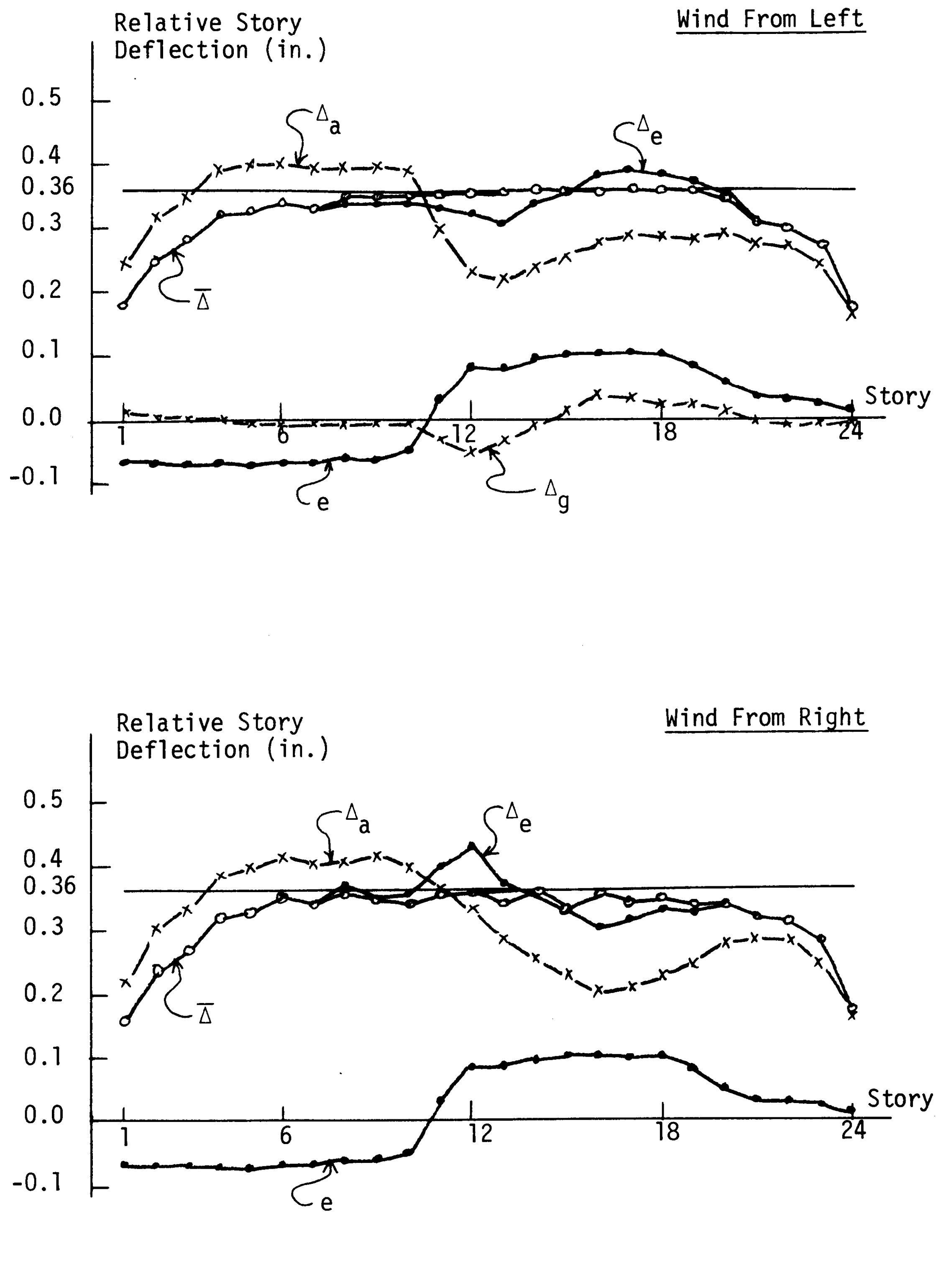


Figure 2.38 Example C9.2A, Elastic Relative Story Deflections After Third Execution of the Elastic Stiffness Design.

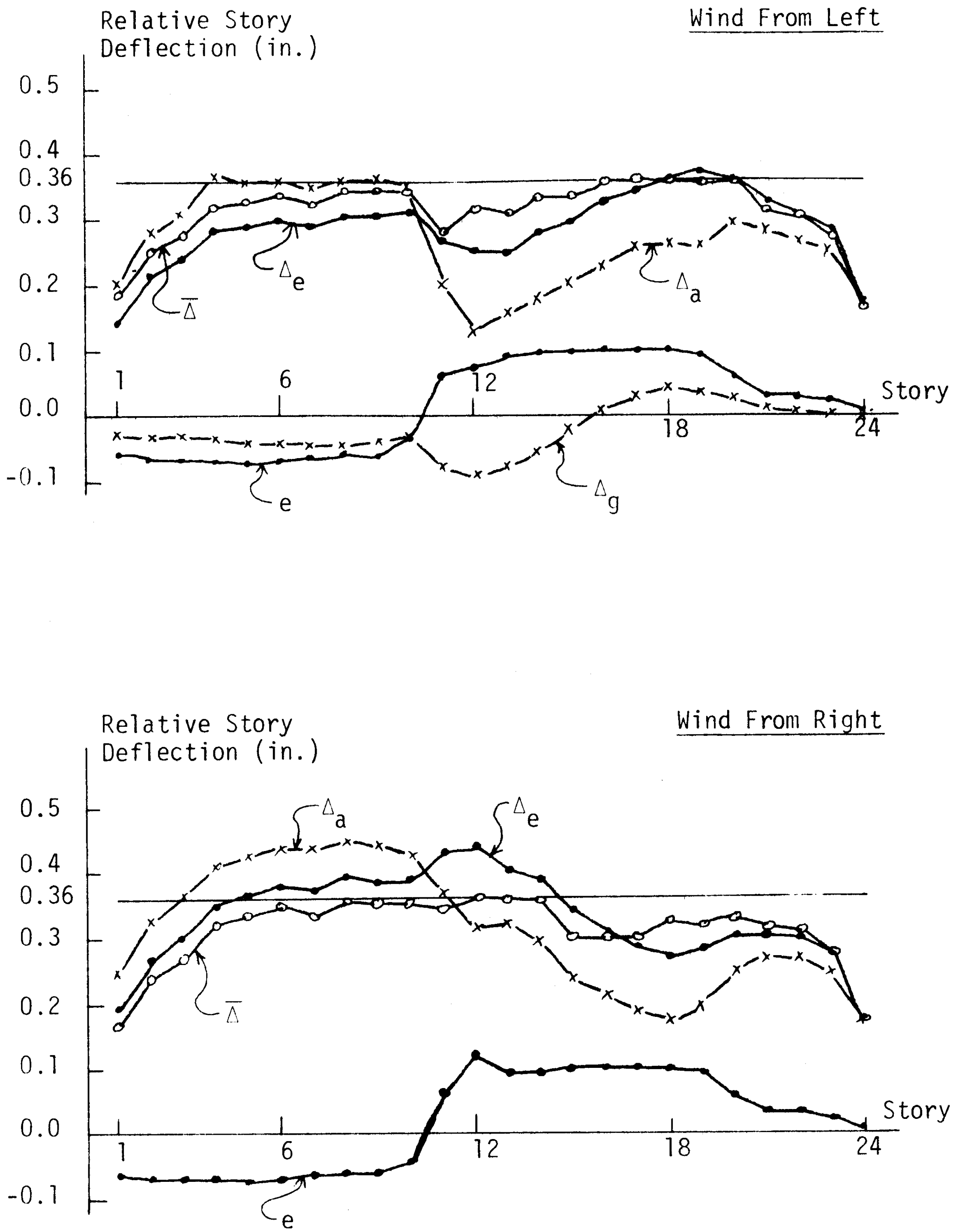


Figure 2.39 Example C9.2A, Elastic Relative Story Deflections After Fourth Execution of the Elastic Stiffness Design.

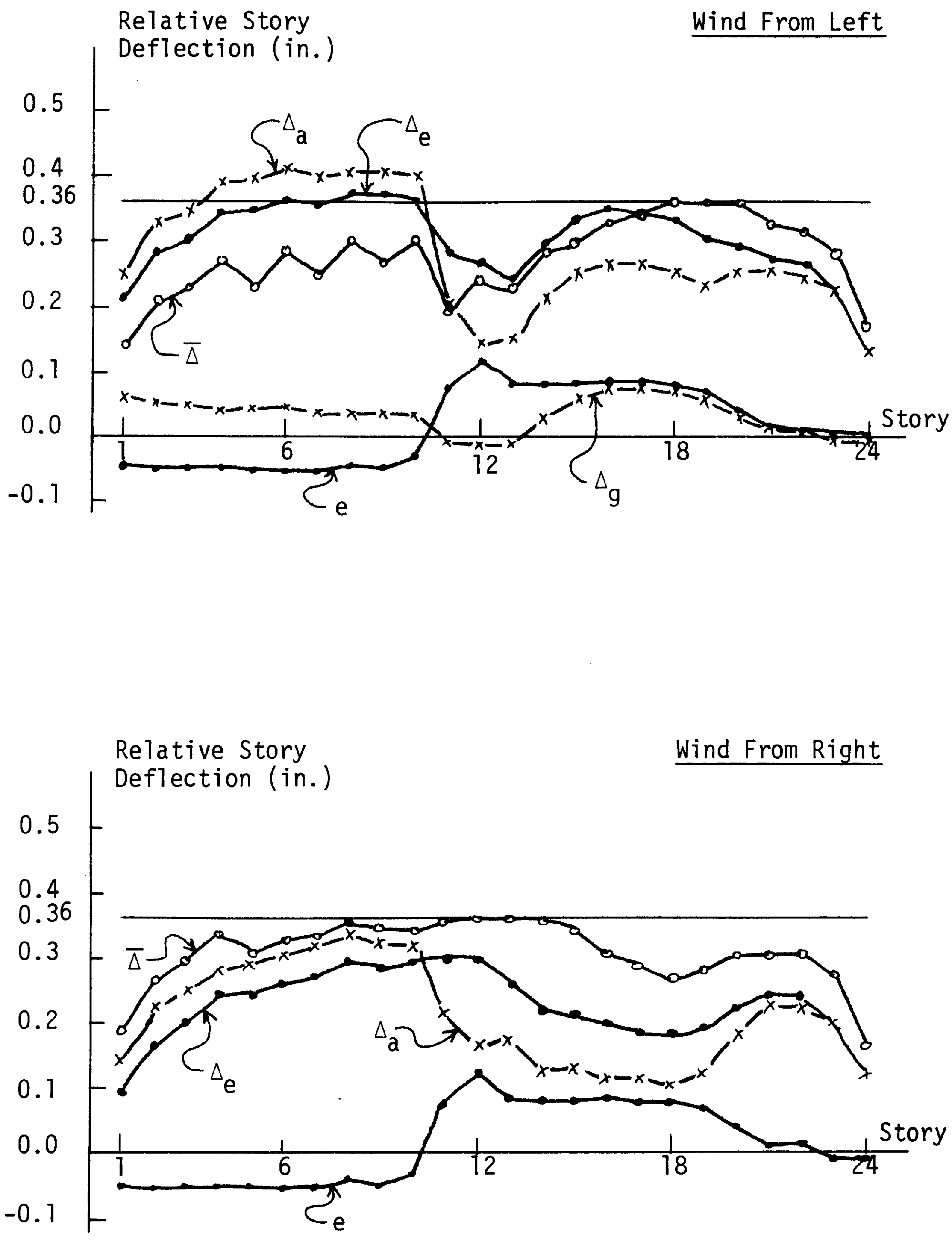
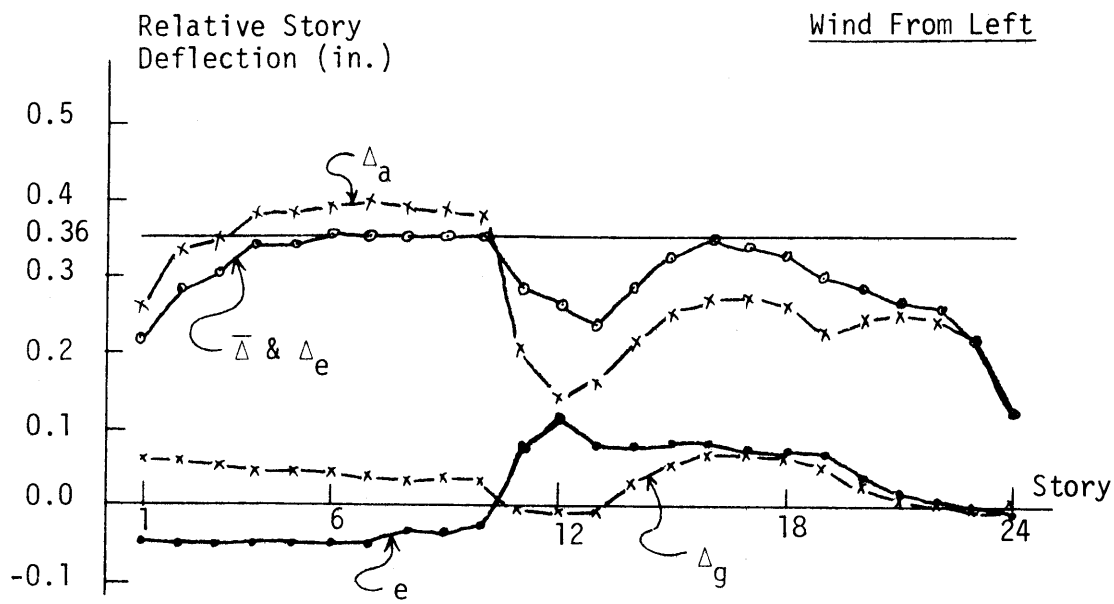


Figure 2.40 Example C9.2A, Elastic Relative Story Deflections After Fifth Execution of the Elastic Stiffness Design.



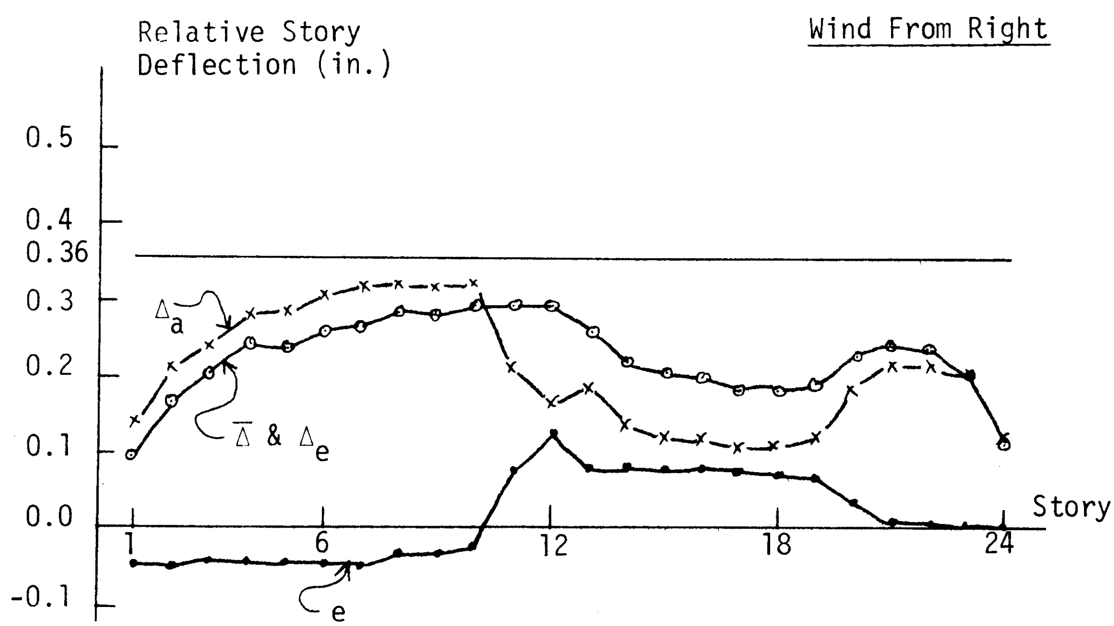


Figure 2.41 Example C9.2A, Elastic Relative Story Deflections After Sixth and Final Execution of the Elastic Stiffness Design.

CHAPTER 3

PLASTIC ANALYSIS AND DESIGN METHOD

3.1 Introduction

The plastic analysis and design procedures begin with the calculation of a minimum member section property configuration as well as a moment and axial force distribution determined from the factored gravity load condition. This is then followed by a calculation of the required additional member properties that are needed to resist the factored combination wind plus gravity load condition. An attempt is made to determine an optimum distribution of additional member properties in a least weight sense and subject to the constraints and limitations of the procedures to be described. These limitations include the fact that no guarantee is given that the global optimum is ever found. However, as is shown in the summary of results, very satisfactory designs are determined.

Numerous investigators[(3),(4),(5)] have attempted and succeeded in formulating and solving the problem of the optimization of unbraced multi-story steel planar frames subject to the constraints of plastic design theory using rigorous mathematical optimization techniques. However, after reviewing many such investigations, it became apparent to the author that the methods developed were extremely time consuming and expensive with respect to their use in the design office. Since one of the intentions of this dissertation is to present a design method that may be economically used in an engineering design office, it was decided that rigorous mathematical optimization techniques would not be used. Instead, it was decided to use heuristic optimization techniques in the solution of the minimum weight problem. In particular, the concept of a story by story design utilizing sensitivity coefficients has been adopted from the doctoral dissertation of Y. Nakamura(6). This concept makes use of the lower Bound Theorem of Plastic Design which states that a lower bound, or safe side solution has been found when two conditions have been satisfied. The first condition is that force equilibrium must be satisfied. The second condition is that the member sizes selected have moment and axial force capacities that are everywhere greater than or equal to the required capacities dictated by the equilibrium condition. Various design programs are used to select member sizes in accordance with the 1969 AISC(1) code specifications and thus satisfy the second condition. The first condition, namely, the equilibrium condition, may be satisfied in an infinity of ways. That is, there is an infinite number of force distributions that will satisfy equilibrium. The problem then becomes one of determining the best distribution of forces that satisfy equilibrium. This problem is solved by determining the appropriate force distribution on a story by story basis where the actual distribution is based on values of the sensitivity coefficients calculated for each panel in the story under consideration. The definition of these sensitivity coefficients will be given in the description of the design procedure which follows.

3.2 Summary of Proposed Method

The minimum section property configuration is determined from the force distribution resulting from the assumption of beam-mechanism failures for each beam under factored gravity loads. The assumed beam-mechanism failures result in required plastic moment capacities for each beam which are then selected from the beam section tables using the appropriate design formulae. Utilizing member and joint equilibrium equations the required axial force and plastic moment capacities of each column are then determined. Again using the appropriate design formulae, all columns are designed and selected from the column section tables. The beams and columns thus selected become the minimum sections to be used in the design process.

The design continues with a consideration of the factored combination wind plus gravity load condition. Associated with each story in the multi-story plane frame is a total required story shear capacity determined from simple global equilibrium conditions. It is composed of two parts. The first part consists of the sum of the factored lateral wind loads at each story level from the top story down to and including the story under consideration. The second part consists of the P-Δ effect. The P-Δ effect as considered here is the effect of the additional story overturning moment due to the gravity loads acting in the laterally displaced position of the structure at the ultimate load. These additional story moments are expressed as equivalent story shears as will be shown in Section 3.4. Thus the total required story shear capacity consists of a summation of factored lateral wind loads plus the equivalent P-Δ story shear.

The design proceeds on a story-by-story basis starting at the top most story and continuing down to the bottom story. At each story level the total story shear is distributed into the panels (i.e. bays) of the story in an incremental fashion. Wind from the left and wind from the right are considered simultaneously in the sense that each increment of lateral shear distributed in a story is first taken as wind from the left and then as wind from the right. After each increment of load is distributed into the story, a new force distribution is determined as well as a redesign of all members which experience force changes.

The essence of the distribution procedure is the sensitivity coefficient. The sensitivity coefficient is defined as the increase in cost of a panel due to an increase in lateral shear capacity of the panel. Two sensitivity coefficients are calculated for each panel in the story. One is associated with a moment resisting panel with no brace to resist the next increment of lateral shear while the second is associated with a truss resisting panel with a tension brace. All sensitivity coefficients calculated in the story are compared. The panel which has the smallest sensitivity coefficient is selected as the one to provide the next increment of lateral shear capacity. The mode of resistance may be either by a moment resisting panel with no brace or a truss panel with a tension brace, depending on which sensitivity coefficient was least. After the increment of lateral load is applied to the panel selected and after a new force distribution and redesign is executed, new sensitivity coefficients are calculated and the above procedure is repeated. This process is continued until all of the required story shear capacity is distributed into the story under consideration. The design iteration then proceeds to the next story where the distribution procedure is repeated. This process continues until all stories have been designed.

In summary, a minimum rolled section configuration is determined for the frame on the basis of beam mechanism failures due to factored gravity loads. Factored lateral loads including the equivalent shears due to the P-Δ effect are then applied to the frame on a story-by-story basis in an incremental fashion. The distribution of an increment of total story shear is a function of the values of the panel sensitivity coefficients. After each application of an increment of story shear a new force distribution is determined and a redesign of those members which experience force changes is executed. The method summarized is essentially a heuristic optimization of material cost or equivalently a heuristic least weight optimization with respect to material cost.

Following the plastic analysis and design method, the elastic analysis and elastic stress design method is executed. This method will be described in Chapter 4.

The following sections of this chapter include detailed descriptions of the calculation of the minimum section property configuration, the formulation of the sensitivity coefficient, the calculation of the incremental value of the lateral story shear applied to a panel, the calculation of the new force distribution, and the basis of the design of individual members.

3.3 Notation and Sign Convention

The sign convention adopted is illustrated in Fig. 3.1. All forces drawn on the column, beam, and bracing elements are in their positive directions. Moment forces acting on the ends of columns and beams are positive when acting in the clockwise direction. Moment at the mid-length of beams is positive when producing tensile stresses on the bottom fibres. Axial forces in beams, columns, and braces are positive when acting in compression. Finally, end shears of beams and columns are positive when producing counter-clockwise member rotation.

The notation used in this design method is illustrated in Figs. 3.2, 3.3, and 3.4. Note that M represents a moment at a joint center (i.e. the intersection point of a beam and column center line), while M' represents a moment at a beam or column end (i.e. the intersection point of a beam center-line with a column flange or a column center-line with a beam flange).

3.4 Basic Equilibrium Relations

Before proceeding on to the detailed description of the force distribution procedure under factored gravity loads and the story shear distribution procedure under the factored combination loads certain basic equilibrium relations will be presented. This section will present equilibrium relations for an unbraced story. Later sections will present equilibrium equations for a braced story.

Firstly, joint moment equilibrium is formulated by summing all moments acting at a joint. Since the joints of the frame are not

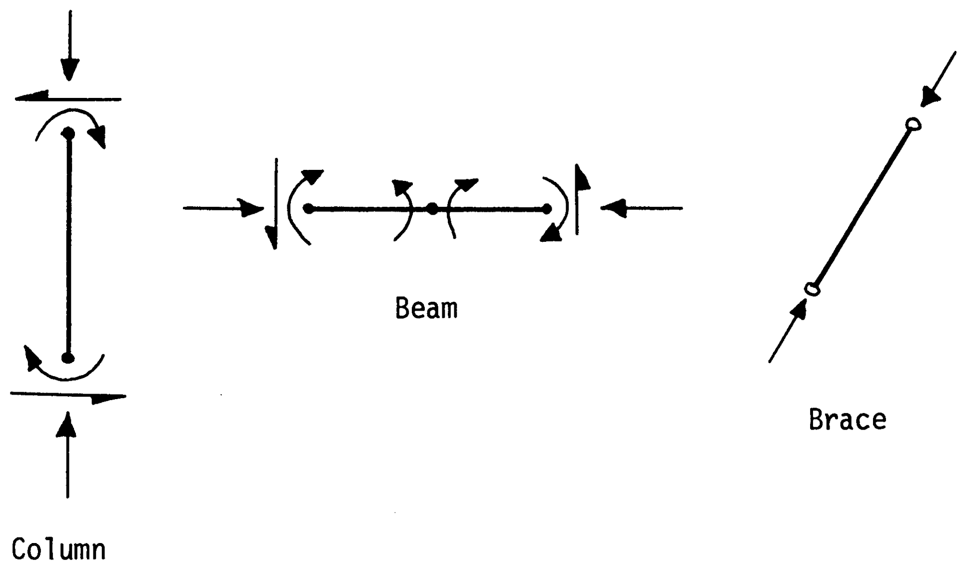


Figure 3.1 Positive Sign Convention for Moment, Axial Force and Shear.

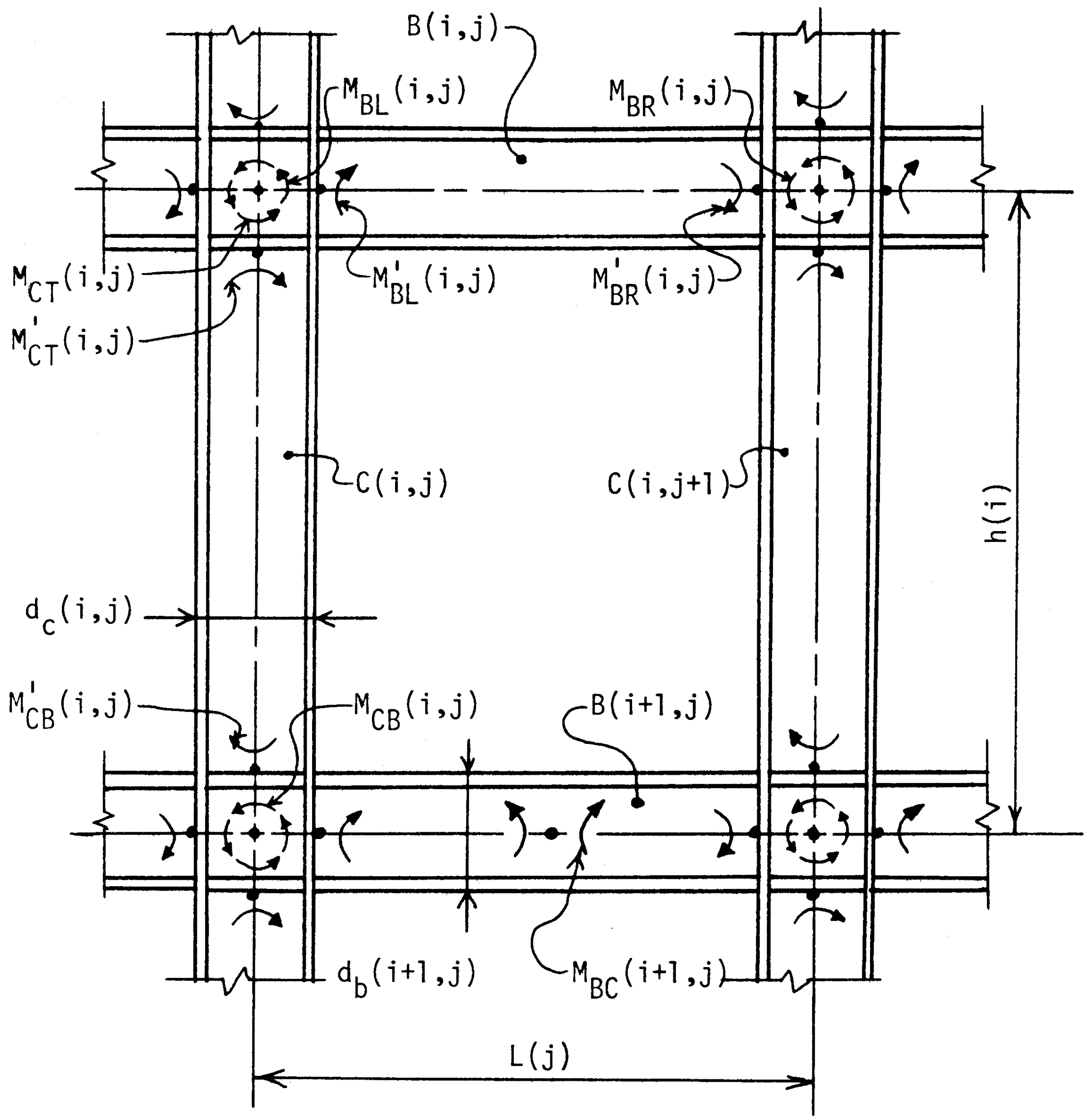


Figure 3.2 Notation

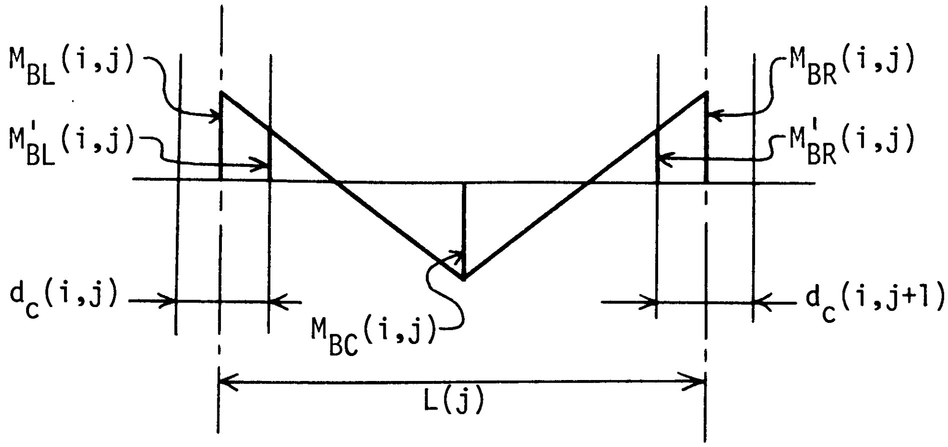


Figure 3.3 Beam Moment Diagram.

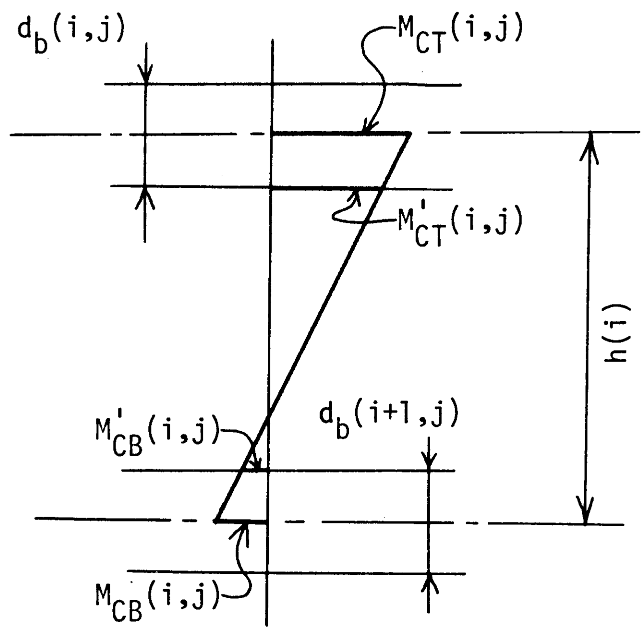


Figure 3.4 Column Moment Diagram.

subject to externally applied moments, the sum of the member joint moments is zero at each joint. Using the notation of Figs. 3.3 and 3.4 the joint moment equilibrium equation is

$$M_{CT}(i,j) + M_{BL}(i,j) + M_{CB}(i-1,j) + M_{BR}(i,j-1) = 0 \qquad (3.1)$$

When Eq. (3.1) is applied at an external joint, the terms associated with the missing members are set to zero.

Beam moment equilibrium will now be considered with respect to the loads acting directly on a beam. In this design method uniformly distributed gravity loads acting on beams are assumed to be replaced by three concentrated vertical loads acting at the center and each end of a beam. The magnitude of the concentrated load at the center of a beam is

$$P_W(i,j) = W(i,j)L(j)/2 \qquad (3.2)$$

This assumption is conservative for the upper stories where the vertical loads are dominant, but it does not influence the results for the lower stories where the horizontal loads become dominant.[(7),(8)] In addition, this assumption results in a significant reduction in analysis execution time since the location of possible beam plastic hinges are fixed.

Now, consider the forces acting on the beam illustrated in Fig. 3.5. Writing a moment equilibrium equation for the free body diagram of the right half of the beam and rearranging terms results in the beam moment equilibrium equation expressed as follows:

$$(M_{BR}(i,j) - M_{BL}(i,j))/2 + M_{BC}(i,j) = \lambda P_W(i,j)L(j)/4 \qquad (3.3)$$

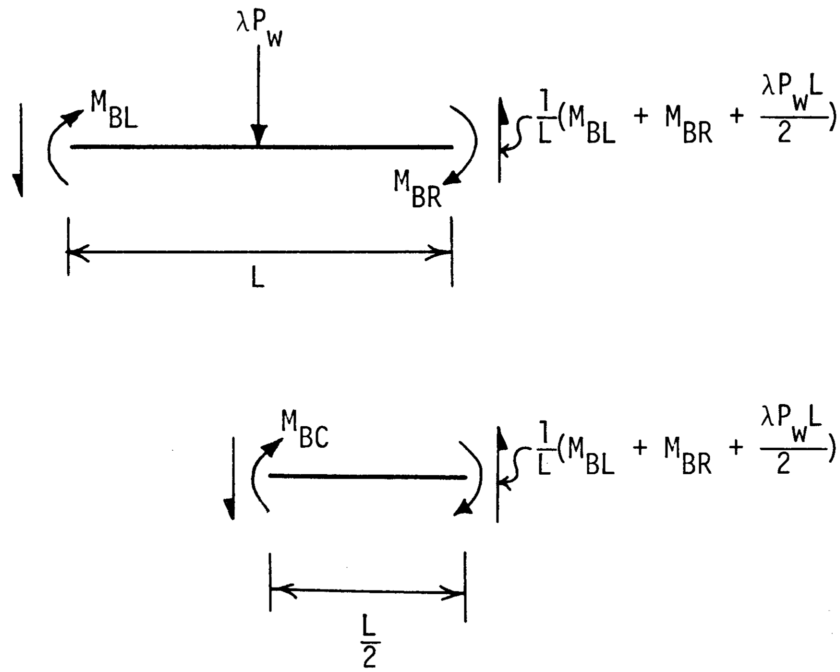


Figure 3.5 Beam Force Equilibrium.

where,

$\lambda = \lambda_1$ for the gravity load condition;

$\lambda = \lambda_2$ for the combination load condition.

The relationship between increments of beam moments will now be formulated. Under the application of lateral loads to the frame, beam moments will change. However, since no change in gravity load occurs when the lateral loads are applied, Eq. (3.3) must be satisfied at all times. Now, designating increments of beam moments by ΔM and using a simplified notation we have the following relationship among the final beam moments under some application of lateral load.

$$((M_{BR}+\Delta M_{BR}) - (M_{BL}+\Delta M_{BL}))/2 + (M_{BC}+\Delta M_{BC}) = \lambda P_W L/4 \tag{3.4}$$

Rearranging terms by separating the initial moments from the changes in moments leads to

$$(M_{BR}-M_{BL})/2 + M_{BC} + (\Delta M_{BR}-\Delta M_{BL})/2 + \Delta M_{BC} = \lambda P_W L/4 \tag{3.5}$$

But, by Eq. (3.3) we have

$$(M_{BR}-M_{BL})/2 + M_{BC} = \lambda P_W L/4$$

Consequently, the relationship between increments of beam moments is

$$\Delta M_{BR}(i,j) - \Delta M_{BL}(i,j) + 2\Delta M_{BC}(i,j) = 0 \tag{3.6}$$

Story moment equilibrium will now be considered. This design method accounts for overall frame instability by formulating the story moment equilibrium on the deformed state of the frame. The total story moment is equal to the sum of the joint moments of all columns in the story. This total moment is composed of two parts. The first part is the overturning moment due to the sum of the factored lateral wind loads applied to all stories from the top story down to and including the story under consideration. The second part is the additional overturning moment due to the P-Δ effect or the moment due to the gravity loads acting in the laterally displaced position of the story.

Taking $\lambda_2 \Sigma F_c$ to be the total of the story column axial forces due to the factored gravity loads and $\Delta(i)$ to be the relative story i deflection at the collapse mechanism, we have

$$H_T(i)h(i) + \Delta(i)\lambda_2 \sum_{j=1}^{N+1} F_c(i,j) + \sum_{j=1}^{N+1} (M_{CT}(i,j) + M_{CB}(i,j)) = 0 \tag{3.7}$$

where,

$$H_T(i) = \lambda_2 \sum_{k=1}^{i} H(k) \tag{3.8}$$

and,

$H(k)$ = story k wind load

N = number of bays

The required total story moment is therefore

$$\sum_{j=1}^{N+1} (M_{CT}(i,j) + M_{CB}(i,j)) = -(H_T(i)h(i) + \lambda_2\Delta(i) \sum_{j=1}^{N+1} F_c(i,j)) \tag{3.9}$$

From Eq. (3.9) it is seen that the additional overturning moment due to formulating the story equilibrium on the deformed state of the frame is equal to $\lambda_2 \Delta(i) \sum_{j=1}^{N+1} F_c(i,j)$. Now, defining $S_T(i)$ to be the equivalent required total story shear to be resisted by story i, we have

$$S_T(i)h(i) = - \sum_{j=1}^{N+1} (M_{CT}(i,j) + M_{CB}(i,j)) \tag{3.10}$$

Consequently, from Eq.'s (3.9) and (3.10) the equivalent required total shear that must be resisted by story i will be calculated as

$$S_T(i) = H_T(i) + \lambda_2 \frac{\Delta(i)}{h(i)} \sum_{j=1}^{N+1} F_c(i,j) \tag{3.11}$$

Note that the story deflection $\Delta(i)$ at a collapse mechanism is unknown during the first iteration of design. Therefore, an assumed value of $\Delta(i)$ may be specified at the beginning of the design. They are usually from 0.01 h(i) to 0.04 h(i) for unbraced frames and 0.001 h(i) to 0.004 h(i) for braced frames. After member proportioning, $\Delta(i)$ will be estimated by the subassemblage method which will be explained in Section 3.10. This new calculated value of $\Delta(i)$ is then used in the next iteration of design to obtain a better approximation to the final P-Δ effects. Note that $\Delta(i)$ may be initially specified as zero. However, more cycles of iteration may be required before convergence is reached. In any case, if $\Delta(i)$ is specified as zero, the program will assume an initial value of $\Delta(i)$ equal to 0.0005 h(i).

3.5 Joint Size Effect

Experiment has shown that the deformed shape of the frame demonstrates that the plastic hinges from outside the joints. That is, plastic hinges usually form at the intersection of beam centerlines with column flanges or column center lines with beam flanges. Therefore the actual strength of the frame is larger than what is predicted by using the centerline dimensions. These results are reflected in this design method by assuming that the plastic hinges occur just outside the joint boundaries. Referring to Fig. 3.3, the relationship between the beam moment at the face of the column and the moment at the joint centerline is

$$
\begin{aligned}
M'_{BR}(i,j) &= M_{BR}(i,j)\left(1 - \frac{d'_c(i,j)}{L(j)}\right) - M_{BC}(i,j)\,\frac{d'_c(i,j)}{L(j)} \\
M'_{BL}(i,j) &= M_{BL}(i,j)\left(1 - \frac{d'_c(i,j)}{L(j)}\right) + M_{BC}(i,j)\,\frac{d'_c(i,j)}{L(j)}
\end{aligned}
\qquad (3.12)
$$

where,

$d'_c(i,j)$ = the average column depth of the left and right column in the panel.

Thus,

$$d'_c(i,j) = \frac{1}{2}\left(d_c(i,j) + d_c(i,j+1)\right) \qquad (3.13)$$

Referring to Fig. 3.4, a similar relationship can be obtained for columns as

$$M'_{CT}(i,j) = M_{CT}(i,j)(1- \frac{d'_b(i,j)}{2h(i)}) - M_{CB}(i,j) \frac{d'_b(i,j)}{2h(i)}$$
$$M'_{CB}(i,j) = M_{CB}(i,j)(1- \frac{d'_b(i,j)}{2h(i)}) - M_{CT}(i,j) \frac{d'_b(i,j)}{2h(i)} \quad (3.14)$$

where,

$d'_b(i,j)$ = the average beam depth of the top and bottom beam.

Thus,

$$d'_b(i,j) = \frac{1}{2} (d_b(i,j) + d_b(i+1,j)) \quad (3.15)$$

Note that the average beam and column depths have been used in the above. This has been done for the sake of simplicity and gives results essentially the same as would occur with the actual depths. Furthermore, since the beam and column depths are unknown during the first iteration of design, they are initially set to zero.

3.6 Force Distribution Under Factored Gravity Loads

The first step in the plastic analysis and design method is that of determining a minimum section property configuration. Minimum section properties are determined using the appropriate code formulae as will be discussed in Section 3.8. The force distribution used to obtain these minimum section properties will now be described. First note that the proposed design method considers braced frames with only diagonal type bracing. It will therefore be assumed that the braces are not to be considered as load carrying members when the frame is subjected only to gravity loads.

The gravity load condition consists of the gravity dead and live loads multiplied by the load factor λ_1. The gravity dead and live loads consist of the equivalent concentrated load $P_W(i,j)$ applied to the center of beams and the equivalent and applied concentrated loads $P_J(i,j)$ applied to the joints. The bending moment distribution in the beams is obtained by assuming a beam mechanism mode of failure. Figure 3.6 illustrates the beam mechanism failure.

The virtual work equation corresponding to this mechanism is

$$M_{BP}(4\theta) = \lambda_1 P_W L' \theta/2$$

$$= \lambda_1 P_W (L - d_c') \theta/2 \tag{3.16}$$

Consequently, the required plastic moment capacity of beams is

$$M_{BP}(i,j) = \lambda_1 P_W(i,j)(L(j) - d_c'(i,j))/8 \tag{3.17}$$

Furthermore, from Fig. 3.6,

$$M_{BR}'(i,j) = M_{BC}(i,j) = -M_{BL}'(i,j) = M_{BP}(i,j) \tag{3.18}$$

Substituting this result into Eq. (3.12) leads to the resulting beam joint moments or

$$M_{BR}(i,j) = \frac{L(j) + d_c'(i,j)}{L(j) - d_c'(i,j)} M_{BR}'(i,j) \tag{3.19}$$

$$M_{BL}(i,j) = \frac{L(j) + d_c'(i,j)}{L(j) - d_c'(i,j)} M_{BL}'(i,j) \tag{3.20}$$

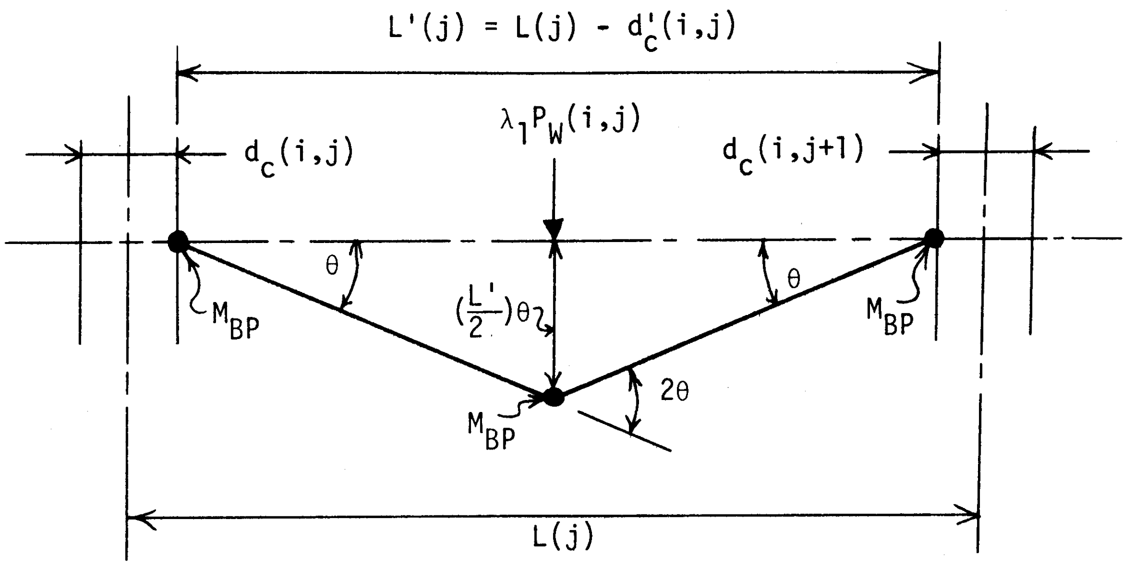


Figure 3.6 Beam Mechanism Failure.

Substituting Eq.'s (3.17) and (3.18) into Eq.'s (3.19) and (3.20) leads to

$$M_{BR}(i,j) = -M_{BL}(i,j) = \lambda_1 P_W(i,j)(L(j) + d_c^{'}(i,j))/8 \qquad (3.21)$$

The bending moment distribution in the columns may now be obtained utilizing the joint equilibrium equation. From Eq. (3.1), the bending moment at the top joint of the top story columns are

$$M_{CT}(i,j) = -(M_{BL}(i,j) + M_{BR}(i,j-1)) \qquad (3.22)$$

In addition, the joint column moments at the remaining joints are obtained from Eq. (3.1) and the assumption that the unbalanced beam moments distribute equally to the upper and lower columns. Thus,

$$M_{CT}(i,j) = M_{CB}(i-1,j) = -\frac{1}{2}(M_{BL}(i,j) + M_{BR}(i,j-1)) \qquad (3.23)$$

The column end moment at the face of a beam, $M_{CT}^{'}(i,j)$ and $M_{CB}^{'}(i,j)$, can be determined by substituting Eq. (3.23) into Eq. (3.14). Finally, the required plastic moment capacity for columns is the larger of the absolute values of $M_{CT}^{'}(i,j)$ and $M_{CB}^{'}(i,j)$.

$$M_{CP}(i,j) = \text{Max.} \left\{ |M_{CT}^{'}(i,j)|, \ |M_{CB}^{'}(i,j)| \right\} \qquad (3.24)$$

Required axial force capacities of the columns is calculated by statics from the bending moments in the beam and then adding the concentrated vertical joint loads plus the axial loads coming from the columns above.

Axial forces in beams due to factored gravity loads alone are negligible and are taken as zero.

An example of the resulting moment diagram due to the factored gravity loads is shown in Fig. 3.7. The initial force distribution due to the factored combination loads, with wind taken as zero, is obtained by multiplying the factored gravity load force distribution by λ_2/λ_1 where λ_2 is the load factor used for the combination load condition.

3.7 Story Shear Distribution with Sensitivity Coefficients

The next step in the proposed plastic analysis and design method is to provide the necessary shear capacity to resist the imposed lateral loads. To accomplish this the design proceeds on a story-by-story basis. Within each story the total required shear capacity, as defined by Eq. (3.11), is distributed into the bays or panels of the story in such a way as to minimize the cost increase of additional material that may be required. Note however that the minimum section configuration determined from the factored gravity load condition has some inherent shear capacity. The proposed method takes full advantage of this inherent capacity as will be shown.

The total required story shear is distributed into the story in an incremental fashion. The decision as to which panel in the story is selected to resist the next increment of story shear is based on the sensitivity coefficient. The sensitivity coefficient is defined as the cost increase of a panel due to an increase in panel shear capacity. Two sensitivity coefficients are calculated for each panel in the story. One coefficient is associated with the panel resisting the next increment of shear by increased moments and axial forces in

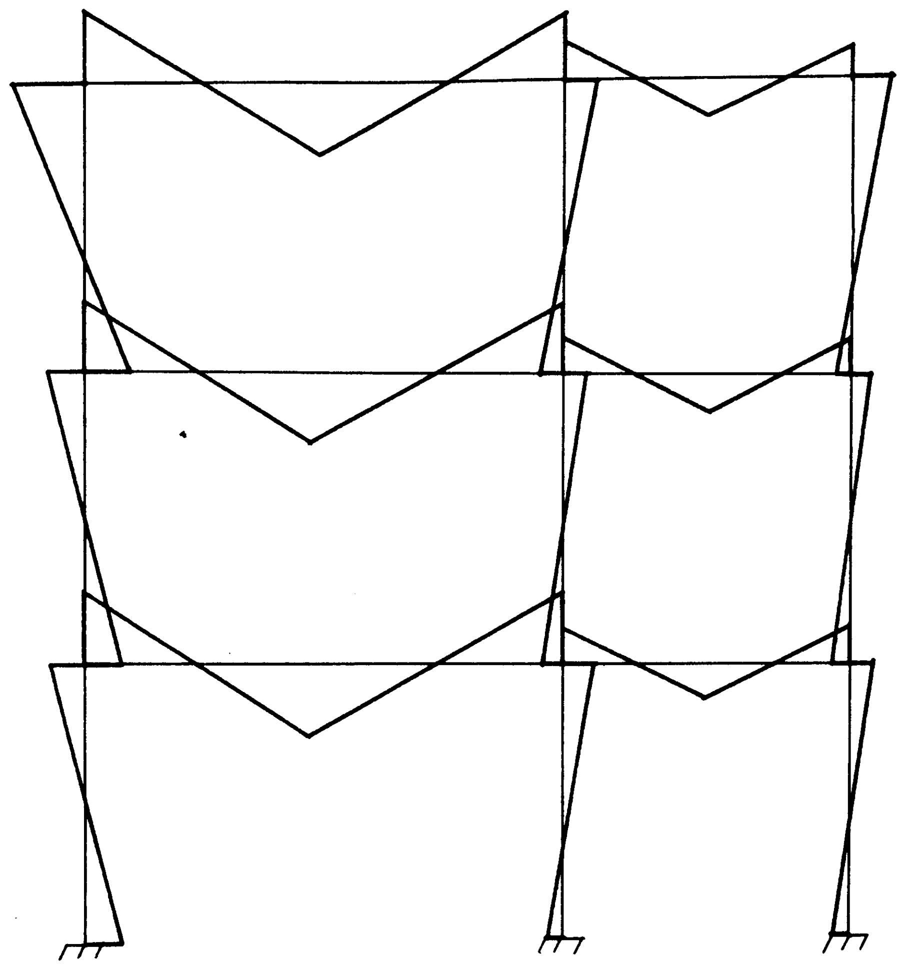


Figure 3.7 Typical Moment Distribution Due To Factored Gravity Loads.

the panel beams and columns as well as increased axial forces in certain members external to the panel. This mode of panel resistance is called moment action and the panel is called a moment resisting panel for some increment of lateral shear application. The second coefficient is associated with the panel resisting the next increment of shear by increased axial forces alone in the panel beams, columns and diagonal tension brace as well as in certain members external to the panel. This mode of panel resistance is called truss action and the panel is called a truss resisting panel for some increment of lateral shear application.

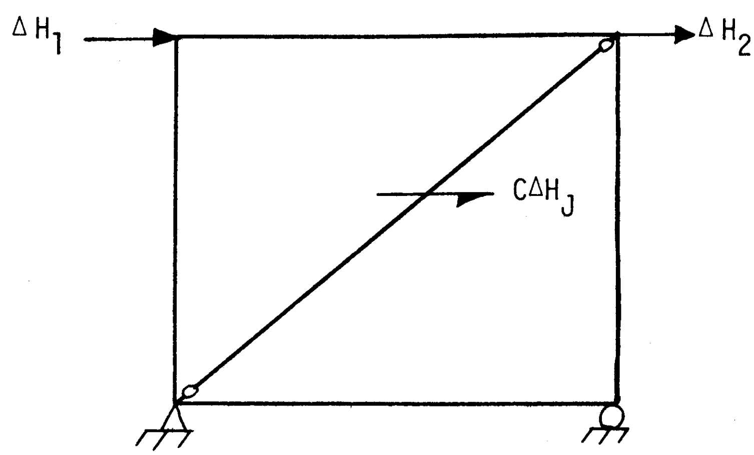
The user of the computer program can specify that a panel may resist lateral shear either by moment action or by truss action. In this case the sensitivity coefficients are calculated as will be described in Section 3.7.1. However, the user can specify that a panel must be a truss resisting panel at all times. In this case, the sensitivity coefficient corresponding to moment action is set to a high value of 100. On the other hand, the user can specify that a panel must be a moment resisting panel at all times. In this case, the sensitivity coefficient corresponding to truss action is set to a high value of 100. This will prevent bracing from being inserted in the panel. One last option is available. The user can specify that a panel may not resist any lateral load by either truss or moment action. In this case, both sensitivity coefficients of the panel are set to a high value of 100. When this last option is used, the user must be careful to allow at least one panel in the story to resist lateral load. Otherwise the design will terminate prematurely. This last option is particularly useful when the user wishes to specify that all stories of a particular bay or bays is to resist all lateral wind loads as a vertical cantilever truss. Note that the above options are applicable only in the plastic design part.

After the two sensitivity coefficients are determined for each panel in a story, the panel with the least valued sensitivity coefficient is selected to resist the next increment of required story shear capacity. The type of resistance, moment or truss action, depends on which of the panel's sensitivity coefficients is least. Note that the values of the sensitivity coefficients usually range from 0.0 to 5.0. Consequently, the values of 100.0 set to prevent a particular type of panel resistance will guarantee that the corresponding truss or moment action will not occur. After a panel and corresponding mode of resistance is selected, the value of the increment of lateral shear is calculated and then applied to the panel. Following this, the corresponding redistribution of forces is determined and a new member design performed on those members which experienced force changes.

3.7.1 Formulation of the Sensitivity Coefficients

At this time it is important to note that all succeeding discussion will be with respect to shear applied to the frame from the left. Completely analogous arguments are valid for shear from the right.

Each panel in a story will have two sensitivity coefficients calculated for it. These sensitivity coefficients are based in part on the current shear capacity of the corresponding panel. Consider the model of a panel used in this design method as illustrated in Fig. 3.8. Since only diagonal bracing is considered in this design



ΔH_J = Panel incremental shear capacity.

$= \Delta H_1 + \Delta H_2$

$C\Delta H_J$ = Horizontal component of tension brace force.

$$C = \begin{cases} 0.0, & \text{Moment resisting panel.} \\ -1.0, & \text{Truss resisting panel.} \end{cases}$$

Figure 3.8 Panel Model.

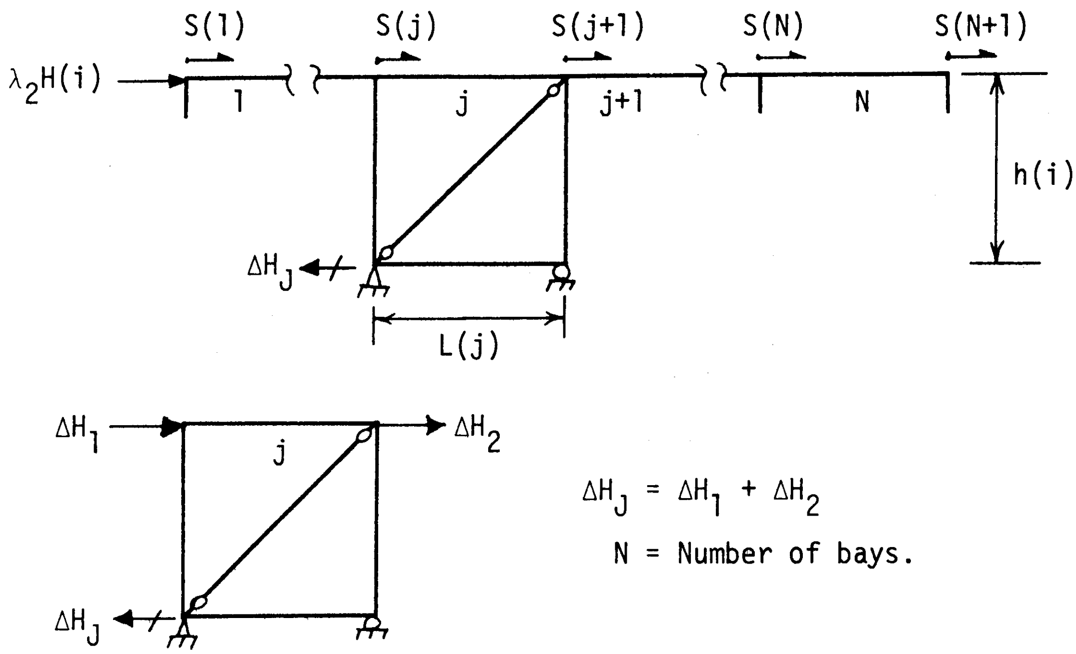
method, it will be assumed that compression bracing has achieved a buckled configuration and will not be considered as resisting any lateral shear. Thus, only tension bracing is included.

The additional or increased shear capacity of panel j is designated by ΔH_J and is equal to the sum of the incremental lateral shears applied to the upper left and right joints of the panel. So,

$$\Delta H_J = \Delta H_1 + \Delta H_2 \tag{3.25}$$

Each increment of lateral shear, ΔH_J, applied to the panel may be resisted either by moment action or by truss action. If the value of the horizontal component of brace axial force due to ΔH_J is represented by $C\Delta H_J$, it is obvious that the panel moment resisting mode is specified by setting C=0.0 and the panel truss resisting mode is specified by setting C=-1.0. Note that the value of C or equivalently the mode of resistance of the panel is not fixed for all applications of lateral shear to the panel. On the contrary, the mode of panel resistance is determined for each application of ΔH_J. Depending on the value of the appropriate sensitivity coefficient at the time of application of ΔH_J, the mode of resistance may change from one incremental shear application to another. Also note that ΔH_J of panel j in story i is assumed to be transmitted to the story below thru the panel's bottom left support point to the upper left joint of panel j in story i+1.

The calculation of ΔH_J will now be considered for shear from the left. Consider the model illustrated in Fig. 3.9. The factored lateral wind load $\lambda_2 H(i)$ is applied at the left most joint of story level i. Also, at each joint k in story level i a lateral shear load



$\Delta H_J = \Delta H_1 + \Delta H_2$

N = Number of bays.

Figure 3.9 Model for the Calculation of ΔH_J.

is applied and designated by S(k). This lateral joint load may be composed of one or two of the following two parts. The first part is due to the P-Δ effect associated with the axial force in column k of story i. The second part is due to the lateral shear ΔH_J being transmitted from the corresponding panels in the story above. So,

$$S(k) = \begin{cases} \Delta H_k^{i-1} + \lambda_2 F_c(i,k) \dfrac{\Delta(i)}{h(i)}, & k = 1,2,\ldots,N \\ \lambda_2 F_c(i,k) \dfrac{\Delta(i)}{h(i)}, & k = N+1 \end{cases} \tag{3.26}$$

where,

$\Delta(i)$ = relative story i deflection at the failure mechanism.

$\lambda_2 F_c(i,k)$ = factored gravity load column axial force.

ΔH_k^{i-1} = the total shear applied to panel k in story i-1.

and,

$$\sum_{k=1}^{N} \Delta H_k^{i-1} = \lambda_2 \sum_{p=1}^{i-1} H(p)$$

From Eq.'s (3.8), (3.11) and (3.26) an equivalent expression for the total required story shear is

$$S_T(i) = \lambda_2 H(i) + \sum_{k=1}^{N+1} S(k) \tag{3.27}$$

Now, dividing Eq. (3.25) by ΔH_J results in

$$1 = \frac{1}{\Delta H_J} (\Delta H_1 + \Delta H_2) \tag{3.28}$$

Also, dividing both sides of Eq. (3.27) by $S_T(i)$ results in

$$1 = \frac{1}{S_T(i)}\left(\lambda_2 H(i) + \sum_{k=1}^{N+1} S(k)\right) \tag{3.29}$$

Consequently, after equating Eq.'s (3.28) and (3.29), multiplying both sides by ΔH_J, and substituting $\sum_{k=1}^{N+1} S(k) = \sum_{k=1}^{j} S(k) + \sum_{k=j+1}^{N+1} S(k)$, an expression for the sum $\Delta H_1+\Delta H_2$ results where,

$$\Delta H_1 + \Delta H_2 = \frac{\Delta H_J}{S_T(i)}\left(\lambda_2 H(i) + \sum_{k=1}^{j} S(k) + \sum_{k=j+1}^{N+1} S(k)\right) \tag{3.30}$$

ΔH_1 is taken as a proportion of the lateral joint loads to the left of panel j and ΔH_2 is taken as a proportion of the lateral joint loads to the right of panel j. Thus,

$$\begin{aligned} \Delta H_1 &= \frac{\Delta H_J}{S_T(i)}\left(\lambda_2 H(i) + \sum_{k=1}^{j} S(k)\right) \\ \Delta H_2 &= \frac{\Delta H_J}{S_T(i)} \sum_{k=j+1}^{N+1} S(k) \end{aligned} \tag{3.31}$$

The calculation of ΔH_J is described in Section 3.7.2.

A formal definition of the sensitivity coefficient will now be considered. Let f_T represent the cost of all members that experience force changes due to the application of ΔH_J to panel j. It will therefore be equal to the sum of the cost of the beams, columns, and tension brace in panel j as well as the cost of the beams in story i external to panel j and all columns below panel j lying on column lines j and j+1. In particular,

$$f_T = (f)_{\substack{\text{panel}\\\text{beams}}} + (f)_{\substack{\text{panel}\\\text{columns}}} + (f)_{\substack{\text{panel}\\\text{tension brace}}} + (f)_{\substack{\text{beams adjacent}\\\text{to panel}}} + (f)_{\substack{\text{columns}\\\text{below panel}}} \tag{3.32}$$

The sensitivity coefficient is now defined as $\frac{\partial f_T}{\partial H_J}$ or the change in cost of panel j due to an increase in panel shear capacity ΔH_J. Note that the phrase 'cost of panel j' as used here means f_T. So, from Eq. (3.32),

$$\frac{\partial f_T}{\partial H_J} = \left(\frac{\partial f}{\partial H_J}\right)_{\substack{\text{panel}\\\text{beams}}} + \left(\frac{\partial f}{\partial H_J}\right)_{\substack{\text{panel}\\\text{columns}}} + \left(\frac{\partial f}{\partial H_J}\right)_{\substack{\text{panel}\\\text{tension brace}}} + \left(\frac{\partial f}{\partial H_J}\right)_{\substack{\text{beams adjacent}\\\text{to panel}}} + \left(\frac{\partial f}{\partial H_J}\right)_{\substack{\text{columns below}\\\text{panel}}} \tag{3.33}$$

Thus, the sensitivity coefficient of a panel is a sum of sub-sensitivity coefficients associated with those members that experience force changes due to the application of ΔH_J.

The following Sections 3.7.1.1 to 3.7.1.3 describe the formulation of these sub-sensitivity coefficients.

3.7.1.1 Sub-Sensitivity Coefficient of Columns Below Panel j.

The sub-sensitivity coefficient associated with all columns directly below panel j will be designated by $\left(\frac{\partial f}{\partial H_J}\right)_{CBP}$. Now, consider the model illustrated in Fig. 3.10. It is assumed that the vertical reactions at the support points in the model of panel j due to ΔH_J are transmitted directly to the foundation thru the columns directly below

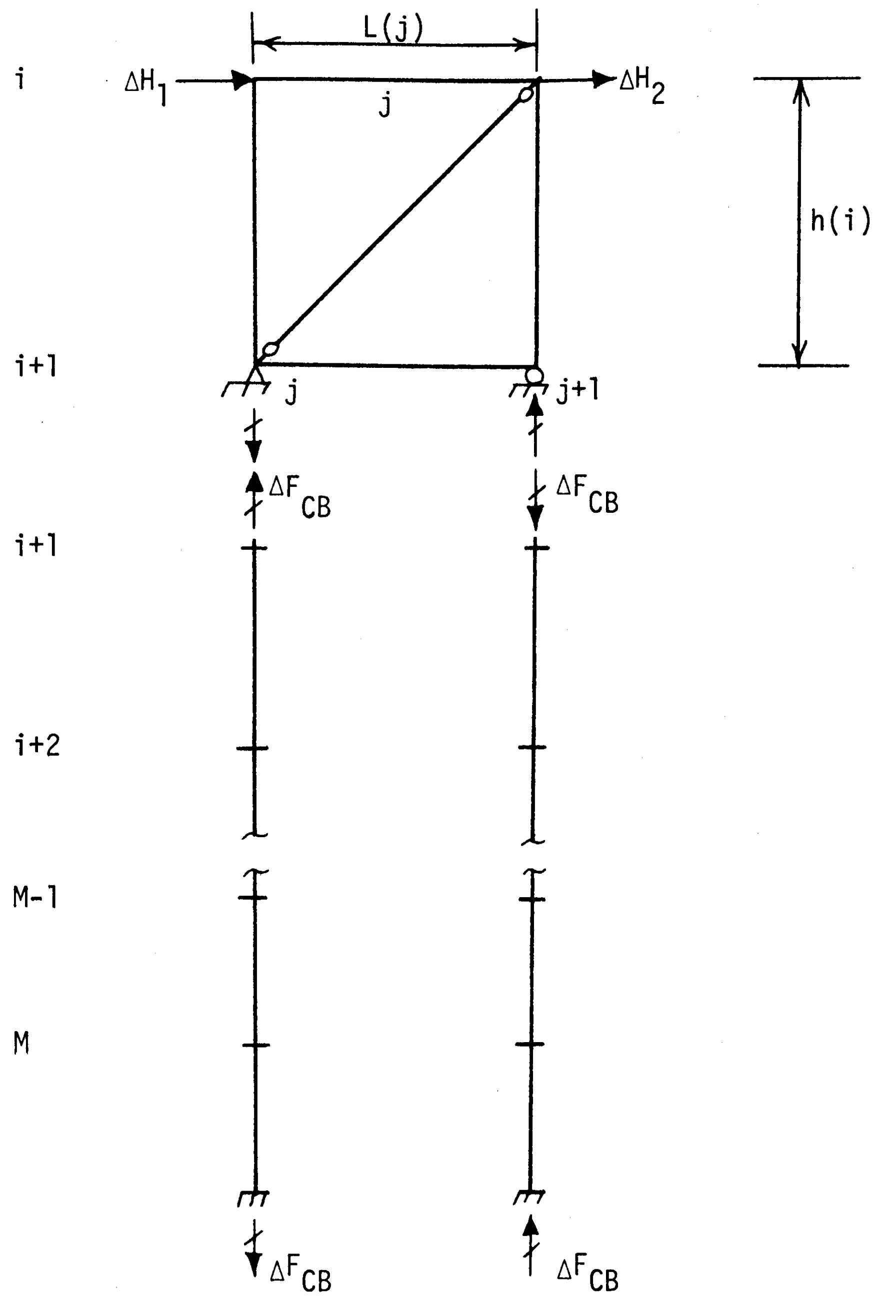


Figure 3.10 Model for the Calculation of the Sub-Sensitivity Coefficient of Columns Below Panel j.

the panel. The cost of one column directly below panel j is defined by f_{CB}. So,

$$f_{CB} = \rho u_c(k,p)h(k)A_c(k,p) \tag{3.34}$$

where,

$p = j, j+1$

ρ = mass density of steel

$u_c(k,p)$ = unit material cost of column p in story k

$h(k)$ = height of story k

$A_c(k,p)$ = area of column p in story k.

Thus, the change in cost of the column due to a change in column axial force is

$$\partial f_{CB} = \rho u_c(k,p)h(k) \frac{\partial A_c(k,p)}{\partial F_{CB}} \partial F_{CB} \tag{3.35}$$

where,

∂F_{CB} = the change in column axial force.

The sub-sensitivity coefficient is determined by dividing Eq. (3.35) by ∂H_J and summing over all columns below panel j. The result is

$$\left(\frac{\partial f}{\partial H_J}\right)_{CBP} = \rho \frac{\partial F_{CB}}{\partial H_J} \left\{ \sum_{k=i+1}^{M} \left(u_c(k,j+1)h(k) \frac{\partial A_c(k,j+1)}{\partial F_{CB}}\right) - \sum_{k=i+1}^{M} \left(u_c(k,j)h(k) \frac{\partial A_c(k,j)}{\partial F_{CB}}\right) \right\} \tag{3.36}$$

where,

M = number of stories

Note that the first summation is over all columns in column line j+1 below the panel. This summation increases the value of $(\frac{\partial f}{\partial H_J})_{CBP}$ since the corresponding columns experience an increase in compression force. The second summation is over all columns in column line j below the panel. This summation decreases the value of $(\frac{\partial f}{\partial H_J})_{CBP}$ since the corresponding columns experience a decrease in compression force.

The factor $\frac{\partial A_c(k,p)}{\partial F_{CB}}$ will be described in Section 3.7.1.4. Note that there are certain conditions under which this factor is zero as will also be described in Section 3.7.1.4. The factor $\frac{\partial F_{CB}}{\partial H_J}$ will be be described next.

From global moment equilibrium of panel j in Fig. 3.10,

$$(\Delta H_1 + \Delta H_2)\, h(i) = \Delta F_{CB}\, L(j)$$

But,

$$\Delta H_1 + \Delta H_2 = \Delta H_J$$

Therefore,

$$\frac{\Delta F_{CB}}{\Delta H_J} = \frac{h(i)}{L(j)} = \text{constant}$$

Thus,

$$\frac{\partial F_{CB}}{\partial H_J} = \frac{h(i)}{L(j)} \qquad (3.37)$$

3.7.1.2 Sub-Sensitivity Coefficient of Beams Adjacent to Panel j.

The sub-sensitivity coefficient associated with all beams in story i to the left of panel j will be designated by $(\frac{\partial f}{\partial H_J})_{BLP}$ and that associated with all beams in story i to the right of panel j will be designated by $(\frac{\partial f}{\partial H_J})_{BRP}$. Consider the model illustrated in Fig. 3.11. The cost of a single beam to the left of panel j is

$$f_{BL} = \rho u_B(i,k)\ L(k)\ A_B(i,k) \tag{3.38}$$

where,

$k = 1, 2, \ldots, j-1$

$u_B(i,k)$ = unit material cost of beam k in story i

$L(k)$ = length of bay k

$A_B(i,k)$ = area of beam k in story i

Thus, the change in cost of the beam due to a change in beam axial force is

$$\partial f_{BL} = \rho u_B(i,k)\ L(k)\ \left(\frac{\partial A_B(i,k)}{\partial F_{BL}(i,k)}\right)\ \partial F_{BL}(i,k) \tag{3.39}$$

where,

$\partial F_{BL}(i,k)$ = the change in beam axial force to the left of panel j.

The change in beam axial force may be calculated in a manner which is analogous to the calculation of ΔH_1. The result is

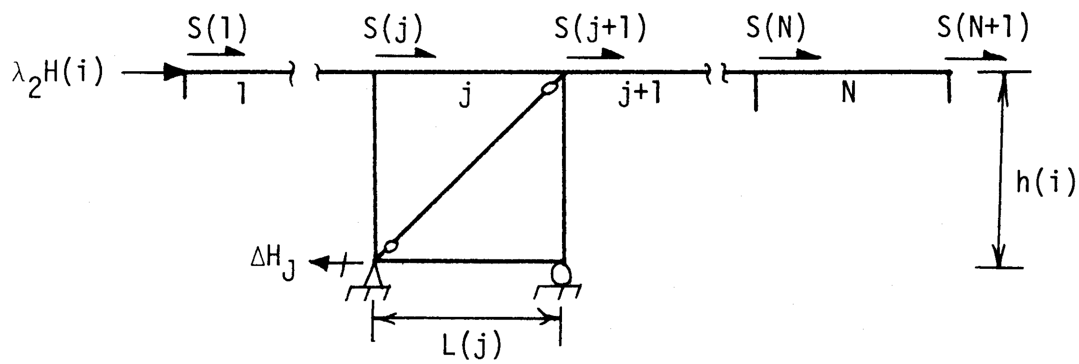


Figure 3.11 Model for the Calculation of the Sub-Sensitivity Coefficient of Beams Adjacent to Panel j.

$$\partial F_{BL}(i,k) = \frac{\Delta H_J}{S_T(i)} (\lambda_2 H(i) + \sum_{p=1}^{k} S(p)) \tag{3.40}$$

Now, the sub-sensitivity coefficient for all beams in story i to the left of panel j may be calculated by substituting Eq. (3.40) into Eq. (3.39), dividing by ∂H_J, and summing over the beams. The result is

$$(\frac{\partial f}{\partial H_J})_{BLP} = \rho \left\{ \sum_{k=1}^{j-1} [u_B(i,k)L(k) \frac{\partial A_B(i,k)}{\partial F_{BL}(i,k)} \cdot \frac{1}{S_T(i)} (\lambda_2 H(i) + \sum_{p=1}^{k} S(p))] \right\} \tag{3.41}$$

The sign of $(\frac{\partial f}{\partial H_J})_{BLP}$ is positive since the beams to the left of panel j experience increases in axial compression forces.

A completely analogous argument is valid for the sub-sensitivity coefficient for all beams in story i to the right of panel j. The results are,

$$\partial F_{BR}(i,k) = - \frac{\Delta H_J}{S_T(i)} \sum_{p=k+1}^{N+1} S(p) \tag{3.42}$$

where,

$\partial F_{BR}(i,k)$ = the change in beam axial force to the right of panel j.

and,

$$\left(\frac{\partial f}{\partial H_J}\right)_{BRP} = -\rho \left\{ \sum_{k=j+1}^{N} [u_B(i,k)L(k) \frac{\partial A_B(i,k)}{\partial F_{BR}(i,k)} \cdot \frac{1}{S_T(i)} \cdot \sum_{p=k+1}^{N+1} S(p)] \right\} \quad (3.43)$$

As shown in Eq. (3.43), the sign of $\left(\frac{\partial f}{\partial H_J}\right)_{BRP}$ is negative when the beams to the right of panel j experience decreases in axial compressive forces. However, there are times when beams to the right of panel j experience increases in axial tension force. If this is the case, the factor $\left(\frac{\partial f}{\partial H_J}\right)_{BRP}$ is set to zero reflecting the fact that beams will not change size under the application of axial tension forces since the combination moment plus axial tension force condition will be designed on the basis of the moment condition only.

The factors $\frac{\partial A_B(i,k)}{\partial F_{BL}(i,k)}$ and $\frac{\partial A_B(i,k)}{\partial F_{BR}(i,k)}$ will be described in Section 3.7.1.4. Note that there are certain conditions under which these two factors are zero as will also be described in Section 3.7.1.4.

Finally, the sub-sensitivity coefficient of all beams adjacent to panel j is

$$\left(\frac{\partial f}{\partial H_J}\right)_{\substack{\text{beams adjacent}\\ \text{to panel}}} = \left(\frac{\partial f}{\partial H_J}\right)_{BLP} + \left(\frac{\partial f}{\partial H_J}\right)_{BRP} \quad (3.44)$$

3.7.1.3 <u>Sub-Sensitivity Coefficient of Panel j Members</u>

Sections 3.7.1.1 and 3.7.1.2 have formulated the sub-sensitivity coefficients associated with those members external to panel j that have experienced force changes due to ΔH_J. It is important to note three conditions that exist for these members. Firstly, these external members experience changes in axial forces only. Secondly, it is known beforehand which members experience increases in compressive axial forces and which members experience decreases in compressive axial forces due to applications of ΔH_J. Thirdly, the changes in axial forces in the external members are independent of the mode of resistance (truss or moment action) of the panel.

The above three conditions are not valid for all panel members. On the contrary, changes in axial forces and moments are very dependent on the mode of panel resistance as well as on the relative values of ΔH_1 and ΔH_2 applied to the upper left and right joints of panel j respectively. For this reason it is first necessary to develop the sub-sensitivity coefficients for the panel members in a more general way than the development for the external panel members. Certain preliminary information will now be developed.

Consider the model panel as illustrated in Fig. (3.8). Define the factor K as,

$$K = \frac{\Delta H_2}{\Delta H_1} \tag{3.45}$$

Note that $\Delta H_1 \neq 0$ since it is required that a concentrated wind load be applied at the left most joint of every story (right most joint when wind applied from the right). Substituting Eq. (3.31) into Eq. (3.45) results in the equation used to calculate K. Thus

$$K = \frac{\sum_{k=j+1}^{N+1} S(k)}{\lambda_2 H(i) + \sum_{k=1}^{j} S(k)} \tag{3.46}$$

Now, dividing Eq. (3.25) by ΔH_1 leads to

$$1 + \frac{\Delta H_2}{\Delta H_1} = \frac{\Delta H_J}{\Delta H_1} \tag{3.47}$$

Consequently, using Eq.'s (3.45) and (3.47), the following relations may be developed.

$$\frac{\Delta H_1}{\Delta H_J} = \frac{1}{1+K}$$
$$\frac{\Delta H_2}{\Delta H_J} = \frac{K}{1+K} \tag{3.48}$$

Note that $\partial H_1/\partial H_J = \Delta H_1/\Delta H_J$ and $\partial H_2/\partial H_J = \Delta H_2/\Delta H_J$.

Define f_{PM} as the cost of a panel member.

So,

$$f_{PM} = \rho u_{PM} L_{PM} A_{PM} \tag{3.49}$$

where,

u_{PM} = unit material cost of a panel member

L_{PM} = length of a panel member

A_{PM} = area of a panel member

Also, the change in cost of a panel member, ∂f_{PM}, due to changes in ΔH_1 and ΔH_2 is expressed as

$$\partial f_{PM} = (\frac{\partial f_{PM}}{\partial H_1})\ \Delta H_1 + (\frac{\partial f_{PM}}{\partial H_2})\ \Delta H_2 \tag{3.50}$$

The sub-sensitivity coefficient for a panel member will now be defined as $(\frac{\partial f}{\partial H_J})_{\substack{panel\\member}}$. After dividing Eq. (3.50) by ∂H_J and using Eq. (3.48), $(\frac{\partial f}{\partial H_J})_{\substack{panel\\member}}$ can be expressed as

$$(\frac{\partial f}{\partial H_J})_{\substack{panel\\member}} = (\frac{\partial f_{PM}}{\partial H_1})\ (\frac{1}{1+K}) + (\frac{\partial f_{PM}}{\partial H_2})\ (\frac{K}{1+K}) \tag{3.51}$$

Since the panel members may be subject to changes in axial force or changes in moment or both, the factors $\frac{\partial f_{PM}}{\partial H_1}$ and $\frac{\partial f_{PM}}{\partial H_2}$ can be expanded as follows,

$$\begin{aligned} \frac{\partial f_{PM}}{\partial H_1} &= (\frac{\partial f_{PM}}{\partial M_p})\ (\frac{\partial M_p}{\partial H_1}) + (\frac{\partial f_{PM}}{\partial F})\ (\frac{\partial F}{\partial H_1}) \\ \frac{\partial f_{PM}}{\partial H_2} &= (\frac{\partial f_{PM}}{\partial M_p})\ (\frac{\partial M_p}{\partial H_2}) + (\frac{\partial f_{PM}}{\partial F})\ (\frac{\partial F}{\partial H_2}) \end{aligned} \tag{3.52}$$

where,

M_p = required plastic moment capacity;

F = required axial force capacity.

The change in cost of a panel member with respect to a change in required force capacity will now be investigated. Dividing Eq. (3.49) by ∂M_p and ∂F, the change in cost of beams, columns and tension brace of panel j in story i with respect to changes in required plastic moment and axial force capacities are expressed respectively as

$$\left(\frac{\partial f}{\partial M_p}\right)_{\text{Beam}} = \rho\, u_B(k,j)L(j)\, \frac{\partial A_B(k,j)}{\partial M_{BP}(k,j)}, \quad k = i,i+1$$

$$\left(\frac{\partial f}{\partial F}\right)_{\text{Beam}} = \rho\, u_B(k,j)L(j)\, \frac{\partial A_B(k,j)}{\partial F_B(k,j)}, \quad k = i,i+1$$

$$\left(\frac{\partial f}{\partial M_p}\right)_{\text{Column}} = \rho\, u_C(i,k)h(i)\, \frac{\partial A_C(i,k)}{\partial M_{CP}(i,k)}, \quad k = j,j+1 \qquad (3.53)$$

$$\left(\frac{\partial f}{\partial F}\right)_{\text{Column}} = \rho\, u_C(i,k)h(i)\, \frac{\partial A_C(i,k)}{\partial F_C(i,k)}, \quad k = j,j+1$$

$$\left(\frac{\partial f}{\partial F}\right)_{\substack{\text{Tension}\\ \text{Brace}}} = \rho\, u_{BR}(i,j)L_B(j)\, \frac{\partial A_{BR}(i,j)}{\partial F_{BR}(i,j)}$$

The factors $\frac{\partial A}{\partial M_p}$ and $\frac{\partial A}{\partial F}$ will be described in Section 3.7.1.4. Note that there are certain conditions under which these two factors are zero which will also be described in Section 3.7.1.4.

The changes in required force capacity of the panel members with respect to changes in ΔH_1 and ΔH_2 will now be described. Since the factors $\frac{\partial M_p}{\partial H_k}$ and $\frac{\partial F}{\partial H_k}$, k = 1,2, are highly dependent on the mode of resistance of the panel (moment or truss action), the behavior of the panel in these modes must be determined. To begin with, the behavior of the panel as a moment resisting panel will first be described. The model used to describe this behavior will be based on the moment distributions in the panel beams and columns.

Under the gravity load condition, a moment distribution exists in the beams and columns where the load factor used is λ_1. Under the combination gravity plus wind load condition the load factor λ_2 is used where $\lambda_2<\lambda_1$. Consequently, the initial moment distributions under the factored combination loads, that is to say, the moment distributions that exist when the total value of incremental shear applied to a panel is zero, may be calculated by multiplying the factored gravity load moments by the factor $\frac{\lambda_2}{\lambda_1} < 1.0$. Furthermore, the initial values of gravity moments considered in the beams are equal to one-half the actual gravity moments except in the top story beams where the full value is used. In other words, one-half of a beam's initial moments are associated with the panel above the beam while the other half is associated with the panel below the beam. Under the application of incremental lateral shear forces to a panel, the beams and columns will experience incremental moment changes. Four moment states of a panel will be defined as functions of the beam moment values resulting from these incremental moments being added to the initial moment diagram. Furthermore, each moment state corresponds to a particular type of failure mechanism defined for the panel. Note that member design will always be based on the full value of moments. For beams, the full value of moment is the sum of beam moments associated with the panel above and panel below the beam. For columns, the full value of moment is the sum of column moments associated with the panel to the left and panel to the right of the column.

The factored gravity load moments used in the moment state model of a panel is shown in Fig. 3.12 where the panel failure mechanism is a beam mechanism failure. Note that the initial moment distribution implies the following values of P_1' and P_2'

$$P_1' = \begin{cases} P_W(i,j), & i = 1 \\ \frac{1}{2} P_W(i,j), & i \geq 2 \end{cases} \tag{3.54}$$

$$P_2' = \frac{1}{2} P_W(i,j), \quad i \geq 2$$

Fig. 3.13 illustrates the initial moment diagram under the factored combination loads.

Moment State 1 is now defined as that state in which lateral load is applied to a panel until the value of the initial right end beam moment under factored combination loads increases up to the value of the initial right end beam moment under the factored gravity loads. The failure mechanism of this state is a combination beam and panel mechanism failure. During the application of ΔH_J in Moment State 1 the value of the left end beam moment decreases in a counter-clockwise sense (or increases in a clockwise sense). The final moments in Moment State 1 are the initial moments of Moment State 2 as illustrated in Fig. 3.14. Moment State 2 is defined as that state in which lateral load is applied to the panel until the value of the left end beam moment equals the value of moment at the center and right end of the beam. The failure mechanism of this state is still the combination failure mechanism. The final moments in Moment State 2 are the initial moments of Moment State 3 as illustrated in Fig. 3.15. Moment State 3 is defined as that state in which lateral load is applied to the panel until the smaller of the upper and lower beam end moments become equal to the larger of the upper or lower beam moments. This state is a transition state from the combination failure mechanism to a full sway failure mechanism. The final moments of State 3 are the initial moments of Moment State 4 as illustrated in Fig. 3.16. Note that Moment State 3 is skipped if the factor $C_v(j)$ defined as

$$C_v(j) \quad = \quad P_2'/P_1' \tag{3.55}$$

has the value $C_v(j)=1.0$. Moment State 4 is defined as that state in which the additional application of lateral load causes equal increments of additional moment at the beam and column ends. This is the full sway failure mechanism state.

The above brief definitions of the four moment states, or failure mechanism states, will serve as an introduction. Detailed definitions including the formulation of the factors $\frac{\partial M_p}{\partial H_k}$ and $\frac{\partial F}{\partial H_k}$, k = 1,2, will now be described. Note again that the case of wind from the left will be formulated. Completely analogous arguments are valid for wind from the right. Also, the joint size effect will be included. When the joint sizes are not known, as in the first cycle of design, this effect is neglected by assuming beam and column depths to be zero.

1. Moment State 1 - Failure Mechanism State 1

The initial moment diagram of this state corresponds to the factored combination load condition with $H_J=0$ as illustrated in

Fig. 3.13. However, as discussed above, for the purposes of defining the failure mechanism states, the initial values in the beams are set to one-half the actual initial values. If the panel is selected to resist the application of lateral load as a moment resisting panel, sufficient lateral load will be applied in order to increase the leeward, or right end beam moments in the panel up to the value of M^* shown in Fig. 3.12.

In the remainder of this chapter, the following notation is adopted: $\Delta M_{BR}(k,j)$, $\Delta M_{BL}(k,j)$, and $\Delta M_{BC}(k,j)$ are the beam right joint, left joint, and center moment increments respectively in panel j of story i. The subscript k=1 designates the upper panel beam and k=2 designates the lower panel beam. Also, $\Delta M_{CT}(j)$ and $\Delta M_{CB}(j)$ are the column top and bottom joint moment increments respectively in column j of story i. Also note that a prime designates a member end moment considering the joint size effect.

Now, suppose the increment of right joint moment of the upper beam due to wind from the left is ΔM. Thus,

$$\Delta M_{BR}(1,j) = \Delta M \qquad (3.56)$$

The increments of moments of the other parts of the beams are now obtained by applying the collapse mechanism condition and the moment equilibriums. The collapse mechanism condition is a combination beam and panel mechanism with plastic hinge locations at the center and right end of the upper and lower panel beams. Consequently,

$$\Delta M_{BC}(k,j) = \Delta M'_{BR}(k,j), \quad k = 1,2, \qquad (3.57)$$

Substituting Eq.'s (3.56) and (3.57) into Eq. (3.12) leads to the upper beam center moment increment. Thus,

$$\Delta M_{BC}(1,j) = \frac{L(j) - d_c'(i,j)}{L(j) + d_c'(i,j)} \Delta M \tag{3.58}$$

The upper left joint beam moment increment is calculated by substituting Eq.'s (3.56) and (3.58) into the equilibrium equation for increments of beam moment, Eq. (3.6), and solving for $\Delta M_{BL}(1,j)$. So,

$$\Delta M_{BL}(1,j) = \frac{3L(j) - d_c'(i,j)}{L(j) + d_c'(i,j)} \Delta M \tag{3.59}$$

Noting the definition of $C_v(j)$ by Eq. (3.55), the lower beam moment increments are calculated as

$$\begin{aligned} \Delta M_{BR}(2,j) &= C_v(j)\,\Delta M \\ \Delta M_{BC}(2,j) &= C_v(j)\,\frac{L(j) - d_c'(i,j)}{L(j) + d_c'(i,j)} \Delta M \\ \Delta M_{BL}(2,j) &= C_v(j)\,\frac{3L(j) - d_c'(i,j)}{L(j) + d_c'(i,j)} \Delta M \end{aligned} \tag{3.60}$$

The increments of joint moments of the columns are obtained by using the joint equilibrium equation. They are,

$$\Delta M_{CT}(j) = \frac{-3L(j) + d_c'(i,j)}{L(j) + d_c'(i,j)} \Delta M$$

$$\Delta M_{CB}(j) = -C_v(j) \frac{3L(j) - d_c'(i,j)}{L(j) + d_c'(i,j)} \Delta M \qquad (3.61)$$

$$\Delta M_{CT}(j+1) = -\Delta M$$

$$\Delta M_{CB}(j+1) = -C_v(j)\ \Delta M$$

The calculation of ΔM will now be described. Consider a free body diagram of the upper half of panel j as illustrated in Fig. 3.17. From the lateral force equilibrium requirement,

$$\Delta H_J + \Delta S_1 + \Delta S_2 + C\Delta H_J = 0$$

Thus,

$$\Delta S_1 + \Delta S_2 = -\Delta H_J(1+C) \qquad (3.62)$$

Also, from the moment equilibrium requirement,

$$\Delta M_{CT}(j)+\Delta M_{CB}(j)+\Delta M_{CT}(j+1)+\Delta M_{CB}(j+1) = (\Delta S_1+\Delta S_2)h(i) \qquad (3.63)$$

Consequently, from Eq.'s (3.62) and (3.63),

$$\Delta M_{CT}(j)+\Delta M_{CB}(j)+\Delta M_{CT}(j+1)+\Delta M_{CB}(j+1) = -\Delta H_J h(i)(1+C) \qquad (3.64)$$

Now, substituting Eq. (3.61) into Eq. (3.64) and collecting terms results in the relation between ΔM and ΔH_J. Thus,

$$\Delta M = \frac{L(j) + d_c'(i,j)}{4L(j)} \cdot \frac{\Delta H_J h(i)(1+C)}{1+C_v(j)} \qquad (3.65)$$

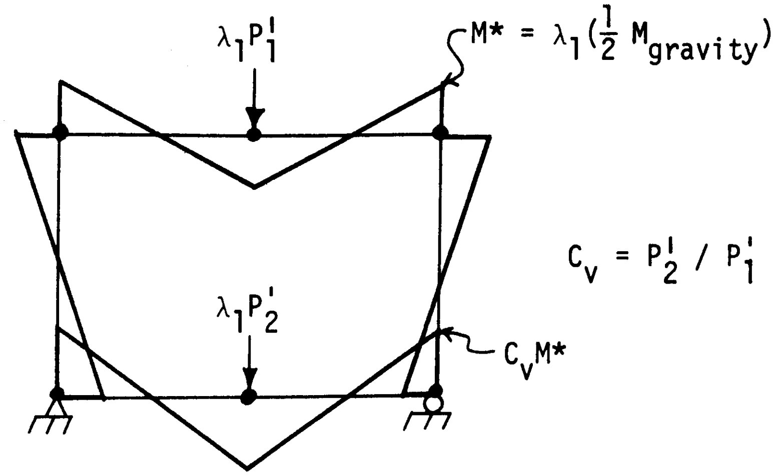


Figure 3.12 Moment Diagram Under Factored Gravity Loads in the Moment State Model Panel.

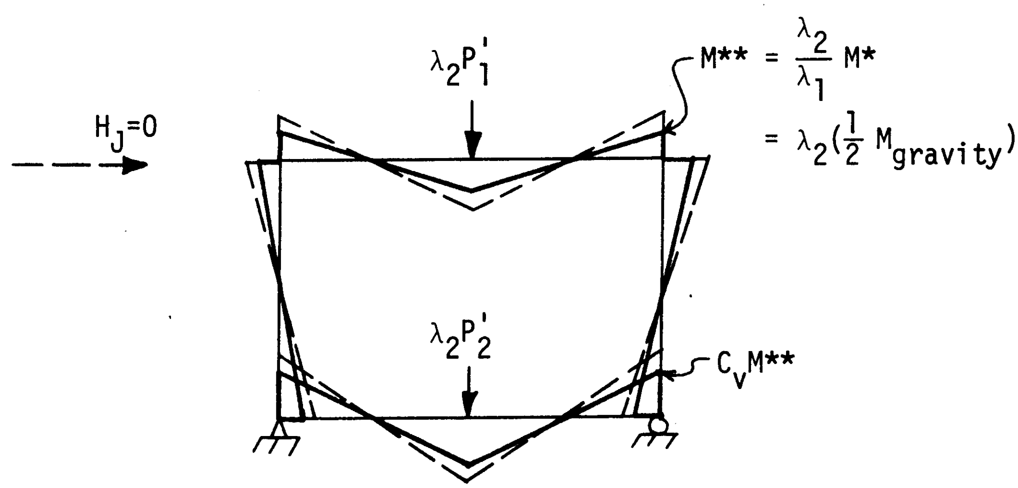


Figure 3.13 Initial Moment Diagram Under Factored Combination Loads in the Moment State Model Panel.

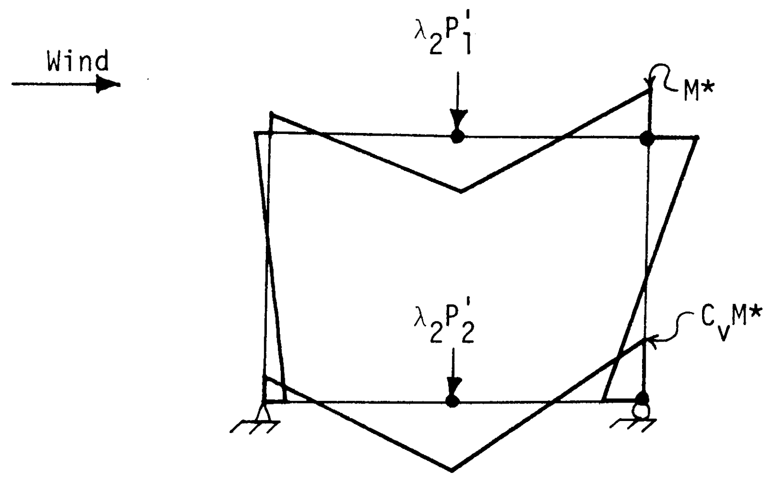


Figure 3.14 Initial Moment Diagram of Moment State 2.

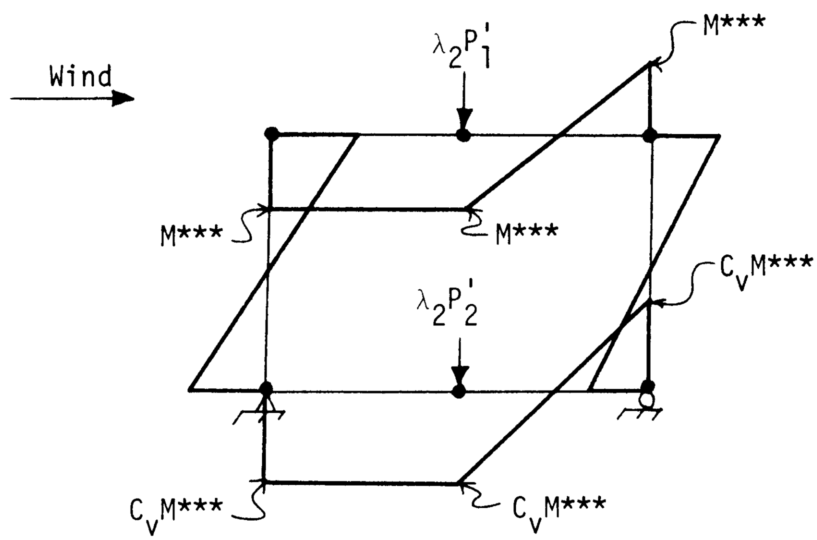


Figure 3.15 Initial Moment Diagram of Moment State 3.

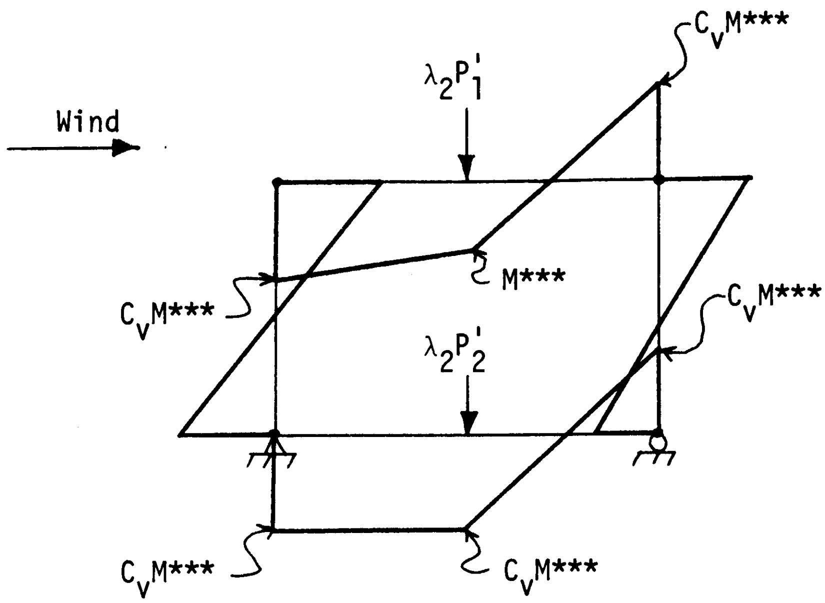


Figure 3.16 Initial Moment Diagram of Moment State 4.

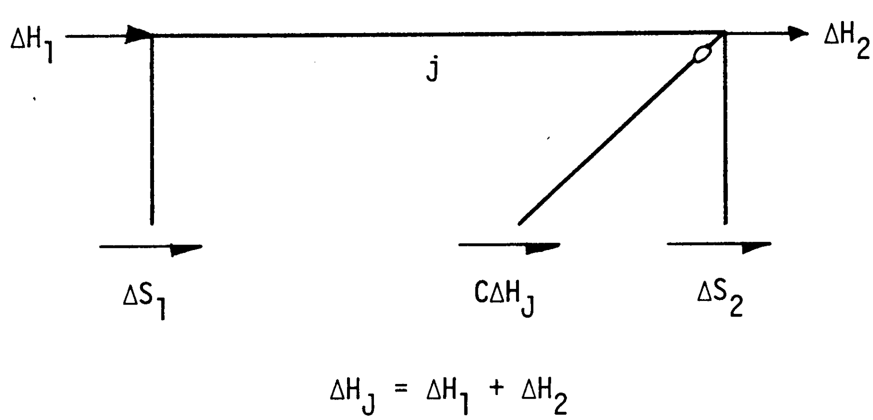


Figure 3.17 Free Body Diagram of Upper Half of Panel j.

In addition, ΔM can be expressed as the sum of ΔM_1 and ΔM_2 due to ΔH_1 and ΔH_2 respectively. So,

$$\Delta M = \Delta M_1 + \Delta M_2$$

$$\Delta M_1 = \frac{L(j) + d_c'(i,j)}{4L(j)} \cdot \frac{\Delta H_1 h(i)(1+C)}{1+C_v(j)} \qquad (3.66)$$

$$\Delta M_2 = \frac{L(j) + d_c'(i,j)}{4L(j)} \cdot \frac{\Delta H_2 h(i)(1+C)}{1+C_v(j)}$$

Consequently, for a given value of $\Delta H_J = \Delta H_1 + \Delta H_2$, the moment distribution in this moment state can be determined.

Furthermore, the terminal value of the upper right joint beam moment, $M_{BR,max.}$ for this state corresponding to Eq. (3.21) and the definition of Moment State 1 is

$$M_{BR,max.} = \lambda_1 P_1'(L(j) + d_c'(i,j))/8 \qquad (3.67)$$

where P_1' is defined by Eq. (3.54). Thus, the total change in the upper right joint beam moment, ΔM_T, is

$$\Delta M_T = (\lambda_1 - \lambda_2)\ P_1'(L(j) + d_c'(i,j))/8 \qquad (3.68)$$

Since the maximum value of beam moment in this state is that due to the factored gravity load condition, the required plastic moment capacity of beams, M_{BP}, due to ΔH_1 and ΔH_2 need not be increased. So,

$$\frac{\partial M_{BP}}{\partial H_1} = \frac{\partial M_{BP}}{\partial H_2} = 0 \qquad (3.69)$$

However, for the columns, the required plastic moment capacities, M_{CP}, are functions of the moment distributions in the adjacent panels. Therefore, $\frac{\partial M_{CP}}{\partial H_1}$ and $\frac{\partial M_{CP}}{\partial H_2}$ will be calculated in this state as follows.

$$\frac{\partial M_{CP}}{\partial H_k} = \frac{\Delta M_{CP}}{\Delta H_k} = \frac{M_{CP,new} - M_{CP,old}}{\Delta H_k}, \quad k = 1,2 \tag{3.70}$$

If by Eq. (3.70) the factor $\frac{\Delta M_{CP}}{\Delta H_k} \leq 0$ it will be set to zero implying no increase in required column plastic moment capacity in Moment State 1.

Changes in required panel member axial forces in Moment State 1 will now be described. The notation used is as follows:

$\Delta F_B(i,j)$ and $\Delta F_B(i+1,j)$ designate changes in the upper and lower beam axial forces respectively while $\Delta F_C(i,j)$ and $\Delta F_C(i,j+1)$ designate changes in the left and right column axial forces. Furthermore, $\Delta F_{BR}(i,j,2)$ designates the change in the tension brace force when wind is applied from the left and $\Delta F_{BR}(i,j,1)$ designates the change in the tension brace force when wind is applied from the right. Note again that the following formulations are with respect to wind from the left.

Consider the free body diagram of the upper left joint of panel j illustrated in Fig. 3.18. The net axial force in the upper beam will be taken as the sum: $\Delta F^t_{B1} + \Delta F^t_{B2}$. Horizontal force equilibrium and column moment equilibrium require that,

$$\begin{aligned} \Delta F^t_{B1} &= \Delta H_1 + \Delta S_1 = \Delta H_1 + \frac{\Delta M_{CT}(j) + \Delta M_{CB}(j)}{h(i)} \\ \Delta F^t_{B2} &= \Delta S_2 = \frac{\Delta M_{CT}(j) + \Delta M_{CB}(j)}{h(i)} \end{aligned} \tag{3.71}$$

Substituting Eq. (3.61) into Eq. (3.71) where $\Delta M=\Delta M_1$ or $\Delta M=\Delta M_2$ defined by Eq. (3.66) results in

$$\frac{\Delta F^t_{B1}}{\Delta H_1} = \frac{1}{4}\left(1-3C + \frac{d'_c(i,j)}{L(j)}(1+C)\right)$$

$$\frac{\Delta F^t_{B2}}{\Delta H_2} = -\frac{3}{4}\left(1+C - \frac{d'_c(i,j)}{3L(j)}(1+C)\right) \tag{3.72}$$

Since $\frac{\Delta F^t_{B1}}{\Delta H_1}$ and $\frac{\Delta F^t_{B2}}{\Delta H_2}$ are constants, the changes in required beam axial force capacities with respect to ΔH_1 and ΔH_2 are

$$\frac{\partial F_{B1}(i,j)}{\partial H_1} = \frac{\Delta F^t_{B1}}{\Delta H_1}$$

$$\frac{\partial F_{B2}(i,j)}{\partial H_2} = \frac{\Delta F^t_{B2}}{\Delta H_2} \tag{3.73}$$

Finally,

$$\Delta F_B(i,j) = \Delta F^t_{B1} + \Delta F^t_{B2} \tag{3.74}$$

Note that when the panel is selected to resist ΔH_J as a truss resisting panel (C=-1.0), $\Delta F^t_{B1} = \Delta H_1$ and $\Delta F^t_{B2} = 0$. This shows that ΔH_1 passes through the upper beam to the upper right joint where it and ΔH_2 are transferred to the bottom support points through the tension brace and the right column.

Consider now a free body diagram of the right half of panel j as illustrated in Fig. 3.19. The net axial force in the lower beam will

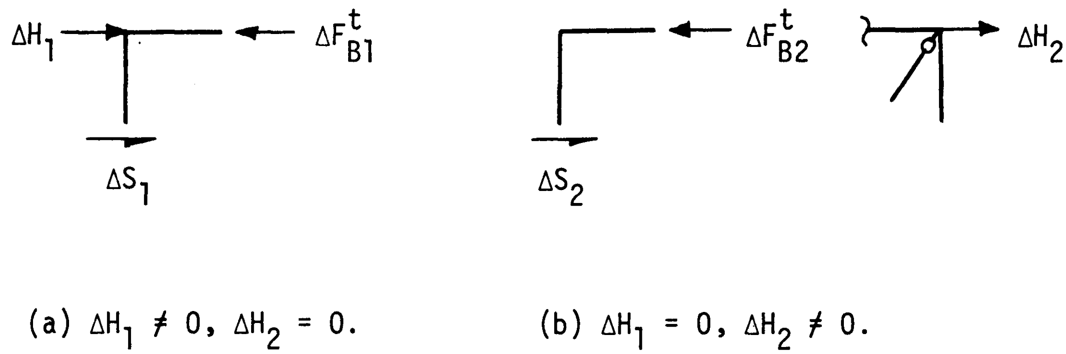


Figure 3.18 Free Body Diagram of Upper Left Joint of Panel j.

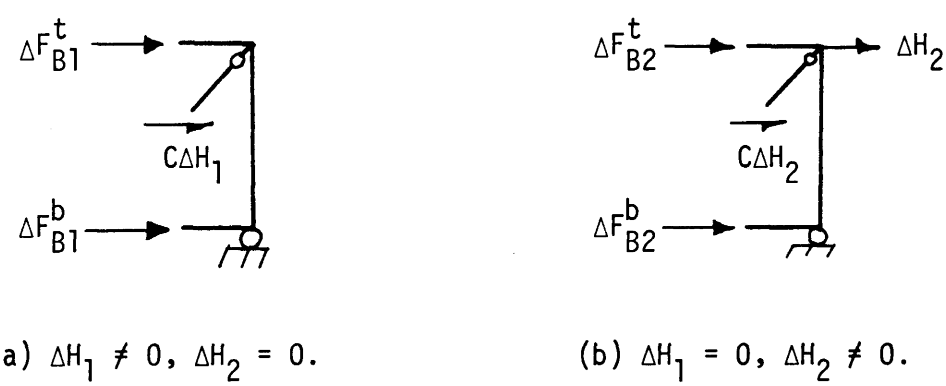


Figure 3.19 Free Body Diagram of Right Half of Panel j.

be taken as the sum: $\Delta F^b_{B1} + \Delta F^b_{B2}$. Horizontal force equilibrium requires that,

$$\begin{aligned} \Delta F^b_{B1} &= -(\Delta F^t_{B1} + C\Delta H_1) \\ \Delta F^b_{B2} &= -(\Delta H_2 + \Delta F^t_{B2} + C\Delta H_2) \end{aligned} \qquad (3.75)$$

Substituting Eq. (3.72) into Eq. (3.75) results in

$$\frac{\Delta F^b_{B1}}{\Delta H_1} = \frac{\Delta F^b_{B2}}{\Delta H_2} = -\frac{1}{4}\left(1+C+\frac{d'_c(i,j)}{L(j)}(1+C)\right) \qquad (3.76)$$

Since $\frac{\Delta F^b_{B1}}{\Delta H_1}$ and $\frac{\Delta F^b_{B2}}{\Delta H_2}$ are constants and equal,

$$\frac{\partial F_B(i+1,j)}{\partial H_1} = \frac{\partial F_B(i+1,j)}{\partial H_2} = \frac{\Delta F^b_{B1}}{\Delta H_1} \qquad (3.77)$$

Finally,

$$\Delta F_B(i+1,j) = \Delta F^b_{B1} + \Delta F^b_{B2} = -\frac{\Delta H_J}{4}\left(1+C+\frac{d'_c(i,j)}{L(j)}(1+C)\right) \qquad (3.78)$$

Consider now a free body diagram of the upper right joint of panel j as illustrated in Fig. 3.20. The net axial force in the right column is taken as the sum: $\Delta F^r_{C1} + \Delta F^r_{C2}$. Vertical force equilibrium requires that,

$$\Delta F^r_{C1} = \Delta S_{B1} - C\Delta H_1 \frac{h(i)}{L(j)}$$

$$= \frac{\Delta M_{BL}(1,j) + \Delta M_{BR}(1,j)}{L(j)} - C\Delta H_1 \frac{h(i)}{L(j)}$$

$$\Delta F^r_{C2} = \Delta S_{B2} - C\Delta H_2 \frac{h(i)}{L(j)} \tag{3.79}$$

$$= \frac{\Delta M_{BL}(1,j) + \Delta M_{BR}(1,j)}{L(j)} - C\Delta H_2 \frac{h(i)}{L(j)}$$

Using Eq.'s (3.56), (3.59), (3.66), and (3.79) leads to,

$$\frac{\Delta F^r_{C1}}{\Delta H_1} = \frac{\Delta F^r_{C2}}{\Delta H_2} = \frac{h(i)}{L(j)} \; \frac{1 - C\cdot C_v(j)}{1 + C_v(j)} \tag{3.80}$$

Since these factors are constant and equal,

$$\frac{\partial F_C(i,j+1)}{\partial H_1} = \frac{\partial F_C(i,j+1)}{\partial H_2} = \frac{\Delta F^r_{C1}}{\Delta H_1} \tag{3.81}$$

Finally,

$$\Delta F_C(i,j+1) = \Delta F^r_{C1} + \Delta F^r_{C2} = \frac{h(i)}{L(j)} \left[\frac{1 - C\cdot C_v(j)}{1 + C_v(j)}\right] \Delta H_J \tag{3.82}$$

Consider now a free body diagram of the upper half of panel j as illustrated in Fig. 3.21. The net axial force in the left column will be taken as the sum: $\Delta F^L_{C1} + \Delta F^L_{C2}$. Vertical force equilibrium requires that,

$$\Delta F^L_{C1} = -(\Delta F^r_{C1} + C\Delta H_1 \frac{h(i)}{L(j)})$$

$$\Delta F^L_{C2} = -(\Delta F^r_{C2} + C\Delta H_2 \frac{h(i)}{L(j)}) \tag{3.83}$$

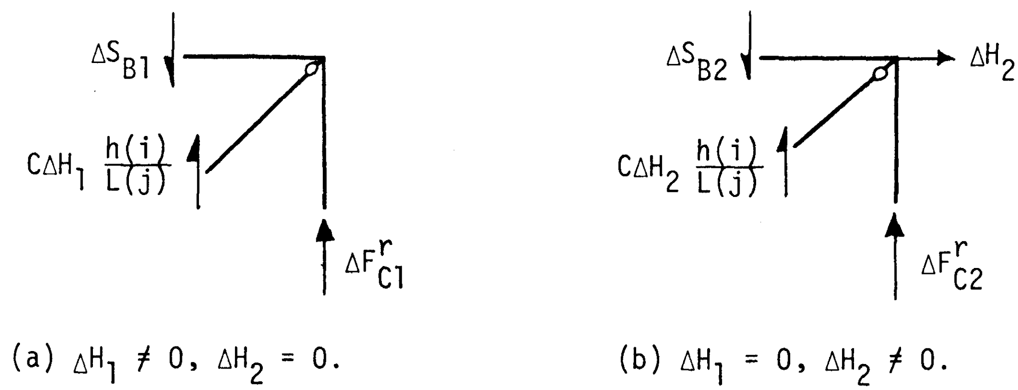


Figure 3.20 Free Body Diagram of Upper Right Joint of Panel j.

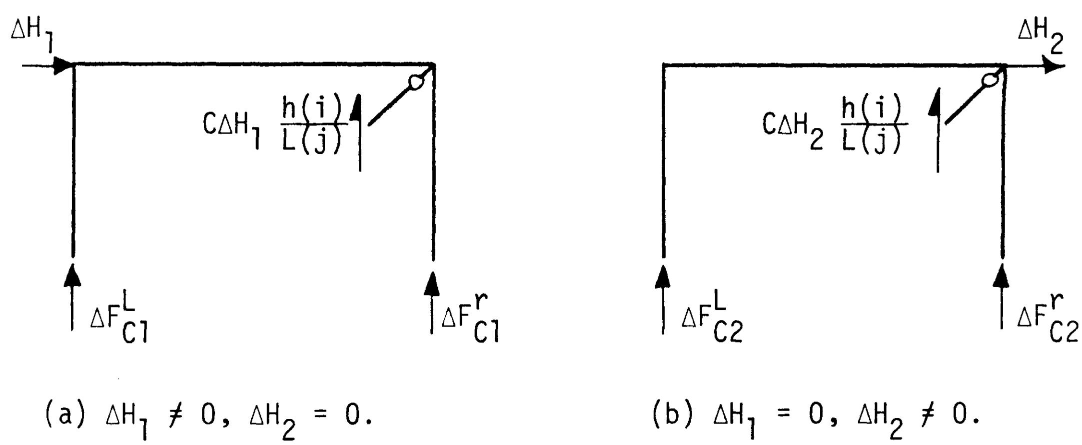


Figure 3.21 Free Body Diagram of Upper Half of Panel j.

Substituting Eq. (3.80) into Eq. (3.83) results in,

$$\frac{\Delta F^L_{C1}}{\Delta H_1} = \frac{\Delta F^L_{C2}}{\Delta H_2} = -\frac{h(i)}{L(j)} \frac{1 + C}{1 + C_v(j)} \tag{3.84}$$

Since these factors are constant and equal,

$$\frac{\partial F_C(i,j)}{\partial H_1} = \frac{\partial F_C(i,j)}{\partial H_2} = \frac{\Delta F^L_{C1}}{\Delta H_1} \tag{3.85}$$

Finally,

$$\Delta F_C(i,j) = \Delta F^L_{C1} + \Delta F^L_{C2} = -\frac{h(i)}{L(j)} \left[\frac{1 + C}{1 + C_v(j)}\right] \Delta H_J \tag{3.86}$$

Consider now a free body diagram of panel j as illustrated in Fig. 3.22. The right and left reactions respectively are ΔR_r and ΔR_L. Also, ΔR_r and ΔR_L are taken as increments of axial force in all columns below the panel in column lines j+1 and j respectively. Now, moment and vertical force equilibrium require that,

$$\begin{aligned} \Delta R_r &= \frac{h(i)}{L(j)} \Delta H_J \\ \Delta R_L &= -\frac{h(i)}{L(j)} \Delta H_J \end{aligned} \tag{3.87}$$

In addition,

$$\begin{aligned} \Delta F_C(k,j) &= \Delta R_L, \; k = i+1, i+2, \ldots, M \\ \Delta F_C(k,j+1) &= \Delta R_r, \; k = i+1, i+2, \ldots, M \end{aligned} \tag{3.88}$$

Also,

$$\frac{\partial F_C(k,j+1)}{\partial H_J} = \frac{\Delta F_C(k,j+1)}{\Delta H_J} = \frac{h(i)}{L(j)}, \quad k = i+1, \ldots, M$$

$$\frac{\partial F_C(k,j)}{\partial H_J} = \frac{\Delta F_C(k,j)}{\Delta H_J} = -\frac{h(i)}{L(j)}, \quad k = i+1, \ldots, M \qquad (3.89)$$

Consider finally a free body diagram of the tension brace as illustrated in Fig. 3.23. The horizontal component of brace force is $C\Delta H_J$. Thus,

$$\Delta F_{BR}(i,j,2) = \frac{C\Delta H_J}{\cos\theta} = C\,\frac{L_B(i,j)}{L(j)}\,\Delta H_J \qquad (3.90)$$

where,

$L_B(i,j)$ = diagonal brace length.

Furthermore,

$$\frac{\partial F_{BR}(i,j,2)}{\partial H_J} = \frac{\Delta F_{BR}(i,j,2)}{\Delta H_J} = C\,\frac{L_B(i,j)}{L(j)} \qquad (3.91)$$

This completes the detailed description of Moment State 1 or Failure Mechanism State 1.

2. Moment State 2 - Failure Mechanism State 2

The initial moment diagram of this state is shown in Fig. 3.14. The leeward end moments of the beams under the factored combination load condition are equal to the corresponding moments under the factored gravity loads. After this point, the required plastic moment capacities of the beams must be increased due to additional horizontal loads.

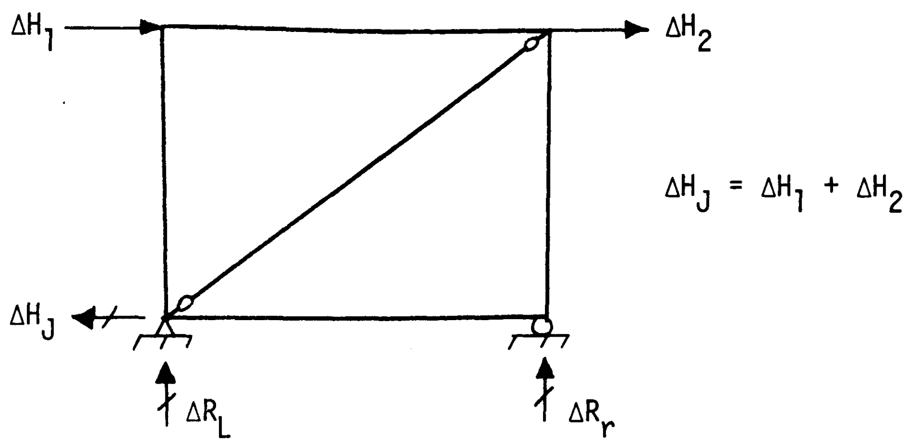


Figure 3.22 Free Body Diagram of Panel j.

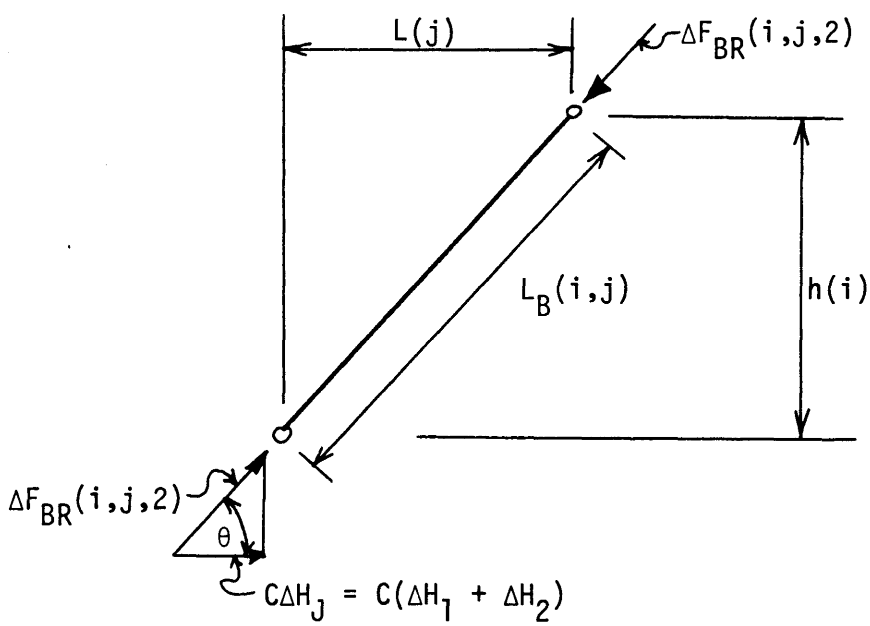


Figure 3.23 Free Body Diagram of Tension Brace.

The failure collapse mechanism in this state is the same as in the first state, that is, the combination beam and panel failure mechanism. Consequently, all equations for increments of moments and axial forces in this state are the same as in the first state. Furthermore, the change in required plastic moment capacities of the beams are

$$\Delta M_{BP}(1,j) = \Delta M'_{BR}(1,j)$$
$$\Delta M_{BP}(2,j) = \Delta M'_{BR}(2,j) \qquad (3.92)$$

So, from Eq.'s (3.57), (3.58), (3.60), (3.66), and (3.92) the change in required beam plastic moment capacities with respect to ΔH_1 and ΔH_2 are

$$\frac{\Delta M_{BP}(1,j)}{\Delta H_k} = \frac{L(j) - d'_c(i,j)}{4L(j)} \; \frac{h(i)(1+C)}{1 + C_v(j)} \; , \; k = 1,2$$
$$\frac{\Delta M_{BP}(2,j)}{\Delta H_k} = C_v(j) \; \frac{\Delta M_{BP}(1,j)}{\Delta H_J} \qquad (3.93)$$

These factors are constant. So,

$$\frac{\partial M_{BP}(1,j)}{\partial H_k} = \frac{\Delta M_{BP}(1,j)}{\Delta H_k} \; , \; k = 1,2$$
$$\frac{\partial M_{BP}(2,j)}{\partial H_k} = \frac{\Delta M_{BP}(2,j)}{\Delta H_k} \; , \; k = 1,2 \qquad (3.94)$$

For columns, due to the same reason as in the first state,

$$\frac{\partial M_{CP}}{\partial H_k} = \frac{\Delta M_{CP}}{\Delta H_k} = \frac{M_{CP,new} - M_{CP,old}}{\Delta H_k} \; , \; k = 1,2 \qquad (3.95)$$

This state terminates at the point where the windward end beam moments become equal to the beam moments at the center and right end. Beyond this point the sway mechanism becomes the exact failure mechanism.

The terminal leeward joint moment of the upper beam in this state, $M_{BR,max.}$, is easily obtainable by considering the terminal moment state as illustrated in Fig. 3.24. The virtual work equation states that,

$$M_{BC} = M'_{BR,max.} = \frac{1}{2\theta} \frac{L(j) - d'_c(i,j)}{2} \theta \lambda_2 P'_1 = \lambda_2 P'_1 \frac{L(j) - d'_c(i,j)}{4}$$

Considering the joint size effect,

$$M_{BR,max.} = \frac{L(j) + d'_c(i,j)}{L(j) - d'_c(i,j)} M'_{BR,max.} \tag{3.96}$$

Thus,

$$M_{BR,max.} = \lambda_2 P'_1 (L(j) + d'_c(i,j))/4 \tag{3.97}$$

Subtracting Eq. (3.67) from Eq. (3.97) results in the total change in the upper right joint beam moment ΔM_T. So,

$$\Delta M_T = (2\lambda_2 - \lambda_1) P'_1 (L(j) + d'_c(i,j))/8 \tag{3.98}$$

This completes the detailed description of Moment State 2 or Failure Mechanism State 2.

3. <u>Moment State 3 - Failure Mechanism State 3</u>

If the vertical loads P'_1 and P'_2 on the upper and lower panel beams are equal, $C_v(j)=1.0$, and this state is skipped. Consequently,

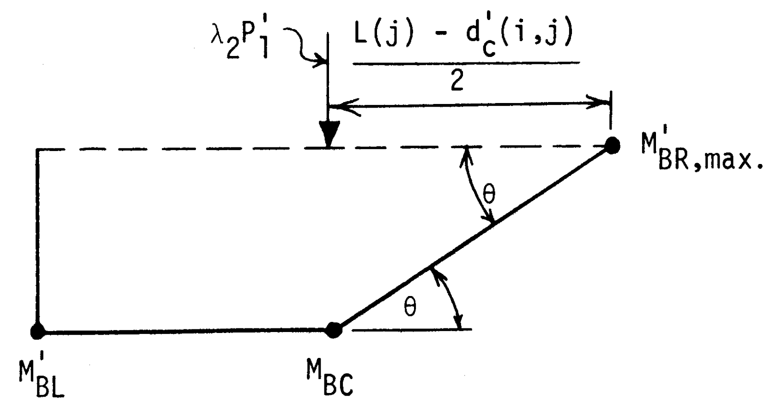


Figure 3.24 Terminal Upper Beam Moments in Moment State 2.

when $C_v(j) \neq 1.0$, this state is considered as the transition state from the combination failure mechanism to the full sway failure mechanism.

Now, when $C_v(j) \neq 1.0$, the beam joint moments are not all equal to each other at the initial point of this state as illustrated in Fig. 3.15. The ratio of the joint moment of the lower beam to the upper beam is $C_v(j)$. So, in order to equate the upper and lower beam joint moments, the additional joint moments will be applied to the beam which has the smaller joint moment.

If $C_v(j) < 1.0$:

$$\begin{aligned} \Delta M_{BR}(2,j) &= \Delta M_{BL}(2,j) = \Delta M \\ \Delta M_{BC}(2,j) &= \Delta M_{BR}(1,j) = \Delta M_{BL}(1,j) = \Delta M_{BC}(1,j) = 0 \\ \Delta M_{CB}(j) &= \Delta M_{CB}(j+1) = -\Delta M \\ \Delta M_{CT}(j) &= \Delta M_{CT}(j+1) = 0 \end{aligned} \tag{3.99}$$

If $C_v(j) > 1.0$:

$$\begin{aligned} \Delta M_{BR}(1,j) &= \Delta M_{BL}(1,j) = \Delta M \\ \Delta M_{BC}(1,j) &= \Delta M_{BR}(2,j) = \Delta M_{BL}(2,j) = \Delta M_{BC}(2,j) = 0 \\ \Delta M_{CT}(j) &= \Delta M_{CT}(j+1) = -\Delta M \\ \Delta M_{CB}(j) &= \Delta M_{CB}(j+1) = 0 \end{aligned} \tag{3.100}$$

Substituting Eq. (3.99) or (3.100) into the moment equilibrium Eq. (3.64) results in

$$\Delta M = \frac{\Delta H_J \; h(i)}{2} (1+C) \tag{3.101}$$

The change in beam end moment, $\Delta M'$, considering the joint size effect may be obtained by substituting Eq. (3.99) or (3.100) into Eq. (3.12). The result is,

$$\Delta M' = (1 - \frac{d_c'(i,j)}{L(j)}) \Delta M \tag{3.102}$$

Now, since the change in required plastic moment capacity of a beam is equal to $\Delta M'$, $\frac{\partial M_{BP}}{\partial H_k}$, k=1,2 for a beam is, for $C_v(j) < 1.0$,

$$\frac{\partial M_{BP}(2,j)}{\partial H_k} = \frac{\Delta M'(2,j)}{\Delta H_k} = \frac{h(i)}{2} (1 - \frac{d_c'(i,j)}{L(j)}) (1+C), \quad k = 1,2$$
$$\frac{\partial M_{BP}(1,j)}{\partial H_k} = \frac{\Delta M'(1,j)}{\Delta H_k} = 0, \quad k = 1,2 \tag{3.103}$$

and for $C_v(j) > 1.0$,

$$\frac{\partial M_{BP}(1,j)}{\partial H_k} = \frac{\Delta M'(1,j)}{\Delta H_k} = \frac{h(i)}{2} (1 - \frac{d_c'(i,j)}{L(j)}) (1+C), \quad k = 1,2$$
$$\frac{\partial M_{BP}(2,j)}{\partial H_k} = \frac{\Delta M'(2,j)}{\Delta H_k} = 0, \quad k = 1,2 \tag{3.104}$$

The required plastic moment capacities of columns obviously do not change in this state. Thus,

$$\frac{\partial M_{CP}(i,j)}{\partial H_k} = \frac{\partial M_{CP}(i,j+1)}{\partial H_k} = 0, \quad k = 1,2 \tag{3.105}$$

The terminal leeward upper joint beam moment, $M_{BR,max.}$, in this state is,

$$M_{BR,max.} = \text{Max.} \quad \{M_{BR}(1,j),\ M_{BR}(2,j)\} \tag{3.106}$$

Since this state seeks to equate upper and lower right joint beam moments, the total change in right joint beam moment, ΔM_T, is

$$\Delta M_T \;=\; |M_{BR}(1,j) - M_{BR}(2,j)| \tag{3.107}$$

Changes in axial forces with respect to changes in ΔH_1 and ΔH_2 are obtained using the same free body diagrams as were used in the description of Moment State 1. The results follow:

For $C_v < 1.0$,

$$\left.\begin{aligned} \frac{\partial F_B(i,j)}{\partial H_1} &= \frac{\Delta F^t_{B1}}{\Delta H_1} = \frac{1}{2}\,(1-C) \\ \frac{\partial F_B(i,j)}{\partial H_2} &= \frac{\Delta F^t_{B2}}{\Delta H_2} = -\frac{1}{2}\,(1+C) \\ \Delta F_B(i,j) &= \Delta F^t_{B1} + \Delta F^t_{B2} \end{aligned}\right\} \tag{3.108}$$

$$\left.\begin{aligned} \frac{\partial F_B(i+1,j)}{\partial H_k} &= \frac{\Delta F^b_{Bk}}{\Delta H_k} = -\frac{1}{2}\,(1+C), \quad k = 1,2 \\ \Delta F_B(i+1,j) &= \Delta F^b_{B1} + \Delta F^b_{B2} = -\frac{\Delta H_J}{2}\,(1+C) \end{aligned}\right\} \tag{3.109}$$

$$\left.\begin{aligned} \frac{\partial F_C(i,j+1)}{\partial H_k} &= \frac{\Delta F^r_{Ck}}{\Delta H_k} = -\frac{Ch(i)}{L(j)}, \quad k = 1,2 \\ \Delta F_C(i,j+1) &= \Delta F^r_{C1} + \Delta F^r_{C2} = -\frac{Ch(i)}{L(j)} \Delta H_J \end{aligned}\right\} \qquad (3.110)$$

$$\left.\begin{aligned} \frac{\partial F_C(i,j)}{\partial H_k} &= \frac{\Delta F^L_{Ck}}{\Delta H_k} = \frac{F^L_{Ck}}{H_k} = 0, \quad k = 1,2 \\ \Delta F_C(i,j) &= \Delta F^L_{C1} + \Delta F^L_{C2} = 0 \end{aligned}\right\} \qquad (3.111)$$

$$\left.\begin{aligned} \frac{\partial F_C(k,j+1)}{\partial H_J} &= \frac{\Delta R_r}{\Delta H_J} = \frac{h(i)}{L(j)} \\ \Delta F_C(k,j+1) &= \frac{h(i)}{L(j)} \Delta H_J \\ \frac{\partial F_C(k,j)}{\partial H_J} &= \frac{\Delta R_L}{\Delta H_J} = -\frac{h(i)}{L(j)} \\ \Delta F_C(k,j) &= -\frac{h(i)}{L(j)} \Delta H_J \end{aligned}\right\} k = i+1, \ldots, M \qquad (3.112)$$

$$\left.\begin{aligned} \frac{\partial F_{BR}(i,j,2)}{\partial H_J} &= \frac{\Delta F_{BR}(i,j,2)}{\Delta H_J} = C\,\frac{L_B(i,j)}{L(j)} \\ \Delta F_{BR}(i,j,2) &= C\,\frac{L_B(i,j)}{L(j)} \Delta H_J \end{aligned}\right\} \qquad (3.113)$$

For $C_v(j) > 1.0$, Eq.'s (3.108), (3.109), (3.112), and (3.113) are still valid. In addition,

$$\begin{aligned} \frac{\partial F_C(i,j+1)}{\partial H_k} &= \frac{\Delta F^r_{Ck}}{\Delta H_k} = \frac{h(i)}{L(j)}, \quad k = 1,2 \\ \Delta F_C(i,j+1) &= \Delta F^r_{C1} + \Delta F^r_{C2} = \frac{h(i)}{L(j)} \Delta H_J \end{aligned} \qquad (3.114)$$

$$\frac{\partial F_C(i,j)}{\partial H_k} = \frac{\Delta F^L_{Ck}}{\Delta H_k} = -\frac{h(i)}{L(j)}(1+C), \quad k = 1,2$$
$$\Delta F_C(i,j) = \Delta F^L_{C1} + \Delta F^L_{C2} = -\frac{h(i)}{L(j)}\Delta H_J\ (1+C), \quad k = 1,2 \tag{3.115}$$

This completes the detailed description of Moment State 3 or Failure Mechanism State 3.

4. <u>Moment State 4 - Failure Mechanism State 4</u>

This moment state is associated with the full sway failure mechanism. Consequently, the increments of moments at the beam and column joints are equal at all four panel joints as illustrated in Fig. 3.16. So,

$$\Delta M_{BL}(1,j) = \Delta M_{BR}(1,j) = \Delta M_{BL}(2,j) = \Delta M_{BR}(2,j) = \Delta M$$
$$\Delta M_{BC}(1,j) = \Delta M_{BC}(2,j) = 0 \tag{3.116}$$
$$\Delta M_{CT}(j) = \Delta M_{CB}(j) = \Delta M_{CT}(j+1) = \Delta M_{CB}(j+1) = -\Delta M$$

Substituting Eq. (3.116) into the moment equilibrium Eq. (3.64) results in,

$$\Delta M = \frac{\Delta H_J h(i)}{4}(1+C) \tag{3.117}$$

The changes in beam and column end moments, $\Delta M_B'$ and $\Delta M_C'$, are obtained by substituting Eq. (3.116) into Eq.'s (3.12) and (3.14). The result is,

$$\Delta M_B' = \left(1 - \frac{d_c'(i,j)}{L(j)}\right) \Delta M$$

$$\Delta M_C' = \left(1 - \frac{d_b'(i,j)}{h(i)}\right) \Delta M \qquad (3.118)$$

Furthermore, $\frac{\partial M_{BP}}{\partial H_k}$ and $\frac{\partial M_{CP}}{\partial H_k}$, $k = 1,2$ are,

$$\frac{\partial M_{BP}(1,j)}{\partial H_k} = \frac{\partial M_{BP}(2,j)}{\partial H_k} = \frac{\Delta M_B'}{\Delta H_k} = \frac{h(i)}{4}\left(1 - \frac{d_c'(i,j)}{L(j)}\right)(1+C)$$

$$\frac{\partial M_{CP}(i,j)}{\partial H_k} = \frac{\partial M_{CP}(i,j+1)}{\partial H_k} = \frac{\Delta M_C'}{\Delta H_k} = \frac{h(i)}{4}\left(1 - \frac{d_b'(i,j)}{h(i)}\right)(1+C) \qquad (3.119)$$

The terminal leeward beam joint moment, $M_{BR,max.}$, in this state depends on the maximum moment capacity of the given wide flange sections. If $M_{BR}(i,j)$ becomes equal to $M_{BR,max.}$, $\frac{\partial M_{BP}}{\partial H_k}$, $k = 1,2$, is set to a very large value in order to yield the lowest priority to the corresponding bay.

Changes in axial forces with respect to changes in ΔH_1 and ΔH_2 are obtained using the same free body diagrams as were used in the description of Moment State 1.

The results follow,

$$\left.\begin{aligned}
\frac{\partial F_B(i,j)}{\partial H_1} &= \frac{\Delta F^t_{B1}}{\Delta H_1} = \frac{1}{2}(1-C) \\
\frac{\partial F_B(i,j)}{\partial H_2} &= \frac{\Delta F^t_{B2}}{\Delta H_2} = -\frac{1}{2}(1+C) \\
\Delta F_B(i,j) &= \Delta F^t_{B1} + \Delta F^t_{B2}
\end{aligned}\right\} \qquad (3.120)$$

$$\left.\begin{aligned}
\frac{\partial F_B(i+1,j)}{\partial H_k} &= \frac{\Delta F^b_{Bk}}{\Delta H_k} = -\frac{1}{2}(1+C), \quad k = 1,2 \\
\Delta F_B(i+1,j) &= \Delta F^b_{B1} + \Delta F^b_{B2} = -\frac{\Delta H_J}{2}(1+C)
\end{aligned}\right\} \qquad (3.121)$$

$$\left.\begin{aligned}
\frac{\partial F_C(i,j+1)}{\partial H_k} &= \frac{\Delta F^r_{Ck}}{\Delta H_k} = \frac{h(i)}{2L(j)}(1-C), \quad k = 1,2 \\
\Delta F_C(i,j+1) &= \Delta F^r_{C1} + \Delta F^r_{C2} = \frac{\Delta H_J\, h(i)}{2L(j)}(1-C)
\end{aligned}\right\} \qquad (3.122)$$

$$\left.\begin{aligned}
\frac{\partial F_C(i,j)}{\partial H_k} &= \frac{\Delta F^L_{Ck}}{\Delta H_k} = -\frac{h(i)}{2L(j)}(1+C), \quad k = 1,2 \\
\Delta F_C(i,j) &= \Delta F^L_{C1} + \Delta F^L_{C2} = -\frac{\Delta H_J\, h(i)}{2L(j)}(1+C)
\end{aligned}\right\} \qquad (3.123)$$

$$
\left.
\begin{aligned}
\frac{\partial F_C(k,j+1)}{\partial H_J} &= \frac{\Delta R_r}{\Delta H_J} = \frac{h(i)}{L(j)} \\
\Delta F_C(k,j+1) &= \frac{h(i)}{L(j)}\,\Delta H_J \\
\frac{\partial F_C(k,j)}{\partial H_J} &= \frac{\Delta R_L}{\Delta H_J} = -\frac{h(i)}{L(j)} \\
\Delta F_C(k,j) &= -\frac{h(i)}{L(j)}\,\Delta H_J
\end{aligned}
\right\} k = i+1,\ \ldots,\ M \qquad (3.124)
$$

$$
\left.
\begin{aligned}
\frac{\partial F_{BR}(i,j,2)}{\partial H_J} &= \frac{\Delta F_{BR}(i,j,2)}{\Delta H_J} = C\,\frac{L_B(i,j)}{L(j)} \\
\Delta F_{BR}(i,j,2) &= C\,\frac{L_B(i,j)}{L(j)}\,\Delta H_J
\end{aligned}
\right\} \qquad (3.125)
$$

This completes the detailed description of Moment State 4 or Failure Mechanism State 4.

3.7.1.4 Change in Required Member Area with Respect to Changes in Required Force Capacities

All changes in required member force capacities with respect to changes in ΔH_1 and ΔH_2 have now been formulated. Only the changes in required member area with respect to changes in required plastic moment and axial force capacity need to be formulated in order to have all the necessary factors to calculate the sub-sensitivity coefficients as defined in previous sections of this chapter.

All member selection will be based on the 1969 AISC[1] plastic design code formulae. Consequently, the changes in required member area with respect to changes in required plastic moment and axial force capacity are formulated on the basis of these code formulae. Note that in what follows, any design formula introduced will not be fully described in this section. Instead, all design formulae used in this plastic design method are described in detail in Section 3.8.

To begin with, the change in required beam area with respect to changes in required plastic moment and axial force capacities respectively are designated by $\frac{\partial A_B(i,j)}{\partial M_{BP}(i,j)}$ and $\frac{\partial A_B(i,j)}{\partial F_B(i,j)}$. For columns, the corresponding factors are $\frac{\partial A_C(i,j)}{\partial M_{CP}(i,j)}$ and $\frac{\partial A_C(i,j)}{\partial F_C(i,j)}$. For a tension brace the factor is $\frac{\partial A_{BR}(i,j,k)}{\partial F_{BR}(i,j,k)}$, $k = 1,2$.

1. Beams

The design of beams may be based on either a moment condition only or a combination moment plus axial compression force condition when lateral torsional buckling is a controlling factor. Note that when a beam experiences a combination moment plus axial tension force condition, it will be designed on the basis of the moment condition only.

When the current beam design is controlled by a moment condition alone, the required plastic section modulus is determined from the equation,

$$Z_B = M_{BR}/\sigma_y \tag{3.126}$$

The empirical relation between beam area and beam plastic section modulus as described in Section 3.9 will now be employed. So,

$$A_B = C_2 Z_B^2 + C_1 Z_B + C_0 \tag{3.127}$$

Now, substituting Eq. (3.126) into Eq. (3.127) leads to,

$$A_B = \left(\frac{C_2}{\sigma_y^2}\right) M_{BP}^2 + \left(\frac{C_1}{\sigma_y}\right) M_{BP} + C_0 \tag{3.128}$$

Finally, differentiating Eq. (3.128) with respect to M_{BP} and F_B respectively results in the desired factors.

$$\frac{\partial A_B(i,j)}{\partial M_{BP}(i,j)} = \left(\frac{2C_2}{\sigma_y^2}\right) M_{BP}(i,j) + \frac{C_1}{\sigma_y}$$

$$\frac{\partial A_B(i,j)}{\partial F_B(i,j)} = 0 \tag{3.129}$$

When the current beam design is controlled by a combination moment plus axial compression force condition, the member properties of a satisfactory section must satisfy the following AISC Formula (22),

$$\frac{F_B}{P_{cr}} + \frac{C_m M_{BP}}{\left(1 - \frac{F_B}{P_e}\right) M_m} \leq 1.0 \tag{3.130}$$

where P_{cr}, C_m, M_m, and P_e are functions of the member properties as described in Section 3.8. A numerical technique is used to find $\frac{\partial A_B}{\partial M_{BP}}$ and $\frac{\partial A_B}{\partial F_B}$. First, changing $\leq$ to = and solving Eq. (3.130) for M_{BP} and F_B respectively results in,

$$M_{BP} = \frac{M_m}{C_m}\left(1 - \frac{F_B}{P_e}\right)\left(1 - \frac{F_B}{P_{cr}}\right)$$

$$F_B = \frac{1}{2}\left[(P_e+P_{cr}) - \sqrt{(P_e-P_{cr})^2 + 4P_eP_{cr}\,C_m\frac{M_{BP}}{M_m}}\right] \tag{3.131}$$

The numerical procedure used is as follows:

i. Select two values of plastic section modulus, Z_1 and Z_2, where Z_0 represents the current value of plastic section modulus and

$$Z_1 = Z_0 + \Delta Z$$

$$Z_2 = Z_0 - \Delta Z$$

ii. Using the empirical relation Eq. (3.127) leads to two corresponding values of member area, A_1 and A_2.

iii. Using another empirical relation described in Section 3.9, calculate two corresponding values of radius of gyration, r_1 and r_2.

iv. All the necessary factors in Eq. (3.131) may now be evaluated. At points 1 and 2, M_{BP1} and M_{BP2} are calculated and $\frac{\partial A_B}{\partial M_{BP}}$ is taken as,

$$\frac{\partial A_B(i,j)}{\partial M_{BP}(i,j)} = \frac{A_1 - A_2}{M_{BP1} - M_{BP2}} \qquad (3.132)$$

Similarly, at points 1 and 2 calculate F_{B1} and F_{B2} respectively and take $\frac{\partial A_B}{\partial F_B}$ as,

$$\frac{\partial A_B(i,j)}{\partial F_B(i,j)} = \frac{A_1 - A_2}{F_{B1} - F_{B2}} \qquad (3.133)$$

2. Columns

The design of columns is based on either the strength interaction equation, AISC Formula (21), or the buckling interaction equation, AISC Formula (22), whichever is critical.

When AISC Formula (21) controls,

$$\frac{F_C}{P_y} + \frac{M_{CP}}{1.18\, M_p} \leq 1.0, \quad M_{CP} \leq M_P \tag{3.134}$$

Changing $\leq$ to = and solving for M_{CP} and F_C results in,

$$\begin{aligned} M_{CP} &= 1.18\, M_p \left(1.0 - \frac{F_C}{P_y}\right) \\ F_C &= P_y \left(1 - \frac{M_{CP}}{1.18\, M_p}\right) \end{aligned} \tag{3.135}$$

Using a similar numerical procedure for columns as described above for beams,

$$\begin{aligned} \frac{\partial A_C(i,j)}{\partial M_{CP}(i,j)} &= \frac{A_1 - A_2}{M_{CP1} - M_{CP2}} \\ \frac{\partial A_C(i,j)}{\partial F_C(i,j)} &= \frac{A_1 - A_2}{F_{C1} - F_{C2}} \end{aligned} \tag{3.136}$$

When AISC Formula (22) controls,

$$\frac{F_C}{P_{cr}} + \frac{C_m M_{CP}}{\left(1 - \frac{F_C}{P_e}\right) M_m} \leq 1.0 \tag{3.137}$$

Again, solving for M_{CP} and F_C results in,

$$M_{CP} = \frac{M_m}{C_m} \left(1 - \frac{F_C}{P_e}\right) \left(1 - \frac{F_C}{P_{cr}}\right)$$

$$F_C = \frac{1}{2} \left[(P_e + P_{cr}) - \sqrt{(P_e - P_{cr})^2 + 4 P_e P_{cr} C_m \frac{M_{CP}}{M_m}} \right] \qquad (3.138)$$

Applying the above described numerical procedure to Eq. (3.138), the resulting Eq. (3.136) is again applicable.

Note that when a column experiences a decrease in axial compressive force or moment to the extent that the section size is controlled by the minimum section requirement from the factored gravity load condition, the factors $\frac{\partial A_C(i,j)}{\partial M_{CP}(i,j)}$ and $\frac{\partial A_C(i,j)}{\partial F_C(i,j)}$ are set to zero.

3. Tension Brace

The design of tension braces is based on the relation,

$$F_{BR}(i,j,k) \leq 0.85\ P_y \qquad (3.139)$$

where,

$$k = \begin{cases} 1, \text{ wind from right} \\ 2, \text{ wind from left} \end{cases}$$

$$P_y = A_{BR}(i,j,k)\ \sigma_y$$

Changing $\leq$ to = and substituting for P_y leads to,

$$\frac{\partial A_{BR}(i,j,k)}{\partial F_{BR}(i,j,k)} = \frac{1}{0.85\ \sigma_y}, \quad k = 1,2 \qquad (3.140)$$

when a panel is being considered as resisting the next increment of lateral force, ΔH_J, by truss action. However, if the panel is being considered as resisting ΔH_J by moment action, $\partial A_{BR}/\partial F_{BR}$ is set to zero. Furthermore, ΔH_J may not be large enough to induce a force greater than 0.85 P_y in the brace when truss action is under consideration. Thus, the factor $\partial A_{BR}/\partial F_{BR}$ is set to zero until the full capacity of the current brace size is utilized.

In conclusion, all factors used in the calculation of the sensitivity coefficients of panel j in story i have been formulated. Recall that there are two sensitivity coefficients calculated for each panel. One is associated with a moment resisting panel (C = 0.0), while the second is associated with a truss resisting panel (C = -1.0). Before an increment of lateral load is applied to a story, the two sensitivity coefficients are calculated for each panel and compared. The panel with the least valued sensitivity coefficient is selected to receive the next increment of lateral story shear. After the panel and mode of resistance are selected, the value of incremental story shear, ΔH_J, to be applied must be determined. The calculations of ΔH_J will be described in the following section.

3.7.2 Calculation of Applied Incremental Story Shear, ΔH_J

The proposed method for distributing lateral loads into a story is essentially a gradient search technique where the gradients of the objective function are the sensitivity coefficients. As in most gradient search methods the problem of determining how far to move along a gradient is, to say the least, no small problem. In the proposed method, the calculation of how far to advance along a gradient, that is to say, the value of the next increment of story shear ΔH_J to be applied to a panel, is based on the current state with respect to the controlling code design formula of those members which will experience force changes due to the application of ΔH_J. Two simplifying assumptions are made in order to calculate the value of ΔH_J. The first assumption is that the gradient, or sensitivity coefficient, is composed of a series of straight line increments where each straight line does not change slope over the application of ΔH_J. Now, the sensitivity coefficient $\partial f/\partial H_J$ can be represented as (G) $(\frac{\partial A}{\partial F})$ $(\frac{\partial F}{\partial H_J})$ where G is some constant, $\partial A/\partial F$ is the change in area with respect to changes in required member force capacities, and $\partial F/\partial H_J$ is the change in required member force capacity with respect to changes in panel shear capacity. The factor $\partial F/\partial H_J$ has been shown to be a constant in Sections 3.7.1.1 to 3.7.1.3. The second assumption is now made. It is assumed that although the factor $\partial A/\partial F$ corresponding to each controlling member design equation is not constant, its value may be taken as constant with small error on the final sensitivity coefficient. In other words, it is assumed that a different constant value of $\partial A/\partial F$ exists for each member design equation. The fact that $\partial A/\partial F$ is not in reality a constant is accounted for only to the extent that new values of $\partial A/\partial F$ are calculated after each ΔH_J is applied to a panel and a redesign of the members is executed.

Several potential values of ΔH_J will be calculated. ΔH_J is calculated for each of the beams in story i to the left and right of panel j as well as the upper beam in panel j with respect to changes in axial force. In addition, ΔH_J is calculated for each column in the panel and below the panel in column lines j and j+1 again with respect to changes in axial forces. If the panel has been selected to resist the next increment of lateral load as a moment resisting panel, ΔH_J is calculated that will just cause the moment state of the panel to change. Finally, if the panel is selected to resist the next increment of lateral shear by truss action, ΔH_J is calculated for the tension brace with respect to changes in axial tension force. The least value of ΔH_J is selected and applied to the panel.

3.7.2.1 ΔH_J Due to Story i Beams to Left of Panel j

This value of ΔH_J is calculated either for panel moment or truss action. Beams in story i to the left of panel j experience increases in axial compressive force. This change in beam axial force is calculated by Eq. (3.40) as,

$$\Delta F_{BL}(i,k) = \frac{\Delta H_J}{S_T(i)} \left(\lambda_2 H(i) + \sum_{p=1}^{k} S(p)\right)$$

Solving this equation for ΔH_J results in,

$$\Delta H_J = \frac{S_T(i)}{\lambda_2 H(i) + \sum_{p=1}^{k} S(p)} \Delta F_{BL}(i,k) \tag{3.141}$$

The equivalent form of Eq. (3.141) used in the computer programs is obtained with the aid of Eq. (3.29) as,

$$\Delta H_J = \frac{\Delta F_{BL}(i,k)}{\left(1 - \frac{1}{S_T(i)} \sum_{p=k+1}^{N+1} S(p)\right)} \tag{3.142}$$

When the beam under consideration has been designed on the basis of a moment condition only, $\Delta F_{BL}(i,k)$ will be taken as the additional axial compressive force that will just cause the current beam size to be controlled by a combination moment and axial compression force condition. Equation (3.142) is then used to solve for ΔH_J. On the other hand, when the beam under consideration is controlled by the combination moment and axial compression force condition, ΔH_J will be taken as 20% of $S_T(i)$.

3.7.2.2 ΔH_J Due to Story i Beams to Right of Panel j

This value of ΔH_J is calculated either for panel moment or truss action. Beams in story i to the right of panel j experience decreases in axial compressive force or increases in axial tension force. This change in beam axial force is calculated by Eq. (3.42) as,

$$\Delta F_{BR}(i,k) = - \frac{\Delta H_J}{S_T(i)} \sum_{p=k+1}^{N+1} S(p)$$

Solving this equation for ΔH_J and using Eq. (3.29) results in,

$$\Delta H_J = - \frac{\Delta F_{BR}(i,k)}{(1 - \frac{1}{S_T(i)} [\lambda_2 H(i) + \sum_{p=1}^{k} S(p)])} \tag{3.143}$$

Now, when the beam under consideration has been designed on the basis of a moment condition only or the combination moment and axial tension force condition, ΔH_J is taken as 20% of $S_T(i)$. However, when the beam under consideration is controlled by the combination moment and axial compression force condition, ΔH_J will be calculated by Eq. (3.143).

In this case, $\Delta F_{BR}(i,k)$ is calculated as follows. Let F_1 be the value of axial compressive force in the current beam. Another beam size is calculated on the basis of the beam moment only. Let F_2 be the value of axial compressive force that will just cause this new beam size to be controlled by the combination moment and axial compression force (F_2) condition. The value of $\Delta F_{BR}(i,k)$ will be taken as $F_2 - F_1$.

3.7.2.3 ΔH_J Due to Upper Beam in Panel j

This value of ΔH_J is calculated either for panel moment or truss action. It is not known a priori whether the upper beam in panel j experiences an increase or decrease in axial compressive force. This must be determined before ΔH_J is calculated. First, using a similar argument that led to Eq. (3.51), the following relation may be derived.

$$\frac{\Delta F_B(i,j)}{\Delta H_J} = \left(\frac{\Delta F_B(i,j)}{\Delta H_1}\right)\left(\frac{1}{1+K}\right) + \left(\frac{\Delta F_B(i,j)}{\Delta H_2}\right)\left(\frac{K}{1+K}\right) \qquad (3.144)$$

where K is calculated by Eq. (3.46), and $\frac{\Delta F_B(i,j)}{\Delta H_1}$ and $\frac{\Delta F_B(i,j)}{\Delta H_2}$ depend on the current moment state as described in Section 3.7.1.3. Solving Eq. (3.144) for ΔH_J results in,

$$\Delta H_J = \frac{\Delta F_B(i,j)}{\left(\frac{\Delta F_B(i,j)}{\Delta H_1}\right)\left(\frac{1}{1+K}\right) + \left(\frac{\Delta F_B(i,j)}{\Delta H_2}\right)\left(\frac{K}{1+K}\right)} \qquad (3.145)$$

Now, when $\frac{\Delta F_B(i,j)}{\Delta H_J} > 0$, the beam experiences an increase in axial compressive force. In this case $\Delta F_B(i,j)$ is calculated in the same way as $\Delta F_{BL}(i,k)$ and Eq. (3.145) is used to calculate ΔH_J. When $\frac{\Delta F_B(i,j)}{\Delta H_J} < 0$, the beam experiences a decrease in axial compressive force. In this case $\Delta F_B(i,j)$ is calculated in the same way as $\Delta F_{BR}(i,k)$ and again Eq. (3.145) is used to calculate ΔH_J.

3.7.2.4 ΔH_J Due to Lower Beam in Panel j

This value of ΔH_J is calculated only for panel moment action. The lower beam in panel j experiences a decrease in axial compression or increase in axial tension under the application of ΔH_1 and ΔH_2. In addition, for the same reasons that led to Eq. (3.144), the following relation may be developed.

$$\frac{\Delta F_B(i+1,j)}{\Delta H_J} = \left(\frac{\Delta F_B(i+1,j)}{\Delta H_1}\right)\left(\frac{1}{1+K}\right) + \left(\frac{\Delta F_B(i+1,j)}{\Delta H_2}\right)\left(\frac{K}{1+K}\right) \tag{3.146}$$

Solving this equation for ΔH_J results in,

$$\Delta H_J = \frac{\Delta F_B(i+1,j)}{\left(\frac{\Delta F_B(i+1,j)}{\Delta H_1}\right)\left(\frac{1}{1+K}\right) + \left(\frac{\Delta F_B(i+1,j)}{\Delta H_2}\right)\left(\frac{K}{1+K}\right)} \tag{3.147}$$

Now, since $\frac{\Delta F_B(i+1,j)}{\Delta H_1} < 0$ and $\frac{\Delta F_B(i+1,j)}{\Delta H_2} < 0$ as discussed in Section 3.7.1.3, then $\frac{\Delta F_B(i+1,j)}{\Delta H_J} < 0$ and the lower panel beam only experiences decreases in axial compressive force or increases in axial tension force. Thus, $\Delta F_B(i+1,j)$ is calculated in the same way as $\Delta F_{BR}(i,k)$ and Eq. (3.147) is used to calculate ΔH_J.

3.7.2.5 ΔH_J Due to Columns Below Panel j

This value of ΔH_J is calculated either for panel moment or truss action. Columns in line j+1 below panel j experience an increase in axial compressive force while the columns in column line j below panel j experience a decrease in axial compressive force. These changes in axial force are obtained from Eq. (3.89) as

$$\begin{aligned} \Delta F_C(k,j+1) &= +\frac{h(i)}{L(j)}\ \Delta H_J, \quad k = i+1,\ \ldots,\ M \\ \Delta F_C(k,j) &= -\frac{h(i)}{L(j)}\ \Delta H_J \quad ,\quad k = i+1,\ \ldots,\ M \end{aligned} \tag{3.148}$$

Solving for ΔH_J in each equation results in,

$$\begin{aligned} \Delta H_J &= \frac{L(j)}{h(i)}\ \Delta F_C(k,j+1), \quad k = i+1,\ \ldots,\ M \\ \Delta H_J &= -\frac{L(j)}{h(i)}\ \Delta F_C(k,j), \quad k = i+1,\ \ldots,\ M \end{aligned} \tag{3.149}$$

When a column size below panel j and in column line j+1 is controlled by the minimum section constraint, $\Delta F_C(k,j+1)$ is taken as the additional axial compressive force that this minimum section can support. If instead this column is controlled either by design equation Eq. (3.134) or Eq. (3.137), $\Delta F_C(k,j+1)$ is taken as the additional axial compressive force that can be supported by the maximum column section size in the column section table.

When a column size below panel j and in column line j is controlled by the minimum section constraint, $\Delta F_C(k,j)$ is taken to be a value that will result in a value of ΔH_J equal to 20% of $S_T(i)$. If instead this same column is controlled by design equation Eq. (3.134) or Eq. (3.137), $\Delta F_C(k,j)$ is taken as the difference between the current column axial force and $F_{min.}$ The term $F_{min.}$ is the maximum axial force that can be supported by a column section selected on the basis of the column moment only. Thus, $\Delta F_C(k,j) = F_{min.} - F_C(k,j)$.

3.7.2.6 ΔH_J Due to Columns in Panel j

This value of ΔH_J is calculated either for panel moment or truss action. The right column in panel j experiences an increase in axial compressive force while the left column experiences a decrease in axial compressive force under the application of ΔH_J. For the same reasons that led to Eq. (3.51), the following relation is obtained.

$$\frac{\Delta F_C(i,k)}{\Delta H_J} = \left(\frac{\Delta F_C(i,k)}{\Delta H_1}\right)\left(\frac{1}{1+K}\right) + \left(\frac{\Delta F_C(i,k)}{\Delta H_2}\right)\left(\frac{K}{1+K}\right) \qquad (3.150)$$

$k = j,\ j+1$

where K is calculated by Eq. (3.46) and $\frac{\Delta F_C(i,k)}{\Delta H_1}$ and $\frac{\Delta F_C(i,k)}{\Delta H_2}$ depend on the current moment state as discussed in Section 3.7.1.3. Solving Eq. (3.150) for ΔH_J results in,

$$\Delta H_J = \frac{\Delta F_C(i,k)}{\left(\frac{\Delta F_C(i,k)}{\Delta H_1}\right)\left(\frac{1}{1+K}\right) + \left(\frac{\Delta F_C(i,k)}{\Delta H_2}\right)\left(\frac{K}{1+K}\right)} \qquad (3.151)$$

where $\Delta F_C(i,j+1)$ and $\Delta F_C(i,j)$ are calculated in the same way as $\Delta F_C(p,j+1)$ and $\Delta F_C(p,j)$ were calculated for columns below the panel ($p = i+1, \ldots, M$).

3.7.2.7 ΔH_J Due to Tension Brace in Panel j

This value of ΔH_J is calculated only for panel truss action. The incremental story shear associated with a tension brace is that value of ΔH_J that will increase the force in the current tension brace size up to the maximum capacity of the brace without causing a change in brace area. So, solving Eq. (3.90), (3.113), or (3.125) for ΔH_J where C=-1.0 results in,

$$\Delta H_J = -\frac{L(j)}{L_B(i,j)} \Delta F_{BR}(i,j,k) \tag{3.152}$$

where,

$$\Delta F_{BR}(i,j,k) = -0.85\, P_y - F_{BR}(i,j,k)$$

$$P_y = \sigma_y A_{BR}(i,j,k)$$

$k = 1,2$ (wind from right, left)

Note that $\Delta F_{BR}(i,j,k)$ is negative in accordance with the sign convention for tension force defined in Section 3.3.

3.7.2.8 ΔH_J Due to Moment State Changes in Panel j

This value of ΔH_J is calculated only for panel moment action.

1. Moment State 1

Solving Eq. (3.65) for ΔH_J with C=0.0 results in,

$$\Delta H_J = \frac{4L(j)}{L(j) + d_c^{'}(i,j)} \frac{1 + C_v(j)}{h(i)} \Delta M \quad (3.153)$$

where ΔM represents the additional moment required to increase the current upper right joint beam moment to the value, $M_{BR,max.}$, given by Eq. (3.67) as,

$$M_{BR,max.} = \lambda_1 P_1^{'} (L(j) + d_c^{'}(i,j))/8$$

Thus,

$$\Delta M = \lambda_1 P_1^{'} (L(j) + d_c^{'}(i,j))/8 - M_{BR}(1,j) \quad (3.154)$$

2. Moment State 2

Since Eq. (3.65) is also valid for this state, Eq. (3.153) is again applicable for the calculation of ΔH_J. The increment of moment ΔM for this state represents the additional moment required to increase the current upper right joint beam moment to the value, $M_{BR,max.}$, given by Eq. (3.97) as

$$M_{BR,max.} = \lambda_2 P_1^{'} (L(j) + d_c^{'}(i,j))/4$$

Thus,

$$\Delta M = \lambda_2 P_1^{'} (L(j) + d_c^{'}(i,j))/4 - M_{BR}(1,j) \quad (3.155)$$

3. Moment State 3

Solving Eq. (3.101) for ΔH_J with C=0.0 results in,

$$\Delta H_J = \frac{2\Delta M}{h(i)} \quad (3.156)$$

where ΔM represents the additional moment required to increase the current lower valued right joint beam moment up to the higher valued right joint beam moment. Thus,

$$\Delta M = \max. \{M_{BR}(1,j), M_{BR}(2,j)\} - \min. \{M_{BR}(1,j), M_{BR}(2,j)\} \tag{3.157}$$

4. Moment State 4

Solving Eq. (3.117) for ΔH_J with C=0.0 results in,

$$\Delta H_J = \frac{4\Delta M}{h(i)} \tag{3.158}$$

where ΔM represents the additional moment required to increase the current right joint beam moment up to the maximum moment carrying capacity, $M_{B,max.}$, of the current beam size. Thus,

$$\Delta M = M_{B,max.} - M_{BR}(i,j) \tag{3.159}$$

Before proceeding on to the next section, an additional point must be clarified related to the calculation of ΔH_J. Suppose the selected value of ΔH_J is one that is calculated on the basis of exhausting the remaining capacity of a given section size. The next time ΔH_J is calculated for this same member, its value must be zero since the member has no remaining reserve capacity. In order to overcome this difficulty, all ΔH_J values calculated on the basis of reserve member capacity are multiplied by the factor 1.10 in order to guarantee at least one member size increase. Thus, the factor 1.10 is applied to ΔH_J values calculated for the following cases:

a. Beam to the left of panel j controlled by the moment condition only.

b. Beam to the right of panel j controlled by the combination moment plus axial compression force condition.

c. Upper panel beam experiencing increases in axial compressive force and controlled by the moment condition only.

d. Upper panel beam experiencing decreases in axial compression force and controlled by the combination moment and axial compression force condition.

e. Lower panel beam controlled by the combination moment plus axial compression force condition.

f. Columns in column line j+1 controlled by the minimum section constraint.

g. Columns in column line j controlled by design equation Eq. (3.134) or Eq. (3.137).

h. Tension brace.

i. Moment State 4.

This concludes the description of the calculation of the incremental story shear, ΔH_J, to be applied to the panel with the least valued sensitivity coefficient.

3.8 Member Selection

The selection of beams, columns and tension braces to resist a given set of forces is in accordance with the 1969 AISC[1] code specifications on Plastic Design of Multi-Story Frames. The appropriate formulae will be listed here for completeness.

1. Beams

$F_B \leq 0.85\ P_y$, Combination gravity plus wind load condition.

$M_{BP} \leq M_P$ (3.160)

$$\frac{F_B}{P_{cr}} + \frac{C_m\ M_{BP}}{(1 - \frac{F_B}{P_e})\ M_m} \leq 1.0$$

where,

F_B = axial force in beam

M_{BP} = maximum beam moment

A_B = beam area

Z_B = beam plastic section modulus

F_y = yield stress of steel

r_b = radius of gyration in plane of bending

r_y = radius of gyration perpendicular to plane of bending

L_b = unbraced length in plane of bending

L_y = unbraced length perpendicular to plane of bending

E = modulus of elasticity

K = column length factor

L/r = largest (length/radius of gyration) factor.

$P_y = A_B F_y$

$M_P = Z_B F_y$

$C_m = 0.85$

$P_{cr} = 1.70\ A_B F_A$

$$P_e = 1.92\, A_B F_e'$$

$$M_m = (1.07 - \frac{\sqrt{F_y}\,(L_y/r_y)}{3160})\, M_P \leq M_P$$

$$F_e' = \frac{149{,}000{,}000}{(KL_b/r_b)^2}$$

$$F_A = \begin{cases} 149{,}000{,}000/(KL/r)^2, & KL/r > C_c \\ \frac{F_y}{F.S.}(1 - \frac{(KL/r)^2}{2C_c^2}), & KL/r \leq C_c \end{cases}$$

$$F.S. = \frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^3}$$

$$C_c = \sqrt{\frac{2\ \pi^2 E}{F_y}}$$

$$KL_b/r_b \leq C_c$$

2. Columns

$F_C \leq 0.85\ P_y$, Combination gravity plus wind load condition.

$$M_{CP} \leq M_P$$

$$\frac{F_C}{P_y} + \frac{M_{CP}}{1.18\ M_P} \leq 1.0 \qquad (3.161)$$

$$\frac{F_C}{P_{cr}} + \frac{C_m M_{CP}}{(1 - \frac{F_C}{P_e})\ M_m} \leq 1.0$$

where all definitions of terms for beams apply to columns with the following additions and changes:

F_C = column axial force

M_{CP} = maximum column moment

A_C = column area

Z_C = column plastic section modulus

$P_y = A_C F_y$

$M_P = Z_C F_y$

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4$$

$|M_1| \leq |M_2|$

M_1/M_2 = ratio of column end moments

$M_1/M_2 \leq 0$ when column bent in double curvature

$M_1/M_2 > 0$ when column bent in single curvature

$P_{cr} = 1.70\ A_C F_A$

$P_e = 1.92\ A_C F_e'$

3. <u>Tension Brace</u>

$|F_{BR}| \leq 0.85\ P_y$, Combination gravity plus wind load condition. (3.162)

where,

F_{BR} = tension brace axial force (negative).

A_{BR} = tension brace area

F_y = yield stress of steel

P_y = $A_{BR}F_y$

After each application of ΔH_J to a panel a new force distribution is determined. All members experiencing force changes will be checked against the above code formulae. If a current member size satisfied the code formulae an attempt is made to decrease its size if it experiences a decrease in member force. If a current member size violates the code formulae the member size is increased. All changes in member sizes are made in an incremental fashion, that is to say, by selecting the next larger or smaller member size in the appropriate section table. After each increment of member size, the code formulae are checked. When increasing member sizes, the first section satisfying the code formulae is selected. When decreasing member sizes, the last section satisfying the code formulae is selected. Note that the section size selected on the basis of the factored gravity load condition always represents the minimum or lower bound member size. Furthermore, the beam and column section tables are composed of two parts. The first part of each represent the economy sections without regard to depth and where sections are arranged in order of increasing section area or, equivalently, increasing section weight. Consequently, each section selected from the first part of the beam or column section table corresponds to a least weight section satisfying the appropriate code design formulae. In addition, bracing sections are also ordered on increasing area (weight) and sections selected also represent least weight sections satisfying the code formulae. On the other hand, the second part of the beam and column section tables represent the non-economy sections used to satisfy imposed depth constraints. The non-economy columns are also ordered on increasing area (weight) and thus section by section incrementation still leads to a least weight section. On the other hand, non-economy beams are ordered on increasing plastic section modulus. Thus, in order to select least weight sections, rather than incrementing one section at a time, a special program is used to select the least weight non-economy beam section that satisfies all design constraints.

3.9 Empirical Relations between Section Properties

Various kinds of hot-rolled sections are available to structural engineers. Although any series of sections may be used in this design method by simply specifying the appropriate section tables to the computer program, rolled sections listed in the AISC Manual(1) are used in the illustrative examples. In particular, the rolled sections used in the illustrative examples are listed in Appendix A.

As discussed in previous sections, the calculation procedure for the sub-sensitivity coefficients associated with beams and columns utilize continuous functional relations between various section properties. Since the sub-sensitivity coefficients are highly sensitive to changes in section area (A) with respect to changes in plastic section modulus (Z), quadratic polynomials are used to closely approximate the A vs. Z relations. On the other hand, the sub-sensitivity coefficients are very insensitive to changes in radius of gyration (r) with respect to changes in plastic section modulus. Consequently, only linear polynomials are necessary to approximate the r vs. Z relations.

In practical design, two series of rolled sections are usually used. They are a column series and a beam series. For example, wide flange sections of the nominal depth of 14 inches and of varying flange thicknesses may be used exclusively for columns while sections of varying depths, but of the most economical shapes, may be used for beams. As discussed in Sections 3.8 and 3.11, column and beam series of these types must be input as the first part of the column and beam section tables respectively. Note that the empirical relations are derived only for these types of sections. Figures 3.25 and 3.26 illustrate the empirical relations and the corresponding economy beam and column section properties used in the illustrative examples. Note that when beams are selected to satisfy depth constraints (from the second part of the beam section table), empirical relations are not used to calculate the corresponding sub-sensitivity coefficients (see Section 3.11). On the other hand, since column depth constraints are not satisfied until the end of the design method, the empirical relations for the first part of the column section table are always used in the calculation of the appropriate sub-sensitivity coefficient.

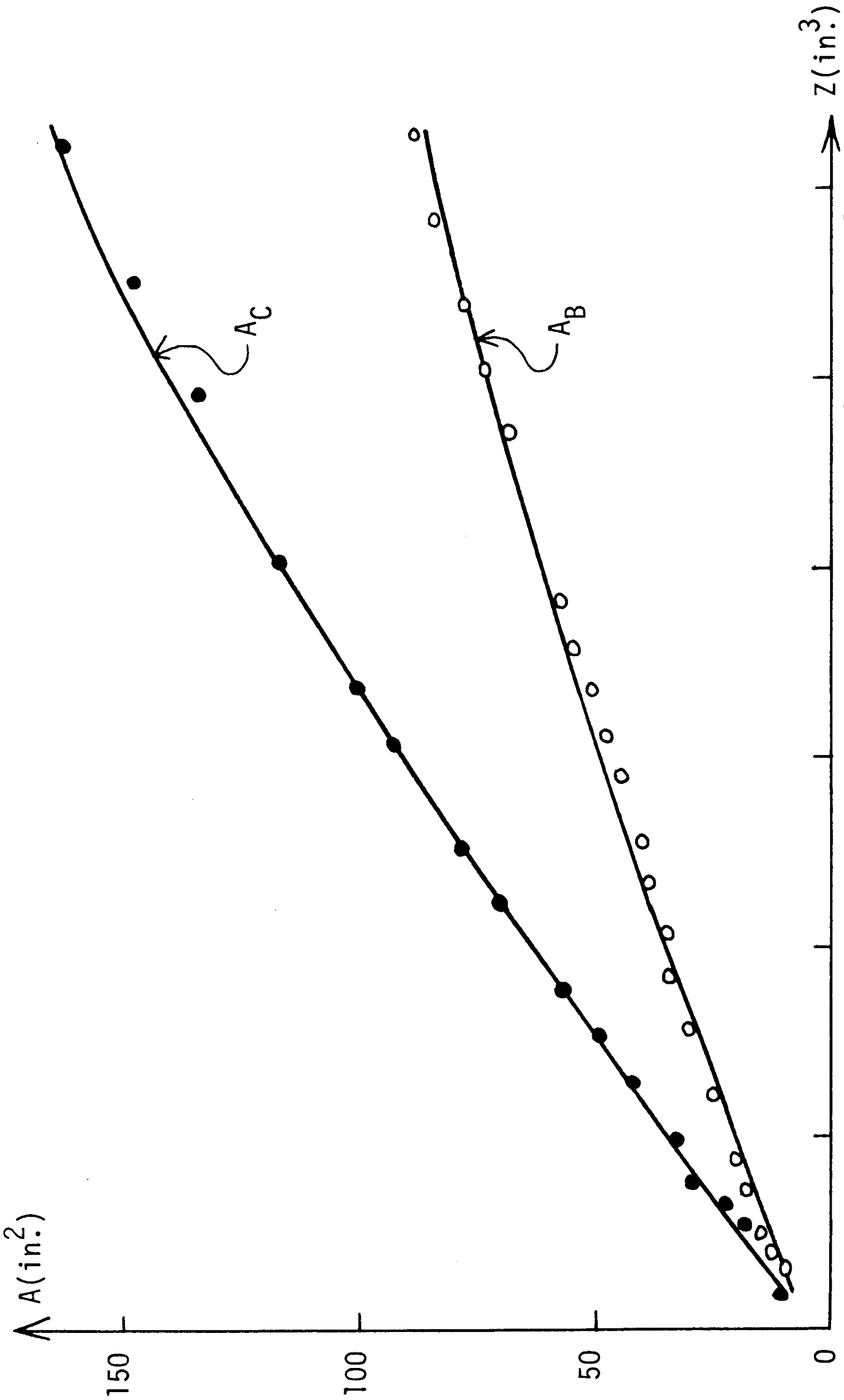


Figure 3.25 Empirical Relations Between Section Area (A) and Plastic Section Modulus (Z) for the Economy Beams and Columns Used in the Illustrative Examples.

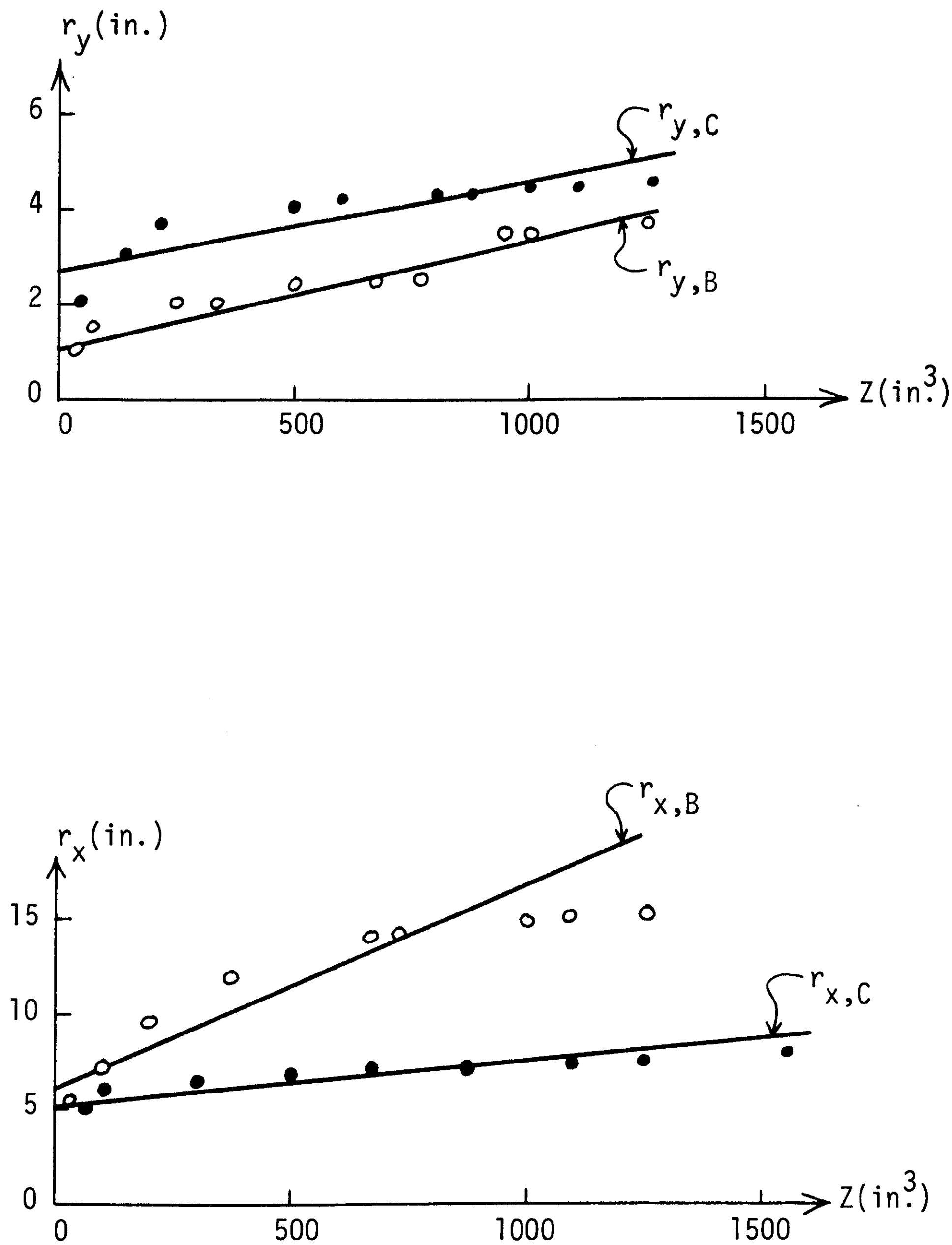


Figure 3.26 Empirical Relations Between Radius of Gyration (r_x & r_y) and Plastic Section Modulus (Z) for the Economy Beams and Columns Used in the Illustrative Examples.

The coefficients of all-empirical relations are calculated by the Method of Least Squares[9] at the beginning of each design, based on the input section tables. The empirical relations for the economy beam and column sections used in the illustrative examples are as follows.

1. Section Area (A) vs. Plastic Section Modulus (Z)

For economy beams,

$$A_B = 4.486 + 0.0794\ Z - 0.0000124\ Z^2 \tag{3.163}$$

For economy columns,

$$A_C = 2.972 + 0.1609\ Z - 0.0000253\ Z^2 \tag{3.164}$$

2. Radius of Gyration (r_x) vs. Plastic Section Modulus (Z)

For economy beams,

$$r_{x,B} = 6.02 + 0.0109\ Z \tag{3.165}$$

For economy columns,

$$r_{x,C} = 5.172 + 0.00239\ Z \tag{3.166}$$

3. Radius of Gyration (r_y) vs. Plastic Section Modulus (Z)

For economy beams,

$$r_{y,B} = 1.022 + 0.00247\ Z \tag{3.167}$$

For economy columns,

$$r_{y,C} = 2.683 + 0.00194\ Z \tag{3.168}$$

3.10 Calculation of Δ for the P-Δ Effect

In the proposed design method, when each panel in a story is required to be an unbraced panel, Δ is taken as the maximum of the mechanism deflections for each panel in the story. While the influence of column and beam elongation and shortening is neglected such influences would be relatively small and the procedure should be conservative even for very tall frames.

When one or more panels in a given story may contain braces the calculation of a Δ value, which would be applicable for all possible bracing patterns, becomes much more difficult. The procedure used in this dissertation is greatly simplified and may be unconservative. It is anticipated that future work (see Chapter 6) will incorporate a more sophisticated calculation procedure for this situation.

What is done at this time is to calculate for each panel where bracing is permitted the relative panel deflection due to brace elongation at the yield strain. Then Δ is taken as the minimum of these values, $\Delta_{min.}$. In addition, the relative deflection is calculated for each unbraced panel after each application of ΔH_J. When this unbraced panel deflection becomes equal to $\Delta_{min.}$ all further applications of ΔH_J to the panel under consideration is required to be resisted by panel truss action.

There are certain aspects of this procedure which are conservative and others which are unconservative. It is conservative in the sense that the maximum shear capacity of the panel is not yet reached when the shortest brace yields. On the other, and unconservative, hand the effects of column and beam elongation and shortening are neglected in the calculation of Δ. For braced stories Δ is an order of magnitude smaller than for unbraced stories and, accordingly, axial deformation of columns and beams can be relatively more important. The AISC Specification, in fact, asks that such deformations be considered. For the situation where all lateral forces are assumed to be resisted by a statically determinate vertical cantilever truss, the calculation of that part of Δ due to axial deformation of columns and beams is easy. However, for the general case being considered here, such calculation is more difficult.

It is important to note that the example problem comparisons between the author's and Lehigh University's braced frame solutions (see Chapter 2) still are considered to be valid. While it is not exactly clear how Δ was calculated in the Lehigh solutions, it appears to have been done in the same way as in this dissertation. In any case inclusion of column and beam axial deformations for these particular cases would have only a minor influence.

3.10.1 Braced Panel

The relative deflection for a braced panel is calculated as the deflection occurring at the time the brace reaches a yielded state. This is also conservative since the design equations require the maximum brace stress to be less than or equal to 85 percent of the yield stress. Referring to Reference (10), Chapter 7, the relative deflection of a braced panel is calculated as follows.

$$\dot{\Delta}(i,j) = \frac{\sigma_y L_B^2(i,j)}{E\,L(j)} \tag{3.169}$$

where,

σ_y = Brace yield stress

$L_B(i,j)$ = Brace length

$L(j)$ = Bay length

E = Modulus of Elasticity

3.10.2 Unbraced Panel

The relative deflection for an unbraced panel is calculated on the basis of the plastic moment diagrams and current beam and column section properties. The method used is the slope deflection method applied to the subassemblages of the story (see Reference (10), Chapter 14). Each subassemblage consists of an upper story panel beam and a windward panel column. The relative deflection will be calculated at the time immediately after the collapse mechanism formation. Before this time, it is assumed that no plastic hinges have formed in the members of the subassemblage. Consequently, the slope deflection method can be applied to the deflection calculation. The results follow.

For wind from the left,

$$\frac{\Delta(i,j)}{h(i)} = \theta(i,j) - \frac{h(i)}{3E\ I_C(i,j)}\ [M_{CT}(i,j) - \frac{M_{CB}(i,j)}{2}]$$

where,

$$\theta(i,j) = \frac{M_{BL}(i,j)L'(j)}{3E\ I_B(i,j)}\ [1 - \frac{d_c'(i,j)}{4L(j)}] - \frac{M_{BR}(i,j)L'(j)}{6E\ I_B(i,j)}[1 + \frac{d_c'(i,j)}{2L(j)}]$$

$$+ \frac{\lambda_2 P_W(i,j)(L'(j))^2}{8E\ I_B(i,j)}\ [1 + \frac{d_c'(i,j)}{L'(j)}] \qquad (3.170)$$

$$L'(j) = L(j) - d_c'(i,j)$$

For wind from the right,

$$\frac{\Delta(i,j)}{h(i)} = \theta(i,j+1) - \frac{h(i)}{3E\ I_C(i,j+1)}\ [M_{CT}(i,j+1) - \frac{M_{CB}(i,j+1)}{2}]$$

where,

$$\theta(i,j+1) = \frac{M_{BR}(i,j)L'(j)}{3E\ I_B(i,j)}\ [1 - \frac{d_c'(i,j)}{4L(j)}] - \frac{M_{BL}(i,j)L'(j)}{6E\ I_B(i,j)}[1 + \frac{d_c'(i,j)}{2L(j)}]$$

$$- \frac{\lambda_2 P_W(i,j)(L'(j))^2}{8E\ I_B(i,j)}\ [1 + \frac{d_c'(i,j)}{L'(j)}] \qquad (3.171)$$

3.11 Beam and Column Depth Constraints

Recognizing that beam and column depth constraints can be very important in practical design, the proposed design method takes these constraints into consideration. Beams and columns intended to be used to satisfy these depth constraints are input as the second part of the respective section table. Whereas the first part of the beam and column section tables represent economy sections without regard to depth, the second part of these tables are intended to provide sections with the necessary section properties to satisfy the appropriate code formulae as well as depths small enough to satisfy various depth constraints.

Beam depth constraints are taken into consideration throughout the total design process. Since beams are primarily bending members, it is easier to account for beam depth constraints in the sensitivity coefficient calculations. Note that the empirical relations between section properties are not derived for the non-economy beams where depth constraints are important. Instead, the appropriate derivatives for the sensitivity coefficient calculations are determined on a section by section basis in the plastic design. In the elastic stiffness design, the deflection sensitivity coefficient calculation is no different for non-economy beams than for economy beams.

Column depth constraints are satisfied at the conclusion of the total design process. There are several reasons for doing this. The first reason is that column depth constraints occur much less frequently and are usually much less restrictive than beam depth constraints. The second reason is that column depths used in a practical design usually have small variations in nominal depth as reflected by the fact that sections with a nominal depth of 14 inches are very often used as column sections. A third reason lies in the nature of the ordering of the section properties as they occur in the non-economy section table for columns. While routines could have been written which would have allowed for the plastic design sensitivity factors to properly account for combined bending and compression, the effort was not deemed worthwhile.

CHAPTER 4

ELASTIC ANALYSIS AND ELASTIC STRESS DESIGN METHOD

4.1 Introduction.

The proposed design system allows the user to specify the maximum elastic stress which will be calculated for an unfactored, or service load condition. If no elastic stress constraint is specified, the programs will assume that the yield stress of the specified steels cannot be exceeded under either the gravity or combined load conditions. Obviously, the elastic stress design can be bipassed by specifying a very high value of allowable elastic stress. An elastic analysis and design is executed to satisfy the elastic stress constraints. In addition to stresses, the elastic analysis provides relative story deflections based on 'exact' analysis techniques which may then be compared to the maximum allowable relative story deflections. If these 'exact' relative story deflections exceed the maximum specified, an elastic stiffness design is executed as described in Chapter 5.

A basic assumption in the elastic analysis and design method consistant with the plastic analysis and design method is as follows. Only diagonal tension bracing will be assumed to be acting under the combination gravity plus wind load condition while no bracing will be considered acting under the gravity load condition alone.

The method of solution selected for the braced or unbraced plane frame is the Stiffness Method of Analysis[11]. This method leads directly to the joint displacements of the frame which are then used to determine the corresponding member end forces. The stiffness matrix developed for the frame corresponds to a consideration of three degrees of kinematic freedom per joint. That is to say, lateral, vertical, and rotational displacements for each joint are considered as the unknown joint displacements. Furthermore, elements of the stiffness matrix neglect shear deformations, but consider both axial and bending deformations of the members.

The joints of the frame are numbered in a way that minimizes the band width of the structure stiffness matrix. The method of matrix reduction selected is a modified form of the Square Root Method [12] whereby full advantage is taken of the symmetrical and banded properties of the structure stiffness matrix.

4.2 Notation and Sign Convention

Global joint displacements and corresponding joint forces (applied forces as well as fixed-end forces) are shown in their positive directions in Fig. 4.1. Member forces calculated from the joint displacements will conform to the same member force sign convention used in the plastic design method.

The joint numbering convention is illustrated in Fig. 4.2 where the joint numbers increase consecutively from the leftmost story joint to the rightmost story joint. Note that the support joints are not numbered since only the free joints with displacement

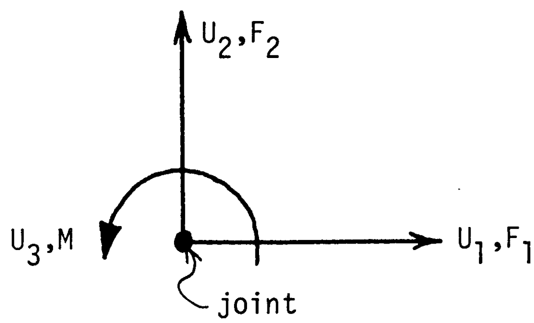


Figure 4.1 Global Joint Displacements and Forces.

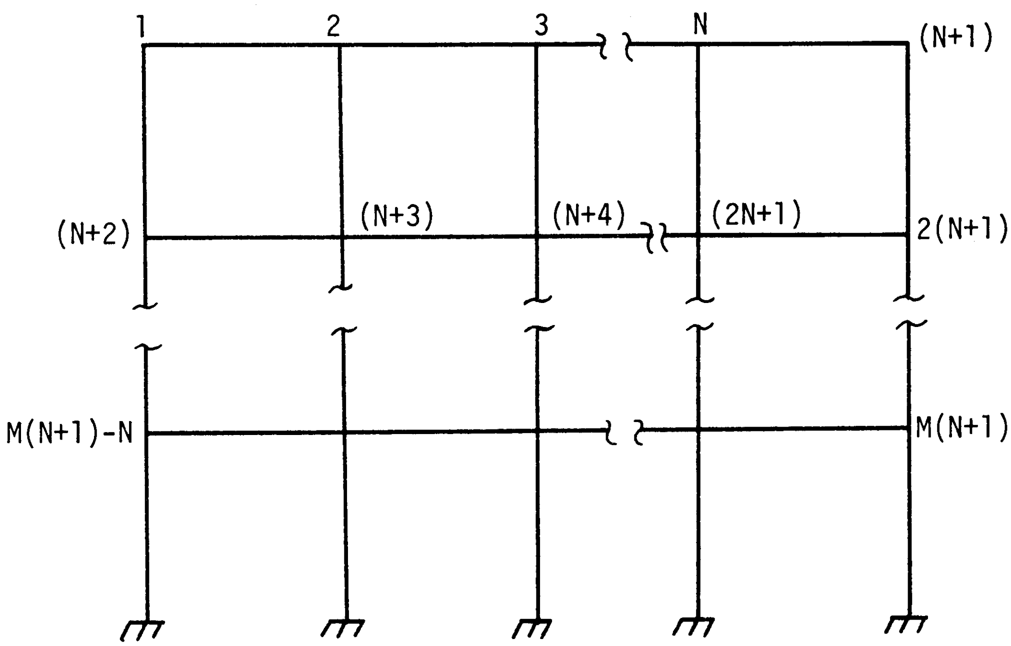


Figure 4.2 Joint Numbering Convention.

unknowns are numbered.

4.3 Member Global Stiffness Matrices.

In order to minimize the time required to construct the structure stiffness matrix as well as minimize the computer core storage required to store the necessary data, member stiffness matrices are formulated directly in terms of the global displacements.

4.3.1 Beams.

The low joint number of a beam designated by 'a', is the left joint and the high joint number, designated by 'b', is the right joint. The beam joint displacement vector, $\underset{\sim}{U}_B$ is taken as,

$$\underset{\sim}{U}_B = \left\{ \begin{array}{c} \underset{\sim}{U}_{B,a} \\ \hdashline \underset{\sim}{U}_{B,b} \end{array} \right\} = \left\{ \begin{array}{c} u_{B,a1} \\ u_{B,a2} \\ u_{B,a3} \\ \hdashline u_{B,b1} \\ u_{B,b2} \\ u_{B,b3} \end{array} \right\} \tag{4.1}$$

Corresponding to the partitioning of Eq. (4.1) the beam global stiffness matrix, $\underset{\sim}{K}_B$, is formulated by noting that each element of $\underset{\sim}{K}_B$, $k_{B,ij} = k_{B,ji}$, represents the force in the i direction due to a unit displacement in the j direction with all other displacements fixed. Fig. 4.3 shows the unit global displacements applied to the

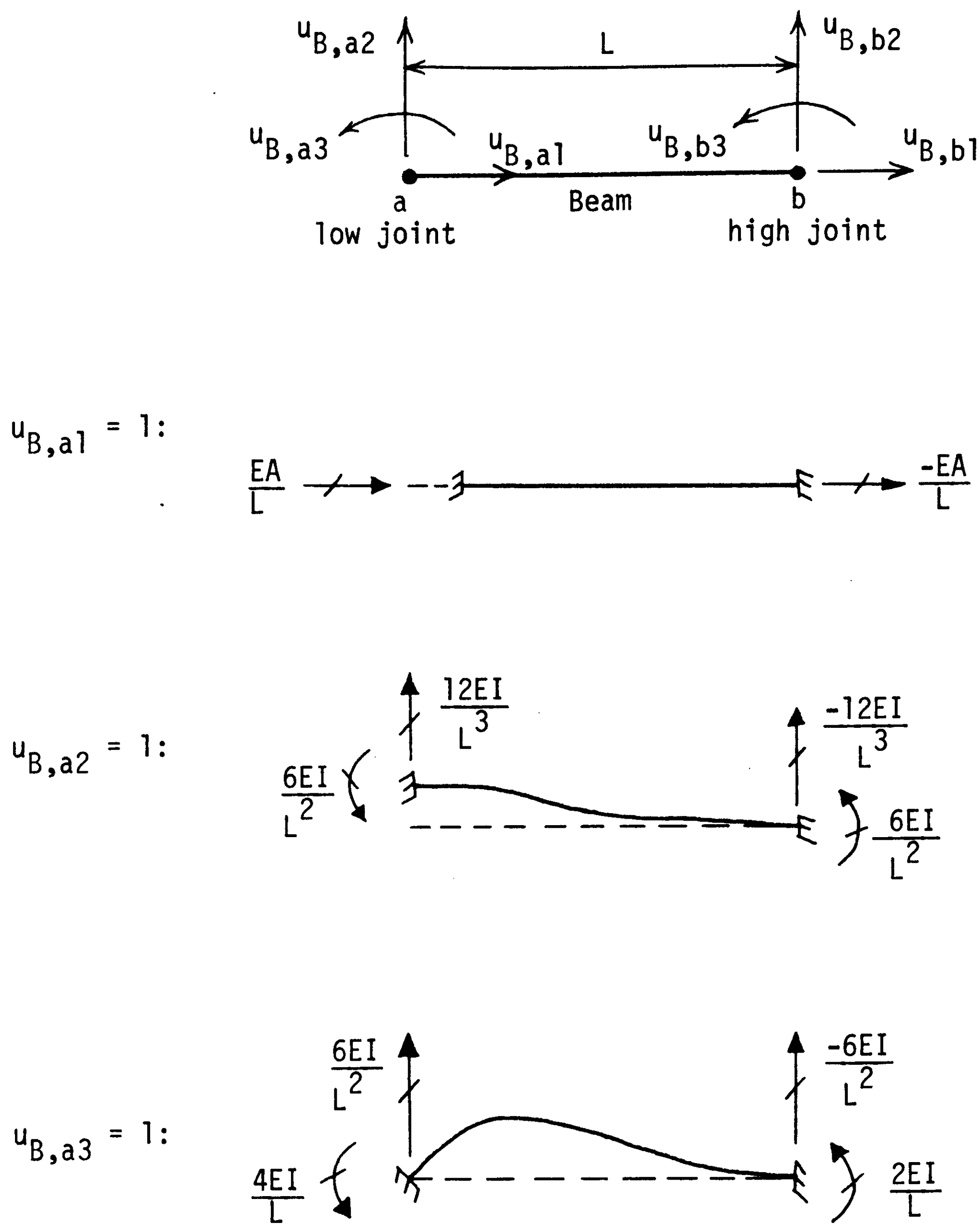


Figure 4.3 Beam Left Joint Unit Displacements and Resulting End Forces.

left joint and the resulting end forces (elements of $\underset{\sim}{K}_{B,aa}$ and $\underset{\sim}{K}_{B,ba}$). Fig. 4.4 shows the unit global displacements applied to the right joint and the resulting end forces (elements of $\underset{\sim}{K}_{B,ab}$ and $\underset{\sim}{K}_{B,bb}$). The resulting 6 x 6 beam global stiffness matrix is as follows where only the upper triangular part is shown. The elements below the diagonal are obvious since $\underset{\sim}{K}_B$ is symmetrical. So,

$$\underset{\sim}{K}_B = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & \underset{\sim}{SYM}. & & \frac{EA}{L} & 0 & 0 \\ & & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix}$$

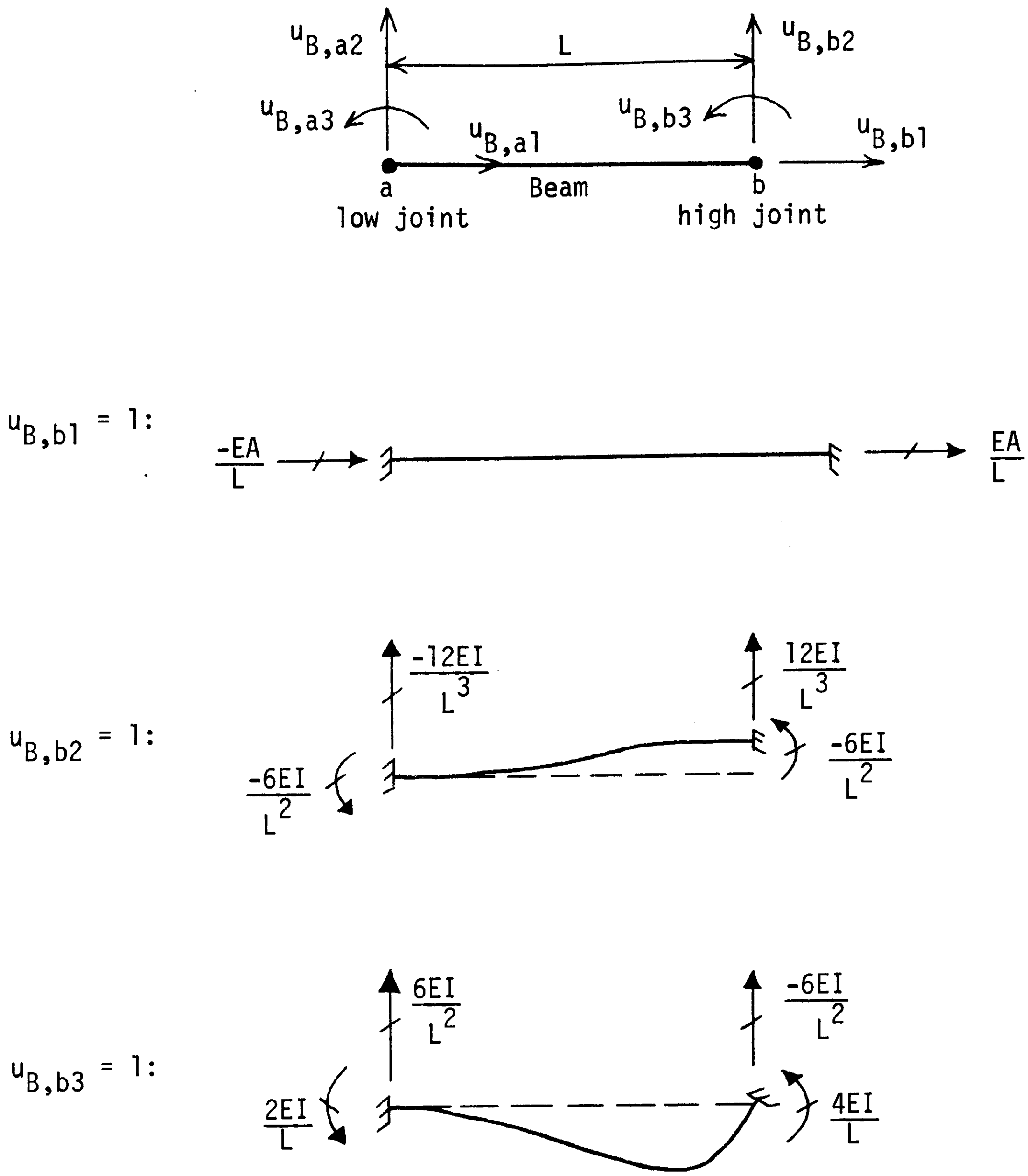


Figure 4.4 Beam Right Joint Displacements and Resulting End Forces.

$$= \begin{bmatrix} \underset{\sim}{K}_{B,aa} & \underset{\sim}{K}_{B,ab} \\ \underset{\sim}{K}_{B,ba} & \underset{\sim}{K}_{B,bb} \end{bmatrix} \quad (4.2)$$

where,

E = modulus of elasticity

A = beam area

I = beam moment of inertia

L = beam length

4.3.2 Columns.

The low joint number of a column, designated by 'b', is the top joint and the high number, designated by 'a', is the bottom joint. The column joint displacement vector, $\underset{\sim}{U}_C$, is taken as,

$$\underset{\sim}{U}_c = \begin{Bmatrix} \underset{\sim}{U}_{c,a} \\ \hdashline \underset{\sim}{U}_{c,b} \end{Bmatrix} = \begin{Bmatrix} u_{c,a1} \\ u_{c,a2} \\ u_{c,a3} \\ \hdashline u_{c,b1} \\ u_{c,b2} \\ u_{c,b3} \end{Bmatrix} \qquad (4.3)$$

Corresponding to the above partitioning the column global stiffness matrix is formulated in a similar way as that for beams. Fig. 4.5 shows the unit global displacements applied to the bottom joint and the resulting end forces (elements of $\underset{\sim}{K}_{c,aa}$ and $\underset{\sim}{K}_{c,ba}$). Fig. 4.6 shows the unit global displacements applied to the top joint and the resulting end forces (elements of $\underset{\sim}{K}_{c,ab}$ and $\underset{\sim}{K}_{c,bb}$). The resulting 6 x 6 column global stiffness matrix is as follows where only the upper triangular part is shown. The elements below the diagonal are obvious since $\underset{\sim}{K}_c$ is symmetrical. So,

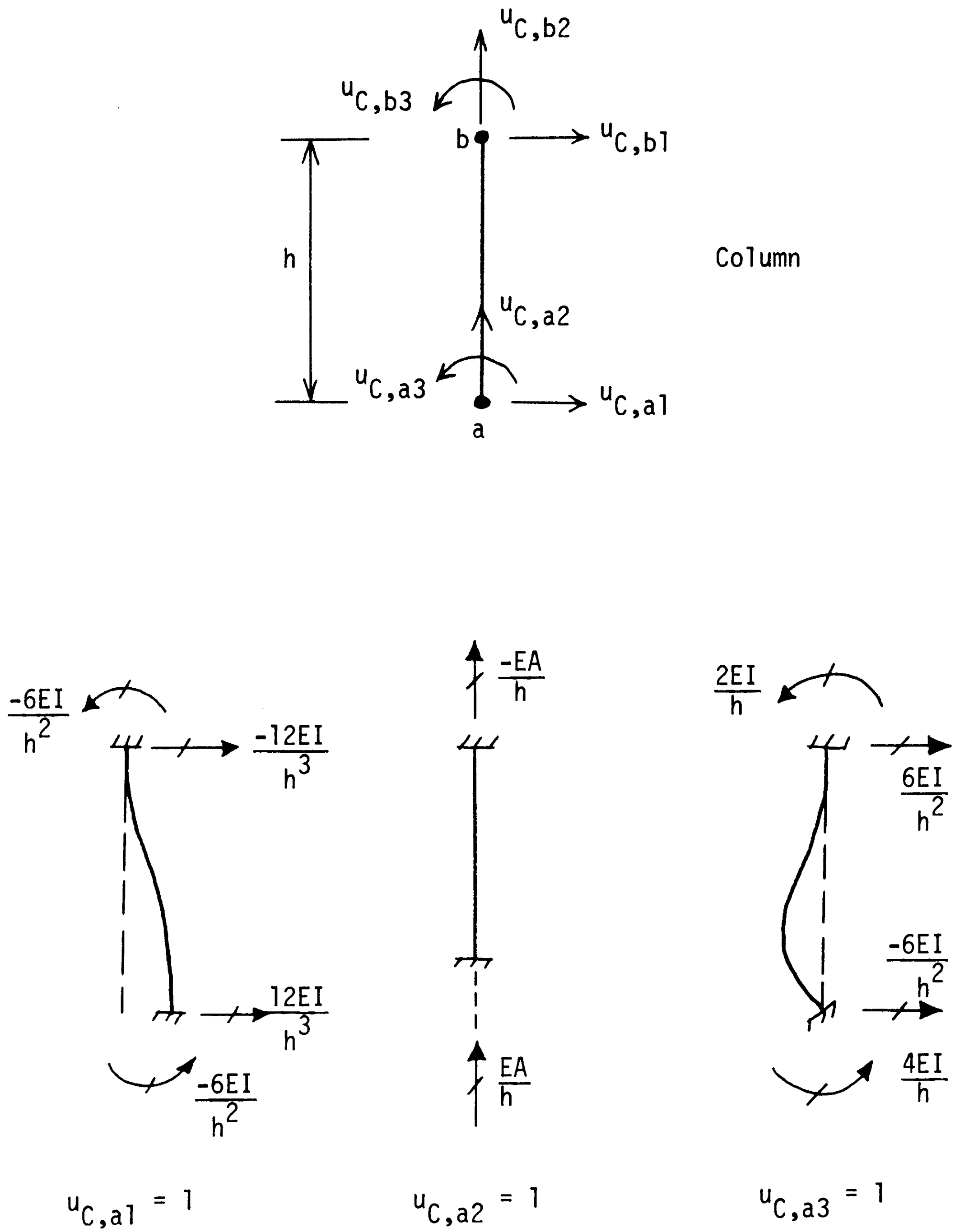


Figure 4.5 Column Bottom Joint Unit Displacements and Resulting End Forces.

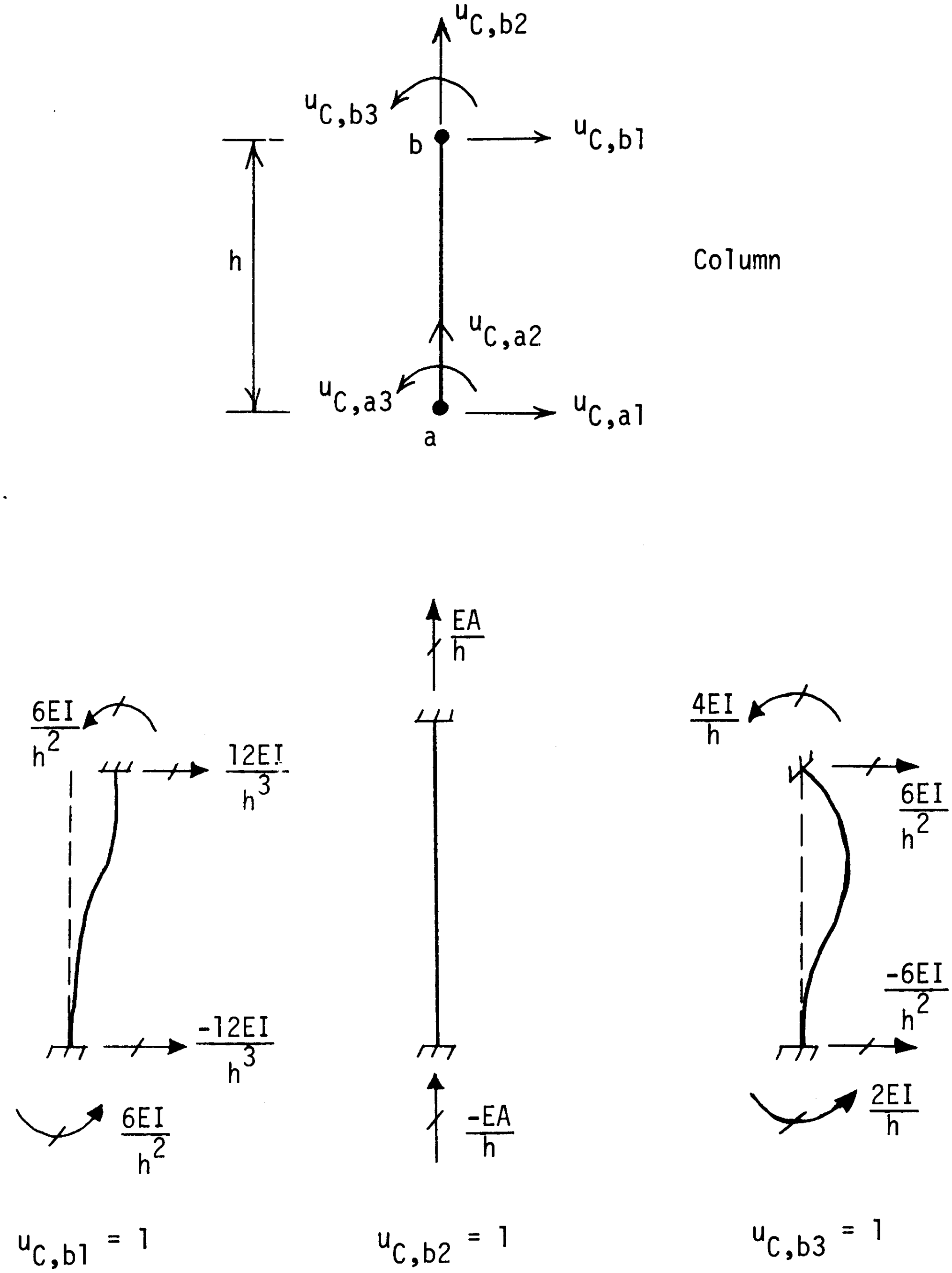


Figure 4.6 Column Top Joint Unit Displacements and Resulting End Forces.

$$\underset{\sim}{K}_c = \begin{bmatrix} \frac{12EI}{h^3} & 0 & -\frac{6EI}{h^2} & -\frac{12EI}{h^3} & 0 & -\frac{6EI}{h^2} \\ & \frac{EA}{h} & 0 & 0 & -\frac{EA}{h} & 0 \\ & & \frac{4EI}{h} & \frac{6EI}{h^2} & 0 & \frac{2EI}{h} \\ & \underset{\sim}{\text{SYM.}} & & \frac{12EI}{h^3} & 0 & \frac{6EI}{h^2} \\ & & & & \frac{EA}{h} & 0 \\ & & & & & \frac{4EI}{h} \end{bmatrix}$$

$$= \begin{bmatrix} \underset{\sim}{K}_{C,aa} & \underset{\sim}{K}_{C,ab} \\ \underset{\sim}{K}_{C,ba} & \underset{\sim}{K}_{C,bb} \end{bmatrix} \qquad (4.4)$$

where,

A = column area

I = column moment of inertia

h = column height

4.3.3 Tension Bracing for Wind from Right (Brace Type 1).

The low joint number, designated by 'a', of the tension brace for wind from the right (brace type 1) is the upper left joint and the high joint number, designated by 'b', is the lower right joint. The brace type 1 joint displacement vector, $\underset{\sim}{U}_{BR1}$, is taken as,

$$\underset{\sim}{U}_{BR1} = \left\{ \begin{array}{c} \underset{\sim}{U}_{BR1,a} \\ \hdashline \underset{\sim}{U}_{BR1,b} \end{array} \right\} = \left\{ \begin{array}{c} u_{BR1,a1} \\ u_{BR1,a2} \\ u_{BR1,a3} \\ \hdashline u_{BR1,b1} \\ u_{BR1,b2} \\ u_{BR1,b3} \end{array} \right\} \tag{4.5}$$

Corresponding to the above partitioning the brace type 1 global stiffness matrix is formulated in a similar way as for beams. Fig. 4.7 shows the unit global linear displacements applied to both joints and the resulting end forces. Rotational joint displacements do not effect the braces since the brace ends are pin connected to the

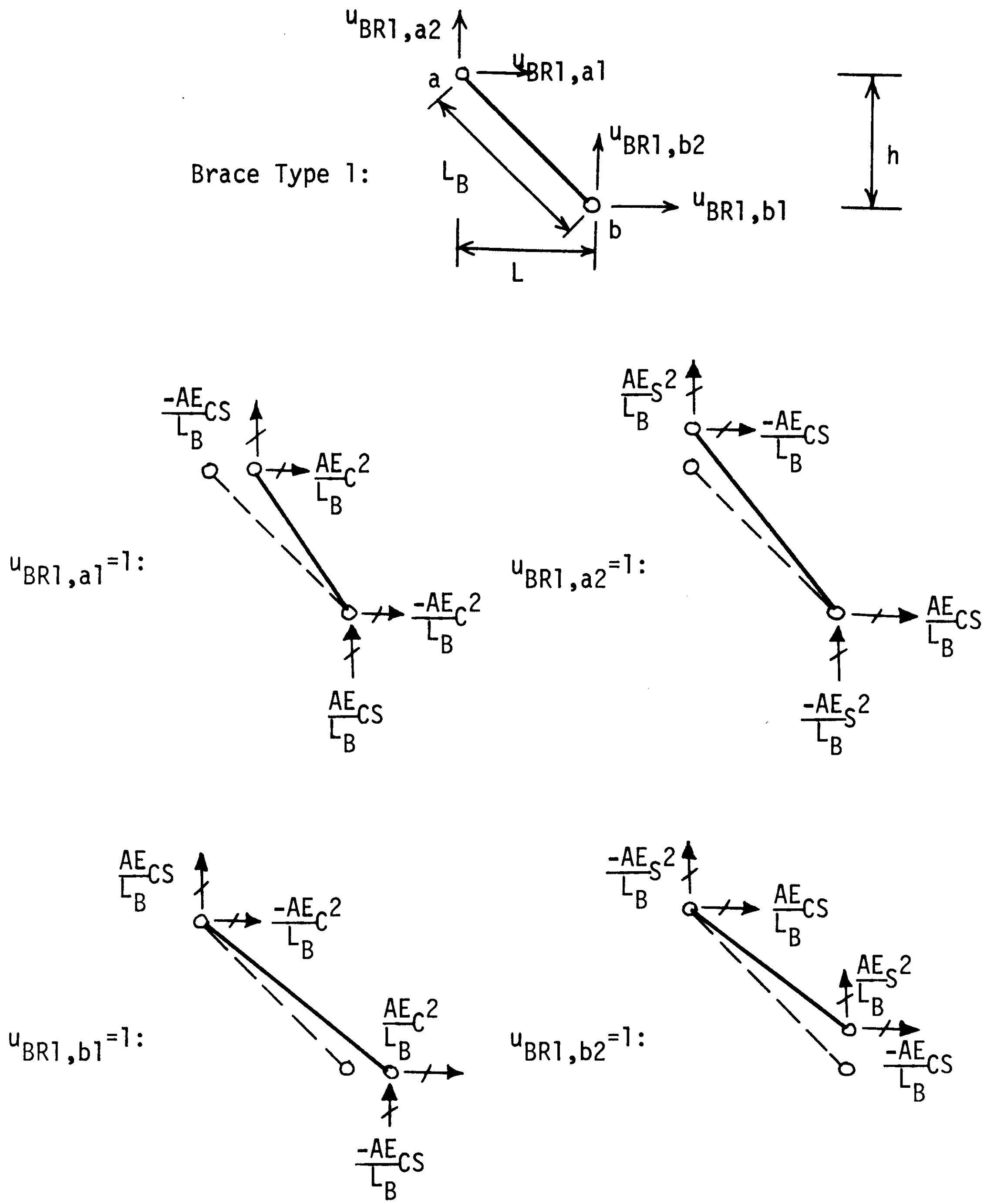


Figure 4.7 Tension Brace Type 1 Joint Unit Displacements and Resulting End Forces.

frame. The resulting 6 x 6 brace type 1 global stiffness matrix is as follows where only the upper triangular part is shown. The elements below the diagonal are obvious since $\underset{\sim}{K}_{BR1}$ is symmetrical. So,

$$\underset{\sim}{K}_{BR1} = \frac{AE}{L_B} \begin{bmatrix} C^2 & -CS & 0 & -C^2 & CS & 0 \\ & S^2 & 0 & CS & -S^2 & 0 \\ & & 0 & 0 & 0 & 0 \\ & \underset{\sim}{SYM.} & & C^2 & -CS & 0 \\ & & & & S^2 & 0 \\ & & & & & 0 \end{bmatrix}$$

$$= \frac{AE}{L_B} \begin{bmatrix} \underset{\sim}{K}_{BR1,aa} & \underset{\sim}{K}_{BR1,ab} \\ \underset{\sim}{K}_{BR1,ba} & \underset{\sim}{K}_{BR1,bb} \end{bmatrix} \quad (4.6)$$

where,

$C = L/L_B$

$S = h/L_B$

L = bay length

L_B = brace length

h = story height

A = brace area

4.3.4 Tension Bracing for Wind from Left (Brace Type 2).

The low joint number, designated by 'b', of the tension brace for wind from the left (brace type 2) is the upper right joint and the high joint number, designated by 'a', is the lower left joint. The brace type 2 joint displacement vector, $\underset{\sim}{U}_{BR2}$, is taken as,

$$\underset{\sim}{U}_{BR2} = \left\{ \begin{array}{c} \underset{\sim}{U}_{BR2,a} \\ \hline \underset{\sim}{U}_{BR2,b} \end{array} \right\} = \left\{ \begin{array}{c} u_{BR2,a1} \\ u_{BR2,a2} \\ u_{BR2,a3} \\ \hline u_{BR2,b1} \\ u_{BR2,b2} \\ u_{BR2,b3} \end{array} \right\} \qquad (4.7)$$

Corresponding to the above partitioning the brace type 2 global stiffness matrix is formulated in a similar way as for beams. Figure

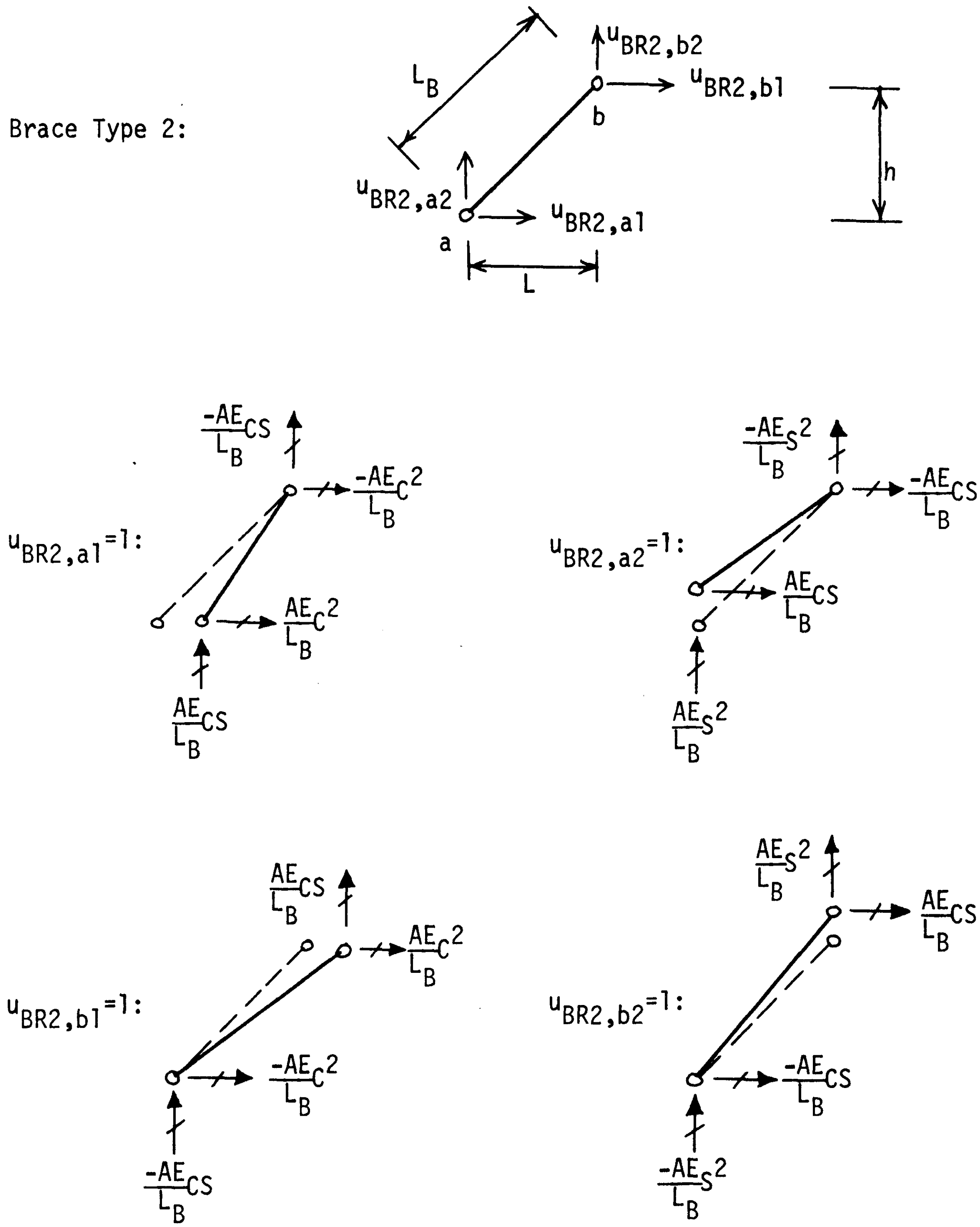


Figure 4.8 Tension Brace Type 2 Joint Unit Displacements and Resulting End Forces.

4.8 shows the unit global linear displacements applied to both joints and the resulting end forces. Rotational joint displacements do not effect the braces since the brace ends are pin connected to the frame. The resulting 6 x 6 brace type 2 global stiffness matrix is as follows where only the upper triangular part is shown. The elements below the diagonal are obvious since $\underset{\sim}{K}_{BR2}$ is symmetrical. So,

$$\underset{\sim}{K}_{BR2} = \frac{AE}{L_B} \begin{bmatrix} C^2 & CS & 0 & -C^2 & -CS & 0 \\ & S^2 & 0 & -CS & -S^2 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & C^2 & CS & 0 \\ & & & & S^2 & 0 \\ & & & & & 0 \end{bmatrix}$$

$$= \frac{AE}{L_B} \begin{bmatrix} \underset{\sim}{K}_{BR2,aa} & \underset{\sim}{K}_{BR2,ab} \\ \underset{\sim}{K}_{BR2,ba} & \underset{\sim}{K}_{BR2,bb} \end{bmatrix} \qquad (4.8)$$

where,

C, S, A, L, L_B, and h are defined in Eq. (4.6)

4.4 Structure Stiffness Matrix.

The formulation of the method used to directly construct the structure stiffness matrix from the member global stiffness matrices is well known(11). Consequently, only the details of the method will be described here. In what follows, the construction of the structure stiffness matrix will be considered followed by a description of the advantages taken of the symmetry and banded properties of the matrix.

To begin with, consider an arbitrary member in the plane frame where the low joint number is designated by 'L' and the high joint number by 'H'. The member stiffness matrix, $\underset{\sim}{K}_M$, in terms of global displacements and partitioned according to joint number for this member may be represented as,

$$
\underset{\sim}{K}_M = \begin{bmatrix} \underset{\sim}{K}_{M,LL} & \underset{\sim}{K}_{M,LH} \\ \underset{\sim}{K}_{M,HL} & \underset{\sim}{K}_{M,HH} \end{bmatrix} \tag{4.9}
$$

The matrix $\underset{\sim}{K}_M$ is a 6 x 6 matrix while each of the submatrices $\underset{\sim}{K}_{M,LL}$, $\underset{\sim}{K}_{M,LH} = (\underset{\sim}{K}_{M,HL})^T$, and $\underset{\sim}{K}_{M,HH}$ is a 3 x 3 matrix.

Consider now the full structure stiffness matrix, K_S, partitioned according to the joint numbers of the frame.

$$
\underset{\sim}{K}_S = \begin{bmatrix} \underset{\sim}{K}_{S,11} & \underset{\sim}{K}_{S,12} & \cdots & \underset{\sim}{K}_{S,1,NJ} \\ \underset{\sim}{K}_{S,21} & \underset{\sim}{K}_{S,22} & & \underset{\sim}{K}_{S,2,NJ} \\ \vdots & & \ddots & \\ \underset{\sim}{K}_{S,NJ,1} & & & \underset{\sim}{K}_{S,NJ,NJ} \end{bmatrix} \tag{4.10}
$$

where,

NJ = total number of joints

= (M)(N+1)

M = number of stories

N = number of bays

The matrix $\underset{\sim}{K}_S$ is a (3NJ) x (3NJ) matrix while each of its submatrices as shown in Eq. (4.10) is a 3 x 3 matrix. In addition, $\underset{\sim}{K}_S$ is symmetric so that,

$$\underset{\sim}{K}_{S,IJ} = (\underset{\sim}{K}_{S,JI})^T \tag{4.11}$$

where the T means matrix transposition.

Finally, the rule for constructing $\underset{\sim}{K}_S$ can be stated as follows. Each submatrix of $\underset{\sim}{K}_S$, say $\underset{\sim}{K}_{S,IJ}$, is equal to the sum of all member submatrices $\underset{\sim}{K}_{M,IJ}$.

A simple example will be used to illustrate the procedure. Fig. 4.9 shows a two-story, one-bay plane frame with all members and free joints labeled.

The partitioned member global stiffness matrices for beams B1 and B2 are,

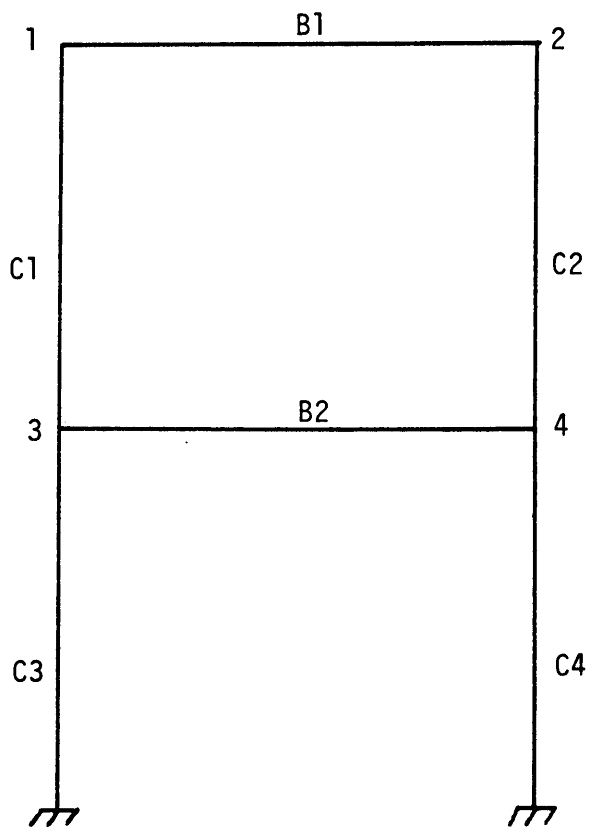


Figure 4.9 Two-Story, One-Bay Frame.

$$\underset{\sim}{K}_{B1} = \begin{bmatrix} \underset{\sim}{K}_{B1,11} & \underset{\sim}{K}_{B1,12} \\ \underset{\sim}{K}_{B1,21} & \underset{\sim}{K}_{BL,22} \end{bmatrix}$$

$$\underset{\sim}{K}_{B2} = \begin{bmatrix} \underset{\sim}{K}_{B2,33} & \underset{\sim}{K}_{B2,34} \\ \underset{\sim}{K}_{B2,43} & \underset{\sim}{K}_{B2,44} \end{bmatrix}$$

The partitioned member global stiffness matrices for columns C1 and C2 are,

$$\underset{\sim}{K}_{C1} = \begin{bmatrix} \underset{\sim}{K}_{C1,11} & \underset{\sim}{K}_{C1,13} \\ \underset{\sim}{K}_{C1,31} & \underset{\sim}{K}_{C1,33} \end{bmatrix}$$

$$\underset{\sim}{K}_{C2} = \begin{bmatrix} \underset{\sim}{K}_{C2,22} & \underset{\sim}{K}_{C2,24} \\ \underset{\sim}{K}_{C2,42} & \underset{\sim}{K}_{C2,44} \end{bmatrix}$$

Only the member global submatrix associated with the free joints of columns C3 and C4 need be used since the support joint displacements are zero. So,

$$\underset{\sim}{K}_{C3} = \underset{\sim}{K}_{C3,33}$$

$$\underset{\sim}{K}_{C4} = \underset{\sim}{K}_{C4,44}$$

Finally, the partitioned global structure stiffness matrix is,

$$
\underset{\sim}{K}_S = \begin{bmatrix} \underset{\sim}{K}_{S,11} & \underset{\sim}{K}_{S,12} & \underset{\sim}{K}_{S,13} & \underset{\sim}{K}_{S,14} \\ & \underset{\sim}{K}_{S,22} & \underset{\sim}{K}_{S,23} & \underset{\sim}{K}_{S,24} \\ & \underset{\sim}{\text{SYM.}} & \underset{\sim}{K}_{S,33} & \underset{\sim}{K}_{S,34} \\ & & & \underset{\sim}{K}_{S,44} \end{bmatrix}
$$

where,

$$\underset{\sim}{K}_{S,11} = \underset{\sim}{K}_{B1,11} + \underset{\sim}{K}_{C1,11}$$

$$\underset{\sim}{K}_{S,22} = \underset{\sim}{K}_{B1,22} + \underset{\sim}{K}_{C2,22}$$

$$\underset{\sim}{K}_{S,33} = \underset{\sim}{K}_{B2,33} + \underset{\sim}{K}_{C1,33} + \underset{\sim}{K}_{C3,33}$$

$$\underset{\sim}{K}_{S,44} = \underset{\sim}{K}_{B2,44} + \underset{\sim}{K}_{C2,44} + \underset{\sim}{K}_{C4,44}$$

$$\underset{\sim}{K}_{S,12} = (\underset{\sim}{K}_{S,21})^T = \underset{\sim}{K}_{B1,12}$$

$$\underset{\sim}{K}_{S,13} = (\underset{\sim}{K}_{S,31})^T = \underset{\sim}{K}_{C1,13}$$

$$\underset{\sim}{K}_{S,14} = (\underset{\sim}{K}_{S,41})^T = \underset{\sim}{0}$$

$$\underset{\sim}{K}_{S,23} = (\underset{\sim}{K}_{S,32})^T = \underset{\sim}{0}$$

$$\underset{\sim}{K}_{S,34} = (\underset{\sim}{K}_{S,43})^T = \underset{\sim}{K}_{B2,34}$$

The advantages taken of the symmetry and banded properties of the global structure stiffness matrix, $\underset{\sim}{K}_S$, will now be described.

Since $\underset{\sim}{K}_S$ is symmetrical and banded, only those elements of $\underset{\sim}{K}_S$ on the diagonal and above the diagonal but within the band width are calculated and stored in the computer. The band width of $\underset{\sim}{K}_S$ is defined here as the maximum number of elements in a row in the set of elements to the right of the diagonal and including the diagonal element in which a non-zero element exists. Thus, the band width is directly related to the maximum difference between **two** joint numbers connected by a member. Since diagonal bracing is considered, the maximum number of joints between two connected joints including the two connected joints is NIC + 2, where NIC designates the number of columns in a story or N + 1. Consequently, the structure of the upper triangle of $\underset{\sim}{K}_S$ including the diagonal elements is illustrated in Fig. 4.10. Now, the elements of $\underset{\sim}{K}_S$ are stored by the computer programs as a one-level array, $\overline{AK}(p_k)$. The one-level array stores the elements within the band of $\underset{\sim}{K}_S$ in a column-by-column fashion. That is to say, each column in the band of $\underset{\sim}{K}_S$ is stored in $\overline{AK}$ as follows.

$$\begin{aligned} \overline{AK} &= \{k_{s,11}, k_{s,12}, k_{s,22}, k_{s,13}, k_{s,23}, k_{s,33}, \ldots\} \\ &= \{p_1, p_2, p_3, p_4, p_5, p_6, \ldots\} \end{aligned} \tag{4.12}$$

Furthermore, the computer programs calculate the position of $k_{s,ij}$ in $\overline{AK}(p_k)$ by calculating the value of k for given values of i and j. In particular,

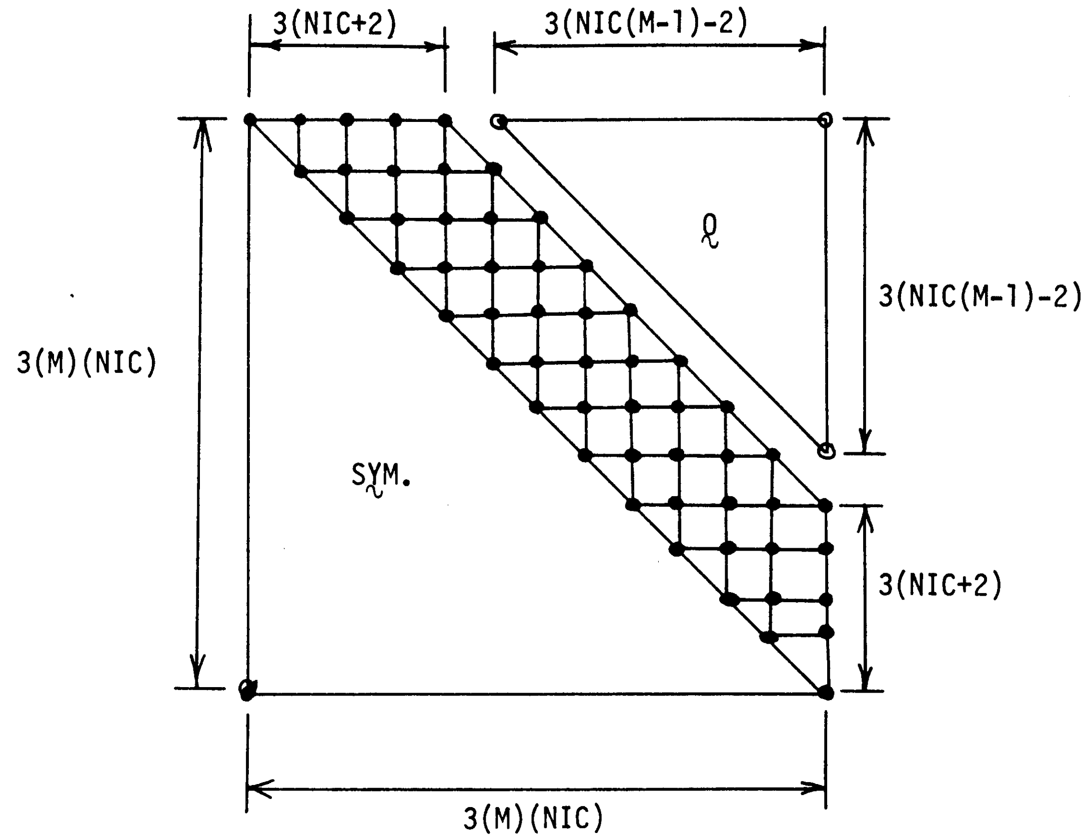


Figure 4.10 Structure of the Structure Stiffness Matrix.

$$\text{when, } 1 \leq j \leq 3(NIC+2)$$
$$\text{and, } 1 \leq i \leq j \qquad (4.13)$$
$$\text{then, } k = (j-1)\, j/2 + i$$

or

$$\text{when, } 3(M)(NIC) \geq j \geq 3\,(NIC+2) + 1$$
$$\text{and, } j - 3(NIC+2) + 1 \leq i \leq j \qquad (4.14)$$
$$\text{then, } k = (j-1)\, j/2 + i - [j-3(NIC+2) + (j-3(NIC÷2) - 1)(j-3(NIC+2))\,/2]$$

4.5 Square Root Method.

The derivation of the Square Root Method of matrix reduction is presented in numerous texts (12),(13) and thus will not be presented here. Instead, only the details of the method will be described. Furthermore, the following description will not include the consideration of the banded property of the stiffness matrix. However, the computer programs dc take full advantage of the banded properties by executing the reduction procedures only on the elements within the band.

The system of equations to be solved may be represented by the matrix equation

$$\underset{\sim}{K}\,\underset{\sim}{X} = \underset{\sim}{F} \tag{4.15}$$

Where,

$\underset{\sim}{K}$ = global structure stiffness matrix

$\underset{\sim}{X}$ = column vector of unknown joint displacements

$\underset{\sim}{F}$ = column vector of applied joint forces plus fixed-end forces due to loads applied directly to the frame members.

Also, $\underset{\sim}{K}$ is symmetrical which permits the application of the Square Root Method. The order of $\underset{\sim}{K}$ will be designated here as p x p. Thus, $\underset{\sim}{X}$ and $\underset{\sim}{F}$ are of order p x 1.

The Square Root Method decomposes $\underset{\sim}{K}$ into the product of a lower and upper triangular matrix such that,

$$\underset{\sim}{K} = \underset{\sim}{S}^T\,\underset{\sim}{S} \tag{4.16}$$

The form of $\underset{\sim}{S}$ is

$$\underset{\sim}{S} = \begin{bmatrix} s_{11} & s_{12} \;.\;.\;. & s_{1p} \\ & s_{22} \;.\;.\;. & s_{2p} \\ \underset{\sim}{0} & & s_{pp} \end{bmatrix} \tag{4.17}$$

where s_{ij} is a function of the elements k_{ij} of $\underset{\sim}{K}$. In addition, an intermediate column vector, $\underset{\sim}{Q}$, is constructed and is of the form,

$$\underset{\sim}{Q} = \begin{Bmatrix} q_1 \\ \cdot \\ \cdot \\ \cdot \\ q_p \end{Bmatrix} \tag{4.18}$$

where q_i is a function of s_{ij} and the elements of f_i of $\underset{\sim}{F}$. Finally, the solution for each x_i of $\underset{\sim}{X}$ uses values of q_i and s_{ij}. In particular,

$$s_{11} = \sqrt{k_{11}}$$

$$s_{ii} = \sqrt{k_{ii} - \sum_{r=1}^{i-1} s^2_{ri}} \tag{4.19}$$

$$s_{ij} = \frac{k_{ij} - \sum_{r=1}^{i-1} s_{ri}s_{rj}}{s_{ii}} \text{ , } i<j$$

$$q_1 = f_1/s_{11}$$

$$q_i = \frac{f_i - \sum_{r=1}^{i-1} s_{ri}q_r}{s_{ii}} \text{ , } i\geq 2 \tag{4.20}$$

$$X_p = q_p/s_{pp}$$

$$X_i = \frac{q_i - \sum_{r=i+1}^{p} s_{ir}X_r}{s_{ii}} \text{ , } i\leq p - 1 \tag{4.21}$$

Note that each time a value of k_{ij} or f_i is used in a particular calculation it is no longer needed in any succeeding calculation. Consequently, in order to minimize the computer core storage needed to store the data, each calculated value of s_{ij} and q_i is stored in the same computer core location as the no longer needed values of k_{ij} and f_i respectively.

4.6 Force Vector.

The force vector $\underset{\sim}{F}$ used in the previous section is defined as,

$$\underset{\sim}{F} = \underset{\sim}{P} + \overline{\underset{\sim}{FEF}} \tag{4.22}$$

where,

$\underset{\sim}{P}$ = applied joint loads

$\overline{\underset{\sim}{FEF}}$ = fixed-end forces due to loads applied directly to the frame members.

The applied joint loads are specified thru input to the computer programs and consist of concentrated vertical gravity loads applied to each joint of the frame and concentrated lateral wind loads applied to the external joints of the frame. Member loads consist only of uniform gravity loads $P_w(i,j)$ applied to the beams of the frame. The fixed-end forces applied to the joints due to $P_w(i,j)$

are illustrated in Fig. 4.11. The values of the fixed-end forces consistent with the sign convention illustrated in Fig. 4.1 are,

$$\begin{aligned} \overline{F}(i,j) &= \overline{F}(i,j+1) = -P_w(i,j)L(j)/2 \\ \overline{M}(i,j) &= -P_w(i,j)L^2(j)/12 \\ \overline{M}(i,j+1) &= P_w(i,j)L^2(j)/12 \end{aligned} \tag{4.23}$$

4.7 Elastic Member Design.

Member end forces are determined from the joint displacements by multiplying the member global stiffness matrix times the member joint displacement vector and adding the appropriate fixed-end forces. The resulting member end forces are in terms of the global sign convention. The computer programs convert the global member end forces into the local member sign convention and store the results.

Elastic member design simply consists of satisfying an elastic stress constraint which states that the maximum calculated elastic member stress, S_E, must be less than or equal to the specified maximum allowable stress, $S_{E,max.}$. So,

$$S_E \leq S_{E,max.} \tag{4.24}$$

The maximum elastic member stress is calculated for beams and columns as,

$$S_E = P/A + M/S \tag{4.25}$$

where,

P = magnitude of axial force

M = maximum magnitude of moment

A = member area

S = member elastic section modulus

For tension braces,

$$S_E = P/A \tag{4.26}$$

Now, the maximum magnitude of moment for columns is one of the two column end moments. However, since uniform loads are applied directly to beams, the maximum magnitude of moment may occur anywhere along the beam and thus must be calculated. The three values of moment that must be compared in order to determine the maximum are the absolute values of the two end moments, M_{BL} and M_{BR}, and the absolute value of the moment, $\overline{M}$, at the place where the slope of the beam moment diagram is zero if any. Consider the free body diagram of a beam in Fig. 4.12 where the end moments and interior moment are shown in their positive directions according to the local member sign convention. The location, $\overline{X}$, and the moment, $\overline{M}$, at the place where the slope of the beam moment diagram is zero is,

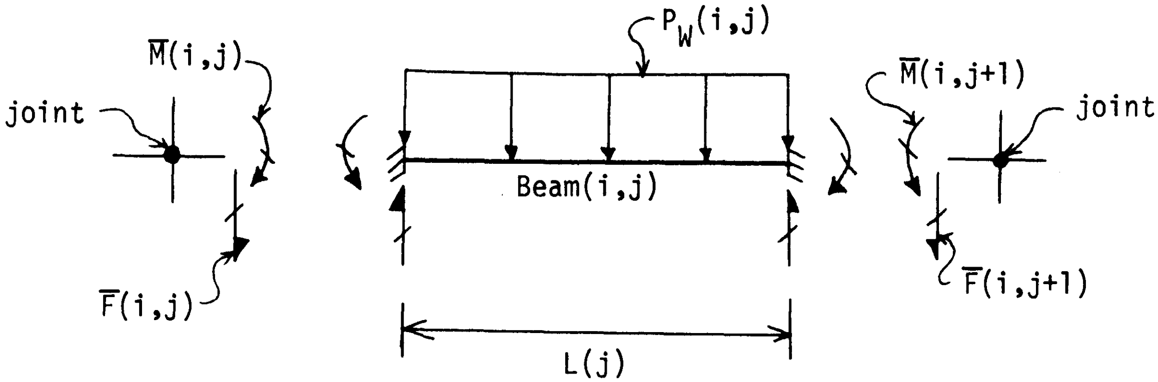


Figure 4.11 Fixed-End Forces Due To Uniform Beam Loads.

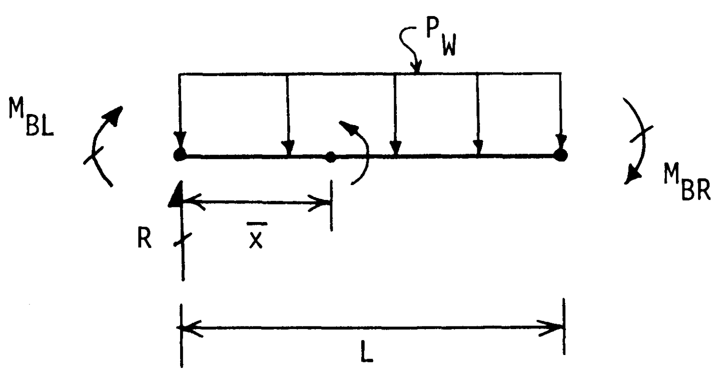


Figure 4.12 Maximum Beam Moment.

$$\overline{M} = R\,\overline{X} + \overline{M}_{BL} - P_w X^2/2$$
$$\overline{X} = L/2 - (M_{BL}+M_{BR})/(P_w L) \qquad (4.27)$$

where,

$$R = P_w L/2 - (M_{BL}+M_{BR})/L$$

If $\overline{X} \leq 0$ or $\overline{X} \geq L$ only the magnitudes of M_{BL} and M_{BR} are compared to determine the maximum moment to be used in the stress calculation.

Finally, if the current member size violates the elastic stress constraint the next larger section in the member section table is checked. The first section satisfying Eq. (4.24) is selected.

If member sizes are changed in order to satisfy the elastic stress constraint, the internal member force distribution will change. However, since the force distribution is relatively insensitive to changes in stiffness distributions, it is assumed that the actual member stresses have not changed sufficiently to cause the elastic stress constraint to be violated. Thus, a new elastic stress design is not performed in order to check this new force distribution. However, referring to Fig. 1.2, note that whenever the elastic stiffness design method is executed, a new elastic stress design is performed.

CHAPTER 5

ELASTIC STIFFNESS DESIGN METHOD

5.1 Introduction.

The proposed design method includes an elastic stiffness design in order to satisfy imposed lateral deflection constraints. The user of the program specifies maximum relative story deflections which he wishes to allow under service (or working) gravity plus wind loading. Exact lateral joint displacements are calculated by the stiffness method as described in Chapter 4. Relative story deflections are then defined as the difference between the average lateral joint displacements of upper and lower story joints. If the relative story deflections thus calculated exceed the specified maximum by more than three per-cent, various member section properties are increased in order to reduce the calculated deflections. In addition, it is obvious that when member properties are to be increased it is very desirable to effect such a modification in a way which minimizes the cost increase for an incremental decrease of relative story deflection. However, the stiffness method does not provide an efficient way to perform such an optimization procedure. Consequently, it is desirable to formulate an approximate method to perform the elastic stiffness design which is amenable to optimization techniques.

Note that the assumption made in Chapter 4 with respect to the diagonal bracing behavior is also made in this Chapter. That is to say, it is assumed that the compression bracing takes on a buckled configuration and thus only the tension bracing contributes to frame stiffness.

5.2 Summary of the Elastic Stiffness Design Method.

The elastic stiffness design method is executed if one or more of the 'exact' relative story deflections, Δ_e, calculated by the matrix stiffness method described in Chapter 4, violate the deflection constraints. During the elastic stiffness design, an optimization procedure is used to modify member properties. In addition approximate relative story deflections, Δ_a, are calculated.

The approximate deflection calculation assumes that relative story deflections are equal to the sum of four basic types of frame deflection. The first three deflection types are taken as relative deflections due to wind load alone while the fourth deflection type is taken as that due to gravity loads. The first type, Δ_s, is due to beam and column bending as well as brace elongation. The second type, Δ_c, is due to column elongation and shortening of the columns below the story under consideration. The third type, Δ_b, is due to beam elongation and shortening effects. Finally, the fourth type, Δ_g, is due to the sway deflections which result from unsymmetrical gravity loads or from gravity loads acting on a geometrically unsymmetrical structure.

Explicit algebraic equations are formulated to approximate the relative story deflections in the braced or unbraced multi-story frame due to the first two deflection types, namely, Δ_s and Δ_c. On the other hand, the remaining two deflection types, Δ_b and Δ_g, do not lend themselves very easily to approximate calculation. In addition, the calculation of Δ_s and Δ_c are approximate and consequently may contain errors in their calculation. A new deflection measure therefore is introduced which accounts for the second two deflection types in addition to the errors involved in the Δ_s and Δ_c calculation. This new measure, $\bar{E}$, is defined as the difference between the exact relative story deflection, Δ_e, calculated at the beginning of the elastic stiffness design and the sum of Δ_{s0} and Δ_{c0} where Δ_{s0} and Δ_{c0} are equal to Δ_s and Δ_c respectively, when calculated at the beginning of the elastic stiffness design before member properties are modified. Thus,

$$\bar{E} = \Delta_e - (\Delta_{s0}+\Delta_{c0}) \tag{5.1}$$

A different value of $\bar{E}$ would exist for each story in the frame. The approximate relative story deflection, Δ_a, calculated during the design optimization procedure is as follows.

$$\Delta_a = \Delta_s + \Delta_c + \bar{E} \tag{5.2}$$

Note that before any member properties are changed, $\Delta_a = \Delta_e$. However, during the design optimization procedure, member properties are increased in order to reduce the calculated Δ_a relative deflections until the deflection constraints are satisfied. The reduction in Δ_a is realized by reductions in Δ_s and Δ_c while the current value of $\bar{E}$ is taken as a constant throughout each cycle of the design optimization procedure.

The design optimization procedure will now be summarized. This procedure is used to determine an optimum distribution of additional member properties needed to satisfy the deflection constraints. The procedure used is the same in principle as that used in the plastic analysis and design method. In particular, the method is based on values of deflection sensitivity coefficients. The deflection sensitivity coefficient of a member reflects the increase in cost of the member with respect to the member's effect on decreasing the relative story deflection under consideration. The member with the least deflection sensitivity coefficient (i.e. most negative) is selected to increase in size by one section in the section table. This selected member will experience the least increase in cost due to an incremental decrease in relative story defection. After each change in member size, new values of Δ_s and Δ_c are calculated by the approximate equations and Δ_a is calculated by Eq. (5.2) where $\bar{E}$ is still taken to be based on the initial Δ_s, Δ_c (i.e. Δ_{s0}, Δ_{c0}) and Δ_e calculation. If the new relative story deflection, Δ_a, still violates the deflection constraint, new deflection sensitivity coefficients are calculated, a new member is selected and increased in size, and Δ_a is recalculated. The procedure is repeated until Δ_a satisfies the deflection constraint. After all stories that initially violated the deflection contraints have been redesigned in order to satisfy these constraints according to values of Δ_a, a new matrix stiffness analysis is executed and new 'exact' relative deflections, Δ_e, are determined. If the new values of Δ_e satisfy the deflection constraints, the elastic stiffness design is terminated. Otherwise, new values of $\bar{E}$ are calculated by Eq. (5.1) based on the latest approximate values of Δ_s and Δ_c to be interpreted as Δ_{s0} and Δ_{c0} and also the latest values of Δ_e. The design optimization procedure is then repeated. The above iteration continues until the 'exact' relative story deflections, Δ_e, satisfy the deflection constraints.

The following sections will describe the formulation of the approximate deflection equations and the design optimization procedure.

5.3 Relative Story Deflection Due to Beam and Column Bending and Tension Brace Elongation.

The calculation of relative story deflections due to beam and column bending will first be formulated for the unbraced plane frame. The formulation will then be extended to include the effects of diagonal brace elongation in a braced story.

5.3.1 Unbraced Plane Frame.

Consider a section of an unbraced plane frame in its deflected position as illustrated in Fig. 5.1. The following assumptions are made with respect to the behavior of the frame:

i . The joint rotations $\theta(k)$ are assumed to be equal for each joint in story level k.

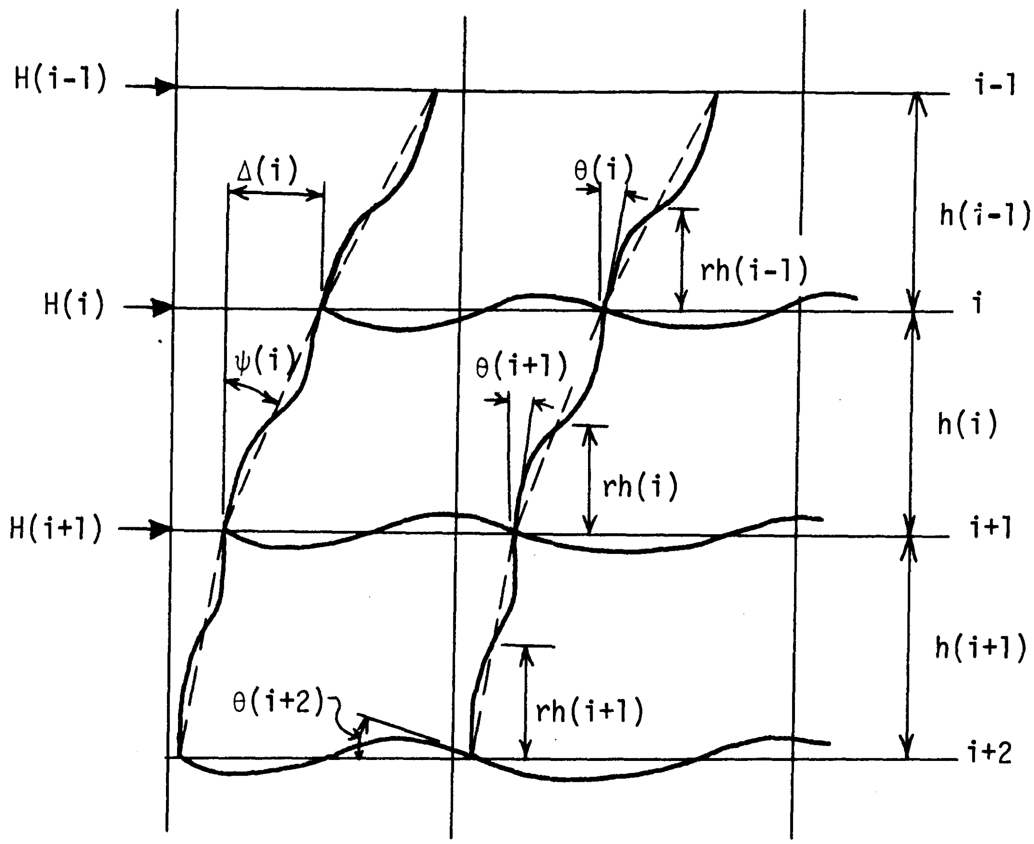


Figure 5.1 Relative Story Deflections.

ii . Rigid body column rotations $\psi(k)$ are assumed to be equal for each column in story k.

iii. The inflection point of each column in story k is assumed to be located at the distance rh(k) from the botton of the column, where $0.0<r<1.0$.

iv . Beam and column elongation and shortening are neglected in this part.

Now, column moment equilibrium for column j in story i is expressed by the slope- deflection equation as,

$$
\begin{aligned}
M_{CT}(i,j) &= 4E\,K_c(i,j)\theta(i) + 2E\,K_c(i,j)\theta(i+1) \\
&\quad - 6E\,K_c(i,j)\psi(i) \\
M_{CB}(i,j) &= 2E\,K_c(i,j)\theta(i) + 4E\,K_c(i,j)\theta(i+1) \\
&\quad - 6E\,K_c(i,j)\psi(i)
\end{aligned}
\tag{5.3}
$$

where,

$K_c(i,j) = I_c(i,j)/h(i)$

$I_c(i,j)$ = column moment of inertia

By assumptions (i) and (ii) and Eq. (5.3) the sum of top and bottom column end moments respectively in story i are,

$$
\begin{aligned}
\sum_{j=1}^{N+1} M_{CT}(i,j) &= [4E\theta(i) + 2E\theta(i+1) - 6E\psi(i)] \sum_{j=1}^{N+1} K_c(i,j) \\
\sum_{j=1}^{N+1} M_{CB}(i,j) &= [2E\theta(i) + 4E\theta(i+1) - 6E\psi(i)] \sum_{j=1}^{N+1} K_c(i,j)
\end{aligned}
\tag{5.4}
$$

Story moment equilibrium is expressed in terms of the sums of column end moments and end shears as,

$$\sum_{j=1}^{N+1} M_{CT}(i,j) + \sum_{j=1}^{N+1} M_{CB}(i,j) + h(i) \sum_{j=1}^{N+1} V_c(i,j) = 0 \qquad (5.5)$$

The sum of column end shears is calculated by considering the story shear equilibrium. So,

$$\sum_{j=1}^{N+1} V_c(i,j) = S(i) \qquad (5.6)$$

where,

$$S(i) = \sum_{k=1}^{i} H(k)$$

$H(k)$ = story k wind load.

Substituting Eq.'s (5.4) and (5.6) into Eq. (5.5) and solving for $\psi(i)$ results in,

$$\psi(i) = \frac{S(i)h(i)}{12E\sum_j K_c(i,j)} + \frac{\theta(i)}{2} + \frac{\theta(i+1)}{2} \qquad (5.7)$$

Beam moment equilibrium for beam j in story i is also expressed by the slope-deflection equation. So,

$$M_{BL}(i,j) = 4E\ K_B(i,j)\theta_L + 2E\ K_B(i,j)\theta_R \qquad (5.8)$$

$$M_{BR}(i,j) = 2E\ K_B(i,j)\theta_L + 4E\ K_B(i,j)\theta_R$$

where,

$K_B(i,j) = I_B(i,j)/L(j)$

$I_B(i,j)$ = beam moment of inertia.

θ_L = left joint beam rotation.

θ_R = right joint beam rotation.

However, by assumption (i),

$$\theta_L = \theta_R = \theta(i) \tag{5.9}$$

Therefore,

$$M_{BL}(i,j) = M_{BR}(i,j) = 6E\ K_B(i,j)\theta(i) \tag{5.10}$$

Consider now the joint moment equilibriums in story level i. The sum of the column end moments at joint j in story i is defined by $\overline{M}(j)$. So,

$$\overline{M}(j) = M_{CB}(i-1,j) + M_{CT}(i,j) \tag{5.11}$$

Thus, the joint moment equilibrium condition requires,

$$\overline{M}(j) = M_{BR}(i,j-1) + M_{BL}(i,j) \tag{5.12}$$

Note that any term referring to a member external to the frame is merely neglected. Substituting Eq. (5.10) into Eq. (5.12) and summing over over all joints in story level i leads to the sum of all column end moments at story level i in terms of the joint rotations and the sum of beam stiffnesses. Thus,

$$\sum_{j=1}^{N+1} \overline{M}(j) = 12E\theta(i) \sum_{j=1}^{N} K_B(i,j) \tag{5.13}$$

Finally, consider the equilibrium of the free body diagram of story level i illustrated in Fig. 5.2. Each column end moment equals the product of the column shear and the distance between the column inflection point and column end. Therefore, the sum of the column end moments can be expressed in terms of the sum of column shears. So,

$$\sum_{j=1}^{N+1} \overline{M}(j) = rh(i-1) \sum_{j=1}^{N+1} V_c(i-1,j) + (1-r)h(i) \sum_{j=1}^{N+1} V_c(i,j) \tag{5.14}$$

Substituting Eq. (5.6) into Eq. (5.14) results in the relation between the sum of column end moments at story level i and the total story shears. Thus,

$$\sum_{j=1}^{N+1} \overline{M}(j) = S(i-1)rh(i-1) + S(i)(1-r)h(i) \tag{5.15}$$

The joint rotations are now determined by equating Eq.'s (5.13) and (5.15) and solving for $\theta(i)$. Thus,

$$\theta(i) = \frac{S(i-1)rh(i-1) + S(i)(1-r)h(i)}{12E\sum_{j} K_B(i,j)} \tag{5.16}$$

Similarly, for story level i + 1,

$$\theta(i+1) = \frac{S(i)rh(i) + S(i+1)(1-r)h(i+1)}{12E\sum_{j} K_B(i+1,j)} \tag{5.17}$$

The assumption is now made that the inflection point of the columns occur at their mid-height or assume,

$$r = 1/2 \tag{5.18}$$

This assumption is satisfactory since it was found that variations of r between 0.0 and 1.0 had little effect on the final calculated results. Thus, by this assumption, $\theta(i)$ and $\theta(i+1)$ become,

$$\theta(i) = \frac{S(i)h(i) + S(i-1)h(i-1)}{24E\sum_j K_B(i,j)} \tag{5.19}$$

The rigid body column rotations $\psi(i)$ are now calculated by substituting Eq. (5.19) into Eq. (5.7). The result is,

$$\psi(i) = \frac{S(i)h(i)}{12E\sum_j K_C(i,j)} + \frac{S(i)h(i) + S(i-1)h(i-1)}{48E\sum_j K_B(i,j)} + \frac{S(i)h(i) + S(i+1)h(i+1)}{48E\sum_j K_B(i+1,j)} \tag{5.20}$$

Finally, the relative story deflection $\Delta_s(i)$ is simply the product of the story height and the column rotation. So,

$$\Delta_s(i) = h(i)\psi(i) \tag{5.21}$$

Thus, the relative story deflection in an unbraced multi-story plane frame is approximated as,

$$\Delta_s(i) = \frac{S(i)h^2(i)}{48E}\left[\frac{4}{\sum_j K_C(i,j)} + \frac{1}{\sum_j K_B(i,j)} + \frac{1}{\sum_j K_B(i+1,j)}\right]$$

$$+ \frac{S(i-1)h(i-1)h(i)}{48E}\left[\frac{1}{\sum_j K_B(i,j)}\right] \qquad (5.22)$$

$$+ \frac{S(i+1)h(i+1)h(i)}{48E}\left[\frac{1}{\sum_j K_B(i+1,j)}\right]$$

The following section extends the above formulation to the case of a diagonally braced plane frame.

5.3.2 Diagonally Braced Plane Frame.

Consider the equilibrium of the free body diagram of story level i illustrated in Fig. 5.3. In the braced story, the sum of the column end shears is no longer equal to the total story shear. Instead, it equals the total story shear less the sum of the horizontal components of brace force. Thus,

$$\sum_{j=1}^{N+1} V_C(i-1,j) = S(i-1) - R(i-1)$$

$$\sum_{j=1}^{N+1} V_C(i,j) = S(i) - R(i) \qquad (5.23)$$

and,

$$R(i) = \sum_{j=1}^{N} \overline{R}(i,j) \qquad (5.24)$$

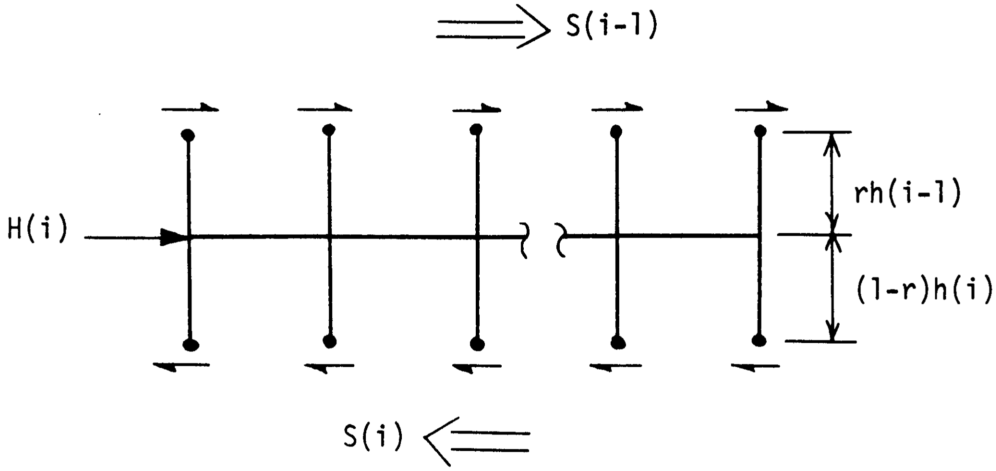


Figure 5.2 Unbraced Story Level Equilibrium.

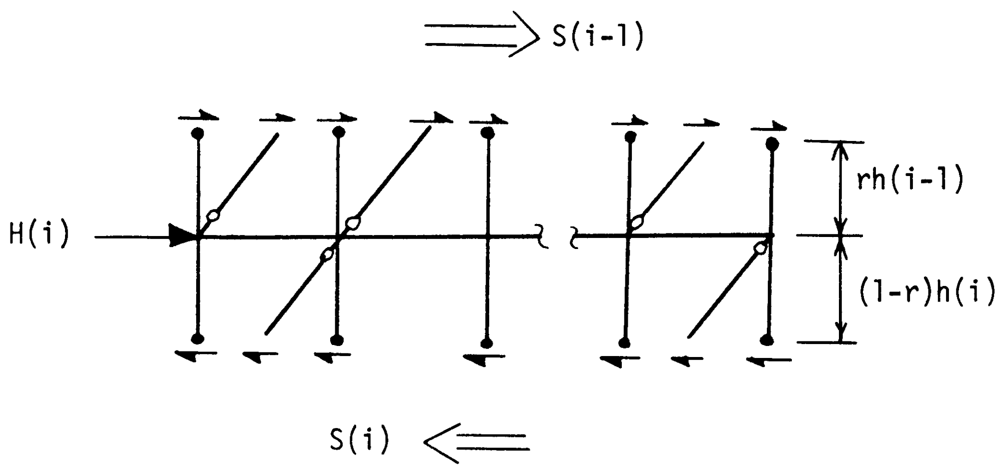


Figure 5.3 Braced Story Level Equilibrium.

where,

N = number of bays in story i.

$\overline{R}(i,j)$ = horizontal component of tension force in brace j of story i.

Now, the principle equations that were developed for the unbraced frame which are applicable for the braced frame are Eq.'s (5.3), (5.4), (5.5), (5.10), (5.13), (5.14), and (5.18). The column rigid body rotations in the braced story are calculated by substituting Eq.'s (5.4) and (5.23) into Eq. (5.5) and solving for $\psi(i)$. The result is,

$$\psi(i) = \frac{[S(i) - R(i)]h(i)}{12E\sum_{j} K_C(i,j)} + \frac{\theta(i)}{2} + \frac{\theta(i+1)}{2} \tag{5.25}$$

The relation between the sum of column end moments and total story shear is determined by substituting Eq. (5.23) into Eq. (5.14).

$$\sum_{j=1}^{N+1} \overline{M}(j) = [S(i-1) - R(i-1)]rh(i-1) + [S(i) - R(i)](1-r)h(i) \tag{5.26}$$

Joint rotations are calculated by equating Eq. (5.13) and Eq. (5.26), applying the assumption that r = 1/2, and solving. The results are,

$$\theta(i) = \frac{S(i)h(i) + S(i-1)h(i-1) - R(i)h(i) - R(i-1)h(i-1)}{24E\sum_{j} K_B(i,j)}$$

$$\theta(i+1) = \frac{S(i)h(i) + S(i+1)h(i+1) - R(i)h(i) - R(i+1)h(i+1)}{24E\sum_{j} K_B(i+1,j)} \tag{5.27}$$

Now, the relative story deflection is calculated by substituting Eq.'s (5.25) and (5.27) into Eq. (5.21) and solving for $\Delta_s(i)$. Thus,

$$\begin{aligned} \Delta_s(i) = {} & \frac{S(i)h^2(i)}{48E} \left[\frac{4}{\sum_j K_c(i,j)} + \frac{1}{\sum_j K_B(i,j)} + \frac{1}{\sum_j K_B(i+1,j)} \right] \\ & + \frac{S(i-1)h(i-1)h(i)}{48E\sum_j K_B(i,j)} + \frac{S(i+1)h(i+1)h(i)}{48E\sum_j K_B(i+1,j)} \\ & - \frac{R(i)h^2(i)}{48E} \left[\frac{4}{\sum_j K_c(i,j)} + \frac{1}{\sum_j K_B(i,j)} + \frac{1}{\sum_j K_B(i+1,j)} \right] \\ & - \frac{R(i-1)h(i-1)h(i)}{48E\sum_j K_B(i,j)} - \frac{R(i+1)h(i+1)h(i)}{48E\sum_j K_B(i+1,j)} \end{aligned} \tag{5.28}$$

The terms R(k) will now be formulated in terms of the bracing areas in story k. Fig. 5.4 illustrates the deflected position of a panel where by assumption (iv), beam and column elongation and shortening is neglected. From the geometry of the deflected position, and considering small deformations, the elongation, e, of the brace is,

$$e = \Delta_s(i)L(j)/L_B(i,j) \tag{5.29}$$

In addition,

$$e = \frac{F_{BR}L_B(i,j)}{A_{BR}(i,j)E} \tag{5.30}$$

where,

F_{BR} = brace force.

$A_{BR}(i,j)$ = brace area.

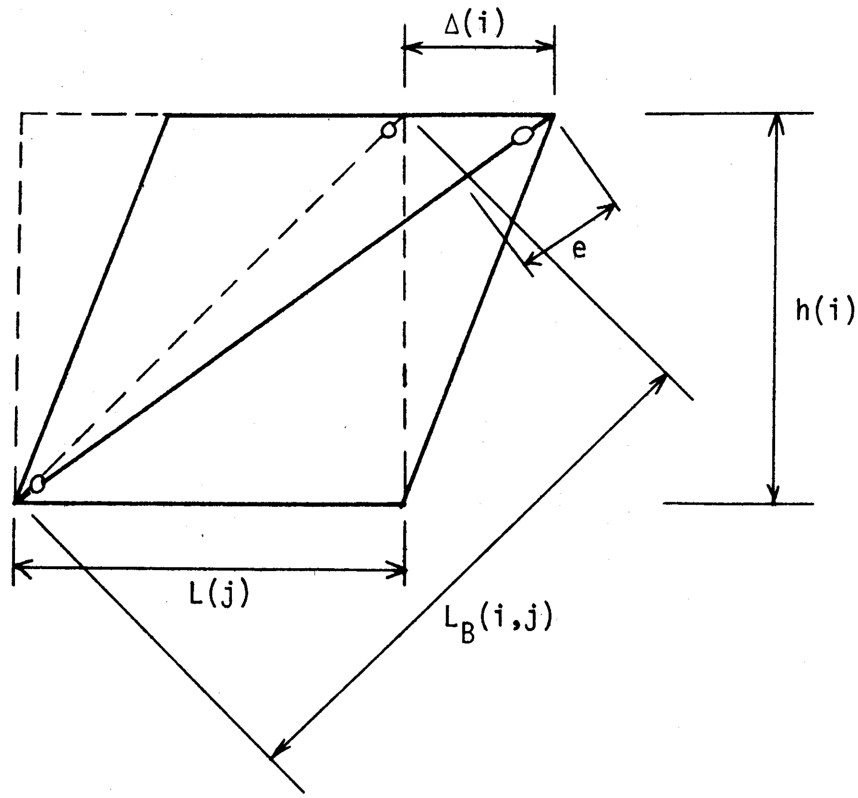


Figure 5.4 Brace Deformation.

The horizontal component of brace force, $\overline{R}(i,j)$, is,

$$\overline{R}(i,j) = F_{BR}L(j)/L_B(i,j) \tag{5.31}$$

From Eq.'s (5.29), (5.30) and (5.31),

$$\overline{R}(i,j) = E\Delta_s(i)\,\frac{L^2(j)}{L_B^3(i,j)}\,A_{BR}(i,j) \tag{5.32}$$

Substituting Eq. (5.32) into Eq. (5.24) results in,

$$R(i) = E\Delta_s(i)\sum_{j=1}^{N}\frac{L^2(j)}{L_B^3(i,j)}\,A_{BR}(i,j) \tag{5.33}$$

Before substituting Eq. (5.33) into Eq. (5.28), the following notation is adopted in the interest of concise notation. So, let

$$K_{iS0} = \frac{4}{\sum_j K_c(i,j)} + \frac{1}{\sum_j K_B(i,j)} + \frac{1}{\sum_j K_B(i+1,j)}$$

$$K_{iS1} = 1/\sum_j K_B(i,j)$$

$$K_{iS2} = 1/\sum_j K_B(i+1,j)$$

$$Q_i = \sum_j \frac{L^2(j)}{L_B^3(i,j)}\,A_{BR}(i,j)$$

$$A_i = S(i)h^2(i)/(48E)$$

$$B_i = S(i-1)h(i-1)h(i)/(48E)$$

$$C_i = S(i+1)h(i+1)h(i)/(48E)$$

$$A_i' = h^2(i)/(48E)$$

$$B_i' = h(i-1)h(i)/(48E)$$

$$C_i' = h(i+1)h(i)/(48E)$$

With this notation,

$$R(i) = \Delta_s(i)EQ_i \tag{5.34}$$

$$\Delta_s(i) = A_iK_{iS0} + B_iK_{iS1} + C_iK_{iS2} - \Delta_s(i)EQ_iA_i'K_{iS0}$$
$$- \Delta_s(i-1)EQ_{i-1}B_i'K_{iS1} - \Delta_s(i+1)EQ_{i+1}C_i'K_{iS2} \tag{5.35}$$

Solving Eq. (5.35) for $\Delta_s(i)$ leads to,

$$\Delta_s(i) = \frac{1}{[1 + EA_i'K_{iS0}Q_i]} [A_iK_{iS0} + B_iK_{iS1} + C_iK_{iS2}$$
$$- \Delta_s(i-1)EB_i'K_{iS1}Q_{i-1} - \Delta_s(i+1)EC_i'K_{iS2}Q_{i+1}] \tag{5.36}$$

Note that when Eq. (5.36) is applied to the top story, i = 1, $\Delta_s(0)$ is taken as zero. Also, when Eq. (5.36) is applied to the bottom story, i = M, $\Delta_s(M+1)$ is taken as zero.

Eq. (5.36) is applied to each story in the frame. The result is a system of M equations in the M unknown relative story deflections. This set of equations is solved by a method of successive substitutions as follows.

i . Eq. (5.36) written for story 1 results in $\Delta_s(1)$ as a function of $\Delta_s(2)$ as well as member properties in story 1.

ii . Eq. (5.36) written for story 2 results in $\Delta_s(2)$ as a function of $\Delta_s(1)$, $\Delta_s(3)$ and member properties in story 2.

iii . $\Delta_s(1)$ is substituted into Eq. (5.36) written for story 2 which is then solved for $\Delta_s(2)$. The result is $\Delta_s(2)$ as a function of $\Delta_s(3)$ and member properties of stories 1, 2.

iv . Eq. (5.36) written for story 3 results in $\Delta_s(3)$ as a function of $\Delta_s(2)$, $\Delta_s(4)$ and member properties in story 3.

v . $\Delta_s(2)$ is substituted into Eq. (5.36) written for story 3 which is then solved for $\Delta_s(3)$. The result is $\Delta_s(3)$ as a function of $\Delta_s(4)$ and member properties of stories 1, 2, and 3.

vi . At this point the assumption is made that terms relating to influences three or more stories away are negligable and thus may be neglected. Consequently, $\Delta_s(3)$ is a function of $\Delta_s(4)$ and member properties of only stories 2 and 3.

vii . Eq. (5.36) written for story 4 results in $\Delta_s(4)$ as a function of $\Delta_s(3)$, $\Delta_s(5)$ and member properties in story 4.

viii. $\Delta_s(3)$ is substituted into Eq. (5.36) written for story 4 which is then solved for $\Delta_s(4)$. The result is $\Delta_s(4)$ as a function of $\Delta_s(5)$ and member properties of stories 2, 3 and 4.

ix . The assumption made in (vi) is applied to $\Delta_s(4)$. Consequently, $\Delta_s(4)$ is expressed as a function of $\Delta_s(5)$ and member properties only in stories 3 and 4.

x . The procedure is continued story by story down the frame to the bottom story where $\Delta_s(M)$ is expressed only as a function of member properties in stories M - 1 and M.

By back substitution all other story deflections may be obtained. The final result for the general story relative deflection in a braced frame due to beam and column bending and brace elongation is as follows.

$$\Delta_s(i) = \frac{A6 - A7 - \frac{(A8)(A13)}{(A14)}}{1 + A1 - \frac{(A2)}{(A14)}} \tag{5.37}$$

where,

$A1 = EA_i'K_{iS0}Q_i$

$A2 = E^2B_i'C_{i-1}'K_{iS1}K_{i-1,S2}Q_iQ_{i-1}$

$A3 = EA_{i-1}'K_{i-1,S0}Q_{i-1}$

$A4 = E^2B_{i-1}'C_{i-2}'K_{i-1,S1}K_{i-2,S2}Q_{i-1}Q_{i-2}$

$A5 = EA_{i-2}'K_{i-2,S0}Q_{i-2}$

$A6 = A_iK_{iS0} + B_iK_{iS1} + C_iK_{iS2}$

$A7 = \Delta_s(i+1)EC_i'K_{iS2}Q_{i+1}$

$A8 = EB_i'K_{iS1}Q_{i-1}$

$A9 = A_{i-1}K_{i-1,S0} + B_{i-1}K_{i-1,S1} + C_{i-1}K_{i-1,S2}$

$A10 = EB_{i-1}'K_{i-1,S1}Q_{i-2}$

$A11 = EA_{i-2}'K_{i-2,S0}Q_{i-2}$

$A12 = A_{i-2}K_{i-2,S0} + B_{i-2}K_{i-2,S1} + C_{i-2}K_{i-2,S2}$

$$A13 = A9 - \frac{(A10)(A12)}{(1+A11)}$$

$$A14 = 1 + A3 - \frac{A4}{(1+A5)}$$

Note that in an unbraced frame, all terms involving Q are zero. Thus, Eq. (5.37) reduces to Eq. (5.22) in the unbraced case or in terms of the new notation

$$\Delta_S(i) = A6 \tag{5.38}$$

Also note that Eq. (5.37) requires knowledge of the relative story deflection in the story below the story under consideration. Therefore, the calculation of relative story deflections begin with the bottom story, i = M, where $\Delta_S(M+1) = 0.0$, and proceed story by story up the frame to the top story. Furthermore, when i = 1, all terms involving the subscripts i-1 and i-2 are taken as zero and when i = 2, all terms involving the subscripts i-2 are taken as zero.

5.4 Relative Story Deflection Due to Column Elongation and Shortening.

The relative story deflection due to column elongation and shortening, $\Delta_c(i)$, is based on an elastic force distribution in the columns due to lateral wind loads alone applied to the braced frame with only tension bracing acting. This force distribution is the most recent one calculated by the matrix stiffness analysis as described in Chapter 4. It is assumed that these elastic forces remain constant during each execution of the elastic stiffness design method. This assumption is reasonable since the elastic force distribution is relatively insensitive to changes in stiffness distributions. However, as described in Section 5.2, at the end of each elastic stiffness design, new exact deflections and forces are recalculated by the matrix stiffness analysis. If the new exact deflections still violate the deflection constraints, the elastic stiffness design method is repeated. If this is the case, the new (i.e., most recent) elastic force distribution is used in the calculation of the relative story deflections due to column elongation and shortening.

The calculation of $\Delta_c(i)$ is based on an accumulation of average story rotations. Thus,

$$\Delta_c(i) = h(i) \sum_{k=i+1}^{M} \psi(k-1) \tag{5.39}$$

where,

$\psi(k-1)$ = additional story k-1 rotation due to column elongation and shortening in story k.

Referring to Fig. 5.5, the additional story p rotation, $\psi(p)$, where $p = k-1$, is calculated as a straight average of column rotations, $\phi(p,j)$, over all interior columns in the story. In particular,

$$\psi(p) = \frac{1}{N-1} \sum_{j=2}^{N} \phi(p,j) \tag{5.40}$$

Each interior column rotation in story p is calculated on the basis of a weighted average of the two beam rotations in level p + 1 at the bottom of the story p column. Thus,

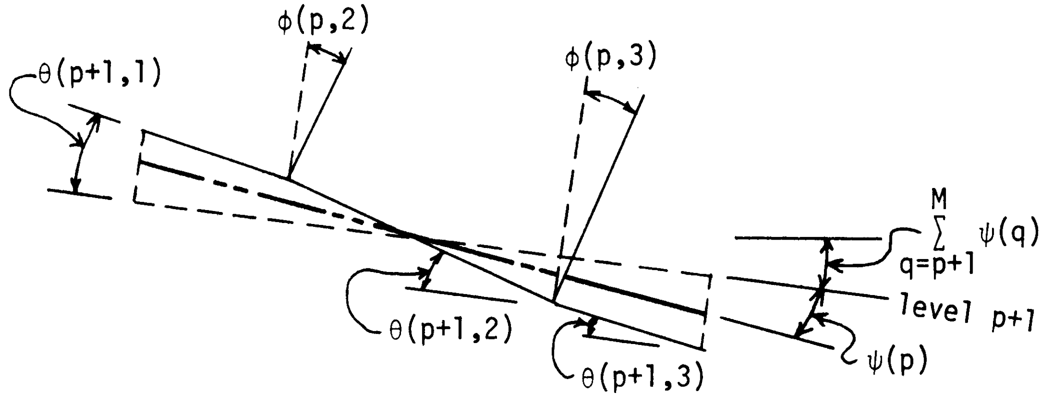


Figure 5.5 Story Rotations.

$$\phi(p,j) = \frac{W(p+1,j)\theta(p+1,j) + W(p+1,j-1)\theta(p+1,j-1)}{W(p+1,j) + W(p+1,j-1)} \qquad (5.41)$$

A weighted average is used rather than a straight average since it is felt that the average story rotation should in some way relfect the shear distribution in the story. This procedure turns out to be quite significant when bracing exists in the p + 1 story. Thus, when story p + 1 contains bracing, the weighting factors are calculated as follows.

$$W(p+1,j) = \frac{1}{2}\left[V_c(p+1,j) + V_c(p+1,j+1) \right] + \overline{R}(p+1,j) \qquad (5.42)$$

where,

$\overline{R}(p+1,j)$ = Horizontal component of tension brace force in the panel from the exact matrix stiffness analysis described in Chapter 4.

$V_c(p+1,j)$ = Column end shear from the Chapter 4 matrix stiffness analysis.

When no bracing exists anywhere in story p + 1, it was found that a better procedure for calculating the weights was to assign W(p+1,j) the value of 1.0. This is equivalent to calculating interior column rotations in story p by straight averages of the beam rotations in story level p + 1.

Finally, the calculation of the beam rotations, $\theta(p+1,j)$, are calculated relative to an assumed straight (but rotated) story level p + 2. These beam rotations are directly related to column elongations and shortenings in story p + 1. Thus,

$$\theta(p+1,j) = \frac{e_c(p+1,j+1) - e_c(p+1,j)}{L(j)} \qquad (5.43)$$

where the change in length of column j in story p+1 is,

$$e_c(p+1,j) = \frac{F_c(p+1,j)h(p+1)}{A_c(p+1,j)E} \qquad (5.44)$$

where,

$F_c(p+1,j)$ = Column axial force.

$h(p+1)$ = Column length.

$A_c(p+1,j)$ = Column area.

E = Modulus of Elasticity.

Note that when $\theta(i+1,j)$ is negative, it was found that setting its value to 0.0 led to better results.

Now, substituting Eq.'s (5.44), (5.43), (5.41) and (5.40) into Eq. (5.39) and defining $T(k,j)$ by,

$$T(k,j) = F_c(k,j)h(k)/E \qquad (5.45)$$

the relative story deflection due to column elongation and shortening may be expressed as,

$$\Delta_c(i) = \frac{h(i)}{N-1} \sum_{k=i+1}^{M} \sum_{j=2}^{N} \left[\frac{H_1 + H_2 + H_3}{W(k,j-1) + W(k,j)} \right] \qquad (5.46)$$

where,

$$H_1 = \frac{-T(k,j-1)}{A_c(k,j-1)} \; \frac{W(k,j-1)}{L(j-1)}$$

$$H_2 = \frac{I(k,j)}{A_c(k,j)} \left[\frac{W(k,j-1)}{L(j-1)} - \frac{W(k,j)}{L(j)} \right]$$

$$H_3 = \frac{I(k,j+1)}{A_c(k,j+1)} \frac{W(k,j)}{L(j)}$$

5.5 Method of Optimization for Elastic Deflection Constraints.

When the total relative story deflection due to combined gravity and wind loads (with $\lambda_2 = 1.0$) exceeds the maximum allowable relative story deflection, member sizes must be increased. The procedure used to increase member sizes should select those members which will increase in cost the least for a unit decrease in relative story deflection. The method selected is a gradient search technique that is similar in principle to the one used in the plastic design method.

The cost, f, of all members effecting the relative story deflection is represented as,

$$f = \sum_{\substack{\text{all} \\ \text{members}}} (u\rho LA) \tag{5.47}$$

where u, ρ,L, and A represent respectively the unit material cost, mass density, member length, and member area. The increase in material cost due to a decrease in relative story deflection is calculated by differentiating Eq. (5.47) with respect to $\Delta(i)$. So,

$$\frac{\partial f}{\partial \Delta(i)} = \Sigma u\rho L \quad \frac{\partial A}{\partial \Delta(i)} \tag{5.48}$$

Since the change in cost, ∂f, is positive while the change in deflection, $\partial\Delta(i)$, is negative, the sign of $\frac{\partial f}{\partial\Delta(i)}$ is negative. Consequently, member size changes are made which maximize $\frac{\partial f}{\partial\Delta(i)}$ (i.e. make least negative or minimize the magnitude of $\frac{\partial f}{\partial\Delta(i)}$).

Now, the deflection sensitivity coefficient, Q is defined as follows.

$$\frac{\partial f}{\partial\Delta(i)} = \Sigma\ u\rho L\ \frac{\partial A}{\partial\Delta(i)} = \Sigma(\frac{1}{Q}) \tag{5.49}$$

Thus, for beams,

$$Q_B(i,j) = \frac{1}{\rho u_B(i,j)L(j)}\ \frac{\partial\Delta(i)}{\partial A_B(i,j)} \tag{5.50}$$

for columns,

$$Q_C(k,j) = \frac{1}{\rho u_C(k,j)\bar{h}(k)}\ \frac{\partial\Delta(i)}{\partial A_C(k,j)} \tag{5.51}$$

for braces,

$$Q_{BR}(i,j) = \frac{1}{\rho u_{BR}(i,j)L_B(i,j)}\ \frac{\partial\Delta(i)}{\partial A_{BR}(i,j)} \tag{5.52}$$

Since $\frac{\partial f}{\partial\Delta(i)}$ must be maximized or made least negative, the value of Q must be minimized or made most negative.

The procedure to increase member sizes consists of first calculating the deflection sensitivity coefficient for each member that effects the relative story deflection to be reduced. The member with the most negative (minimum) deflection sensitivity coefficient is selected to increase in size by one section in the appropriate section table.

After each increase in member size, the new relative story deflection is calculated. If $\Delta(i)$ still exceeds the maximum allowable, new deflection sensitivity coefficients are calculated and the process repeated. Furthermore, the method proceeds on a story by story basis from the bottom story to the top story. This takes full advantage of the most current column sizes below the story under consideration. Obviously this method is feasible only due to the fact that the approximate method of calculation of $\Delta(i)$ is extremely efficient on the computer.

The factor $\frac{\partial \Delta(i)}{\partial A}$ will now be formulated. Since beam and column bending, tension brace elongation, and column elongation and shortening make the most significant contributions to the relative story deflection, only these effects will be considered in the calculation of $\frac{\partial \Delta(i)}{\partial A}$.

5.5.1 Beams, Columns, and Tension Brace in Story i.

The factor $\frac{\partial \Delta(i)}{\partial A}$ for beam and column bending and tension brace elongation in story i could be calculated by differentiating Eq. (5.37) with respect to each member area in story i. However, due to the complexity of effecting an exact differentiation on Eq. (5.37), a simple numerical differentiation will be performed. In particular, each beam, column, and tension brace area is increased separately keeping all other story member areas at their current values. The change in member area is designated by ΔA. The new relative story deflection, $\Delta_s(i)_{new}$, after a change in member area is then calculated by

Eq. (5.37). The factor $\frac{\partial\Delta(i)}{\partial A}$ for the particular member area change is then taken as,

$$\frac{\partial\Delta(i)}{\partial A} = \frac{\partial\Delta_s(i)}{\partial A} = \frac{\Delta_s(i)_{new.} - \Delta_s(i)_{current}}{\Delta A} \qquad (5.53)$$

5.5.2 Columns Below Story i.

The factor $\frac{\partial\Delta(i)}{\partial A}$ for columns below story i is obtained by differentiating Eq. (5.46) with respect to each column area below story i. The result is as follows.

$$\frac{\partial\Delta(i)}{\partial A_c(k,j)} = \frac{\partial\Delta_c(i)}{\partial A_c(k,j)} = \frac{h(i)h(k)F_c(k,j)}{A_c^2(k,j)(N-1)L(j)E}\left[G_1 - G_2 + G_3\right] \qquad (5.54)$$

where,

$$i+1 \le k \le M$$

$$G_1 = \frac{-W(k,j-1)L(j)}{L(j-1)[W(k,j-2) + W(k,j-1)]}$$

$$G_2 = \frac{W(k,j-1)L(j) - W(k,j)L(j-1)}{L(j-1)[W(k,j-1) + W(k,j)]}$$

$$G_3 = \frac{W(k,j)}{W(k,j) + W(k,j+1)}$$

Note that G_1, G_2, or G_3 is taken as zero when any one of them contain any terms involving subscripts less than or equal to zero or greater than N.

CHAPTER 6

RECOMMENDATIONS

Recommendations for future extensions to the design system are as follows:

1. Rewrite the input format making it more user oriented. The use of a problem oriented language is recommended.

2. Develop a more general and flexible loading specification in order to:

   (a) account for live load reduction coefficients automatically;

   (b) allow for the specification of more general loading types such as concentrated loads applied directly to members.

3. Extend the design system to include:

   (a) a consideration of checkerboard loading patterns;

   (b) a consideration of a vertical deflection of beams constraint;

   (c) a consideration of more general bracing types (i.e. K-bracing, etc.)

   (d) a consideration of more general loading configurations such as concentrated loads applied at various points along the beams.

4. Extend the elastic stiffness design method to include a procedure to eliminate the poor elastic relative story deflection

convergence characteristics in those situations where the effects of gravity sway deflections become significant. This might be accomplished by modifying the procedure that selects members to increase in size so that the tendency to increase gravity sway deflections is minimized.

5. It is recommended that the effects of column and beam elongation and shortening effects be included in the calculation of Δ for the P-Δ effect. The effects of column elongation and shortening might be accounted for by using the same method as used for the approximate elastic deflection calculation, but where the column axial forces would be those resulting from the plastic design part. This addition would fit into the current design procedure whereby an iterative procedure is used to satisfy the ultimate relative story deflection convergence criterion since the Δ calculation is based on the final member properties following each cycle of the plastic design part.

6. Extend the method to three-dimensional structures including:

(a) a procedure to distribute lateral loads to the bents of a building according to relative bent stiffnesses;

(b) a consideration of biaxial column bending;

(c) a formulation of an approximate three-dimensional deflection calculation to include the effects of overall frame torsion;

(d) a major revision of data storage capability in order to handle the enormous quantity of data associated with a three-dimensional structure.

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APPENDIX A

SECTION PROPERTIES OF ROLLED STEEL SHAPES USED IN THE ILLUSTRATIVE EXAMPLES

Rolled sections*and their section properties used in the illustrative examples are tabulated in the following Tables A1 to A5.

Table A1 presents the economy beam sections.

Table A2 presents the non-economy beam sections used when beam depth constraints controlled beam sizes.

Table A3 presents the economy column sections. Since column depth constraints were not considered in the design examples, a table of non-economy column sections is not presented.

Table A4 presents the equal leg double angle bracing sections when used.

Table A5 presents the unequal leg double angle bracing sections when used.

*Note that all sections used are taken from the AISC Manual[(1)].

TABLE A1. Economy Beam Sections.

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	r_y (in.)
6JR4.4	4.4	1.30	6.00	7.3	0.17	2.4	2.8	2.37	0.36
8JR6.5	6.5	1.92	8.00	18.7	0.34	4.7	5.4	3.12	0.42
10JR9	9.0	2.64	10.00	39.0	0.61	7.8	9.2	3.85	0.48
12JR11.8	11.8	3.45	12.00	72.2	0.98	12.0	14.3	4.57	0.53
10B15	15.0	4.40	10.00	68.8	2.79	13.8	16.0	3.95	0.80
12B16.5	16.5	4.86	12.00	105.3	2.79	17.5	20.6	4.65	0.76
14B17.2	17.2	5.05	14.00	147.3	2.65	21.0	24.7	5.40	0.72
14B22	22.0	6.47	13.72	197.4	6.40	28.8	33.0	5.52	0.99
16B26	26.0	7.65	15.65	298.1	8.71	38.1	43.9	6.24	1.07
14WF30	30.0	8.81	13.86	289.6	17.5	41.8	47.1	5.73	1.41
16B31	31.0	9.12	15.84	372.5	11.57	47.0	53.8	6.39	1.13
14WF34	34.0	10.00	14.00	339.2	21.3	48.5	54.5	5.83	1.46
16WF36	36.0	10.59	15.85	446.3	22.1	56.3	63.9	6.49	1.45

TABLE A1. Continued

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	r_y (in.)
16WF40	40.0	11.77	16.00	515.5	26.5	64.4	72.7	6.62	1.50
18WF45	45.0	13.24	17.86	704.5	31.9	78.9	89.6	7.30	1.55
18WF50	50.0	14.71	18.00	800.6	37.2	89.0	100.8	7.38	1.59
21WF55	55.0	16.18	20.80	1140.7	44.0	109.7	125.4	8.40	1.65
21WF62	62.0	18.23	20.99	1326.8	53.1	126.4	144.1	8.53	1.71
24WF68	68.0	20.00	23.71	1814.5	63.8	153.1	175.5	9.53	1.79
24WF76	76.0	22.37	23.91	2096.4	76.5	175.4	200.1	9.68	1.85
27WF84	84.0	24.71	26.69	2824.8	95.7	211.7	243.2	10.69	1.97
27WF94	94.0	27.65	26.91	3266.7	115.1	242.8	277.7	10.87	2.04
30WF99	99.0	29.11	29.64	3988.6	116.9	269.1	312.0	11.70	2.00
30WF108	108.0	31.77	29.82	4461.0	135.1	299.2	345.5	11.85	2.06
30WF116	116.0	34.13	30.00	4919.1	153.2	327.9	377.6	12.00	2.12
33WF118	118.0	34.71	32.86	5886.9	170.3	358.3	414.3	13.02	2.2

TABLE A1. Continued

SECTION	Wt./Ft.	AREA	DEPTH	MOMENT OF INERTIA		SECTION MODULUS	PLASTIC MODULUS	RADIUS OF GYRATION	
	(lb/ft)	(in.^{2})	(in.)	I_x (in.^{4})	I_y (in.^{4})	S_x (in.^{3})	Z_x (in.^{3})	r_x (in.)	r_y (in.)
33WF130	130.0	38.26	33.10	6699.0	201.4	404.8	466.0	13.23	2.29
36WF135	135.0	39.70	35.55	7796.1	207.1	438.6	509.1	14.01	2.28
36WF150	150.0	44.16	35.84	9012.1	250.4	502.9	579.8	14.29	2.38
36WF160	160.0	47.09	36.00	9738.8	275.4	541.0	623.3	14.38	2.42
36WF170	170.0	49.98	36.16	10470.0	300.6	579.1	666.7	14.47	2.45
36WF182	182.0	53.54	36.32	11281.5	327.7	621.2	716.9	14.52	2.47
36WF194	194.0	57.11	36.48	12103.4	355.4	663.6	767.2	14.56	2.49
36WF230	230.0	67.73	35.88	14988.4	870.9	835.5	942.7	14.88	3.59
36WF245	245.0	72.03	36.06	16092.2	944.7	892.5	1008.0	14.95	3.62
36WF260	260.0	76.56	36.24	17233.8	1020.6	951.1	1076.0	15.00	3.65
36WF280	280.0	82.32	36.50	18819.3	1127.5	1031.2	1167.0	15.12	3.70
36WF300	300.0	88.17	36.72	20290.2	1225.2	1105.1	1255.0	15.17	3.73

TABLE A2. NON-ECONOMY BEAM SECTIONS.									
SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA		SECTION MODULUS	PLASTIC MODULUS	RADIUS OF GYRATION	
				I_x (in.^{4})	I_y (in.^{4})	S_x (in.^{3})	Z_x (in.^{3})	r_x (in.)	r_y (in.)
12WF45	45.0	13.24	12.06	350.8	50.0	58.2	64.9	5.15	1.94
10WF54	54.0	15.88	10.12	305.7	103.9	60.4	67.0	4.39	2.56
12WF50	50.0	14.71	12.19	394.5	56.4	64.7	72.6	5.18	1.96
10WF72	72.0	21.18	10.50	420.7	141.8	80.1	90.7	4.46	2.59
10WF89	89.0	26.19	10.88	542.4	180.6	99.7	114.4	4.55	2.63
12WF79	79.0	23.22	12.38	663.0	216.4	107.1	119.3	5.34	3.05
18WF60	60.0	17.64	18.25	984.0	47.1	107.8	122.6	7.47	1.63
14WF74	74.0	21.76	14.19	796.8	133.5	112.3	125.6	6.05	2.48
10WF100	100.0	29.43	11.12	625.0	206.6	112.4	130.1	4.61	2.65
18WF64	64.0	18.80	17.87	1045.8	70.3	117.0	131.8	7.46	1.93
14WF78	78.0	22.94	14.06	851.2	206.9	121.1	134.0	6.09	3.00
12WF92	92.0	27.06	12.62	788.9	256.4	125.0	140.2	5.40	3.08
18WF70	70.0	20.56	18.00	1153.9	78.5	128.2	144.7	7.49	1.95

TABLE A2. Continued

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	r_y (in.)
16WF78	78.0	22.92	16.32	1042.6	87.5	127.8	145.5	6.74	1.95
10WF112	112.0	32.92	11.38	718.7	235.4	126.3	147.5	4.67	2.67
12WF99	99.0	29.09	12.75	858.5	278.2	134.7	151.8	5.43	3.09
18WF77	77.0	22.63	18.16	1286.8	88.6	141.7	160.5	7.54	1.98
12WF106	106.0	31.19	12.88	930.7	300.9	144.5	163.4	5.46	3.11
16WF88	88.0	25.87	16.16	1222.6	185.2	151.3	169.0	6.87	2.67
18WF85	85.0	24.97	18.32	1429.9	99.4	156.1	177.6	7.57	2.00
12WF120	120.0	35.31	13.12	1071.7	345.1	163.4	186.4	5.51	3.13
21WF82	82.0	24.10	20.86	1752.4	89.6	168.0	191.6	8.53	1.93
18WF96	96.0	28.22	18.16	1674.7	206.8	184.4	206.0	7.70	2.71
18WF105	105.0	30.86	18.32	1852.5	231.0	202.2	226.5	7.75	2.73
18WF114	114.0	33.51	18.48	2033.8	255.6	220.1	247.9	7.79	2.76
21WF112	112.0	32.93	21.0	2620.6	289.7	249.6	278.0	8.92	2.96

TABLE A2. Continued

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	r_y (in.)
14WF158	158.0	46.47	15.00	1900.6	745.0	253.4	286.3	6.40	4.00
14WF167	167.0	49.09	15.12	2020.8	790.2	267.3	302.9	6.42	4.01
24WF110	110.0	32.36	24.16	3315.0	229.1	274.4	307.7	10.12	2.66
12WF190	190.0	55.86	14.38	1892.5	589.7	263.2	311.5	5.82	3.25
21WF127	127.0	37.34	21.24	3017.2	338.6	284.1	317.8	8.99	3.01
24WF120	120.0	35.29	24.31	3635.3	254.0	299.1	336.6	10.15	2.68
21WF142	142.0	41.76	21.46	3403.1	385.9	317.2	357.0	9.03	3.04
14WF202	202.0	59.39	15.63	2538.8	979.7	324.9	373.6	6.54	4.06
24WF145	145.0	42.62	24.49	4561.0	434.3	372.5	416.0	10.34	3.19
24WF160	160.0	47.04	24.72	5110.3	492.6	413.5	463.7	10.45	3.23
14WF264	264.0	77.63	16.50	3526.0	1331.2	427.4	502.4	6.74	4.14
27WF160	160.0	47.04	27.08	6018.6	458.0	444.5	504.3	11.31	3.12
14WF287	287.0	84.37	16.81	3912.1	1466.5	465.5	551.6	6.81	4.17

TABLE A2. Continued

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	MOMENT OF INERTIA I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	RADIUS OF GYRATION r_y (in.)
27WF177	177.0	52.10	27.31	6728.6	518.9	492.8	556.9	11.36	3.16
33WF152	152.0	44.71	33.50	8147.6	256.1	486.4	558.3	13.36	2.39
30WF172	172.0	50.65	29.88	7891.5	550.1	528.2	593.0	12.48	3.30
30WF190	190.0	55.90	30.12	8825.9	624.6	586.1	659.6	12.57	3.34
30WF210	210.0	61.78	30.38	9872.4	707.9	649.9	733.9	12.64	3.38
14WF398	398.0	116.98	18.31	6013.7	2169.7	656.9	803.0	7.17	4.31
33WF240	240.0	70.52	33.50	13585.1	874.3	811.1	918.2	13.88	3.52
36WF230	230.0	67.73	35.88	14988.4	870.9	835.5	942.7	14.88	3.59
36WF280	280.0	82.32	36.50	18819.3	1127.5	1031.2	1167.0	15.12	3.70
36WF300	300.0	88.17	36.72	20290.2	1225.2	1105.1	1255.0	15.17	3.73

TABLE A3. ECONOMY COLUMN SECTIONS.

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	r_y (in.)
6WF20	20.0	5.90	6.20	41.7	13.3	13.4	15.0	2.66	1.50
8WF24	24.0	7.06	7.93	82.5	18.2	20.8	23.1	3.42	1.61
8WF28	28.0	8.23	8.06	97.8	21.6	24.3	27.1	3.45	1.62
8WF31	31.0	9.12	8.00	109.7	37.0	27.4	30.9	3.47	2.01
8WF35	35.0	10.30	8.12	126.5	42.5	31.1	34.7	3.50	2.03
10WF39	39.0	11.48	9.94	209.7	44.9	42.2	47.0	4.27	1.98
12WF40	40.0	11.77	11.94	310.1	44.1	51.9	57.6	5.13	1.94
14WF43	43.0	12.65	13.68	429.0	45.1	62.7	69.7	5.82	1.89
14WF48	48.0	14.11	13.81	484.9	51.3	70.2	78.5	5.86	1.91
14WF53	53.0	15.59	13.94	542.1	57.5	77.8	87.1	5.90	1.92
12WF58	58.0	17.06	12.19	476.1	107.4	78.1	86.5	5.28	2.51
14WF61	61.0	17.94	13.91	641.5	107.3	92.2	102.4	5.98	2.45

TABLE A3. Continued									
SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA		SECTION MODULUS	PLASTIC MODULUS	RADIUS OF GYRATION	
				I_x (in.^{4})	I_y (in.^{4})	S_x (in.^{3})	Z_x (in.^{3})	r_x (in.)	r_x (in.)
14WF74	74.0	21.76	14.19	796.8	133.5	112.3	125.6	6.05	2.48
14WF78	78.0	22.94	14.06	851.2	206.9	121.1	134.0	6.09	3.00
12WF79	79.0	23.22	12.38	663.0	216.4	107.1	119.3	5.34	3.05
14WF84	84.0	24.71	14.18	928.4	225.5	130.9	145.4	6.13	3.02
12WF99	99.0	29.09	12.75	858.5	278.2	134.7	151.8	5.43	3.09
14WF111	111.0	32.65	14.37	1266.5	454.9	176.3	196.0	6.23	3.73
14WF119	119.0	34.99	14.50	1373.1	491.8	189.4	210.9	6.26	3.75
14WF127	127.0	37.33	14.62	1476.7	527.6	202.0	225.9	6.29	3.76
14WF136	136.0	39.98	14.75	1593.0	567.7	216.0	242.7	6.31	3.77
14WF142	142.0	41.85	14.75	1672.2	660.1	226.7	254.8	6.32	3.97
14WF150	150.0	44.08	14.88	1786.9	702.5	240.2	270.2	6.37	3.99
14WF158	158.0	46.47	15.00	1900.6	745.0	253.4	286.3	6.40	4.00
14WF167	167.0	49.09	15.12	2020.8	790.2	267.3	302.9	6.42	4.01
14WF176	176.0	51.73	15.25	2149.6	837.9	281.9	321.3	6.45	4.02

TABLE A3. Continued									
SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	r_y (in.)
14WF184	184.0	54.07	15.38	2274.8	882.7	295.8	337.5	6.49	4.04
14WF193	193.0	56.73	15.50	2402.4	930.1	310.0	355.1	6.51	4.05
14WF202	202.0	59.39	15.63	2538.8	979.7	324.9	373.6	6.54	4.06
14WF211	211.0	62.07	15.75	2671.4	1028.6	339.2	391.7	6.56	4.07
14WF219	219.0	64.36	15.87	2798.2	1073.2	352.6	408.0	6.59	4.08
14WF228	228.0	67.06	16.00	2942.4	1124.8	367.8	427.2	6.62	4.10
14WF237	237.0	69.69	16.12	3080.9	1174.8	382.2	445.4	6.65	4.11
14WF246	246.0	72.33	16.25	3228.9	1226.6	397.4	464.5	6.68	4.12
14WF264	264.0	77.63	16.50	3526.0	1331.2	427.4	502.4	6.74	4.14
14WF287	287.0	84.37	16.81	3912.1	1466.5	465.5	551.6	6.81	4.17
14WF314	314.0	92.30	17.19	4399.4	1631.4	511.9	611.5	6.90	4.20

TABLE A3. Continued

SECTION	Wt./Ft. (lb/ft)	AREA (in.^{2})	DEPTH (in.)	MOMENT OF INERTIA I_x (in.^{4})	MOMENT OF INERTIA I_y (in.^{4})	SECTION MODULUS S_x (in.^{3})	PLASTIC MODULUS Z_x (in.^{3})	RADIUS OF GYRATION r_x (in.)	RADIUS OF GYRATION r_y (in.)
14WF320	320.0	94.12	16.81	4141.7	1635.1	492.8	592.2	6.63	4.17
14WF342	342.0	100.59	17.56	4911.5	1806.9	559.4	673.0	6.99	4.24
14WF370	370.0	108.78	17.94	5454.2	1986.0	608.1	737.3	7.08	4.27
14WF398	398.0	116.98	18.31	6013.7	2169.7	656.9	803.0	7.17	4.31
14WF426	426.0	125.25	18.69	6610.3	2359.5	707.4	869.3	7.26	4.34
14WF455*	455.0	133.73	19.05	7214.9	2561.2	757.5	986.0	7.35	4.38
14WF500*	500.0	146.95	19.63	8234.1	2882.7	839.1	1099.0	7.48	4.43
14WF550*	550.0	161.75	20.26	9443.1	3256.7	932.2	1241.0	7.64	4.49
14WF605*	605.0	177.85	20.94	10842.3	3680.9	1035.7	1403.0	7.81	4.55
14WF665*	665.0	195.51	21.67	12477.7	4166.2	1151.7	1567.0	7.99	4.62
14WF730*	730.0	214.65	22.44	14371.4	4716.8	1280.6	1770.0	8.18	4.69

* = "USS Shapes and Plates"

TABLE A4. EQUAL LEG DOUBLE ANGLE BRACING SECTIONS.			
AISC DESIGNATION	SECTION NAME INPUT	Wt./Ft. (lb./ft.	AREA (in.^{2})
$2\text{x}2\text{x}\frac{1}{4}$	2AN6	6.38	1.88
$2\text{x}2\text{x}\frac{5}{16}$	2AN7	7.84	2.30
$2\frac{1}{2}\text{x}2\frac{1}{2}\text{x}\frac{5}{16}$	2.5AN10	10.0	2.94
$3\text{x}3\text{x}\frac{3}{8}$	3 AN14	14.4	4.22
$3\frac{1}{2}\text{x}3\frac{1}{2}\text{x}\frac{3}{8}$	3.5AN17	17.0	4.96
$4\text{x}4\text{x}\frac{3}{8}$	4AN19	19.6	5.72
$3\frac{1}{2}\text{x}3\frac{1}{2}\text{x}\frac{1}{2}$	3.5AN22	22.2	6.50
$4\text{x}4\text{x}\frac{1}{2}$	4AN25	25.6	7.50
$5\text{x}5\text{x}\frac{7}{16}$	5AN28	28.6	8.36
$5\text{x}5\text{x}\frac{1}{2}$	5AN32	32.4	9.50
$6\text{x}6\text{x}\frac{7}{16}$	6AN34	34.4	10.12
$6\text{x}6\text{x}\frac{1}{2}$	6AN39	39.2	11.50
$8\text{x}8\text{x}\frac{1}{2}$	8AN52	52.8	15.50
$6\text{x}6\text{x}\frac{3}{4}$	6AN57	57.4	16.88
$8\text{x}8\text{x}\frac{5}{8}$	8AN65	65.4	19.22

TABLE A4. Continued			
AISC DESIGNATION	SECTION NAME INPUT	Wt./Ft. (lb./ft.)	AREA (in.^{2})
6x6x1	6AN74	74.8	22.00
8x8x$\frac{7}{8}$	8AN90	90.0	26.46
8x8x1	8AN102	102.0	30.00
8x8x$\frac{9}{8}$	8AN113	113.8	33.46

TABLE A5. UNEQUAL LEG DOUBLE ANGLE BRACING SECTIONS.			
AISC DESIGNATION	SECTION NAME INPUT	Wt./Ft. (lb./ft.)	AREA (in.^{2})
$3 \times 2 \times \frac{3}{16}$	3UAN6.1	6.1	1.80
$3 \times 2\frac{1}{2} \times \frac{1}{4}$	3UAN9.0	9.0	2.62
$4 \times 3 \times \frac{1}{4}$	4UAN11.6	11.6	3.38
$3 \times 2\frac{1}{2} \times \frac{3}{8}$	3UAN13.2	13.2	3.84
$4 \times 3 \times \frac{5}{16}$	4UAN14.4	14.4	4.18
$4 \times 3\frac{1}{2} \times \frac{5}{16}$	4UAN15.4	15.4	4.50
$4 \times 3 \times \frac{3}{8}$	4UAN17.0	17.0	4.96
$4 \times 3\frac{1}{2} \times \frac{3}{8}$	4UAN18.2	18.2	5.34
$4 \times 3 \times \frac{7}{16}$	4UAN19.6	19.6	5.74
$4 \times 3\frac{1}{2} \times \frac{7}{16}$	4UAN21.2	21.2	6.18
$4 \times 3 \times \frac{1}{2}$	4UAN22.2	22.2	6.50
$6 \times 3\frac{1}{2} \times \frac{3}{8}$	6UAN23.4	23.4	6.84
$6 \times 4 \times \frac{3}{8}$	6UAN24.6	24.6	7.22
$5 \times 3 \times \frac{1}{2}$	5UAN25.6	25.6	7.50
$5 \times 3\frac{1}{2} \times \frac{1}{2}$	5UAN27.2	27.2	8.00

TABLE A5. Continued

AISC DESIGNATION	SECTION NAME INPUT	Wt./Ft. (lb./ft.)	AREA (in.^{2})
6x4x$\frac{7}{16}$	6UAN28.6	28.6	8.36
7x4x$\frac{7}{16}$	7UAN31.6	31.6	9.24
8x4x$\frac{7}{16}$	8UAN34.4	34.4	10.12
8x4x$\frac{1}{2}$	8UAN39.2	39.2	11.50
8x4x$\frac{3}{4}$	8UAN57.4	57.4	16.88

APPENDIX B

FRAME B LOADING DATA

General design data applicable to all Frame B example problems are presented in Section 2.1. Details of the applied loading also applicable to all Frame B example problems are presented here as follows.

i. Wind Load INPUT

Top story (Level 1):	2.88 kips
General story (levels 2-9):	5.76 kips
Bottom story (level 10):	6.33 kips

ii. Girder Loads.

Roof girders (Level 1):

L.L. = 0.03 ksf x 24 ft.	= 0.72 k/ft.
D.L. = 0.06 ksf x 24 ft.	= 1.44 k/ft.
Total INPUT to program	= 2.16 k/ft.

Floor girders (Levels 2-10):

Percent L.L. reduction by ASA A58.1:

Bays 1,2,3: 46.1%

L.L. = (1-0.461) x 0,08 ksf x 24 ft.	= 1,03 k/ft.
D.L. = 0,08 ksf x 24 ft.	= 1,92 k/ft.
Total INPUT to program	= 2,95 k/ft.

iii. Column Loads.

Wall loads on exterior columns (Levels 2-10) = 13.0 kips

Dead weight of columns (average) = 0.20 k/ft.

Dead weight of fireproofing = 0.05 k/ft.

Percent L.L. reduction by ASA A58.1:

Story 1, Col. A,B,C,D	=	0.0%
Story 2, Col. A	=	2.8%
B,C	=	46.1%
D	=	23.0%
Stories 3-10, Col. A,B,C,D	=	46.1%

A special procedure must now be used to determine the actual applied joint loads to be INPUT to the computer program. Chapter 3 describes the method used to calculate the gravity load condition column axial forces from the joint and girder loads INPUT to the computer program. In effect, the method used distributes one-half the girder load to each joint connected by the girder and sums the total joint loads above the column. However, when the live load reduction coefficient for the beams and columns are not all equal, the applied joint loads INPUT to the computer program should take on values that will account for the differences in live load reduction coefficients. Consequently, the joint loads to be INPUT are calculated as follows. Let $F_C(i-1,j)$ represent a story i-1 column axial force due to gravity loads only and based on the beam live load reduction coefficients. Let $\overline{F}_C(i,j)$ represent a story i column axial force due to gravity loads only, but based on the column live load reduction coefficients. The joint load, $P_j(i,j)$ applied to the top joint of the column is simply,

$$P_j(i,j) = \overline{F}_C(i,j) - F_C(i-1,j) - \frac{W(i,j)L(j)}{2}$$

where W(i,j) is the uniform load applied to beam (i,j) using the beam live load reduction coefficient.

An example of the calculation of $P_j(2,1)$ will be shown as follows.

1. In this case, i = 2, j = 1.
2. The axial force in column (1,1) due to gravity loads only and based on the beam live load reduction coefficient is:

$$F_C(1,1) = (\tfrac{30}{2})(2.16) + (12)(.25)$$
$$= 35.4 \text{ kips}$$

3. The axial force in column (2,1) due to gravity loads only and based on the column live load reduction coefficient is:

$$\overline{F}_C(2,1) = (\tfrac{30}{2})(2.16+(1-.288)(1.92)+1.92)$$
$$+ (12)(.25+.25)+13.0$$
$$= 100.7 \text{ kips}$$

4. Thus, joint load (2,1) INPUT is,

$$P_j(2,1) = 100.7 - 35.40 - \frac{(2.95)(30)}{2}$$
$$= \underline{\underline{21.05}} \text{ kips}$$

This procedure is easily tabulated by proceding joint by joint down the frame in any column line. The resulting joint loads INPUT for Frame B are as follows.

		Joint A	Joint B	Joint C	Joint D
Level 1	:	3.0 k.	3.0 k.	3.0 k.	3.0 k.
Level 2	:	21.05	3.25	3.20	21.40
Level 3	:	11.05	3.15	3.20	10.80
Levels 4-9	:	16.05	3.20	3.20	16.10
Level 10	:	16.80	3.95	3.95	16.85

APPENDIX C

FRAME C LOADING DATA

General design data applicable to all Frame C example problems are presented in Section 2.2. Details of the applied loading also applicable to all Frame C example problems are presented here as follows.

i. Wind Load INPUT.

Top story (Level 1):	4.8 kips
General story (Levels 2-24):	5.76 kips

ii. Girder Loads.

Roof Girders (Level 1):

L.L. = 0.030 ksf x 24 ft.	=	0.72 k/ft.
D.L. = 0.095 ksf x 24 ft	=	2.28 k/ft.
Total INPUT to program	=	3.00 k/ft.

Floor Girders (Levels 2-24):

Percent L.L. reduction by ASA A58.1

Bay 1 : 38.4%

Bay 2 : 23.0%

Bay 3 : 50.9%

Uniformly distributed loads on floor girders;

Bay 1:	L.L. = (1-0.384) x 0.100 ksf x 24 ft.	=	1.48 k/ft.
	D.L. = 0.120 ksf x 24 ft.	=	2.88 k/ft.
Total INPUT to program		=	4.36 k/ft.

Bay 2:	L.L. = (1-0.230) x 0.100 ksf x 24 ft.	=	1.85 k/ft.
	D.L. = 0.120 ksf x 24 ft.	=	2.88 k/ft.
Total INPUT to program		=	4.73 k/ft.
Bay 3:	L.L. = (1-0.509)x 0.100 ksf x 24 ft.	=	1.18 k/ft.
	D.L. = 0.100 ksf x 24 ft.	=	2.88 k/ft.
Total INPUT to program		=	4.06 k/ft.

iii. Column Loads.

Wall loads on exterior columns:

Level 1 (4 ft. parapet wall) = 8.2 kips

Levels 2-24 = 24.5 kips

Estimated dead load of column plus fireproofing (average) = 0.625 k/ft.

Percent L.L. reduction by ASA A58.1:

Story 1, Col.	A,B,C,D	=	0.0%
Story 2, Col.	A	=	19.2%
	B	=	30.7%
	C	=	38.4%
	D	=	26.9%
Story 3, Col.	A	=	38.4%
	B,C,D	=	50.9%

Stories 4-24, Col. A,B,C,D = 50.9%

The procedure used to calculate the actual applied joint loads to be INPUT to the computer program and to account for the difference between beam and column live load reduction coefficients is the same as the one described for Frame B in Appendix B. Consequently, only the final results will be presented. So, the joint loads INPUT for Frame C are as follows.

	Joint A	Joint B	Joint C	Joint D
Level 1 :	15.7 k.	7.5 k.	7.5 k.	15.7 k.
Level 2 :	36.6	8.22	9.48	40.06
Level 3 :	27.4	-7.38	-2.52	23.86
Level 4 :	23.0	0.42	3.48	31.96
Levels 5-24 :	29.0	0.42	3.48	31.96

APPENDIX D

DESCRIPTION OF COMPUTER PROGRAM INPUT FORMAT

The following description of the computer program INPUT format assumes the reader is familiar with the Fortran IV Read statement and FORMAT statement. The INPUT is described in the order they are required to appear.

1. READ IRD, IWR
   FORMAT (2I10)

   Where, IRD = the read code associated with the computer card reading device.

   IWR = the write code associated with the computer printer output device.

2. READ NPROB
   FORMAT (I10)

   Where, NPROB = the total number of problems to be input (NPROB $\geq$ 1).

3. READ LN, LPR1, LPR
   FORMAT (3I10)

   Where, LN = maximum number of plastic design cycles (LN $\geq$ 1).

LPR1 = 1 = maximum amount of output for each design part.

= 0 = minimum amount of output for each design part.

LPR = 1 = maximum amount of debugging output.

= 0 = no debugging output.

The following values are recommended for normal problem execution:

LN $\geq$ 2

LPR1 = 1

LPR = 0

4. READ (TITLE(I), I=1, 18)
FORMAT (18A4)

Where, TITLE = a maximum of 72 alphanumeric characters composing the title of the current problem.

5. READ NB, NBT
FORMAT (2I10)

Where, NB = number of economy beam sections to be input.

NBT = total number of beam sections to be input.

Note that when NBT = NB, no non-economy beams are to be input.

Note also that NB $\geq$ 3 and 3 $\leq$ NBT $\leq$ 250.

6. For each beam section to be input (I=1 to NBT):

READ BMID1(I), BMID2(I), WFWB(I),
WFAB(I), WFDB(I), WFIXB(I),
WFIYB(I), WFSB(I), WFZB(I),

WFRXB(I), WFRYB(I)
FORMAT (2A4, F8.1, F6.2, F6.2,
2F11.1, 2F8.1, 2F7.2)

Where, BMID1 = first four alphanumeric characters in the beam name.

BMID2 = last four alphanumeric characters in the beam name.

WFWB = beam weight (lb./ft.).

WFAB = beam area ($in.^2$).

WFDB = beam depth (in.)

WFIXB = beam major axis moment of inertia ($in.^4$).

WFIYB = beam minor axis moment of inertia ($in.^4$).

WFSB = beam elastic section modulus ($in.^3$).

WFZB = beam plastic section modulus($in.^3$).

WFRXB = beam major axis radius of gyration (in.).

WFRYB = beam minor axis radius of gyration (in.).

Note the first NB beam sections input (economy beams), are ordered on increasing area. The next NBT-NB beam sections (non-economy beam sections) are ordered on increasing plastic section modulus. Also note that since NBT may equal NB, the non-economy beam sections are optional.

7. READ NC, NCT
FORMAT (2I10)

Where, NC = number of economy column sections to be input.

NCT = total number of column sections to be input.

Note that when NCT = NC, no non-economy columns are to be input.

Note also that NC $\geq$ 3 and 3 $\leq$ NCT $\leq$ 90.

8. For each column section to be input (I=1 to NCT):

   Input the same type data for the columns as for beams and in the same format as for Beams.

   All columns input are ordered on increasing area.

9. READ NBR
   FORMAT (I10)

   Where, NBR = number of bracing sections to be input.

   Note that 1 $\leq$ NBR $\leq$ 20.

10. For each bracing section to be input (I=1 to NBR):

    READ BRID1(I). BRID2(I), WFWBR(I), WFABR(I).
    FORMAT (2A4, F8.1, F8.2)

    Where, BRID1 = first four alphanumeric characters in the brace name.

    BRID2 = last four alphanumeric characters in the brace name.

    WFWBR = brace weight (lb./ft.)

    WFABR = brace area ($in.^2$).

11. READ NSTRY, NBAY
    FORMAT (2I10)

    Where, NSTRY = number of stories

    NBAY = number of bays.

Note that $2 \leq$ NSTRY ≤ 30
$2 \leq$ NBAY ≤ 5.

12. READ (RL(J), J=1, NBAY)
FORMAT (8F10.3)

Where, RL(J) = length of bay J, (in.).

13. READ (RH(I), I=1, NSTRY)
FORMAT (8F10.3)

Where, RH(I) = height of story I, (in.).

14. READ((PW(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)

Where, PW(I,J) = uniformly applied unfactored gravity load applied to beam (I,J), (kips/in.).

15 READ ((PJVD(I,J), J=1, N1), I=1, NSTRY)
FORMAT (8F10.3)

Where, PJVD(I,J) = applied unfactored concentrated gravity load applied to joint (I,J), (kips).

N1 = NBAY + 1

16. READ (PH(I), I=1, NSTRY)
FORMAT (8F10.3)

Where PH(I) = lateral unfactored wind load applied to story level I, (kips).

17. READ (DA(I), I=1, NSTRY)
FORMAT (8F10.3)

Where, DA(I) = initially assumed relative story I deflection at the ultimate load (i.e. initial Δ of the P-Δ effect) in inches.

18. READ (DP(I), I=1, NSTRY)
FORMAT (8F10.3)

Where, DP(I) = maximum permissible elastic relative story I deflection in inches for unfactored loads.

19. READ RLD1, RLD2
FORMAT(2F10.3)

Where, RLD1 = load factor for the gravity load condition (λ_1).

RLD2 = load factor for the combination gravity plus wind load condition (λ_2).

20. READ ((SYB(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)

Where, SYB(I,J) = beam (I,J) steel yield stress, (ksi.).

21. READ ((SYC(I,J), J=1, N1), I=1, NSTRY)
FORMAT (8F10.3)

Where, SYC(I,J) = column (I,J) steel yield stress, (ksi.).

N1 = NBAY + 1.

22. READ ((TSYBR(I,J), J=1, NBAY),I=1, NSTRY)
FORMAT (8F10.3)

Where, TSYBR(I,J) = steel yield stress of the pair of diagonal braces in panel (I,J) assuming braces were permitted, (ksi.).

23. READ ((UCB(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)

Where, UCB(I,J) = unit material cost (cents/lb.) corresponding to the beam (I,J) steel type.

24. READ ((UCC(I,J), J=1, N1), I=1, NSTRY)
FORMAT (8F10.3)

Where, UCC(I,J) = unit material cost (cents/lb.) corresponding to the column (I,J) steel type.

N1 = NBAY + 1.

25. READ ((TUCBR(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)

Where, TUCBR(I,J) = unit material cost (cents/lb.) corresponding to the steel used for the pair of braces in panel (I,J) assuming braces were permitted.

26. READ ICDEB
FORMAT (I10)

Where, ICDEB = flag indicating whether or not the maximum laterally unsuported beam (I,J) length, RLYB(I,J), is to be specified for all beams (0=no; 1=yes).

If ICDEB = 0, RLYB(I,J) is set to RL(J) for all beams. GO TO 27.

If ICDEB = 1, GO TO 26a.

26a. This data input only when ICDEB = 1.

READ ((RLYB(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)

Where, RLYB(I,J) = maximum laterally unsupported beam (I,J) length, (in.).

27. READ ICDEC
FORMAT (I10)

Where, ICDEC = flag indicating whether or not the maximum laterally unsupported column (I,J) length, RLYC(I,J), is to be specified for all columns (0=no; 1=yes).

If ICDEC = 0, RLYC(I,J) is set to RH(I) for all columns. GO TO 28.

If ICDEC = 1, GO TO 27a.

27a. This data input only when ICDEC = 1:

READ ((RLYC(I,J), J=1, N1), I=1, NSTRY)
FORMAT (8F10.3)

Where, RLYC(I,J) = maximum laterally unsupported column (I,J) length (in.).

N1 = NBAY + 1.

28. READ IMAXD
FORMAT (I10)

Where, IMAXD = flag indicating whether or not maximum permissible beam (I,J) depth, BMAXD(I,J), is to be specified for all beams (0=no; 1=yes).

If IMAXD = 0, BMAXD(I,J) is set to 10000.0 in. for all beams.

GO TO 29.

If IMAXD = 1, GO TO 28a.

28a. This data input only when IMAXD = 1:

READ ((BMAXD(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)

Where, BMAXD(I,J) = maximum permissible depth of beam (I,J), (in.).

29. READ JMAXD
FORMAT (I10)

Where, JMAXD = flag indicating whether or not maximum permissible column (I,J) depth, CMAXD(I,J), is to be specified for all columns (0=no; 1=yes).

If JMAXD = 0, CMAXD(I,J) is set to 10000.0 in. for all columns.

GO TO 30.

If JMAXD = 1, GO TO 29a.

29a. This data input only when JMAXD = 1:

READ ((CMAXD(I,J), J=1, N1), I=1, NSTRY)
FORMAT (8F10.3)

Where, CMAXD(I,J) = maximum permissible depth of column (I,J) (in.)

N1 = NBAY + 1.

30. READ IBST
FORMAT (I10)

Where, IBST = flag indicating whether or not maximum permissible beam (I,J) elastic stress STBMX(I,J), is to be specified for all beams (0=no; 1=yes).

If IBST = 0, STBMX(I,J) is set to SYB(I,J) for all beams. GO TO 31.

If IBST = 1, GO TO 30a.

30a. This data input only when IBST = 1:

```
READ ((STBMX(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (8F10.3)
```

Where, STBMX(I,J) = maximum permissible beam (i,j) elastic stress (ksi.) under unfactored loads.

31. READ ICST
FORMAT (I10)

Where, ICST = flag indicating whether or not maximum permissible column (I,J) elastic stress, STCMX(I,J), is to be specified for all columns (0=no; 1=yes).

If ICST = 0, STCMX(I,J) is set to SYC(I,J) for all columns.

GO TO 32.

If ICST = 1, GO TO 31a.

31a. This data input only when ICST = 1:

```
READ ((STCMX(I,J), J=1, N1), I=1, NSTRY)
FORMAT (8F10.3)
```

Where, STCMX(I,J) = maximum permissible column (I,J) elastic stress (ksi.) under unfactored loads.

N1 = NBAY + 1.

32. READ ((IBRP(I,J), J=1, NBAY), I=1, NSTRY)
FORMAT (40I2)

Where, IBRP(I,J) = flag indicating the permissible modes of panel resistance for plastic design only.

= 1 = panel moment and truss action permitted.

= 0 = only panel moment action permitted.

=-1 = only panel truss action permitted.

= -2 = neither panel moment nor truss action permitted.

BIOGRAPHY

The author was born in Brooklyn, New York on August 15, 1943.

He entered the Georgia Institute of Technology, Atlanta, Georgia, in September, 1961 and received the degrees of Bachelor of Civil Engineering in June, 1965 and Master of Science in Civil Engineering in September, 1966.

He entered the Massachusetts Institute of Technology, Cambridge, Massachusetts, in September, 1966. During his doctoral program, he was appointed as a Research Assistant during the academic years 1966-1967, 1967-1968, and 1968-1969.

His practical experience includes a part-time position as research engineer with the Georgia Highway Department, Atlanta, Georgia, and summer positions as engineer with the Esso Research and Engineering Company, Florham Park, New Jersey; as engineer with the Humble Oil and Refining Company, Linden, New Jersey; as research engineer with the M.I.T. Division of Sponsored Research, Cambridge, Massachusetts; and as structural engineer, Jackson and Moreland Consulting Engineers, Boston, Massachusetts.

He will begin teaching and conducting research as an Assistant Professor of Civil Engineering at the Georgia Institute of Technology, Atlanta, Georgia, in October, 1969.

He is an Associate Member of the American Society of Civil Engineers. He is also a member of Chi Epsilon (Georgia Tech Chapter), Sigma Xi (M.I.T. Chapter), Tau Beta Pi (Georgia Tech Chapter), and Phi Kappa Phi (Georgia Tech Chapter).