AN EXPERIMENTAL INVESTIGATION OF OIL SPREADING OVER WATER

by

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ABSTRACT

One-dimensional experiments were performed to determine the spreading rates of oil over water. The viscosity of the water was modified to enable both gravity-inertia and gravity-viscous spreading to be observed in a 7' tank. The resulting data was in excellent agreement with Fay's order of magnitude theory, and the numerical spreading coefficients were determined.

Thesis Supervisor: David P. Hoult

Title: Associate Professor of Mechanical Engineering

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List of Symbols

d	channel depth
g	gravity, 32.2 ft./sec.^{2}
h	average oil thickness
h_o	initial oil thickness
h_F	oil thickness in front of dam prior to release
ℓ	overall slick length
ℓ_o	initial slick length
ℓ_t	slick length at transition, $= (\Delta g L^2)^{1/3}\, T_t^{2/3}$
L	length
T	time
T_t	time to transition, $= L^{8/7}/(\Delta g)^{2/7} \cdot \nu_w^{3/7}$
u	oil front velocity
V	oil volume
w	channel width
δ_w	boundary layer thickness of base fluid
Δ	percentage density difference
μ_o	absolute viscosity of oil
μ_w	absolute viscosity of base fluid
ν_w	kinematic viscosity of base fluid
ρ	density of base fluid
σ	surface tension

I. Introduction

Oil spills are the object of deep concern these days as major accidents, such as the Torrey Canyon in England and the well blowouts both in the Santa Barbara Channel and the Gulf of Mexico, have brought to the public's attention the damage resulting both to the local marine life and to the shoreline itself. A troubling fact is, however, that the threat of potential spills grows with the current construction of deep sea drilling platforms and the advent of supertankers plowing dangerous Artic waters, while our present technology is still in the state of using primitive booms or even straw to sweep up the oil.

Of vital importance in advancing this technology is a knowledge of the spreading rates of an oil slick. With this information, clean up personnel could have on hand an estimate of the area of the slick at any time after the spill occured, and thus would know what quantity of boom or chemical dispersant to deploy. Three previous papers have dealt with this problem. Abbott (1,2) deals only with the inertial spreading of the oil, and treats the advancing front like a dam break flow. He also conducted several steady state experiments. Fay (3) considers three different spreading states of the oil. The first state is like that studied by Abbott, in which the driving gravity force is balanced by inertia forces. In the second state the gravity force is balanced with viscous forces in the water. And in the last state, we have surface tension, tending to spread the oil, opposed by viscous forces.

The intent of this thesis was to verify the results of Fay for the first two spreading regimes, i.e. the gravity-inertia and the gravity-viscous. In the following pages, first the order of magnitude theory of Fay is presented for the one-dimensional flow of oil over calm water (Chapter II); next the details of the experiments conducted in the M.I.T. Fluid Mechanics Laboratory are described (Chapter III); and finally, the results for both regimes are presented (Chapter IV).

II. Theory

Here we rederive the results of Fay for a one-dimensional flow of a volume of oil released from a point source in still water. If the water is of density ρ, then the oil has density $(1 - \Delta)\rho$, where Δ is typically 1/10. Initially we assume the important forces will be the driving gravity force and the retarding inertial force, or,

$$F_i \sim F_g \tag{1}$$

Using a characteristic dimension ℓ, the slick length, and a characteristic time, T, we have,

$$F_i \sim \rho \frac{\ell}{T^2} \cdot \ell h w \tag{2}$$

and,

$$F_g \sim \{\rho - (1 - \Delta)\rho\} \; gh \cdot \ell w \tag{3}$$

where

$$h = V/\ell w \tag{4}$$

the average thickness of the oil. If we set

$$L^2 = V/w \quad , \tag{5}$$

then equation (1) reduces to,

$$\ell \sim (\Delta g L^2)^{1/3} \; T^{2/3} \quad . \tag{6}$$

This model of the first regime of spreading assumes that there exists no energy coupling between the two fluids, i.e. the oil might be considered a non-viscous fluid of density $\Delta\rho$ proprogating over a flat plate.

As the slick continues to spread, one would expect the friction or viscous forces to become important in flow retardation, or

$$F_v \sim F_g \quad . \tag{7}$$

With $\mu_o/\mu_w >> 1$, we assume that all the shear will be in the water and that the velocity profile in the oil is constant (see Fig. 1b). A characteristic y-dimension is the boundary layer thickness δ_w, which we take to be $\sqrt{\nu_w T}$. Then we have

$$F_v \sim (\rho\nu_w) \frac{\ell/T}{(\nu_w T)^{1/2}} \cdot \ell w \quad . \tag{8}$$

Combining equations (3), (8), (7), we arrive at the second spreading equation,

$$\ell \sim \frac{(\Delta g L^4)^{1/4}}{\nu_w^{1/8}} T^{3/8} \quad . \tag{9}$$

In this second regime we have assumed that there does exist a coupling between oil and water, the gravitational potential energy of the oil being dissipated by the water.

III. Experiment

A series of experiments were performed to see if the theory presented in the preceding section gives a reasonable representation of the flow. The most essential features of the theory to be verified were:

a) the power of T with which ℓ, the slick length, varies in each spreading regime,

b) the dependence of ℓ on the volume of oil, wL^2,

c) the transition from the inertial to viscous spreading regime.

The experiments consisted basically of releasing a known volume of oil dammed up at one end of a long, uniform channel and timing its progress as it moved across the water's surface. The apparatus employed was a 7' by 4' tank with 1' high glass walls along its length. A third, movable plywood wall was placed inside narrowing the tank width w and thereby reducing oil volumes and the size of the mechanical dam. This dam, located 1' from the channel's end, was an 1/8" aluminum plate mounted in guides, carefully fitted to prevent any seepage of oil and yet loose enough to allow quick manual removal to start the flow. When needed, another barrier was fixed in place behind the dam to alter the starting length ℓ_o of the pool of oil. In a typical experiment, oil was poured on the water behind the dam; the volume was then determined by taking the product of ℓ_o, w, and h_o, the initial depth of the oil pool as measured with a rule through the glass outer wall. To record the oil's movement down the channel, an X-Y chart recorder with a time base was employed. After activating the horizontal time sweep, a small blip could be left on the paper by closing a push button switch.

Dark vertical lines had been drawn on the outside glass and inside plywood tank walls at 6" intervals from the dam front; as the dam was withdrawn, releasing the oil, a timing mark was recorded, and subsequently as the oil wave front reached each 6" line (see Fig. 1).

The primary consideration in selecting the fluids appropriate for the experiment was that it be possible to conduct both gravity-inertia and gravity-viscous tests within the limitations imposed by the tank's length. It was proposed to have the transition point, i.e. the spreading distance at which viscous rather than inertial forces become important in flow retardation, occur approximately half-way down the channel, or 3.5' with $\ell_o = 1'$ and $h_o = 1''$. Since an estimate for the transition time may be obtained by finding the time at which equations (6) and (9) predict the same length,

$$T_t = L^{8/7}/(\Delta g)^{2/7} \nu_w^{3/7} \tag{10}$$

and for the transition length,

$$\ell_t = (\Delta g L^2)^{1/3} T_t^{2/3} \tag{11}$$

by increasing the viscosity ν_w of the base fluid it should be possible to induce viscous shear, and hence transition, sooner in the flow. A 75% glycerine-water solution was used, with $\mu_w = 30$ centipoises, sp.g. = 1.2. This was maintained at a 8.5" depth. A highly viscous black oil, Mobilube 900W, sp.g. = 0.9, $\mu_o = 900$ cp. was selected to insure that $\mu_o/\mu_w > 10$, so that the velocity gradients in the oil might be ignored. All viscosity measurements were made with a Brookfield rotating viscometer.

With the channel thus designed, it was possible to run either an entirely gravity-inertia experiment or a gravity-viscous experiment by setting the oil volume such that transition occurred either near the tank's far end or in the vicinity of the dam. With ℓ_o = 1', a typical h_o for the first regime was 2" and for the second regime, 0.30".

Several preliminary tests were done to determine how the flow was effected by the viscous drag of the oil on the walls. The data indicated that a channel width w = 24" was sufficiently wide enough to neglect the wall drag; this parameter was held constant for all experiments. Likewise, surface tension forces at the oil-water-air interface were minimized by, before a run, precoating the glycerine solution with a thin film of the oil, making certain that the walls, too, were wetted with oil. Wall and surface tension effects are discussed in more detail in Appendices I and II.

IV. Results

A. The Gravity-Inertia Spreading Phase. To examine the gravity-inertia section of the theory, the transition point was moved towards the channel's far end by adjusting the volume accordingly. For the largest volumes, i.e. ~ 600 in.^{3} the advancing front of the slick was not smooth, but rather turbulent as viewed on the surface. As viewed from the side, the profile of the slick was somewhat like the side view of a spoon, the head being about 2" in length and twice the apparently uniform thickness of the remaining oil behind it. This profile gradually smoothed out to give the front a prow-lie appearance (Fig. 2.a).

The data for these experiments are plotted in Fig. 3 with ℓ non-dimensionalized with L, and T with $L^{1/2}/(\Delta g)^{1/2}$, where ℓ is the total length of the slick. There is good agreement between the predicted (i.e. 2/3) and experimental slopes. The final empirical relation derived from the data is

$$\ell = 1.5\ (\Delta g L^2)^{1/3}\ T^{2/3} \quad . \tag{12}$$

B. The Gravity-Viscous Spreading Phase. With the smallest volume of oil used, 40 in.^{3}, the transition point occurred at the first marker, or 6" from the dam front. Smaller volumes were not practical since surface tension became an important factor in the flow due to the relative thinness of the oil layer (Appendix II). The front was smooth and the profile showed a sharp leading edge several inches long advancing across the water with, again, the rest of the oil behind it at seemingly uniform thickness (Fig. 2.b). Thickness measurements were not made.

Fig. 4 shows the data for these experiments plotted with ℓ non-dimensionalized with L and T with $\nu^{1/3}/(\Delta g)^{2/3}$. Here again there is good agreement with the predicted slope, i.e. 3/8. The resulting spreading function is

$$\ell = 1.5\ [(\Delta g L^4)/\nu_w^{\ 1/2}]^{1/4}\ T^{3/8} \qquad (13)$$

Thus the empirically determined coefficient in equation (9) is 1.5.

C. Transition from Inertial to Viscous Spreading Phases. In order to examine the nature of the transition, the data from both regimes had to be displayed on a single plot. It can be seen that a length and a time characteristic to both regimes are ℓ_t and T_t, the estimated transition length and time. Plotting ℓ/ℓ_t vs T/T_t, and using the empirical spreading functions, transition is predicted to occur at (1.0, 1.5), Fig. 5. The data clearly exhibit a shift in slope from 2/3 to 3/8 at this point, and in fact, the break is relatively sharp.

D. Errors. The largest error associated with the experiments was in recording the arrival times of the front over the first few intervals of the larger volume runs. The first 6" was covered in 1/2 second, as was the next interval. An estimate of the error in elapsed time due to the visual-manual response of the experimenter is 50% for the first 6" travelled and 25% for the next. This deviation decreases to negligible amounts for subsequent positions. There is also a small error induced by the finite starting length of the slick, as the theory assumes a point release. This starting effect is lost, however, soon after the flow is established. If one replots the data defining ℓ as the distance from the front of the dam rather than the total length, it can be seen that as ℓ_o is decreased, the data converges towards the theoretical line of Figs. 3, 4.

Tests were also done to verify that no secondary flows in the base solution were disturbing the progress of the oil. These were done by varying the channel depth over a range of 2 1/2" to 12", using water. Only the minimum depth slowed the oil appreciably.

Finally, the viscosity ratio was varied (Fig. 6). The consistency of the data obtained in these tests with that obtained from the runs using the glycerine solution and highly viscous oil indicates that the choice of this fluid combination was reasonable, in that it apparently induced no significant side effects. Table 1 contains a summary of the data.

V. Conclusions

An important fact about the order of magnitude theory is that it does not give any of the details of the flow. It cannot, for example, predict velocities along the slick length, nor can it distinguish between a pool of oil propagating out in one direction, x, as in these experiments, and a pool of identical volume spreading in both the x and -x directions. Thus the spreading coefficients (1.5 and 1.5 for these trials) must be determined empirically. A more complete theory by Hoult (4) indicates that the coefficient for the gravity-inertia flow is 3.0. It is not known at this time why the discrepancy arises.

One must also realize that these two regimes of spreading are, in general, only transient in that most of the spreading of a spill occurs when a transition to the third, or surface tension-viscous regime, is reached, provided the net surface tension tends to spread the oil. It turns out that for large oil spills the inertial phase lasts a few hours, whereas the viscous-gravitational phase lasts about one day. Thus it is likely that if methods to contain oil slicks are successful, they will impede the oil spread in the middle of the viscous-gravitational phase of spreading.

Appendix I

Initially it was intended to run the experiments along a 12" wide channel. However, sizeable discrepancies between the experimental and predicted slopes led one to suspect that other forces were tending to slow the spreading. It was noticed that the wave front, as it reached the end of the tank, had assumed a parabolic, tongue-like shape. This strongly suggested that a boundary layer was developing across the surface of the oil due to the shear at the walls. An estimate of when these boundary layers should meet is,

$$T = \delta^2/\nu_o$$

For δ = 15 cm, or half the channel width, this gives T = 28 sec. The runs in which this shaped front appeared, in fact, took on the order of 30 sec to reach the far end of the tank.

To eliminate these wall effects, it was proposed to widen the channel, at constant oil volume and starting length, until the experimental data converged. Runs were taken at 6, 12, 18, and 24 inch widths. The results were that the effect of increasing the width from 18 to 24 inches was sufficiently small to warrent running all the experiments at 24". (Fig. 7)

For both the gravity-inertia and the gravity-viscous experiments, a flat profile, except in the immediate vicinity of the walls, was obtained. The centerline velocity, then, of the wave front was most likely not much different from a free stream velocity in the absence of wall shear.

Appendix II

In order to insure that the primary driving forces in the spreading experiments were either, respectively, gravity-inertia or gravity-viscous, it was necessary to eliminate, or at least minimize, the surface tension at the oil-air-water interface. To accomplish this a thin layer of oil was placed along the length of the channel in front of the dam. The thickness of this layer was varied from several thousandths to a tenth of an inch. Observations revealed that the thicker films tended to retard the spread, while a maximum spreading rate was obtained for films less than .01 inches (Fig. 8). With the deeper layer in front, the released oil presumably was adsorbing a sizable amount of mass to its bulk, thereby increasing the inertia of the flow. With this in mind, it was decided to use the thin, "dirty" film for all the experiments. Though this "dirty" layer was not as noticeably homogeneous as the one 0.10" thick, the general reproducibility of experiments from day to day led one to believe that this film maintained a reasonably constant surface tension.

As to the absolute value of the surface tension, one can estimate (Ref. Fay) a critical thickness h_c at which the surface tension forces are of the same magnitude of the gravity forces, by:

$$h_c = (\sigma/\Delta\rho g)^{1/2}$$

In several small volume runs, a transition from gravity-viscous flow was observed, as evidenced by a decrease in the 3/8 slope.

This break occured at $h_c = h_1 = 0.05''$. Typical values for ocean conditions are $h_c = h_2 = 0.25''$, $\sigma_2 = 30$ dynes, $(\Delta\rho)_2 = .1\ \mathrm{gm/cm^3}$, which gives:

$$\sigma_1 = [(h\Delta\rho)_1/(h\Delta\rho)_2]^2\ \sigma_2$$

or

$$\sigma_1 \approx 3 \text{ dynes.}$$

In most cases, h_c was much less than .05 inches, if it was observed at all, which leads to the conclusion that for the experiments,

$$\sigma \leq 3 \text{ dynes,}$$

an order magnitude smaller than usual sea-like conditions.

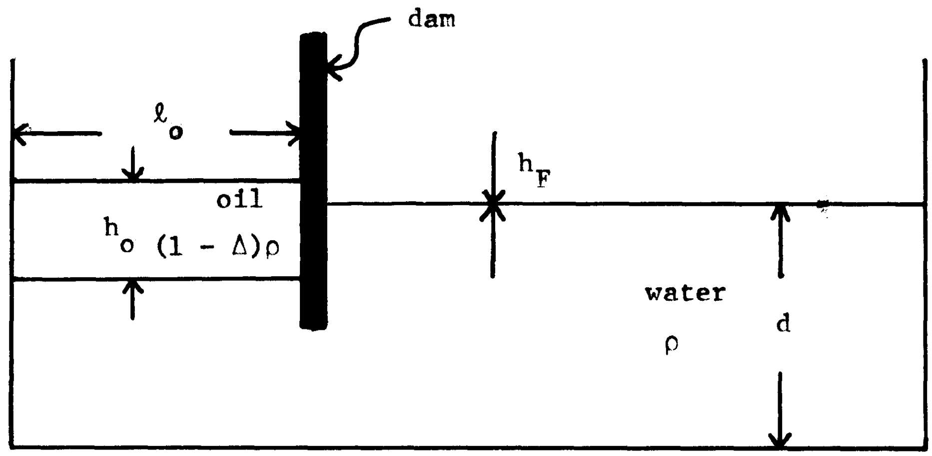
References

1. Abbott - La Houille Blanche, Dec. 1961, No. 6.

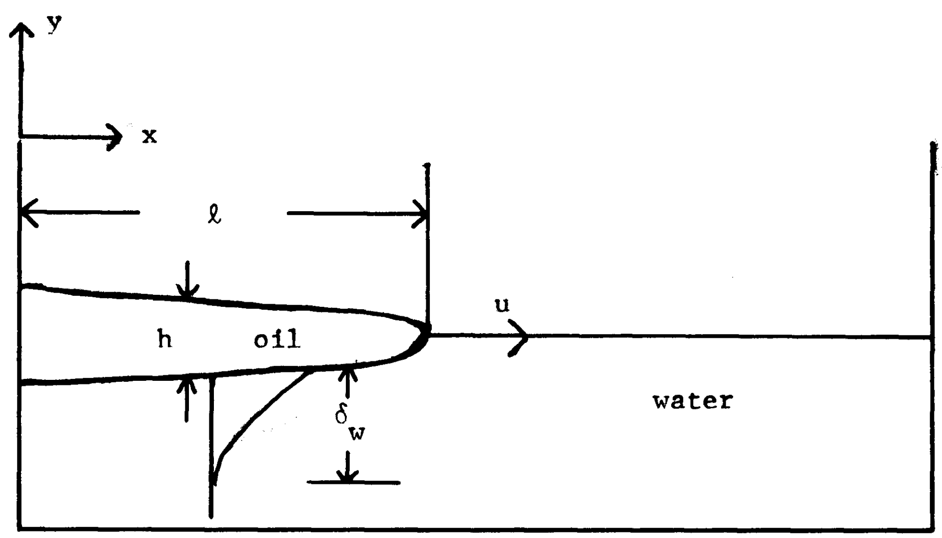
2. Abbott - La Houille Blanche, Oct. 1961, No. 5.

3. Fay - Fluid Mechanics Laboratory Publication (M.I.T.) No. 69-6.

4. Hoult - Private communication.

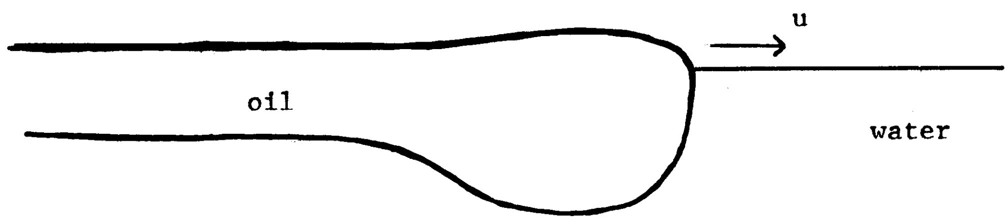


(a) Oil before release

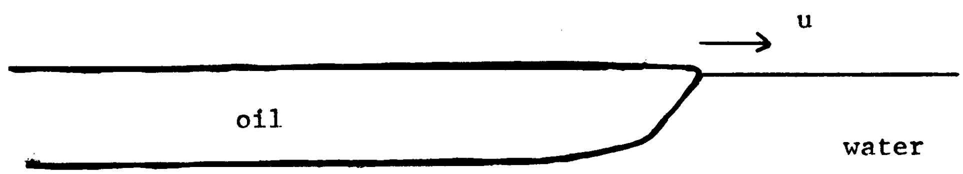


(b) Oil after release with boundary layer in water.

Fig. 1



(i)



(ii)

(a) Gravity-inertia wave front (i) early and (ii) later in flow

(b) Gravity-viscous wedge shape wave front

Fig. 2

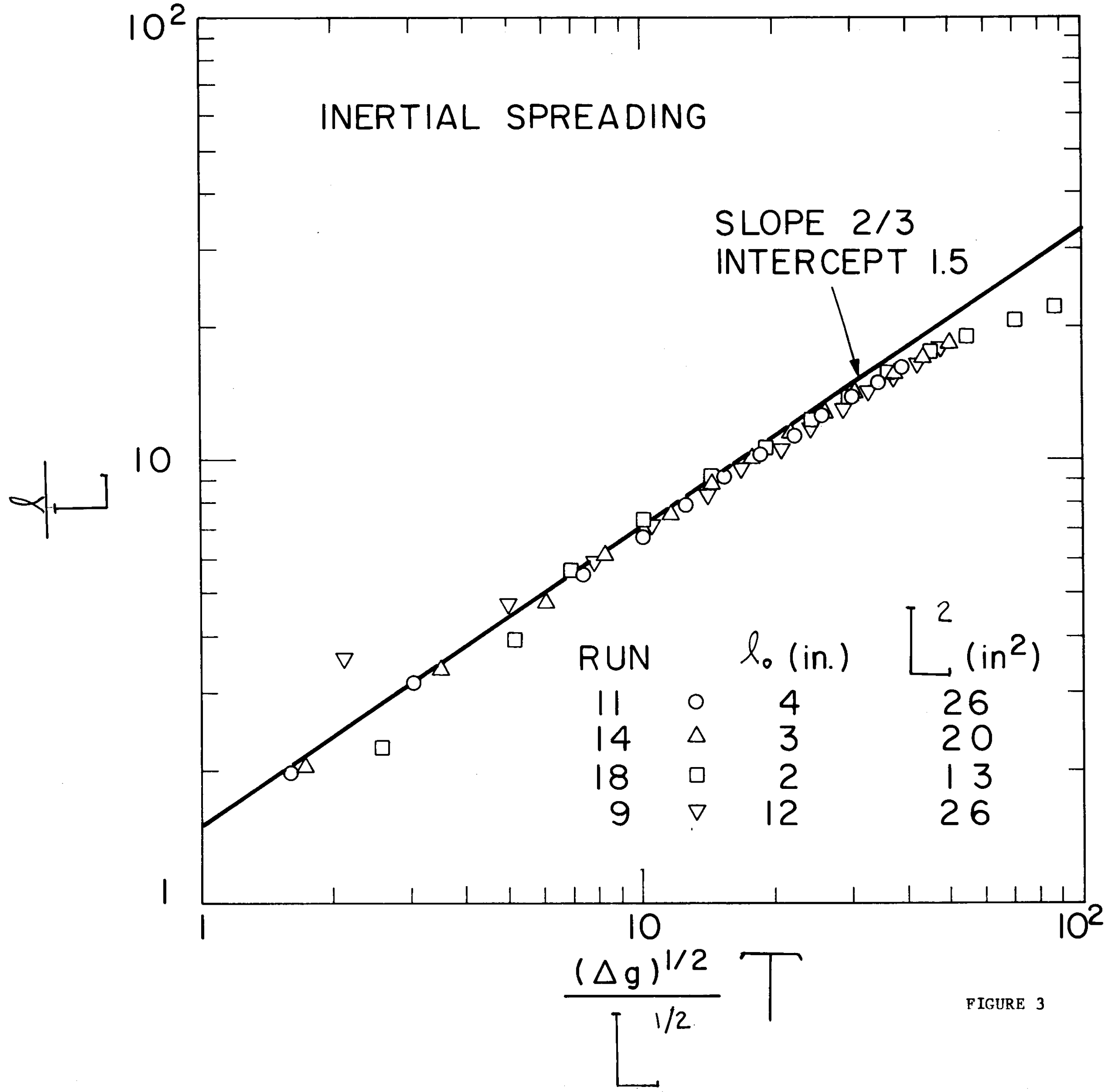

INERTIAL SPREADING
SLOPE 2/3
INTERCEPT 1.5
RUN
ℓ_o (in.)
L^2 (in^2)
11 4 26
14 3 20
18 2 13
9 12 26
ℓ / L
$\frac{(\Delta g)^{1/2}}{L^{1/2}} T$
1
10
10^2

FIGURE 3

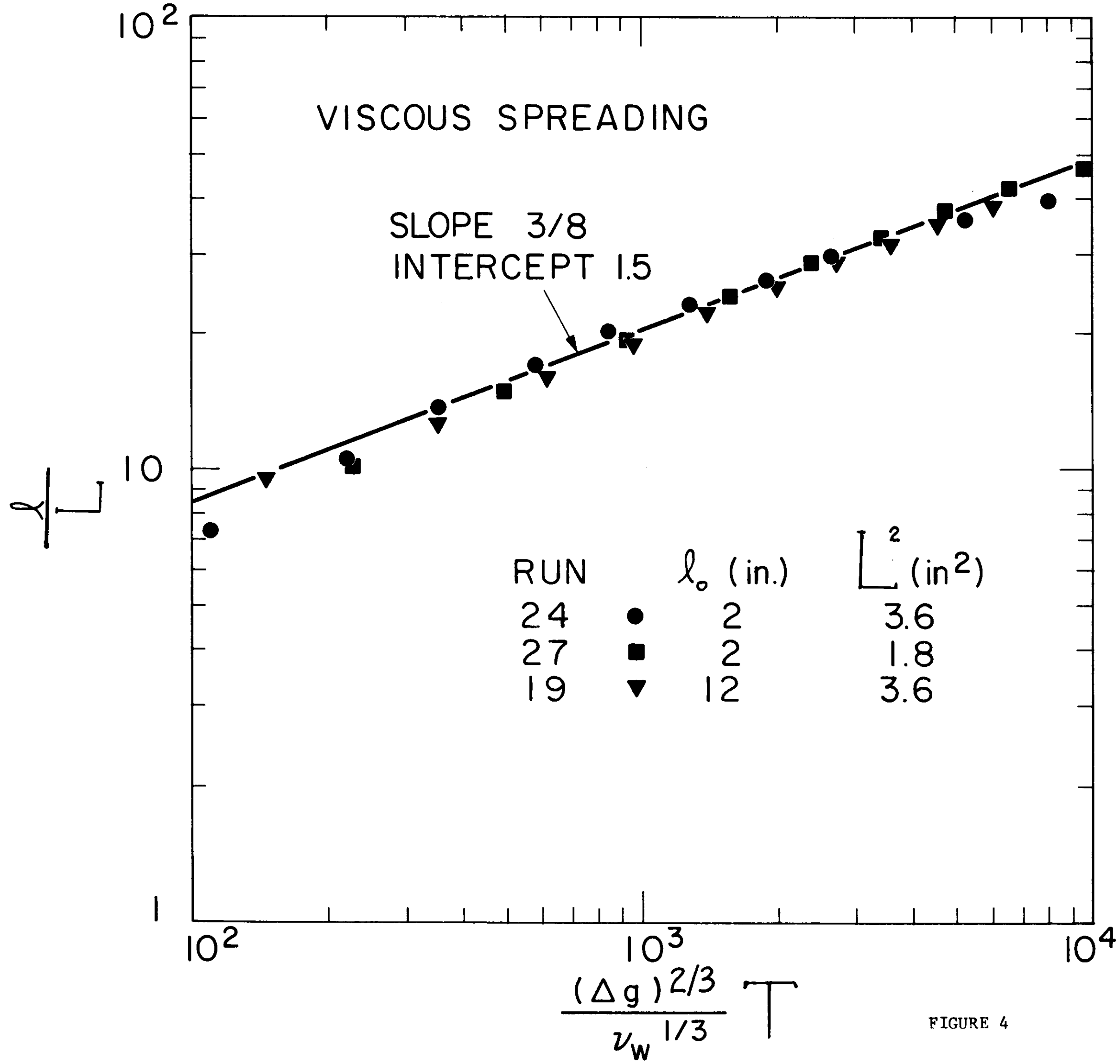

VISCOUS SPREADING
SLOPE 3/8
INTERCEPT 1.5
RUN
24
27
19
2
2
12
3.6
1.8
3.6
10²
10
1
10²
10³
10⁴

FIGURE 4

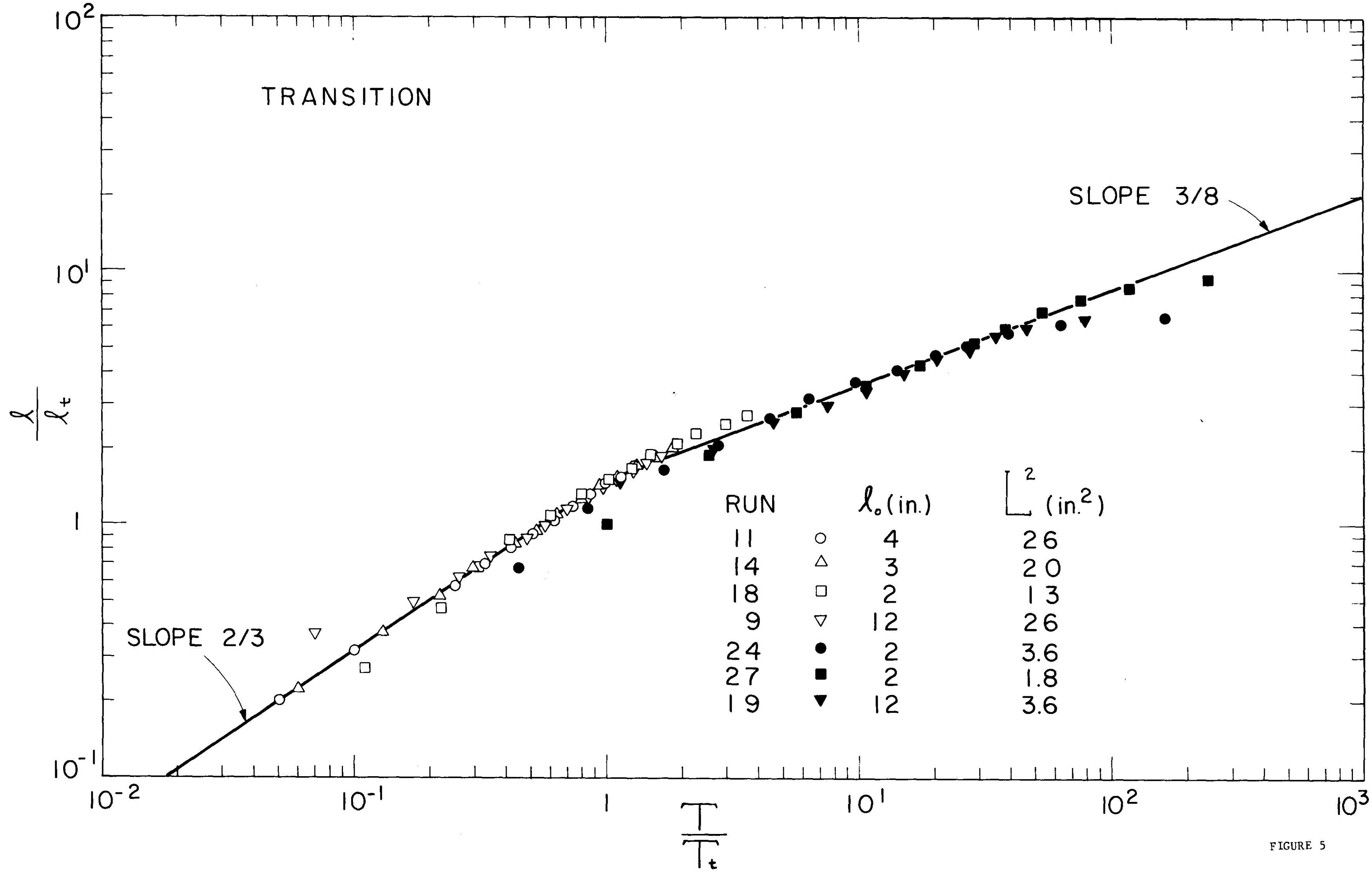

TRANSITION
SLOPE 3/8
SLOPE 2/3
RUN ℓo (in.) L2 (in.2)
11 ○ 4 26
14 △ 3 20
18 □ 2 13
9 ▽ 12 26
24 ● 2 3.6
27 ■ 2 1.8
19 ▼ 12 3.6
ℓ/ℓt
T/Tt
10^-1
1
10^1
10^2
10^-2
10^3

FIGURE 5

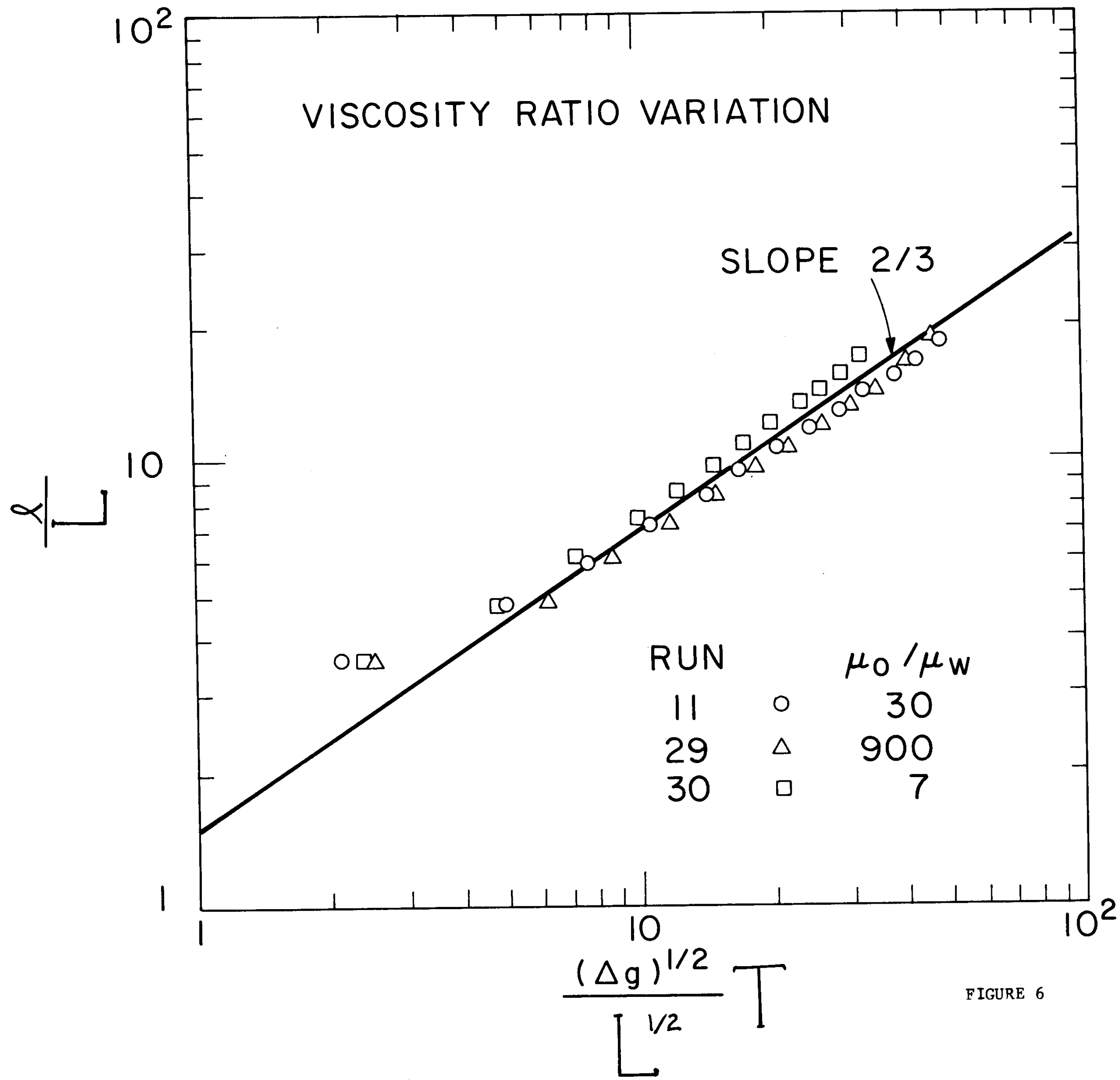

VISCOSITY RATIO VARIATION
SLOPE 2/3
RUN
μ_o / μ_w
11 30
29 900
30 7
10^2
10
1
ℓ / L
$(\Delta g)^{1/2} T / L^{1/2}$

FIGURE 6

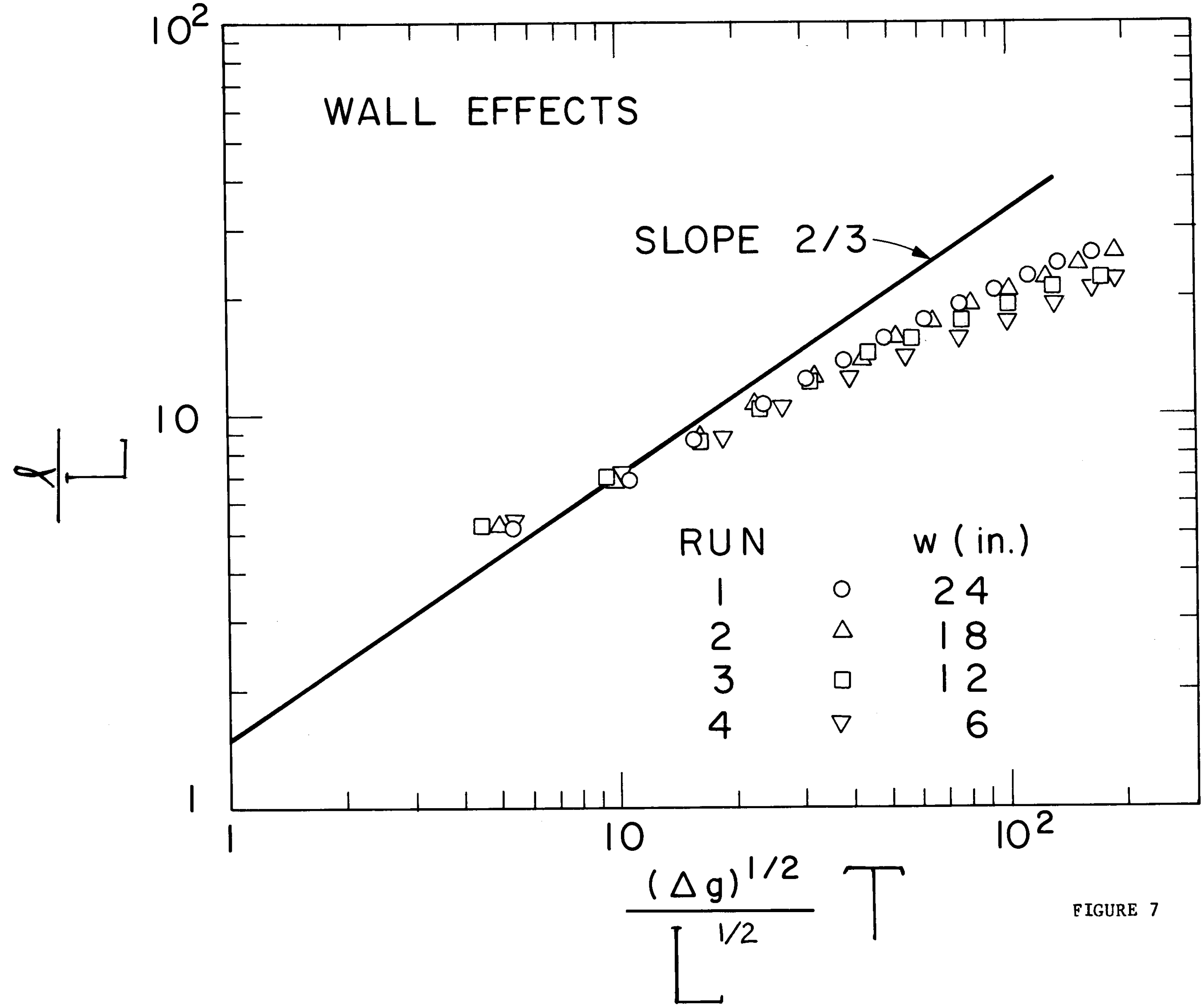
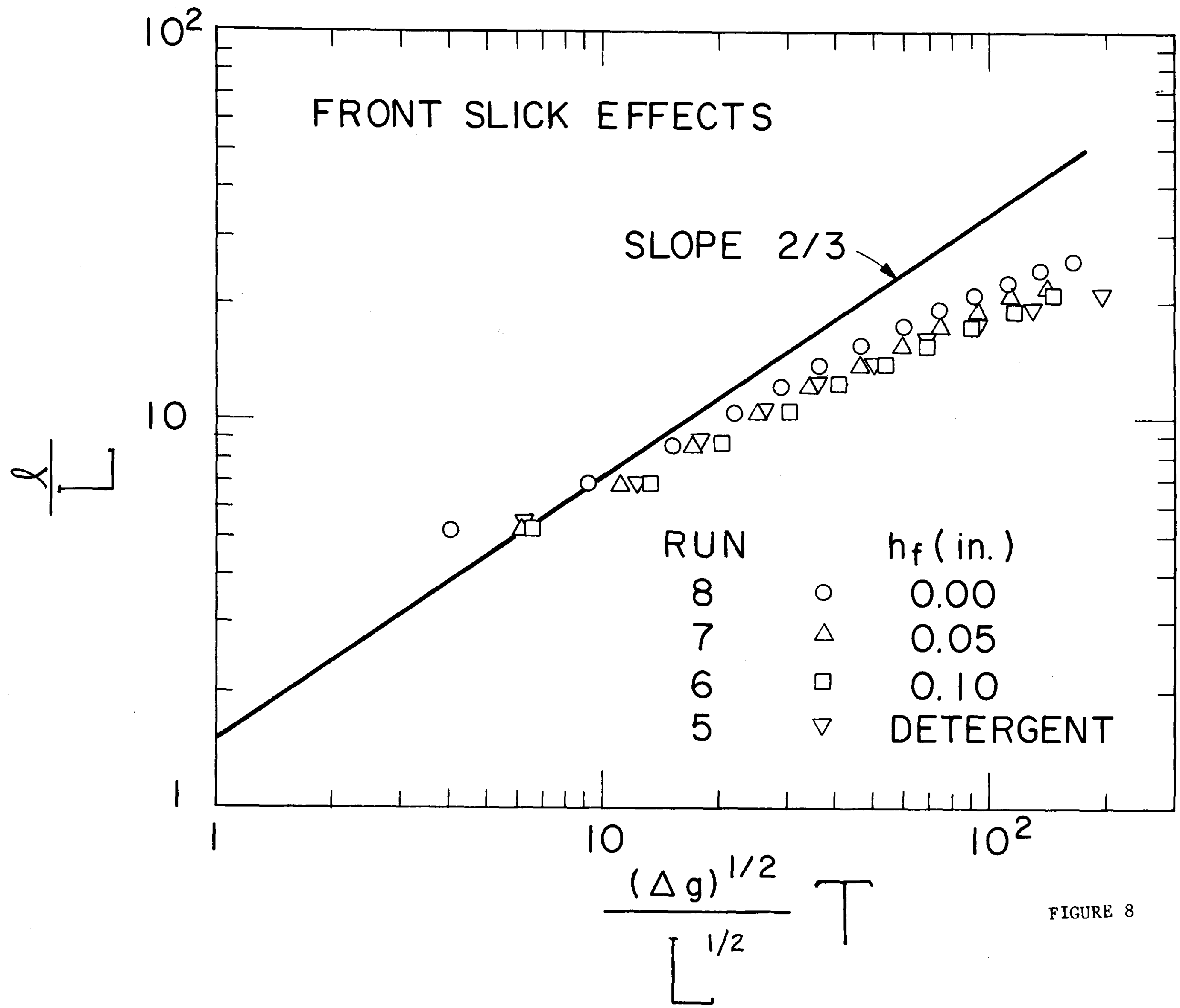

WALL EFFECTS
SLOPE 2/3
RUN w (in.)
1 ○ 24
2 △ 18
3 □ 12
4 ▽ 6
10^2
10
1
1
10
10^2
ℓ / L
(Δg)^1/2 / L^1/2 T

FIGURE 7



FRONT SLICK EFFECTS
SLOPE 2/3
RUN
h_f (in.)
8 ○ 0.00
7 △ 0.05
6 □ 0.10
5 ▽ DETERGENT
$\frac{\ell}{L}$
$\frac{(\Delta g)^{1/2}}{L^{1/2}} T$
10^2
10
1
1
10
10^2

FIGURE 8

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
1	1.5	1.0	12.	1.00	24.
	2.0	1.9			
	2.5	3.0		T_t = 4.44	
	3.0	4.4			
	3.5	5.8			
	4.0	7.4			
	4.5	9.4			
	5.0	11.8			
	5.5	14.5			
	6.0	17.7			
	6.5	21.6			
	7.0	26.1			
	7.5	31.8			
2	1.5	0.9	12.	1.00	18.
	2.0	1.9			
	2.5	3.0		T_t = 4.44	
	3.0	4.3			
	3.5	5.8			
	4.0	8.0			
	4.5	10.0			
	5.0	12.5			
	5.5	15.6			
	6.0	19.5			
	6.5	24.0			
3	1.5	0.9	12.	1.00	12.
	2.0	1.8			
	2.5	3.1		T_t = 4.44	
	3.0	4.3			
	3.5	6.1			
	4.0	8.0			
	4.5	10.8			
	5.0	14.7			
	5.5	19.1			
	6.0	24.9			
	6.5	32.3			
4	1.5	1.0	12.	1.00	6.
	2.0	1.9			
	2.5	3.5		T_t = 4.44	
	3.0	5.1			
	3.5	7.5			
	4.0	10.4			
	4.5	14.3			
	5.0	19.1			
	5.5	24.9			
	6.0	31.2			
	6.5	37.2			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
5	1.5	1.2	12.	1.00	24.
	2.0	2.4			
	2.5	3.3		$T_t = 4.44$	
	3.0	5.0			
	3.5	7.0			
	4.0	9.8			
	4.5	13.1			
	5.0	18.3			
	5.5	24.7			
	6.0	37.8			
6	1.5	1.3	12.	1.00	24.
	2.0	2.5			
	2.5	3.9		$T_t = 4.44$	
	3.0	5.8			
	3.5	7.8			
	4.0	10.3			
	4.5	13.2			
	5.0	17.3			
	5.5	22.5			
	6.0	28.2			
7	1.5	1.2	12.	1.00	24.
	2.0	2.1			
	2.5	3.3		$T_t = 4.44$	
	3.0	4.9			
	3.5	6.5			
	4.0	8.8			
	4.5	11.4			
	5.0	14.4			
	5.5	17.9			
	6.0	21.8			
	6.5	26.9			
8	1.5	0.8	12.	1.00	24.
	2.0	1.8			
	2.5	2.9		$T_t = 4.44$	
	3.0	4.2			
	3.5	5.5			
	4.0	6.9			
	4.5	9.0			
	5.0	11.5			
	5.5	14.3			
	6.0	17.7			
	6.5	21.2			
	7.0	25.8			
	7.5	31.5			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
9	1.5	0.5	12.	2.17	24.
	2.0	1.1			
	2.5	1.8		T_t = 6.90	
	3.0	2.4			
	3.5	3.3			
	4.0	3.9			
	4.5	4.8			
	5.0	5.6			
	5.5	6.6			
	6.0	7.5			
	6.5	8.8			
	7.0	9.8			
	7.5	11.1			
10	1.0	0.0	6.	4.33	24.
	1.5	0.8			
	2.0	1.3		T_t = 6.90	
	2.5	1.9			
	3.0	2.5			
	3.5	3.1			
	4.0	3.8			
	4.5	4.6			
	5.0	5.4			
	5.5	6.4			
	6.0	7.3			

Run	ℓ	T	ℓ_o	h_o	w
11	0.8	0.4	4.	6.50	24.
	1.3	0.7			
	1.8	0.9		T_t = 6.90	
	2.3	1.7			
	2.8	2.3			
	3.3	2.9			
	3.8	3.6			
	4.3	4.3			
	4.8	5.1			
	5.3	5.9			
	5.8	6.9			
	6.3	7.9			
	6.8	9.0			
12	1.5	0.6	12.	1.62	24.
	2.0	1.3			
	2.5	2.1		T_t = 5.84	
	3.0	2.9			
	3.5	3.8			
	4.0	4.8			
	4.5	5.8			
	5.0	6.7			
	5.5	8.1			
	6.0	9.7			
	6.5	11.2			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
13	1.0	0.4	6.	3.25	24.
	1.5	0.9			
	2.0	1.5		T_t = 5.86	
	2.5	2.2			
	3.0	2.9			
	3.5	3.6			
	4.0	4.5			
	4.5	5.4			
	5.0	6.4			
	5.5	7.6			
	6.0	9.0			
	6.5	10.6			
	7.0	12.5			
14	0.8	0.4	3.	6.55	24.
	1.3	0.8			
	1.8	1.3		T_t = 5.88	
	2.3	1.8			
	2.8	2.5			
	3.3	3.1			
	3.8	3.9			
	4.3	4.6			
	4.8	5.6			
	5.3	6.6			
	5.8	7.9			

Run	ℓ	T	ℓ_o	h_o	w
15	1.5	0.8	12.	1.08	24.
	2.0	1.6			
	2.5	2.6		T_t = 4.64	
	3.0	3.8			
	3.5	5.1			
	4.0	6.6			
	4.5	8.3			
	5.0	10.4			
	5.5	12.6			
	6.0	15.6			
	6.5	18.8			
16	1.0	0.5	6.	2.17	24.
	1.5	1.3			
	2.0	1.9		T_t = 4.65	
	2.5	2.6			
	3.0	3.6			
	3.5	4.5			
	4.0	5.6			
	4.5	6.8			
	5.0	8.5			
	5.5	10.3			
	6.0	12.9			
	6.5	16.0			
	7.0	18.8			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
17	0.8	0.5	3.	4.33	24.
	1.3	1.0			
	1.8	1.7		T_t = 4.64	
	2.3	2.4			
	2.8	3.1			
	3.3	4.0			
	3.8	5.2			
	4.3	6.3			
	4.8	7.8			
	5.3	9.9			
	5.8	12.3			
	6.3	15.4			
	6.8	18.8			
18	0.7	0.5	2.	6.50	24.
	1.2	1.0			
	1.7	1.4		T_t = 4.65	
	2.2	1.9			
	2.7	2.8			
	3.2	3.7			
	3.7	4.7			
	4.2	5.7			
	4.7	7.0			
	5.2	8.7			
	5.7	10.6			

Run	ℓ	T	ℓ_o	h_o	w
19	1.5	2.5	12.	0.30	24.
	2.0	6.0			
	2.5	10.5		T_t = 2.24	
	3.0	16.5			
	3.5	24.0			
	4.0	34.0			
	4.5	46.0			
	5.0	61.0			
	5.5	77.0			
	6.0	104.0			
	6.5	197.0			
20	1.2	1.5	8.	0.45	24.
	1.7	3.8			
	2.2	6.5		T_t = 2.24	
	2.7	10.5			
	3.2	16.0			
	3.7	22.0			
	4.2	31.0			
	4.7	40.0			
	5.2	50.5			
	5.7	63.0			
	6.2	83.0			
	6.7	139.0			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
21	1.0	1.2	6.	0.60	24.
	1.5	2.7			
	2.0	4.8		$T_t = 2.24$	
	2.5	7.8			
	3.0	11.7			
	3.5	16.8			
	4.0	23.5			
	4.5	30.8			
	5.0	39.1			
	5.5	47.5			
	6.0	59.5			
	6.5	92.5			
22	0.8	1.5	4.	0.90	24.
	1.3	2.7			
	1.8	4.7		$T_t = 2.24$	
	2.3	8.2			
	2.8	13.0			
	3.3	20.0			
	3.8	29.0			
	4.3	40.0			
	4.8	52.2			
	5.3	67.2			
	5.8	89.0			
	6.3	157.0			
23	0.8	0.8	3.	1.20	24.
	1.3	1.8			
	1.8	3.3		$T_t = 2.24$	
	2.3	5.2			
	2.8	8.1			
	3.3	12.5			
	3.8	18.8			
	4.3	26.3			
	4.8	36.0			
	5.3	47.0			
	5.8	65.4			
24	0.7	1.0	2.	1.80	24.
	1.2	1.9			
	1.7	3.8		$T_t = 2.24$	
	2.2	6.0			
	2.7	9.9			
	3.2	14.4			
	3.7	22.0			
	4.2	32.5			
	4.7	44.8			
	5.2	60.5			
	5.7	89.0			
	6.2	137.0			
	6.7	367.0			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w
25	0.6	0.7	1.	3.60	24.
	1.1	1.9			
	1.6	3.7		T_t = 2.24	
	2.1	5.9			
	2.6	10.0			
	3.1	16.3			
	3.6	25.0			
	4.1	36.5			
	4.6	51.2			
	5.1	68.8			
	5.6	93.5			
	6.1	188.0			
26	0.8	2.8	4.	0.45	24.
	1.3	7.3			
	1.8	17.0		T_t = 1.51	
	2.3	35.0			
	2.8	61.8			
	3.3	105.0			
	3.8	185.5			
	4.3	379.5			
	4.8	863.5			

Run	ℓ	T	ℓ_o	h_o	w
27	0.7	1.5	2.	0.90	24.
	1.2	3.9			
	1.7	8.5		T_t = 1.51	
	2.2	16.0			
	2.7	26.8			
	3.2	40.8			
	3.7	58.0			
	4.2	80.5			
	4.7	112.0			
	5.2	176.0			
	5.7	364.0			
28	0.6	1.5	1.	1.80	24.
	1.1	3.5			
	1.6	8.8		T_t = 1.51	
	2.1	16.1			
	2.6	29.5			
	3.1	50.5			
	3.6	82.0			
	4.1	129.0			
	4.6	234.0			
	5.1	428.0			

TABLE 1

RAW SPREADING DATA

Run	ℓ	T	ℓ_o	h_o	w	Run	ℓ	T	ℓ_o	h_o	w
29	1.5	0.9	12.	2.00	24.	30	1.5	.7	12.	2.00	24.
	2.0	2.2					2.0	1.4			
	2.5	3.1		T_t = 6.80			2.5	2.1		T_t = 6.80	
	3.0	4.2					3.0	2.9			
	3.5	5.4					3.5	3.6			
	4.0	6.6					4.0	4.3			
	4.5	7.9					4.5	5.0			
	5.0	9.3					5.0	5.8			
	5.5	10.8					5.5	6.8			
	6.0	12.3					6.0	7.6			
	6.5	14.2					6.5	8.4			
	7.0	16.2					7.0	9.2			

N.B.

Units are as follows:

ℓ_o, h_o, w: inches

T, T_t: seconds

ℓ: feet

The 30 centipoise water-glygerine solution was used in all runs except runs 29 and 30, in which water only was used.

The 900 centipoise oil was used in all runs except for run 30, in which no. 2 fuel oil was used.