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# A Robust Optimization Approach to Backup Network Design with Random Failures

Matthew Johnston, Hyang-Won Lee, and Eytan Modiano

Laboratory for Information and Decision Systems

Massachusetts Institute of Technology

Cambridge, MA

Email: {mrj, hwlee, modiano}@mit.edu

**Abstract**—This paper presents a scheme in which a dedicated backup network is designed to provide protection from random link failures. Upon a link failure in the primary network, traffic is rerouted through a preplanned path in the backup network. We introduce a novel approach for dealing with random link failures, in which probabilistic survivability guarantees are provided to limit capacity over-provisioning. We show that the optimal backup routing strategy in this respect depends on the reliability of the primary network. Specifically, as primary links become less likely to fail, the optimal backup networks employ more resource sharing amongst backup paths. We apply results from the field of robust optimization to formulate an ILP for the design and capacity provisioning of these backup networks. We then propose a simulated annealing heuristic to solve this problem for large-scale networks, and present simulation results that verify our analysis and approach.

## I. INTRODUCTION

Today's backbone networks are designed to operate at very high data rates, now exceeding 10 Gbit/s [1]. Consequently, any link failure can lead to catastrophic data loss. In order to ensure fast recovery from failures, protection resources must be allocated prior to any network failures. This paper deals with providing protection in networks from multiple random link failures.

A widely used approach for recovery from a link failure is *preplanned link restoration* [2], where a backup path between the end nodes of a link is chosen for every link during the network configuration stage. In the event of a link failure, the disrupted traffic can be rerouted onto its backup path. Preplanned methods of link restoration offer benefits over other methods in terms of speed and simplicity of failure recovery, as no additional dynamic routing is necessary at the time of a failure [3]. In addition to designing a backup path for each link, preplanned link restoration requires provisioning of sufficient spare capacity along each backup path to carry the load of failed links. Backup paths can share spare capacity and network resources to reduce the total cost of protection.

Communication networks can suffer from multiple simultaneous failures, for example, if a second link fails before a first failed link is repaired. Furthermore, natural disasters or

large scale attacks can destroy several links in the vicinity of such events. Preplanning backup paths for combinations of multiple failures can be complex and impractical, and can lead to significant capacity over-provisioning. Consequently, new approaches must be considered to offer protection against multiple failures.

Spare capacity allocation for link-based protection has been studied extensively in the context of single-link failures [1], [4], [5], [6]. The objective of these works is to allocate sufficient protection resources to recover from any single link failure. Recently, the authors in [7] proposed the use of a dedicated backup network to protect against a single failure on the primary network. Upon such a failure, the load on the failed link is routed on a predetermined path on the backup network. The authors provide an Integer Linear Program (ILP) to design an optimal backup network with minimal cost. They show that the cost of the optimal backup network is small relative to that of a large primary network. Specifically, they show that the ratio between the total backup capacity and the total primary capacity tends to zero as the network size grows large for certain classes of networks.

For many applications, it is insufficient to protect against only single-link failure events. Several authors have extended the results of survivability for single link failures to dual-link failures [2], [8], [9]. The work in [10] considers protecting against up to three link failures. Most of these works require the primary network to have multiple disjoint paths between node pairs to survive multiple failures. This assumption is too restrictive when considering a large number of failures. Additionally, [11] provides a spare capacity allocation approach based on a specific set of failure events and restricted backup path lengths. However, in all of these works, large amounts of spare capacity are required if many links can fail simultaneously.

Survivability amidst multiple failures has also been addressed in the form of a Shared Risk Link Group (SRLG) [12]. An SRLG is a set of links sharing a common network resource, such that a failure of that resource could lead to a failure of all links in the SRLG. Many authors have proposed routing strategies for path-based protection against SRLG failures [13], [14], [15], [16]. These works assume that links in a SRLG all fail simultaneously and deterministically. However, this line of work does not extend to uncorrelated, nondeterministic failures.

In this paper, we introduce a new framework for providing

protection from multiple random link failures involving probabilistic survivability guarantees. Since large-scale attacks and natural disasters can result in multiple links failing randomly, providing protection from any single failure is insufficient, and networks designed for protection against single-link failures often cannot protect against multiple failures. The straightforward approach of offering guaranteed protection against any random failure scenario is to allocate capacity such that every failure event is protected. However, this approach is impractical as it requires enormous amounts of capacity to protect against potentially unlikely events. By allocating capacity to offer protection with high probability, the total cost of protection is greatly reduced.

Motivated by the results of [7] and the simplicity of their approach, we extend the use of a dedicated backup network to deal with multiple random link failures. We show that a dedicated backup network is a low-cost method of providing protection against random failures, relative to large primary networks. Additionally, we show that the structure of the minimum-cost backup network changes with the reliability of the primary network. Specifically, optimal backup networks for primary networks with a low link-failure probability employ a high level of link sharing amongst backup paths. On the other hand, optimal backup networks for primary networks with a high link-failure probability emphasize shorter backup paths, and less capacity sharing.

To design a backup network under random link failures, we develop a robust optimization approach to backup capacity provisioning. Robust optimization finds a solution to a problem that is robust to uncertainty in the optimization parameters [17], [18], [19]. In [19], Bertsimas and Sim propose a novel linear formulation with an adjustable level of robustness. These techniques have previously been successfully applied to network flow problems [20]. We apply these results to design backup networks that are robust to the uncertainty in link failures. This leads to an ILP formulation for backup capacity provisioning. We also present a simulated annealing approach to solve the ILP for large-scale networks.

The remainder of this paper is organized as follows. In Section II, we present the network model and formulate the problem of backup network design. In Section III, we consider protection for uniform-load primary networks to investigate the impact of link failure probability on backup network design and the cost of protection. Robust optimization is introduced in Section IV to formulate an ILP for general primary link loads, and a heuristic based on simulated annealing is presented to solve it for large networks. We provide simulation results in Section V and concluding remarks in Section VI.

## II. NETWORK MODEL

Consider a primary network made up of a set of nodes  $\mathcal{V}$  and a set of directed links  $\mathcal{L}$  connecting these nodes. We assume throughout that the links are directed, as the undirected case is a specific instance of the directed link case.

Each link  $(s, d) \in \mathcal{L}$  has a given primary link capacity  $C_{sd}^P$ , and a positive probability of failure  $p$ , independent of all other links. Let the random variables  $X_{sd}$  equal 1 if link  $(s, d)$  fails and 0 otherwise. This probabilistic failure model represents a snapshot of a network where links fail and are repaired according to some Markovian process. Hence,  $p$  represents the steady-state probability that a physical link is in a failed state. This model has been adopted by several previous works [6], [21], [22], [23]. A backup network is to be constructed over the same set of nodes  $\mathcal{V}$  and a new set of links  $\mathcal{L}_B$ , by routing a backup path for each primary link over the backup network and allocating capacity to every backup link. We assume that  $\mathcal{L}_B$  can consist only of links  $(i, j)$  if there is a primary link connecting nodes  $i$  and  $j$ . An example backup network is shown in Figure 1. Furthermore, the backup links are designed such that failures can only occur in the primary network. For each primary link  $(s, d) \in \mathcal{L}$ , a path on the backup network is chosen such that in the event that  $(s, d)$  fails, the traffic load on  $(s, d)$  is rerouted over the backup path. Let  $b_{ij}^{sd} = 1$  if link  $(s, d) \in \mathcal{L}$  uses backup link  $(i, j) \in \mathcal{L}_B$  in its backup path. Hence,  $b^{sd} = \{b_{ij}^{sd} | \forall (i, j) \in \mathcal{L}_B\}$  represents the backup path for the primary link  $(s, d) \in \mathcal{L}$ .

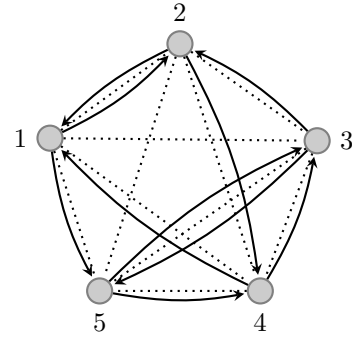


Fig. 1: Example backup network shown as solid directed links over dotted bidirectional primary network.

A capacity  $C_{ij}^B$  is allocated to each backup link  $(i, j) \in \mathcal{L}_B$  such that  $(i, j)$  can support the increased load due to a random failure scenario with probability  $1 - \epsilon$ , where  $\epsilon > 0$  is a design parameter. Naturally, as  $\epsilon$  becomes smaller, more capacity is required on the backup network. Throughout this work we only consider the case where  $p \geq \epsilon$ , since no backup capacity is required for  $p < \epsilon$ .

Each primary link has exactly one path in the backup network for protection, and the links in this path can be shared amongst backup paths for multiple primary links. The goal is to construct a minimal cost dedicated backup network. The problem can be formulated as follows:

Minimize:

$$\sum_{(i,j) \in \mathcal{L}_B} C_{ij}^B \quad (1)$$

Subject To:

$$\mathbf{P}\left(\sum_{(s,d)\in\mathcal{L}} X_{sd} b_{ij}^{sd} C_{sd}^P > C_{ij}^B\right) \leq \epsilon \quad \forall (i,j) \in \mathcal{L}_B \quad (2)$$

$$\sum_j b_{ij}^{sd} - \sum_j b_{ji}^{sd} = \begin{cases} 1, & \text{if } s = i \\ -1, & \text{if } d = i \\ 0, & \text{o.w.} \end{cases} \quad \forall (s,d) \in \mathcal{L}, i \in \mathcal{V} \quad (3)$$

$$b_{ij}^{sd} \in \{0, 1\} \quad \forall (s,d) \in \mathcal{L}, (i,j) \in \mathcal{L}_B \quad (4)$$

The constraint in (3) is a standard flow conservation constraint for the routing of a single backup path for each primary link. The probabilistic constraint (2) is the capacity constraint, from which the backup capacities are computed. Backup link  $(i,j)$  must carry the load of each failed primary link that it protects. Constraint (2) restricts the probability that the load on  $(i,j)$  due to failures exceeds the backup capacity provisioned on  $(i,j)$ . This survivability metric, which considers the reliability of each backup link independently, is referred to as the backup-link survivability metric. There are a number of possible survivability metrics that can be considered in this setting; the choice of which will impact the network design. One can consider survivability from a primary link perspective. In this case, one constrains the joint probability that a primary link fails and its backup path has insufficient capacity. Alternatively, one can consider a survivability constraint on the entire backup network, rather than on each backup link independently. The backup-network constraint restricts the probability that *any* of the backup links have insufficient capacity. It is straightforward to show that the primary-link and backup-network constraints can be written in the form of the backup-link constraint in (2) using a union-bound argument. Therefore, we will use the backup-link constraint of (2) throughout this paper.

We start by considering the backup network design problem for networks with uniform primary link loads. In Section IV, this is generalized to primary networks with arbitrary primary link capacities.

### III. UNIFORM-LOAD NETWORKS

Any primary network can be represented by a fully connected graph, with  $C_{sd}^P = 0$  for links that are not in the primary network. However, in order to form an intuitive understanding of the general problem, we first explore the backup-network design problem for the special case where each primary link has unit capacity, i.e.  $C_{sd}^P = 1 \quad \forall (s,d) \in \mathcal{L}$ . The capacity required on each backup link is dictated by the reliability constraint in (2). Let  $n_{ij}$  be the number of primary links for which backup link  $(i,j)$  is part of the backup path. In other words,

$$n_{ij} = \sum_{(s,d)\in\mathcal{L}} b_{ij}^{sd} \quad (5)$$

Let  $Y_{ij}$  be a random variable representing the number of failed primary links using  $(i,j)$  as part of their backup paths, i.e.,

$$Y_{ij} = \sum_{(s,d)\in\mathcal{L}} b_{ij}^{sd} X_{sd}. \quad (6)$$

Since each  $X_{sd}$  is an i.i.d bernoulli random variable with parameter  $p$ ,  $Y_{ij}$  is a binomial random variable with parameters  $n_{ij}$  and  $p$ . Furthermore, as all the primary links have unit capacity, equation (2) can be rewritten as

$$\mathbf{P}\left(\sum_{(s,d)\in\mathcal{L}} X_{sd} b_{ij}^{sd} C_{sd}^P > C_{ij}^B\right) = \mathbf{P}\left(Y_{ij} > C_{ij}^B\right) \quad (7)$$

$$= \sum_{y=[C_{ij}^B]+1}^{n_{ij}} \binom{n_{ij}}{y} p^y (1-p)^{n_{ij}-y} \leq \epsilon \quad \forall (i,j) \in \mathcal{L}_B \quad (8)$$

Equation (8) uses the cumulative distribution function (CDF) of the binomial distribution. For each link  $(i,j)$ , let  $G(n_{ij}, p, \epsilon)$  be the minimum value of  $C_{ij}^B$  satisfying (8). Clearly, the capacity required on a backup link increases with the number of primary links it protects, and it decreases as the probability of failure decreases. Additionally, as  $\epsilon$  decreases, more capacity is required on each backup link.

#### A. Impact of Link Failure Probability

To gain intuition about the optimal backup network design, we compare three backup routing schemes, shown in Figure 2, and show that backup network performance depends on the link failure probability. In the cycle protection scheme of Figure 2a, each primary link  $(s,d)$  has a backup path lying in a single Hamiltonian cycle through the network. This is the minimum-cost backup network providing protection against a single link failure [7]. Each backup link in this cycle requires unit capacity to protect against a single link failure, resulting in a total cost of  $N$  for an  $N$ -node network. Due to network symmetry, each backup link protects half of the primary links. Therefore, in order to use this scheme to provide protection from a random number of failures with high probability, a total backup capacity of  $C_{\text{total}}^B = N \cdot G\left(\frac{N(N-1)}{2}, p, \epsilon\right)$  is required, where  $G(n, p, \epsilon)$  is the smallest value of  $C_{ij}^B$  satisfying (8).

For large values of  $p$ , this capacity is  $\frac{N^2(N-1)}{2}$ , since  $G(n_{ij}, p, \epsilon) = n_{ij}$  for  $p$  close to 1. This capacity can be reduced by considering the scheme in Figure 2c, where the backup network is a mirror of the primary network, and the backup path for  $(s,d)$  is the one-hop path from  $s$  to  $d$ . Since each backup link offers protection to a single primary link, the total capacity required is  $C_{\text{total}}^B = N(N-1) \cdot G(1, p, \epsilon)$ . For all values of  $p \geq \epsilon$ ,  $C_{\text{total}}^B = N(N-1)$ . Thus, the mirror scheme requires a factor of  $N$  less capacity than the cycle scheme for primary networks with a high probability of link failure.

It is clear that for values of  $p$  close to 1, each link requires dedicated backup capacity to protect against its probable

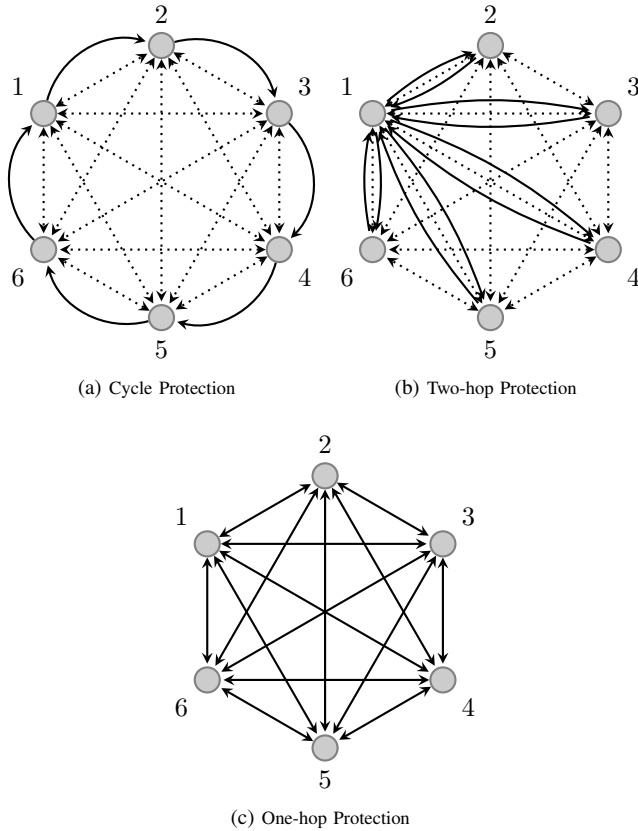


Fig. 2: Sample backup network link placement to protect a 6-node, fully-connected primary network. The dotted lines represent the primary network, and the solid lines represent the backup links.

failure. Consequently, a shortest-path backup routing scheme, as provided by the one-hop protection scheme, minimizes the total backup capacity. While the one-hop protection scheme is preferred in the high- $p$  regime, other schemes are more capacity-efficient for smaller values of  $p$ . Consider the two-hop scheme in Figure 2b, where node 1 serves as a relay node for every backup path. The primary links from node 2 to every other node share the backup link  $(2, 1)$  and similarly the primary links from all nodes to node 2 share the backup link  $(1, 2)$ . Extending this to an  $N$ -node network, each backup link protects  $N - 1$  primaries, and there are  $2(N - 1)$  backup links. Thus,  $C_{\text{total}}^B = 2(N - 1) \cdot G(N - 1, p, \epsilon)$ .

The three aforementioned routing schemes are compared in Figure 3 for a fully connected network with 50 nodes and varying probability of link failure. The cycle-protection scheme, which is optimal in the single failure scenario, requires excessive capacity when considering multiple failures. For values of  $p$  close to  $\epsilon$ , the two-hop routing strategy outperforms the other two strategies. Once  $p$  exceeds roughly 0.4, there is no longer a benefit to sharing backup resources, and the one-hop starts to outperform the two-hop schemes. Hence, it is clear that the optimal backup network topology depends on the reliability of the primary network. This is further analyzed

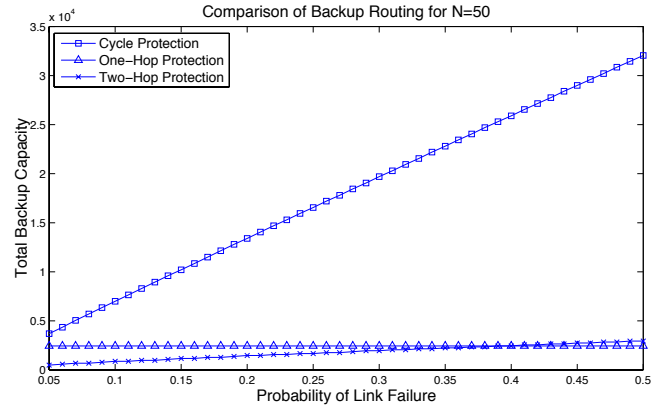


Fig. 3: Comparison of three protection schemes for an  $N = 50$  fully-connected network with unit load.

in Section IV where the problem is formulated for general primary link capacities.

### B. Scaling Properties of Backup Network Capacity

Consider the cost of the backup network with respect to that of the primary network. Let  $\rho$  be defined as

$$\rho \triangleq \frac{\sum_{(i,j) \in \mathcal{L}_B} C_{ij}^B}{\sum_{(i,j) \in \mathcal{L}} C_{ij}^P}. \quad (9)$$

I.e.,  $\rho$  is the ratio of the total capacity of the optimal backup network to that of the primary network. In [7], the authors show that this ratio tends to 0 asymptotically as the network size gets very large for specific networks and single-failure protection. For fully-connected, uniform-load networks, the optimal backup network under single-failure protection is shown in Figure 2a, and for this topology

$$\rho = \frac{N}{N(N-1)} = \frac{1}{N-1}. \quad (10)$$

Conversely, for protection against random failures, the ratio in (9) can be upper bounded using the following proposition.

**Proposition 1.** *Assuming a fully connected primary network with unit-capacity on each link and probability of link failure  $p$ , the ratio between the total capacity of the optimal backup network and that of the primary network can be upper bounded as the primary network size grows large by the following:*

$$\rho \leq 2p \quad (11)$$

*Proof:* The optimal total backup capacity is bounded by that of the two-hop scheme considered in Figure 2b.

$$\rho = \frac{\sum_{(i,j) \in \mathcal{L}_B} C_{ij}^B}{\sum_{(i,j) \in \mathcal{L}} C_{ij}^P} \leq \frac{2(N-1)G(N-1, p, \epsilon)}{N(N-1)} \quad (12)$$

Consider the behavior of  $G(n, p, \epsilon)$  when  $n$  is large. Recall that  $G(n, p, \epsilon)$  is the required number of primary links out of

$n$  that need to be protected to ensure sufficient capacity with probability  $1 - \epsilon$ . The Weak Law of Large Numbers (WLLN) can be used to show that for large  $n$ ,  $G(n, p, \epsilon) \approx n \cdot p$  is sufficient to meet the probability requirements (for any positive  $\epsilon$ ). In the limit of large  $N$ , the inequality in (12) reduces to

$$\rho \leq \frac{2(N-1)Np}{N(N-1)} = 2p \quad (13)$$

■

Therefore, the size of the backup network is a small fraction of the size of the primary network, since  $p$  is usually small. Consequently, a backup network designed using the backup-link survivability constraint is a low-cost method of providing protection against random failures in addition to single-link failures. This result is consistent with [7], in that as the primary network size grows large,  $p$  approaches zero under the single-failure model.

#### IV. GENERAL-LOAD NETWORKS

Next, we develop a formulation for general primary link loads. First, we apply the robust optimization results from [19] to formulate a non-linear program for backup capacity provisioning, and develop an equivalent integer linear formulation in terms of new parameters  $\Gamma_{ij}$ . We show that the choice of these parameters affects the amount of capacity provisioned, and hence the probability of insufficient backup capacity. Then, we add constraints to directly compute these parameters, yielding a solution satisfying the probabilistic constraint in (2).

##### A. Robust Optimization Formulation

In the case of uniform link loads, capacity is allocated to the backup network by computing  $G(n_{ij}, p, \epsilon)$  for each link  $(i, j)$ . The backup capacity provisioned is the number of primary link failures protected against, as a function of  $n_{ij}$ ,  $p$ , and  $\epsilon$ . However, this approach does not apply directly to non-uniform primary link loads, as different links will require different capacities to provide protection. In order to mathematically formulate the problem for general link loads, we will use techniques from the field of robust optimization.

Robust optimization finds a solution to a problem that best fits all possible realizations of the data, when that data is subject to uncertainty. In [19], the authors propose a novel formulation with an adjustable level of conservatism for such problems. Their approach is to introduce an optimization parameter  $\Gamma$ , and provide sufficient capacity to support all scenarios in which *any*  $\Gamma$  of the demands exceed their mean. The solution is guaranteed to be robust for those scenarios, and is shown to be robust for all other scenarios with high probability, determined by  $\Gamma$ .

A similar approach can be applied to the problem of backup network design for general link loads, where the uncertainty is in the number of primary links that fail. Consider allocating capacity on link  $(i, j)$  to protect against any scenario where up to  $0 \leq \Gamma_{ij} \leq n_{ij}$  of the primary links utilizing  $(i, j)$

for protection fail. Clearly, for the specific case of uniform loads, the required backup capacity  $C_{ij}^B$  is just  $\Gamma_{ij}$ , and as shown in the previous section,  $\Gamma_{ij}$  is given by  $G(n_{ij}, p, \epsilon)$  under the constraint in (2). For general loads,  $G(n_{ij}, p, \epsilon)$  is not the bandwidth that needs to be allocated, as in Section III, but rather the number of primary links to provide protection for. To extend this idea, let  $L_{ij}$  be the set of primary links protected by backup link  $(i, j)$ , i.e.  $L_{ij} = \{(s, d) | b_{ij}^{sd} = 1\}$ . Let  $S_{ij}$  be a set of  $\Gamma_{ij}$  primary links in  $L_{ij}$  with the largest capacities. Thus, for any  $(s, d) \in S_{ij}$ , we have

$$C_{sd}^P \geq C_{s'd'}^P \quad \forall (s', d') \in L_{ij} \setminus S_{ij}. \quad (14)$$

The backup capacity required to protect against any  $\Gamma_{ij}$  primary link failures is given by

$$C_{ij}^B = \sum_{(s,d) \in S_{ij}} C_{sd}^P. \quad (15)$$

In a complete form, this constraint can be expressed as

$$C_{ij}^B \geq \max_{S_{ij} | S_{ij} \subseteq \mathcal{L}, |S_{ij}| = \Gamma_{ij}} \left\{ \sum_{(s,d) \in S_{ij}} C_{sd}^P b_{ij}^{sd} \right\} \quad \forall (i, j). \quad (16)$$

The value of  $\Gamma_{ij}$  determines the probability of protection. While  $\Gamma_{ij}$  should be chosen such that (2) is satisfied, for now we fix the value of  $\Gamma_{ij}$  for each link. The capacity constraint in (16) replaces the probabilistic constraint in (2), leading to the following non-linear optimization problem.

Minimize:

$$\sum_{(i,j) \in \mathcal{L}_B} C_{ij}^B$$

Subject To:

$$C_{ij}^B \geq \max_{S_{ij} | S_{ij} \subseteq \mathcal{L}, |S_{ij}| = \Gamma_{ij}} \left\{ \sum_{(s,d) \in S_{ij}} C_{sd}^P b_{ij}^{sd} \right\} \quad \forall (i, j)$$

$$\sum_j b_{ij}^{sd} - \sum_j b_{ji}^{sd} = \begin{cases} 1, & \text{if } s = i \\ -1, & \text{if } d = i \\ 0, & \text{o.w.} \end{cases} \quad \forall (s, d) \in \mathcal{L}, i \in \mathcal{V}$$

$$b_{ij}^{sd} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{L}_B \quad (17)$$

The above is non-linear due to the backup capacity constraint in (16), but it can be reformulated as an ILP using duality techniques similar to [19], detailed in the Appendix. The following is an equivalent formulation to (17).

Minimize:

$$\sum_{(i,j) \in \mathcal{L}_B} C_{ij}^B$$

Subject To:

$$\begin{aligned}
C_{ij}^B &\geq \nu_{ij}\Gamma_{ij} + \sum_{(s,d)\in\mathcal{L}} \theta_{ij}^{sd} \quad \forall (i,j) \in \mathcal{L}_B \\
\nu_{ij} + \theta_{ij}^{sd} &\geq C_{sd}^P b_{ij}^{sd} \quad \forall (s,d) \in \mathcal{L}, (i,j) \in \mathcal{L}_B \\
\sum_j b_{ij}^{sd} - \sum_j b_{ji}^{sd} &= \begin{cases} 1, & \text{if } s = i \\ -1, & \text{if } d = i \\ 0, & \text{o.w.} \end{cases} \quad \forall (s,d) \in \mathcal{L}, i \in \mathcal{V} \\
b_{ij}^{sd} &\in \{0,1\} \quad \forall (i,j) \in \mathcal{L}_B \\
\nu_{ij}, \theta_{ij}^{sd} &\geq 0 \quad \forall (s,d) \in \mathcal{L}, (i,j) \in \mathcal{L}_B
\end{aligned} \tag{18}$$

Clearly, if fewer than  $\Gamma_{ij}$  links in  $L_{ij}$  fail, the capacity allocated in (16) will be sufficient. Therefore, the probability of insufficient backup capacity can be upper bounded using the tail probability of a binomial random variable.

$$\mathbf{P}\left(\sum_{(s,d)\in\mathcal{L}} X_{sd} b_{ij}^{sd} C_{sd}^P > C_{ij}^B\right) \leq \mathbf{P}\left(Y_{ij} > \Gamma_{ij}\right) \tag{19}$$

The capacity allocated in (15) is sufficient to meet the reliability constraint in (19) with probability  $\epsilon$  if  $\Gamma_{ij} = G(n_{ij}, p, \epsilon)$ . However,  $n_{ij}$  is an optimization variable, on which  $\Gamma_{ij}$  depends. Thus, the remaining task is to modify (18) to directly compute the value of  $\Gamma_{ij}$  for each link using an ILP formulation.

### B. Complete Formulation

Since  $\Gamma_{ij}$  cannot be computed analytically, we create a table a priori in which the  $m$ th entry  $\Gamma(m)$  equals  $G(m, p, \epsilon)$ , computed numerically. We develop an ILP that leads to the direct computation of  $n_{ij}$  in order to index the table.

To compute  $n_{ij}$ , let  $x_{ij}^m = 1$  if  $n_{ij} = m$ , and 0 otherwise. The following constraints are introduced.

$$\sum_{m=0}^{N(N-1)} x_{ij}^m = 1 \quad \forall (i,j) \in \mathcal{L}_B \tag{20}$$

Constraint (20) enforces  $x_{ij}^m$  to be equal to 1 for only one value of  $m$  for each backup link.

$$\sum_{(s,d)\in\mathcal{L}} b_{ij}^{sd} = \sum_{m=0}^{N(N-1)} m \cdot x_{ij}^m \quad \forall (i,j) \in \mathcal{L}_B \tag{21}$$

Constraint (21) ensures that the number of primary links utilizing a backup link  $(i,j)$  is equal to the value of  $m$  for which  $x_{ij}^m = 1$ . Consequently,  $\Gamma_{ij}$  can be represented by the following.

$$\Gamma_{ij} = G(n_{ij}, p, \epsilon) = \sum_{m=0}^{N(N-1)} \Gamma(m) x_{ij}^m \tag{22}$$

The capacity constraint of (18) is rewritten as

$$C_{ij}^B \geq \sum_{m=0}^{N(N-1)} \Gamma(m) \nu_{ij} x_{ij}^m + \sum_{(s,d)\in\mathcal{L}} \theta_{ij}^{sd} \tag{23}$$

Since the product  $\nu_{ij} x_{ij}^m$  is non-linear, another set of optimization variables is added to represent this product in linear form. Let  $y_{ij}^m$  be a nonnegative variable satisfying the following constraints:

$$y_{ij}^m \geq \nu_{ij} + K(x_{ij}^m - 1) \quad \forall (i,j), m \tag{24}$$

$$y_{ij}^m \leq K x_{ij}^m \quad \forall (i,j), m \tag{25}$$

$$y_{ij}^m \geq 0 \quad \forall (i,j), m \tag{26}$$

In the above equations,  $K$  is a large number such that  $K > \max_{sd} C_{sd}^P$ . When  $x_{ij}^m = 0$ , then  $x_{ij}^m \nu_{ij} = 0$ , and constraints (25) and (26) force  $y_{ij}^m$  to 0. On the other hand, if  $x_{ij}^m = 1$ , constraint (24) will force  $y_{ij}^m \geq \nu_{ij}$ , which at the optimal solution will be satisfied with equality. These constraints lead to an ILP formulation for backup network design, given in (27).

The following is an ILP formulation for the design of a dedicated backup network to protect against random failures.

Minimize:

$$\sum_{(i,j)\in\mathcal{L}_B} C_{ij}^B$$

Subject To:

$$C_{ij}^B \geq \sum_{m=0}^{N(N-1)} y_{ij}^m \Gamma(m) + \sum_{(s,d)\in\mathcal{L}} \theta_{ij}^{sd} \quad \forall (i,j) \in \mathcal{L}_B$$

$$\nu_{ij} + \theta_{ij}^{sd} \geq C_{sd}^P b_{ij}^{sd} \quad \forall (s,d) \in \mathcal{L}, (i,j) \in \mathcal{L}_B$$

$$\sum_{m=0}^{N(N-1)} x_{ij}^m = 1 \quad \forall (i,j) \in \mathcal{L}_B$$

$$\sum_{(s,d)\in\mathcal{L}} b_{ij}^{sd} \leq \sum_{m=0}^{N(N-1)} m \cdot x_{ij}^m \quad \forall (i,j) \in \mathcal{L}_B$$

$$y_{ij}^m \geq \nu_{ij} + K(x_{ij}^m - 1) \quad \forall (i,j) \in \mathcal{L}_B, m$$

$$y_{ij}^m \leq K x_{ij}^m \quad \forall (i,j) \in \mathcal{L}_B, m$$

$$\sum_j b_{ij}^{sd} - \sum_j b_{ji}^{sd} = \begin{cases} 1, & \text{if } s = i \\ -1, & \text{if } d = i \\ 0, & \text{o.w.} \end{cases} \quad \forall (s,d) \in \mathcal{L}, i \in \mathcal{V}$$

$$b_{ij}^{sd}, x_{ij}^m \in \{0,1\} \quad \forall (i,j) \in \mathcal{L}_B, m$$

$$\theta_{ij}^{sd} \geq 0, \nu_{ij} \geq 0, y_{ij}^m \geq 0 \quad \forall (s,d) \in \mathcal{L}, (i,j) \in \mathcal{L}_B, m \tag{27}$$

This formulation calculates the backup paths and capacity allocation for a dedicated backup network satisfying the survivability constraint in (2).

### C. Simulated Annealing

The ILP in (27) can be directly solved for small instances, but becomes intractable for large networks. There are a number of heuristic approaches to solving ILPs, such as randomized rounding, tabu search, and simulated annealing. Here, we employ a simulated annealing approach to estimate the backup path routing in (27).

Simulated annealing (SA) is a random search heuristic which can be used to find near optimal solutions to optimization problems [24]. The algorithm begins with an arbitrary feasible solution, with a cost computed with respect to an objective function. Then, a random perturbation is applied to the solution, and the cost is reevaluated. The new solution is probabilistically accepted based on the relationship between the two costs. A positive probability of moving to a worse solution avoids the problem of being trapped in a local minima. SA has been used previously on network survivability problems [25].

For a fixed backup path routing, the computation of the optimal backup capacity  $C_{ij}^B$  is straightforward. Therefore, we use simulated annealing to estimate the backup path routing. For the problem in (27), the solution at each SA iteration is the backup path for each primary link, and the cost is the total backup capacity, computed using (16). Perturbations are applied to this solution by randomly recomputing the backup path for a randomly chosen primary link. The current network with cost  $C_{total}^B$  is modified by changing a single backup path, and the network cost  $C_{total}^{B'}$  is recomputed. The new backup network is accepted with probability  $\min(q, 1)$  where

$$q = \exp\left(\frac{C_{total}^B - C_{total}^{B'}}{T}\right). \quad (28)$$

Hence, better solutions are unconditionally accepted and worse solutions are accepted with probability  $q$ . The parameter  $T$  in equation (28) represents the "temperature" of the system. At high temperatures, there is a high probability of accepting a solution with a larger cost than the current solution. This prevents the algorithm from getting trapped in a local minima. The temperature decreases after a number of iterations depending on the network size by  $T' = \rho T$ , for  $0 < \rho < 1$ . SA cannot escape local minima if  $\rho$  is too small, but high values of  $\rho$  result in long computation times. Eventually,  $T$  becomes small enough that the probability of accepting a worse solution approaches zero. At this point, the algorithm terminates and returns the resulting backup network.

There are only limited theoretical results on the convergence time of SA, which is known to be highly problem dependent. Regardless, SA approaches are widely used in practice [24]. The choice of parameters leads to an inherent tradeoff between the accuracy of SA and its convergence time. As the number of iterations before a temperature reduction increases, the accuracy of the SA approach improves at the expense of increased convergence time.

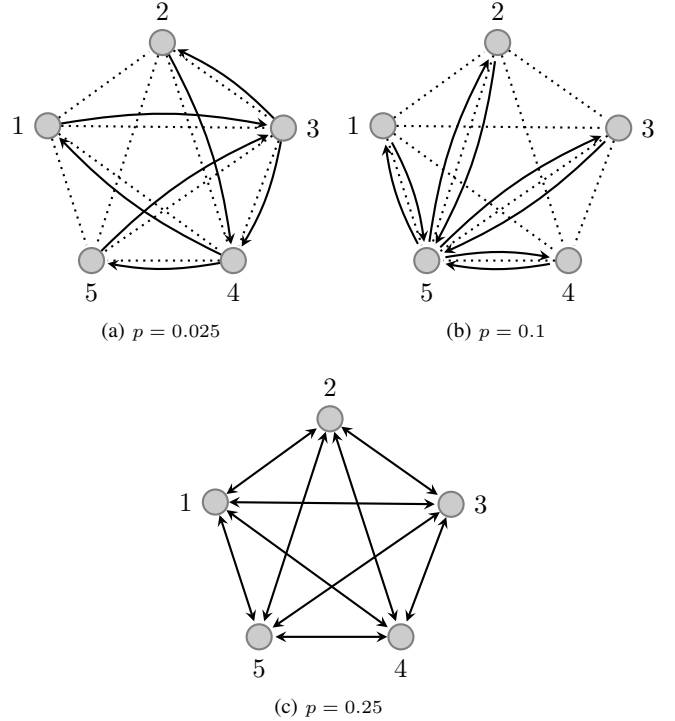


Fig. 4: Optimal backup networks shown as solid links over dotted primary networks for different probabilities of link failure. Designed using  $\epsilon = 0.01$ .

### V. SIMULATION RESULTS

To begin with, consider a five-node, fully-connected topology where each primary link has unit-capacity. Due to the small size of this network, the ILP in (27) can be solved to compute the optimal backup topologies for different values of  $p$ . These backup networks are shown in Figure 4. For small values of  $p$ , the backup topology consists of few links, whereas for large values of  $p$ , the backup network resembles the primary network. Table I summarizes the results of the backup networks for different values of  $p$ , using all of the design heuristics discussed. Cycle protection, two-hop protection, and one-hop protection refer to the strategies analyzed in Section III. The optimal column refers to the solution returned by solving the ILP in (27) using CPLEX, and the SA column refers to an approach where simulated annealing is used to solve the ILP.

Strategy	$p = 0.025$	$p = 0.05$	$p = 0.075$	$p = 0.1$	$p = 0.25$
optimal	7	10	13	16	20
cycle	10	15	15	20	30
two-hop	8	16	16	16	24
one-hop	20	20	20	20	20
SA	7	11	13	16	20

TABLE I: Backup network capacity required for topologies designed using different strategies.  $\epsilon = 0.01$  in each design.

The table shows that for  $p = 0.1$ , the two-hop protection



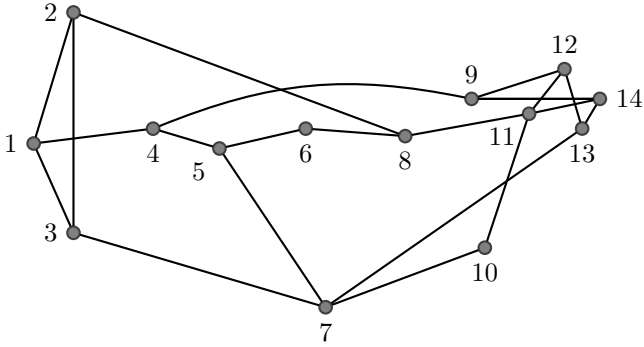


Fig. 5: 14 Node NSFNET backbone network (1991)

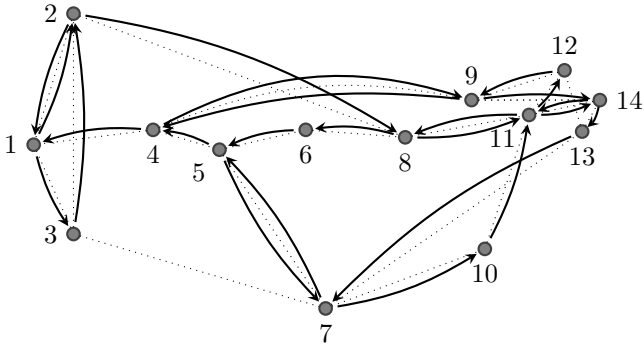


Fig. 6: Backup network (solid) shown for the NSFNET (dotted) with the restriction that the backup network must be a subset of the primary network. The primary network here assumes a probability of link failure of 0.075, and the backup network is designed for  $\epsilon = 0.05$ .

scheme is optimal, and for  $p = 0.25$ , the one-hop protection scheme is optimal. Furthermore, the simulated annealing heuristic performs very close to optimal for different values of  $p$ . Clearly, the optimal topology depends on the probability of link failure, and it is therefore necessary to use a different backup routing scheme depending on the link failure probabilities.

The heuristics can be extended to larger networks, but the ILP in (27) cannot be solved directly for large networks. Thus, we use the SA approach to solve the ILP for backup network design for large primary networks.

Consider the NSFNET primary network shown in Figure 5. Each link is bidirectional, with unit capacity in each direction. Our goal is to construct a backup network consisting of links  $(i, j) \in \mathcal{L}_B$ , where  $i$  and  $j$  are connected by a link in the NSFNET. The survivability constraint in (2) must be satisfied with probability  $\epsilon = 0.05$ . The SA algorithm, shown to be near-optimal for smaller networks, is used to compute the backup network for this larger example. The resulting backup networks for probability of link failure  $p = 0.075$  and  $p = 0.10$  are shown in Figures 6 and 7 respectively.

In the backup network of Figure 6, a total capacity of 24 is required. Most backup links protect up to 5 primary links. In the case of Figure 7, where the probability of link failure

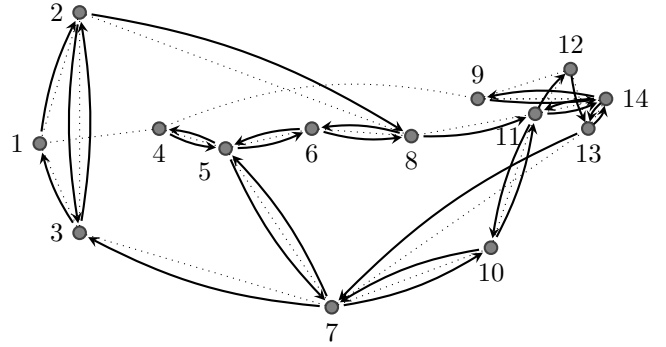


Fig. 7: Backup network (solid) shown for the NSFNET (dotted) with the restriction that the backup network must be a subset of the primary network. The primary network here assumes a probability of link failure of 0.1, and the backup network is designed for  $\epsilon = 0.05$ .

is higher, a total capacity of 28 is needed. The backup links in this example protect an average of 3 primary links. If the probability of link failure increases to  $p = 0.25$ , the resulting backup topology is a mirror of the primary topology, requiring a capacity of 42. As  $p$  increases, the number of backup links needed rises, and similarly the number of primary links being protected by each backup link falls, until the network follows the one-hop protection scheme. These results are summarized in Table II.

Link Failure Probability	$\sum_{(i,j) \in \mathcal{L}_B} C_{ij}^B$	Average $n_{ij}$
$p = 0.06$	22	4.87
$p = 0.075$	24	4.42
$p = 0.085$	27	3.59
$p = 0.10$	28	3
$p = 0.175$	34	1.88
$p = 0.25$	42	1

TABLE II: Comparison of backup networks for NSFNET with different probabilities of primary link failure. Networks were designed using  $\epsilon = 0.05$ . Average  $n_{ij}$  refers to the average number of primary links being protected by a backup link.

## VI. CONCLUSIONS

Dedicated backup networks are a low-cost and efficient method for providing protection against multiple (random) failures. In the event of a failure, the load on the failed link can be automatically rerouted over a predetermined path in the backup network, providing fast recovery from network failures. We formulated the backup network design problem as an ILP for primary networks with general link capacities and independent, identically distributed probabilities of link failure. For primary networks with rare failures, backup networks are shown to use fewer links, with more resource sharing among backup paths. Conversely, when the primary network has a high probability of link failure, the backup network consists of

shorter backup paths. For larger primary networks, a simulated annealing approach was presented to solve the backup network design ILP. This approach has been shown to perform near optimally in designing dedicated backup networks. The SA algorithm can be adjusted to trade-off between computation time and accuracy.

Throughout this work, it has been assumed that the backup network is free from failures. This assumption holds if the backup links are physically designed such that they are more robust to failures. It would be interesting to extend the approach presented in this paper to a failure model in which the backup links are also susceptible to failure.

## VII. APPENDIX

The following steps are used to convert Formulation (17) to Formulation (18) using a duality approach. For a fixed  $b_{ij}^{sd}$  and  $\Gamma_{ij}$ , the backup capacity of link  $(i, j)$ ,

$$\beta_{ij}(\mathbf{b}_{ij}, \Gamma_{ij}) = \max_{S_{ij} | S_{ij} \subseteq \mathcal{L}, |S_{ij}| = \Gamma_{ij}} \left\{ \sum_{(s,d) \in S_{ij}} C_{sd}^P b_{ij}^{sd} \right\}, \quad (29)$$

can be written as the solution to the following LP.

$$\begin{aligned} \beta_{ij}(\mathbf{b}_{ij}, \Gamma_{ij}) = \text{maximize} \quad & \sum_{(s,d) \in \mathcal{L}} C_{sd}^P b_{ij}^{sd} z_{ij}^{sd} \\ \text{subject to} \quad & \sum_{(s,d) \in \mathcal{L}} z_{ij}^{sd} \leq \Gamma_{ij} \\ & 0 \leq z_{ij}^{sd} \leq 1 \quad \forall (s,d) \in \mathcal{L} \end{aligned} \quad (30)$$

Assuming the number of primary links  $(s, d)$  satisfying  $b_{ij}^{sd} = 1$  is larger than or equal to  $\Gamma_{ij}$ , the LP will choose the  $\Gamma_{ij}$  of them with the largest capacities, by setting  $z_{ij}^{sd} = 1$  for those links  $(s, d)$ . This corresponds to choosing the set  $S_{ij}$  in (15). If there are fewer than  $\Gamma_{ij}$  primary links  $(s, d)$  satisfying  $b_{ij}^{sd} = 1$ , then for each of these links  $z_{ij}^{sd} = 1$  and the other  $(s, d)$  satisfying  $z_{ij}^{sd} = 1$  are chosen arbitrarily.

Let  $\nu_{ij}$  be the dual variable for the first constraint in (30), and let  $\theta_{ij}^{sd}$  be the dual variables for the second set of constraints. The dual problem of (30) is formulated below.

$$\begin{aligned} \text{minimize} \quad & \nu_{ij} \Gamma_{ij} + \sum_{(s,d) \in \mathcal{L}} \theta_{ij}^{sd} \\ \text{subject to} \quad & \nu_{ij} + \theta_{ij}^{sd} \geq C_{sd}^P b_{ij}^{sd} \quad \forall (s,d) \in \mathcal{L} \quad (31) \\ & \nu_{ij} \geq 0 \\ & \theta_{ij}^{sd} \geq 0 \quad \forall (s,d) \in \mathcal{L} \end{aligned}$$

Since there is zero duality gap between problem (30) and its dual (31), then the optimal value of the objective function in (31) is equal to  $\beta_{ij}(\mathbf{b}_{ij}, \Gamma_{ij})$ . Additionally, since problem

(17) minimizes  $\beta_{ij}(\mathbf{b}_{ij}, \Gamma_{ij})$  for each  $(i, j)$ , problem (31) can be substituted into (17) to arrive at the formulation in (18).

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