Experimental Investigation and Constitutive Modeling of the Large Deformation Behavior of Anisotropic Steel Sheets Undergoing Strain-Induced Phase Transformation

by

Allison M. Beese

S.M., Massachusetts Institute of Technology (2008)

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of

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Abstract

The strain-induced phase transformation from austenite to martensite is responsible for the high strength and ductility of TRansformation-Induced Plasticity (TRIP)-assisted steels. The large deformation behavior of conventional steels is governed by crystallographic slip. In the case of TRIP steels, the phase transformation provides an additional microstructural deformation mechanism, which has a particularly strong effect on the strain hardening response at the macroscopic level. This thesis work develops a new plasticity model for TRIP steels that accounts for the effect of phase transformation. In particular, the large deformation behavior of 1.5mm thick stainless steel 301LN sheets at room temperature is studied in detail. Several techniques for quantifying the martensite volume fraction are evaluated including micrography, X-ray diffraction, neutron diffraction, magnetic saturation, and magnetic permeability measurements. The latter is then used to measure the evolution of the martensite content throughout mechanical experiments. The experimental program for different stress states includes experiments for uniaxial tension, uniaxial compression, equi-biaxial tension, pure shear, and transverse plane strain tension. The resulting experimental data demonstrate the influence of both the stress triaxiality and Lode angle parameter on the austenite-to-martensite transformation kinetics. A stress-state dependent transformation kinetics evolution equation is proposed which describes the martensite content as a function of plastic strain, the stress triaxiality, and the Lode angle parameter. Furthermore, a phenomenological plasticity model is developed comprising an anisotropic yield function, an isotropic hardening law, and a nonlinear kinematic hardening law with initial back stress. The isotropic hardening law expresses the increase in deformation resistances as a function of the plastic strain and the martensite content and is directly coupled with the stress-state dependent transformation kinetics equation. As a result, the model is able to describe the experimentally observed effect of stress state on the macroscopic hardening response. The constitutive model is implemented into a finite element program and used to simulate all experiments performed. The model predictions agree well with the experimental results for a wide range of stress states and for both specimens with homogeneous and heterogeneous stress and strain fields.

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Title: CNRS Research Associate Professor
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>Cauchy stress tensor</td>
</tr>
<tr>
<td>( s )</td>
<td>Deviatoric Cauchy stress tensor</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Deviatoric back stress</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Deviatoric back stress for plane stress</td>
</tr>
<tr>
<td>( I_1, I_2, I_3 )</td>
<td>Invariants of the Cauchy stress</td>
</tr>
<tr>
<td>( J_1, J_2, J_3 )</td>
<td>Invariants of the deviatoric Cauchy stress</td>
</tr>
<tr>
<td>( \sigma_m )</td>
<td>Mean stress</td>
</tr>
<tr>
<td>( p )</td>
<td>Pressure</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Stress triaxiality</td>
</tr>
<tr>
<td>( \bar{\theta} )</td>
<td>Lode angle parameter</td>
</tr>
<tr>
<td>( \varepsilon^p )</td>
<td>Plastic strain tensor</td>
</tr>
<tr>
<td>( \bar{\varepsilon}^p )</td>
<td>Anisotropic equivalent plastic strain</td>
</tr>
<tr>
<td>( \varepsilon_{VM}^p )</td>
<td>Von Mises equivalent plastic strain</td>
</tr>
</tbody>
</table>

### Martensite evolution

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi )</td>
<td>Martensite volume fraction</td>
</tr>
<tr>
<td>( \chi_0, \chi_{\text{max}} )</td>
<td>Initial, maximum martensite volume fraction</td>
</tr>
<tr>
<td>( D_0, m, a_\delta, a_\eta )</td>
<td>Transformation kinetics material parameters</td>
</tr>
</tbody>
</table>

### Anisotropy

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>Yield function</td>
</tr>
<tr>
<td>( \overline{\sigma} )</td>
<td>Anisotropic equivalent stress (Hill 1948 definition)</td>
</tr>
<tr>
<td>( r_\alpha )</td>
<td>Lankford ratios</td>
</tr>
<tr>
<td>( F, G, H, L, M, N )</td>
<td>Anisotropy parameters</td>
</tr>
<tr>
<td>( P_{11}, P_{12}, P_{22}, P_{33} )</td>
<td>Anisotropic plane stress yield surface coefficients</td>
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</table>

### Hardening parameters

<table>
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<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_0, A, H_0, H_x )</td>
<td>Isotropic hardening parameters</td>
</tr>
<tr>
<td>( c_L, c_{NL} )</td>
<td>Kinematic hardening parameters</td>
</tr>
<tr>
<td>( k )</td>
<td>Deformation resistance</td>
</tr>
<tr>
<td>( \beta_1^0, \beta_2^0, \beta_3^0 )</td>
<td>Initial back stress</td>
</tr>
</tbody>
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Chapter 1

Introduction

Advanced High Strength Steels (AHSS) comprise a class of materials developed by the steel industry in order to produce steels with superior strength characteristics with respect to more traditional steels (e.g., mild or low carbon steels). One practical application for AHSS is use in automobile components. As AHSS offer high specific strength, less material can be used without sacrificing the overall structural safety of a component leading to weight savings, and consequently reduced fuel consumption. However, the plasticity and failure properties of these AHSS must be understood in order for these materials to be used efficiently.

The mechanical properties of many multi-phase steels depend strongly on the martensite content. An increase in martensite content is typically associated with an increase in both initial yield and ultimate strength. TRansformation Induced Plasticity (TRIP) steels present complex multi-phase microstructures consisting of a ferritic matrix and a dispersion of multiphase grains of bainite, martensite, and metastable retained austenite (Jacques et al., 2007 [39]). The austenitic phase transforms to martensite when subject to mechanical or thermal loading (e.g., Angel, 1954 [89]; Lecroisey and Pineau, 1972 [49]; Olson and Cohen, 1975 [67]; Stringfellow et al., 1992 [86]). The austenite-to-martensite formation is displacive, which gives rise to internal stresses that may cause the yielding of the surrounding austenite matrix (e.g., Greenwood and Johnson, 1965 [30]). The active formation of martensite substantially
increases the macroscopic work hardening rate, while the associated transformation strain contributes to the ductility of TRIP steels. Austenitic stainless steels fall into this category, as do TRIP-assisted steels, which contain only a small volume fraction of retained austenite.

The development of plasticity and fracture models for TRIP steels requires reliable measurements of the martensite content evolution in mechanical experiments. The most widely used techniques for martensite content measurements are X-ray diffraction and quantitative optical metallography. The ferromagnetic martensite can also be detected through magnetic methods (e.g., Zhao et al., 2001 [98]; Smaga et al., 2008 [85]; Post et al., 2008 [77]). In the present study, we make use of a ferritescope probe that allows for in situ measurements of the magnetic permeability of a mechanically deformed specimen.

1.1 Phase transformation

The stainless steel sheets contain face centered cubic (FCC) austenite, which is a relatively ductile phase, and body centered cubic (BCC) martensite, which has a higher deformation resistance than the austenitic phase (Santacreu et al., 2006). As the steel is deformed, the austenite transforms to martensite, as seen in other austenitic steels (e.g., Angel, 1954 [89]; Olson and Cohen, 1975 [67]; Stringfellow et al., 1992 [86]). The phase transformation in austenitic stainless steels is usually strain-induced. Strain-induced phase transformation occurs when the transformation stress is higher than the austenite yield stress; the austenitic phase is plastically deformed, creating intersecting slip bands that serve as nucleation sites for strain-induced $\alpha'$-martensite (e.g., Diani and Parks, 1998 [22]), see Figure (1-1). Transformation at stresses below the yield point of austenite are referred to as stress-induced phase transformation. Stress-induced phase transformation is responsible for the shape-memory effect in many alloys (e.g., Abeyaratne and Knowles, 2006 [5]). For the stainless steel 301LN
material, it is typically assumed that the large deformation response at room temperature is governed by strain-induced transformation. Furthermore, it has been shown experimentally that the phase transformation effect in the 301LN alloy decreases dramatically at temperatures above 80°C (Santacreu et al., 2006 [82]).

The transformation kinetics of metastable austenitic steels depend on several factors, including chemical composition, strain, stress state, strain rate, and temperature (e.g., Angel 1954 [89]; Lecroisey and Pineau 1972 [49]; Olson and Cohen, 1975 [67]; Hecker et al., 1982 [34]; DeMania, 1995 [21]; Diani and Parks, 1998 [22]; Stringfellow et al., 1992 [86]). In the work of Mohr and Jacquemin (2008) [56], it was assumed that the transformation kinetics of this anisotropic temper-rolled sheet material depends on the loading direction. However, recent martensite content evolution measurements during uniaxial tension experiments on the stainless steel type 301LN (SS301LN) in three material directions contradict this assumption: the results show that there is no direction-dependence of the transformation kinetics; i.e., the martensite content is just a function of the axial plastic strain, irrespective of the specimen orientation under uniaxial tension (Beese and Mohr, 2010 [9]).

1.1.1 Effect of chemical composition

The effect of chemical composition on austenite stability is discussed briefly in Chapter 2. A full description of the effect of alloy chemistry on the behavior of stainless steels is given in Peckner and Bernstein, 1977 [71]. The alloy chemistry of a particular steel has a strong effect on the stability of the austenite phase. This has been demonstrated by several researchers who have developed empirical expressions for the effect of alloys on the martensite start temperature, \(M_s\). Eichelman and Hull ([25]) describe the \(M_s\) temperature as

\[
M_s(°F) = 75(14.6 - Cr) + 110(8.9 - Ni) + 60(1.33 - Mn) + 50(0.47 - Si) + 3000(0.068 - (C + N)),
\]

(1.1)
Figure 1-1: Schematic differentiating stress-induced and strain-induced martensite phase transformation. Below $M_s$ no martensitic transformation occurs, while between $M_s$ and $M_s'$, the phase transformation is stress-assisted, and between $M_s'$ and $M_d$, the phase transformation is strain-induced. From Olson and Cohen, 1972 [66].

where the elemental weight percentages are used.

Monkman et al. ([58]) describe the $M_s$ temperature with the equation

$$M_s(°F) = 2160 - 66(Cr) - 102(Ni) - 2620(C + N).$$

The $M_d$ temperature, above which there is no austenite to martensite phase transformation is difficult to determine experimentally. Hence, Angel described an $M_{d30}$ temperature, defined as the temperature at which 50% of the austenite transforms to martensite under an applied true strain of 30% (Angel, 1954 [89]). The expression for the $M_d$ temperature proposed by Angel is given as

$$M_{d30}(°C) = 413 - 462(C + N) - 9.2Si - 8.1Mn - 13.7Cr - 9.5Ni - 18.5Mo. \quad (1.3)$$

These empirical expressions describe the temperature range over which strain-induced phase transformation occurs in an austenitic stainless steel.
1.1.2 Effect of plastic deformation

The stress-strain response of austenitic steels and multi-phase steels containing retained austenite is governed by both crystallographic slip and phase transformation. When subject to mechanical loading, the austenitic phase may transform into martensite, which is accompanied by an increase in volume at the microscale. The activation of the TRansformation Induced Plasticity (TRIP) effect usually increases the ductility of steels as well as their strain hardening capacity (e.g., Angel, 1954 [89]). As explained by Olson and Cohen (1972) [66], stress-induced transformation takes place when the stress level in the austenite does not exceed the deformation resistance for crystallographic slip. Here, we focus on the modeling of the elasto-plastic response of solids that undergo strain-induced transformation, which involves the plastic deformation of the austenitic phase (Olson and Cohen, 1972 [66]).

1.1.3 Effect of stress and strain state

There exists a considerable amount of literature addressing the effect of stress and strain state on martensite transformation kinetics and material hardening behavior (e.g., Cina, (1954) [20]; Hecker et al., (1982) [34]; Murr et al., (1982) [60]; Young, (1988) [97]; Kosarchuk et al., (1989) [42]; Okutani et al., (1995) [65]; DeMania, (1995) [21]; Miller and McDowell, (1996) [54]; Iwamoto et al., (1998) [38]; and Lebedev and Kosarchuk, (2000) [44]). However, the experimental data on the effect of stress state are sparse and contain contradicting results. A review of the existing literature on the effect of stress state on transformation kinetics is presented in Chapter 4, along with our experimental results and analysis over a wide range of stress states.

1.1.4 Effect of strain rate and temperature

The strain rate effect is strongly dominated by a temperature effect on the martensitic transformation kinetics. Under high strain rates, the heating conditions are usually
adiabatic, resulting in an increase in temperature that stabilizes the austenitic phase, thus preventing further martensitic evolution. (e.g., Hecker et al., 1982 [34]; Murr et al., 1982 [60]; Talonen et al., 2005 [90]; Nanga et al., 2008 [61]).

Hecker et al. (1982) [34] and Murr et al. (1982) [60] studied the strain rate effect on the stress-strain response of stainless steel 304 over a range of strain rates of $10^{-3}$ to $10^{3}$ s$^{-1}$. They observed that the number of shear band intersections, as well as the $\alpha'$-martensite content at low strains increased with increasing strain rate. However, the adiabatic heating in high strain rates experiments resulted in lower martensite content at strains above 0.25. Talonen et al. (2005) [90] and Nanga et al. (2008) [61] studied the effect of strain rate on martensitic transformation in stainless steel 301LN over a range of strain rates from $3\times10^{-4}$ to 200 s$^{-1}$. Both teams of authors reported that increasing the strain rate halts the martensite transformation because of stabilizing austenite. The strain rates during the crushing of automotive components may be up to approximately 1000 s$^{-1}$, while forming operations involve lower, intermediate strain rates (e.g., Talonen et al., 2005 [90]).

1.2 Thesis outline

The goal of this thesis is to develop a finite-strain rate-independent isothermal constitutive model to describe the large deformation behavior of a metastable austenitic stainless steel that undergoes deformation-induced phase transformation. A procedure for studying the macroscopic constitutive behavior over a wide range of stress states is developed first. To create a micromechanics-inspired model, we implement a technique for quantifying the evolving martensite content in situ during mechanical experiments under different stress states. The resulting data are used to develop a stress-state dependent transformation kinetics evolution law, describing the amount of martensite formation as a function of plastic strain and stress state. Finally, the transformation kinetics evolution equation is coupled with an anisotropic plasticity
model to incorporate the first order effect of martensite content on macroscopic strain hardening behavior, resulting in a finite-strain constitutive model. The model is programmed for use in a finite element package; good agreement is found between the experimentally observed stress-strain response and that predicted by the model. In addition, structural validation experiments are performed.

The thesis is arranged as follows:

In Chapter 2, information about the material studied in this thesis is provided, and in particular its anisotropic and stress-state dependent plasticity behavior is shown.

In Chapter 3, several methods for monitoring the evolution of the martensite content in mechanical experiments are discussed. In particular, a technique for quantifying the martensite content in situ using magnetic permeability measurements is investigated including the important Villari effect, or inverse magnetostriction.

In Chapter 4, the evolution of martensite as a function of deformation (quantified by the von Mises equivalent plastic strain) and stress/strain state is experimentally studied. A stress-state dependent transformation kinetics law is developed, where the martensite transformation is determined to depend on both the stress triaxiality and the Lode angle parameter. The effect of stress state on the transformation kinetics of stainless steel 301LN sheets at room temperature is investigated using newly developed experimental techniques for simple shear and large strain in-plane compression. In addition, uniaxial and equi-biaxial tension experiments are performed. Planar and stereo Digital Image Correlation (DIC) techniques are used to measure the surface strain fields. In situ magnetic permeability measurements are performed to monitor the martensite content evolution throughout each experiment. The experimental results indicate that the martensitic transformation kinetics cannot be described solely by a monotonically increasing function of stress triaxiality: for instance, less martensite is developed under equi-biaxial tension than under uniaxial tension for the same increment in equivalent plastic strain. A stress-state dependent
transformation kinetics law is proposed that incorporates the effect of the Lode angle parameter in addition to the stress triaxiality. In the proposed model, the rate of martensite formation increases monotonically with the stress triaxiality and the Lode angle parameter. The comparison with the experimental data demonstrates that the proposed transformation kinetics law provides an accurate description of the evolution of the martensite content in stainless steel 301LN over a wide range of stress states.

In Chapter 5, a phenomenological plasticity model is developed for steels that exhibit strain-induced austenite-to-martensite transformation. The model makes use of a stress-state dependent transformation kinetics law that accounts for both the effects of the stress triaxiality and the Lode angle on the rate of transformation. The macroscopic strain hardening is due to nonlinear kinematic hardening as well as isotropic hardening. The latter contribution is assumed to depend on the equivalent plastic strain as well as the current martensite volume fraction. The constitutive equations are embedded in the framework of finite strain isothermal rate-independent anisotropic plasticity. Experimental data for an anisotropic austenitic stainless steel 301LN are presented for uniaxial tension, uniaxial compression, transverse plane strain tension, and pure shear. The model parameters are identified using a combined analytical-numerical Monte Carlo approach. Numerical simulations of all calibration experiments are performed, and excellent agreement is observed. Moreover, we make use of experimental data from ten combined tension and shear experiments to validate the proposed constitutive model.

In Chapter 6, structural validation of the plasticity model is presented. One important application of the plasticity model developed in this thesis is its use in predicting the global force versus displacement behavior during forming of sheet metal components. During these forming operations, the material is subject to inhomogeneous stress and strain fields. Additionally, the individual material points may be subject to complex loading histories. The validity of the plasticity model is illustrated
using punch loading and notched tension tests.

Chapter 7 gives a summary and the major conclusions of the present thesis, along with recommendations for future research.
Chapter 2

Material Description

Metastable austenitic stainless steel sheets of the specification 18-7L C1000 (-full hard with UTS ≥ 1000MPa) provided by ArcelorMittal are used for this study. This type of stainless steel corresponds to the austenitic stainless steel type 301LN according to the AISI standard. It contains 17.5% chromium, 6.5% nickel, 0.025% carbon and 0.15% nitrogen. The temper-rolled sheets have a thickness of $t_0 = 1.5mm$. Using the empirical expressions in Chapter 1 (Equations 1.1, 1.2, and 1.3), the $M_s$ temperature is estimated to be between -113°C and -83°C, while the $M_{d30}$ temperature is estimated to be about 30°C. Therefore, at room temperature, the phase transformation is strain-induced. The measured material mass density is $7.9g/cm^3$. Figure (2-1) shows a micrograph of the initial material microstructure. It is composed of face-centered cubic (FCC) $\gamma$-austenite (white), about 20 vol-% of body-centered cubic (BCC) $\alpha'$-martensite (black/brown) and a small fraction of hexagonal close-packed (HCP) $\epsilon$-martensite.

Stainless steels are used in lieu of carbon steels or other alloy steels because of their resistance to corrosion, due to their composition of at least 10% chromium by weight. Adding nickel, nitrogen, or manganese to steel stabilizes the austenitic phase of iron, making the steel less brittle at low homologous temperatures (e.g., Post and Eberly, 1947 [75]; Irvine et al., 1959 [37]; Griffiths and Wright, 1969 [31]; Peckner and Bernstein, 1977 [71]). Carbon is added to increase the hardness and strength of the
steel. According to the AISI standards, the 100- and 200-series of stainless steel are comprised of austenitic chromium-nickel-manganese alloys, where manganese is a less costly alternative to adding nickel. The 300-series of stainless steel contains austenitic chromium-nickel alloys, including the widely used grades 304 and 316. The 400-series refers to hard ferritic and martensitic chromium alloys, which contain at least 10% chromium, less than 2.5% nickel, and typically relatively high amounts of carbon. The 500-series contains heat resistant chromium alloys, and the 600-series contains martensitic precipitation hardening alloys.

Stainless steel 301 differs from the grades 304 and 316 in the amount of chromium, nickel, and carbon. Stainless steel 301 has the same range of chromium composition (16-18%) as grade 316, but less than grade 304 (18-20%); grade 301 has the least amount of nickel (6-8%) with grade 304 having 8-10.5% and grade 316 having 10-14%; and grade 301 has the highest amount of carbon (0.15%) as grade 316 and grade 304 each have 0.08% (ASTM standard A666-03 [1]). The specifications “L” and “N” refer to a low carbon content (less than 0.03%) to improve welding properties, and the fact that nitrogen is added to increase strength and compensate the low nickel for austenite stabilization, respectively.

2.1 Anisotropic plasticity behavior in uniaxial tension

The uniaxial stress-strain curves for tensile loading along the 0° (rolling), 45° and 90° (cross-rolling) directions are shown in Figure (2-2). It can be inferred from the differences in stress level and the corresponding Lankford ratios ($r_0 = 0.67$, $r_{45} = 0.67$ and $r_{90} = 0.89$) that the polycrystalline material features an anisotropic microstructure, which is due to the temper-rolled processing history. In most anisotropic metals, the true stress versus plastic strain curves, $\sigma_a = \sigma_a(\epsilon_a)$, for different loading directions,
\( \alpha \), are similar in shape and may be reconstructed from the same reference curve, 
\( \sigma_r = \sigma_r(\epsilon_r^p) \), using the similarity relationship

\[
\sigma_\alpha = \delta_\alpha \sigma_r \quad \text{and} \quad \epsilon_\alpha^p = \frac{\epsilon_r^p}{\delta_\alpha}
\]  

(2.1)

with the loading direction dependent factor \( \delta_\alpha \). However, this relationship breaks down for the stress-strain curves shown in Figure (2-2). Due to the lack of similarity, Mohr and Jacquemin (2008) [56] assumed in their model that the directional dependence of the martensitic phase transformation differs from that of the plastic deformation response. It is the objective of the present study to generate experimental evidence to support or contradict this modeling assumption. This requires the availability of an \textit{in situ} martensite content measurement technique, which will be discussed in the next section.

### 2.2 Differential strain hardening behavior

The strain hardening behavior of the stainless steel 301LN sheets not only depends on material direction, but also stress or strain state. Therefore, if we study the equivalent stress versus strain behavior with the applied loading axis aligned with a single material direction, we observe different yield stresses and strain hardening behavior under the various stress states. An example is shown in Figure (2-3), where the applied loading axis is aligned with the 90° direction for tests in uniaxial tension, uniaxial compression, transverse plane strain tension, and pure shear. In this thesis, a hypothesis is made that the stress-state dependent transformation kinetics have a first order effect on the macroscopic stress-state dependent strain hardening behavior.
Figure 2-1: Micrograph of the initial material microstructure; the bright and dark regions correspond to austenite and martensite, respectively.

Figure 2-2: Stress-strain curves for uniaxial tension specimens strained 15% in the material rolling direction, cross-rolling direction, and 45° direction.
Figure 2-3: Von Mises equivalent stress-strain curves for uniaxial tension, uniaxial compression, plane strain tension, and shear specimens whose applied loading axis is aligned with the $90^\circ$ direction.
Chapter 3

Measurement of the Martensite Volume Fraction

3.1 Introduction

The evolution of the martensite content is monitored throughout uniaxial tensile experiments on anisotropic temper-rolled stainless steel 301LN. Several martensite content measurement techniques are discussed. It is found that micrography, basic X-ray diffraction, and EBSD provide good qualitative results, but the absolute errors in the estimated absolute martensite content can be greater than 10%. Magnetic saturation induction measurements provide the spatial average of the martensite content over a large volume, which eliminates inaccuracies associated with metallographic surface preparation. Inverse magnetostriction of the ferromagnetic martensitic phase has a strong effect on the results from magnetic permeability measurements. It is critically important to remove all elastic strains before measuring the magnetic permeability. Neutron diffraction is used to quantify the residual lattice strains in the martensite after removing all macroscopic elastic strains. The results demonstrate that the linear relationship between the magnetic permeability and the martensite content holds true.

despite the presence of small residual strains. *In situ* measurements of the martensite content evolution during tensile tests along the rolling, the cross-rolling, and the 45° direction of the anisotropic sheet material reveal that the transformation kinetics are independent of the loading direction in stainless steel 301LN under uniaxial tension.

In the present study, we make use of a ferritescope probe that allows for *in situ* measurements of the magnetic permeability of a mechanically deformed specimen. Uniaxial tensile tests are performed to investigate the effect of texture on the austenite-to-martensite transformation in stainless steel 301LN. The experimental results demonstrate that the rate of martensitic phase transformation in textured austenite does not depend on the loading direction under uniaxial tension.

### 3.2 Martensite content measurement techniques

There exist several techniques to determine the martensite content in steels composed of austenite and martensite. The most widely used techniques for martensite content measurements are X-ray diffraction and quantitative optical metallography. The ferromagnetic martensite can also be detected through magnetic methods (e.g., Zhao et al., 2001 [98]; Smaga et al., 2008 [85]; Post et al., 2008 [77]). Radu et al. (2005) [81] designed a magnetic saturation device that could be used *in situ* during uniaxial tension tests to monitor the evolving austenite and martensite content in TRIP steels with an initial retained austenite volume of about 13%. As discussed by Talonen et al. (2004) [90], magnetic balance and magnetic saturation methods provide the most reliable measurements of the martensite content because of the inherent averaging over a large sample volume.

Four different techniques have been employed in the present study to quantify the martensite content in stainless steel 301LN at different levels of plastic strain.

**Micrography:** For this approach, a sample of the material is extracted, polished,
and etched with a solution that preferentially attacks martensite grains. An image of the sample is then obtained with digital camera integrated into an optical light microscope, and austenite appears bright while martensite is dark. A quantitative analysis is then performed to calculate the fraction of dark martensite pixels with respect to the total number of pixels in the micrograph. This technique proves to have poor sensitivity, and is a destructive testing technique.

**X-ray diffraction:** In X-ray diffraction, phases are differentiated by their lattice parameters. A sample is positioned in an X-ray beam, and the X-rays penetrate a thin surface layer of the specimen, and are scattered and reflected back to a detection device. Based on the reflected signal recorded, the amount of each phase can be calculated. The analysis of the results from this technique is complicated by the rolling texture present in this material, and it is also a destructive testing technique.

**Magnetic saturation:** Magnetic measurement methods may be used because martensite and austenite have disparate magnetic properties: martensite is ferromagnetic while austenite is paramagnetic. In this approach, a sample of the material is extracted and placed in a magnetic saturation device. The device subjects the sample to a high magnetic field so that the sample reaches its saturation magnetization. The volume fraction of the two phases is then determined using the known saturation magnetization of the martensite phase alone, along with the weight of the specimen. This is a destructive testing technique, as the sample must be completely enclosed in the chamber for accurate magnetic saturation measurements.

**Magnetic permeability:** For this approach, a commercially available Ferritescope is used to measure the evolving magnetic permeability of the specimen. The measurement probe of this device is held in contact with the surface of the test specimen, and a low frequency alternating current is sent through an input wire coiled around the iron core of the measurement probe. This creates a magnetic field around the probe, which interacts with the material. The presence of
ferromagnetic material creates a perturbation in the magnetic field, which is measured as a voltage through an output coil around the probe. This technique requires calibration to quantify the absolute martensite content from the output signal, which is discussed in Section 3.3.1. This approach can be used \textit{in situ} during mechanical tests to monitor the martensite evolution.

### 3.2.1 Micrography

A small sample is extracted from the mechanical specimen and prepared for metallography. After mechanical grinding and polishing with diamond paste, the sample surface is etched with Beraha's tint \((50\,ml\,H_2O, 10\,ml\,HCl, 0.15g\,K_2S_2O_5)\) for about 10sec. An optical compound microscope with a built-in digital camera is then used to take a micrograph at a magnification of 120x, as shown in Figures (3-1a) and (3-1b). The martensite corresponds to the dark phase in Figures (3-1a) and (3-1b), while the bright phase corresponds to austenite. Thus, the martensite area fraction could possibly be determined from the gray scale value histogram of the micrograph (Figure (3-1c)). However, it is found that the determined martensite content depends strongly on the choice of the gray scale threshold value, \(\lambda_{\text{thres}}\). For example, as shown in Figure (3-1c), a threshold value of \(\lambda_{\text{thres}} = 120\) yields a martensite area fraction of 55\% for the micrograph in Figure (3-1b), while a fraction of 70\% is obtained for \(\lambda_{\text{thres}} = 170\). Furthermore, it has been observed that the gray scale histogram for a given microstructure depends on the duration of the etching procedure as well as the camera settings. Thus, it is concluded that micrography is not suitable for the accurate determination of the martensite content in stainless steel 301LN.

### 3.2.2 X-ray diffraction and Electron backscatter diffraction

A PANalytical X-Pert Pro Multipurpose Diffractometer with \(CuK_\alpha\) radiation is used to perform all X-ray diffraction measurements. The \(CuK_\alpha\) radiation is both absorbed and fluoresced by the iron in the stainless steel sample, resulting in a relatively high
(a) Micrograph of the initial material microstructure; the bright and dark regions correspond to austenite and martensite, respectively.

(b) Micrograph of the material microstructure after uniaxial tension in the rolling direction up to 15% plastic strain.

(c) Histogram of micrograph of the initial microstructure showing the number of pixels as a function of the integer gray scale value.

Figure 3-1: Micrography for quantifying martensite content.

and complex background in the produced diffraction pattern. In order to reduce the effect of texture on the quantification of the phases, a wobble scan is performed, during which the sample is tilted incrementally from 0° to 5° to collect data from grains that are not parallel to the surface of the sample. However, the maximum tilt of 5° is not sufficient to eliminate the effect of texture. Comparing the ratios of the resulting diffraction peaks using the Reference Intensity Ratio (RIR) method (Jenkins and Sny-
der, 1996 [40]), the martensite content in the original sample is calculated to be 70%, increasing to 86% or 85% after straining the material in the rolling or cross-rolling direction, respectively (see Figure(3-2)). It is concluded that the X-ray diffraction measurements do detect increasing martensite content with plastic strain, but the absolute values calculated with the techniques used here are not reliable. Therefore, this measurement technique is not considered further. It is speculated that the significant uncertainty in the present X-ray diffraction measurements is attributable to the presence of material texture and sample surface effects.

Electron backscatter diffraction (EBSD) is also explored for measuring the martensite content. The samples for EBSD are prepared using the same mechanical polishing procedure as for micrography. However, instead of subsequent etching, they are electrochemically polished. Scans are run for approximately 48 hours to analyze an area of 500µm x 600µm. About 20-30% of all measurement points needed to be removed from the data sets as the analysis software could not identify the type of crystal structure. Therefore, the martensite fraction of the valid data points can no longer be interpreted as a representative measurement.

Figure 3-2: X-ray diffraction patterns for: specimen in initial temper-rolled condition (black), specimen strained 15% in the material rolling direction (red), and specimen strained 15% in the material cross-rolling direction (blue).
3.2.3 Magnetic permeability

Changes in the magnetic permeability of metals can be detected with a ferritescope. We make use of a commercially available ferritescope (Model MP30E-S, Fischer, Germany [27]) that has been developed for the measurement of the ferrite content in austenitic and duplex steels. A low frequency alternating magnetic field is generated around a cylindrically shaped iron probe (5mm diameter). A coil wound around this probe is used to measure changes in the surrounding magnetic field due to the presence of the sample. There is a linear relationship between the output voltage (amplified eddy current) and the magnetic permeability of the sample. Analogous to ferrite content measurements, the magnetic permeability measurement can then be related to the martensite content through the rule of mixtures. Details on this calibration procedure will be discussed in Section 3.3. A ferritescope is employed in the present study as it can easily be used to perform in situ measurements during tensile experiments. Its disadvantages are possible systematic measurement errors associated with the Villari effect and changes in specimen geometry.

3.2.4 Magnetic saturation

The determination of the martensite content based on magnetic saturation measurements may be considered as the most accurate measurement technique. The measurements of micrography, X-ray diffraction and EBSD are limited to a thin surface layer, while the magnetic saturation technique is measuring the volume average of the magnetic saturation induction for a large sample (about 125 mm$^3$). Thus, effects of surface preparation or material texture are eliminated from magnetic saturation measurement results. Furthermore, as discussed in Hecker et al. (1982) [34], the magnetic saturation induction is not affected by elastic strains in the martensite phase. In the present study, magnetic induction measurements are used to establish a relationship between the magnetic permeability measurements and the martensite content.
3.3 Determination of the martensite content based on magnetic permeability measurements

The magnetic permeability is measured by putting the tip of the ferritescope probe in contact with the specimen surface; at the same time, the probe axis needs to be held perpendicular to the specimen surface. A minimum specimen size of 10mm x 10mm is required in order to avoid boundary effects. In the following subsections, we discuss the calibration of the measurement device as well as potential sources of measurement error.

3.3.1 Calibration based on magnetic saturation measurements

Let \( S \) denote the dimensionless output signal\(^2\) of the ferritescope when the probe tip is touching the surface of a homogeneous unstressed specimen at an angle of 90°. The martensite volume fraction \( \chi \) is then estimated based on the relationship

\[
\chi = k_0 \chi_t S
\]

after calibrating the device- and geometry-specific factors \( k_0 \) and \( \chi_t \). The output signal is assumed to be proportional to the magnetic permeability of an unstressed and infinitely thick sample. This proportionality is described by the calibration factor \( k_0 \). It is noted that the magnetic permeability of ferromagnetic materials depends on the magnetic induction. This effect is expected to be negligible since the same magnetic field strength is used to perform the measurements. The thickness correction factor \( \chi_t \) is introduced to account for the effect of specimen thickness on the probe output. The measured perturbation of the magnetic field around the ferritescope probe decreases as the specimen becomes thinner than a critical thickness. The ferritescope manufacturer provides a set of thickness correction curves that describe

---

\(^2\)This output signal corresponds to the ferrite content when ferrite is the only ferromagnetic phase of the sample.
the effect of the specimen thickness on the ferritescope measurements, shown in Figure (3-3a). Denoting the current specimen thickness in \( \text{mm} \) by \( t \), we approximate the thickness correction factor through the empirical relationship

\[
\chi_t(S, t) = \begin{cases} 
1 - 0.008t + (0.0804 - 0.0017S)t^{-1.25} & \text{for } t < t_0 \\
1 & \text{for } t > t_0 
\end{cases}, \tag{3.2}
\]

where \( t_0 = t_0(S) \) is the maximum thickness for which the measured signals are thickness dependent (\( \chi_t > 1 \)).

Macroscopically unstressed samples of different martensite content are used to identify the device-specific calibration factor \( k_0 \). First, we perform ferritescope measurements on all samples. Second, for each sample, a magnetic saturation induction measurements is made by ArcelorMittal. For this, a sigmameter is used in order to determine the absolute martensite content in each sample. Figure (3-3b) compares the sigmameter measurements with the thickness-corrected ferritescope output signals, \( \chi_tS \). A linear fit of these data yields a calibration factor of \( k_0 = 1.67 \). The same calibration factor of 1.7 was found independently by Talonen et al. (2004) [90] using Satmagan (magnetic balance) and density measurements.

### 3.3.2 Effect of magneto-mechanical couplings

Due to inverse magnetostriction, the magnetization of ferromagnetic materials changes when subject to mechanical loading (e.g., Bozorth, 1951 [15]). This Villari effect (Villari, 1865 [94]), or magnetostriction, may be explained by the mechanically induced rotation of the domains of uniform polarization within a ferromagnetic material. Thus, the magnetic permeability of our tensile specimens is a function not only of the martensite volume fraction, but also of the elastic and plastic deformation. Morishita et al. (1998) [59] measured magnetic flux density as a function of applied magnetic field (B-H curves) of A533B steel under different amounts of stress (residual strain
(a) Thickness correction factor $\chi_t$ as a function of material thickness $t$. Manufacturer data (dots) are shown along with the empirical curves described by Equation (3.2).

(b) Martensite content measured with sigmameter versus thickness-adjusted ferritescope signal $\chi_t S$ for identification of the calibration factor $k_0 = 1.67$.

Figure 3-3: Calibration of magnetic magnetic permeability measurements.
ranging from 0 to 7.45%); their results indicate that the saturated magnetic flux density does not depend on the applied stress. Hecker et al. (1982) [34] explained that the applied magnetic field in magnetic saturation measurements is sufficiently strong to impose its orientation on all dipole moments within the ferromagnetic phase. Thus, we assume that the magnetic saturation measurements are not affected by magneto-mechanical couplings.

However, the present experiments show that the measured magnetic permeability is affected by the elastic strains. Figure (3-4b) depicts the ferritescope recordings as a function of the axial strain in a uniaxial tension experiment. As illustrated by the corresponding stress-strain curve in Figure (3-4a), the specimen has been periodically unloaded to zero stress after applying a plastic strain increment of about 1%. We observe that the ferritescope signal increases upon elastic unloading. For example, at an axial strain of $\epsilon = 0.11$, the ferritescope signal is 54 prior to unloading ($\sigma = 1130\text{MPa}$), but increases to 71 as the specimen is unloaded to zero stress ($\sigma = 0\text{MPa}$). Due to the assumed linear relationship between the ferritescope measurement signal and the martensite content, neglecting the effect of elastic strains in the present example would result in an underestimation of the martensite content by 24%.

The unloading of a tensile specimen eliminates the effect of macroscopic elastic strains. However, the martensitic phase may still be subject to elastic residual strains at the grain level. Neutron diffraction measurements are performed to measure the residual elastic strains in the martensitic phase. Neutron diffraction could also be used to identify the martensite content, but similar to X-ray diffraction, reliable measurements have not been possible due to the effect of crystallographic texture (and the high cost associated with neutron diffraction measurements of long duration). This issue could possibly be resolved through the use of more advanced neutron diffraction techniques, which are not considered in the present study.
Figure 3-4: Experimental illustration of significance of magnetostriction, or Villari effect.

3.3.3 Neutron diffraction residual strain measurements

While X-ray diffraction provides information on properties very close to the material surface due to the fact that the penetration depth of X-rays is on the order of μm,
neutron beams penetrate on the order of \textit{mm} through a material, providing information about a much larger material volume. X-ray diffraction and neutron diffraction both utilize Bragg's law of diffraction to measure the lattice spacing of a material,

\[ 2d_{hkl} \sin \theta = \lambda, \quad (3.3) \]

where \( d_{hkl} \) is the distance between planes in the atomic lattice being measured, \( \theta \) is the angle between the incident ray and the detected scattered beam, and \( \lambda \) is the wavelength of radiation.

In X-ray diffraction, the radiation wavelength is fixed, and \( \theta \) is varied to measure the lattice spacing. In neutron diffraction, \( \lambda \) and \( \theta \) can be varied. As discussed in Allen et al. (1985) \[7\], the wavelength can be adjusted with a crystal monochromator or filter, and kept constant during a measurement to measure the lattice spacing. Alternatively, using a linear particle accelerator (LINAC) or spallation neutron source (SNS), a pulsed beam of neutrons, composed of a range of wavelengths can be obtained, resulting in the measurement of multiple lattice spacings during one measurement (e.g., Allen et al., 1985 \[7\]; Allen et al., 1992 \[6\]; Brown et al., 2008 \[16\]; Noyan et al., 2010 \[64\]; Proust et al., 2010 \[80\]).

Here, \textit{in situ} neutron diffraction data are collected during uniaxial tension tests using the High Flux Isotope Reactor's (HFIR) Neutron Residual Stress Mapping Facility (HB-2B) at Oak Ridge National Laboratory (ORNL). Two experiments are performed under force control at room temperature. The tensile axis of the first specimen is aligned with the rolling direction, while the second specimen is subject to uniaxial tension along the cross-rolling direction. All experiments are set up such that the tensile axis of the specimen bisects the angle between the incident neutron beam and the detector bank. Using this configuration, the spatial average of the lattice strain along the [211] direction is measured for all martensite crystals comprised in a volume of 2.5\textit{mm} x 0.7\textit{mm} x 0.7\textit{mm} (centered through the thickness of the specimen) and
for which the [211] direction coincides with the tensile axis of the specimen.Eighteen
minute long neutron diffraction scans are performed for each lattice strain measure-
ment. The blue squares in Figure (3-5a) show the thickness-corrected ferritescope
signal as a function of the lattice strain before and after elastic unloading of a speci-
men that has been subject to a plastic strain of about 0.1 along the rolling direction.
The corresponding red circles in Figure (3-5a) are obtained for a specimen that has
been subject to a plastic strain of about 0.13 along the cross-rolling direction. The
measured lattice strain is small but not zero after unloading.

Assuming that the Villari effect is proportional to the lattice strain, we can then
estimate the ferritescope signal for zero residual strain from the linear extrapolation
of the data shown in Figure (3-5a). The same procedure is repeated for different
levels of plastic strain and loading directions. Figure (3-5b) shows the ratio of the
extrapolated ferritescope signal to that after unloading as a function of martensite
content. It corresponds to the error in the martensite content when the effect of resid-
ual strains is neglected. It varies between 4% and 8% for low and high martensite
contents, respectively. Observe that this error is independent of the loading direction
and depends on the martensite content only. In other words, despite the presence
of small residual strains, it is still possible to establish a calibration curve between
the martensite content and the ferritescope readout. Therefore, it is concluded that
the calibration relationship given by Equation (3.1) with $k_0 = 1.67$ is valid for the
determination of the absolute martensite content from ferritescope measurements in
tension experiments. This conclusion is also supported by the fact that all points for
calibration lie on a straight line in Figure (3-3b), even though points for specimens
deformed in both the rolling and transverse directions are included.

3.3.4 Effects of material heterogeneity and anisotropy

The magnetic permeability is a tensor property that may depend on the direction of
measurement. At the same time, spatial variations of the magnetic permeability may
(a) Thickness-adjusted ferritescope reading as a function of elastic lattice strains parallel to the BCC [211] direction in the specimen before and after elastic unloading. The blue squares correspond to the ferritescope readings of a specimen that is subjected to 10% strain in the rolling direction, while the red circles correspond to those of a specimen subjected to 13% strain in the material cross-rolling direction.

(b) Ratio of the extrapolated ferritescope reading (zero residual strain) to the unloaded ferritescope reading (with residual strains) as a function of measured martensite content for specimens strained in the rolling direction (red dotted line) and specimens strained in the cross-rolling direction (blue solid line).

Figure 3-5: Experimental neutron diffraction results.
potentially influence the measurement results. Ferritescope measurements are performed on the undeformed sheet material prior to tensile testing to quantify the effect of material heterogeneity. For a ferritescope readout of $S = 13.50$, the signal varies by about ±0.02 at the same location. Among different locations within a vicinity of $10mm \times 50mm$, the standard deviation of the recorded signal variation is ±0.25. Furthermore, we incline the ferritescope axis at an angle of 45° with respect to the normal of the sheet surface and take measurements while rotating the ferritescope around the surface normal without changing the contact point with the sheet. The ferritescope readout remains constant throughout this procedure, which is considered as a partial validation of the assumption of isotropic magnetic permeability.

3.4 Results for uniaxial tension

3.4.1 Experimental procedure

Uniaxial tensile tests are performed on flat dogbone-shaped specimens with a gauge width of 20$mm$, a gauge length of 50$mm$, and a thickness of 1.52$mm$ (in accordance with ASTM-E8M [2]). The specimens are extracted such that their tensile axis is aligned with the rolling (0°), 45°, or cross-rolling (90°) direction. A random speckle pattern is painted onto the gauge section of each specimen, and a digital camera (QImaging Retiga 1300i) along with digital image correlation software (VIC2D, Correlated Solutions) is used to determine the strains through a virtual extensometer of about 10$mm$ length. The ferritescope probe is placed perpendicular to the center of the specimen gauge section and held in contact with the paint-free specimen surface throughout the entire experiment. The ferritescope readout is recorded at a frequency of about 1Hz. Each specimen is loaded under displacement control at a crosshead velocity of about 1$mm$/min during elasto-plastic loading. After each 1% strain increment, an elastic unloading/reloading cycle is performed to reduce the effect of inverse magnetostriction on the ferritescope measurements.
3.4.2 Martensite transformation kinetics

The black curves in Figure (3-4a) show the measured stress-strain history for two uni-
axial experiments along the rolling direction. The corresponding thickness-corrected
ferritescope signal measurements are shown in Figure (3-4b). The results are only
shown up to the point of necking. The comparison of the solid and dotted curves
in Figures (3-4a) and (3-4b) indicates good repeatability of this experiment. The
martensite content versus plastic strain curves are shown as solid dots of different
color in Figure (3-6) for the specimens loaded along the $0^\circ$, $45^\circ$ and $90^\circ$ directions.
The experimental results indicate that the martensitic phase transformation in the
textured austenitic stainless steel does not depend on the loading direction under
uniaxial tension. Irrespective of the specimen orientation, we measured almost the
same martensite content versus axial plastic strain curves.

Santacreu et al. (2006) [82] proposed a simple phenomenological transformation
kinetics law to describe the deformation-induced martensite formation in stainless
steel,

\[ \frac{\chi}{\chi_{\text{max}}} = 1 - e^{\exp\{-[D(\bar{\varepsilon}_0 + \bar{\varepsilon}_{VM})]^{m}\}}, \tag{3.4} \]

where $\chi$ denotes the current martensite volume fraction. Mohr and Jacquemin (2008)
[56] made use of this kinematics law, but used the Hill 1948 equivalent plastic strain
as the deformation measure. Here, the von Mises equivalent plastic strain, $\bar{\varepsilon}_{VM}$, is
used instead because of the apparent isotropic nature of the relationship between the
martensite content and the plastic strain. Based on our experimental data, we find
that the model parameters $m = 3.5$, $D = 3.7$, $\chi_{\text{max}} = 100\%$, and $\bar{\varepsilon}_0 = 0.18$ provide a
good approximation of the transformation kinetics (solid black line in Figure (3-6)).
We note that the transformation kinetics may be stress-state dependent, especially at
low temperatures, which may then require more advanced transformation laws (e.g.,
Stringfellow et al., 1992 [86]).
3.4.3 Texture evolution

The EBSD measurements provide some information on the texture in both the austenitic and martensitic phases after different amounts of deformation. Figure (3-7) gives the pole figures for the FCC austenite in a specimen in the original temper-rolled condition, as well as for specimens that have been strained up to 15% under uniaxial tension in either the rolling or cross-rolling direction. Figure (3-8) depicts the corresponding pole figures for the BCC martensite. The color contours in Figures (3-7a) and (3-8a) indicate that both the austenite and martensite phases are textured in the initial condition. As the material is deformed to 15% plastic strain, the amount of austenite decreases from about 76 vol-% to about 15 vol-%. This greatly decreases the number of analyzed data points, and therefore it is difficult to draw conclusions on the texture of the small volume of austenite remaining in the plastically deformed samples (Figures (3-7b) and (3-7c)). However, we do note that the FCC austenite pole figure contours for the deformed specimens are similar to those of the material
in its initial state. In all samples, the normals of the FCC \{110\} planes are aligned preferentially with the sheet thickness direction.

The texture in the BCC martensite phase is shown in Figure (3-8). The contours of the BCC \{100\} and \{110\} pole figures for both the temper-rolled specimen and that strained by 15% in the rolling direction qualitatively agree with the typical BCC texture of cold-rolled low carbon steels (e.g., compare with results in Kocks et al., 1998 [41]). As the material is deformed under tension along the rolling direction, the texture becomes more defined, and martensite grains are either newly formed or rotated such that the normals of the \{111\}-planes are approximately aligned with the sheet thickness direction. For tension along the cross-rolling direction, the resulting texture in the martensite differs noticeably from both of the above mentioned deformation states, as the contours become much less defined.

3.5 Conclusions

A ferritescope has been calibrated based on magnetic saturation induction measurements to monitor the martensite content evolution throughout uniaxial tensile experiments. X-ray diffraction, optical metallography, and EBSD did not provide satisfactory measurement accuracy. The results show that the Villari effect may lead to major errors during \textit{in situ} ferritescope measurements on macroscopically loaded specimens. Therefore, specimens are periodically unloaded throughout each test to determine the magnetic permeability under macroscopically stress-free conditions.

Previous studies have shown that the kinetics of the austenite-to-martensite transformation in metastable stainless steels are stress-state dependent (e.g., Stringfellow et al., 1992 [86]). Furthermore, Mohr and Jacquemin (2008) [56] have recently shown that stainless steels feature non-associated anisotropic hardening, i.e., the strain hardening under monotonic loading cannot be described by a single equivalent stress ver-
sus work conjugate equivalent plastic strain curve. Here, we have performed in situ martensite content measurements throughout uniaxial tensile tests to demonstrate that the initial texture has no measurable effect on the martensitic phase transformation kinetics. The initial texture causes anisotropy in the strain hardening response, while the evolution of the martensite content under uniaxial tension appears to be direction independent.
Figure 3-7: Pole figures generated from EBSD analysis, where the rolling direction is vertical, and cross-rolling direction is horizontal. Pole figures of the FCC austenite phase in: (a) a specimen in the initial temper-rolled condition, (b) a specimen subjected to 15% plastic strain under uniaxial tension in the material rolling direction, and (c) a specimen strained 15% under uniaxial tension in the material cross-rolling direction.
Figure 3-8: Pole figures generated from EBSD analysis, where the rolling direction is vertical, and cross-rolling direction is horizontal. Pole figures of the BCC martensite phase in: (a) a specimen in the initial temper-rolled condition, (b) a specimen subjected to 15% plastic strain under uniaxial tension in the material rolling direction, and (c) a specimen strained 15% under uniaxial tension in the material cross-rolling direction.
Chapter 4

Stress Triaxiality and Lode Angle Dependent Transformation Kinetics Law

4.1 Introduction

Olson and Cohen (1975) [67] proposed a transformation kinetics law that describes the martensite content evolution as a function of equivalent plastic strain and temperature. To calibrate their model, they used data from Angel (1954) [89], who studied the martensite evolution and mechanical properties of an annealed 304 stainless steel under tension and a wide range of temperatures. Stringfellow et al. (1992) [86] expanded on Olson and Cohen’s law by incorporating the effect of stress triaxiality on the phase transformation. Using data from Young (1988), which indicated that more martensite developed under uniaxial tension than under uniaxial compression in metastable steel bars, they postulated that the martensite evolution rate and saturation value increased with increasing stress triaxiality. Tomita and Iwamoto (1995) [91] expanded on the Stringfellow et al. model (1992) [86] by incorporating the ef-

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fect of strain rate. They also introduced a heat conduction equation to account for the latent heat due to phase transformation. They present numerical simulations to investigate the combined effect of strain rate and temperature on the evolution of martensite and the mechanical behavior of SUS304 cylinders under uniaxial tension.

Despite the considerable amount of literature dealing with the effect of stress state on martensite transformation kinetics and overall hardening behavior, the experimental data on the effect of stress state are sparse and contain contradicting results. Cina (1954) [20] performed tension and compression tests on alloys containing 18-25% Cr and 8-12% Ni. Using X-ray diffraction and a magnetic balance to measure the martensite volume, Cina (1954) [20] found that more martensite is developed under tension than under compression at the same strain. Powell et al. (1958) performed tension, torsion, and compression tests on austenitic stainless steel 301 and 304, and observed that more martensite was formed under tension than under shear or compression. Hecker et al. (1982) [34] and Murr et al. (1982) [60] studied the effect of strain state on phase transformation in stainless steel 304 sheets, using both magnetic permeability and saturation measurements, subjecting specimens to uniaxial tension and equi-biaxial tension. They found that the martensite volume fraction after biaxial tension was up to five times higher than under uniaxial tension at the same maximum principal strain. Comparing the martensite content as a function of the von Mises equivalent plastic strain reduced the discrepancy in the results. However, as the temperature of loading decreased, the martensite content developed under equi-biaxial tension exceeded that developed under uniaxial tension at any given equivalent strain.

Young (1988) [97] performed tension and compression tests on round bars of a 0.5Mn overaged phosphocarbide strengthened alloy. By performing magnetization measurements using a vibrating sample magnetometer (VSM), he determined that more martensite developed under uniaxial tension than under compression at the same temperature and same equivalent strain. These data were then used to incorporate the effect of stress triaxiality on the martensite transformation kinetics in Stringfellow
et al. (1992) [86]. Kosarchuk et al. (1989) [42] studied two grades of stainless steel thin-walled tubes under various combinations of axial tension and internal pressure. Their data show that the transformation under equi-biaxial tension is less than under uniaxial tension.

Okutani et al. (1995) [65] performed tension, compression, equi-biaxial compression, and deep drawing experiments on 304 austenitic stainless steels, reporting that more martensite is developed under compression than under tension. DeMania (1995) [21] studied stainless steel 304L sheets, and found that the martensite content was higher in a specimen subjected to uniaxial tension than to plane strain tension at the same equivalent strain at -40°C. DeMania noted that the stress triaxiality is higher in plane strain tension than in uniaxial tension, and suggested that the strain state should also be incorporated into the martensite transformation kinetics law as fewer shear band intersections, and therefore martensite nucleation sites, are created in plane strain tension than in uniaxial tension due to geometrical constraints. There was little to no difference in the martensite developed under uniaxial tension and plane strain tension at 20°C.

Miller and McDowell (1996) [54] studied the mechanical behavior of stainless steel 304L under compression, torsion, and sequential loading of these two deformation states. They measured a higher martensite fraction after a specimen had been subjected to compression than after it had been tested in torsion, and attribute this to a higher number of possible planes of maximum shear stress in compression as compared to torsion. Iwamoto et al. (1998) [38] used X-ray diffraction with MoKα radiation to measure the martensite content after deformation of type 304 austenitic stainless steel under uniaxial tension and uniaxial compression. They observed that initially more martensite developed under uniaxial compression than under uniaxial tension; however, in the high strain region, this is reversed, and the martensite developed under uniaxial tension exceeds that developed under compression.
Lebedev and Kosarchuk (2000) [44] performed experiments on austenitic stainless steel 18-10, studying the effect of temperature and stress state on the martensite transformation kinetics. Using X-ray diffraction and optical micrography to measure the martensite content, they determined that the highest $\alpha'$-martensite developed under uniaxial tension, followed by torsion, with the lowest $\alpha'$-martensite content developed under compression. They conclude that the stress triaxiality is not the only parameter controlling the phase transformation, and suggest a dependence on the stress deviator by incorporating the Lode parameter. However, they do not provide an explicit definition of how the martensite evolution depends on the Lode parameter and stress triaxiality.

Shin et al. (2003) [84] performed tension, compression, and pure shear by equal channel angular pressing (ECAP) experiments. They reported that shear loading by ECAP resulted in the highest amount of martensite developed, followed by compression. Tension resulted in the lowest volume fraction of transformed martensite.

Yan et al. (2006) [95] performed uniaxial tension, biaxial stretching, and plane strain tension experiments on TRIP 600 cold-rolled sheet. Using X-ray diffraction to calculate the amount of retained austenite, they determined that plane strain tension was the deformation mode resulting in the fastest phase transformation, followed by biaxial tension, and finally uniaxial tension.

Perdahcioglu et al. (2008) [72] performed plane stress biaxial tests on ASTM A564 austenitic stainless steel, measuring the martensite fraction with a magnetic sensor. They report that more martensite is developed under plane strain tension than under simple shear. In addition, they present their data by plotting the strain required to develop 40% of martensite versus the tensile stress at that point under different loading conditions. This shows that the strain required to create 40% martensite decreases with increasing tensile stress. Nanga et al. (2009) [62] studied AISI 301LN and 201 stainless steel sheets under various temperatures and strain rates, and their
experimental results indicate the highest transformation to martensite occurs under uniaxial tension and equi-biaxial tension, followed by plane strain tension, with the least amount of martensite developing under shear loading. A summary of the experimental results reported in the above-mentioned literature is provided in Table (4.1).

The goal of present work is to produce a comprehensive set of data that illustrates the effect of stress state on the martensitic transformation kinetics in an austenitic stainless steel. It is speculated that some of the contradictions in the data in the literature are due to inaccurate experimental measurements. In particular, the effects of inverse magnetostriction and sample surface preparation techniques on martensite content measurements (e.g., Beese and Mohr, 2010 [9]) were seldom taken into account. Sections 4.2 through 4.6 of this chapter present a detailed description of selected experiments for different stress states. Based on the experimental results, a new austenite-to-martensite transformation kinetics law is developed in Section 4.7, which accounts for the effect of the Lode angle parameter in addition to stress triaxiality. Section 4.8.1 discusses the underlying micromechanical arguments regarding the Lode angle dependence. It is shown that the proposed transformation kinetics model for isothermal conditions provides an accurate description of the evolution of the martensite content in stainless steel 301LN sheets for uniaxial tension, uniaxial compression, simple shear, and equi-biaxial tension.

4.2 Experimental procedures

A ferritescope (Model MP30E-S, Fischer, Germany) is used to measure the martensite content evolution in situ. A low frequency alternating magnetic field is generated around a cylindrically shaped iron probe (5mm diameter). A coil wound around this probe is used to measure changes in the surrounding magnetic field due to the presence of the sample. There is a linear relationship between the output voltage (amplified eddy current) and the magnetic permeability of the sample. Analogous
Table 4.1: Summary of results reported in existing literature on the effect of stress state on martensite content development in steels undergoing deformation-induced phase transformation. The number of plus symbols represents the relative amount of martensite developed under the stress states studied by the respective research groups.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Uniaxial Compression</th>
<th>Shear</th>
<th>Plane Strain Tension</th>
<th>Equi-Biaxial Tension</th>
<th>Uniaxial Tension</th>
<th>Martensite Measurement Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Powell et al., 1958 [78]</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td>++</td>
<td>Density measurements</td>
</tr>
<tr>
<td>Hecker et al., 1982 [34]</td>
<td></td>
<td>++</td>
<td>+</td>
<td></td>
<td></td>
<td>Magnetic permeability, magnetic saturation</td>
</tr>
<tr>
<td>Young, 1988 [97]</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td>++</td>
<td>Magnetic moment</td>
</tr>
<tr>
<td>Kosarchuk et al., 1989 [42]</td>
<td></td>
<td></td>
<td></td>
<td>+</td>
<td>++</td>
<td>X-ray diffraction</td>
</tr>
<tr>
<td>Miller and McDowell, 1996 [54]</td>
<td>++</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td>Magnetization studies</td>
</tr>
<tr>
<td>Iwamoto et al., 1998 [38]</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td>++</td>
<td>X-ray diffraction, optical micrography</td>
</tr>
<tr>
<td>Lebedev and Kosarchuk, 2000 [44]</td>
<td>+</td>
<td>++</td>
<td></td>
<td></td>
<td>+++</td>
<td>X-ray diffraction, optical micrography</td>
</tr>
<tr>
<td>Shin et al., 2003 [84]</td>
<td>++</td>
<td>+++</td>
<td></td>
<td></td>
<td>+</td>
<td>Ferritescope (do not mention Villari effect)</td>
</tr>
<tr>
<td>Yan et al., 2006 [95]</td>
<td></td>
<td>+++</td>
<td>++</td>
<td></td>
<td>+</td>
<td>X-ray diffraction</td>
</tr>
<tr>
<td>Perdahcioglu et al., 2008 [72]</td>
<td>+</td>
<td>++</td>
<td></td>
<td></td>
<td></td>
<td>Magnetic sensor</td>
</tr>
<tr>
<td>Nanga et al., 2009 [61]</td>
<td>+</td>
<td>++</td>
<td>+++</td>
<td>+++</td>
<td></td>
<td>Saturation magnetization</td>
</tr>
</tbody>
</table>
to ferrite content measurements, the magnetic permeability measurement can then be related to the martensite content through the rule of mixtures. Details on this calibration procedure are presented in Beese and Mohr (2010) [9]. The ferritescope is employed as it can easily be used to perform in situ measurements during mechanical experiments. Its disadvantages are possible systematic measurement errors associated with the Villari effect and changes in specimen geometry, as discussed in Beese and Mohr (2010) [9]. Magnetic saturation measurements are performed for selected deformed specimens to confirm the ferritescope measurements.

Two mechanical testing machines are used in this study. The first is a hydraulic testing machine (Instron Model 8800) with a maximum axial loading capacity of 50kN, while the second is a uniaxial servo-mechanical testing machine with a load capacity of 200kN (MTS Model 45G). During all experiments, the deformation field in the gauge section is determined from optical strain measurements using digital image correlation (Vic2D and Vic3D, Correlated Solutions, West Columbia, SC). The gauge section of each specimen is painted white with a black speckle pattern on top for DIC. For planar DIC, a single camera is focused on the gauge section, while for stereo DIC, two cameras are employed simultaneously. During the experiments, the digital cameras (QImaging Retiga 1300i) capture images at a rate of 1Hz. The cubic B-spline interpolation algorithms of the software packages Vic2D (Version 4.4.1) and Vic3D (Version 2007.1.0) are used to determine the displacement fields on the specimen surfaces.

4.3 Uniaxial tension

Uniaxial tension tests are performed on dogbone specimens with a gauge length of 50mm and width of 20mm in accordance with ASTM-E8M [2]. Tensile specimens are cut such that their tensile axes are aligned with the rolling direction (0°) or cross-rolling direction (90°). Each specimen is loaded under displacement control at a speed
of $1\,mm/min$, resulting in a strain rate on the order of $10^{-4}\,s^{-1}$. The axial and width strains are measured using a DIC based digital extensometer; the evolution of the martensite volume fraction is measured by keeping the ferritescope probe in contact with the gauge section throughout the entire experiment. The specimens are unloaded to zero stress and subsequently reloaded after each increment of $1\%$ of strain to reduce the effect of inverse magnetostriction on the ferritescope measurements. The resulting true stress-strain curves for tests in the rolling and cross-rolling direction are shown in Figure (4-1), while the martensite evolution is shown in Figure (4-2).

Although there is a notable difference between the hardening behavior under uniaxial tension along the rolling and cross-rolling directions (Figure (4-1)), there is no significant difference in the martensite evolution with respect to the direction of loading (see Figure (4-2)). Based on a careful validation of the experimental measurements (Beese and Mohr, 2010 [9]), it can be concluded that the martensite transformation kinetics are not directionally dependent under uniaxial tensile loading.

## 4.4 Uniaxial compression

The behavior of sheet metal has been extensively studied for uniaxial and multi-axial tensile loading while the compressive behavior at large strains is seldom investigated. This is due to experimental difficulties associated with the compression testing of sheet materials. Here, we employ a new anti-buckling device that can be used in conjunction with an optical strain measurement system and a ferritescope for magnetic permeability measurements.

### 4.4.1 In-plane uniaxial compression background

In-plane compression specimens extracted from sheet materials buckle either elastically or plastically at small strains; consequently, anti-buckling devices need to be
Figure 4-1: Absolute true stress-strain curves for uniaxial tension (*dotted lines*) and compression (*solid lines*) along the rolling (*red*) and cross-rolling (*blue*) directions of stainless steel 301LN sheets.

used to achieve large compressive strains. Based on the original design of Dietrich and Turski (1978) [24], Kuwabara et al. (1995) [43] proposed a comb-shaped anti-buckling device that can be used in conjunction with a lateral blank holder pressure of about 1% of the yield stress. In order to prevent buckling, Yoshida et al. (2002) [96] made use of a stack of five adhesively bonded dogbone specimens in conjunction with an anti-buckling device to apply compressive strains of up to 10%. They report a surface pressure of 0.1MPa. The technique of Boger et al. (2005) [14] comprises a hydraulic system that applies a lateral force of about 10kN through a set of solid clamping plates. They also optimized their specimen geometry and proposed a procedure to correct for frictional effects. More recently, Cao et al. (2009) [17] presented a double-wedge system to prevent buckling. The distance between the two rigid support surfaces is fixed, while 0.5mm thick Teflon sheets are placed between the device and the specimen to act as an elastic support system.

Most previous techniques require the use of lateral extensometers as the view of
Figure 4-2: Measured evolution of the martensite volume fraction for uniaxial tension (solid symbols) and compression (open symbols) along the rolling (red circles) and cross-rolling (blue squares) directions of stainless steel 301LN sheets.

the specimen front surface is obstructed by the anti-buckling device. We make use of a single thickness sheet specimen and design the anti-buckling device such that it is capable of stabilizing the thin specimen in a membrane dominated deformation mode at large strains. Experiments are performed on short cylindrical specimens in addition to thin sheet specimens to validate the proposed experimental technique. Furthermore, experiments on stainless steel 301LN are performed to measure martensite content evolution under uniaxial compression.

4.4.2 Validation of the compression testing technique

The series of photographs in Figure (4-3) show how the anti-buckling device is applied to a compression specimen, while a photograph of the assembled device is given in Figure (4-4). A set of fourteen springs is used to apply a surface pressure of about 1.5 MPa through a pair of aluminum plates. The spring lengths are chosen such that
the pressure remains approximately constant even when the specimen thickness increases during the experiments. The gauge section dimensions are similar to those used by Yoshida et al. (2009) [96]. A rectangular window is machined into the front plate of the anti-buckling device. The visible part of the gauge section is painted white with a black speckle pattern for DIC measurements.

To validate the proposed experimental technique, we compare the results from conventional compression tests on cylindrical specimens with those obtained from testing flat specimens with the anti-buckling device. Both types of specimens have been extracted from the same piece of bulk 4140 hardened alloy steel. The cylindrical specimens have a diameter of 12.7mm and a height of 25.4mm (in accordance with ASTM E9 [3]). The servo-mechanical testing machine is used to perform the compression tests on the cylindrical specimens, while the flat specimens are tested on the hydraulic testing machine, which is equipped with precisely aligned high pressure clamps (Mohr and Oswald, 2008 [57]). Both types of experiments are performed under displacement control at a plastic strain rate of about $10^{-4}s^{-1}$.

The true compressive stress-strain curves for a cylindrical specimen and a sheet specimen are shown in Figure (4-5). We observe good agreement up to a true compressive strain of about 13%. Beyond this point, bending deformation becomes dominant and the sheet specimen eventually fails because of plastic buckling despite the lateral pressure applied by the anti-buckling device. It is concluded from this comparison that the proposed experimental technique is approximately valid up to the maximum true stress. It is emphasized that friction between the specimen and the anti-buckling device is negligibly small because of the low lateral pressures. Furthermore, surface strains are measured through DIC, which eliminates the need for approximations with respect to the effective gauge section length.

Note that these validation experiments could not be performed on the stainless steel 301LN sheets since a bulk material was needed for the extraction of the cylindrical specimens.
4.4.3 Experimental Results

In a second series of experiments, we make use of the anti-buckling device to indentify the compressive response of the 1.5mm thick stainless steel 301LN sheets. Figure (4-1) shows the measured compressive stress-strain curves. The comparison with the material response for uniaxial tension reveals a significant difference. The strain hardening under compression is significantly lower than in tension. In situ magnetic permeability measurements during compression and tensile testing indicate that the rate of austenite-to-martensite transformation is significantly higher under tension.
than under compression (Figure (4-2)). This observation is consistent with results reported on 18Cr-10Ni austenitic stainless steel (e.g., Young, 1988 [97]; Lebedev and Kosarchuk, 2000 [44]). We note that there is no strong evidence of a directional dependence of the martensite formation under uniaxial tension. However, the data indicate that there is a measureable difference of the martensite transformation kinetics between uniaxial compression along the rolling and cross-rolling directions. At any given strain, more martensite is developed when the material is compressed along the cross-rolling direction than along the rolling direction.

4.5 Simple shear

A symmetric double shear specimen with two identical gauge sections is used to characterize the sheet material response under simple shear loading (Figure (4-6)). The specimen has two gauge sections in order to ensure symmetric shear loading,
as suggested by Gary and Nowacki (1994) [29]. Mohr and Oswald (2008) [57] recommend reducing the thickness of the gauge sections with respect to the specimen shoulders to avoid plastic deformation within the clamping area. However, since the ferritescope measurement accuracy increases as a function of the gauge section thickness, the gauge section thickness reduction is omitted for the present experiment. Figure (4-7) shows the custom-made shear testing device. The center of the double shear specimen is clamped between the two components of the inner fixture. The outer specimen shoulders are then clamped between the two components of the outer fixture. A torque of 120N·m is applied to each of the twenty 7/16-20 steel cap screws to provide an estimated clamping force of about 350kN per specimen shoulder to prevent slipping of the specimen during the test. The assembly is positioned between the flat loading platens of the servo-mechanical testing machine. The top loading platen of the testing machine applies a compressive load on the inner part of the fixture, which converts the compressive loading into shear loading of the two gauge sections. Four windows are milled into the outer fixture to (1) provide a window through which the digital camera can capture images for DIC, and to (2) allow the ferritescope probe to be in contact with the specimen during the test to measure the martensite evolution.

The DIC software is used to determine the engineering shear strain, $\gamma$, within the specimen gauge section. The corresponding deformation gradient may be written as

$$ F = RU = \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix}, \quad (4.1) $$

with $R^T R = 1$ and $U = \sum_{i=1}^{2} \lambda_i u_i \otimes u_i$. $R$ is a rotation and $U$ is the right stretch tensor with the principal stretches

$$ \lambda_1 = \frac{1}{2} \left( \sqrt{4 + \gamma^2} + \gamma \right) \text{ and } \lambda_2 = \frac{1}{2} \left( \sqrt{4 + \gamma^2} - \gamma \right). \quad (4.2) $$
Using the logarithmic strain definition, the equivalent von Mises strain for simple shear loading is calculated as

$$\bar{\varepsilon}_{VM} = \int_0^\gamma \sqrt{\frac{2}{3} \left[ \frac{1}{\lambda^2_1} \left( \frac{d\lambda_1}{d\gamma} \right)^2 + \frac{1}{\lambda^2_2} \left( \frac{d\lambda_2}{d\gamma} \right)^2 \right]} d\gamma, \quad (4.3)$$

which reduces to the well known expression $\bar{\varepsilon}_{VM} \approx \frac{\gamma}{\sqrt{3}}$ in the case of small and moderate engineering shear strains.

![Figure 4-6: Geometry of full-thickness double shear specimen used in this study (dimensions in mm).](image)

The green diamond-shaped open dots in Figure (4-8) show the measured martensite content as a function of the equivalent plastic strain for simple shear. The austenite-to-martensite transformation rate under shear loading is slower than that for uniaxial tension, but faster than for uniaxial compression. A martensite volume fraction of 90% is reached at an equivalent plastic strain of 0.3, which is almost twice as large as the strain reached under uniaxial tension. The experiment has been stopped at 0.3 due to the formation of cracks at the corners of the specimen gauge section.
4.6 Equi-biaxial tension

Displacement driven punch tests are performed to subject the SS301LN sheets to equi-biaxial tension. A circular disk of diameter 126mm is machined from the material, and then bolted on top of a 24.5mm-radius circular die. The plate is horizontal
during the test, and a hemispherical punch with a radius of 22.2mm moves downward to load the specimen. Five 0.05mm-thick Teflon layers, with grease in between all layers, are placed between the top of the specimen and the punch to reduce frictional effects. The bottom of the disc specimen is painted white with a black speckle pattern. Since the specimen deforms out of its initial plane during punch testing, stereo DIC is required to capture the deformation fields, and therefore two cameras are employed simultaneously. A mirror is positioned underneath the specimen at a 45° angle to monitor the deforming specimen with two digital cameras.

The punch is loaded under displacement control at a speed of 2mm/min, which corresponds to a local equivalent strain rate of the order of $10^{-4}\text{s}^{-1}$. The specimen is unloaded and reloaded at increments of about 4% of equivalent strain to perform
ferritescope measurements on the unloaded specimen. However, due to the residual stresses present during punch testing, these data are not reliable because of the effect of inverse magnetostriction on the apparent magnetic permeability. To ensure reliable ferritescope measurements, three interrupted tests are performed. Samples measuring 10\text{mm} \times 10\text{mm} are extracted from the middle of the deformed punch specimen to relieve the residual stresses before making the ferritescope measurement. The effect of the sample curvature on the ferritescope measurements has been taken into account through a correction procedure provided by the ferritescope manufacturer (Fischer, 2006).

Figure (4-9a) shows a contour plot of the maximum principal logarithmic strains on the deformed specimen surface as measured by stereo DIC for the maximum punch displacement. The corresponding evolutions of the two principal surface strains $\epsilon_I$ and $\epsilon_{II}$ at the apex of the punched specimen are shown in Figure (4-9b). The ratio of these two strains is close to 1 throughout the entire experiment. Thus, the equivalent plastic strain (von Mises definition) is approximated as

$$
\epsilon_{VM} \cong \epsilon_I + \epsilon_{II}.
$$

(4.4)

The corresponding data points ($\bar{\epsilon}_{VM}$, \chi) for the three interrupted tests are shown as black cross symbols in Figure (4-8). Since equivalent plastic strains as high as 0.43 can be achieved under equi-biaxial tension, the austenitic phase is almost completely transformed ($\chi > 95\%$) toward the end of the punch test. It is worth noting that the data points for equi-biaxial loading correspond well to the data for simple shear.
(a) Maximum logarithmic principal strains measured on the deformed specimen at the maximum punch displacement of a test to an equivalent strain of about 32%.

(b) Evolution of first two logarithmic principal strains during the same test shown in the figure above.

Figure 4-9: Stereo digital image correlation results during equi-biaxial tension experiment.

4.7 Modeling of the transformation kinetics

4.7.1 Background

Several transformation kinetics laws have been proposed in the literature to describe the martensite transformation as a function of strain, strain rate, temperature, and
stress state. One of the first proposed models, developed by Olson and Cohen (1975) [67], is an isotropic phase transformation law that describes the martensite content evolution as a function of plastic strain and temperature. Stringfellow et al. (1992) [86] expanded on the Olson and Cohen (1975) [67] law by incorporating the dependency of the martensitic transformation kinetics on stress triaxiality. The stress triaxiality, \( \eta \), is defined as the ratio of the hydrostatic stress, \( \sigma_m \), and the von Mises stress, \( \sigma_{VM} \). The hydrostatic stress is proportional to the first invariant, \( I_1 \), of the Cauchy stress tensor, \( \sigma \), while the von Mises stress is a monotonic function of the second invariant, \( J_2 \), of the deviatoric Cauchy stress tensor, \( s \). The stress triaxiality is given as

\[
\eta = \frac{\sigma_m}{\sigma_{VM}},
\]

where

\[
\sigma_m = \frac{\text{tr}(\sigma)}{3} \quad \text{and} \quad \sigma_{VM} = \sqrt{3J_2} \quad \text{with} \quad J_2 = \frac{1}{2} s \cdot s.
\]

Denoting the rate of martensite formation as \( \dot{\chi} \), the evolution law of Stringfellow et al. (1992) may be written as

\[
\dot{\chi} = (1 - \chi) (A \dot{\gamma}_a + B \dot{\eta}),
\]

where \( \dot{\gamma}_a \) is the plastic shear strain rate in the austenite, \( \dot{\eta} \) is the rate of change of stress triaxiality, and \( A \) and \( B \) are functions that depend on both temperature and stress state. Under isothermal conditions, the model by Stringfellow et al. (1992) [86] describes a monotonic relationship between the rate of transformation and the stress triaxiality. Figure (4-10) shows the predictions of Stringfellow’s model after calibration based on the data by Young (1988) [97] for uniaxial compression and tension on a phosphocarbide-strengthened austenitic steel.

However, our present experimental data reveal that the rate of transformation cannot be described as a monotonic function of the stress triaxiality only. The experimental curves (Figure (4-8)) show the following order with respect to transformation speed
(fast to slow):

- uniaxial tension ($\eta = 0.33$),
- equi-biaxial tension ($\eta = 0.66$) and simple shear ($\eta = 0$),
- uniaxial compression ($\eta = -0.33$).

The data for equi-biaxial tension clearly contradicts the assumption that the rate of martensitic transformation is a monotonic function of the stress triaxiality only. The above observation is supported by the experimental results of DeMania (1995) [21] who shows that under plane strain tension ($\eta = 0.58$), less martensite is developed than under uniaxial tension ($\eta = 0.33$).

![Graph showing predicted evolution of martensite content](image)

Figure 4-10: Predicted evolution of martensite content as shown in Stringfellow et al. (1992) after calibration based on the experiments by Young (1998) on a phosphocarbide-strengthened austenitic steel.

### 4.7.2 Lode angle dependent model

The assumption of a monotonic relationship between the stress triaxiality and the rate of transformation is supported by strong mechanism-based arguments as the
transformation from austenite to martensite is dilatational (see Stringfellow et al., 1992 [86]). Thus, we propose a new model that maintains this property, but makes use of the Lode angle parameter in addition to the stress triaxiality to account for the effect of stress state on the transformation kinetics. The dimensionless Lode angle parameter \( \bar{\theta} \) \((-1 \leq \bar{\theta} \leq 1)\) is a function of the normalized third invariant, \( J_3 \), of the deviatoric Cauchy stress tensor,

\[
\bar{\theta} = 1 - \frac{2}{\pi} \arccos \left( \frac{3\sqrt{3}}{2} \frac{J_3}{\sqrt{J_3^2}} \right) \quad \text{with} \quad J_3 := det(s).
\] (4.8)

The Lode angle parameter may be interpreted as a measure of the magnitude of the second principal stress, \( \sigma_{II} \), with respect to the maximum and minimum principal stresses, \( \sigma_I \) and \( \sigma_{III} \), respectively. It is a monotonically decreasing function of the ratio of the second and first principal deviatoric stresses (see Figure (4-11)). For example, we have

- \( \bar{\theta} = -1 \) when \( \sigma_{II} = \sigma_I \) (e.g., equi-biaxial tension or uniaxial compression),
- \( \bar{\theta} = 0 \) when \( \sigma_{II} = \frac{1}{2}(\sigma_I + \sigma_{III}) \) (e.g., pure shear),
- \( \bar{\theta} = 1 \) when \( \sigma_{II} = \sigma_{III} \) (e.g., uniaxial tension).

We choose the stress-state independent transformation kinetics model by Santacreu et al. (2006) [82] as starting point for our model development. For isothermal conditions, the rate form of Santacreu’s evolution law reads

\[
\dot{\chi} = (\chi_{max} - \chi) m D (D\dot{\varepsilon})^{m-1} \dot{\varepsilon},
\] (4.9)

where \( \chi_{max} \) is the maximum martensite volume fraction that can be developed in the material at a given temperature, and \( D \) and \( m \) are material parameters. Due to the apparent isotropic nature of the relationship between the martensite content evolution and the plastic strain (Beese and Mohr, 2010 [9]), we use the von Mises equivalent plastic strain \( \dot{\varepsilon}_{VM} \) as a scalar measure of the deformation in the sheet material. The
Figure 4-11: Lode angle parameter as a function of the ratio of second principal deviatoric stress to first principal deviatoric stress.

apparent anisotropy of the martensite formation under uniaxial compression will be neglected.

In order to account for the stress-state dependency of the martensitic transformation, we assume that $D$ is a linear function of the stress triaxiality and Lode angle parameter,

$$D = D(\eta, \bar{\theta}) = (D_0 + a_\eta \eta + a_\bar{\theta} \bar{\theta})_+$$

where the maximum operator $z_+ = \max(z, 0)$ is used to ensure that $D \geq 0$ for any $\eta$ and $\bar{\theta}$. The rate of martensite transformation increases with $D$, and therefore with higher stress triaxiality and Lode angle parameter when using $a_\bar{\theta} \geq 0$ and $a_\eta > 0$.

The comparison with the model by Stringfellow et al. (1992) [86] shows that the terms $(1 - \chi)$ and $(\chi_{max} - \chi)$ in Equations (4.7) and (3.4), respectively, prescribe a ceasing rate of transformation as the martensite reaches its maximum value. The only difference is that Stringfellow's model assumes a maximum martensite content of $c_{max} = 100\%$ while the present model (and Santacreu's model) can also be adjusted.
Table 4.2: Calibrated transformation kinetics material parameters for 1.5mm-thick temper-rolled stainless steel 301LN sheets.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\chi_{max}$</td>
<td>1</td>
</tr>
<tr>
<td>$D_0$</td>
<td>6</td>
</tr>
<tr>
<td>$a_\eta$</td>
<td>3</td>
</tr>
<tr>
<td>$a_\delta$</td>
<td>2</td>
</tr>
</tbody>
</table>

for smaller values of $\chi_{max}$. However, there is a major difference in the way the effect of stress triaxiality is incorporated. Stringfellow et al. (1992) [86] consider the stress triaxiality as a thermodynamic force that can actually change the martensite content. According to Equation (4.7), the martensite content may change ($\dot{\chi} > 0$) by changing the stress triaxiality ($\dot{\eta} \neq 0$) without deforming the austenite plastically. The mathematical structure of the present model rules out the possibility of martensitic transformation due to changes in stress triaxiality. Furthermore, the martensitic transformation is irreversible since $\dot{\gamma}_p \geq 0$ and thus $\dot{\chi} \geq 0$ for any increment of loading.

### 4.7.3 Experimental validation

The calibration of the evolution law, described by Equations (3.4) and (4.10) based on the present material data (Figure (4-8)), yields the model parameters shown in Table (4.2). The predicted martensite content versus equivalent plastic strain curves are shown in Figure (4-8). The comparison with the experimental data (dots) for different stress states indicates a good agreement between the model and the shown experimental results. The model would work less well for uniaxial compression along the cross-rolling direction since the effect of anisotropy on the transformation kinetics has been neglected. Note that the model prediction for uniaxial compression coincides with that for compression at $0^\circ$. Consequently, the difference between the experiment and model for $90^\circ$ is the same as that between the two experimental results shown in Figure (4-2).
4.8 Discussion

4.8.1 Effect of the Lode angle parameter

In the context of stress-induced martensitic transformation, Patel and Cohen (1953) [70] explain that both normal and shear stresses act as thermodynamic driving forces for martensitic transformation. Among the stresses on an austenite habit plane, shear and tensile normal stresses facilitate phase transformation while compressive normal stresses are hindering. In the special case of zero stress triaxiality, we have $\sigma = s$ and hence the maximum shear and normal stresses are determined by the stress deviator. In the principal stress coordinate system, we have

$$s = \begin{bmatrix} s_l & 0 & 0 \\ 0 & s_{II} & 0 \\ 0 & 0 & -s_l - s_{II} \end{bmatrix} \quad \text{with} \quad s_l \geq s_{II} \geq -s_l - s_{II}. \quad (4.11)$$

The maximum shear stress for the stress state described by Equation (4.11) is

$$\tau = s_l + \frac{s_{II}}{2} \geq 0, \quad (4.12)$$

while the normal stress that is acting on the corresponding plane of maximum shear reads

$$\sigma_N = -\frac{s_{II}}{2}. \quad (4.13)$$

We may thus rewrite the stress deviator in terms of the maximum shear stress and the corresponding normal stress,

$$s = \begin{bmatrix} \tau + \sigma_N & 0 & 0 \\ 0 & -2\sigma_N & 0 \\ 0 & 0 & -\tau - \sigma_N \end{bmatrix} \quad \text{with} \quad \tau \geq 0 \quad \text{and} \quad |\sigma_N| \leq \frac{\tau}{3}. \quad (4.14)$$

Applying the definitions of Equation (4.8), related to the third deviatoric stress invariant, and Equation (4.6), related to the second deviatoric stress invariant, we
obtain
\[
\frac{J_3}{\sqrt{J_2^2}} = 2 \frac{|\sigma_N|}{\sigma_N} \left( \frac{\tau}{\sigma_N} \right)^2 - 1 \left[ \frac{3 + \left( \frac{\tau}{\sigma_N} \right)^2}{3} \right].
\]  
(4.15)

Using Equation (4.8) in Equation (4.15), we find that the normal to shear stress ratio, \( \frac{\sigma_N}{\tau} \), is a monotonically increasing function of the Lode angle parameter (Figure (4-12)). In the case of zero stress triaxiality, the Lode angle parameter is negative when \( \sigma_N \) is compressive, increases to zero as \( \sigma_N \to \pm 0 \), and increases further as \( \sigma_N \) becomes tensile. Note that the above definition is only valid for \( \frac{|\sigma_N|}{\tau} < 0.33 \). Higher normal to shear stress ratios can only be achieved by increasing the stress triaxiality.

The above calculations show that the Lode angle parameter is monotonically related to the ratio of the normal and shear stress on the plane of maximum shear. Thus, the micromechanical argument by Patel and Cohen (1953) [70] explains the present experimental observation and modeling assumption that the rate of austenite-to-martensite transformation increases as the stress triaxiality and Lode angle parameter increase.

### 4.8.2 Effect of the model parameters

The influence of the model parameters \( m, \chi_{\text{max}}, D_0, a_\parallel \) and \( a_\perp \) on the transformation kinetics is shown in Figure (4-13). If \( D \) is equal to zero, the rate of martensite transformation is zero. As \( \eta \to \infty \) we have \( D \to \infty \), and therefore, the martensite transformation rate increases with an infinite stress triaxiality regardless of the normalized Lode angle parameter value. For \( m \) increases beyond 1, the rate of transformation increases. Finally, Figure (4-13) shows the resulting curves as \( \chi_{\text{max}} \) increases from \( \chi_0 \) (indicating that the initial material already contains a saturated volume of martensite) to 1.0 (meaning the maximum martensite content is 100%).
4.8.3 Effect of texture

The proposed model neglects the effect of texture on phase transformation. At the crystal level, the austenite-to-martensite transformation is accompanied by a shear strain and a positive normal strain on the habit plane (Machlin and Cohen, 1951 [51]), which is defined by the interface between the crystal lattices of the two phases (Blank and Kulnitskiy, 1997 [13]). The material before temper-rolling is solely polycrystalline austenite. During the rolling process, the crystal orientations change and texture is introduced (see pole figures in Beese and Mohr, 2010 [9]). At the same time, some of the austenite transforms into martensite, thereby producing the transformation habit planes. The orientation of the habit planes after temper-rolling has not been measured, but it is expected that their distribution is not isotropic, thereby creating preferential directions for phase transformation in the temper-rolled sheet material.

The experimental results shown in Figure (4-2) suggest that the rate of martensite
formation under uniaxial compression is higher when the compression axis is aligned with the 90° direction than when the compression axis is aligned with the 0° direction. Less notably, there appears to be a slightly higher rate of martensite formation under uniaxial tension when the tension is applied to the 0° direction compared to when tension is applied to the cross-rolling direction. Note that the in-plane deformation state under uniaxial tension along the rolling direction is similar to that for uniaxial compression along the 90° direction: in both cases, we have \( dc_{00} > 0 \) and \( dc_{90} < 0 \). Conversely, for uniaxial tension along the 90° direction and uniaxial compression along the 0° direction, we have \( dc_{90} > 0 \) and \( dc_{00} < 0 \).

Based on these experimental observations, and the fact that the austenite-to-martensite transformation requires a positive normal strain on the habit plane, it is speculated that habit planes in the current temper-rolled sheet material are preferentially oriented along the cross-rolling direction (i.e., the habit plane normals are preferentially aligned with the rolling direction). This would result in a higher probability, and hence rate, of transformation for loading states with tensile strains along the rolling direction.

### 4.9 Conclusions

Two newly developed experimental techniques for the in-plane shear and finite strain compression testing of sheet materials are used to investigate the effect of stress state on the strain-induced austenite-to-martensite phase transformation in stainless steel 301LN. The comparison of the evolution of the martensite volume fraction for uniaxial tension, uniaxial compression, simple shear, and equi-biaxial tension reveals that the rate of martensite formation cannot be described as a monotonic function of the stress triaxiality.

A stress-state dependent transformation kinetics law is proposed that accounts for
the effect of the Lode angle parameter in addition to stress triaxiality. The normal stress acting on the plane of maximum shear is monotonically related to both the stress triaxiality and the Lode angle parameter; it increases with increasing stress triaxiality and increasing Lode angle parameter. Based on the present experimental observations, it is thus argued that tensile normal stresses on the plane of maximum shear facilitate the martensitic transformation in stainless steel.

The proposed isotropic transformation kinetics law is presented for isothermal conditions. It can describe the transformation kinetics over a wide range of stress states, but cannot account for the apparent direction dependency of the phase transformation under uniaxial compression.
Figure 4-13: Parametric study of transformation kinetics law given in Equation (4.9) for equi-biaxial tension, where \( \eta = \frac{2}{3} \) and \( \theta = -1 \). In (a) \( m = \{0.5, 1, 2, 5, 10\} \); in (b) \( \chi_{\text{max}} = \{0.2, 0.4, 0.6, 0.8, 1.0\} \); in (c) \( D_0 = \{0, 1, 5, 10, 100\} \); in (d) \( a_\eta = \{0, 2, 4, 5, 8\} \), and in (e) \( a_\eta = \{0, 2, 4, 6, 8\} \). The parameters held constant in all graphs are \( m = 1.5 \), \( \chi_{\text{max}} = 1.0 \), \( D_0 = 6 \), \( a_\eta = 2 \), and \( a_\eta = 3 \).
Chapter 5

Anisotropic Plasticity Model
Coupled with Stress-State
Dependent Transformation
Kinetics Law

5.1 Introduction

Both micromechanics-based and phenomenological macroscopic constitutive models have been developed for metastable austenitic steels. The micromechanics-based models make use of separate constitutive equations for the austenitic and martensitic phases, and possibly others, while an evolution law is used to describe the changing phase volume fractions; the effective behavior of the multi-phase material is then computed through homogenization. Several homogenization techniques have been adopted ranging from simple rules of mixtures (e.g., Lecroisey and Pineau, 1972 [49]; Hänsel et al., 1998 [33]; Hallberg et al., 2007 [32]; Santacreu et al., 2006 [82]; and Post et al., 2008 [76]) to more complex homogenization techniques that account for field fluctuations within the phases (e.g., Leblond et al., 1986a [47]; Leblond et al.,

\[Reproduced\ from: \ AM\ Beese\ and\ D\ Mohr,\ 2011.\ Anisotropic\ plasticity\ model\ coupled\ with\ Lode\ angle\ dependent\ strain-induced\ transformation\ kinetics\ law.\ Submitted\ for\ publication\ [10].\]
1986b [48]; Leblond, 1989 [45]; Leblond et al., 1989 [46]; Stringfellow and Parks, 1991 [87]; Stringfellow et al., 1992 [86]; Bhattacharyya and Weng, 1994 [12]; Diani et al., 1995 [23]; Cherkaoui et al., 1998 [19]; Papatriantafillou et al., 2004 [69]; Turteltaub and Suiker, 2005 [93]; and Papatriantafillou et al., 2006 [68]).

The micromechanical model by Hallberg et al. (2007) [32] for transforming solids makes use of a yield potential and a transformation potential. The transformation potential incorporates the stress state by including both the second and third invariants of the deviatoric stress, while a nonlinear rule of mixtures is employed to calculate the macroscopic flow stress of the evolving two-phase composite. Their model accounts for both the Greenwood-Johnson (Greenwood and Johnson, 1965 [30]) and the Magee (Magee, 1966 [52]) effects in a phenomenological manner. Post et al. (2008) [76] propose a model for both stress-assisted and strain-induced martensite formation along with aging effects. The backbone of their model is a description of the evolving dislocation densities in each phase (Estrin, 1996 [26]), assuming that the newly formed martensite will inherit the dislocation properties of the parent austenite. Leblond et al. (1986a) [47] demonstrate through the use of Mandel-Hill homogenization (Mandel, 1964 [53]; Hill, 1967 [36]) that the macroscopic strain rate in a transforming material may be decomposed into a first term related to classical plasticity and a second related to transformation plasticity. Later, Leblond et al. (1989) [46] developed a model that describes the transformation strain in terms of the stress deviator, the martensite content, and the martensite transformation rate for ideal plastic phases. Leblond (1989) [45] further expands this model by developing evolution equations for isotropic and kinematic hardening in the two phases.

Stringfellow and Parks (1991) [87] use a self-consistent homogenization scheme to predict the inelastic stress-strain behavior of multiphase materials, assuming isotropic and viscoplastic constituent phases. This model does not account for an evolving volume content of the phases, and its applicability is limited to materials of high rate sensitivity and small hardness differences between the constituent phases. Bhat-
tacharyya and Weng (1994) [12] propose an energy-based criterion to describe the constitutive behavior of ductile materials undergoing austenite-to-martensite phase transformation. Instead of assuming an explicit transformation kinetics law, the evolving martensite volume fraction is calculated incrementally from the change in Gibbs' free energy, and the strains are estimated using the lattice parameters of the parent and transformed phases. Diani et al. (1995) [23] use a self-consistent homogenization scheme to come up with a micromechanics-based model that describes the macroscopic transformation strain rate in terms of an effective tangent modulus, the mechanical properties of the martensite, and the rate of martensite formation.

Motivated in part by the experimentally observed lower equivalent stress under torsion as compared with compression (Miller and McDowell, 1996a [54]), Miller and McDowell (1996b) [55] propose a plasticity model that incorporates the third invariant of the overstress tensor, where overstress is defined as the difference between the Cauchy stress and the back stress. They conclude that the difference in strain hardening behavior is due to both geometric effects, as shear deformation results in a lower Taylor factor than compression, as well as stress-state dependent material hardening, as there are more possible planes of maximum shear stress in compression than torsion. Cherkaoui et al. (1998) [19] develop a thermomechanical model to describe the behavior of a single crystal of austenite undergoing strain-induced phase transformation. They propose a specific form of the Helmholtz free energy and make use of the corresponding thermodynamic driving forces to model the phase transformation and plastic flow. The thermomechanical multiscale model of Turtletaub and Suiker (2005) [93] describes the stress-induced phase transformations from a cubic to a tetragonal crystal structure. The model is developed for a single crystal of austenite, considering the kinetics and thermodynamics of transformation at multiple length scales. Papatriantafillou et al. (2006) [68] present a constitutive model for a four-phase TRIP steel. They decompose the total strain into an elastic strain, a plastic strain, and a transformation strain, where the transformation strain contains all of the inelastic dilatation, as well as a deviatoric component (modeled after Stringfellow
et al., 1992 [86]). They develop hardening laws for the individual phases and use a homogenization technique for nonlinear composites (Ponte Castañeda, 1991 [73]; Ponte Castañeda, 1992 [74]; and Suquet, 1996 [88]) to estimate the strain distribution in the individual phases as well as the resulting macroscopic properties of the material.

The computational cost of most micromechanics-based models is still too high for use in industrial applications. Phenomenological models have thus been developed as a computationally efficient alternative to micromechanics-based models. For example, Mohr and Jacquemin (2008) [56] proposed a plasticity model that makes use of a Hill 1948 (Hill, 1948 [35]) yield surface along with a non-associated hardening model to describe the direction dependency of the strain hardening in an anisotropic stainless steel of type 301LN. Hänsel et al. (1998) [33] developed a temperature dependent isotropic constitutive model for TRIP steels. It makes use of the isothermal transformation kinetics law proposed by Tsuta and Cortes (1993) [92], in which the martensite volume content is defined as a function of the von Mises equivalent plastic strain. Hänsel et al. (1998) [33] introduce an empirical weighting function of temperature to account for the temperature-dependency of the phase transformation. The macroscopic behavior is described through a von Mises yield surface with an isotropic hardening law that depends both on the equivalent plastic strain and the martensite content. Schedin et al. (2004) [83] slightly modified the implementation of the model by Hänsel et al. (1998) [33] through the introduction of an anisotropic yield function. This empirical model is implemented in the commercial finite element software LS-DYNA, but it does not account for the effect of stress state on martensite evolution.

Many experimentalists have shown that the phase transformation in austenitic steels is stress-state dependent (e.g., Cina, 1954 [20]; Powell et al., 1958 [78]; Hecker et al., 1982 [34]; Murr et al., 1982 [60]; Young, 1988 [97]; Kosarchuk et al., 1989 [42]; Okutani et al., 1995 [65]; DeMania, 1995 [21]; Miller and McDowell, 1996a [54]; Iwamoto et al., 1998 [38]; Lebedev and Kosarchuk, 2000 [44]; Shin et al., 2003 [84]; Yan et al., 2006 [95]; Perdahcioglu et al., 2008 [72]; and Nanga et al., 2009 [61]). In other
words, the transformation kinetics cannot be described as a function of the von Mises equivalent plastic strain only. Stringfellow et al. (1992) [86] addressed this issue by incorporating the effect of stress triaxiality into their micromechanics-based plasticity model. However, the recent experimental analysis of Beese and Mohr (2011) [11] has explicitly shown that the third invariant of the deviatoric Cauchy stress tensor (Lode angle) affects the rate of phase transformation under isothermal conditions in addition to the stress triaxiality.

It is the objective of the present work to develop an isothermal rate independent phenomenological finite strain plasticity model for solids undergoing strain-induced austenite-to-martensitic transformation that incorporates the effect of stress triaxiality and Lode angle on the transformation kinetics. The proposed constitutive model is composed of an anisotropic yield function with a nonlinear kinematic hardening law of the Frederick-Armstrong type (e.g., Chaboche, 2008 [18]), and an isotropic hardening function that is coupled with a stress-state dependent transformation kinetics law. The model is implemented into a finite element software, calibrated based on selected experiments, and used to predict the stress-strain response for various loading conditions. It is found that the proposed model is able to describe the material’s constitutive response over a wide range of stress states including uniaxial tension, uniaxial compression, transverse plane strain tension, simple shear, and equi-biaxial tension.

5.2 Experimental procedures and results

The results from static experiments performed by Beese and Mohr (2011) [11] on a stainless steel 301LN are briefly summarized. In particular, we focus on the experiments for uniaxial tension, in-plane uniaxial compression, shear, and equi-biaxial tension. During each experiment, the applied force and the displacement fields on the specimen surface are measured using digital image correlation. In addition, a
ferritescope is used to measure the martensite evolution as a function of the strain and stress state (Beese and Mohr, 2010 [9]).

5.2.1 Uniaxial loading

Static uniaxial tension and in-plane uniaxial compression tests are performed. During both types of experiments, the gauge section is painted white with a black speckle pattern and DIC is used to determine the displacement and strain fields on the specimen gauge section surfaces. The uniaxial tension experiments are standard and adhere to ASTM E8 [2]. However, the in-plane compression tests require a custom-made anti-buckling device to prolong the initial range of membrane-dominated deformation. As detailed in Beese and Mohr (2011) [11], this device is composed of two \( \frac{1}{4} \text{in} \) thick aluminum plates, which sandwich the gauge section of the in-plane compression specimen. A set of fourteen springs holds the two face plates together by applying a compressive pressure of about 1.5MPa. This pressure is sufficiently high to delay the transition from membrane- to bending-dominated loading due to buckling. At the same time, it is sufficiently low such that the measured stress-strain curve is not affected by the friction between the specimen and the aluminum plates. Teflon between the face plates and the gauge section renders the frictional effects negligible. Using this procedure, the uniaxial stress-strain curve for compression could be determined for true strains of up to 10%. The measured stress-strain curves for uniaxial tension and compression in the cross-rolling direction are shown in Figure (5-1), which emphasizes the differential yield and hardening behavior in these two stress states.

5.2.2 Combined tension and shear

Mohr and Jacquemin (2008) [56] performed combined tension and shear experiments on the present sheet material. They used a custom made hydraulic dual-actuator loading machine and followed the experimental procedures developed by Mohr and
Figure 5-1: Absolute true stress-strain curves for uniaxial tension (dotted line) and compression (solid line) along the cross-rolling direction of stainless steel 301LN sheets.

Oswald (2008) [57]. The specimens used are flat sheet specimens with a reduced thickness gauge section (Figure 5-2), resulting in plane strain conditions along the horizontal direction of the specimen and plane stress through the gauge thickness of the specimen. The tests are performed under force control, and various combinations of shear and tensile loading are applied to each specimen by varying the biaxial loading angle, $\beta$ (Figure 5-2), which describes the ratio of the vertical (normal) force to the horizontal (shear) force. Here, we make use of their results from a first series of experiments where the horizontal axis (plane strain direction) is aligned with the material rolling direction, and a second series where the horizontal axis coincides with the cross-rolling direction.

### 5.3 Plasticity model

A phenomenological model is developed to describe the large deformation behavior of the austenitic stainless steel under static loading at room temperature. In addition
to the equivalent plastic strain, we introduce the martensite content as an internal state variable because of its first order effect on the rate of strain hardening. In the following, we outline the rate independent finite strain constitutive equations, which involve the yield surface, flow rule, isotropic hardening law, kinematic hardening law, and the martensite transformation kinetics law. Bold upper case letters (e.g., $B$) and double underscored lower case bold letters (e.g., $b$) are used to denote matrices and tensors, while bold lowercase letters without underscore (e.g., $b$) are used to denote vectors. Square brackets are used exclusively to indicate the arguments of a function, while round and curly brackets are employed to signify the precedence of mathematical operations.

5.3.1 Kinematics of finite strain

The constitutive model is implemented in the commercial finite element software Abaqus/explicit. Therefore, the standard finite strain formulation for plane stress shell elements with co-rotational coordinate frames is used (Abaqus, 2008 [4]). The
Cauchy stress tensor in the current configuration is denoted as $\bar{\sigma}$, while $d\bar{\epsilon}$ denotes the work conjugate strain increment. Stress and strain components, $\sigma_{ij}$ and $\epsilon_{ij}$, are reported in the current material coordinate systems, assuming that the orthotropic material symmetry is preserved throughout loading. Formally, we write

$$\bar{\sigma} = \sigma_{ij} (R e_i \otimes R e_j),$$

(5.1)

with $R$ denoting the rotation of the co-rotational material coordinate system; the unit vectors $e_i$ and $e_j$ are aligned with the initial rolling and cross-rolling directions.

### 5.3.2 Yield surface

The results from multi-axial experiments (Mohr and Jacquemin, 2008 [56]) have demonstrated that the quadratic anisotropic yield function by Hill (1948) [35] provides a good approximation of the initial yield surface of the stainless steel 301LN sheet material. It is defined as

$$f = \bar{\sigma} - k = 0,$$

(5.2)

where $\bar{\sigma} = \bar{\sigma}[\bar{\sigma}]$ defines the equivalent stress, $k$ is the deformation resistance, and $\bar{\sigma}$ is the Cauchy stress tensor. The Hill 1948 equivalent stress is typically given in the form

$$\bar{\sigma} = \left\{ F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 
+ 2L\sigma_{12}^2 + 2M\sigma_{13}^2 + 2N\sigma_{32}^2 \right\}^{\frac{1}{2}},$$

(5.3)

where $F$, $G$, $H$, $L$, $M$, and $N$ are the coefficients describing the material anisotropy. In the present model, we introduce a deviatoric back stress tensor $\alpha$, and replace the components $\sigma_{ij}$ in Equation (5.3) by $\sigma_{ij} - \alpha_{ij}$.
5.3.3 Associated flow rule

An associated flow rule is chosen to describe the evolution of the plastic strain tensor. Therefore, the increment in plastic strains, \( d\mathbf{\varepsilon}^p \), is proportional to the derivative of the equivalent stress,

\[
d\mathbf{\varepsilon}^p = d\lambda \frac{\partial \bar{\sigma}}{\partial \bar{\sigma}},
\]

where \( d\lambda \geq 0 \) is the plastic multiplier. The integral \( \bar{\varepsilon}_p = \int d\lambda \) is referred to as anisotropic equivalent plastic strain.

5.3.4 Kinematic hardening

The nonlinear kinematic hardening rule is written as

\[
d\mathbf{a} = \frac{2}{3} c_L d\mathbf{\varepsilon}^p - c_{NL} \mathbf{a} d\lambda,
\]

where \( c_L \) and \( c_{NL} \) are material parameters. For \( c_{NL} = 0 \), Equation (5.5) reduces to the linear kinematic hardening law by Prager (1949) [79]; in this case, the back stress evolution is unbounded and evolves along the direction of the plastic strain increment. For \( c_{NL} \neq 0 \), we activate the dynamic recovery term proposed by Armstrong and Frederick (1966 [8], 2007 [28]). As a result, the evolution of the back stress is no longer unbounded and converges towards a saturation value under monotonic loading. As discussed by Lemaitre and Chaboche (1994) [50], the dynamic recovery term may be interpreted as a description of the "fading memory effect of the strain path." For example, in the case of uniaxial tension, the back stress evolution asymptotically approaches a saturation value.

5.3.5 Isotropic hardening

In addition to a kinematic hardening law, an isotropic hardening law is used to describe the evolution of the deformation resistance, \( k \), during plastic loading. It is as-
sumed that the deformation resistance depends on the evolution of the plastic strain and the martensite volume fraction. We decouple the mechanisms of increased deformation resistance due to dislocation pile-up from that due to the formation of martensite in an additive manner. We write

\[ dk = H_c d\varepsilon_{VM} + H_x d\chi, \]  

(5.6)

where \( d\varepsilon_{VM} \) is the increment in the isotropic equivalent plastic strain (von Mises plastic strain), and \( d\chi \) is the increment in the martensite volume fraction. Under the absence of phase transformation, it is assumed that the deformation resistance is an exponential function of the von Mises equivalent plastic strain,

\[ k = k_0 + H_0 \left\{ 1 - \exp \left[ -A\varepsilon_{VM}^p \right] \right\}, \]  

(5.7)

with the model parameters \( k_0, H_0, \) and \( A. \) Consequently, the relationship for the isotropic strain hardening modulus reads

\[ H_c [\varepsilon_{VM}^p] = \frac{dk}{d\varepsilon_{VM}^p} = AH_0 \exp \left[ -A\varepsilon_{VM}^p \right]. \]  

(5.8)

The second hardening modulus, \( H_x, \) characterizes the rate of strain hardening due to an increasing martensite volume fraction, and is assumed to be constant.

5.3.6 Transformation kinetics law

The isotropic transformation kinetics law developed in Beese and Mohr (2011) [11] is used to describe the martensite evolution as a function of plastic strain and stress state. The latter will be characterized by the stress triaxiality and the Lode angle parameter. The stress triaxiality, \( \eta, \) is proportional to the ratio of the first invariant of the Cauchy stress tensor, \( I_1, \) and the second invariant of the deviatoric stress tensor,
where \( \mathbf{s} \) is the deviatoric stress tensor. The dimensionless Lode angle parameter, \( \tilde{\theta} \), depends on the ratio of the second and third invariants of the deviatoric stress tensor, \( J_2 \) and \( J_3 \), respectively. Its definition reads

\[
\tilde{\theta} = 1 - \frac{2}{\pi} \arccos \left( \frac{3 \sqrt{3} J_3}{2 \sqrt{J_2^3}} \right) \quad \text{with} \quad J_3 := \det[\mathbf{s}].
\]

According to the transformation kinetics law by Beese and Mohr (2011) [11], the change of martensite volume fraction is governed by the differential equation

\[
dx = (\chi_{\max} - \chi) m D (D\tau_{YM})^{m-1} (d\tau_{YM}).
\]

The function \( D \) depends on both the stress triaxiality and the Lode angle parameter,

\[
D = (D_0 + \alpha_\eta \eta + \alpha_{\tilde{\theta}} \bar{\theta})_+,
\]

with the material model parameters \( \chi_{\max}, m \geq 0, D_0, \alpha_\eta, \) and \( \alpha_{\tilde{\theta}}. \) The maximum operator \( z_+ = \max(z, 0) \) is used to ensure that \( D \geq 0 \) for any \( \eta \) and \( \tilde{\theta}. \) In the special case of \( \alpha_\eta = \alpha_{\tilde{\theta}} = 0, \) the above relationship reduces to the transformation kinetics law of Santacreu et al. (2006) [82] for isothermal conditions.

### 5.3.7 Complementary conditions

The direction of plastic evolution is unilaterally constrained through the Kuhn-Tucker complementary conditions

\[
d\lambda \geq 0, \quad df \leq 0 \quad \text{and} \quad (d\lambda)(df) = 0.
\]
Consequently, the isotropic strain hardening is irreversible. The evolution direction of the martensite content is also implicitly constrained to \( d\chi \geq 0 \) by the particular choice of the transformation kinetics law. Thus, we only allow for a transformation from austenite-to-martensite, while the reverse transformation is prohibited.

### 5.3.8 Specialization for plane stress

For the case of plane stress, it is worth rewriting the constitutive equations in vector notation. We define the stress vector

\[
\sigma = \begin{bmatrix} \sigma_0 & \sigma_{90} & \tau \end{bmatrix}^T, \tag{5.14}
\]

with the Cauchy stress components \( \sigma_0 = \sigma_{11}, \sigma_{90} = \sigma_{22}, \) and \( \tau = \sigma_{12} \). The strain vector definition reads

\[
\epsilon = \begin{bmatrix} \epsilon_0 & \epsilon_{90} & \gamma \end{bmatrix}^T, \tag{5.15}
\]

where \( \epsilon_0 = \epsilon_{11} \) and \( \epsilon_{90} = \epsilon_{22} \) are the logarithmic strains along the rolling and cross-rolling directions; \( \gamma = 2\epsilon_{12} \) denotes the corresponding in-plane shear strain, which is twice the mathematical shear strain.

The yield function with back stress for plane stress conditions reduces to

\[
\bar{\sigma} = \left\{ F(\sigma_{22} - \alpha_{22} + \alpha_{33})^2 + G(\sigma_{11} - \alpha_{11} + \alpha_{33})^2 \right. \\
+ H(\sigma_{11} - \alpha_{11} - \sigma_{22} + \alpha_{22})^2 + 2L(\sigma_{12} - \alpha_{12})^2 \left\}^{\frac{1}{2}}. \tag{5.16}
\]

We introduce the back stress vector \( \beta \) with the components \( \beta_1 = \alpha_{11} - \alpha_{33}, \beta_2 = \alpha_{22} - \alpha_{33}, \) and \( \beta_3 = \alpha_{12}, \) and rewrite Equation (5.16) as

\[
\bar{\sigma} = \sqrt{P(\sigma \cdot \beta)} \cdot (\sigma \cdot \beta), \tag{5.17}
\]
with the anisotropy coefficient matrix

\[
\begin{bmatrix}
P_{11} & P_{12} & 0 \\
P_{12} & P_{22} & 0 \\
0 & 0 & P_{33}
\end{bmatrix} = \begin{bmatrix}
G + H & -H & 0 \\
-H & F + H & 0 \\
0 & 0 & 2L
\end{bmatrix}.
\] (5.18)

The corresponding flow rule for plane stress reads

\[
d\epsilon^p = \frac{d\lambda}{\sigma} \mathbf{P}(\sigma) \mathbf{r}.
\] (5.19)

while the thickness strain is computed based on the assumption of plastic incompressibility, \(d\epsilon_{33}^p = -(d\epsilon_0^p + d\epsilon_{60})\). The equations for isotropic hardening remain unchanged; however, the nonlinear kinematic hardening rule must be revisited. Evaluation of the kinematic hardening rule, Equation (5.5), for plane stress yields the evolution equation

\[
d\beta = c_L G d\epsilon^p - c_{NL} \beta d\lambda = \left\{ \frac{c_L}{\sigma} \mathbf{G} \mathbf{P}(\sigma) \mathbf{r} \right\} d\lambda,
\] (5.20)

where

\[
\mathbf{G} = \frac{2}{3} \begin{bmatrix}
2 & 1 & 0 \\
1 & 2 & 0 \\
0 & 0 & 0.5
\end{bmatrix}.
\] (5.21)

### 5.4 Computational model for plane stress

The time integration scheme is developed to implement the proposed constitutive model for plane stress into a nonlinear explicit finite element code. For a given strain increment \(\Delta\epsilon = \epsilon_{n+1} - \epsilon_n\), the algorithm provides an approximate solution to the strain-driven problem, where the state variables \(\beta_{n+1}\) and \(\chi_{n+1}\), the equivalent strains \(\tilde{\epsilon}_{n+1}^p\) and \(\tilde{\tau}_{V,n+1}^p\), and stresses \(\sigma_{n+1}\) are computed at time \(t_{n+1} = t_n + \Delta t\) based on the solution at time \(t_n\). If the elastic trial stress,

\[
\sigma^{tr} = C(\epsilon_{n+1} - \epsilon_n),
\] (5.22)
lies outside the elastic domain,

$$ f[\sigma^{tr}, \beta_n, c_n, \lambda_n] > 0, \quad (5.23) $$

a return-mapping scheme is employed to solve the constitutive equations, where $C$ is the elasticity tensor. The equivalent plastic strain, $\lambda$, is used as an evolution parameter to discretize all evolution equations. A backward-Euler (implicit) integration procedure is used to approximate the derivatives with respect to $\lambda$,

$$ y_{n+1} = y_n + \Delta \lambda \left. \frac{dy}{d\lambda} \right|_{n+1}, \quad (5.24) $$

and thus

$$ \frac{\partial y_{n+1}}{\partial (\Delta \lambda)} = \left. \frac{dy}{d\lambda} \right|_{n+1}. \quad (5.25) $$

In order to determine the stress and state variables at time $t_{n+1}$, we solve the discretized consistency condition at time $t_{n+1}$,

$$ \left. \frac{df}{d\lambda} \right|_{n+1} = \frac{f_{n+1} - f_n}{\Delta \lambda} = 0. \quad (5.26) $$

For sufficiently small time steps and smooth loading histories, we have $f_n \approx 0$. Thus, the computational problem reduces to determining $\Delta \lambda$ such that

$$ f_{n+1} = f_{n+1}[\Delta \lambda] = 0. \quad (5.27) $$

An iterative Newton-Raphson scheme is employed to solve Equation (5.27). During the $i^{th}$ iteration, the solution $\Delta \lambda_{i+1}$ is obtained from linearizing Equation (5.27),

$$ \Delta \lambda_{i+1} = \Delta \lambda_i - \frac{f_{n+1}[\Delta \lambda_i]}{\left. \frac{\partial f_{n+1}}{\partial (\Delta \lambda)} \right|_i}. \quad (5.28) $$

Based on the solution $\Delta \lambda_i$ of the previous iteration, we calculate the partial derivatives

$$ \left. \frac{\partial f_{n+1}}{\partial (\Delta \lambda)} \right| = \left. \frac{\partial \sigma_{n+1}}{\partial (\Delta \lambda)} \right| - \left. \frac{\partial k_{n+1}}{\partial (\Delta \lambda)} \right| \quad (5.29) $$

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using the chain rule. Defining $\xi := \sigma - \beta$ the first term of Equation (5.29) reads

$$
\frac{\partial \sigma_{n+1}}{\partial (\Delta \lambda)} = \frac{\partial \sigma}{\partial \xi}_{n+1} 
\left( \frac{\partial \sigma_{n+1}}{\partial (\Delta \lambda)} \frac{\partial \sigma_{n+1}}{\partial (\Delta \lambda)} \right), \tag{5.30}
$$

with

$$
\frac{\partial \sigma}{\partial \xi}_{n+1} = P \left( \frac{\xi_{n+1}}{\sigma_{n+1}} \right) \tag{5.31}
$$

$$
\frac{\partial \sigma_{n+1}}{\partial (\Delta \lambda)} = \frac{\partial \sigma}{\partial \lambda}_{n+1} = -C \frac{\partial \sigma}{\partial \xi}_{n+1} \tag{5.32}
$$

$$
\frac{\partial \beta_{n+1}}{\partial (\Delta \lambda)} = \frac{\partial \beta}{\partial \lambda}_{n+1} = c_L - G \frac{\partial \sigma}{\partial \xi}_{n+1} - c_{NL} \beta_{n+1}. \tag{5.33}
$$

The second term of Equation (5.29) reads

$$
\frac{\partial k_{n+1}}{\partial (\Delta \lambda)} = AH \exp[-A R_{VM,n+1}] \frac{\partial \bar{e}_{VM}}{\partial \lambda}_{n+1} + H_x \frac{\partial \chi}{\partial \lambda}_{n+1}, \tag{5.34}
$$

with

$$
\frac{\partial \bar{e}_{VM}}{\partial \lambda}_{n+1} = \sqrt{B} \frac{\partial \sigma}{\partial \xi}_{n+1} \frac{\partial \sigma}{\partial \xi}_{n+1}, \tag{5.35}
$$

$$
B = (e_1 \otimes e_1 + e_2 \otimes e_2) + \frac{2}{3} (e_1 \otimes e_2 + e_2 \otimes e_1) + \frac{1}{3} I, \tag{5.36}
$$

and

$$
\frac{\partial \chi}{\partial \lambda}_{n+1} = (\chi_{\text{max}} - \chi_{n+1}) m D (D_{VM,n+1})^{m-1} \frac{\partial \bar{e}_{VM}}{\partial \lambda}_{n+1}, \tag{5.37}
$$

with the stress-state dependent function

$$
D = D_0 + a_0 [\sigma_{n+1}] + a_0 [\sigma_{n+1}]. \tag{5.38}
$$

After calculating the new solution $\Delta \lambda_{n+1}$ using Equation (5.28), we update the internal state variables

$$
\beta_{n+1} = \beta_n + \frac{\partial \beta}{\partial \lambda}_{n+1} (\Delta \lambda) \tag{5.39}
$$

$$
\chi_{n+1} = \chi_n + \frac{\partial \chi}{\partial \lambda}_{n+1} (\Delta \lambda) \tag{5.40}
$$

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and the equivalent plastic strains

\[
\bar{\varepsilon}_{VM,n+1}^p = \bar{\varepsilon}_{VM,n}^p + \left. \frac{\partial \bar{\varepsilon}_{VM}^p}{\partial \lambda} \right|_{n+1} (\Delta \lambda) \tag{5.41}
\]

\[
\bar{\varepsilon}_{n+1}^p = \bar{\varepsilon}_n^p + \Delta \lambda. \tag{5.42}
\]

Subsequently, the stresses are updated

\[
\sigma_{n+1} = \sigma^{ir} - C \left. \frac{\partial \bar{\sigma}}{\partial \xi} \right|_{n+1} (\Delta \lambda) \tag{5.43}
\]

\[
\xi_{n+1} = \sigma_{n+1} - \beta_{n+1} \tag{5.44}
\]

\[
\bar{\sigma}_{n+1} = \sqrt{ \mathbf{P} \xi_{n+1} \cdot \xi_{n+1}} \tag{5.45}
\]

\[
k_{n+1} = k_0 + H_0 \left\{ 1 - \exp[-A \bar{\varepsilon}_{VM,n+1}^p] \right\} + H_\chi \chi_{n+1} \tag{5.46}
\]

before evaluating

\[
f_{n+1} = \bar{\sigma}_{n+1} - k_{n+1}. \tag{5.47}
\]

The iterative procedure is continued until the criterion

\[
|f_{n+1}| \leq TOL \tag{5.48}
\]

is met for \( TOL = 10^{-2} \text{MPa} \).

To demonstrate the objectivity of the above time integration scheme, we apply plane strain tension along with a rigid body rotation to a single shell element. For plane strain tension along the cross-rolling direction along with in-plane rigid body rotation, the current position \( \mathbf{x} \) for a point within a single shell element is described by

\[
\mathbf{x}(t) = \mathbf{L}(t) \{ \mathbf{X} + \epsilon_{eq} \mathbf{e}_2 \} - \mathbf{X}, \tag{5.49}
\]
with the rotation

$$\mathbf{L}(t) = (\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2) \cos(\dot{\theta}t) + (\mathbf{e}_1 \otimes \mathbf{e}_2 - \mathbf{e}_2 \otimes \mathbf{e}_1) \sin(\dot{\theta}t),$$

and the position vector $\mathbf{X}$ in the reference configuration. In the simulation, 100,000 explicit time steps are performed to reach a maximum strain of $\epsilon_{90}$ while rotating the shell element by $2\pi$ at a constant loading velocity ($\dot{\theta} = \text{const.}, \dot{\epsilon}_{90} = \text{const.}$). Figure (5-3) shows selected intermediate configurations of the deforming shell element. Figure (5-4) shows the stress and strain evolutions in the single shell element using the user defined plasticity model. The symbols in each figure give the curves for an element undergoing plane strain tension along with rigid body rotation. The data from this deformation are compared with the evolution of stress and strain for an element that undergoes plane strain tension with zero rigid body rotation (solid lines). As expected, we observe the results are identical, which illustrates the objectivity of the proposed time integration scheme.

### 5.5 Model calibration and validation

The above anisotropic plasticity model for metastable austenitic steel sheets requires the identification of seventeen model parameters:

- **Yield surface coefficients:** $P_{11}, P_{22},$ and $P_{33}$
- **Isotropic hardening:** $k_0, A, H_0,$ and $H_x$
- **Kinematic hardening:** $c_L, c_{NL}$
- **Initial back stress:** $\beta_1^0,$ and $\beta_2^0,$ and $\beta_3^0$
- **Transformation kinetics:** $\chi_{\text{max}}, m, D_0, a_\eta,$ and $a_\bar{\eta}.$

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All material model parameters may be identified based on experiments along the rolling and cross-rolling directions for

- uniaxial tension,
- uniaxial compression,
- transverse plane strain tension, and
- pure shear.

Figure 5-3: Deformation history for plane strain tension along the material 90° direction during 360° rigid body rotation of a single shell element (S4R), with local coordinate system shown in white.

5.5.1 Model parameter identification

The identification of the material model parameters associated with the transformation kinetics has been detailed in Beese and Mohr (2011) [11] and will not be discussed.
Figure 5-4: Evolution of stresses and strains during simulation demonstrating the objectivity of the plasticity model.

here. The remaining model parameters are identified in a two-step procedure: a first estimate of the material model parameters is made based on approximate analytical solutions of the constitutive equations for simple loading conditions; subsequently, the analytical estimates are used as seed values for the parameter identification through
Monte Carlo simulations. The computational version of the constitutive model is used for the latter step.

Initial values for the anisotropy matrix components $P_{11}$, $P_{22}$, and $P_{33}$ are obtained from the measured Lankford parameters $r_0 = 0.67$, $r_{45} = 0.67$, and $r_{90} = 0.89$. With $\alpha$ denoting the angle between the tensile direction and the rolling direction, we have

$$r_\alpha = \frac{H + (2L - F - G - 4H) \sin^2 \alpha \cos^2 \alpha}{F \sin^2 \alpha + G \cos^2 \alpha},$$

(5.51)

and thus $P_{11} = 1.17$, $P_{22} = -0.47$, and $P_{33} = 2.88$.

The measured initial yield stresses for tension and compression are used to identify the seed values for the initial back stresses $\beta^0_1$, and $\beta^0_2$, and $\beta^0_3$ and the initial deformation resistance $k_0$. Before determining the initial back stresses, it is worth looking at their expected development throughout temper rolling. For this, we rewrite Equation (5.20) in terms of the increments in plastic strain as

$$d\beta = \frac{2}{3}c_L \begin{bmatrix} 2de^p_0 + de^p_{90} \\ de^p_0 + 2de^p_{90} \\ \frac{1}{2}d\gamma^p \end{bmatrix} - c_{NL}d\lambda \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}.$$  

(5.52)

Throughout rolling, we have $d\gamma^p = 0$, based on the assumption of a homogeneous strain distribution though the sheet thickness, and we therefore assume $\beta^0_3 = 0$.

We denote the yield stress in a material direction $\alpha$ as $s_{y,\alpha}$ with an additional subscript of "T" or "C" to indicate yield under tension or compression, respectively. After back-extrapolating the slope of the respective stress-strain curves at a plastic strain of 0.02 to zero plastic strain, we obtain the initial yield stresses $s_{y,0,T} = 830MPa$, $s_{y,90,T} = 890MPa$, $s_{y,0,C} = -900MPa$, and $s_{y,90,C} = -1100MPa$. The correspond-
ing analytical solutions for uniaxial tension read

\[
\sigma_{11} = \sigma_0 = \beta_1 + \frac{P_{12}}{P_{11}} \beta_2 + \sqrt{\beta_2^2 \left[ \left( \frac{P_{12}}{P_{11}} \right)^2 - \left( \frac{P_{22}}{P_{22}} \right) \right] - \frac{P_{33}}{P_{11}} \beta_3^2 + \frac{k^2}{P_{11}}} \tag{5.53}
\]

and

\[
\sigma_{22} = \sigma_{90} = \frac{P_{12}}{P_{22}} \beta_1 + \beta_2 + \sqrt{\beta_2^2 \left[ \left( \frac{P_{12}}{P_{22}} \right)^2 - \left( \frac{P_{11}}{P_{22}} \right) \right] - \frac{P_{33}}{P_{22}} \beta_3^2 + \frac{k^2}{P_{22}}} \tag{5.54}
\]

For uniaxial compression, we find

\[
\sigma_{11} = \sigma_0 = \beta_1 + \frac{P_{12}}{P_{11}} \beta_2 - \sqrt{\beta_2^2 \left[ \left( \frac{P_{12}}{P_{11}} \right)^2 - \left( \frac{P_{22}}{P_{11}} \right) \right] - \frac{P_{33}}{P_{11}} \beta_3^2 + \frac{k^2}{P_{11}}} \tag{5.55}
\]

and

\[
\sigma_{22} = \sigma_{90} = \frac{P_{12}}{P_{22}} \beta_1 + \beta_2 - \sqrt{\beta_2^2 \left[ \left( \frac{P_{12}}{P_{22}} \right)^2 - \left( \frac{P_{11}}{P_{22}} \right) \right] - \frac{P_{33}}{P_{22}} \beta_3^2 + \frac{k^2}{P_{22}}} \tag{5.56}
\]

After evaluating the above four equations, we use \( \beta_1^0 \approx -95MPa, \beta_2^0 \approx -150MPa \) and \( k_0 \approx 990MPa \) as seed values for our Monte Carlo simulation-based parameter identification.

The stress-strain response for uniaxial tension in the cross-rolling direction is compared with that for uniaxial compression along the rolling direction to obtain a first estimate of the isotropic hardening parameters. The former loading state produces the highest rate of martensite evolution (with respect to axial strain), while the latter produces the lowest (Beese and Mohr, 2011 [11]). For example, at axial strains of about 15% and 11%, respectively, we observe

- \( \sigma_{90|\varepsilon_M=0.15} = 1294MPa \) and \( \chi = 0.85 \) for tension along the cross-rolling direction;
Using an estimated isotropic hardening parameter value of \( A = 5 \), we obtain the moduli \( H_0 \approx 70 \text{MPa} \) and \( H_x \approx 400 \text{MPa} \) from the above results. The values \( c_L = 750 \text{MPa} \) and \( c_{NL} = 2 \) are used as initial guesses for the kinematic hardening parameters.

An interval is defined around each of the material model parameters. The Monte Carlo simulations are then performed based on the assumption of a uniform probability distribution function for each parameter within the respective interval. For a given set of random parameters \( \Pi_i = \{ P_{11}, P_{12}, P_{33}, k_0, A, H_0, H_x, c_L, c_{NL}, \beta_1^0, \beta_2^0, \beta_3^0 \} \), the stress-time curves \( \sigma_{\text{sim},k}(t, \Pi) \) are computed for a prescribed strain history \( \epsilon_{\text{exp},k}(t) \), with \( k = 1, 2, \ldots, 7 \) denoting the calibration experiment number. Subsequently, the dimensionless cost function

\[
\Xi(\Pi_i) = \sum_k \left[ \sum_{n=1}^{N} \left| \frac{\sigma_{\text{sim},k}(t_n, \Pi_i) - \sigma_{\text{exp},k}(t_n)}{N\sigma_{\text{exp},k}(t_n)} \right| \right] \tag{5.57}
\]

is evaluated for \( N = 900 \) equally spaced time steps for each simulation/experiment. The final set of parameters is then chosen from the minimum for more than 1000 random parameter combinations \( \Pi_i \),

\[
\Pi = \arg \Pi_i \min \Xi(\Pi_i) \text{ for } i = 1, \ldots, 1000. \tag{5.58}
\]

The final set of the calibrated model parameters is given in Table (5.1).
### Transformation kinetics parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_{\text{max}}$</td>
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</tr>
<tr>
<td>$m$</td>
<td>1.5</td>
</tr>
<tr>
<td>$D_0$</td>
<td>6</td>
</tr>
<tr>
<td>$a_\eta$</td>
<td>3</td>
</tr>
<tr>
<td>$a_{\bar{\eta}}$</td>
<td>2</td>
</tr>
</tbody>
</table>

### Anisotropy coefficients

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}$</td>
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</tr>
<tr>
<td>$P_{12}$</td>
<td>-0.552</td>
</tr>
<tr>
<td>$P_{33}$</td>
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</tr>
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</table>

### Isotropic hardening parameters

<table>
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</tr>
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</tr>
<tr>
<td>$A$</td>
<td>5</td>
</tr>
<tr>
<td>$H_0$</td>
<td>100 MPa</td>
</tr>
<tr>
<td>$H_x$</td>
<td>400 MPa</td>
</tr>
</tbody>
</table>

### Kinematic hardening parameters

<table>
<thead>
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<th>Value</th>
</tr>
</thead>
<tbody>
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<td>$c_L$</td>
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</tr>
<tr>
<td>$c_{NL}$</td>
<td>2</td>
</tr>
</tbody>
</table>

### Initial back stress

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^0$</td>
<td>-130 MPa</td>
</tr>
<tr>
<td>$\beta_2^0$</td>
<td>-160 MPa</td>
</tr>
<tr>
<td>$\beta_3^0$</td>
<td>0 MPa</td>
</tr>
</tbody>
</table>

Table 5.1: Calibrated material parameters for 1.5mm-thick temper-rolled stainless steel 301LN sheets.
5.5.2 Comparison of numerical and experimental results

The numerical predictions of the calibrated model agree well with the experimentally measured stress-strain curves for all seven experiments used for calibration. For uniaxial tension, the maximum difference between the simulation and the experimental result does not exceed 7% of the stress level (Figure (5-5)). The wide spread in stress level for uniaxial compression along the rolling and cross-rolling directions is predicted correctly by the model (Figure (5-6)). Furthermore, the good agreement of the results shown in Figures (5-5) and (5-6) demonstrates the model’s ability to describe the pronounced difference between the rate of strain hardening for uniaxial tension and compression, as shown in Figure (5-7a). The results for the remaining three calibration experiments (plane strain tension and pure shear) are summarized in Figure (5-8). Again, we observe good agreement between the model predictions and the experimental results, which is due mostly to the adequate modeling of the effect of stress state on the rate of strain hardening illustrated in Figures (5-7)b and (5-8)b. The plastic width strain is shown as a function of the plastic thickness strain in Figure (5-9) for uniaxial tension. The good agreement for both tension along the rolling direction and tension along the cross-rolling direction confirms the model’s predictive capabilities.

We also performed numerical simulations of the combined tension and shear experiments by Mohr and Jacquemin (2008) [56]. The corresponding model predictions (solid lines) are shown next to the experimental data (dashed lines) in Figure (5-10). Here, the good agreement is seen as a partial validation of the assumption of a quadratic yield surface.

5.6 Conclusions

A phenomenological constitutive model is proposed for steels undergoing deformation-induced austenite-to-martensite phase transformation. Several experimental studies
on the evolution of the martensite content in austenitic stainless steels (e.g., Cina, 1954 [20]; Powell et al., 1958 [78]; Kosarchuk et al., 1989 [42]; DeMania, 1995 [21]; and Iwamoto et al., 1998 [38]) have demonstrated that the martensitic transformation is stress-state dependent. Here, we make use of a newly developed transformation kinetics law that accounts for the stress triaxiality, as well as the Lode angle parameter, to describe the stress-state dependency. The transformation kinetics law is coupled with the isotropic hardening part of a rate independent anisotropic finite strain plasticity model with combined nonlinear kinematics and isotropic hardening. After developing the corresponding time integration scheme and implementing the model into a nonlinear explicit finite element code, the material model parameters are calibrated based on isothermal static experiments for uniaxial tension, transverse plane strain tension, uniaxial compression, and pure shear. It is subsequently validated for combined tension and shear experiments. Good agreement of the model predictions and the measured force-displacement curves is observed for all calibration and validation experiments. In particular, the stress-state dependency of the strain hardening is captured by the model due to the coupling with an advanced transformation kinetics law. It is emphasized that the present model is limited to isothermal loading conditions. However, since the effect of temperature on the stress-strain response of austenitic TRIP steels is typically attributed to the effect of temperature on the transformation kinetics (e.g., Stringfellow et al., 1992 [86]; Hänsel et al., 1998 [33]), it is expected that the present modeling framework can be extended into the range of variable temperatures by enhancing the underlying transformation kinetics law.
Figure 5-5: Experimental stress-plastic strain curves (*symbols*) and stress strain curves described by the model (*solid lines*) for uniaxial tension in the rolling (*red*) and cross-rolling (*blue*) directions.

Figure 5-6: Compressive axial plastic strain versus compressive axial stress under uniaxial compression: model prediction (*solid lines*) versus experimental data (*open symbols*).
(a) Experimentally measured stress versus strain curves (symbols) for uniaxial tension (blue squares) versus uniaxial compression (red crosses) along the cross-rolling direction along with the model predictions of stress versus strain in uniaxial tension (blue dashed line) and in uniaxial compression (solid red lines).

(b) Martensite formation under uniaxial tension (blue solid line) and under uniaxial compression (dashed red line) as described by the stress-state dependent transformation kinetics evolution equation presented in Chapter 3.

Figure 5-7: Stress-strain curves measured from experiments and numerical simulations (a) and the evolving martensite formation as a function of strain for uniaxial tension and uniaxial compression.
(a) Plastic axial strain versus axial stress under plane strain tension: model prediction (solid lines) versus experimental data (open symbols). Equivalent plastic strain (von Mises definition) versus von Mises equivalent stress under pure shear: model prediction (solid lines) versus experimental data (open symbols).

(b) Martensite formation under plane strain tension (dashed green line) and under pure shear (dotted gray line) as described by the stress-state dependent transformation kinetics evolution equation presented in Chapter 3.

Figure 5-8: Stress-strain curves for plane strain tension in two directions and shear measured from experiments and numerical simulations (a) and the evolving martensite formation as a function of strain for plane strain tension and shear (b).
Figure 5-9: Plastic width strain versus plastic through-thickness strain (Lankford ratio) under uniaxial tension in the rolling and cross-rolling directions up to an axial strain of 0.2 as found in experiment (open symbols) and calculated by the model (solid lines).
Figure 5-10: Engineering stress-strain curves for model predictions (solid lines) compared with experiments (dashed lines) for $\alpha = 0^\circ$ (top graphs) and $\alpha = 90^\circ$ (bottom graphs) and various values of loading angle $\beta$. 
Chapter 6

Structural Validation of the Plasticity Model

6.1 Introduction

In industrial applications, parts of complex shape are formed from sheet metal. It is important to predict the global force versus displacement behavior in these forming operations. Here, two types of structural experiments are chosen to demonstrate the performance of the constitutive model in applications with heterogeneous stress and strain fields. A punch test is carried out where a clamped sheet is subject to out-of-plane loading. Furthermore, three types of notched tensile tests are performed under quasi-static loading conditions. Simulations of each experiment are performed and compared with the experimental results.

6.2 Punch loading

A circular disk specimen with a diameter of 126mm is extracted from the stainless steel sheets and clamped onto a 49.2mm diameter die. A set of sixteen \( \frac{1}{2} \) - 20 bolts is used to apply the clamping pressure. The specimen is then loaded using a hemispherical punch with a radius of 22.2mm. Five sheets of 0.05mm-thick Teflon with
grease are placed between the specimen and the punch to reduce frictional effects. The experiment is performed on an electro-mechanical testing machine under displacement control. The force is measured using a 200kN load cell, while an LVDT is used to measure the crosshead displacement. The effective punch displacement is determined from the crosshead displacement assuming an overall machine stiffness of 100kN/mm. A representative force versus punch displacement curve is shown as dashed line in Figure (6-1). After a soft bending-dominated initial response, we observe a monotonically increasing force-displacement curve until a crack forms at the apex of the punched specimen.

![Figure 6-1](image)

**Figure 6-1**: Force versus displacement during equi-biaxial punch experiment *(dashed line)* and simulation *(solid line)*.

In the corresponding computational model, both the die and the punch are represented by analytical rigid surfaces. We take advantage of symmetry in the mechanical system and model only one quarter of the disk specimen using four-node reduced-integration plane stress shell elements *(element type S4R from the Abaqus library)*. The elements have a side length of 1mm and feature five thickness integration points. A low friction coefficient of 0.04 is assumed between the punch and specimen, while a
high friction coefficient of 0.5 is used between the die and the specimen in the clamping area. The simulations are performed using about 100,000 explicit time steps. The predicted force-displacement curve is shown as a solid line in Figure (6-1). The comparison with the experimentally measured curve demonstrates the model's ability to predict the force-displacement response in a punch experiment with good accuracy.

6.3 Notched tension

Notched tension is another important structural experiment, which involves a range of stress states that are frequently encountered in sheet metal forming. The geometry of the first notched tensile specimen tested is shown in Figure (6-2a). The circular cutouts have a radius of 6.67\text{mm}, while the minimum width at the notch is 10\text{mm}. Increasing the cutout radius decreases the initial stress triaxiality at the center of the specimen. A DIC-based 34\text{mm} long virtual extensometer is used to measure the relative displacement of the specimen shoulders. The recorded force-displacement is shown as a solid line in Figure (6-2b). It features a force maximum at a relative displacement of about 1.6\text{mm}. Localized necking through the thickness becomes dominant beyond this point, which explains the decrease in force before the point of fracture is reached.

The mesh of a quarter of the specimen is shown in Figure (6-2b). It features 3,200 reduced-integration shell elements (type S4R). The elements in the notch region have a side length of 0.125\text{mm}, while those in the shoulder region have a maximum side length of 0.525\text{mm}. We apply symmetry boundary conditions to the horizontal and vertical axes of symmetry, and a constant vertical velocity to all nodes at the top boundary of the specimen. A total of 100,000 explicit time steps are performed to simulate the static experiment. The computed force-displacement curve (red dotted line in Figure (6-2b)) is compared with that obtained experimentally. We note that there is excellent agreement between the two curves up to the point of maxi-
mum force. Beyond this point, the validity of plane stress assumption of the shell element formulation breaks down as out-of-plane stresses develop throughout necking.

To overcome this limitation associated with the plane stress assumption, we developed a time integration scheme for the three-dimensional constitutive model and programmed a corresponding user material subroutine. Simulations are subsequently performed with eight first-order solid elements (type C3D8R of Abaqus element library) through the thickness direction of a mesh of an eighth of the specimen (exploiting the symmetry with respect to the sheet mid-plane). The corresponding force-displacement curve (dashed green curve in Figure (6-2b)) coincides with that for the shell element model up to the force maximum, and continues in close agreement with the experimental curve up to the point of fracture. The contours of the anisotropic equivalent plastic strain and martensite content are shown in Figures (6-2c) and (6-2d), respectively. Observe that the material has fully transformed into martensite before fracture occurs.

In addition to the notched tensile specimen with a cutout radius of 6.67mm, notched tension specimens with cutout radii of 10mm and 20mm are also performed in order to explore the model’s capabilities under different stress states (see Figures (6-3a) and (6-4a)). The experimental force versus displacement curves are compared with the predicted force versus displacement curves in Figures (6-3b) and (6-4b), showing a good prediction of the displacement to peak force.

6.4 Conclusions

The plasticity model coupled with the stress state dependent transformation kinetics law results in good prediction of the measured force versus displacement for structural validation experiments. In addition, note that the experiments and simulations are under displacement-controlled loading conditions, and therefore after the peak force
is achieved, there is a relatively slow decrease in force up until final fracture. However, in force controlled loading, fracture would occur almost immediately after the peak force develops, and localized plastic flow occurs. Therefore, it is critical to predict the displacement at maximum force. We may take this displacement at the maximum force, and the corresponding strains, as a lower limit for specimen failure.
Figure 6-2: (a) Geometry of notched tensile specimen (all dimensions in mm) with thickness of 1.5mm. (b) Force versus displacement during notched tension experiment (solid blue line), shell element simulation (dotted red line), and solid element simulation (dashed green line) with finite element mesh for plane stress simulation (inset). (c) Contours of surface anisotropic equivalent plastic strain at the displacement to fracture, and (d) corresponding contours of the martensite volume fraction.
(a) Geometry of notched tension specimen with cutout radius of 10\text{mm} (all dimensions in \text{mm}).

(b) Experimentally measured force versus displacement (gauge length of 34\text{mm}).

Figure 6-3: Force versus displacement during R=10\text{mm} notched tension experiment (solid line) compared with solid element simulation (dotted line).
(a) Geometry of notched tension specimen with cutout radius of 20mm (all dimensions in mm).

(b) Experimentally measured force versus displacement (gauge length of 38mm).

Figure 6-4: Force versus displacement during R=20mm notched tension experiment (solid line) compared with solid element simulation (dotted line).
Chapter 7

Summary and Conclusions

Different experimental techniques for measuring the martensite volume fraction in TRIP steels are evaluated. It is shown that X-ray diffraction, optical metallography, and EBSD are not sufficiently accurate in order to monitor the martensite formation in mechanical experiments. A ferritescope is used instead, which measures the magnetic permeability of the sheet material. A relationship between the martensite volume fracture and the ferritescope measurement is established based on magnetic saturation induction measurements. The experimental results show magneto-mechanical couplings at the specimen level have a significant effect on the ferritescope measurements. To minimize possible errors due to the Villari effect, or inverse magnetostriction, the martensite content is measured for a macroscopically stress-free configuration. Therefore, all mechanical loads are removed from the specimen boundaries before performing the ferritescope measurements. Some conflicting data on the effect of stress state on the rate of martensite formation can be found in the literature. Based on the present analysis of the experimental techniques, it is speculated that some results in the open literature on the martensite content evolution are polluted by the effect of macroscopic stress on the measured magnetic permeability.

A comprehensive experimental program is designed and executed to study effect of stress state on the large deformation behavior of TRIP steels. It involves different types of specimens and the simultaneous measurement of local deformation, applied
force, and the martensite content. The comparison of the evolution of the martensite volume fraction for uniaxial tension, uniaxial compression, simple shear, and equibiaxial tension reveals that the rate of martensite formation with respect to plastic strain cannot be described as a monotonic function of the stress triaxiality. Instead, it is demonstrated that the martensite evolution also depends on the Lode angle parameter, which is a function of the third invariant of the deviatoric stress. With this experimental evidence, an isotropic rate-independent isothermal transformation kinetics evolution equation is developed in which the martensite content is described as a function of strain, through an equivalent plastic strain measure, and the stress or strain state, through the stress triaxiality and Lode angle parameter. The normal stress acting on the plane of maximum shear is monotonically related to both the stress triaxiality and the Lode angle parameter; it increases with increasing stress triaxiality and increasing Lode angle parameter. Based on the present experimental observations, it is thus argued that tensile normal stresses on the plane of maximum shear facilitate the martensitic transformation in stainless steel. The corresponding mechanism-inspired transformation kinetics law captures the observed stress-state dependency of the transformation kinetics over a wide range of stress states.

Subsequently, a rate independent isothermal anisotropic finite deformation constitutive model is proposed for steels undergoing deformation-induced austenite-to-martensite phase transformation. This model is composed of an anisotropic yield function, an isotropic hardening law, a nonlinear kinematic hardening law, and a nonzero initial back stress to account for the material's temper-rolled processing history. The key feature of the proposed constitutive model is the coupling of the isotropic hardening rule with the stress-state dependent transformation kinetics law.

A backward-Euler time integration scheme is developed to solve the constitutive equations numerically. The resulting computational material model is implemented into a nonlinear explicit finite element code. The material model parameters are calibrated based on isothermal quasi-static experiments for uniaxial tension, transverse plane
strain tension, uniaxial compression, and pure shear, while the model is validated for combined tension and shear experiments. The stress-state dependency of the strain hardening is successfully captured by the model. Furthermore, the model also describes the experimentally observed plastic anisotropy.

The plasticity model is further validated using structural experiments, where the stress and strain fields are heterogeneous. Good agreement of the model predictions and the measured force-displacement curves is observed for all calibration and validation experiments.

7.1 Future research

There are several remaining issues of practical and scientific importance with respect to microstructural phase transformation and its effect on macroscopic material behavior.

7.1.1 Strain rate and temperature dependency of transformation kinetics

In addition to the quasi-static shear tests discussed previously ($\dot{\gamma} \sim 10^{-3}s^{-1}$), exploratory dynamic shear tests were performed using a newly-developed shear testing device for sheet materials (Figure (7-1)a). This device can be used in conjunction with hydraulic universal testing machines as well as in a Split Hopkinson Pressure Bar (SHPB) (Bordier and Mohr, 2009). Strain rates on the order of $\dot{\gamma} \sim 10^3s^{-1}$ were performed using a SHPB system. In addition to the deformation history during dynamic loading, a newly developed infrared temperature measurement technique was employed in SHPB tests (Negreanu et al., 2009 [63]). This technique provides a unique opportunity to accurately measure the evolving strains and temperature with
time during tests of very short duration (on the order of 200\(\mu\)s).

The preliminary results on the strain rate dependency of the transformation kinetics show a dramatic decrease in martensite developed when comparing the content in a specimen strained at a rate of \(\dot{\gamma} \sim 10^{-3}s^{-1}\) to a specimen strained at a rate on the order of \(\dot{\gamma} \sim 10^3s^{-1}\), as shown in Figure (7-1)b. In addition, we note that the temperature increased by about 35°C during the experiment.

In the present study, quasi-static uniaxial tension tests were performed over a range of temperatures from \(-100°C\) to \(100°C\). The experimentally measured martensite content as a function of plastic strain and temperature is shown in Figure (7-2)a, while the measured stress versus strain curves are shown in Figure (7-2)b. As expected, the martensite content that is developed decreases with increasing temperature. The stress versus strain curves illustrate the complexity of the resulting temperature-dependent macroscopic plasticity behavior.

### 7.1.2 Anisotropy of transformation kinetics

As shown in Figure (7-3), the transformation kinetics under uniaxial tension appear to be isotropic. However, a direction-dependency of the transformation kinetics is observed under in-plane uniaxial compression. The transformation kinetics law presented in this thesis is isotropic, and further research is needed to investigate the possible anisotropy of the transformation kinetics.
Figure 7-1: Device used in performing high strain rate double shear experiments (a), and experimentally measured martensite content as a function of plastic strain for quasi-static tests (*green diamonds*) and for tests at strain rates on the order of $10^3 \text{s}^{-1}$ (*red crosses*).
Figure 7-2: The measured effect of temperatures ranging from $-100^\circ C$ to $100^\circ C$ on the martensite formation (a) and the macroscopic plasticity under quasi-static uniaxial tension along the material rolling direction.
(a) Martensite evolution under uniaxial tension (solid symbols) and uniaxial compression (open symbols) with the loading axis aligned with the rolling direction (red) and the cross-rolling direction (blue).

(b) Absolute stress versus strain curves for uniaxial tension (solid lines) and uniaxial compression (dashed lines) along the rolling (red) and cross-rolling (blue) directions.

Figure 7-3: Anisotropy of martensite evolution and macroscopic plasticity behavior under uniaxial tension and uniaxial compression.
Appendix A

Refereed journal publications related to this thesis


Bibliography


