Effects of Broad-Banded Higher Harmonics on Fatigue Damage of Risers due to Vortex-Induced Vibrations

by

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B.S., Mechanical and Ocean Engineering
Massachusetts Institute of Technology (2010)

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OCEAN ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2011

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Abstract

Recent works have discussed "chaotic" or "Type-II" riser motion and suggested that it is a general feature of VIV riser response. Chaotic riser response contains broad-banded harmonics and a combination of standing and traveling waves, unlike periodic or "Type-I" motion which contains only standing waves and exhibits very narrow harmonics. The extent to which regions of chaotic response increase the damage on the riser had not previously been explored in detail. To facilitate the exploration, a method for separating the effects of the fundamental frequency, higher harmonics, and the chaotic content was developed and applied to four cases from the NDP 38m data set. For test of a bare riser in uniform flow, the damage was increased by a factor of 2 when the higher harmonics of the signal were included and another factor of 2 when the broad-banded harmonics were included. Similar results were obtained for tests of the bare riser in sheared flow as well as a 50% straked riser in uniform and sheared flow. After the results have been supported by theoretical estimations from first principles, it is concluded that the increase in fatigue damage resulting from the chaotic behavior of the riser is as important as that of the higher harmonics of the signal.

Thesis Supervisor: Michael Triantafyllou
Title: Professor of Mechanical and Ocean Engineering
Acknowledgments

I would very much like to thank my mother, father, and brother for their tireless support and patience throughout this project and all previous projects, capers, and assorted misadventures. You are owed a more than usually large thank-you for all of the years of work you put into my schooling, and your continued support and love has meant the world to me.

The Dumbros Scholarship and Fellowship Fund has made the dream of MIT a reality for me, when it would not have otherwise been at all feasible. I am very much indebted to the generosity of the Dumbros family.

I would like to thank Professor Triantafyllou for the opportunity to participate in his work, and for all of his advice and suggestions on this project. I would also like to thank Dr. Yahya Modarres-Sadeghi, who has been an indispensable source of advice and inspiration, and Haining Zheng for his help in navigating the group’s resources. I would also like to thank Norwegian Deepwater Programme (NDP) for the use of their data in this project.

Finally, but certainly not least of all, thanks to Steven for his encouragement, understanding, and willingness to wait while I finished this project. To the rest of my friends and loved ones, especially Caroline and Fiona, thank you for your unwavering friendship and constant doses of much-needed tomfoolery which were a necessary part of the creative process.
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Chapter 1

Introduction

1.1 Motivation: VIV and Fatigue

The vortex-induced vibrations (VIV) and the resulting material fatigue are particularly important considerations in design for ultra-deepwater oil exploration and development. Risers and mooring lines are designed to maximize both strength and capacity while minimizing flow separation effects. As the world’s need for energy continues to increase, interest in the large oil reserves located off the coast of many countries has also increased. Figure 1.1 shows the evolution of such offshore structures, with their increasingly complex riser and mooring line systems.

![Increasing Depth of Offshore Structures](Source: OilandGasProcessing.com)

There are many motivations for advancements in the design of such structures beyond profit and increasing demand, however. Recent unrest in the
Middle East coupled with the dearth of permits for new projects in the Gulf of Mexico in the wake of the Deepwater Horizon accident have increased the prices of oil and may lead to destabilization in the energy supply. Offshore resources offer diversification of the sources of the world’s oil. Additionally, even in moderately unstable countries, offshore resources are less susceptible to political unrest because they are separated from the mainland.

1.2 Mechanism of VIV

VIV events occur when a bluff body is placed in a fluid stream under conditions which lead to large scale separation of the flow. The unsteady wake behind a bluff body, such as a cylinder, creates an alternating pressure gradient on the body by generating vortices on alternating sides with a frequency proportional to the size of the body and speed of the flow. The wake behind the body, called the Von-Karman vortex street, has become a very famous image in fluid mechanics and is seen in Figure 1.2.

![Figure 1.2: Von-Karman Vortex Street](Source: PhD Thesis, Mukundan [17])

If the body is free to move and the frequency of the pressure gradient oscillation is near a natural mode of the body, the body becomes excited and oscillates in a phenomenon known as "lock-in". The motions resulting from vortex excitation are self-limited to about one body diameter because if the body moves more than one diameter outside of its initial position the vortex forces act in the opposite direction.

Figure 1.3 illustrates the self-limitation of VIV. In the image on the left, a newly formed vortex produces a force which causes the body to accelerate...
downward. In the right image, the body has been dragged about one diameter from its original position and is pulled back in line by the suction force of the vortex. Thus, VIV damage results in material fatigue rather than ultimate tensile failure because the stresses it causes are low in magnitude but high in frequency.

![Figure 1.3: Self-limitation of VIV](image)

As a phenomenon, VIV is seen well beyond the oil industry, and these instances often provide more dramatic examples the potential for catastrophic damage. Figure 1.4 shows a picture of pipes from a tube heat exchanger. The visible damage was attributed to VIV which resulted from positioning the long tubes in a high-speed cross-flow of cool water. [12]

![Figure 1.4: VIV Damage on Heat Exchanger Pipes](image)

Dramatic failure of this type is not seen in the offshore industry because of aggressive maintenance and replacement schedules. Early replacement is an effective but costly method of ensuring the safety structural components in an industry where failure of a single component could cause great environmental damage or loss of human life irreparably damaging the reputation
of the companies involved. Developing tools to effectively identify the most damaged components reduces cost and safety risks so that current risers can be replaced on an less aggressive schedule and future risers can be more robustly designed.

1.3 VIV in Industry

In industry, vortex-induced vibrations on marine risers is a topic of much concern. Much of past research has been concerned with the design and testing of various passive devices which seek to mechanically suppress the vibrations. At the conceptual design stage, prediction codes are used to aid in the design of large structures, the most popular including VIVA, VIVIANA, and Shear7.

The most popular VIV suppression device is the helical strake, a piece of rubber or other compliant material which winds around the outside of a cylindrical line like a spiral staircase. Strakes suppress the formation of vortices in the wake of the cylinder by interfering with the pressure gradient around the perimeter of the cylinder and by mechanically interfering with the vortices which do form. [22].

![Strakes used in VIV Suppression Tests](Source: Marintek Test Report [3])
In the past it was common for predictive methods used in the design of the riser to consider only the first harmonic of the riser, as it was thought that these stresses and motions would cause the most damage. 2-3 orders of magnitude were then added to the desired life for the design as safety factors.

More recently, as demonstrated in [15] and other sources, it has become a standard practice to include the third and fifth harmonics of the signal as well. It has been shown that including the higher harmonics increases the predicted damage by an order of magnitude.

Continued work in [4] and [16] has shown that the presence of "chaotic" motions and stresses in the data is a factor which can also greatly increase the damage on a riser. Additional work is needed to determine the size of this change with respect to the previously acknowledged increase in damage due to the higher harmonics of the signal.

1.4 VIV Damage, Stress Power, Higher Harmonic Stress, and Type-II Riser Behavior

This thesis explores the effects of three different factors on the fatigue life on a cylinder in VIV. The majority of the increase in damage on a riser is due to the increase in stress power. Determining the value and cause of the stress power increases allows other, smaller factors to be explored more fully. Although the importance of higher harmonics has already been explored, previous studies have failed to separate its effects from the increase in power associated with using the higher harmonic components. Finally, exploring the effect of including chaotic stresses in the damage calculation in this context will show how important this factor can be for certain flow configurations and riser geometries.

The influence of higher harmonics on the fatigue life of a marine riser has been acknowledged for some time. In general, higher harmonics increase the damage by causing the riser to experience more motion cycles in a given period of time. Using the power spectral density (PSD) as a diagnostic tool, higher harmonic distribution can be discussed directly by examining the amount of stress power (variance of the stress) present at certain frequencies.

By contrast, the behavior of the riser in a "Type-I" (periodic) or "Type-II" (chaotic) fashion has become a factor to be considered in VIV fatigue
studies much more recently. Type-II behavior is thought of as "chaotic" behavior because of its wide-banded power spectral densities and the seemingly random nature of the associated time series. In this thesis, chaotic behavior is discussed in terms of different parameters with empirically determined cutoffs which indicate that Type-II behavior is occurring. Type-II behavior is more damaging to the riser than Type-I behavior because of the presence of many different frequency components. The amplitudes of these components are additive and can be many times larger than the amplitudes for a less chaotic signal.

Experimental data, especially field data, naturally contains both higher harmonics and Type-II behavior. In order to separate the damage which is due to each it was necessary to develop a method to separately analyze the effects of each. Time series generation methods allow for the addition or removal of different effects at the will of the engineer, at the expense of a small amount of added uncertainty due to the loss of the phase angles between frequency components.

Time series with no higher harmonics, with higher harmonic distribution but no Type-II behavior, and with both higher harmonics and Type-II behavior were created in this way. The fatigue lives of these time series were then found using a fatigue calculation method known as the rainflow algorithm which has been previously applied to riser VIV by Mukundan [17]. The results and conclusions presented in this thesis are a comparison of these fatigue lives.

Previous work and theoretical background in this area are discussed in Chapter 2, then a full synopsis of the mathematical methods used to generate time series and calculate fatigue life are discussed in Chapter 3. Chapter 4 discusses some sensitivity analysis methods which were used as independent confirmation of the time series generation results. Chapters 5 presents raw results of the simulations for two different experimental data sets. Finally, Chapter 6 presents and supports conclusions which may be drawn from these results.
Chapter 2

Background and Literature Review

The history of fatigue research is much more extensive than that of VIV research, thus there is a large body of knowledge about fatigue which was developed before VIV fatigue research became popular. Fatigue research began in the 1830s when the first mention of a "tired" material was made by Jean-Victor Poncelet in a series of lectures at a military college in Metz [23]. Research on VIV effects in offshore structures did not begin until the 1980s when it became both economical and technologically feasible to retrieve petroleum from deepwater sources.

This brief overview is at the level of an overview of research, resources, and methods which are outside the scope of engineering textbooks. For a review of the underlying concepts of stress, strain, and fatigue, the reader is directed to Hibbeler’s book on the mechanics of materials [8]. The topic of vibrations of multi-degree-of-freedom systems is discussed in [9] and [21].

Likewise, the summary of VIV research presented here comprises only a basic smattering of information necessary to understand the subsequent work in VIV fatigue. For a more exhaustive review of VIV literature in general, the reader is referred to Sarpkaya’s 2004 paper, [22], which compiles much of what was known about VIV at the time in one reference. Williamson’s 1996 paper is another useful resource. [29]

2.1 Fatigue

The first early recorded research on fatigue was performed by A. Wohler as part of his work as an engineer for a German railway. He began publishing the results of his tests of the stresses and fatigue lives of train axles in 1858, culminating his work in a final report published in 1870. This report detailed a set of principles which have come to be known as "Wohler’s Laws." His original work was published in German, and was translated as part of Schutz’s
1996 paper "A History of Fatigue" [23].

Wohler's three ultimate conclusions were: (1) materials can fail due to repetitions of stresses lower than the tensile strength, (2) stress amplitudes are the largest factor in the time to failure in these cases, and (3) higher maximum stress lowers the repetitive stress needed to cause failure. In summary, although the mean tensile stress can have a small influence, the amplitudes of repetitive stress cycles are the most important factor in determining the fatigue life of a material.

Armed with these conclusions, Wohler went on to develop tables for the number of stress cycles his test objects could withstand at various stress levels. A plot of the values in these tables has come to be known as a stress-number of cycle curve, or an S-N curve for short. Figure 2.1 shows a plot of the S-N Curve for the steel used as a test material for one of the sets of experimental data discussed later. The important feature is the drop in the number of cycles the material can withstand as the amplitude of stress increases.

![S-N Curve for Steel](image)

Figure 2.1: S-N Curve for Steel

Figure 2.1 shows a plot of the S-N Curve for the steel used as a test material for one of the sets of experimental data discussed later. The important feature is the drop in the number of cycles the material can withstand as the amplitude of stress increases.

The next great leap in the development of modern fatigue theory was A. Palmgren's 1924 publication of the "linear damage hypothesis" which
provided a method for calculating the fatigue life of a part undergoing stress cycles of varying frequency and amplitude. Interestingly, the theory did not come into popular use until A. Miner began to express his strident support for the theory in his 1945 paper. [13] In acknowledgement of his crucial role in developing popular support for the theory, the hypothesis has come to be known as the Palmgren-Miner Rule, or in many cases simply Miner’s Rule.

The Palmgren-Miner Rule states that in a spectrum of stresses, \( S \), with \( k \) different stress magnitudes where magnitude \( S(k) \) contributes \( n(k) \) cycles, while \( N(k) \) is the fatigue life in cycles of the material at the level \( k \), then failure occurs when

\[
\sum_k \frac{n(k)}{N(k)} = C.
\]  

(2.1)

\( C \) is an experimentally-determined constant which has been shown to vary between 0.7 and 2.2 depending on the material, while a value of 1 is commonly used for design purposes. In short, the percent of damage done at each stress level is found individually, then these individual damages are summed to find the total damage.

Two major shortfalls exist in the Miner’s Rule methodology. Firstly, fatigue is a fundamentally probabilistic phenomenon and there is no simple way to include that dependence in this formula. In industry, engineers and analysts often adjust their S-N curves downward during the design phase to account for scatter in the \( N(k) \) data. Additionally, this method does nothing to account for the order in which stress cycles occur. Intuition dictates a large cycle late in the life of a sample should be more harmful than the same magnitude stress cycle occurring earlier. However despite its shortcomings, when combined with conservative assumptions, Miner’s rule serves as a reliable way to ensure that no part is pushed past its minimum fatigue life.

Miner’s rule is an especially powerful tool when combined with a method for simplifying complex stress loadings and counting the number of cycles of a set of pre-defined stress amplitudes. One such method is rainflow analysis method debuted by Matsuishi and Endo in their 1968 paper, *Fatigue of Metals Subjected to Varying Stress*. [11]

In practical applications, a body is loaded in a far more complex manner than can be simulated in a laboratory setting. These types of loadings are best described as a random sequence of large and small load cycles. Rainflow analysis offers a bridge between complex loadings and Miner’s Rule by
reducing a complex time history into a series of simple, cyclic loadings.

Figure 2.2 presents a detailed step-by-step walkthrough of the Rainflow Algorithm as it was implemented by Matsuishi and Endo. Many modifications to this algorithm exist, but a thorough understanding of the basic algorithm is a good foundation for understanding the changes which were later added.

Matsuishi and Endo’s algorithm instructs engineers to begin with a stress time series, like the one shown in Figure 2.2(a), then turn it on its side so that the tensile peaks are on the right as in Figure 2.2(b), so that the time series resembles an uneven pagoda. To continue this analogy, think of the stress running down the sloped roofs of the pagoda like rain. A tensile half cycle of stress is the y-axis distance that water flowing down the right side of the building flows before the flow is stopped. Likewise, a compressive half-cycle is the y-axis distance that water flowing down the left side of the pagoda could flow before the flow was stopped.

There are three ways in which the flow can be "stopped." If the water joins the flow of an already flowing stress amplitude then the joining flow does not count as a half-cycle. If an opposing flow (tensile for compressive and vice versa) reaches a value greater than a dripping flow, the dripping is stopped. Finally, if the time series ends, the flow ends.

When the algorithm is applied by hand, it may be necessary to magnify the time series, which has been done to create the plot seen in Figure 2.2(b). From this point, the next step is to identify the turning points of the signal. This has been done to find the points highlighted in Figure 2.2(c).

The next step is to identify the compression and tension half-cycles. These are highlighted for the example time series in Figures 2.2(d) and 2.2(e), respectively. Subsequently, the dripping patterns must be found in order to find the magnitude of the stress cycles.

There are six total compression drip sources, which are identified and numbered in Figure 2.2(f). The first begins with the beginning of the time series and ends when the subsequent tension half-cycle has a larger magnitude. The second and third are ended in the same way. The fourth drips into the fifth and the two form a single half-cycle with a very large magnitude. The sixth is ended by the conclusion of the time series.
Likewise, there are five total tension drip sources, which are identified and numbered in 2.2(g). The first and second run together and form a drip which is ended by a large compression half-cycle. The third and fourth are independently ended in the same way. The fifth and final half-cycle concludes with the end of the time series.

Once the magnitude of the half-cycles has been determined, they are paired into full cycles by matching tension and compression half-cycles of similar magnitude. Since these values do not match exactly, an appropriate bin size must be determined to define ”similar” for each new problem. Choosing an appropriate bin size is one of the fine arts of applying the algorithm properly.

Once this pairing has been accomplished, the fatigue life (in cycles) of each stress level is calculated using the S-N curve. Damage can be calculated using Miner’s rules. Fatigue life is equal to the reciprocal of the average damage per unit time, so the fatigue life can finally be determined.

Table 2.1 shows this process for the full time series depicted in Figure 2.2(a). The upper section shows the calculations which must be performed for each stress level, while the lower section shows the sum of the total damage (complete damage to failure would be 1) and the fatigue life associated with this time series.

<table>
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<th>Stress (MPa)</th>
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<th>Life (cycles)</th>
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<td>43</td>
<td>2.3E17</td>
<td>2.0E(-16)</td>
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<tr>
<td>2.8E7</td>
<td>43</td>
<td>6.5E12</td>
<td>6.6E(-12)</td>
</tr>
<tr>
<td>4.5E7</td>
<td>44</td>
<td>1.4E12</td>
<td>3.1E(-11)</td>
</tr>
<tr>
<td>6.8E7</td>
<td>39</td>
<td>3.9E11</td>
<td>1.0E(-10)</td>
</tr>
<tr>
<td>8.9E7</td>
<td>46</td>
<td>1.6E11</td>
<td>2.8E(-9)</td>
</tr>
<tr>
<td>Total Damage</td>
<td>-</td>
<td>-</td>
<td>4.4E(-10)</td>
</tr>
<tr>
<td>Total Damage</td>
<td>-</td>
<td>-</td>
<td>727 years</td>
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There are many points in this algorithm where small changes can have large effects on the fatigue life result and when unexpected results are obtained it is often a good idea to check the inputs and outputs of the Rainflow Algorithm program. The full implementation of the Rainflow Algorithm as used to obtain the results in this thesis are discussed in Chapter 3.
Figure 2.2: Detailed Explanation of the Rainflow Algorithm
2.2 Modal Analysis of Risers in VIV

Because the frequency domain is utilized by some of the most popular VIV damage prediction codes, modal analysis for risers is an important concept in any discussion of VIV damage analysis. Additionally, much of the research in this thesis has roots in frequency domain analysis so it is beneficial to see how these calculations are performed.

The basic process and assumptions of modal analysis were discussed by G.S. Triantafyllou in a paper written about an early version of the VIV motion and stress prediction program VIVA [24]. At the time the paper was written, it was known that the lift and added mass forces on an oscillating cylinder were quite different from those on a stationary cylinder and the VIV was a self-limiting phenomenon (as was explained in Chapter 1).

These factors added enough complexity to the motion prediction problem that a new program was needed to calculate the vortex-induced response of a flexible structure. Triantafyllou established a somewhat less empirical model than the "semi-empirical" models of the time, relying upon experimental data only to determine the hydrodynamic forces and solving all other aspects of the problem from first principles.

The modal analysis calculations begin by modeling the riser as a long cylindrical structure of length $l$ and diameter $D$ placed in a current with velocity $V$. Additionally, the riser has mass $m$ and stiffness $EI$, viscous damping described by the parameter $b$, and is subjected to a tension $T$. Figure 2.3 shows a diagram of this problem.

![Figure 2.3: Geometric Diagram of Riser Problem](Source: Triantafyllou, 1998 [24])

Self-limitations of VIV mean that the motions have a maximum near one body diameter and that it is therefore valid to simplify the problem by linearizing the equation of motion. In order to discuss the concepts of modal analysis with as few complications as possible, the two-dimensional problem is discussed here rather than the full three-dimensional equation.
The additional algebra required to include the third dimension obscures the process with its multitude of small factor cancellations and does not add value to the ideas expressed in the development of the following equations.

Riser motion can be described by the canonical mass-spring-dashpot equation in two dimensions. When factors such as the tension on the riser are included this equation can be written:

\[
m \frac{\partial^2 y(z,t)}{\partial t^2} + b \frac{\partial y(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( T \frac{\partial y(z,t)}{\partial z} \right) + \frac{\partial^2}{\partial z^2} \left( EI \frac{\partial y(z,t)}{\partial t^2} \right) = f(y,z,t)
\]  

(2.2)

where the z-axis lies axially along the length of the riser and the y-axis is perpendicular to the riser. Triantafyllou was interested in finding the normal modes of the coupled fluid/structure system, so he assumed a trial solution of the form

\[
y(z,t) = \sum_{j=1}^{n} A_{j} e^{i(\omega_{j} t + \phi_{j})} + \Re \left[ Y(z) e^{i\omega t} \right].
\]

(2.3)

Equation 2.3 can be substituted into the riser motion equation to find a solution to the problem where the only unknowns are a set of modal amplitudes contributions and frequencies, \( Y(z) \) and \( \omega \). Triantafyllou made a simplification of similar importance using experimentally obtained lift and drag coefficient databases to define the force term of the original equation. He separated the force into two components, one of which is 180° out of phase with the acceleration (the added mass force) and one which is in phase with velocity (the excitation lift force).

\[
f(y,z,t) = \Re \left[ Y(z) e^{i(\omega t - \psi(z))} \right] = \Re \left[ \left( a^2 C_m(\omega) \omega^2 Y + i C_L(\omega) q D \frac{Y}{|Y|} \right) e^{i\omega t} \right]
\]

(2.4)

In this equation \( \psi(z) \) is a phase lag, \( a \) and \( q \) are the potential flow added mass and lift coefficients of the cross section, and \( C_m \) and \( C_L \) are frequency-dependent non-dimensional coefficients for the velocity and acceleration forces.

The potential flow added mass coefficient, \( a \), can be found using the formula

\[
a = \frac{\pi \rho D^2}{4}.
\]

(2.5)
Similarly, the potential flow lift coefficient of the cross-section, \( q \) is defined by the equation

\[
q = 0.5\rho V^2.
\]  
(2.6)

The frequency-dependent non-dimensional force coefficient for acceleration is defined as

\[
C_m = \frac{F(z, \omega, Y) \cos(\psi(z))}{[0.25\pi \rho D^2][\omega^2 Y(z)]}.
\]  
(2.7)

Finally, the frequency-dependent non-dimensional coefficient for the velocity forces is

\[
C_L = \frac{F(z, \omega, Y) \sin(\psi(z))}{[0.5\pi \rho D V(z)^2][Y(z)]}.
\]  
(2.8)

For convenience, the added mass force can be bundled into the mass term using the expression \( M = m + zC_M \), where \( M \) is the total inertial per unit length, so that the final equation becomes

\[
(-M\omega^2 + i\omega) Y(z) - \frac{d}{dz} \left( T \frac{dY(z)}{dz} \right) + \frac{d^2}{dz^2} \left( EI \frac{d^2Y(z)}{dz^2} \right) = iC_L q D \frac{Y(z)}{|Y(z)|}
\]  
(2.9)

Each side of Equation 2.9 is solved by assuming an initial value for \( Y(z) \) then iterating until the two sides are within a predetermined tolerance. The coefficients of the \( Y(z) \) and \( \psi(z) \) vectors are returned as the modal contributions and phases. The optimal values of \( Y(z) \) and \( \psi(z) \) can be plugged into the trial solution from Equation 2.3 to find the position and stress of any point on the riser at any time.

### 2.3 "Chaotic" VIV Response

With the modal analysis framework firmly in place, the next phase of VIV research involved the development of experimental databases from which the values of the coefficients in the riser equation could be extracted. However, it was found that many real world data sets exhibit a transient or time-changing component, which cannot be analyzed using modal analysis because modal analysis is only mathematically valid for time-invariant systems.
Time series with a significant transient portion are typically described as exhibiting "Type-II" or "chaotic" motion, while signals exhibiting non-transient purely periodic responses are called "Type-I" or "periodic". These types of responses were discussed in detail by Chasparis, who examined experimental data cases with maximum flow velocities ranging from 0.6m/s to 1.6m/s from bare and straked cylinders placed in sheared and uniform flows. Ultimately, he concluded that riser response can exhibit either or both of the response types. [4]

Type I responses are narrow-banded about a single dominant frequency while Type II responses are distributed along a range of frequencies. Both Type I and Type II behavior can be present in a series, and long series often transitions between the two multiple times. Chasparis notes that both flow profiles (sheared and uniform) and geometries (bare and straked cylinders) exhibit Type I and Type II responses with the caveat that there are subtle differences between the datasets.

Additional characteristics of Type I responses include amplified third harmonic displacements, accelerations, and strains; greater correlation lengths; and the presence of monochromatic standing and traveling waves. Conversely, Type II responses are characterized by smaller third harmonic components and correlation lengths, riser motions which appear random, and waves of several distinct frequencies.

Chasparis works through an example data case from the sheared bare data set in detail, showing different types of plots which he used as diagnostic tools in his research. The thesis concludes with seven conclusions which all have consequences for fatigue. These conclusions are outlined by Tables 2.2 and 2.3 which, respectively, outline the characteristics of each type of response and the response characteristics of each data set.

<table>
<thead>
<tr>
<th>Table 2.2: Characteristics of Type-I and Type-II Response</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Response Type</strong></td>
</tr>
<tr>
<td>Type-I</td>
</tr>
<tr>
<td>Type-II</td>
</tr>
</tbody>
</table>
Table 2.3: Response Characteristics by Data Set

<table>
<thead>
<tr>
<th>Traveling Wave Origination End</th>
<th>Uniform Bare &amp; right</th>
<th>Uniform Straked Bare End</th>
<th>Sheared High V End</th>
<th>Sheared Straked Bare End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monochromatic Response (Type-I)</td>
<td>&amp; right</td>
<td>Bare End</td>
<td>High V End</td>
<td>Bare End</td>
</tr>
<tr>
<td>Multi-Frequency Response (Type-II)</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Standing Waves</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Tight Frequency Bands</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Chasparis' analysis of the time series for displacement and acceleration lays the groundwork for similar analysis of the corresponding stress time series and fatigue lives which are performed in this thesis. The "broad frequency bands" he described are included in time series generated from the experimental data and used to estimate the contribution of this factor to the damage on the riser.

A work by Modarres-Sadeghi makes further observations on the nature of the Type-II (chaotic) response in the NDP 38m data set. [16] Through this analysis, it is shown that the vortex-induced vibrations of long flexible risers are characterized by time intervals of chaotic response, followed or preceded by periods of statistically stationary response. This result is significant because regions of chaotic response have been ignored in past analyses, although they contain distinctly different response features and have significant implications on riser fatigue analysis.

Modarres-Sadeghi suggests that a concentration on Strouhal or excitation region of the riser yields adequate results for the statistically stationary response, if the higher force harmonics are also included in the analysis. However, such a focus is inadequate for the chaotic parts of the response, whose fatigue properties are influenced by the entire broadly-banded spectrum. This leads to the immediate conclusion that including the broadly-banded harmonics in fatigue estimates is an important part of accurately predicting the damage on a riser.
2.4 VIV Damage Analysis

Discussions of the transient component of the riser motion require an explanation of the techniques which have been developed for estimating the evolution of riser VIV. In [17] and [18], Mukundan outlines his method for performing this task as a way of improving estimates of the fatigue life of the riser. This method captures the presence of traveling waves, the transient portion of the signal, with relatively few sensors. He also uses a cycle counting algorithm for fatigue life estimation, a method for determining whether a signal is statistically stationary, and a special Van-der-Pol wake oscillator modal. He puts all of these elements together to analyze Norwegian Deepwater Project (NDP) data known to contain traveling wave elements and higher harmonics.

His methods for cycle counting are similar enough to those presented in the discussion of the work of Matsuishi and Endo [11] that it is not necessary to discuss his counting method further. Similarly, his riser model does not differ significantly from that of G.S. Triantafyllou [24].

Part of the challenge of a reconstruction is that, to use modal decomposition, the time series presented to the program must be statistically stationary. Mukundan employs a unique method involving a plot called a scalogram for extracting a statistically stationary portion of experimental data to present to the VIVA program.

A scalogram is defined as a contour plot of the squared magnitude of a continuous wavelet transform describing the frequency content of a signal in time. In both this paper and in Mukundan’s thesis, the scalogram can be though of as a PSD which evolves in time. Since the definition of a statistically stationary signal is a signal for which all statistical moments of the PSD are independent of time, the scalogram can be used to identify sections of a time series with constant statistical moments. These sections are Mukundan’s statistically stationary sections.

Figure 2.4 shows a sample scalogram from NDP Datcase 2110. Vertical slices of the plot are PSDs of the signal over a time period centered at the x-coordinate of the slice. Statistically stationary sections are sets of vertical slices which look approximately the same on this plot, such as the section from $t = 2$-4 seconds. Mukundan’s program identifies these sections electronically and presents them to VIVA for signal reconstruction.

After performing the reconstructions of the data series, Mukundan concludes that removing the higher harmonic components from the VIVA inputs
Mukundan also examines the effect of considering only the statistically stationary portion of the signal. He finds that, in general, the statistically stationary portion of the signal gives a more conservative estimate for the minimum fatigue life than the full signal or the non-stationary portion of the signal alone. However, he also mentions that the use of a stationary segment may overestimate the rms and mean value of the fatigue life along the riser. The statistically stationary portion of the signal is thus not the only important factor in determining the fatigue life of the signal.

Building on this work, Y. Modarres-Sadeghi applied these techniques to a riser experiencing both in-line and cross-flow motion with and without higher harmonic forces. [15] Ultimately, he concluded that the exclusion of the higher harmonic forces in these cases causes the fatigue life of the riser to be overestimated by several orders of magnitude. One notable feature of this paper was that it was the first study to attempt to reconstruct the higher harmonics in the same detail as the first harmonic, finding lift forces associated with the third and fifth harmonics.

Modarres-Sadeghi’s methods for riser motion reconstruction and cycle counting are the same methods used by Mukundan. He also uses Mukundan’s method for using the scalogram to find a statistically stationary portion of the signal to analyze. He then develops his method for extracting the magnitudes
of the higher harmonic components and uses them to find an adjusted method for finding the damage on the riser.

The cross-flow and in-line forces as well as their relative phase angles and the velocity of the riser are known for the first harmonic by using the VIVA program. The ratio of each of the harmonic forces to the total force is derived using formulas and data for rigid cylinders. Thus, Modarres-Sadeghi is able to define the parameters $H_3$ and $H_5$ which are the third and fifth harmonic components of the force. For instance,

$$H_3 = \frac{S_3}{S_1}$$

Where $S_1$ and $S_3$ are the stresses associated with the first and third harmonics, respectively. Since the stress of the first harmonic is known and $H_3$ and $H_5$ can be approximated using the ratio from a rigid cylinder, the higher harmonic stresses can be found. The total stress is then the sum of all of these stresses.

$$S_T = S_1 + S_3 + S_5 = S_1 \left( 1 + \frac{\sqrt{H_3} + \sqrt{H_5}}{\sqrt{1 - H_3 - H_5}} \right)$$

After examining many different cases, Modarres-Sadeghi concluded that an $H_3$ or $H_5$ as low as 0.3 is large enough to cause an order of magnitude reduction in the fatigue life of the riser. These results validate concern for the reduction in fatigue life seen in the present of higher harmonics. However, one aspect which is not examined is the magnitude of this change when the stress power is held constant. It is important to determine whether the presence of the higher harmonic, rather than just the additional stress, is an important factor.

It is in the context of this final work that this thesis was undertaken. The broadly-banded spectra of much of the NDP data suggest the presence of significant Type-II response. To prove the importance of including these periods to accurate fatigue life calculation, the Type-II characteristics will be removed from the signal and the fatigue life of this new series found. Thus, it will be shown that the influence of the chaotic behavior is as important as the higher harmonic components of the stress, the importance of which has been previously recognized.
Chapter 3

Time Series Generation Methods

One of the great confounding factors in the study of VIV is non-linearity. Interactions between frequency components in the time domain (constructive and destructive interference) affect the problem in such a way that frequency domain analysis alone cannot accurately solve the fatigue damage problem. However, higher harmonics and PSD spreading are best described in the frequency domain.

Additionally, both factors are present and constantly changing in the experimental data, so that the results of the changes are confounded upon each other. In order to separate the impacts of each factor, it is necessary to generate data as similar as possible to the experimental data while carefully controlling some specific characteristics. Once the factors are understood on an individual basis, it is much more feasible to understand their combined impact.

Figure 3.1 shows a diagram of the full analysis process which was followed. The bolded section at the bottom shows the main steps of the analysis, with the substeps of each process progressing towards the right. First, the original experimental data was analyzed so that its characteristics could be reproduced in the generated data. Then, new time series were generated using these results. The fatigue damage incurred by these new time series is found and compared with the damage from the original data.

3.1 Data Collection Methods

The first step in analyzing the experimental data is to take the power spectral density (PSD). However, since the data was obtained as a set of strain signal readings in time, it was necessary to use the young's modulus to calculate the stress. The formula for finding the stress, $\sigma$, from the strain, $\varepsilon$, is
Figure 3.1: Fatigue Life Reduction Analysis Process

- Experimental Data Analysis
  - Take PSD
  - Identify Harmonics (starts and ends)
  - Integrate PSD (stress power)
  - Calculate Spreading Parameters

- Time Series Generation
  - Alter PSD (new parameters)
  - \( \sigma(t) = \sum_{1}^{N} \sqrt{2S(f_n) \cos(2\pi f_n t + \phi_n)} \)

- Fatigue Life Calculation
  - Find Turn Points
  - Find Drip Ends
  - Calculate Cycle Totals
\[
\sigma = E\epsilon 
\]  

(3.1)

where \( E \) is the Young’s modulus in \( N/m^2 \). The stress signal is then used to perform the remainder of the calculations.

The validity of this analysis depends on the similarity of the generated time series to the original experimental data, so it is important that the experimental data be carefully analyzed and the prominent features preserved. Figure 3.2 shows a closeup of this stage of the analysis from Figure 3.1. The power spectral density of the stress time series is taken to create a frequency domain picture of the signal. Then, the starting and ending points of significant power content are identified for each harmonic. Finally, by integrating under the PSD, the total stress power and stress power at each of the harmonics is found.

The PSD is found from the stress time series using MATLAB’s ”pwelch” function. In order to ensure proper calculation, both ends of the signal are padded with zeros. ”Pwelch” uses a hamming window technique to calculate the PSD, more information on this function can be found in the MATLAB help manual.

Once the PSD has been found, it is processed manually using a custom script, which plots the PSD and defines the start and end points of the harmonics with the input of the user. The PSD is plotted, then the beginnings and ends of the first, third, and fifth harmonics are identified. Figure 3.3 shows the PSD of a sample time series from the NDP 38m data set where the identified start and end of each harmonic is indicated by vertical lines.

The final experimental data processing step is to find the total stress power and harmonic stress power levels by taking the integral underneath the PSD between the identified endpoints. This step is important for making comparisons between data sets or even between trials in the same data set because the total stress power has a larger impact than the other factors.
3.2 Time Series Creation

The time series creation step is one of the more delicate steps in the analysis process. Three different types of time series are needed for the analysis. The first is a simple one-peaked time series which has a frequency at the peak frequency of the time series where all of the stress power is concentrated. For a time series with a total stress power of $P_1 Pa^2$ and a main frequency of $f_1 hz$, the stress time series is

$$\sigma(t) = \sqrt{2P_1} \cos(2\pi f_1 t). \tag{3.2}$$

Figure 5.1(a) shows a sample PSD of the first harmonic only type. As expected, it only contains one frequency component. In order to get a good
estimate of the fatigue life, the time series must be sufficiently long, on the order of 1000 stress cycles. Since the frequency is $f_1$ hz, the proper length is around $1000/f_1$ seconds.

The second type of time series is one with the proper power distribution between the higher harmonic modes. The PSD of this time series is composed of three delta functions, each of the appropriate power. Figure 3.6 shows a plot of this type of PSD.

If the harmonic stress powers for the first, third, and fifth harmonics found in the first step are $P_1$, $P_3$, and $P_5 P_{a^2}$, respectively, than the equation for the stress time series of this type can be written

$$
\sigma(t) = \sqrt{2P_1 \cos(2\pi f_1 t + \phi_1)} + \sqrt{2P_3 \cos(2\pi f_3 t + \phi_3)} + \sqrt{2P_5 \cos(2\pi f_5 t + \phi_5)}
$$

(3.3)

where $f_1$, $f_3$, and $f_5$ are the first, third, and fifth harmonic frequencies. It is also important to have enough data to capture the full interaction of the harmonics. For these cases $1000/f_5$ seconds of data were created.

Finally, the third type of time series is one with the original shapes of the harmonics intact. This is more complicated because the resolution of the PSD can be too large or too small to capture the right frequency windows, depending on the amount of data which is available. As will be discussed in Chapter 4, the number of peaks used in the simulation can have a large
effect on the overall fatigue life. Therefore, it is vital to make sure that the appropriate number of peaks is used to create the time series.

To ensure that only as many peaks as are needed to fully capture the spreading of the PSD are used, the PSD is filtered so that only the local maxima are used as frequency components. Figure 3.7 shows a plot of the spread PSD of the same NDP 38m data used to create Figures 3.6 and 5.1(a). The local maxima, used as frequency components for the creation of the time series, are indicated by the markers. The filtration reduces the number of frequency components per time series from over 500 to around 90.

Once the local maxima are found, the PSD is integrated between minimum points to find the stress power associated with each maximum point. Then the time series is found using the formula

$$\sigma(t) = \sum_{i=1}^{N} \sqrt{2P_i} \cos(2\pi f_i t + \phi_i)$$  \hspace{1cm} (3.4)

where $\phi_i$ is a randomly-generated phase angle corresponding to frequency component $f_i$. Generating a long enough time series is also important for this case, but often there are too many frequency components to make 1000 full modulation cycles a feasible goal. Instead, 10,000 seconds of data were generated in each case.

To highlight the difference caused by the higher harmonics and spreading, Figure 3.8 shows plots of the time series which are built from the PSDs in
Figures 5.1(a) to 3.7. The first harmonic only time series, shown in Figure 3.8(a), contains many repetitions of the same amplitude at one frequency. The time series containing the higher harmonics, shown in Figure 3.8(b), is somewhat more interesting to look at, with small beating effects varying the maximum amplitude over time. Finally, the time series with the higher harmonics and spreading, shown in 3.8(c), is the most interesting, with the various components weaving in and out in a seemingly random fashion.

![Figure 3.8: Types of Generated Time Series](image)

The validity of the analysis hinges on the ability of the time series with spreading to accurately approximate the original time series, so some checks were performed to ensure this ability. First, the fourier transform of the original data was taken so that the phase angles of the frequency components
could be viewed. These values varied from $-2\pi$ to $2\pi$. In the cases which were checked, there was no significant difference in the variance of the real and generated phase angles.

The full original series was also tested against the most detailed generated time series. Figure 3.9 shows a plot of the original time series which was used to generate the time series in Figure 3.8. It holds up quite well in a comparison with the data in Figure 3.8(c), but there are dissimilarities between the two. For instance, the two signals are quite similar in amplitude, but the original signal appears to evolve in time. The generated signal cannot imitate the time-evolving nature of the original signal because its frequency components are constant in time.

![Figure 3.9: Comparison of Original and Generated Data](image)

Since the original and generated time series cannot be exactly the same, the variance of the generated signals was tested. In the comparisons, the results for the generated time series are the average of 50 trials. Figure 3.10 shows a plot of the variance of the original signal and 50 generated signals at each location along the riser for one of the tests. The average variance of the generated signals approximates the original variance, so the two types of time series are similar enough for the desired comparisons.
3.3 Fatigue Life Calculation

As mentioned in previous sections, the fatigue life of each time series is calculated using the Rainflow Algorithm, which was described in detail in Chapter 2. The scripts used to perform these calculations were obtained from Harish Mukundan, the author of [17] and [18] and a former student of M.S. Triantafyllou. The scripts were altered in two important ways which make the calculation of the drip lengths more in accordance with the way the Rainflow Algorithm was originally described and reduce the number of unusable data points.

The first change was relatively minor and had to do with the sizing of the stress amplitude bins into which the real stress cycles were sorted. The old algorithm began with stress bins of a set size and separated the bins at linearly-scaled pre-determined intervals. This algorithm was changed so that
the bin size and spacing was automatically scaled to fit the minimum and maximum order of magnitude of the stress as endpoints and to divide the intervals between into log-scaled bins with the total number of bins proportional to the total number of cycles for more accurate damage predictions for longer time series.

The second change was more complex and had to do with the way in which the amplitude of the stress cycles is determined. The turning points are found by means of a script which locates the local extrema of the signal by progressively stepping through the time series. It was originally obtained from the MATLAB website, but was then altered by Mukundan to make it more applicable to the project.

A significant alteration in the way the stress amplitudes were calculated was made in order to smooth out some irregularities in the calculations made by the Rainflow Algorithm scripts as they had been applied previously. The problem arose when time series with a third harmonic much smaller than the first harmonic was being tested. A sudden dramatic increase in the fatigue life was seen when the power in the third harmonic passed a critical threshold, which made it difficult to find a reliable approximation for the fatigue life.

![Figure 3.12: Time Series Irregularity Causing Rainflow Algorithm Issue](image)

Figure 3.12 shows two example time series to explain why an alteration of the Rainflow Algorithm was necessary. As previously implemented, the algorithm looked for the local maxima and minima of the signal and simply took the difference between the two as the amplitude of stress cycle. This method is fine for the time series on the left hand side where the power at the third harmonic is not large enough to cause local maxima of its own. As
shown by the vertical bars, the full range of the amplitude is captured by the algorithm in this case.

When the third harmonic becomes large enough that it causes local extrema where none existed before, as is the case for the time series on the right, the algorithm cannot keep pace with the new stress amplitudes and finds twice as many cycles of half stress it found previously along with some smaller cycles.

At first glance the solution to this problem would be to simply count quarter cycles of stress as valued from zero. However, this is not complex enough to handle the variety of time series upon which the Rainflow Algorithm is used. For the time series on the right of Figure 3.12, this solution would be enough. However, this solution would not capture the complexity of the time series shown below in Figure 3.13 which consists of a high-power low-frequency signal superimposed upon a low-power high-frequency signal.

![Sample Complex Time Series](image)

Figure 3.13: Sample Complex Time Series

To capture this level of complexity, one final feature must be added to the cycle counter. After finding the local extrema of the signal, the new Rainflow Algorithm checks to see if two adjacent cycles have the same sign. If they do, it uses the difference between the two as a half-amplitude of stress. If an adjacent maxima and minima have opposite signs, the algorithm counts 1/4 cycle of the absolute value of each. Clearly there are imperfections to this method, for example biased cycles with most of their amplitude on one side of the y-axis, but it is much more sophisticated than the older implementation.
Figure 3.14 shows a comparison of the results from the new and old versions of the rainflow algorithms compared with results obtained using a commercially available fatigue calculation code. While the old rainflow algorithm result, shown in blue with circular markers, is within a decent margin of the commercial code, the new algorithm matches the commercial even more nicely.

![Figure 3.14: Rainflow Algorithm Results Compared with Commercial Code](image)

At the end of the analysis process, the damages, total stress power, and harmonic stress powers are available for plotting, in addition to information about the frequencies at which the riser has been excited. As the following chapters show, this information is sufficient not only to show the importance of including the broadly-banded harmonics in the fatigue damage estimate, but also provide enough information that three of the most important factors contributing to the damage on the riser can be estimated.
Chapter 4

Sensitivity Analysis Methods

To increase confidence in the time series generation results and verify the understanding of how each factor increases the damage on the riser, estimates of the damage increase for both the presence of higher harmonic stresses and the broadly-banded harmonics were found. The influence of relative increases in stress power on fatigue life was also investigated, as this quantity is not standard between all cases.

The integral under the stress power spectral density (PSD), by definition the variance of the stress, is used as the total power. This is done with the understanding that some of the data it will later be applied to is not statistically stationary and as such the mathematical definition of the PSD does not strictly apply. In such cases, the PSD is only used as a diagnostic tool facilitating the discovery of certain features of the time series rather than a mathematically rigorous operation.

4.1 Stress Power and Fatigue Damage

The most basic factor contributing to the fatigue of the riser is the total stress power, here defined as the integral under the stress PSD. Thus, the stress power contribution to the fatigue life is the first thing that must be approximated in order to compare other types of damage between cases where the stress power differs.

If a stress time series, $\sigma(t)$, has a PSD, $S(f)$, defined over a range of frequencies $f$, then $\sigma(t)$ is the sum of a set of frequency components $\sigma_i$ so that

$$\sigma(t) = \sum_{i} \sigma_i \cos(2\pi f_i t + \phi_i)$$

(4.1)

where $\phi_i$ is the phase angle associated with the frequency component $f_i$. The value of the frequency components $\sigma_i$ can be found from the PSD using the equation
\[ \sigma_i^2 = 2S(f)df \] (4.2)

where \( df \) is an arbitrary width which has been associated with each frequency component.

The S-N curve, a plot of the relationship between stress and the number of cycles the material can withstand at each stress, is given by the equation

\[ N_i = \left( \frac{A}{\sigma_i} \right)^B \] (4.3)

where A and B are experimentally determined constants. For the material used in the ND 38m tests \( A = 7.53E9Pa \) and \( B = 3 \). Combining these equations with the Miner’s Rule equation from Chapter 2 yields an expression for the damage.

\[ \text{Damage} = \sum_{i=1}^{m} \frac{n_i}{N_i} = \sum_{i=1}^{m} \frac{f_i t}{\left( \frac{A}{\sigma_i} \right)^B} \] (4.4)

Assuming that the riser is subjected to a single frequency of stress, that both the higher stress and lower stress are imposed for the same length of time and at the same frequency, the proportional increase in the damage is

\[ \frac{D_2}{D_1} = \frac{\sigma_2^B}{\sigma_1^B} = \left( \frac{P_2(f)}{P_1(f)} \right)^{\frac{B}{2}}. \] (4.5)

Thus, increasing the stress power by a factor of 2 increases the damage by a factor of \( \sqrt[2]{8/2} = \sqrt[2]{2} \). Figure 4.1 shows two plots to illustrate this effect. On the left, the fatigue life is shown as a function of the stress power, while the proportional damage is shown as a function of the proportion of stress power on the right.

The inverse of this equation can be used to describe the change in the fatigue life in terms of the change in stress power. This equation will be used to estimate the contributions of stress power to differences in fatigue life which are observed in the experimental data.

\[ \Delta FL = \left( \frac{P_1}{P_2} \right)^{1.5} \] (4.6)
4.2 Higher Harmonic Stress Damage

The second quantity to be estimated is the damage increase expected from distribution of power to the higher harmonics. This can be estimated in much the same way, using the S-N curve and Miner's Rule to estimate the damage increase.

Returning to Equation 4.4, the relationship between the stress level and the PSD can be incorporated so that the equation expands to

\[
\text{Damage} = \frac{f_1 t}{\left(\frac{A}{\sqrt{2S_1df}}\right)^B} + \frac{f_3 t}{\left(\frac{A}{\sqrt{2S_3df}}\right)^B} + \frac{f_5 t}{\left(\frac{A}{\sqrt{2S_5df}}\right)^B} \quad (4.7)
\]

For a beam pinned at both ends, the third harmonic frequency is three times the first harmonic frequency while the fifth harmonic frequency is five times the first harmonic frequency. Further the powers in the first, third, and fifth harmonics can be expressed in terms of power ratios such that \(PR_1, PR_3,\) and \(PR_5\) are the ratios of the power at the first, third, and fifth harmonics to the total power. These values can be substituted into Equation 4.7 to make the equation,

\[
D = \frac{f_1 t}{\left(\frac{A}{\sqrt{2PR_1 Power_{total}}}\right)^B} + \frac{3f_3 t}{\left(\frac{A}{\sqrt{2PR_3 Power_{total}}}\right)^B} + \frac{5f_5 t}{\left(\frac{A}{\sqrt{2PR_5 Power_{total}}}\right)^B} \quad (4.8)
\]
When the damage of a series which includes the higher harmonics is compared with one which does not the ratio of the two can be expressed in terms of only the fraction of the total stress power which is present at each harmonic.

\[
\frac{D_{HH}}{D_{1stOnly}} = PR_1^3 + 3PR_3^3 + 5PR_5^3. \tag{4.9}
\]

where \(PR_1\) is the fraction of the total stress power at the first harmonic, \(PR_3\) is the fraction of total stress power at the third harmonic, and \(PR_5\) is the amount of stress power at the fifth harmonic.

Several interesting observations can be made based on this equation. Firstly, it can be seen that the mechanism by which the higher harmonics cause additional damage with respect to the first harmonic is that the body undergoes more cycles in a given time period when subjected to a higher harmonic stress. Secondly, it can be seen that if all of the power is at the third harmonic the damage will be three times its value with all of the stress power at the first harmonic, and five times its value at the first harmonic if all of the stress power is transferred to the fifth harmonic.

Figure 4.2: Damage Increases from Higher Harmonics

Figure 4.2 shows a plot of the increase in damage which the estimate formula predicts for increasing third and fifth harmonic stress power. On the left, the result of increasing the fraction of stress power at the third harmonic is shown by the solid blue line, while the result of increasing the fraction of the stress power at the fifth harmonic is shown by the dashed red line. On
the right, a three-dimensional surface which spans the range of third and fifth harmonic powers is shown.

One interesting feature of the plot on the left is the small decrease in damage which is seen when low levels of power are transferred to the higher harmonics. This occurs because the fatigue life of the material for low level stresses is extremely high. Transferring a small amount of stress to a higher harmonic is akin to storing the stress in a place which reduces the large amplitudes but does not produce cycles of small amplitude to keep the damage estimate in the same region.

4.3 Spreading/Type-II Behavior Damage

The last factor to be examined in this thesis is the role of Type-II behavior, or rather the relationship of Type-II behavior indicators, in the fatigue of the riser. This factor is harder to estimate because its influence depends on a number of factors, including the width of the spreading, the amount of power which is off of the peak of the PSD, and the number of distinct peaks in the spreading.

First, however, it is important to understand why PSD spreading can cause a reduction in the fatigue life. Consider a time series with power at just one peak and another time series with the same amount of power distributed as two equal peaks positioned 0.1hz apart. These two components will modulate in and out of phase over time, with the smallest amplitude being zero when they cancel out. The largest amplitude, however, would be larger than the maximum amplitude of the single-peak time series. If the original PSD power is $P_1$, then the maximum amplitude of the single-peak time series would be

$$\sigma_{\text{max}} = \sqrt{2P_1}. \quad (4.10)$$

Similarly, the maximum amplitude for the two-peaked PSD would be

$$\sigma_{\text{max}} = 2\sqrt{\frac{2P_1}{2}} = 2\sqrt{P_1}. \quad (4.11)$$

Thus, the two-peaked spectrum has a larger maximum amplitude by a factor of $\sqrt{2}$. When there are many peaks, this effect becomes amplified. For 50 peaks, the maximum amplitude is $\sqrt{50} = 5\sqrt{2}$ times larger than for single-peaked PSD.
It is also important to consider the impact of this effect on the damage increases on the riser. Figure 4.3 shows a plot of the stress resulting from the one-peaked spectrum (non-dimensionalized) and the stress resulting from the two-peaked spectrum. For the one-peaked spectrum the frequency was set at 5hz, while the two-peaked spectrum contained two equal peaks at 4.9hz and 5hz.

![Graphs showing stress from one-peaked and two-peaked spectra](image)

Figure 4.3: PSD Spreading and Stress Amplitude

From the graphs, it can be seen that about half of the two-peaked stress cycles are larger than the one-peaked cycles and about half are smaller. For the two-peaked spectrum to cause more damage, the largest cycle and smallest cycle combined must cause more damage than two of the one-peaked cycles. Stated another way, if the damage on the riser at the maximum amplitude of the two-peaked stress signal is twice the damage on the riser at the amplitude of the one-peaked stress signal, then the two-peaked time series is more damaging than the one-peaked time series.

For the material used in the NDP 38m tests, the S-N curve is

\[ N = \left( \frac{7.53E9}{\sigma} \right)^3. \]  

(4.12)

This means that increasing the stress by a factor of two increases the damage by a factor of \( \sqrt{2} \). If the stress amplitude from the single-peaked example in Figure 4.3 is arbitrarily called \( \sigma_1 \), then the fatigue life in cycles at that stress level is
From the earlier calculations, the maximum amplitude of the two-peaked spectrum time series is about $1/\sqrt{2}$ larger than the amplitude of the one-peaked spectrum time series. Thus, the fatigue life of the material at this stress level is

$$N_2 = \frac{A^3}{\sqrt{2}\sigma_1^3} = \frac{N_1}{2\sqrt{2}}$$

The largest amplitude alone causes enough damage that it compensates for the negligible damage done by the smallest amplitude. This repeats with the 2nd largest and 2nd smallest, so that the two-peaked spectrum is ultimately more damaging than the one-peaked spectrum by a factor of $2\sqrt{2}$. For a spectrum with 50 peaks, the multi-peaked spectrum can be up to $5\sqrt{2}$ more damaging.

This knowledge can be combined with the damage equation to create an estimate for the increase in damage from the first harmonic only to the broadly-banded time series. Simplified, this expression is:

$$\frac{D_{spread}}{D_{1stOnly}} = \left(Peaks_1^{3}\right) + 3 \left(Peaks_3^{3}\right) + 5 \left(Peaks_5^{3}\right)$$

where $Peaks_1$, $Peaks_3$, and $Peaks_5$ are the number of significant peaks at the first, third, and fifth harmonics, respectively. In Chapter 5, it is shown that the number of excited string modes encompassed in the manually identified bounds of the harmonic, weighted by the amount of stress power at the harmonic, gives an appropriate number for the number of peaks at a given harmonic.

### 4.4 Summary

This section has explored the mechanisms by which each of the factors which increase fatigue damage. Additional stress power causes greater damage because it increases the amplitude of the stress. Higher harmonics cause additional damage because the body undergoes a greater number of stress cycles in the same time period when subjected to higher frequencies.
spreading increases the maximum stress amplitudes so much that although the minimum amplitudes are decreased, the damage is still increased.

The insights gained from this exploration will guide further investigations. The estimates are not exact, so deviations of 10-20% from the results of the experimental data and time series generation data are expected. These deviations, caused by interactions between frequency components, can alter the fatigue life a great deal. However, on balance, the estimation techniques offer a useful guideline for interpreting and verifying the results of the other forms of analysis which are discussed in the next section.
Chapter 5

NDP 38m Data Analysis

The analysis methods outlined in previous chapters were applied to an experimental data set to find the relative contributions of the stress power, higher harmonics, and broad-banded power spectra to damage on a riser. For clarity, the data analysis process is briefly recounted.

First, the experimental data was manually processed to identify the beginnings and ends of each of the first, third, and fifth harmonics. After this data had been appropriately filtered, some mathematical calculations were performed. The filtering was used to remove noise so that new time series with properties corresponding to those of the original data could be found.

Three types of time series were calculated: the first contained only the peak first harmonic frequency of the original time series; the second contained the first, third, and fifth modes of the original time series; the third preserved the full broad-banded shape of the first, third, and fifth modes of the original signal. Figure 5.1 visually explains the difference between the components used to construct the time series.

The rainflow fatigue life algorithm was applied to both the generated and original time series to find the fatigue lives. The power characteristics, such as the distribution of power among the harmonic modes, were found so that the calculated damage increases could be compared with the estimate formulas from Chapter 4. The following presents some specifications of the experiments as well as the results of the analysis.

5.1 Test Specifications

The NDP 38m test set consists of data from high-mode VIV experiments conducted by the Norwegian Deepwater Programme (NDP) in 2003. All of the data as well as the final experimental report can be found online at http://www.oe.mit.edu/VIV. Detailed descriptions and photos of the experimental setup are contained within this report.
The experiments were conducted at Marintek's ocean basin in Trondheim. A long fiberglass riser was towed down the length of the basin at varying speeds in two different configurations. One flow configuration ensured a uniform flow velocity along the length of the riser, while the other was designed to create a sheared flow with velocity increasing linearly from zero at one end to the maximum trial velocity at the other. All tests were conducted on both bare and straked risers, for both 50% and 100% coverage of the riser by the strakes. Finally, two different strake configurations were tested.

The tests produced much more data than can be coherently discussed in one work. Thus, the focus will be on the uniform and sheared flow cases for the bare riser and the riser with 50% coverage by strakes with a 17.5D/0.25D pitch to height ratio. This particular strake coverage and geometry was noted as a particularly effective one in the final Marintek report.
Table 5.1: NDP 38m Riser Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length Between Pinned Ends</td>
<td>38</td>
<td>m</td>
</tr>
<tr>
<td>Outer Diameter</td>
<td>0.027</td>
<td>m</td>
</tr>
<tr>
<td>Wall Thickness</td>
<td>0.003</td>
<td>m</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>3.62E10</td>
<td>Pa</td>
</tr>
<tr>
<td>Bending Stiffness(EI)</td>
<td>598</td>
<td>Nm²</td>
</tr>
<tr>
<td>Mass per Unit Length (air-filled)</td>
<td>0.761</td>
<td>kg/m</td>
</tr>
<tr>
<td>Mass Per Unit Length (water)</td>
<td>0.933</td>
<td>kg/m</td>
</tr>
<tr>
<td>Tension</td>
<td>5000</td>
<td>N</td>
</tr>
<tr>
<td>SN Curve A</td>
<td>7.53E9</td>
<td>Pa</td>
</tr>
<tr>
<td>SN Curve B</td>
<td>3</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 5.1 contains a list of basic properties of the riser which were used in the course of the data analysis. It should be noted that the values for the Young's modulus and the bending stiffness are differ from those found in the Marintek final report. In March of 2011, it was announced that the wrong values for these properties had been reported. The values were initially reported as 2.25E9 Pa and 37.2 Nm² and were then corrected to 3.62E10 Pa and 598 Nm² respectively. The Wohler or S-N curve value for A also differs from the one used on previous occasions, and has changed from 4.8E11 Pa to 7.53E9 Pa since the Marintek report was issued.

Table 5.2 contains the key properties of the helical strakes attached as VIV-suppression devices. Note that only one of the two strake configurations which were tested is considered, as the other strake configuration was shown to be less effective for suppressing VIV. The strakes were arranged in a triple helix around the outside of the riser, while periodic breaks in the strake to allow for instrumentation. Increases in the bending stiffness due to the presence of the strake were not considered. A brief calculation shows that since the Young's modulus of polypropylene is 2-3 orders of magnitude below the fiberglass of the riser, the stress caused by a given strain is not significantly altered by the addition of the strake material to the outside of the riser.

Towing speeds varied between 0.3 and 2.3 m/s are used as a convenient index for plotting data. Vortex shedding frequency increases with increased flow velocity, so there are more vibrations and a larger amount of energy as the flow velocity increases. Since most variables plotted are related to stress
Table 5.2: NDP 38m Riser Strake Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer Sleeve Diameter</td>
<td>0.0032</td>
<td>m</td>
</tr>
<tr>
<td>Pitch / Diameter</td>
<td>17.5</td>
<td>--</td>
</tr>
<tr>
<td>Height / Diameter</td>
<td>0.25</td>
<td>--</td>
</tr>
<tr>
<td>Weight in Water</td>
<td>2.79</td>
<td>N/m</td>
</tr>
</tbody>
</table>

power, plotting against velocity is a sensible method of data organization. In the sheared flow cases, the velocity reported is the maximum velocity, while in the uniform cases it is the only velocity. To emphasize this point, the velocity axis is labelled "Maximum Trial Velocity."

5.2 Uniform Flow Bare Riser Results

The uniform bare test case contains tests at towing speeds from 0.3 to 2.4 m/s and represents the trial with no attempts made to suppress the periods of chaotic (Type-II) motion. It is therefore the data set most suited to this investigation, because it showcases the potential size of the increase in damage caused by periods of chaotic response. This section discusses the maximum values of the data over the length of the riser, except where otherwise noted. Plots of the results of each trial over the length of the riser can be found in Appendix A.

Figure 5.2(a) shows a plot of the maximum damage over the span of the riser for the original data, as well as the maximum damages for the time series created using only the first harmonic; the pure first, third, and fifth harmonics; and the full broadly-banded first, third, and fifth harmonics, respectively. While all four series show nearly the same trend, the gain in accuracy which accompanies the use of the broadly-banded harmonics is detectable.

To make the difference in accuracy easier to distinguish, the ratio of the damage for each of the three constructed time series to the damage found for the original time series is shown in Figure 5.2(b). For convenience, a dashed line is plotted at a value of one, indicating the value at which the damage of the recreated data is equal to the value of the damage of the original time series.

The reconstructed signal which includes spreading, the "1st + HH +
Spreading” data, is not exactly the same signal as the original. Therefore we do not expect to see a perfect value of 1 for the comparison with the original data. The ratios are presented in this manner in order to make the differences between the three types of reconstructed data more visible.

Three things are noticeable from this plot, firstly the data representing the broadly-banded harmonics is closer to the fatigue life found for the origi-
inal data than the data representing the purely periodic higher harmonics. Secondly, although the broadly-banded harmonic damage is not always equal to the damage of the original time series it is usually in error in the positive direction so it is more conservative. Finally, and most importantly, the increase in damage attributed to the inclusion of the influence of the chaotic motion and stress is equal or greater in magnitude to the damage increase which can be attributed to the inclusion of the higher harmonic frequencies.

To verify this result and confirm an understanding of the mechanism by which these two factors increase the damage on the riser, the results of the quantities plotted in Figure 5.2 were compared with estimates for the damage which were derived from first principles. These derivations are shown in full in Chapter 3: Fatigue Damage Estimation from First Principles.

As was shown earlier, the stress power has the largest effect in determining the damage on the riser. The total stress power as well as the stress power at the first, third, and fifth modes was calculated and is shown in Figure 5.3. From this plot it can be seen that the bulk of the power is located at the first harmonic. While the third harmonic also makes a substantial contribution to the total power, the fifth harmonic is relatively unimportant. The large increase in the stress power with increasing velocity also supports the dramatic increase in the damage on the riser which was observed in Figure 5.2.

![Figure 5.3: Total Stress Power for Uniform Bare Case](image)

---

"Total" --- 1st Mode --- 3rd Mode --- 5th Mode

- Total
- 1st Mode
- 3rd Mode
- 5th Mode

- Stress Power (Pa²)
- Maximum Trial Velocity (m/s)

- $10^5$
- $10^6$
- $10^7$
- $10^8$
- $10^9$
- $10^10$
- $10^11$
- $10^12$
- $10^13$
- $10^14$

- 0.5
- 1
- 1.5
- 2
- 2.5

Figure 5.3: Total Stress Power for Uniform Bare Case
The relationship between the increase in the stress power and the increase in damage on the riser was derived in Chapter 4, where a formula describing the theoretical relationship between the two was found. This relationship can be used to produce an estimate for the damage increase on the riser based on the stress power.

\[
\frac{\text{Damage}_2}{\text{Damage}_1} = \left(\frac{\text{Power}_2}{\text{Power}_1}\right)^{0.3}
\]  

(5.1)

The resulting value is compared to the damage which was found for the data created using only the first harmonic of the signal in the plot in Figure 5.4. It can be seen that the theoretical calculation is an excellent approximation for the increase in damage which is seen with increasing stress power.

The next important factor in the calculation of the damage on the riser is the distribution of the stress power to the higher harmonics. As stress power shifts from low to high frequency, the damage increases because the riser experiences more cycles of the same amplitude of stress in a given period of time. A mathematical formula describing this relationship was derived in Chapter 4.
Thus, if all of the damage was transferred from the first to the third harmonic the damage would increase by a factor of three. Likewise, if all of the power were transferred from the first to the fifth harmonic the damage would increase by a factor of five.

Figure 5.5 shows a plot of both the damage and increase in damage from the first harmonic only case. On the left, the damage calculated using Equation 1.2 is compared to the damage found for the data labeled "1st Harmonic + HH" in Figure 5.2. At this level, the estimate and approximate results appear nearly identical.

To further explore the accuracy of the derived equation, the plot on the right compares the ratio of the increase in damage from the data labelled "1st Harmonic Only" in Figure 5.2 is shown for the generated time series data and the results of Equation 1.2. This plot more clearly shows the points where there is disagreement between the estimate and the actual data. Although these differences, which arise because of the ways in which different frequency components combine, appear large at some points, the estimate is within 5%.

The third type of damage was discussed in Chapter 4 was the damage increase associated with broadly banded harmonics, which are used as an indicator for Type-II behavior. Because of the way the frequency components add together, a more broadly-banded harmonic translates to a larger maximum amplitude of stress, increasing the damage. The mathematical description of this phenomenon is:

\[
\frac{Damage_{1st+HH}}{Damage_{1stOnly}} = \left( \frac{Power_{1st}}{Power_{Total}} \right)^{\frac{3}{2}} + 3 \left( \frac{Power_{3rd}}{Power_{Total}} \right)^{\frac{3}{2}} + 5 \left( \frac{Power_{5th}}{Power_{Total}} \right)^{\frac{3}{2}}
\]  

(5.2)

More qualitatively, the more broadly-banded the spectrum, the lower the fatigue life, with spreading at the higher harmonics having more influence because of the greater number of cycles experienced at these frequencies.

Figure 5.6 shows a plot of the number of excited string modes at each harmonic as well as a plot showing the locations of the peak frequencies, theoretical shedding frequency, and theoretical string frequencies of the riser.
A specially calculated weighted version of the number of peak frequencies, which utilized the number of string modes, power at each harmonic, and distance from the theoretical shedding mode, was used to calculate the number
of peaks to insert in Equation 1.3 for the spreading damage estimate. Thus, the larger the number of modes the larger the spreading damage increase.

Figure 5.7 shows plots which compare the results obtained using Equation 1.3 with the results of the tests earlier described as "1st Harmonic + Higher Harmonics + Spreading". On the right, the raw values of the damage results are compared. Once again, the purely theoretical calculation provides an estimate which is very close to the real data. On the left, the ratio of the damage with the higher harmonics and spreading to the damage obtained from time series created from only the first harmonic.

As with the distributed damage, the scale of the change over the range of the velocities obscures the differences between the estimate and the actual data. Thus, Figure 5.7(b) shows the difference between the damage found when the power spreading is included and the damage found from the first harmonic only. This plot makes it clear that, while there are some differences between the actual and estimated damage, the estimation captures the general trend in the data as well as the conservative side of the damage. The highs and lows in the data from the recreated time series are likely the result of interactions between frequency components of the time series. These interactions would thus be non-linear in the time domain and incalculable in the frequency domain.

One final line of investigation which was pursued for the test case was to examine the differences between estimates of the damage caused by the broadly-banded harmonics based on the width of the power spectral density and estimates based on the scalogram. It is expected that the scalogram will give a more accurate, less conservative, prediction for the damage because it will have the right number of frequency components building on one another. Spreading damage estimates based on the PSD do not alter their coefficients in time, while the real time series does appear to evolve in time.

Figure 5.8 shows the damage found for the original time series as well as the damage found using spreading estimates. The black line with triangular markers representing the estimate based on the PSD falls above the solid red line representing the original time series. The blue line with square markers representing the estimate found using the scalogram and the procedure described in Chapter 4 falls cleanly between these two lines, confirming that the scalogram is a more accurate method of determining the damage increase caused by the broadly-banded harmonics.

In review, it has been shown that the results which were obtained for the time series with only one frequency, the time series with pure first and higher
(a) Mean Number of Excited String Modes over Riser Length

(b) Peak, Shedding, and String Modes for Uniform Bare Case

Figure 5.6: Frequency Analysis for the Uniform Bare Case
Figure 5.7: Maximum Spreading Damage and Damage Increase from the "First Harmonic Only" Signals for Uniform Bare Case

harmonics, and time series which included the broadly-banded forms of the signal are accurate and in accordance with available methods for estimating
their values. Where the broadly-banded signal is concerned, it is slightly more correct to used the scalogram estimate rather than an estimate based on the PSD because the former captures the transient portions of the signal. The contribution of the broadly-banded spectrum is as important to an accurate result as the contribution of the higher harmonics.

5.3 Sheared Bare and Straked Riser Results

The same analysis which was performed on data from the uniform flow bare riser test set was performed on the test set from a bare riser in sheared flow as well as tests on a 50% straked riser in uniform and sheared flow. The main results were quite similar to those obtained for the uniform bare test case when differences in power distribution and spreading were considered. As for the uniform bare case, the plots shown here represent the average values over the length of the riser. Plots of the results of each individual trial over the length of the riser can be found in Appendix A.

Figure 5.9 shows plots of the total and harmonic stress power for each of the remaining three data cases. All cases exhibit increases in the total stress over the increase in maximum trial velocity. The most marked difference between the three cases is the order of magnitude of the total power. The
uniform bare data which was shown in Figure 5.3 was on the order of $10^{16} Pa^2$, the sheared bare and uniform straked results are on the order of $10^{15} Pa^2$ and the sheared straked result is on the order of $10^{14} Pa^2$. Thus it is expected that the power damage will be greatest for the uniform bare data set, and smallest for the sheared straked data set, with the uniform straked and sheared bare data sets falling somewhere in between.

The stress power-related damage is for the three data cases is shown in Figure 5.10. In all cases the data labelled as "actual" is the damage from time series which were created using only the first harmonic of the original signal. The qualitative predictions which were based on Figure 5.9 are proven correct. The uniform bare data set shows a maximum damage which approaches $10^2 years^{-1}$, both the uniform straked and sheared bare data sets have a maximum damage on the order of $10^1 years^{-1}$, and the sheared straked data set has a maximum damage near $10^{-2} years^{-1}$.
Figure 5.10: First Harmonic Only Damage by Case

Although Figure 5.9 showed the power at each harmonic, it is perhaps more visually helpful to see the fraction of the total power contained at each harmonic for all of the data cases. The data from the sheared flow bare riser case is shown in Figure 5.11(a), where it can be seen that nearly all of the power is positioned at the first harmonic with the third harmonic carrying an amount of the stress which increases with trial velocity. There is a sharp increase in the total stress power above 1.5 m/s.

Figure 5.11(b) shows the total and harmonic stress power for the uniform flow 50% strake coverage case. Even more of the stress power is at the first harmonic with no noticeable trend present in the small amount of power which is present at the first harmonic. Finally, Figure 5.11(c) shows the
total and harmonic stress power for the sheared flow 50% strake coverage data set. In this case, the first harmonic contains virtually all of the stress power.

Given these results, it is expected that the sheared bare case will have an decreasing, but relatively large damage increase when the first harmonic only data is compared with the data which includes the higher harmonics. The uniform straked data will show an smaller but increasing damage increase, and the sheared straked data will show little to no increase.

Figure 5.12 contains plots of the ratio of the damage when the higher harmonics are included in the signal to the damage when only the first harmonic is used to create the signal. These plots confirm the qualitative predictions
made from the distribution of the stress power between harmonics. Firstly, the uniform bare data has the largest amount of power distributed to the higher harmonics, so it was predicted that it would exhibit the largest increase in damage when the higher harmonics were included. This is confirmed by its average value near 2, higher than any other case. It also agreed with predictions that it would show a slight decreasing trend.

![Graphs showing damage increase for different cases.](image)

(a) Estimated and Actual Damage from the "First Harmonic and Higher Harmonics" Signals for Sheared Bare Case  
(b) Estimated and Actual Damage from the "First Harmonic and Higher Harmonics" Signals for the Uniform Straked Case  
(c) Estimated and Actual Damage from the "First Harmonic and Higher Harmonics" Signals for the Sheared Straked Case

Figure 5.12: First Harmonic and Higher Harmonic Damage by Case

Figure 5.12(a) shows the damage increase for the sheared bare data set. This set shows the second largest increase, with a maximum value near 1.6 and also displays the predicted decreasing trend. The results for the uniform straked data set are shown in Figure 5.12(b). This set presents an even
lower power distribution damage increase, with a maximum value near 1.3 and follows the predicted increasing trend. Finally, the sheared straked data, displayed in Figure 5.12(c), shows a relatively flat ratio which is very near 1, as predicted.

To calculate the spreading damage for each case, the number of excited string modes were calculated for each test. Figure 5.13 shows a plot of the number of excited string modes for each case. Naturally, these plots are not adequate to create a full picture of the extent to which the cases display broadly-banded harmonics, but they do provide a useful metric for qualitative predictions for the spreading damage increases.

Figure 5.13: Excited String Modes by Case

Like the uniform bare excite string modes, there are largely the same number of excited string modes for each harmonic in the uniform bare case.
Both of the sheared cases show smaller numbers of excited string modes for the higher harmonics than for the first harmonic. The uniform bare has the most excited string modes followed by the uniform straked, sheared bare, and sheared straked cases in order.

The number of significant peaks among the excited string modes is found using a simple searching algorithm and then weighted by the amount of power present at each harmonic to produce a weighted number of excited string modes at each harmonic. Then, these modes are used as the number of peaks in Equation 1.3 to calculate an estimate for the increase in damage due to spreading. The results of these calculations are shown in Figure 5.14.

The results of the spreading damage increase calculations are somewhat surprising because the sheared straked case has one of the largest increases in damage. On further investigation though, this is due to the lack of significant higher harmonics in this case. Since there are no higher harmonics, all of the increase from the first harmonic only results is due to spreading at the first harmonic. The lack of higher harmonics also explains why the estimation formula underestimates the damage increase only in this case.

To conclude the analysis, Figure 5.15 shows a comparison of the damages for the sheared bare, uniform straked, and sheared straked data cases. For the sheared bare case, shown in Figure 5.15(a), the data which includes the first harmonics and the higher harmonics is visibly lower than the original data, while the data which includes the spreading is visibly above. The inclusion of the spreading clearly accounts for factors which are not encompassed in the higher harmonics alone, yet the inclusion of the spreading does overshoot the damage found in the original data.

This theme is repeated in the sheared straked results which are shown in Figures 5.15(c), but is not seen in the results for the uniform straked case shown in Figure 5.15(b), where the data which includes the spreading is directly over the original data. A possible explanation for this observance is that the uniform flow cases at times exhibit more chaotic behavior because the riser is excited along its full length rather than only in the Strouhal region. Thus, the inclusion of the chaotic points during the full recreation of the time series rather than in only part of it causes less of an overcalculation.

The results of the analysis of data beyond the uniform bare case confirms the statements which were tentatively made at the conclusion of the uniform bare case analysis. Most importantly, the inclusion of the broadly-banded harmonics is as important to an accurate and conservative estimate of the damage on a riser in VIV as the inclusion of the higher harmonics. The
mechanisms by which the damage is increased by increases stress power, distribution of power to higher harmonics, and the widening of the harmonics is understood well enough to produce reasonable estimates of the outcome when one of these factors is changed.
Figure 5.15: All Types of Damage by Case
Chapter 6

Conclusions

The impacts of the total stress power (variance of the stress), distribution of stress power to higher harmonics, and broadly-banded higher harmonics have been discussed conceptually and explored though analysis of experimental data and estimated from first principles. In each case, the mechanism by which the factor increases the total damage has been explored, estimates for the impact have been calculated, and the increase in damage has been calculated using a combination of analytical techniques.

In the case of the total stress power, it was found that the mechanism by which the fatigue life is decreased is that the amplitude of the stress cycles is increased. For the NDP 38m data set, every factor of two increase in the stress amplitude the damage on the riser is increased by a factor of \(2\sqrt{2}\). Differences in the stress power cause much of the fatigue life disparities within and between data sets.

The power distribution among harmonics increased the damage on the riser because it forced the riser to undergo more cycles in a given time span. The impact of the higher harmonics is related to the fraction of the total stress power present at the higher harmonics. If all of the power were moved from the first to the third harmonic, the damage would be increase by a factor of 3, while if all the power were moved to the fifth, the damage would be increased by a factor of 5. Consequently, future studies which attempt to scale the higher harmonic contributions to the fatigue in terms of the first harmonic should scale the contribution of the higher harmonics with the fraction of stress powers rather than the relative stresses.

PSD spreading increases the damage on the riser by increasing the maximum stress amplitude though constructive interference between frequency components. Modulation of the components means that there is also destructive interference, but the damage increase during the constructive interference periods is larger than the decrease during the destructive interference periods. A series with two peaks instead of one has its damage increased by a factor of \(\sqrt{2}\), while a series with 50 peaks has its damage increased by a
factor of $5\sqrt{2}$.

The number of excited string modes within the frequency region identified for a given harmonic weighted by the amount of the total stress power present at that harmonic is an accurate number of peaks with which to estimate the damage increase due to spreading. A reasonable and conservative estimate is obtained if the PSD is used for this estimate, but using the time-averaged values of the scalogram is less conservative but more accurate. Since the number of modes to consider determines the amplitudes created by constructive interference, it is useful to remember this relationship.

Provided that there are significant periods of chaotic behavior, this behavior is capable of causing increases in damage equal or greater than the increases caused by the presence of purely periodic higher harmonic components. The NDP 38m data set provides an example of this situation. Since this data set is so widely used, it is theorized that the capturing chaotic behavior will be a crucial portion of accurate predictions in the future.

Most importantly, it has been definitively shown that including the periods of chaotic response in calculations of the damage on the riser is as important as including the higher harmonic components of the signal. This result was seen across data cases and holds up to comparison with the fatigue life of the original signal.
Bibliography


Appendix A

NDP 38 Results by Data Case
Figure A.1: Total Stress Power for Uniform Bare Case
Figure A.2: Total Stress Power for Sheared Bare Case
Figure A.3: Total Stress Power for Uniform Stacked Case
Figure A.4: Total Stress Power for Sheared Streaked Case
Figure A.2: Fraction of stress power for uniform bare case
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Figure A.25: Power Spreading Damage and Estimate for Uniform Bare Case
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Figure A.28: Power Spreading Damage and Estimate for Sheared Stranded