15.093J/2.098J Optimization Methods

Assignment 1 (100 points)

Due September 22

This assignment covers lectures 1,2,3. Exercises 1.1, 1.2, 1.3, 1.4, 1.6 are from the book, denoted at BT, by D. Bertsimas and J. Tsitsiklis, Introduction to Linear Optimization, Athena Scientific, 1997. In each exercise, I summarize the reason I assigned it and what you should learn from it.

Exercise 1.1 (15 points) BT, Exercise 1.10.

The objective of this exercise is to teach you (a) how to model an interesting production and inventory planning problem, and (b) how to model problems involving absolute values using linear optimization.

Exercise 1.2 (15 points) BT, Exercise 1.14.

The objective of this exercise is to give you geometrical insight on the solution of two dimensional linear optimization problems.

Exercise 1.3 (15 points) BT, Exercise 1.16. After formulating the problem please solve it using either Matlab or Excel.

The objective of this exercise is (a) to improve your formulation skills and (b) to familiarize you with solving linear optimization problems on the computer.

Exercise 1.4 (15 points) BT, Exercise 1.17. After formulating the problem please solve it using either Matlab or Excel using the following data:

s_i	p_i	q_i	r_i
1000	55	70	75
2000	73	60	65
800	40	42	40
1500	80	65	85
600	140	160	170

Table 1: Data for Exercise 1.4.

What would the solution be for $K = 100,000$ and $K = 140,000$?

The objective of this exercise is to show the usefulness of linear optimization in your personal life. Suppose for example you are trying to raise cash for a down-payment in a house.

Exercise 1.5 (20 points) Consider the problem

$$\begin{array}{llll}
\min & c_1x_1+ \; c_2x_2+ \; c_3x_3 & \\
\text{s.t.} & x_1+ \; x_2+ \; x_3 & \leq 4 \\
& x_1 & \leq 2 \\
& x_3 & \leq 3 \\
& 3x_2+ \; x_3 & \leq 6 \\
& x_1, \; x_2, \; x_3 & \geq 0.
\end{array}$$

Note that the coefficients c_1, c_2, c_3 have not yet been specified. The feasible space is shown in Figure 1. In standard form $\mathbf{Ax} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}$ the feasible space is written as follows:

$$\begin{array}{c} \begin{array}{ccccccc} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \mathbf{A}_4 & \mathbf{A}_5 & \mathbf{A}_6 & \mathbf{A}_7 \end{array} \\ \left[\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \\ 6 \end{pmatrix}$$

$$\mathbf{x} \geq \mathbf{0}$$

1. Is the point $(0, 1, 3)$ a basic feasible solution?

2. Is the point $(0, 1, 3)$ a degenerate basic feasible solution? If so, identify the possible basis corresponding to it.

3. Consider the basis:

$$B_1 = [\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_5],$$

$$B_2 = [\mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4, \mathbf{A}_5].$$

   corresponding to the BFS $(0, 1, 3)$. Suppose $\mathbf{c} = (3, -1, -1)'$. Are the optimality conditions satisfied for B_1? For B_2? (Use Matlab to perform matrix calculations as it will make your life very simple.) What do you observe?

4. Suppose again $\mathbf{c} = (3, -1, -1)'$ and you start with basis B_1. What will the simplex method do? What happens geometrically?

Figure 1: The feasible space for Exercise 1.5.

5. Suppose again $\mathbf{c} = (3, -1, -1)'$ and that you start with the BFS $(1, 0, 3)$. What will the simplex method do? What happens geometrically?

6. Suppose $\mathbf{c} = (-2, -2, -2)'$ and that you start with the BFS $(0, 0, 3)$. What will the simplex method do? What happens geometrically? Are there multiple optimal solutions in this case? How does the simplex method detect this?

The objective of this exercise is to teach you the insightful interplay between geometric and algebraic concepts in a concrete example, and to deepen your understanding of degeneracy, optimality conditions, and how the simplex method works. I reiterate my advice to use Matlab for matrix computations.

Exercise 1.6 (20 points) BT, Exercise 2.10.

The purpose of this exercise is to deepen your understanding of the underlying geometric concepts of linear optimization.