15.093J

Midterm Exam

November, 6, 2001

1. This is a 2 hours exam.

2. You are required to submit both the questions and your answers.

3. You can use the textbook for the class, the notes from the lectures, your homeworks and homeworks solutions.

4. Good luck!

Problem 1 (30 points)

Suppose we are responsible for production planning for a factory that can produce four types of products. Each of the four products requires a certain amount of each of three limited resources. We would like to optimize our production by solving a linear program of the form maximize $\mathbf{c}'\mathbf{x} : \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}$. After solving the problem, we obtain the following solution and sensitivity information. (Note that some of the entries have been obscured.)

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Product 1	0	-9	12	(f)	(f)
Product 2	27	(f)	6	Infinite	5
Product 3	0	0	12	8	4
Product 4	0	-7	6	7	Infinite
Name	**Final Value**	**Shadow Price**	**Constraint R.H. Side**	**Allowable Increase**	**Allowable Decrease**
Resource 1	100	14	100	50	50
Resource 2	200	(f)	300	(f)	(f)
Resource 3	100	75	100	80	90

Table 1: Information about variables and constraints.

Please answer the following questions, and explain your reasoning for each.

(a) If we could purchase another 10 units of Resource 3 for a total of \$700, would it be worth it to us to do so?

(b) If we could purchase another 100 units of Resource 3 for a total of \$8000, would it be worth it to us to do so?

(c) Within the range of the allowable increase and allowable decrease for cost coefficient of Product 3, what are the smallest and largest amounts of Product 3 that will be produced?

(d) What is the smallest objective coefficient for which Product 4 will be produced?

(e) If we could increase our supply of any one of our resources by 10 units without incurring any cost, which resource should we choose?

(f) Fill in the correct values for the six shaded entries labeled (f).

(g) Is this solution degenerate?

Problem 2 (30 points)

We consider the production of a single product over T periods. If we decide to produce at period t, a setup cost c_t is incurred. Excess products produced at earlier periods are stored to meet the demand at later periods. Moreover, all the demand in earlier periods must be met during the last period. For $t = 1, \ldots, T$, let d_t be the demand for this product in period t, and let p_t, h_t be the unit production cost and unit storage cost (per period), respectively.

(a) Formulate an integer programming problem in order to minimize the total cost of production, storage, and setup.

(b) Suppose we allow demand to be lost in every period except for period T, at a cost of b_t per unit of lost demand. Show how to modify the model to handle this option.

(c) Suppose that production can occur in at most five periods, but no two such periods can be consecutive. Show how to modify the model to handle this option.

Problem 3 (30 points)

While solving a standard form linear programming problem using the simplex method, we arrive at the following tableau:

		x_1	x_2	x_3	x_4	x_5
		0	0	$\bar{c}_3$	0	$\bar{c}_5$
$x_2 =$	1	0	1	-1	0	β
$x_4 =$	2	0	0	2	1	γ
$x_1 =$	3	1	0	4	0	δ

Suppose also that the last three columns of the matrix $\mathbf{A}$ form an identity matrix.

(a) Give necessary and sufficient conditions for the basis described by this tableau to be optimal (in terms of the coefficients in the tableau).

(b) Assume that this basis is optimal and that $\bar{c}_3 = 0$. Find an optimal basic feasible solution, other than the one described by this tableau.

(c) Suppose that $\gamma \geq 0$. Show that there exists an optimal basic feasible solution, regardless of the values of $\bar{c}_3$ and $\bar{c}_5$.

(d) Assume that the basis associated with this tableau is optimal. Suppose also that b_1 in the original problem is replaced by $b_1 + \epsilon$. Give upper and lower bounds on ϵ so that this basis remains optimal.

(e) Assume that the basis associated with this tableau is optimal. Suppose also that c_1 in the original problem is replaced by $c_1 + \epsilon$. Give upper and lower bounds on ϵ so that this basis remains optimal.

Problem 4 (10 points)

United Airlines has four daily flights from Chicago to Boston. At 10am, 12Pm, 2PM and 4PM. The first two flights have a capacity of 100 passengers and the last two flights can accommodate 150 passengers each. If overbooking results in insufficient room for passengers on a scheduled flight, United can divert a passenger to a later flight. It compensates any passenger delayed by paying \$200 plus \$20 for every hour of delay. United can always accommodate passengers delayed beyond the 4pm flight on the 8pm flight of another airline that always has spare capacity. There is no additional cost to use the other airline. For example, if a passenger scheduled to depart at 2PM is rescheduled for the 8PM flight, he gets \$200+6×20=\$320. Suppose that at the start of a particular day the four United flights have 110, 160, 100, and 140 confirmed reservations. Show how to formulate the problem of determining the most economical passenger routing strategy as a minimum cost flow problem.