2.098/15.093J: Example Solutions 1

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Example 1 Multicommodity path-flow formulation
In a given network, there are K origin-destination(OD) pair. Each OD pair k has a set of paths P^k and a demand d^k. Each edge e in the network is associated with a cost c_e and a capacity u_e. The problem is to minimize the total cost while satisfying the demand.
Solution
Decision variables: x_P, the flow carried on the path P, $P \in \bigcup_{k=1}^K P^k$
Objective function:

$$\sum_{P \in \bigcup_{k=1}^K P^k} (\sum_{e \in P} c_e) x_P \tag{1}$$

Capacity constraint:

$$\sum_{P \in \bigcup_{k=1}^K P^k : e \in P} x_P \leq u_e \qquad \forall e \tag{2}$$

Demand constraint:

$$\sum_{P \in P^k} x_P \geq d^k \qquad \forall k = \overline{1, K} \tag{3}$$

Nonnegative constraint:

$$x_P \geq 0 \qquad \forall P \in \bigcup_{k=1}^K P^k \tag{4}$$

Example 2 Formulation with absolute values

$$\begin{array}{ll} \min & x_1 \\ \text{s.t.} & \\ & |x_1 + 2| + |x_2| \leq 5 \end{array} \tag{5}$$

Solution

$$|x_1 + 2| + |x_2| \leq 5 \Leftrightarrow \begin{cases} x_1 + x_2 \leq 3 & x_1 \geq -2, x_2 \geq 0 \\ x_1 - x_2 \leq 3 & x_1 \geq -2, x_2 \leq 0 \\ -x_1 + x_2 \leq 7 & x_1 \leq -2, x_2 \geq 0 \\ -x_1 - x_2 \leq 7 & x_1 \leq -2, x_2 \leq 0 \end{cases} \tag{6}$$

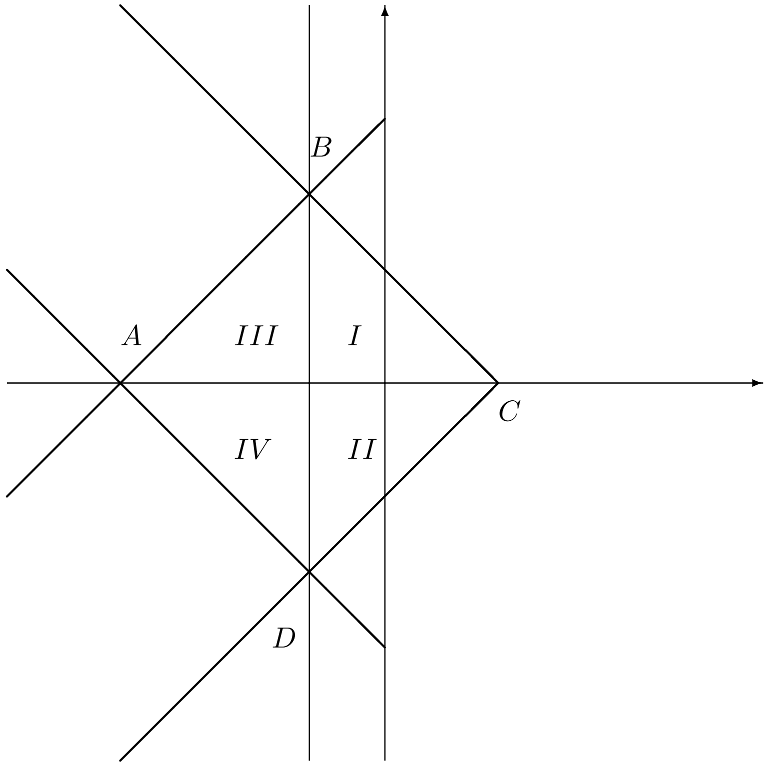

Figure 1 Feasible region of the problem

The feasible region is the square $ABCD$. Thus the constraints can be written as below:

$$x_1 + x_2 \leq 3 \tag{7}$$
$$x_1 - x_2 \leq 3 \tag{8}$$
$$-x_1 + x_2 \leq 7 \tag{9}$$
$$-x_1 - x_2 \leq 7 \tag{10}$$

Example 3 Basic feasible solutions, optimal basis, and degeneracy

$$\begin{aligned} \min \quad & x_1 \\ \text{s.t.} \quad & \\ & x_1 + x_2 \leq 2 \\ & -x_1 + x_2 \leq 0 \\ & x_1, x_2 \geq 0 \end{aligned} \tag{11}$$

Solution
The standard form with slack variables:

$$\begin{aligned} \min \quad & x_1 + x_2 \\ \text{s.t.} \quad & \\ & x_1 + x_2 + s_1 \quad = 2 \\ & -x_1 + x_2 \quad + s_2 = 0 \\ & x_1, x_2, s_1, s_2 \geq 0 \end{aligned} \tag{12}$$

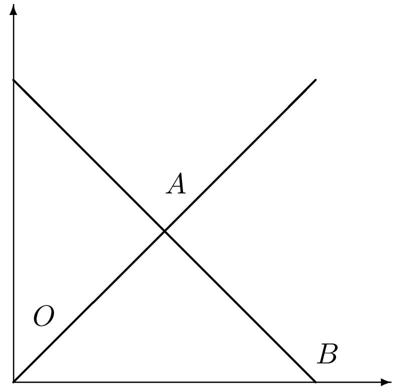

Figure 2 Feasible region of the original problem

From the graph, we have: the feasible region is the triangle OAB with three extreme points $O(0,0)$, $A(1,1)$, and $B(2,0)$. The matrix $\mathbb{A}$ for the problem in standard form is:

$$\mathbb{A} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

$A(1,1)$: basic feasible solution (1, 1, 0, 0), nondegenerate with the basis $\mathbb{B}_1 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

$B(2,0)$: basic feasible solution (2, 0, 0, 2), nondegenerate with the basis $\mathbb{B}_2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$.

$O(0,0)$: basic feasible solution (0, 0, 2, 0), degenerate with three bases $\mathbb{B}_3 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $\mathbb{B}_4 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, and $\mathbb{B}_5 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The optimal solution is (0, 0, 2, 0). In order to check optimal bases, we need to use the optimality condition: $\overline{c}' = c' - c'_B \mathbb{B}^{-1}\mathbb{A} \geq 0'$. For example, the basic $\mathbb{B}_4$ is not an optimal basis: $\overline{c}'_4 = c' - c'_{B_4}\mathbb{B}_4^{-1}\mathbb{A} = (2, 0, 0, -1)$.