2.098/15.093J: Example Solutions 4

Xuan Vinh Doan,

October 8, 2004

Exerise 5.7 BT
Solve the problem with $b_1 = 300,000$, $b_2 = 240,000$, and $b_3 = 30,000$.
Solution
The normal formulation for this problem is as follows:
x_1: number of 1-pad packs produced, x_2: number of 5-pad packs produced, and x_3: number of 20-pad packs produced. The formulation is

$$\begin{array}{ll} \min & -0.2x_1 - 0.7x_2 - 4.7x_3 \\ \text{s.t.} & x_1 + 7x_2 + 23x_3 \leq b_1 \\ & x_1 + 6.75x_2 + 22.5x_3 \leq b_2 \\ & x_2 + 3x_3 \leq b_3 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Introducing three slack variables and we have the initial tableau

0	-0.2	-0.7	-4.7	0	0	0
$x_4 = 300$	1	7	23	1	0	0
$x_5 = 240$	1	6.75	22.5	0	1	0
$x_6 = 30$	0	1	3	0	0	1

The final tableau is

50	0	2.15/3	0	0	0.2	0.2/3
$x_1 = 15$	1	-0.75	0	0	1	-7.5
$x_3 = 10$	0	1/3	1	0	0	1/3
$x_4 = 55$	0	0.25/3	0	1	-1	$-0.5/3$

The formulation we developed in the class is as follows:
x_1: number of pads produced, x_2: number of small notebooks produced, and x_3: number of large notebooks produced. The formulation is then

$$\begin{array}{ll} \min & -0.2x_1 + 0.1x_2 - 0.7x_3 \\ \text{s.t.} & x_1 + 2x_2 + 3x_3 \leq b_1 \\ & x_1 + 1.75x_2 + 2.5x_3 \leq b_2 \\ & x_2 + 3x_3 \leq b_3 \\ & -x_1 + 5x_2 + 20x_3 \leq 0 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Introducing four slack variables and we have the initial tableau

0	-0.2	0.1	-0.7	0	0	0	0
$x_4 = 300$	1	2	3	1	0	0	0
$x_5 = 240$	1	1.75	2.5	0	1	0	0
$x_6 = 30$	0	1	3	0	0	1	0
$x_7 = 0$	-1	5	20	0	0	0	1

The final tableau is

50	0	1.55/3	0	0	0.2	0.2/3	0
$x_1 = 215$	1	2.75/3	0	0	1	$-2.5/3$	0
$x_3 = 10$	0	1/3	1	0	0	1/3	0
$x_4 = 55$	0	0.25/3	0	1	-1	$-0.5/3$	0
$x_7 = 15$	0	-0.75	0	0	1	-7.5	1

The third formulation is using the fact that the objective function can be written as $-0.1x_1 + 1.4x_2 + 6.7x_3 + 0.3x_7$, where x_7 is the surplus variable in the second formulation or the number of 1-pad packs. The formulation is then

$$\begin{aligned} \min \quad & 0.1x_1 - 1.4x_2 - 6.7x_3 - 0.3x_7 \\ \text{s.t.} \quad & x_1 + 2x_2 + 3x_3 \le b_1 \\ & x_1 + 1.75x_2 + 2.5x_3 \le b_2 \\ & x_2 + 3x_3 \le b_3 \\ & -x_1 + 5x_2 + 20x_3 + x_7 = 0 \\ & x_1, x_2, x_3, x_7 \ge 0 \end{aligned}$$

Introducing four slack variables and we have the initial tableau

0	-0.2	0.1	-0.7	0	0	0	0
$x_4 = 300$	1	2	3	1	0	0	0
$x_5 = 240$	1	1.75	2.5	0	1	0	0
$x_6 = 30$	0	1	3	0	0	1	0
$x_7 = 0$	-1	5	20	0	0	0	1

The final tableau is

50	0	1.55/3	0	0	0.2	0.2/3	0
$x_1 = 215$	1	2.75/3	0	0	1	$-2.5/3$	0
$x_3 = 10$	0	1/3	1	0	0	1/3	0
$x_4 = 55$	0	0.25/3	0	1	-1	$-0.5/3$	0
$x_7 = 15$	0	-0.75	0	0	1	-7.5	1

You can see that the third and second final tableaux are the same. However, what changed here are the dual variables (which ones and why?). With this development, please check to see which formulation can be used to answer which questions from the exercise.

In addition, from this example, you can also see that it is not necessary that in the initial tableau, the reduced costs are the same as the objective coefficients. Please remember to use definitions to check everytime you write down a tableau.