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1 Linear Algebra Review

An $m \times n$ matrix $\mathbf{A}$ is a two-dimensional array a_{ij}:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

A vector $\mathbf{x} \in \mathbb{R}^n$:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}$$

The transpose matrix $\mathbf{A}'$ of an $m \times n$ matrix $\mathbf{A}$ is an $n \times m$ matrix:

$$[\mathbf{A}']_{ij} = [\mathbf{A}]_{ji}$$

The inner product of two vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ is the value:

$$\mathbf{x}'\mathbf{y} = \mathbf{y}'\mathbf{x} = \sum_{i=1}^{n} x_i y_i$$

A matrix-vector product of an $m \times n$ matrix $\mathbf{A}$ and a vector $\mathbf{x}$ in $\mathbb{R}^n$ is:

$$\mathbf{A}\mathbf{x} = \begin{bmatrix} a_1'\mathbf{x} \\ a_2'\mathbf{x} \\ \dots \\ a_m'\mathbf{x} \end{bmatrix} = \sum_{j=1}^{n} x_j \mathbf{A}_j$$

The matrix multiplication of an $m \times n$ matrix $\mathbf{A}$ and an $n \times k$ matrix $\mathbf{B}$ results in an $m \times k$ matrix:

$$[\mathbf{A}\mathbf{B}]_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

The identity matrix $\mathbf{I}$ is a square matrix with $1'$s on the main diagonal and $0'$s everywhere else.
$\mathbf{A}$ is a square matrix. If there exists a matrix $\mathbf{B}$ such that $\mathbf{A}\mathbf{B} = \mathbf{I}$ then $\mathbf{A}$ is called invertible and $\mathbf{B}$ is the inverse of $\mathbf{A}$, $\mathbf{B} = \mathbf{A}^{-1}$.

Example 1
$\mathbf{x}$ is a basic feasible solution to the standard form problem with corresponding basis $\mathbf{B}$. Consider a direction $\mathbf{d}$ such that $\mathbf{y} = \mathbf{x} + \mathbf{d}$ is also a feasible solution. Find the change in the objective cost.
Example 2
If $\mathbf{d}_N$ is a unit vector in $\mathbb{R}^{n-m}$. Find the direction $\mathbf{d}$.

2 Full Tableau Simplex

Question 1
How do you check whether there are multiple optimal solutions with the simplex method?
Question 2
Given the initial tableau and the current tableau. How do you find the basis $\mathbf{B}$ and its inverse?
Example 3 Solve the following problem using the full tableau simplex method:

$$\begin{array}{ll} \min & x_1 \\ \text{s.t.} & \\ & x_1 + x_2 \leq 2 \\ & -x_1 + x_2 \leq 0 \\ & x_1, x_2 \geq 0 \end{array} \tag{1}$$

3 Duality Theory I

Question 3
Given the genereal linear programming problem:

$$\begin{array}{ll} \min & \mathbf{c}'\mathbf{x} \\ \text{s.t.} & \\ & \mathbf{A}\mathbf{x} \geq \mathbf{b} \end{array} \tag{2}$$

Construct its dual problem using Lagrange multiplier method.
Example 4 Construct the dual of the following problem and then the dual of its dual:

$$\begin{array}{ll} \min & 2x_1 + 3x_2 + 8x_3 \\ \text{s.t.} & \\ & x_1 - x_2 - x_3 \geq 1 \\ & 3x_2 + x_3 = 4 \\ & x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{array} \tag{3}$$