AN ARITHMETICAL MODEL OF SPATIAL DEFINITION

by Duncan S Kincaid
BSc, BA, Tulane University
New Orleans, Louisiana
May 1984

Submitted to the Department of Architecture on June 6, 1997 in partial fulfillment of the requirements for the Degree of Master of Architecture at the Massachusetts Institute of Technology

September 1997

Signature of Author....... ....,.....-....................
dskincaid

Certified by.......................   —
Maurice K Smith
Professor of Architecture, Emeritus
Thesis Superv:

Accepted by ..
Wellington Reiter
Chairman, Committee on Graduate Students

Aaron Fleisher
Senior Lecturer of Architecture, Emeritus

Fernando Domeyko
Senior Lecturer of Architecture

Readers

AN ARITHMETICAL MODEL OF SPATIAL DEFINITION

by Duncan S Kincaid

Submitted to the Department of Architecture
on June 6, 1997 in partial fulfillment of the
requirements for the Degree of Master of
Architecture

Abstract

The manner in which spatial definition is built
by architectural form is identified and forma-
lised in part. A description is given for the
structure of spatial definition. This description
allows for a mapping from the class of uses to
the class of spatial structures.

Thesis Supervisor: Maurice K Smith
Title: Professor of Architecture, Emeritus

3

[handwritten marks]

I don't know that the aforementioned would appreciate these ideas or their execution. The simplicity of the abstraction and errors in my understanding of form is sure to disappoint any designer and thinker of their caliber. But what we've in common I think more deep than the differences are numerous. And that is the position that architecture is an empirical art and science, one which admits to empirical facts and belongs as much to the human intellect as spirit

if there a difference.

Acknowledgment

and *[handwritten text]*

TABLE OF CONTENTS

Use Model

Concluding Remarks

Appendices

References

CHAPTER 1

I N T R O D U C T I O N

...what did you do after
*you'd made certain that
you'd done nothing at
all?...*

—Sherlock Holmes to Inspector Hop-
kins after the latter's investigation in
The Case of the Golden Pince-Nez
(as told by M K Smith to dskincaid
apropos...)

1.1 Abstract

Architecture practices its art and craft in the articulation of space.
Building elements are necessary components of its efforts. These ele-
ments may be columns, beams, walls, screens, light and assemblies of
the same. Their physical attributes include dimension, colour, mate-
rial, use, and tectonics.

Certain of these attributes admit to formalisation. Indeed, computer
based formalisations of some already exist: CAD encodes geometric
descriptions, raytracing/texture mapping encodes the interaction of
light with geometry/material, structural analysis software encodes
the performance of loaded members.

What remains lacking, however, is a formalisation of the *architectural
behaviour* of these properties. By *architectural* we mean *spatial* or *terri-
torial*. How might structure, material, light and dimension contribute

---

severally and in combination to the definition of territories? What structure is exhibited by the territories so defined? Might there exist a mapping from the class of uses to these spatial structures?

This research programme should be of some interest for two reasons. The first is epistemological. The resulting formalisation may be thought as a set of axioms from which theorems describing territory are derived. As such, the calculus embodies a theory of architectural form which is testable in the built environment—scientific method is given its due in architectural discourse. This programme should also be of interest to those engaged in knowledge-based architectural design systems—for one cannot reason intelligently about architectural form if ignorant of the spatial commitments which lie in that form.

Towards these ends, this work begins to identify and formalise the manner in which physical and organisational properties of architectural elements build territory. It also describes some properties of the territories themselves.

1.2 Related work

1.2.1 Past formalisations of spatial definition

Architecture has not been without its treatises. Vitruvius, Vasari, Palladio, the Spanish Crown, the Ecole de Beaux Arts, Durand, and countless others have offered their theories of architectural form. Most all these, however, have written prescriptive rules for *parti* generation which amount to rules of composition.

This is most unfortunate. If considered more closely, these works reveal shortcomings such as to compromise their worth. For the most part, attention is restricted to the shape of form and shape of its arrangement.[1] Shape in itself is arguably the least interesting property of architectural form. It is uninteresting as it is a property shared with every other object in the physical world. Further, an architect doesn't design with shapes but with materials and their structural properties, with light, access, spatial definition, tectonic expression, and uses.

The prescriptive rules of these treatises are also lacking in architectural substance. One would expect these to address the material and formal behaviours of the architectural entities upon which they operate. Rather than define rules to flag redundancies in spatial defini-

---

1. The *Leyes de Indias* of 1635 may be an exception to this, as considerable attention was given to use organisation (Kincaid 1997).

tions, suggest access given a set of architectural conditions (such as light, public/private zones and section, among others), the rules may simply mirror or rotate a shape.

What architectural knowledge is captured by these formalisations? That bilateral symmetry rules the day, that voids are always built from a module of thus and such ratio, that perforations are axially placed? Again, these address the geometry, not architecture of the piece. They could be describing a tablecloth pattern, a waffle iron or sheet of stamps.

Of course there is much written on the subject of architectural form and space which resists formalisation.[2] A number of these works are in the manner of Ramussen (1964) and Arnheim (1977) who write of architecture's experiential qualities. These at least acknowledge space and use and credit physical form as their partial determinant. Norberg-Schulz (1986) has concerned himself with phenomenological aspects of architectural form, at times relegating physical space for a speculative 'existential space.'

Other works have concentrated on the sociological nature of space as in Hall (1966) and Newman (1973). Herzberger (1993), Alexander (1977) and recent thesis work at MIT address territory explicitly. Herzberger and Alexander promote an architecture of association predicated upon coherent use territory. Chong (1992) identifies terri-

---

2. As there is which resists comprehension, to wit Heidegger.

tories in Schindler's work, but like Herzberger and Alexander, fails explicitly to identify the means by which these are built. Reifenstein (1992) investigates 'positioning rules' which build territory and spatial structure. These rules beg the question, however, as they operate on territories whose definition remains unanswered. None of the aforementioned speak directly as to how physical form builds spatial definitions, nor do they formalise their findings.

1.2.2 Formalisations of spatial definition in knowledge-based systems

Computer-based systems for evaluating aspects of architectural form are many. One may partition the class into those which do not evaluate design with respect to geometric representations of physical form and those which do. The former would include systems reasoning on topological models for adjacency, say, or even systems representing experiential qualities but not relating them to physical form (Mortola 1991).The latter would include systems such as (Carrara 1994) and (Dave 1994) wherein design characteristics are dynamically calculated on the basis of physical form representation.

Those knowledge-based systems which acknowledge physical form would surely appear better equipped. As discussed in (Koile 1997) a physical form model would facilitate computation of circulation paths, enable the computation of visual barriers (Hanson 1994) and allow for designer interaction in a mode with which he is already familiar (Gross 1996). They would be better equipped for an even more fundamental reason, however; a physical form model allows, at

least in principle, for a spatial definition model. As architects reason with spatial definitions—their dimensions, *qualia*, degree of definition, organisation, and supported uses—so too should a computation-based system purporting design intelligence.

Evaluative reasoning on experiential qualities such as *private* or *open*, and use ascriptions such as *bedroom* or *entrance*, say, can only fully be made in the presence of territories which support them. Likewise, ascriptions made of territories—such as dimension or degree of privacy—can best be made in the presence of the physical form which builds them.

Most evaluative systems have either ignored or misused a physical form component. None have made use of a territory model. This is likely a result of not knowing how physical form builds spatial form. This work may aid in correcting the deficiency.[3]

---

3. Kimberle Koile of the MIT Artificial Intelligence Laboratory, with assistance of this author, is implementing some of these ideas (Koile 1997). Physical form is paramount in this work. From it is built a territory model which in turn supports a use-space model and connectivity model. A considerable, if not essential, part of evaluative reasoning takes place at the level of territory. The territory model is constantly updated as changes are made by the user to the physical form model.

1.3 Disclaimer

The work contained herein might best be considered an hypothesis. As with most hypotheses, it is likely wrong and certainly incomplete. There is no excuse for the former. As for the latter, it may well be the case that architecture's infinitudes will always outnumber any formalisation's.

This model of spatial definition makes a number of simplifying assumptions and abstractions, to be sure, though none are thought to compromise the enquiry. The greatest of these is the primacy given to form's spatial extents.[4] Though other attributes of form such as material and tectonics participate in the definition of territory, it is thought that they do so in a manner subsumed by spatial extents. Specifically, these secondary properties are believed to build *ensembles* of form which in turn induce territory as a function of the ensembles' extents. There is considerable evidence to suggest this reasonable as seen in later chapters.

For some time now the subjective and whimsical have found many champions in architectural education and practice. Though one cannot condemn *a priori* the subjective and whimsical, one can condemn the ignorance with which it is often practiced. This ignorance is not of

---

4. Spatial extents are not to be confused with material extents. A structural bay, for instance, will exhibit the spatial extents of a box. These extents are composed of planes, *not surfaces*. Only in the trivial case of a single entity such as a column are the spatial extents coincident with the material extents and the faces rendered as surfaces.

1.3 DISCLAIMER

the latest French literary theory, German *Phenomenologie*, or the latest Wonder of the World Wide Web, but an ignorance of the spatial and organisational commitments which lie in architectural form. It is hoped this work may reinforce the position that architecture operates in the objective, physical world and not in that of invented discourse nor that of cyberspacial fancy.

CHAPTER 2

S P A T I A L   M O D E L

The spatial extents of physical form induce territory, a model of which is given.

Spatial definition or *territory* is built principally through the spatial extents of physical form. This does not discount the many other properties of form which contribute to spatial definition such as material and tectonic character. As shall be argued elsewhere, these properties perform in a manner encompassed by spatial extents.

2.1 Territory

Spatial definition is recognised as an alteration in the isotropy of Space (ζ). Alterations appear as organisational or structural perturbations.

Two such structures are considered below. The first describes a spatial structure induced by *registration from a form's spatial extents*. The second describes a *directional field structure induced by spatial extents*. In both, territory or spatial definition is specified through organisation, much as a mathematical space may be specified by a metric.

Both models are experientially based, which is to say the organisations described are readable in one's experience of architectural form. Further empirical work might well suggest adjustments in the models' particularities.

normal

2.1.1 Registration model

Consider the spatial extents α of some physical demarcation A. Associated with α is a measure of registration described in plan in Figure 2.1.
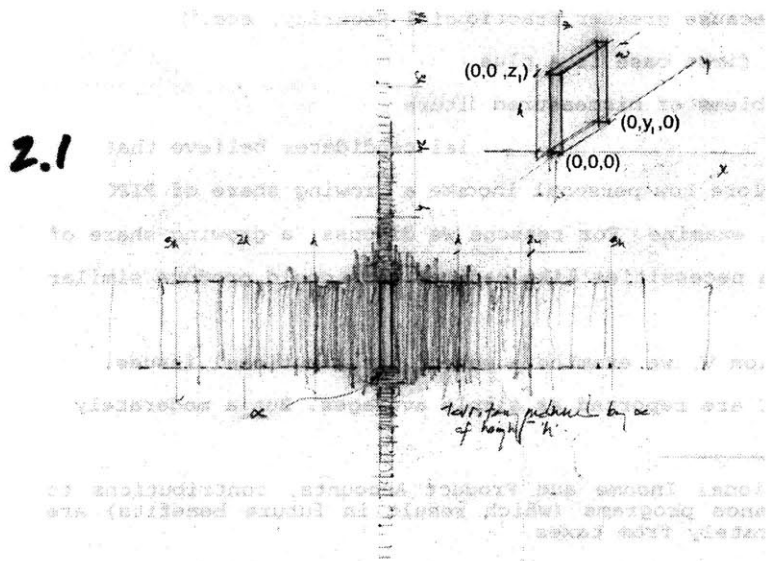

Figure 2-1

The intensity of registration from α diminishes with distance from α. It diminishes even more rapidly outside the projection of α, shown by dotted lines. As this behaviour holds uniformly for every face of α, the following discussion focuses on a single face.

Intensity of registration can be described as a step function:

$$I_r|\ \Re^3 \rightarrow \Re$$

$$
\begin{aligned}
I_r(x, y, z) &= i_r & & 0 \le x \le h, -h \le x \le 0 \\
&= i_r/2 & & h < x \le 2h, -2h \le x < -h \\
&= i_r/4 & & 2h < x \le 3h, -3h \le x < -2h \\
&= 0 & & \begin{cases} 3h < x < -3h \\ y_1 < y \ \text{ or } \ y < 0 \\ z_1 < z \ \text{ or } \ z < 0 \end{cases}
\end{aligned}
$$

for some constant i_r

Registration I_r halves with every displacement h from α. For distances greater than $3h$, I_r is small as to be discounted. Which is to say the registration, hence territorial definition, vanishes to zero outside the projection of α. This does not accord *exactly* with facts, but should prove serviceable.

What is important in this description is that the y and z values can zero the function, but do not otherwise figure in the value of $I_r(x, y, z)$. Indeed, non-zero territorial definition is controlled by h, the extent's height. It follows that a plan representation of architectural form does not adequately reveal territory.

As desired, territories are identified with non-zero values for I_r. They may be understood as a disruption in the isotropy of ζ, where I_ζ is everywhere 0. Territory is bounded by the projection of the form's spatial extents and diminishes with distance from the same.

2.1.2 Field model

Spatial extents as those considered above may be said to induce another type of rent in isotropic ζ. The width and height of α introduce directional biases in ζ as shown for a single face in Figure 2-2.
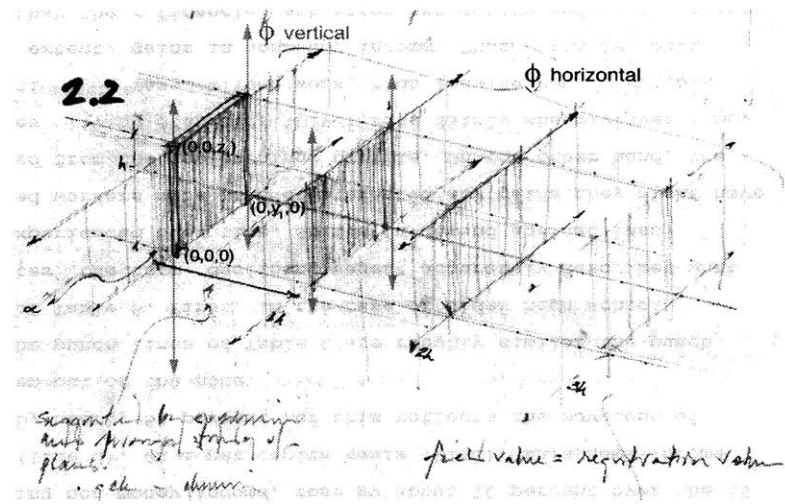

Figure 2-2

Both directional fields $\phi_{\alpha_{vertical}}$ and $\phi_{\alpha_{horizontal}}$ weaken as they extend beyond the width and height of the extents, as they do with increasing distance from the extents. Only one field is realised in experience—the field coincident with direction of movement. As a result, all treatment of territory assumes a single, well-defined direction.

This field behaviour can be described at every point by a *direction* given by the orientation of the inducing extent and an *intensity* I_f given by the step function below:

$$I_f|\ \Re^3 \to \Re$$

$$
\begin{aligned}
I_f(x, y, z) &= \mathbf{i}_f & 0 \le x \le h,\ -h \le x \le 0 \\
&= \mathbf{i}_f/2 & h < x \le 2h,\ -2h \le x < -h \\
&= \mathbf{i}_f/4 & 2h < x \le 3h,\ -3h \le x < -2h \\
&= \mathbf{i}_f/8 & 3h < x \le 4h,\ -4h \le x < -3h \\
&= 0 & \begin{cases} 4h < x < -4h \\ 2y_1 < y \ \text{ or } \ y < -y_1 \\ 2z_1 < z \ \text{ or } \ z < -z_1 \end{cases}
\end{aligned}
$$

$$\text{for some constant } \mathbf{i}_f$$

It is significant again to note that I_f is largely controlled by h; side effects at the projection's extents are small, hence ignored.

2.1.3 Equivalency of the two models

Claim: The registration model given in 2.1.1 and field model given in 2.1.2 are scalar-wise equivalent.

Argument: It need be shown that I_r and I_f are in effect equivalent. With domains restricted to $I_r^{-1} \ni I_r(x, y, z) \ne 0$,

$$
\begin{aligned}
I_f(x, y, z) &= k \cdot I_r(x, y, z) = k \cdot \mathbf{i}_r & k = \text{constant} \\
\mathbf{i}_f &= k\mathbf{i}_r
\end{aligned}
$$

Equivalency correlates a field's intensity with the intensity of registration. Field intensity, however, runs parallel with the face which

induces it whilst registration intensity is normal to the same face. This is an important observation and may stated as a theorem: *registration is always normal to the associated field(s) and is of like scalar value.*

In what follows, preference is given to the field definition model as it subsumes the registration model, i.e. field values bear *direction* as well as degree of intensity. Spatial definition, or territory, induced by extents α may then be given by

$$\Gamma_\alpha \equiv (A, \phi_\alpha(A))$$

where A denotes the set of points within the projection of α and $\phi_\alpha(A)$ denotes the field intensity at these points.

2.1.4 General observations

Several important observations can be made of the spatial model described above. These observations are treated in greater depth in ensuing sections.

- It is evident that spatial extents' faces induce territory. These faces are described geometrically as planes. This greatly facilitates the model's application to spatial extents more complex than those considered thus far. This is done in Section 2.2 below.

- The territory associated with physical form A is described wholly in terms of A's spatial extents α. Outside of these spatial extents,

no more is said of A. The implications are great. For one, the model applies to the full range of physical form, independent of size, material, orientation and complexity. For instance the spatial extents described by the extruded parallelogram in Figure 2-2 might well accommodate a structural bay, a piece of section, a screen or a simple wall.

*   The model also allows for physical form to operate territorially at different sizes and as a part of different extents concurrently. For example, the columns which build a structural bay claim territory at the size of their own extents (material size), whilst also contributing to the spatial extents of the bay (which claims territory at the size of *its* extents, i.e. room size).[1]

2.2 Applicability of spatial model to all spatial extents

It is argued below that the territory model accommodates *all* spatial extents as it accommodates all planar and curved extents.

2.2.1 Planar spatial extents

Claim: All planar spatial extents may be described by a finite number of planes.

---

1. An ensemble is formed by shared control of its constituents' territory. Control is discussed in Chapter 3. Ensembles are treated in detail in Appendix A.

Argument: Planar spatial extents, *by definition*, are comprised of a finite number of planar faces (i.e. planes).
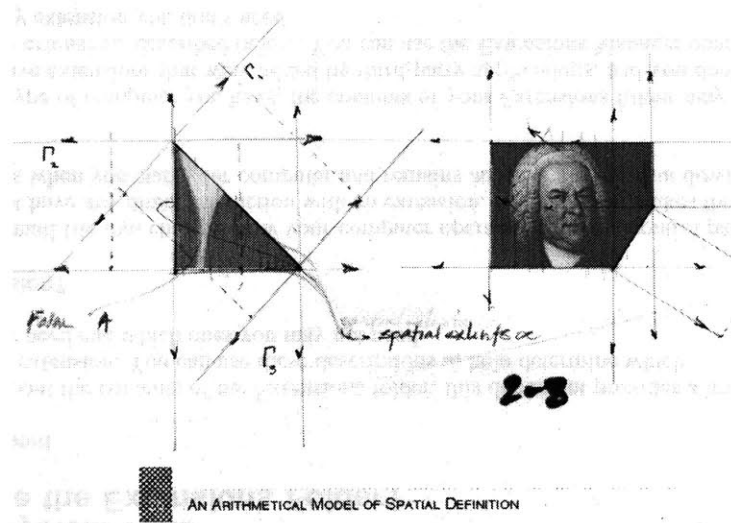
Figure 2-3a/2-3b

Figures 2-3a and 2-3b illustrate the spatial extents and induced territories given by 3 and 5 planar faces respectively. Note that territories overlap within the extents. An addition defined on territory in Chapter 3 shows these to be zones of field intensification. Considered as footprints of buildings, one can readily see why entries should not be placed at corners; there is no territorial demarcation at corners.

2.2.2 Curved spatial extents

Claim: Curved spatial extents can be described adequately as a sum of planes.

Argument: Curved spatial extents of any degree can be described adequately as a sum of arbitrarily small tangent planes given by the set of tangent vectors at any point.
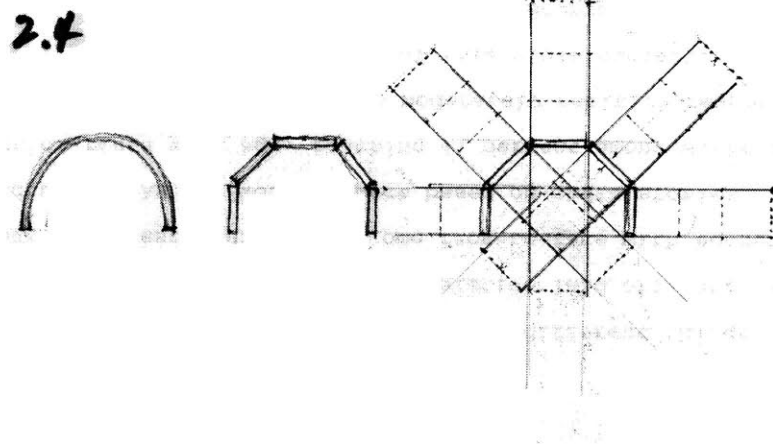
Figure 2-4a/2-4b/2-4c

Consider a simple curvilinear form (Figure 2-4a) with the gross planar approximation of Figure 2-4b. Treating planar segments severally, the territory map is that of Figure 2-4c.

As the planar approximation improves, there is a propensity for the
arc shaped territory to complete itself as a circle. Of course the degree
to which this phenomenon occurs depends on the ratio of height and
diameter of the inducing extents.

The field description of curved territory is a little more complex.
Assuming uniformity in height for the concave extent (which has
been the case throughout), field lines stitch themselves into concen-
tric arcs. Some ratios of height to diameter allow the arcs to complete
as circles with varying I_f. The portion of the circle in closest proxim-
ity to the extent bears a greater I_f than that portion furthest from the
extent. This is in accord with experience; spatial definition is palpably
greater within the bounds of the curvilinear form than without.

With assistance from field arithmetic, Section 3.5.2 demonstrates why
convex extents refuse to build a unified and inclusive territorial defi-
nition, whilst concave extents do.

2.2.3 All spatial extents

Claim: Territory is a necessary consequence of physical form.
Argument: All form bears spatial extents which at the *minimum* may
be identified with its material extents. Spatial extents generate terri-
tory. Therefore, territory is always induced by physical form.

A R I T H M E T I C   M O D E L

An arithmetic is defined for both orthogonal and non-orthogonal overlapping territories.

A spatial model has been sketched in Chapter 2 in which minimally convex form elicits overlapping territories—as does compound assemblies of form of which architecture is usually comprised.

There is no *a priori* reason for overlapping territories to be of any special interest or consequence. However experience suggests otherwise as does the spatial model in which every (x, y, z) in an overlap bears an **i** value from more than one inducer.

3.1 Addition on territory

Definition: Addition on territory

Let Γ_α and Γ_β denote the territories built by demarcations A, B with spatial extents α and β respectively. As per Chapter 2, these territories are described by

$$\Gamma_\alpha \equiv (A, \phi_\alpha(A))$$
$$\Gamma_\beta \equiv (B, \phi_\beta(B))$$

where A and B denote the set of points in the projection of α and β, and $\phi_\alpha(A)$ and $\phi_\beta(B)$ denote directional field definition at points in A and B.

Let ζ denote the set of all territories.

The *addition of* Γ_α *and* Γ_β denoted by the symbol $\Gamma_{\alpha \oplus \beta}$ is defined as a mapping

$$\oplus \mid \zeta, \zeta \rightarrow \zeta$$
$$\oplus \mid \Gamma_\alpha, \Gamma_\beta = \Gamma_{\alpha \oplus \beta} = ((A \oplus B), \phi_{\alpha \oplus \beta}(A \oplus B))$$

This arithmetic maps two territories to a third such that the third is comprised of points $A \oplus B$ and directional field definition $\phi_{\alpha \oplus \beta}(A \oplus B)$. The function $\oplus$ is described in section 3.1.1 whilst $\phi_{\alpha \oplus \beta}$ is described in Section 3.1.2.

3.1.1 Determination of $\oplus$

Definition: Addition of A and B

Let

  Π denote the set of all subsets of $\mathfrak{R}^3$

  A denote the set of points in Γ_α

  B denote the set of points in Γ_β

The *addition of A and B*, denoted by the symbol $A \oplus B$ is defined as a mapping

$$\oplus \mid \Pi, \Pi \rightarrow \Pi$$
$$\oplus \mid A, B = (A \cap B)$$

3.1.1.1 Properties of $\oplus$:

for $\forall A, B, C \in \Pi$:

1. closure

$$A, B \in \Pi, \exists (A \oplus B) = (A \cap B) \in \Pi$$

2. commutative

$$A \oplus B = (A \cap B) = (B \cap A) = B \oplus A$$

3. associative

$$(A \oplus B) \oplus C = (A \oplus B) \cap C = ((A \cap B) \cap C)$$
$$= (A \cap (B \cap C))$$
$$= (A \cap (B \oplus C))$$
$$= (A \oplus (B \oplus C))$$

4. unit

$$A \oplus \Pi = \Pi \oplus A = A$$

5. zero

$$A \oplus \emptyset = \emptyset \oplus A = \emptyset$$

6. no inverse

$$\neg \exists A^{-1} \ni A \oplus A^{-1} = A^{-1} \oplus A = \Pi$$

If there were an inverse defined for $\oplus$ then ($\oplus$, Π) would have formed an abelian group. As it stands, ($\oplus$, Π) forms an abelian semi-group.

3.1.2 Determination of $\phi_{\alpha \oplus \beta}$

Every element in ϕ_α can be treated as a vector with a direction that of its inducer α and with magnitude I_{f_α} as described by the step function in Section 2.1.2. Adding directional fields ϕ_α, ϕ_β —as required by the definition of $\Gamma_{\alpha \oplus \beta}$ —should then amount to vector addition on the shared elements of ϕ_α and ϕ_β.

Definition: Let $\vec{\alpha}$ be a vector in ϕ_α. The *norm* or *length* of $\vec{\alpha}$ is denoted by $\|\vec{\alpha}\|$ and is valued at $I_{f_{\vec{\alpha}}}$.

Definition: Addition of ϕ_α *and* ϕ_β
Let

> ϕ_α denote the field associated with Γ_α
>
> ϕ_β denote the field associated with Γ_β
>
> $\vec{\alpha}$ and $\vec{\beta}$ be vectors in ϕ_α and ϕ_β respectively
>> for some shared (x, y, z)
>
> $\theta_{\vec{\alpha}, \vec{\beta}}$ denote the angle $\vec{\alpha}$ makes with $\vec{\beta}$

The addition of ϕ_α and ϕ_β denoted by the symbol $\phi_{\alpha \oplus \beta}$ is defined as a mapping

$$\oplus \mid \phi_\alpha, \phi_\beta = \phi_{\alpha \oplus \beta}$$
$$= \vec{\alpha} + \vec{\beta}$$
$$= \text{vector of norm } I_{f_{(\vec{\alpha} + \vec{\beta})}}$$
$$\text{and relative direction } \theta_{(\vec{\alpha} + \vec{\beta}), \vec{\beta}}$$

where

$$I_{f_{(\vec{\alpha}+\vec{\beta})}} = \|\vec{\alpha}+\vec{\beta}\| = (\|\vec{\alpha}\|^2 + \|\vec{\beta}\|^2 + 2\vec{\alpha}\bullet\vec{\beta})^{1/2}$$

$$= (\|\alpha\|^2 + \|\vec{\beta}\|^2 + 2\|\vec{\alpha}\|\|\vec{\beta}\|\cos\theta_{\vec{\alpha},\vec{\beta}})^{1/2}$$

and

$$\theta_{(\vec{\alpha}+\vec{\beta}),\vec{\beta}} = \text{asin}\frac{\|\alpha\|\sin\theta_{\vec{\alpha},\vec{\beta}}}{\|\vec{\alpha}+\vec{\beta}\|}$$

Figure 3.1 shows examples of territory addition. Note that $\Gamma_{\alpha\oplus\beta}$ is defined for that region formed by the intersection of A's and B's projected extents. (In the interests of clarity, only that territory associated with the largest extents is drawn.)
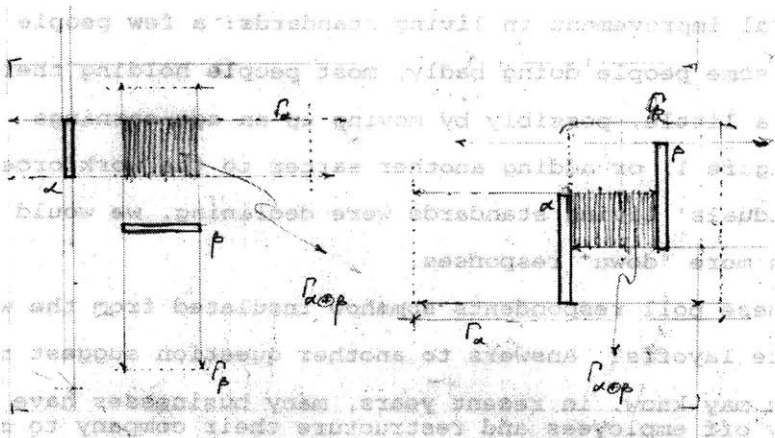
Figure 3-1

3.1.2.1 Properties of $\phi_{\alpha \oplus \beta}$

for $\forall \phi_\alpha, \phi_\beta, \phi_\gamma \in \zeta:$

1. closure

$$\phi_\alpha, \phi_\beta \in \zeta, \exists \phi_{\alpha \oplus \beta} = \vec{\alpha} + \vec{\beta} \in \zeta$$

2. commutative

$$\phi_{\alpha \oplus \beta} = \vec{\alpha} + \vec{\beta} = \vec{\beta} + \vec{\alpha} = \phi_{\alpha \oplus \beta}$$

3. associative

$$\phi_{(\alpha \oplus \beta) \oplus \gamma} = \phi_{\alpha \oplus \beta} + \vec{\gamma} = (\vec{\alpha} + \vec{\beta}) + \vec{\gamma}$$
$$= \vec{\alpha} + (\vec{\beta} + \vec{\gamma})$$
$$= \vec{\alpha} + (\phi_{\beta \oplus \gamma})$$
$$= \phi_{\alpha \oplus (\beta \oplus \gamma)}$$

4. unit

$$\phi_{\alpha \oplus 0} = \phi_{0 \oplus \alpha} = \phi_\alpha \text{ where } \vec{0} \text{ is the 0-vector}$$

5. no inverse

as $\theta_{\alpha, \beta}$ is always normalised to $0 < \theta_{\alpha, \beta} < \dfrac{\pi}{2}$

$(\oplus, \phi)$ forms an abelian semi-group.

3.2 Additional field structure

Analysis of $\phi_{\alpha \oplus \beta}$ reveals additional information as to the field structure of $\Gamma_{\alpha \oplus \beta}$.

3.2.1 Degree of definition

Definition: The *degree of definition* for any territory is the number of overlapping territories which build it. Degrees of definition add as one would expect, i.e.

$$degree\text{-}def_{\alpha \oplus \beta} = degree\text{-}def_{\alpha} + degree\text{-}def_{\beta}.$$

3.2.2 Field bias

Definition: A *field bias* for any territory $\Gamma_{\alpha \oplus \beta}$ is that field associated with (max $\|\vec{\alpha}\| \; \|\vec{\beta}\|$). Intuitively, a field bias is evident when fields are disproportionately strong, thereby favouring association with one inducer over another.

Definition: A *field bias value* for any territory $\Gamma_{\alpha \oplus \beta}$ is given by $\left| \|\vec{\alpha}\| - \|\vec{\beta}\| \right|$. This value is an abstraction with no real world counterpart. It gives some indication as to the strength of the field bias.

Definition: When two fields add, each makes a contribution to the other's I_f. *Field gain* describes this amount and is given by ϕ_α-*gain* and ϕ_β-*gain* below.

$$\phi_\alpha\text{-}gain \equiv \text{projection of } \vec{\beta} \text{ along } \vec{\alpha}$$

$$= \frac{\vec{\alpha} \bullet \vec{\beta}}{\vec{\alpha} \bullet \vec{\alpha}}$$

$$= \frac{\|\vec{\alpha}\| \|\vec{\beta}\| \cos\theta}{\|\vec{\alpha}\|^2}$$

$$= \frac{\|\vec{\beta}\| \cos\theta}{\|\vec{\alpha}\|}$$

$$\phi_\beta\text{-}gain \equiv \text{projection of } \vec{\alpha} \text{ along } \vec{\beta}$$

$$= \frac{\|\vec{\alpha}\| \cos\theta}{\|\vec{\beta}\|}$$

Definition: Let $\theta_{\vec{\alpha}, \vec{\beta}}$ be the angle between $\vec{\alpha}$ and $\vec{\beta}$ and $\theta_{(\vec{\alpha} + \vec{\beta}), \vec{\alpha}}$ be the angle $(\vec{\alpha} + \vec{\beta})$ makes with $\vec{\alpha}$. ϕ_β is said to interfere with ϕ_a *reductively* when $\theta_{(\vec{\alpha} + \vec{\beta}), \vec{\alpha}} > \frac{\theta_{\alpha, \beta}}{2}$. Otherwise ϕ_β interferes *constructively* with ϕ_a. Similarly for ϕ_a interfering with ϕ_β.

Though ϕ_α-*gain* and ϕ_β-*gain* are always positive by definition, the net contribution of components can favour one inducer over another. This leads to the constructive and reductive interference signalled by $\theta_{(\vec{\alpha} + \vec{\beta}), \vec{\alpha}}$ above.

3.3 Control

Planar and curvilinear extents were shown to be reducible to a sum of planes in Chapter 2. There remains, all the same, a fundamental difference in their respective territory. This difference lies in the manner in which the territory is *controlled*.

Definition: A territory $\Gamma_{\alpha \oplus \beta}$ is said to be *controlled* where I_f is an absolute maximum in the region $\Gamma_\alpha \cup \Gamma_\beta$.

Planar extents control their territory with their inducing faces, whilst curvilinear extents often have territory controlled outside the extents, as seen in Example 1b of Section 3.5.2.

For gentle curvature with respect to height, the curvilinear form behaves much as do simple planar extents; the face of the curvilinear form controls the territory with geometry outside the extents. This may be seen in Example 1a of Section 3.5.1.

Definition: Territory which is controlled by inducing extents is said to be *intrinsic*.

Definition: Territory which is controlled by geometry outside the extents is said to be *extrinsic*.

Claim: Intrinsic and extrinsic properties partition the class of territories.

Argument: Every territory bears a maximum I_f value somewhere. This value obtains either at the inducer's extents (intrinsic) or elsewhere (extrinsic).

3.4 Union of territories

Sections 3.1.1 and 3.1.2 showed that intersecting territories build additional territories. A union of territories does not produce any additional field intensification, hence does not build any additional territory. This is confirmed by the arithmetic defined on ϕ.

3.5 Examples

3.5.1 Example 1a: Curved spatial extents: intrinsic control
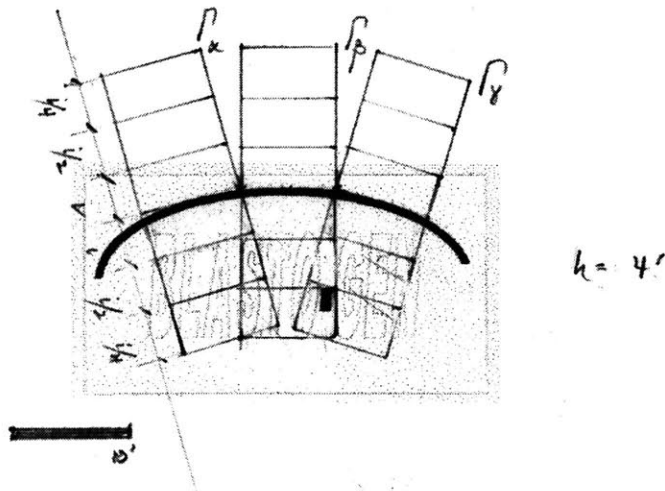
Figure 3-2

$h = 4'$, $\theta = 20°$ between each approximating element α, β, γ

1. Approximating territories Γ_{α}, Γ_{β}, Γ_{γ} fail to overlap owing to a height of only 4 feet for the element.

2. Territorial control remains intrinsic, and the curved extents behave much as a simple planar extent.

3.5.2 Example 1b: Curved spatial extents: extrinsic control
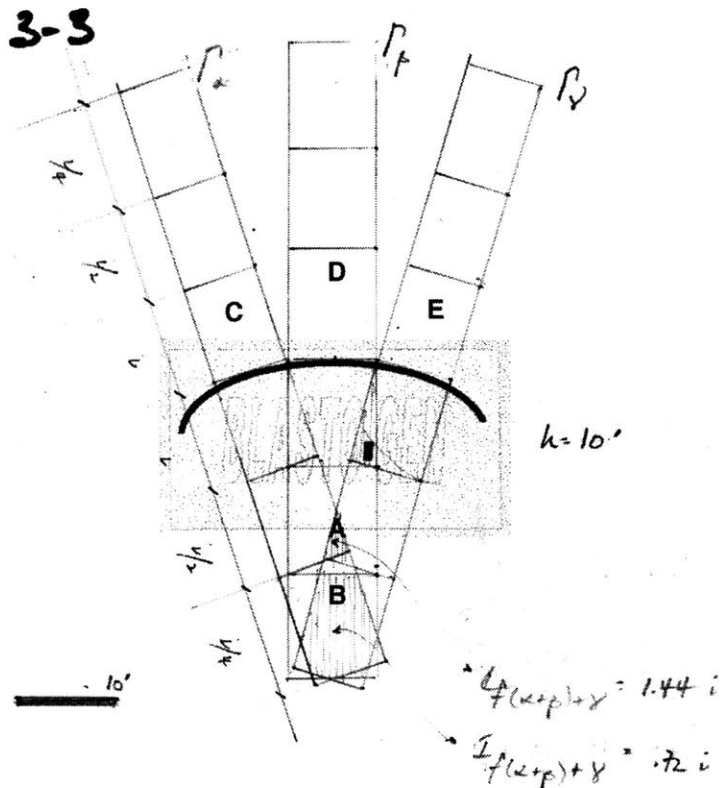
Figure 3-3

h = 10′ , θ = 20° between each approximating element α, β, γ

1. Approximating territories Γ_α, Γ_β, Γ_γ overlap owing to a height of 10 feet.

2. Maximal I_f in region A, hence control is extrinsic.

Values are documented in Table 3-1.

Region	°-def	$\|\vec{\alpha}\|$	$\|\vec{\beta}\|$	$\|\vec{\gamma}\|$	$\|\vec{\alpha} + \vec{\beta} + \vec{\gamma}\|$	field-bias	field-bias-value
A	3	1/2	1/2	1/2	1.44	—	—
B	3	1/4	1/4	1/4	.72	—	—
C	1	1	—	—	1	—	—
D	1	—	1	—	1	—	—
E	1	—	—	1	1	—	—

Table 3-1: Calculated values for Figure 3-3

Notes:

1. Region A displays the greatest degree of definition and i value. As a result, it is said to control that territory generated by the concave face. As region A lies outside the extents of the curvilinear form, the control is said to be extrinsic.

2. As the planar approximation to the curvilinear form improves, region A approaches a line in 3-space.

3. Regions C, D, E do not intersect (as told by degree-def = 1). There is no possibility for field intensification with convex form. Control is strictly intrinsic.

3.5.3 Example 2: Rectilinear extents
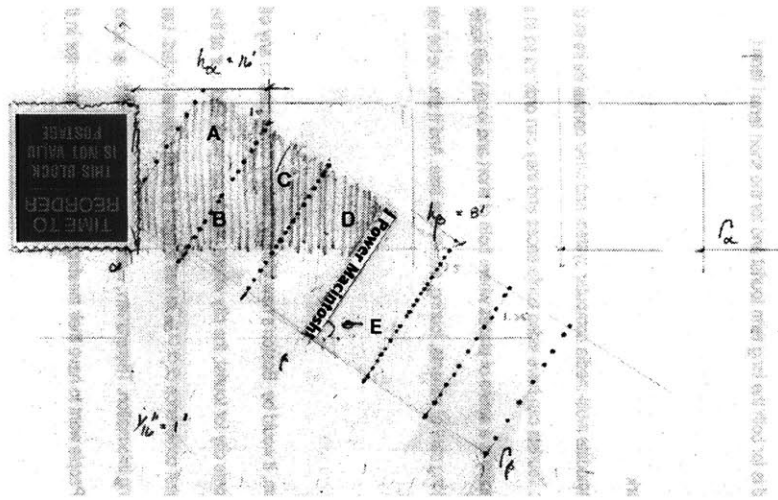
Figure 3-4

$\theta = 60°$, $h_\alpha = 16'$, $h_\beta = 8'$

1. $\Gamma_{\alpha \oplus \beta}$ is represented by the shaded region.
2. I_f values vary within $\Gamma_{\alpha \oplus \beta}$ as it varies within Γ_α and Γ_β .
3. Regions A, B, C, D denote those areas of $\Gamma_{\alpha \oplus \beta}$ bearing discrete I_f values.

These and other related values are documented in Table 3-2.

Region	°-def	$\|\vec{\alpha}\|$	$\|\vec{\beta}\|$	$\|\vec{\alpha}+\vec{\beta}\|$	$\theta_{(\vec{\alpha}+\vec{\beta}),\vec{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_β-gain
A	2	1	1/4	$\dfrac{\sqrt{21}}{4}$	49°	ϕ_α	3/4	1/8	2
B	2	1	1/2	$\dfrac{\sqrt{7}}{2}$	41°	ϕ_α	1/2	1/4	1
C	2	1/2	1/2	$\dfrac{\sqrt{3}}{2}$	30°	—	0	1/2	1/2
D	2	1/2	1	$\dfrac{\sqrt{7}}{2}$	19°	ϕ_β	1/2	1	1/4

Table 3-2: Calculated values for Figure 3-4

Notes:

1. Region C field-bias-value is 0 as both $\vec{\alpha}$ and $\vec{\beta}$ are of the same magnitude.
2. ϕ_α-gain $= \phi_\beta$-gain in Region C as each field reinforces the other equally for reasons given above.

3.5 Examples

An Arithmetical Model of Spatial Definition

U S E   M O D E L

Use ascriptions are made to territories on the basis of their field structure.

As seen in Chapter 2, fields interfere with one another. An arithmetic was defined on these fields in Chapter 3. This arithmetic tells either of constructive or reductive interference. As shown below, it also suggests an assignment of use to certain field definitions.

Field structure is partitioned in three: *go* or *directional* structure, *stop* or *bi-directional* structure and *slack*. A directional field structure is termed a *go* structure as it reinforces direction and movement in that direction. A *stop* on the other hand is built by reductive field interference—no or little directional field bias is evident within the composite field. *Stop*s and *go*s are always relative to one another. A *slack* structure describes a relaxation in the degree of definition and reinforces neither *go* nor *stop*.

Every elemental action is a *go*. As a result, designing use territories amounts to orchestrating field definitions; at times reinforcing *go*s with physical form, at other times building disjunctions in *go* systems for the purpose of *stop*s or even *slack*.

This correlation of use and field structure is well defined. With the partition of the class of uses in three—access, relative privacy and collective—a mapping may be defined from use to field structure.[1]

---

1. The design of buildings as fish or binoculars may frustrate this claim.

4.1 Access/go definitions

Access definitions are built with directional demarcations such that a principal field direction is reinforced. For instance, spatial extents α, β whose territories intersect and whose fields are parallel will strongly build access. Access need always be larger than its relative privacies.

4.1.1 Necessary and sufficient conditions for access/go field

1. $\|\vec{\alpha} + \vec{\beta}\| \geq 1$
2. $30° \geq \theta_{(\vec{\alpha} + \vec{\beta}), \vec{\alpha}} \geq 60°$
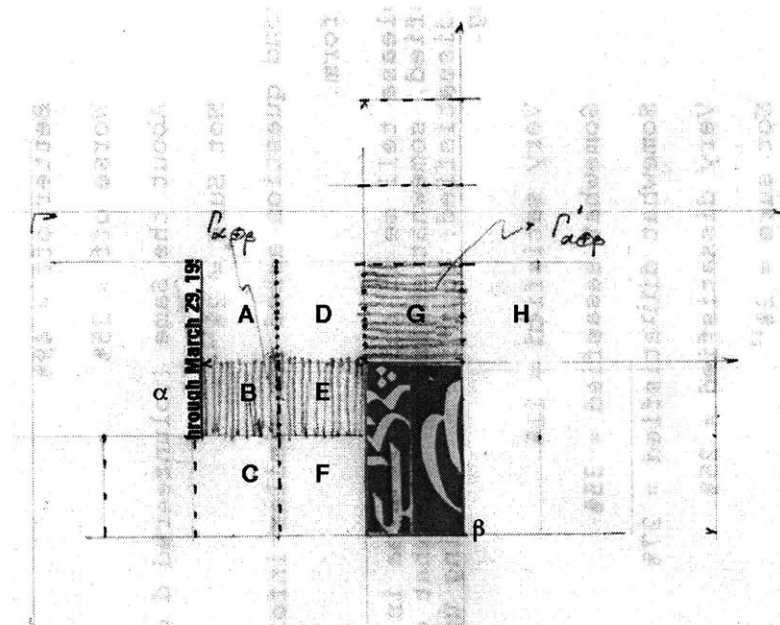
4.1.2 General conditions

1. See Table 4-1.

Figure 4-1

AN ARITHMETICAL MODEL OF SPATIAL DEFINITION

Region	°-def	$\|\dot{\alpha}\|$	$\|\dot{\beta}\|$	$\|\dot{\alpha}+\dot{\beta}\|$	$\theta_{(\dot{\alpha}+\dot{\beta}),\dot{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_β-gain	field/use class
A	1	1	0	1	—	ϕ_α	1	0	—	go/access
B	2	1	1/2	3/2	0°	ϕ_α	1/2	1/2	2	go/access
C	1	0	1/2	1/2	—	ϕ_β	1/2	—	0	go/access
D	1	1/2	0	1/2	—	ϕ_α	1/2	0	—	go/access
E	2	1/2	1	3/2	0°	ϕ_β	1/2	2	1/2	go/access
F	1	0	1	1	—	ϕ_β	1	—	0	go/access
G	2	1/4	1	$\dfrac{\sqrt{17}}{4}$	14°	ϕ_β	3/4	0	0	go/access
H	0	0	0	0	—	—	—	—	—	go/access

Table 4-1: Conditions for Figure 4-1

Notes:

1. Regions B and E display mirrored values for $\|\dot{\alpha}\|$, $\|\dot{\beta}\|$, field-bias, ϕ_α-gain and ϕ_β-gain. There are also large field intensification values in $\Gamma_{\alpha \oplus \beta}$. These conditions are typical of *parallel form*. *Passing form* requires one additional condition: $\phi_{\alpha+\beta} \subset (\phi_\alpha \text{ or } \phi_\beta)$.

2. Passing form can generate access or slack depending on the placement and height of α and β. In region BE it is access as $\phi_{(\dot{\alpha}+\dot{\beta})}$ is uniformly 3/2i. If α and β were further from one another or their height were less, then slack might ensue (cf Section 4.3).

3. Region G displays a *field reversal* (cf Section 4.2).

4.2 Privacy/stop definitions

Privacy definitions are built with directional demarcations such that one of two conditions obtains: there is no dominant field direction for $\Gamma_{\alpha \oplus \beta}$, (i.e. fields interfere destructively or are *zeroed*,) or there is a field *reversal* within $\Gamma_{\alpha \oplus \beta}$.

4.2.1 Necessary and sufficient conditions for privacy/stop *field*
   1. *Zeros*: (field-bias-value $\Gamma_{\alpha \oplus \beta}$) $\leq i/4$, $60° > \theta_{(\alpha + \beta), \alpha} > 30°$
   2. Reversal: (field-bias Γ_β) $\neq$ (field-bias $\Gamma_{\alpha \oplus \beta}$)
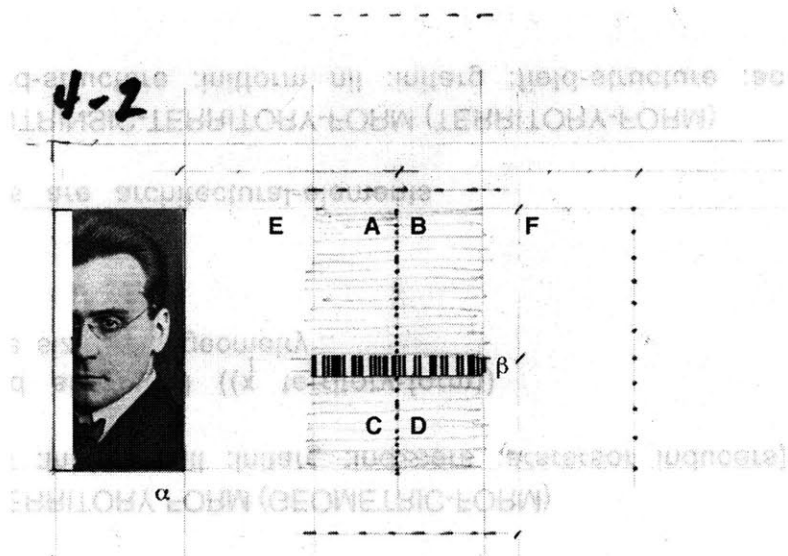
4.2.2 General conditions
   1. See Table 4-2.

Figure 4-2

Region	°-def	$\|\dot{\alpha}\|$	$\|\dot{\beta}\|$	$\|\dot{\alpha} + \dot{\beta}\|$	$\theta_{(\dot{\alpha}+\dot{\beta}),\dot{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_β-gain	field/use class
A	2	1	1	$\sqrt{2}$	45°	—	0	0	0	stop/privacy
B	2	1/2	1	$\dfrac{\sqrt{5}}{2}$	26°	ϕ_β	1/2	0	0	go/access
C	2	1	1	$\sqrt{2}$	45°	—	0	0	0	stop/privacy
D	2	1/2	1	$\dfrac{\sqrt{5}}{2}$	26°	ϕ_β	1/2	0	0	go/access
E	1	1	0	1	90°	ϕ_α	1	0	—	go/access
F	1	1/2	0	1/2	90°	ϕ_α	1/2	0	—	go/access

Table 4-2: Conditions for Figure 4-2

Notes:

1. Regions B, D are identical in all respects. This is characteristic of *T form*, as is the *field reversal* across regions BF, DF. (Field reversal occur when field-bias-values change across contiguous territories.) As β moves further from α, the reversal diminishes. As β moves closer to α, the reversal may be replaced by a *zeroing*, wherein the field-bias is nil. The same reasoning applies for α with respect to β.

2. Regions A, C are also identical in all respects as expected of *T form*. They both display a field-bias of 0, and requisite $\theta_{(\dot{\alpha}+\dot{\beta}),\dot{\alpha}}$ value.

3. All regions exhibit 0 valued field intensifications. This occurs if and only if ϕ_α and ϕ_β are normal to one another as in this example.

4.3 Collective/slack definitions

Field definitions may vary in their degree of definition. Where there is a relaxation in the degree of definition, one finds *slack*. Slack regions support collective uses and even the encroachment of neighbouring uses without forfeiture of overall organisation.

4.3.1 *Necessary and sufficient conditions for* collective/slack *field*
   1. fields with degree-def $> n$ flank some definition of degree-def $= n$
   2. size of the slack zone is at least that of its flanking territories.
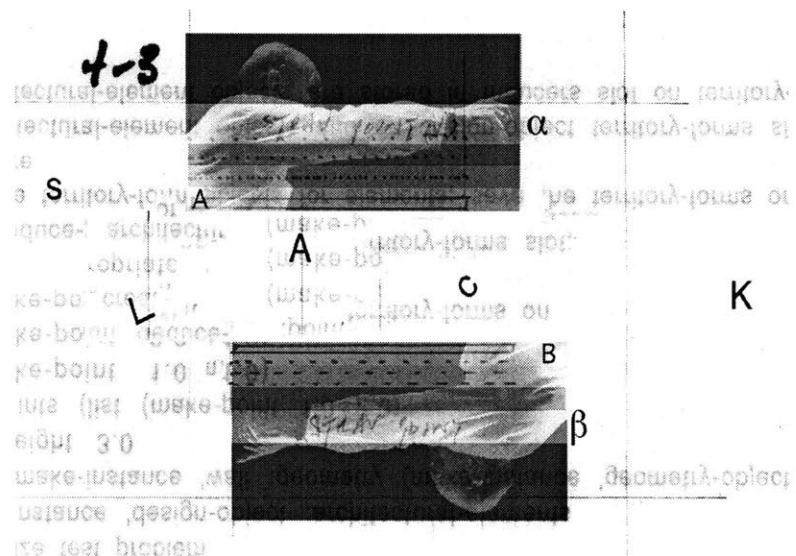
4.3.2 *General Conditions*
   1. See Table 4-3.

Figure 4-3

Region	°-def	$\|\dot{\alpha}\|$	$\|\dot{\beta}\|$	$\|\dot{\alpha}+\dot{\beta}\|$	$\theta_{(\dot{\alpha}+\dot{\beta}),\dot{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_β-gain	field/use class
A	1	1/4	0	1/4	0°	θ_α	1/4	0	—	—
B	1	0	1/4	1/4	0°	θ_β	1/4	—	0	—
C	0	0	0	0	—	—	—	—	—	slack

Table 4-3: Conditions for Figure 4-3

Notes:

1. Figure 4-3 shows Γ_α and Γ_β disjoint and of degree-def = 1. Between them is a zone of degree-def zero—hence an alternation in definitions. This slack zone is not, strictly speaking, a territory as there is no field definition.[2]

2. Regions A, B are neither *stop*s nor *go*s. The field definition is too weak for a go and $\theta_{(\dot{\alpha}+\dot{\beta}),\dot{\alpha}}$ is too small for a *stop*.

3. As slack is of a lesser degree of definition it most often belongs to a larger sized territory which α and β have partially intensified. As a vestige of the larger definition, slack plays an important role; it allows concurrent territorial associations across sizes.

---

2. This is not entirely true. Appendix A shows that α and β induce a territory owing to dimensional stability.

4.4 Case Studies

4.4.1 Access/Go

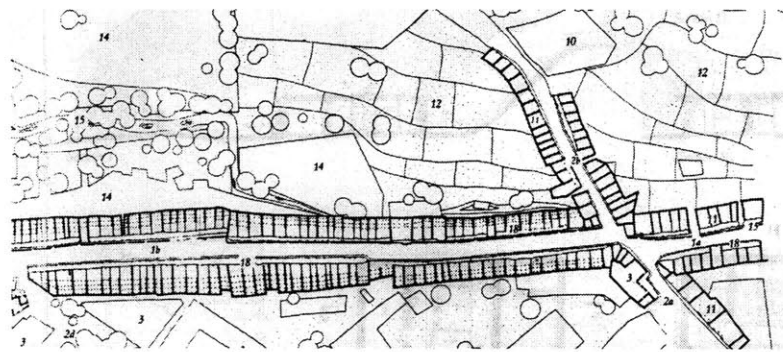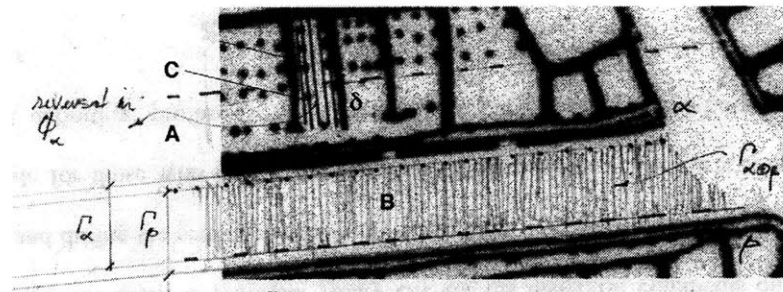#### 4.4.1.1 Bazaar Road: Chitral, Northwest Frontier Province, Pakistan

Figure 4-4

Figure 4-5

Parallel shop fronts produce a strong directional field occupied by access. This field, ϕ_α, is further intensified by a covered walk. The shops themselves reverse ϕ_α owing to their great depth. This results in relative privacies or *stop*s. See Table 4-4 for computed values.

Region	°-def	$\|\dot{\alpha}\|$	$\|\dot{\beta}\|$	$\|\dot{\alpha}+\dot{\beta}\|$	$\theta_{(\dot{\alpha}+\dot{\beta}),\,\dot{\alpha}}$	field-bias	ϕ_α-gain	ϕ_β-gain	field/use class
A	2	1	1/2	3/2	0°	ϕ_α	1/2	2	go/access
B	2	1	1	2	0°	—	1	1	go/access

Region	°-def	$\|(\dot{\alpha}+\dot{\beta})\|$	$\|\dot{\delta}\|$	$\|(\dot{\alpha}+\dot{\beta})+\dot{\delta}\|$	$\theta_{((\dot{\alpha}+\dot{\beta})+\dot{\delta}),\,(\dot{\alpha}+\dot{\beta})}$	field-bias	ϕ_α-gain	ϕ_δ-gain	field/use class
C	4	3/2	2	5/2	37°	ϕ_δ	0	0	stop/privacy

Table 4-4: Conditions for Figure 4-4 and 4-5

Notes:

1. Very large $\|\dot{\alpha}+\dot{\beta}\|$ suggests access in region B.
2. Region A also displays large I_f and serves pedestrian access.
3. Region C shows a *stop*. This corresponds with the anteroom of stores.
4. Region D displays a field reversal in ϕ_α. This area forms the back rooms for shops.

4.4.1.2 Gale House: Frank Lloyd Wright
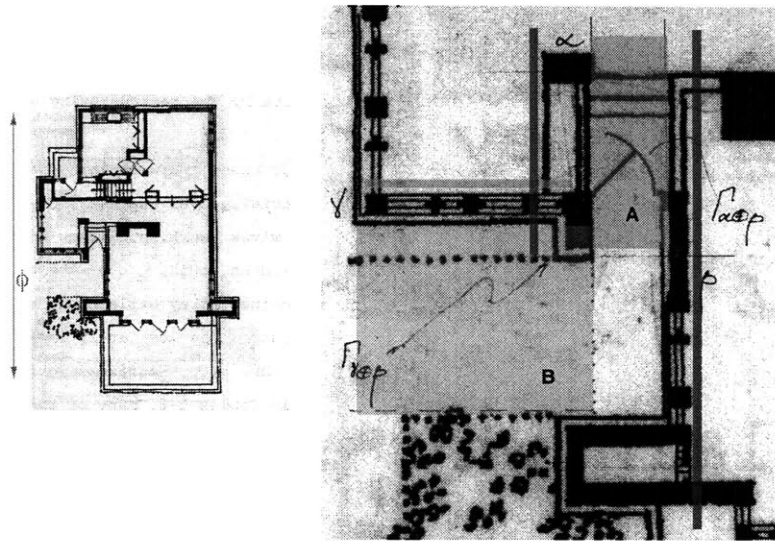
Figure 4-6a/4-6b

Walls α and β reinforce the building sized field ϕ and build an access zone $\Gamma_{\alpha \oplus \beta}$. Note that β passes α, thus simultaneously contributing to the *stop* definition $\Gamma_{\gamma \oplus \beta}$.

See Table 4-5 for computed values.

Region	°-def	$\|\dot{\alpha}\|$	$\|\dot{\beta}\|$	$\|\dot{\alpha}+\dot{\beta}\|$	$\theta_{(\dot{\alpha}+\dot{\beta}),\dot{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_β-gain	field/use class
A	2	1	1	2	0°	—	0	1	1	go/access

Region	°-def	$\|\dot{\gamma}\|$	$\|\dot{\beta}\|$	$\|\dot{\gamma}+\dot{\beta}\|$	$\theta_{(\dot{\gamma}+\dot{\beta}),\dot{\gamma}}$	field-bias	field-bias-value	ϕ_γ-gain	ϕ_β-gain	use class
B	2	1	1	$\sqrt{2}$	45°	—	0	0	0	stop/privacy

Table 4-5: Conditions for Figure 4-6

Notes:

1. No field bias in region B is indicative of a *stop* in the larger building-sized field ϕ. Closer inspection reveals that *all* use territories are generated as *stop*s (lateral displacements) in ϕ.

4.4.2 Privacy/Stop

4.4.2.1 Verandah, Village of Taqma, Salang Mountains, Afghanistan
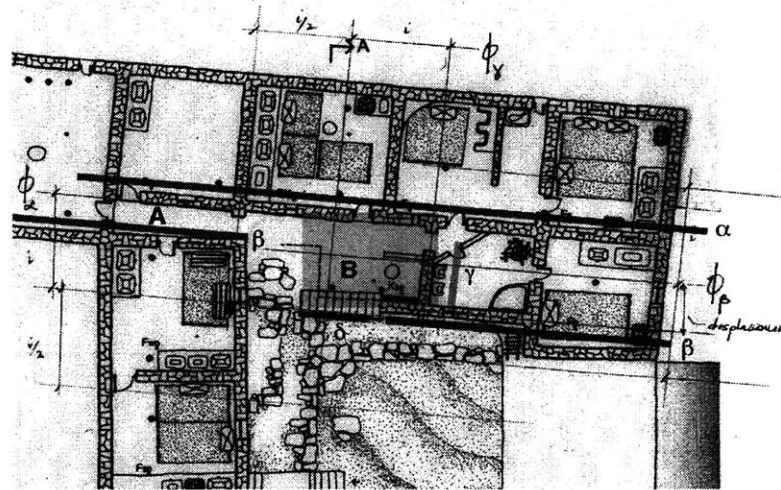
Figure 4-7

Wall element α induces a strong directional field to which access has been assigned. This direction is reinforced by β and sectional access δ. The lateral displacement of β weakens $\phi_{\alpha \oplus \beta}$ allowing for γ to generate a full *stop*, $\Gamma_{\gamma \oplus (\alpha \oplus \beta)}$. As expected of field *stops*, a relative privacy is built—in this case a summer kitchen.

See Table 4-6 for computed values.

Region	°-def	$\|\vec{\alpha}\|$	$\|\vec{\beta}\|$	$\|\vec{\alpha}+\vec{\beta}\|$	$\theta_{(\vec{\alpha}+\vec{\beta}),\vec{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_β-gain	use class
A	2	1	1	2	0°	—	0	1	1	go/access
Region	°-def	$\|\vec{\alpha}\|$	$\|\vec{\gamma}\|$	$\|\vec{\alpha}+\vec{\gamma}\|$	$\theta_{(\vec{\alpha}+\vec{\gamma}),\vec{\alpha}}$	field-bias	field-bias-value	ϕ_α-gain	ϕ_γ-gain	use class
B	3	1	1	$\sqrt{2}$	45°	—	0	0	0	stop/privacy

Table 4-6: Conditions for Figure 4-7

Notes:

1. Region A: large intensity value, large field-gain and θ equal to zero signals severe access

2. Region B: values specify necessary and sufficient conditions for the strongest field interference.

4.4.2.2 Gale House: Frank Lloyd Wright
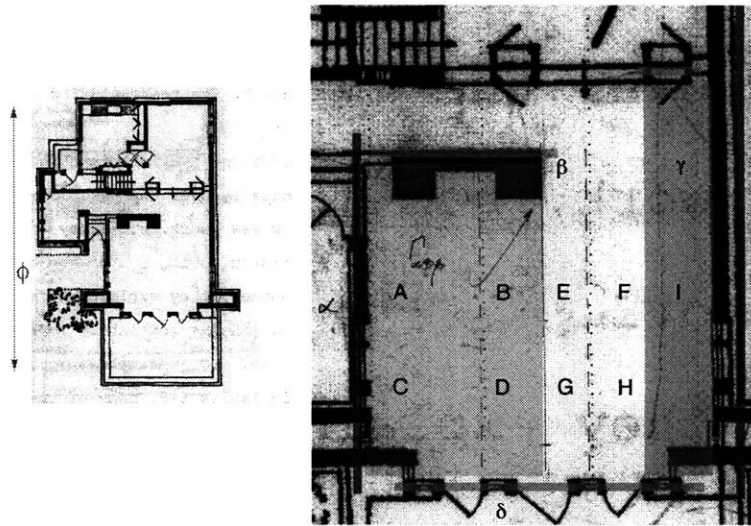
Figure 4-8a/4-8b

Walls α and γ, build a *go* definition $\Gamma_{\alpha \oplus \gamma}$. Fireplace β and closure δ introduce $\Gamma_{\beta \oplus \delta}$ with $\phi_{\beta \oplus \delta}$ normal to $\phi_{\alpha \oplus \gamma}$. These opposing fields lead to a zeroing at $\Gamma_{(\beta \oplus \delta) \oplus (\alpha \oplus \gamma)}$ —sufficient to build a privacy which is clearly in evidence by use. Note that slack is built between territories ABCD and I, denoted by EFGH.

See Table 4-7 for computed values.

Region	°-def	$\|\vec{\alpha} + \vec{\gamma}\| \equiv A$	$\|\vec{\beta} + \vec{\delta}\| \equiv B$	$\|A + B\|$	θ_{A+B}	field-bias	field-bias value	field/use class
A	4	5/4	5/4	$\dfrac{5\sqrt{2}}{4}$	45°	—	0	stop/privacy
B	4	1	5/4	$\dfrac{\sqrt{41}}{4}$	38°	$\phi_{\vec{\beta}+\vec{\delta}}$	1/4	stop/privacy
C	4	5/4	5/4	$\dfrac{5\sqrt{2}}{4}$	45°	—	0	stop/privacy
D	4	1	5/4	$\dfrac{\sqrt{41}}{4}$	38°	$\phi_{\vec{\beta}+\vec{\delta}}$	1/4	stop/privacy
E	3	1	1/2	$\dfrac{\sqrt{5}}{2}$	65°	$\phi_{\vec{\alpha}+\vec{\gamma}}$	1/2	go/access
F	3	5/4	1/2	$\dfrac{\sqrt{29}}{4}$	68°	$\phi_{\vec{\alpha}+\vec{\gamma}}$	3/4	go/access
G	3	1	1	1	90°	—	0	go/access
H	3	5/4	1	$\dfrac{\sqrt{41}}{4}$	51°	$\phi_{\vec{\alpha}+\vec{\gamma}}$	1/4	stop/privacy
I	4	5/4	0	$\dfrac{\sqrt{5}}{2}$	90°	$\phi_{\vec{\alpha}+\vec{\gamma}}$	5/4	go/access

Table 4-7: Conditions for Figure 4-8

Notes:

1. Region EFGH is slack as it is flanked by territories of greater degree-def.
2. Region EFGH is also rendered as access.

4.4.3 Collective/Slack

4.4.3.1 Hysolar Institute, Universität Stuttgart: Günter Behnisch

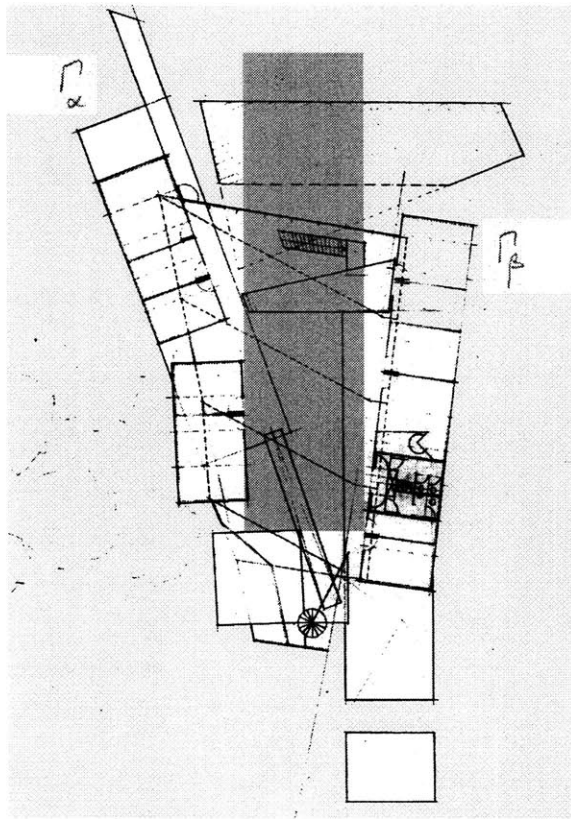**Figure 4-9**

Between Γ_α and Γ_β exists a territory of roughly equal size and of degree-def less than both Γ_α and Γ_β. As expected, this slack zone accommodates collective use, and allows for association with larger sized definitions (site and landscape).
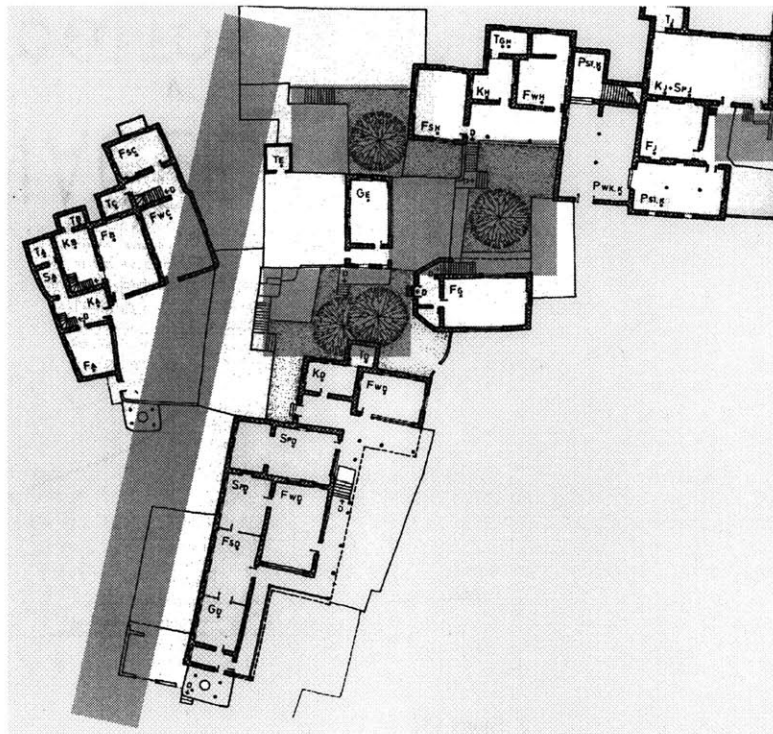
4.4.3.2 Village of Kolalan, Koh Daman Valley, Afghanistan

Figure 4-10

Slack zones are clearly in evidence at varying sizes. The uses ascribed to these are always public.

4.4.3.3 Fallingwater (ground floor): Frank Lloyd Wright
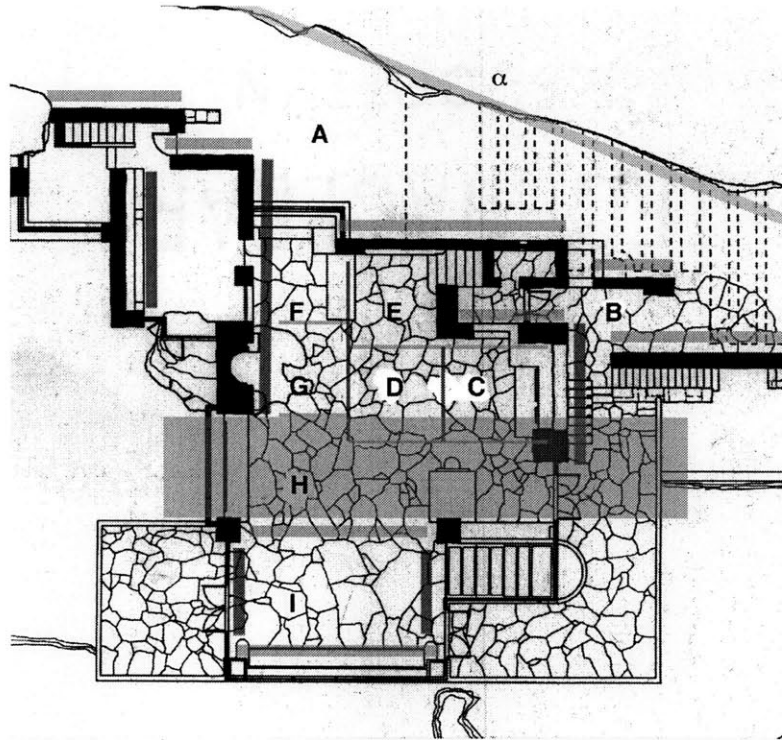
Figure 4-11

The site's directional field is built by the sectional element α and Bear Run Creek. Wright reinforces this directional field through the registration of elements shown by horizontal bands. Those elements shown by vertical bands build an opposing directional field which forms variable *zero*s and reversals. The territories exhibiting these *zero*s and reversals are occupied as relative privacies. Note the slack in zone H, thus reinforcing association at the largest (landscape) size.

Region	°-def	$\|\overrightarrow{region}\|$	θ_{region}	field-bias	field/use class
A	2	1.90	0°	—	go/access
A′	2	1.45	20°	vertical band	go/access
B	2	2.00	0°	vertical band	go/access
C	3	1.60	38°	horizontal band	stop/privacy
D	4	1.41	45°	—	stop/privacy
E	4	1.95	56°	horizontal band	stop/privacy
F	4	1.95	56°	horizontal band	stop/privacy
G	4	1.6	39°	horizontal band	stop/privacy
H	2	1.25	0°	—	go/access/slack
I	4	4.00	45°	—	stop/privacy

Table 4-8

Notes:

1. An important component in the generation of the slack zone H is the screen assembled from two principle columns. Appendix A illustrates the manner in which territorial control assembles these columns into the larger sized definition of a screen.

4.5 Size

Very little mention has been made of particular use sizes. This has been deliberate as *go, stop* and *slack* behaviours can be found at all sizes. They operate *across* sizes as well, though these are limited to interactions with the next larger and smaller size.

The foregoing has been content to describe privacy, access and collective in purely relative terms. Of course in the full determination of use, reference *must* be made to size. Introducing size as an additional condition to those of Sections 4.1, 4.2 and 4.3 is sufficient for this purpose.

Table 4-4 ranks the seven major sizes within which architectural form most commonly operates. Each size accommodates access at that size.

Size	approximate dimension
material	< 3′
personal	3-8′
room	8-14′
collective	14-25′
building	25-100′
site	100′+
landscape	200′+
access	at each of the sizes above

Table 4-9

CHAPTER 5

C O N C L U D I N G R E M A R K S

The arithmetical model sketched in Chapters 2 and 3 found applica-
tion in the use model of Chapter 4. Other investigations of architec-
tural form lend themselves to the model. Several are suggested
below.

5.1 Other applications

5.1.1 Form families and organisations

Some generic form organisations such as the *T form, parallel form* and
passing form were identified in terms of their shared territories' field
structure (Tables 4-1, 4-2). Other form families should be as easy to
describe. One advantage of describing these in terms of their territory
is that the particularities of the physical form (outside of the determi-
nation of its spatial extents) need never be addressed. Form is under-
stood as an organisation, not shape.

5.1.2 Containment

Containment can be expressed as a function of degree of definition
and field intensity. The greater the number of inducers and intensity
of their resulting territories, the greater the containment.

5.1.3 Light

Just as territories have been said to be built by physical demarcations, so too is light. The definition of spatial extents, particularly as applied to ensembles, describes the building of light. (See Section 4.4.3.3 and Appendix A.)

5.1.4 Computational model

The spatial model's scope of applicability and mathematical formalisation invites computational implementation.[1] Analytical engines might map the territorial commitments of any architectural form. Knowledge-based systems such as that proposed by Koile (1997) may reason with these computed territories.

Alternatively, application programs may allow for user interaction at the level of territory; given a desired territorial definition, the application informs the user of those physical form properties necessary for its inducement. Note that in this last case the mapping from territory to physical form is not injective; uncountably many instances of physical form can build a particular territory.[2]

---

1. Appendix B diagrams a set of Classes and Methods for the arithmetical model of spatial definition presented herein.
2. The interest in building application programs to *generate* physical form is wrong-headed. At best its results will remain uninteresting as arbitrary rules of shape are introduced to limit that which is unbounded. The far more interesting and difficult question is: what spatial understanding gives rise to desired properties of physical form, and how does the latter accommodate the former.

5.2 Concluding remarks

Spatial definition admits to an infinitude of articulations; material, light, use, dimension, tectonics and sundry combine in rendering territory. Given the many properties of which it is comprised—and presumably manners in which it is defined—there would appear little hope of saying much more (or less) than has been said by countless others.

In response, this model of spatial definition reduces the aforementioned infinitudes to a modest three: dimension, direction and intensity. Those manners by which territory is induced are reduced to one: the projection of physical form's spatial extents. Though the abstraction dispenses with much of the phenomenological, it remains firmly rooted in experience—not as a model *of* experience, but one *informed* by the same.

The arithmetical model of spatial definition presented herein makes precise hypotheses of the empirical world. No doubt these will soon shipwreck upon the large and dispassionate world of empirical fact, and take with them these 82 pages. However the method which invited the formalisation—one which Jerome Wiener had once described as 'represent[ing] nothing less than the good manners of

the mind'[3]—will survive the errors of this work; as it will survive the charlatans of Heidelberg; as it will the sophists heralding 'new architectural promenades.'

May architecture, and those whom it is to serve, fare as well.

---

3. Communicated to the Department of Architecture on February 14, 1963 when serving on the MIT Visiting Committee to the School of Architecture and Planning. (Told the author by M K Smith).

APPENDIX A

E N S E M B L E S

Necessary and sufficient conditions are considered for the determination of ensembles.

The spatial and arithmetic models of Chapters 2 and 3 accommodated ensembles as though they were any other form of like spatial extents.

In experiencing architecture our understanding takes seemingly disparate elements and fashions them into coherent assemblies—much as does the mind with discrete notes of music, building harmonic, melodic and contrapuntal assemblies from them. What properties of form are addressed in constructing these architectural assemblies? There are any number, to be sure. One class may be described as physical attributes, which includes material, colour, dimension and tectonic type. Another class of properties may be termed formal, which includes spatial organisation. These are present at all sizes.

Physical attributes build ensembles through association; elements are grouped on the basis of shared properties. Elements in concrete may be deployed in such a way that the material builds associations at the building size. A secondary structure, if made legible, would constitute an association at a smaller size than the primary structure. Repeated use of dimension, or material might allow for the assembly of a screen, a structural bay or village and so on. For the most part, a class as this would tie the elements together by virtue of shared intrinsic properties, and cast the ensemble as the extents of those elements.

Spatial organisation performs somewhat differently. The association built is not through a shared physical property, but a spatial organisation in which each element is a necessary part. For instance a set of columns may be read or assembled as a bay by virtue of the columns' spacing and height. Similarly, a piece of landscape may be claimed through a dimensional displacement at a building size. This appendix considers only those ensembles built from spatial organisation. It describes necessary and sufficient conditions for an organisationally built ensemble.

Evidence of spatial organisation can be found in the constituent elements' mutual control of shared territory. If $\Gamma_{\alpha \oplus \beta}$ exists for some elements A and B, then A and B control one another as each intensifies the other's territory in $\Gamma_{\alpha \oplus \beta}$. Spatial organisation can also be found in the consistent dimensional deployment of elements. If an element's dimensions are also recorded in its placement *vis-a-vis* other elements, then a recognisable association is made, one which is termed *self-stability*. Each will be treated in turn.

Definition: Consider elements A and B and their respective territories Γ_{α} and Γ_{β}. An *ensemble built from territorial control* is the set of inducers for $\Gamma_{\alpha \oplus \beta}$ such that:

1. $\Gamma_{\alpha \oplus \beta} \neq \varnothing$
2. $i \geq \|\dot{\alpha}\| \geq i/2$ in region $(A \oplus B)$
   $i \geq \|\dot{\beta}\| \geq i/2$ in region $(A \oplus B)$

3.

$$(\min \, (\text{size } \alpha)(\text{size } \beta)) \geq (\text{size } \Gamma_{\alpha \oplus \beta}) \geq (-(\min \, (\text{size } \alpha)(\text{size } \beta))1)$$

4. $\|\dot{\alpha}\| \cong \|\dot{\beta}\|$

The rationale for the definition is as follows:

> *Condition 1* ensures that $\Gamma_{\alpha \oplus \beta}$ exists. Recall that a necessary condition of an ensemble is that its constituents participate in the control of one-another's territory.
>
> *Condition 2* ensures that $\Gamma_{\alpha \oplus \beta}$ occurs where Γ_α and Γ_β are strongly defined. If $\Gamma_{\alpha \oplus \beta}$ were elsewhere in Γ_α and Γ_β, the mutual territorial control would be too weak to register experientially.
>
> *Condition 3* ensures that $\Gamma_{\alpha \oplus \beta}$ is not trivially small nor large.
>
> *Condition 4* ensures that α and β are roughly equal in their control of the other's territory.

In sum, one might claim that our reading of $\Gamma_{\alpha \oplus \beta}$ acknowledges the inducers and joins them in experience. Examples are given in Sections A.0.1 and A.0.2.

Definition: An *ensemble built from dimensional stability* is one in which for elements A, B the element A is displaced its own dimension from B. An example is given in Section A.0.3.

It is important to note again the manner in which these ensembles generate their territory *as ensembles*. The territory generated by these ensembles is simply that given by the projection of their extents. The

only quantitative difference between the territory associated with a structural bay element and a wall element, say, is that the bay's projection of extents renders a footprint of usable territory, whilst the wall yields a footprint occupied by material.

A.0.1 Study: Ensemble built from territorial control: screen definition in Fallingwater (ground floor)
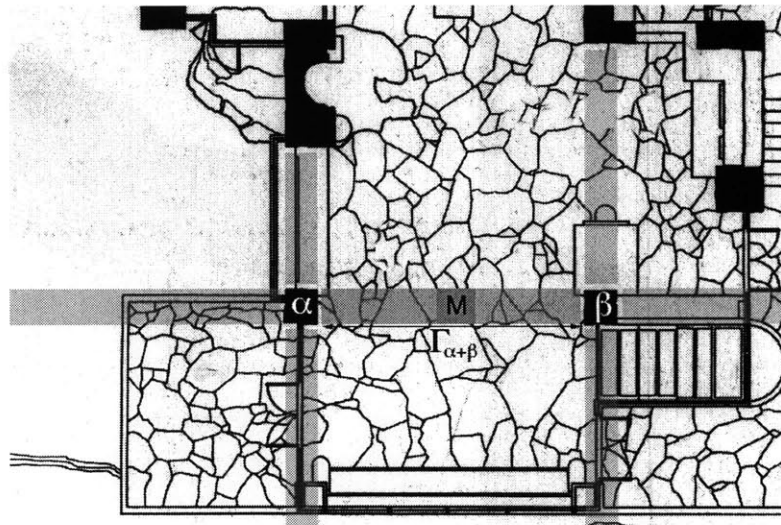

Figure A-1

Figure A-1 depicts Γ_α, Γ_β and $\Gamma_{\alpha \oplus \beta}$. The following conditions obtain:

1. $\Gamma_{\alpha \oplus \beta}$ is non empty and occurs where I_{f_α} and I_{f_β} are greatest.
2. Region M assumes the maximum I_f value for $\Gamma_\alpha \cup \Gamma_\beta$. Hence $\Gamma_{\alpha \oplus \beta}$ is extrinsically controlled.

3. M is at least of size α and β.

4. $\|\ddot{\alpha}\| \cong \|\ddot{\beta}\|$ in region M.

All four conditions obtain for α and β to build an ensemble $\alpha\beta$. In Figure A-2 below, the spatial extents and resulting territories for $\alpha\beta$ are drawn. Note that these extents partake in slack and stop definitions described in Chapter 4, Section 4.4.3.3.
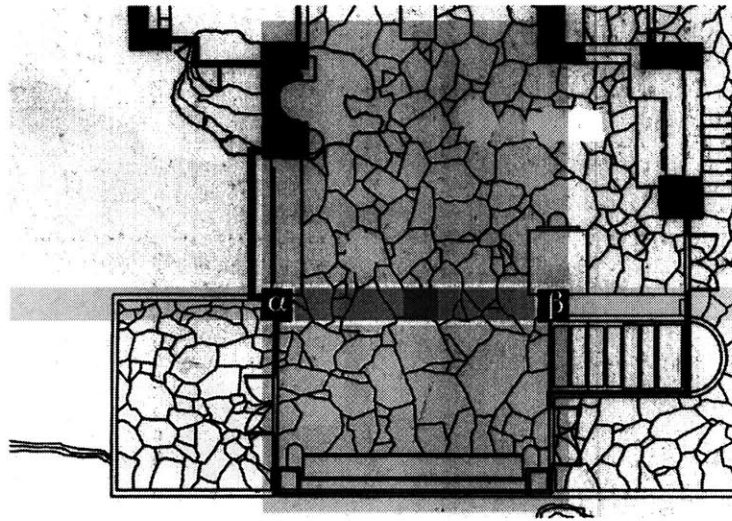

Figure A-2

It should be noted that column α and the fireplace do not form an ensemble as condition 3 fails.

A.0.2 Study: Ensemble built from territorial control: the Ha of Hunza Valley, Northern Areas, Pakistan

The *ha* is the most important room in the domestic architecture of the Hindu Kush and Karakoram of Central Asia. A room of approximately 20' x 17' need accommodate: women's sleeping quarters, men's sleeping quarters, a place for musicians, a kitchen and storage.
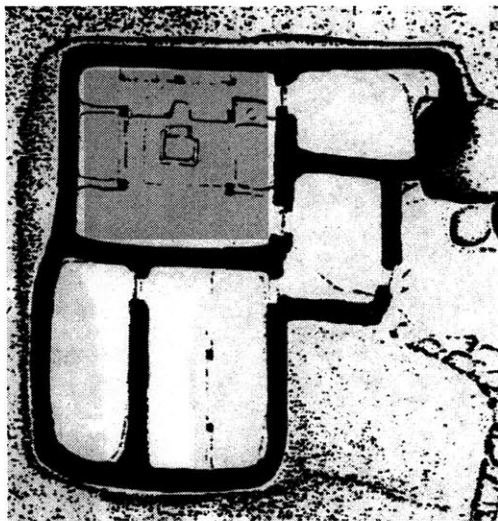

Figure A-3

Outside of its four walls, the only physical form present is that of four posts. Because of their height and spacing, an organisation obtains which is akin to that seen in Fallingwater. Applied four times over, the requisite spatial definitions are made. These are illustrated in Figure A-4.
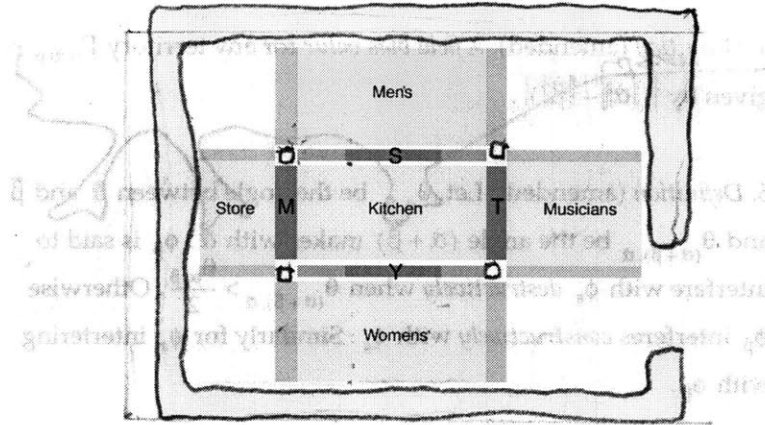
Figure A-4

Regions M, S, T, Y boast the largest I_f values for their respective columns. As conditions for an ensemble are met for each pairing, four ensembles are built. (In point of fact, these very ensembles build a fifth ensemble which is that of the four column bay.) NEW Territories are built with every ensemble.

Note that the columns do not form diagonal pairings. This is a result of their geometry (their spatial extents do not project diagonally) and height and spacing (conditions 2 and 3 above fail).

A.0.3 Study: Ensemble built from dimensional stability: Parliament Building, Bonn, Deutschland, Günter Behnisch
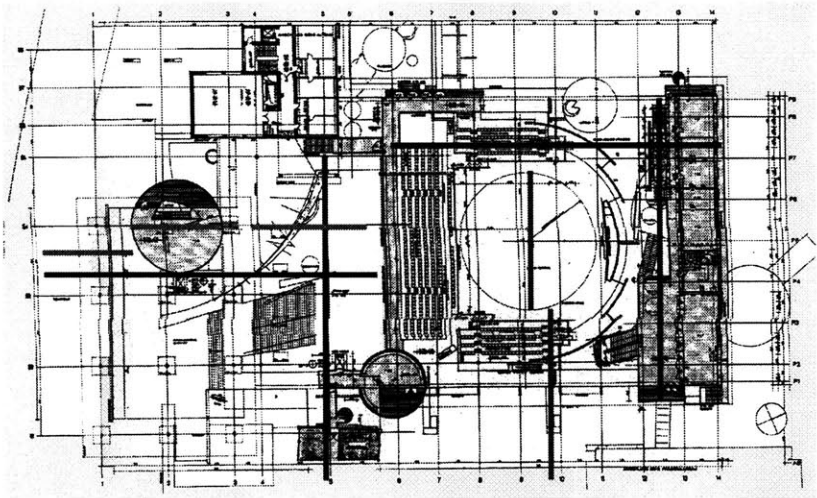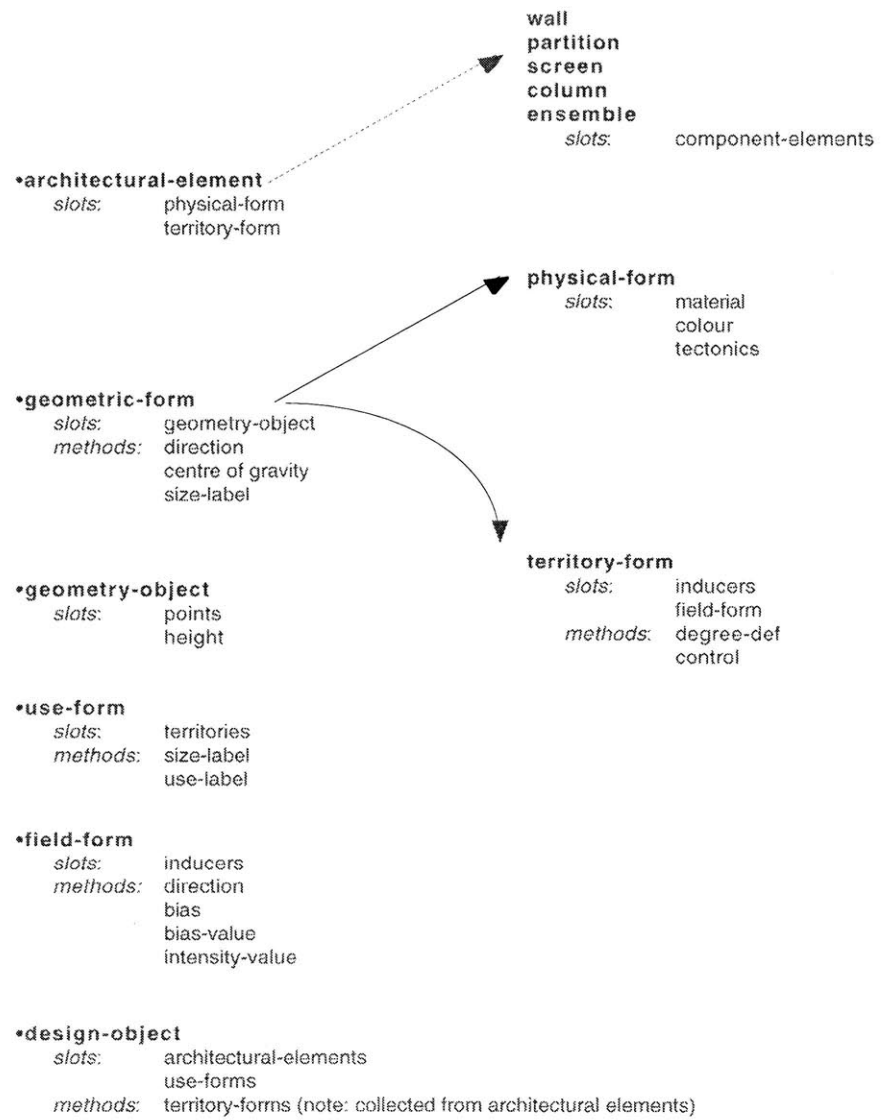

Figure A-5

Lines denote dimensions of various elements. Note that the elements responsible for these dimensions are in turn displaced from other physical demarcations by the same dimension. Territory is built without resort to a strict, hierarchically arranged plan.

CLASSES AND METHODS

wall
partition
screen
column
ensemble
    slots: component-elements

•architectural-element
    slots: physical-form
             territory-form

physical-form
    slots: material
             colour
             tectonics

•geometric-form
    slots: geometry-object
    methods: direction
               centre of gravity
               size-label

territory-form
    slots: inducers
             field-form
    methods: degree-def
               control

•geometry-object
    slots: points
             height

•use-form
    slots: territories
    methods: size-label
               use-label

•field-form
    slots: inducers
    methods: direction
               bias
               bias-value
               intensity-value

•design-object
    slots: architectural-elements
             use-forms
    methods: territory-forms (note: collected from architectural elements)

AN ARITHMETICAL MODEL OF SPATIAL DEFINITION

REFERENCES

R.1 Bibliography

Alexander, C., Ishikawa, S., Silverstein, M., Jacobsen, M., Fiksdahl-King, I., and Angel, S. (1977). A Pattern Language. New York, Oxford University Press.

Arnheim, Rudolf. (1977). The Dynamics of Architectural Form. Berkeley, University of California Press.

Behnisch & Partner. (1975). Bauten und Entwürfe 1952-1974. Stuttgart, Verlag Gerd Haitje.

Carrara, G., Kalay, Y. E., and Novembri, G. (1994). Knowledge-based computational support for architectural design. Knowledge-Based Computer-Aided Architectural Design. New York, Elsevier: 147-201.

Chong, J. H. (1992). Territory and Dwelling: habitation, access and light. School of Architecture and Planning. Cambridge, Massachusetts Institute of Technology.

Dave, B., Schmitt, B., Faltings, B., and Smith, I. (1994). Case-based design in architecture. Artificial Intelligence in Design 1994, Proceedings of the Third International Conference on Artificial Intelligence in Design, Norwell, MA, Kluwer.

Gross, M. D. (1996). The Electronic Cocktail Napkin—a computational environment for working with design diagrams. Design Studies 17:53-69.

Hall, E. T. (1966). The Hidden Dimension. Garden City, Doubleday and Company, Inc.

Hanson, J. (1994). Deconstructing architects' houses. Environmental and Planning B 21: 675-704.

Hassan, I. (1985). The Indigenous Architecture of Chitral Pakistan. Mimar 17.

Hertzberger, H. (1993). Lessons for Students in Architecture. 2nd ed, Uitgeverij 010 Publishers, Rotterdam.

Illi, Dieter W. (1991). Das Hindukush Haus. Stuttgart, Franz Steiner Verlag.

Kincaid, D. (1997). Urban Form of Colonial Iberoamerica. (Unpublished).

Koile, K. (1997). Design Conversations with your Computer. (To appear in CAAD Futures 1997, Proceedings of the Seventh International Conference on Computer-Aided Architectural Design Futures, August 3-7, 1997, Munich, Germany).

Koschmann, T.D. (1990). Common Lisp Companion. John Wiley & Sons.

McCarter, R. (1994). Fallingwater: Frank Lloyd Wright. London, Phaidon.

Mortola, E. a. G., A. (1991). An evaluation module for 'An Interface for Designing (AID)': a procedure based on trichotomic segmentation. CAAD Futures 1991, Proceedings of the Fourth International Conference on Computer-Aided Architectural Design Futures, Wiesbaden, Vieweg.

Newman, O. (1973). Defensible Space. New York, Macmillan Publishing Co., Inc.

Norberg-Schulz, C. (1986). Architecture: Meaning and Place. Milano, Rizzoli.

Popper, K. R. (1959). The Logic of Scientific Discovery. New York. Harper & Row.

Rasmussen, S. E. (1964). Experiencing Architecture. Cambridge, MIT Press.

Reifenstein, E. C. (1992). Understanding Spatial Structure: Identification, Transformation, Evaluation. School of Architecture and Planning. Cambridge, Massachusetts Institute of Technology.

Storrer, William A. (1993). The Frank Lloyd Wright Companion. Chicago, University of Chicago Press.

Smith, Maurice K. (1992). The Comic Book. (Course reader consisting of Maurice K Smith writings for Maurice K Smith Design Studio, Spring 1992 at the Massachusetts Institute of Technology).

Szabo, A. a. Barfield, T. J. (1991). Afghanistan: An Atlas of Indigenous Domestic Architecture. Austin, University of Texas Press.

R.2 Illustration credits

Note: Credits are for underlays. All annotations are by the author. All figures not specified below are by the author.

Chapter 3
Figure 3-2: Blasto-Gen logo courtesy of the Blasto-Gen Corporation
Figure 3-3: Blasto-Gen logo courtesy of the Blasto-Gen Corporation

Chapter 4
Figure 4-4: Hassan, I. (1985)
Figure 4-5: Hassan, I. (1985)
Figure 4-6: Storrer, William A. (1993)
Figure 4-7: Szabo, A. a. Barfield, T. J. (1991)
Figure 4-8: Storrer, William A. (1993)
Figure 4-9: Behnisch & Partner (1975)

Figure 4-10: Szabo, A. a. Barfield, T. J. (1991)

Figure 4-11: McCarter, R. (1994)

Appendix A

Figure A-1: McCarter, R. (1994)

Figure A-2: McCarter, R. (1994)

Figure A-3: Illi, Dieter W. (1991)

Figure A-5: Behnisch & Partner (1975)

R.2 Illustration credits

An Arithmetical Model of Spatial Definition