

# Temporal Integration Of Seismic Traveltime Tomography

Jonathan B. Ajo-Franklin,  
Earth Resources Laboratory  
Dept. of Earth, Atmospheric, and Planetary Sciences  
Massachusetts Institute of Technology  
Cambridge, MA 02142,

Jaime Urban, Jerry M. Harris, Stanford University

June 1, 2005

## Abstract

Time-lapse geophysical measurements and seismic imaging methods in particular are powerful techniques for monitoring changes in reservoir properties. Traditional time-lapse processing methods treat each dataset as an independent unit and estimate changes in reservoir state through differencing these separate inversions. We present a general least-squares approach to jointly inverting time-varying property models through use of spatio-temporal coupling operators. Originally developed within the medical imaging community, this extension of traditional Tikhonov regularization allows us to constrain the way in which models vary in time, thereby reducing artifacts observed in traditional time-lapse imaging formulations. The same methodology can also accommodate changes in experiment geometry as a function of time thus allowing inversion of incremental or incomplete surveys. In this case, temporal resolution is traded for improved spatial coverage at individual timesteps. We use seismic traveltime tomography as a model problem although almost any geophysical inversion task can be posed within this formalism. We apply the developed time-lapse inversion algorithm to a synthetic crosswell dataset designed to replicate a CO<sub>2</sub> sequestration monitoring experiment.

## 1 Introduction

Time-lapse geophysical measurements provide a powerful toolbox of techniques for monitoring subsurface flow processes including the enhanced recovery of oil and gas (6), CO<sub>2</sub> sequestration (2) (5), and contaminant remediation (4). Seismic methods in particular are increasingly used to generate quantitative maps of variations in fluid saturation or pressure over spatial domains ranging from 10s of meters for environmental problems to 10s of kilometers for field-wide production monitoring.

Most traditional time-lapse processing techniques treat each acquired dataset as an independent measurement. Temporal variations in subsurface properties are typically determined by subtraction of image pairs. Naive image subtraction tends to be sensitive to survey-to-survey changes in S/N ratio and variations in acquisition geometry, both of which can generate artifacts in the resulting time-lapse images. This conceptual approach also fails to acknowledge a key component of prior knowledge, mainly that the changes in subsurface seismic response in these environments are due to variations in fluid properties or effective stress and not changes in geologic structure. Significant artifacts in time-lapse processing results have spurred development of cross-equalization algorithms (8) designed to match the geometry and signal characteristics of repeated surveys.

Joint inversion of a series of time-lapse surveys seems to be a natural approach to decreasing image artifacts which result from naive differencing. Through use of constraints on how the model can change in time, we can suppress non-repeatable noise while providing a more consistent reconstruction of real variations

in subsurface properties. Day-Lewis et.al.(3) demonstrated a technique for integrating time-lapse imaging based on using smooth basis functions with a time dimension for tomographic inversion. They effectively applied their method to dynamically image the flow of a saline tracer through a large aperture fracture using borehole radar. We adopt an approach developed within the medical imaging community based on a straight-forward extension of Tikhonov regularization to include a time dimension. Brooks et.al. (1) apply a temporal constraint operator within a least-squares framework for the inversion of electrocardiography data. Zhang et.al. (9) perform a comparison of spatio-temporal inversion approaches including both the Tikhonov approach and Kalman filtering. We largely follow the formulation of Brooks et.al. with the exception of our choice of regularization operator and the techniques used for solving the coupled system.

## 2 Linear Formulation

We will initially consider the general linear inverse problem where a linear operator  $G$ , maps a model ( $m$ ) to a dataset ( $d$ ),

$$G m = d. \quad (1)$$

Consider a series of  $n$  datasets ( $d_i$ ) acquired at different times ( $t_i$ ), possibly with different source/receiver geometries as encapsulated by variations in the kernel ( $G_i$ ) but equivalent model parameterizations. In the naive case, the inverse problem would be solved independently for each dataset by minimizing an objective function of the form

$$\Phi(m_1, \dots, m_n) = \sum_{i=1}^n \|G_i m_i - d_i\|_2^2 + \lambda_s^2 \sum_{i=1}^n \|D m_i\|_2^2 \quad (2)$$

where  $D$  is a weighting operator and  $\lambda_s$  is a regularization parameter. The first term in equation 2 measures data misfit while the second measures model length as modified by  $D$ . If  $D = I$  than the resulting minimization of equation 2 is simply the damped least-squares solution. Neither of the terms in equation 2 couple solutions across multiple time steps since modification to  $m_i$  does not effect either the data misfit or length of model  $m_j$  ( $i \neq j$ ); this approach seems most applicable in cases where models are not correlated in time.

In the case of our time-lapse seismic problem, models have a strong temporal correlation since the underlying geologic structure is clearly not changing in the relatively short (geologically speaking) time between repeat surveys. We can modify equation 2 to include a temporal cross-coupling term which minimizes the time-lapse change in some model attribute in addition to decreasing data misfit and model norm for individual surveys. Consider a combined objective function of the form

$$\begin{aligned} \Phi(m_1, \dots, m_n) = & \sum_{i=1}^n \|G_i m_i - d_i\|_2^2 + \\ & \lambda_s^2 \sum_{i=1}^n \|D m_i\|_2^2 + \\ & \lambda_t^2 \sum_{i=1}^{n-1} \frac{\|D m_{i+1} - D m_i\|_2^2}{\Delta t_i}. \end{aligned} \quad (3)$$

where  $\Delta t_i = t_{i+1} - t_i$ . Minimizing equation 3 is equivalent to the least-squares solution of an augmented

system with operator  $G_c$ ,

$$G_c = \begin{bmatrix} G_1 & 0 & \cdots & 0 \\ 0 & G_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \vdots & 0 & G_n \\ \lambda_s D & 0 & \vdots & 0 \\ 0 & \lambda_s D & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_s D \\ \frac{\lambda_t}{\Delta t_1} D & -\frac{\lambda_t}{\Delta t_1} D & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & \frac{\lambda_t}{\Delta t_{n-1}} D & -\frac{\lambda_t}{\Delta t_{n-1}} D \end{bmatrix} \quad (4)$$

$$G_c \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \quad (5)$$

The numerator in the third term of equation 3 penalizes the differences between models which are temporal neighbors while the denominator scales this weight by the time elapsed between surveys. The second regularization parameter,  $\lambda_t$ , controls the strength of the temporal constraint. If  $D = I$  this component can be thought of as a time damping term.

For the simple case of two surveys ( $n = 2$ ) and 0th order Tikhonov regularization for both the spatial and spatio-temporal terms ( $D = I$ ), equation 4 can be reduced to

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \\ \lambda_s I & 0 \\ 0 & \lambda_s I \\ \frac{\lambda_t}{\Delta t} I & -\frac{\lambda_t}{\Delta t} I \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

Although damping is the simplest type of temporal regularization to add, different types of operators should also be considered. Minimizing change in the first or second spatial derivatives of the model are two alternatives. We can imagine computing the associated pseudo-inverse inverse of our augmented operator,  $G_c^{-g}$ , which allows us to write the model resolution matrix as,  $R = G_c^{-g} G_c$  where  $R$  can be seen as a filter

which shows how the imaging experiment and the choice of  $G_c^{-g}$  modify the true model. In this case  $R$  gives us insight into not only the spatial aspects of model resolution but also shows us temporal “smearing”, the process by which information is spread between our multiple experiments.

This inversion formalism also provides a good approach to incrementally acquired seismic surveys where the survey geometry at any particular time step  $n$  is relatively sparse. Although independent inversion of a single survey might yield an image with very low spatial resolution, by jointly inverting a series of surveys we can effectively add spatial aperture in exchange for losing temporal resolution.

### 3 Traveltime Tomography

Up to this point, our formulation has been relatively general with no assumption regarding the operation which  $G$  performs, the model parametrization represented by  $m$ , or the type of data stored as  $d$ . We will now apply our formulation to the concrete example of seismic traveltime tomography with one temporal dimension and two spatial dimensions. In this case we choose each  $m$  to be a rectangular mesh of homogeneous slowness cells while  $d$  is a vector of picked first-arrival traveltimes and  $G$  is the ray-path matrix. For the examples presented in this paper, we will use straight rays to preserve the linearity of the coupled inverse problem but extensions to the non-linear case seem quite feasible. We use a split laplacian operator ( $D_{xx}, D_{zz}$ ) to allow anisotropic regularization with two spatial parameters ( $\lambda_{sx}, \lambda_{sz}$ ) and two spatio-temporal parameters ( $\lambda_{tx}, \lambda_{tz}$ ) for the respective terms in equation 4. Regularization parameters are chosen by observation although use of the L-surface technique advocated by Brooks et.al. (1) would decrease the amount of manual tuning required in the inversion process. The resulting coupled systems were solved using the LSQR algorithm (7).

#### 3.1 A Synthetic CO<sub>2</sub> Monitoring Problem

Several field experiments to date have attempted to monitor the subsurface extent of active CO<sub>2</sub> injections including the McElroy (5) and Weyburn (2) EOR projects. For our synthetic experiment we have generated four time-lapse images of a CO<sub>2</sub> flood progressing through a permeable layer shown in row (A) of figure 1. Data was synthesized for a crosswell geometry with 40 sources and 40 receivers evenly spaced near the boundary of the model; the resulting arrays allowed tomographic imaging within a 40 x 98 m subdomain of the 115 x 49 m initial model. Gaussian noise ( $\sim 3\%$ ) was added to the traveltime picks for all four synthetic surveys.

The lower 3 rows of figure 1 show the inversion results for the uncoupled case (B) and the effect of including the spatio-temporal coupling term (C and D) using a 30 x 70 cell mesh. In all three cases the same spatial regularization parameters were used [ $\lambda_{sx} = 10, \lambda_{sz} = 8$ ] while the weak and strongly coupled examples used spatio-temporal coupling parameters of [ $\lambda_{tx} = 16, \lambda_{tz} = 12$ ] and [ $\lambda_{tx} = 64, \lambda_{tz} = 48$ ] respectively. While all three inversions successfully imaged the target CO<sub>2</sub> flood, visible at approximately 70 m depth, stronger spatio-temporal constraints appear to suppress velocity artifacts. Figure 2 shows the result of differencing the tomograms between the four sequential surveys with the same ordering of conditions. Similar trends in noise levels are visible in the difference images. In the uncoupled case (row B) the artifacts are of the same order of magnitude as the CO<sub>2</sub> flooded region making confident interpretation difficult. The use of strong spatio-temporal coupling clearly decreases this interpretation ambiguity at the expense of accurate difference velocity estimates and spatial resolution.

### 4 Conclusion

Temporal regularization provides a natural approach for jointly inverting time-lapse datasets. A key topic of future research is understanding the trade-offs between spatial and temporal resolution displayed within our synthetic example. While we have not discussed the non-linear tomography problem in this paper, the same techniques could be easily integrated within iterative imaging algorithms. Recent developments in the medical imaging community suggest several routes to improving these methods including the use

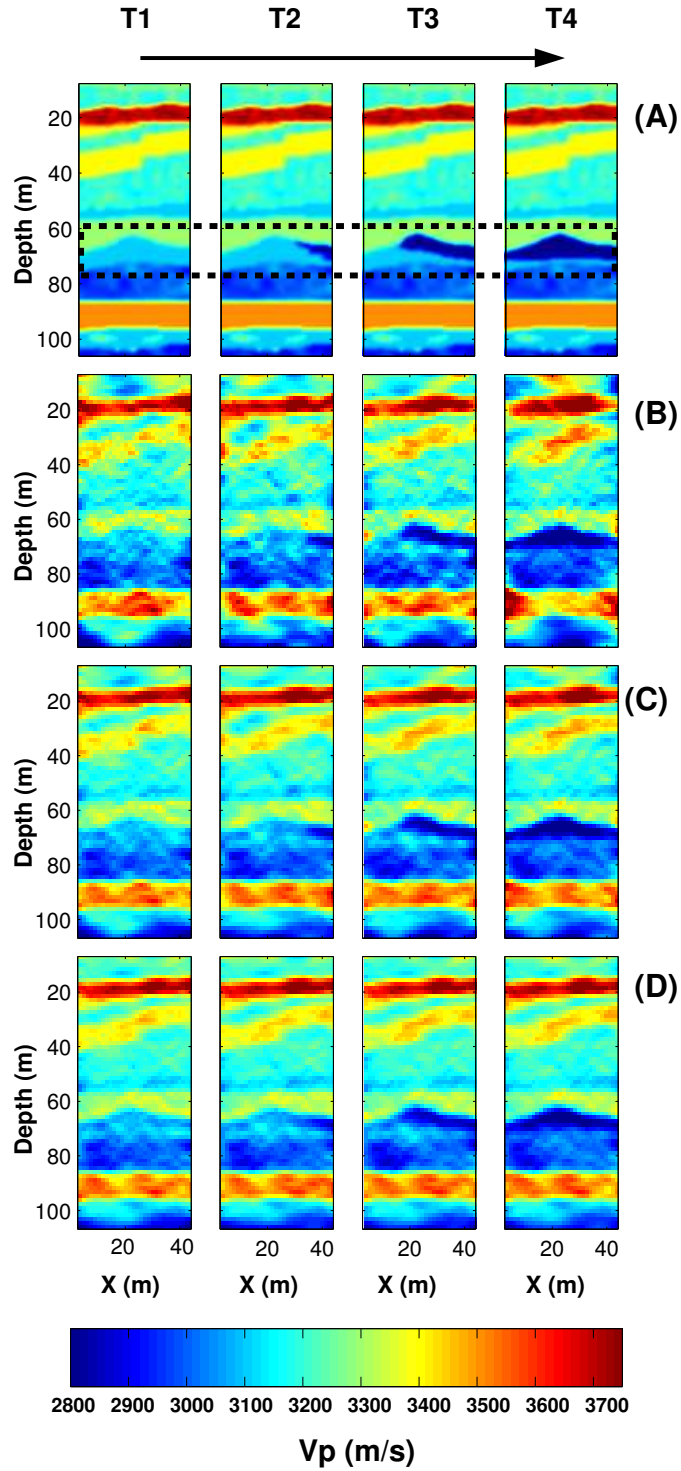


Figure 1: Tomography results with variable temporal coupling : (A) True model used to generate synthetics, (B) Uncoupled, (C) Weak temporal coupling, (D) Strong temporal coupling (regularization parameters in text)

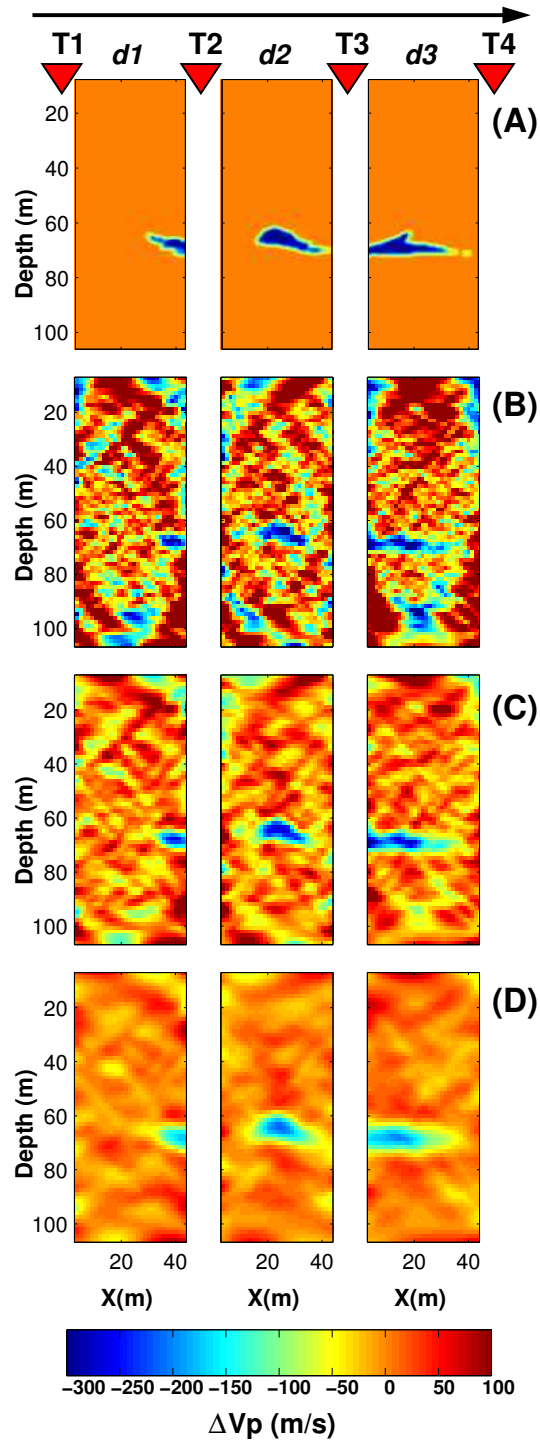


Figure 2: Difference tomograms with variable temporal coupling corresponding to the tomograms in figure 1 : (A) True model used, (B) Uncoupled tomograms, (C) Weak temporal coupling, (D) Strong temporal coupling

of L-surface techniques for regularization parameter selection (1) and the incorporation of prior temporal covariance models (9). Our future work will attempt to integrate and extend these techniques to the seismic monitoring problem within the context of full wave-equation tomography.

## References

- [1] BROOKS, D., AHMAD, G., MACLEOD, R., AND MARATOS, G. Inverse electrocardiography by simultaneous imposition of multiple constraints. *IEEE Transactions On Biomedical Engineering* 46, 1 (January 1999), 3–18.
- [2] DAVIS, T., TERREL, M., R. B., CARDONA, R., KENDALL, R., AND WINARSKY, R. Multicomponent seismic characterization and monitoring of the CO<sub>2</sub> flood at Weyburn Field, Saskatchewan. *Leading Edge* 22 (July 2003), 696–697.
- [3] DAY-LEWIS, F., HARRIS, J., AND GORELICK, S. Time-lapse inversion of crosswell radar data. *Geophysics* 67, 6 (November-December 2002), 1740–1752.
- [4] HUBBARD, S., CHEN, J., PETERSON, J., MAJER, E., WILLIAMS, K., SWIFT, D., MAILLOUX, B., AND RUBIN, Y. Hydrogeological characterization of the South Oyster bacterial transport site using geophysical data. *Water Resources Research* 37, 10 (October 2001), 2431–2456.
- [5] LAZARATOS, S., AND MARION, B. Crosswell seismic imaging of reservoir changes caused by CO<sub>2</sub> injection. *The Leading Edge* 16 (September 1997), 1300–1306.
- [6] LUMLEY, D. Time-lapse seismic reservoir monitoring. *Geophysics* 66, 1 (January-February 2001), 50–53.
- [7] PAIGE, C., AND SAUNDERS, M. An algorithm for sparse linear equations and sparse least squares. *ACM Transactions in Mathematical Software* 8 (1982), 43–71.
- [8] RICKETT, J., AND LUMLEY, D. Cross-equalization data processing for time-lapse seismic reservoir monitoring: A case study from the Gulf of Mexico. *Geophysics* 66, 4 (2001), 1015–1025.
- [9] ZHANG, Y., GHODRATI, A., AND BROOKS, D. An analytical comparison of three spatio-temporal regularization methods for dynamic linear inverse problems in a common statistical framework. *Inverse Problems* 21 (2005), 357–382.