FLIGHT TRANSPORTATION LABORATORY
REPORT R 88-2

COMPARISON OF OPTIMIZATION TECHNIQUES FOR ORIGIN-DESTINATION SEAT INVENTORY CONTROL

ELIZABETH L. WILLIAMSON

MAY 1988

ABSTRACT

Airlines have recently realized the importance of an effective seat inventory control system on revenues and profits. Yet, at the same time, there is a lack of practical optimization models for determining the number of seats to allocate to each origin-destination and fare class itinerary in an airline's network. In this thesis, several different mathematical models and optimization techniques for origin-destination seat inventory control are evaluated and compared.

Each technique is applied to a small network with assumed demand levels and fares for each O-D/fare class combination. The techniques are then compared with respect to the differences in seat allocations and booking limits, fare class nesting order and total potential system revenue. The "optimal" seat allocation solution is found by the probabilistic linear programming technique, but actual use of such a method is impractical due to the size of its formulation and its distinct inventory solution, which is not compatible with the nested reservations systems of most major airlines today. The technique that seems to have the most potential as an efficient origin-destination seat inventory control method is a network based deterministic linear programming technique, with seat allocations nested according to shadow prices.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation for Thesis

In the airline industry today it is common practice for a carrier to offer a wide range of fares for any given seat in the same cabin. On a nonstop flight from Boston to Miami it is possible to find a passenger traveling on a round-trip discounted ticket for $228, and at the same time, a passenger sitting in the very next seat paying a full coach fare of $548, round trip. Occurrences such as this have become commonplace since deregulation of the U.S. airline industry.

Prior to deregulation, fares were set by the Civil Aeronautics Board (CAB) for the industry as a whole. Fares were established according to industry average costs and based on a dollars per passenger mile structure. Carriers that operated at lower costs were not permitted to offer a lower fare which would be uneconomical to the rest of the industry. Besides determining the industry's prices, the CAB also governed each carrier's route structure, controlling which carriers could and could not serve a given market.

With deregulation in the U.S. in 1978, both pricing and market entry changed immensely. Carriers began offering seats that would otherwise be empty to low-fare passengers. Ticketing and travel restrictions were imposed on these low fares in order to limit diversion of those passengers willing to pay higher fares. These restrictions included such requirements as advance purchase, minimum stays, and round-trip travel. More recently, total and partial non-refundability restrictions have also been affixed to low-fare seats.

Through the practice of differential pricing, airlines have been able to increase total revenue. The marginal cost of carrying an additional passenger in an otherwise empty seat is very low. Therefore, seats can be offered at low fares in order to induce extra demand. As long as the lower fares are more than the marginal cost of carrying the extra passenger, these passengers will be contributing to the fixed costs of operating the flight and to profits. Not only do the airlines benefit, but high fare passengers may also benefit. With the extra revenue from the additional low fare passengers, airlines may actually be able to reduce the fares of the higher full fare passengers who would be travelling regardless of the cost.

Restrictions on the purchase and use of the low fare tickets limited the diversion of high fare passengers, but airlines were soon faced with another problem. The seats sold to low fare passengers were not necessarily seats which would otherwise be empty. Besides the restrictions on the low fare tickets, capacity controls, or limits, on the number of available seats were needed. It was important to determine the number of seats which would be empty on a flight. These seats could then be made available for low-fare passengers while leaving an adequate number of seats for full-fare passengers so they would not be displaced.

Once restrictions and regulations on market entry were removed through deregulation, regional carriers, formerly limited to certain routes by the CAB, expanded into high density markets, offering lower fares on multi-stop and connecting flights. At the same time, many new airlines began operating across the country. These airlines were not tied to labor union agreements, meaning their labor costs, which make up as much as 35% of an airline's operating costs, were much lower. In turn, these new low-cost carriers could afford to provide service at much lower prices.

In order for the major airlines to compete with the lower fares offered by the regional and new carriers, they had to offer a limited number of seats at these low fares. By offering seats at low fares, major airlines were able to advertise low prices and compete with low fare carriers. At the same time, by limiting the number of low fare seats available, these higher cost airlines would still be able to cover direct operating costs on a flight. This made the capacity control problem much more complex. Rather than simply determining the number of seats that would otherwise remain empty, seats had to be provided for a range of different low-fare passengers, as well as higher-fare passengers. In order to control capacities effectively, demands for each fare class had to be forecasted and seats allocated to each depending on the overall contribution to total revenue and the chances that seats would ultimately go unsold.

This concept of capacity control is also known as seat inventory control. Seat inventory control is the process of balancing the number of seats sold at each fare level so as to maximize total system revenues. The problem has become more complicated for several reasons:

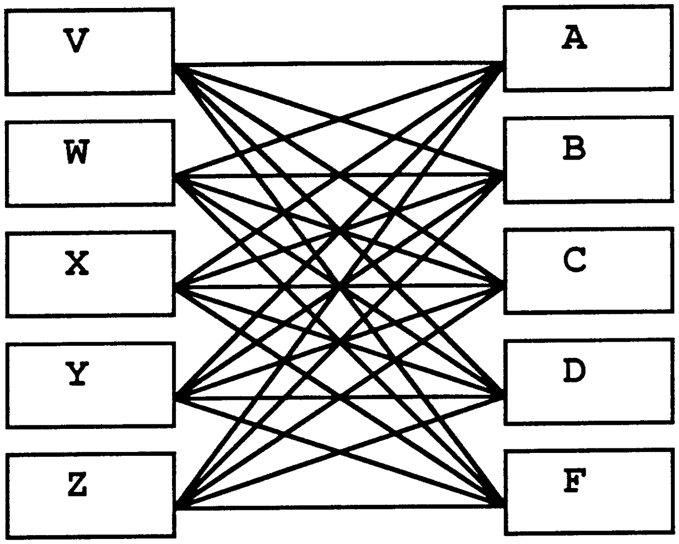
1. The number of different fares offered for any given origin-destination pair has increased dramatically, creating a multitude of fare class/O-D combinations. For each combination, the expected demand level needs to be forecasted, the

number of seats made available to each O-D/fare class needs to be determined, and the respective booking limits need to be calculated.

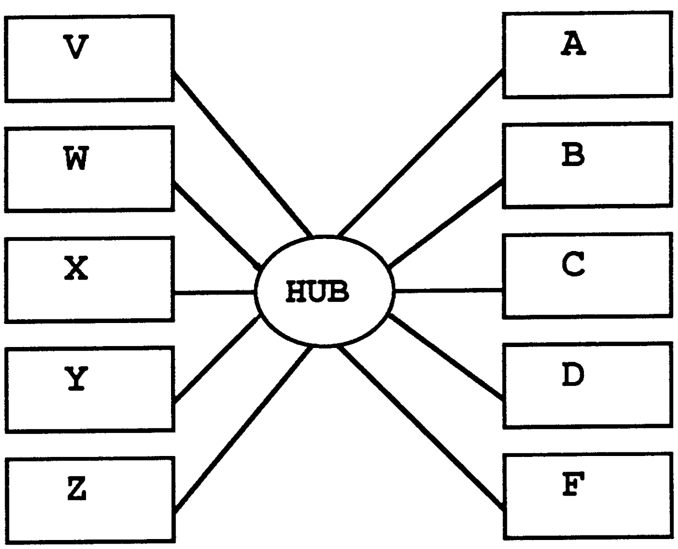
2. The number of connecting flight segments operated by a single airline has increased. This makes it possible for passengers to get to their destination over a wide variety of routes and combinations of routes. Not only is a carrier's network itself bigger, but due to the number of interacting flights and connection options which must be considered, the process of determining the number of seats to allocate to a particular fare class and O-D pair is very large and complicated.

3. The level of demand varies over time. Demand is a function of the season of the year, the day of the week, the time of the day, the number of ultimate destinations which can be reached by a particular flight, whether the flight is non-stop or one-stop, and the alternative flights offered by the same carrier or competitive carriers. With the number of variables that can affect demand, it can be difficult to derive good forecasts of expected demand levels needed to determine booking limits.

Complexity has increased even more with the development of hub-and-spoke networks. A hub-and-spoke system is an efficient way to provide service to many different markets while minimizing the resources needed to do so. In such a network, an airline picks a centrally located city to serve as a connecting hub for its flights. Then, instead of flying an aircraft between each individual city pair the airline serves, aircraft from cities around the country converge into and out of the hub airport as seen in Figure 1.1, which illustrates the basic concept of a point-to-point network versus a hub-and-spoke network. A hub-and-spoke system reduces the number of



POINT TO POINT



HUB AND SPOKE

Figure 1.1: Point-to-Point versus Hub-and-Spoke

aircraft needed to provide service between a set of cities since any given flight in the network can be a part of many origin-destination itineraries. The demand for a flight is no longer simply the number of passengers traveling from city A to city Z, but includes passengers going from A to city V, to city W, and so on, as well as to such cities as B and C.

A seat inventory control system designed to maximize total revenues for the airline thus involves more than simple decisions about which seats to sell to which fare classes. For a given flight into a hub, the number of alternatives for selling a specific seat is significantly greater that the comparatively small number of fare classes offered on the flight. Thus, in the process of allocating seats in a cabin, decisions as to which seats should be sold to which origin-destination pair as well as fare class can also be included. In order to make such decisions, total system revenue generated from selling a seat on a flight to a multi-leg passenger at a higher fare should be compared to the system revenue which could be generated by selling that same seat to a single-leg passenger, but at a lower fare. Selling the seat to the single-leg passenger may make it possible to sell the seat on the second leg of the multi-leg passenger's itinerary to another single-leg passenger. The combined revenue of the two single-leg, lower fare passengers may be greater than the revenue of the multi-leg passenger who is paying a higher fare than each of the single-leg passengers individually.

Most airline reservations systems currently maintain seat inventories and manage seat availability by fare class for each individual flight leg. If a seat is available in the low fare class, it can be reserved regardless of ultimate destination, overall itinerary and total revenue contribution. Because of the development of hub-and-spoke systems with many different itineraries on any given flight, there is a great amount of interest in the possibility of practicing inventory control in an origin-destination based environment. Methods are being explored to control seat inventories by passenger itinerary and/or origin-destination revenue. That is, on any given flight leg from some point P to some other point Q, it is possible to take passengers who are travelling to point Q or continuing on and connecting to a variety of other destinations, all with different revenue potential for the carrier. Airlines would like to be able to determine which of the many passenger itineraries should have seats allocated to them on the initial flight leg (P to Q) in order to maximize system-wide revenues.

1.2 Objective of Thesis

Airlines have recognized that there are advantages in using statistical tools and mathematical analysis in controlling seat inventories and, in turn, managing revenue. Most airlines have decision support tools which retrieve, summarize and analyze historical reservations and traffic data. The more advanced airlines in seat inventory control are in a transitional phase between a system dependent on human judgement and some form of automated booking limit system. Automated booking limit systems use historical reservations data and actual bookings to forecast demands. The demand estimates are then used as inputs, along with revenue information, in a seat allocation model, which determines optimal booking limits. The seat allocation model can include mathematical approaches and algorithms for setting and revising booking limits.

Most of the seat allocation models currently used by airlines are leg based approaches directed at maximizing flight leg revenues. Some models try to account for differences in passenger itinerary revenues by using "virtual" inventory classes, although the model itself is leg based. A virtual inventory class is a seat inventory allocation category which can be used to control the number of seats available for sale within a given fare range. Although the virtual inventories are based on total passenger itinerary ticket revenue, seat allocations for the virtual classes are determined on the basis of maximizing revenues on individual flight legs. The interaction of different flight legs in a network system is not taken into account.

Maximizing flight leg revenues is not necessarily the same as maximizing total system revenues. An origin-destination optimization approach is needed to overcome such problems. True O-D approaches are network formulations which determine optimal seat allocations based on distinct fare class and itinerary combinations, yet such formulations can be quite complex. At the same time, a solution from such an O-D model is not directly compatible with the nested leg-based structure of most airline reservations systems today.

Critical questions for airlines considering the development of origin-destination seat inventory control systems are: How effective are different approaches to controlling seat inventories and maximizing revenues? Is the potential revenue of an origin-destination approach greater than that of leg-based approaches? If so, is the difference in potential revenue significant enough to offset the ease and simplicity of using a leg-based approach? How do the different methods currently being considered for use vary? Is it possible to effectively use origin-destination methods and their solutions in existing airline reservations systems?

The objective of this thesis is to compare several different mathematical models and optimization techniques relevent to airline seat inventory control. The evaluation is based on a small hub-based network of connecting flights. Six different techniques, which include both leg based and origin-destination approaches, are applied to the network. O-D demand levels and fares for a four coach fare class structure are assumed and the alternatives are compared with respect to the differences in seat allocations and booking limits, fare class nesting order and total system revenues. Through this comparison, the issues mentioned above are addressed and discussed.

The intent here is to present a comparison of inventory management systems. A number of factors such as overbooking, probabilities of passenger upgrade, diversion, and loss of denied requests are ignored. Such factors must be considered in a seat inventory management system, but by not including them in this analysis it will be possible to identify more clearly the differences between the methods themselves. Once a basic seat inventory control system is developed, these factors can be dealt with and incorporated into the system.

1.3 Structure of Thesis

The remainder of this thesis is divided into five chapters. Chapter Two serves as a formal introduction to the seat inventory control problem. The characteristics of airline operations and reservations systems which contribute to the size, complexity, and definition of the problem are discussed. The second section of the chapter summarizes current methods and practices used by major domestic airlines for controlling seat inventories.

Chapter Three is a brief overview of past research. Mathematical approaches that have been considered for use on the seat inventory control problem are presented. Methods for solving simple representations of the problem, as well as operations research models which determine "optimal" seat allocations are introduced and reviewed.

The techniques being evaluated as origin-destination seat inventory control alternatives are decribed in Chapter Four. Six techniques are presented, including leg-based and origin-destination network formulations, as well as deterministic and probabilistic optimization algorithms. The concepts behind each technique are described in detail.

Chapter Five presents the analysis of the six alternatives. Each alternative is applied to a small network model. The actual steps involved in the application of each technique are explained and the results from these applications are provided. Comparisons of the different alternatives, in terms of seat allocations, nesting order and total system revenues, are discussed.

Chapter Six concludes this thesis by presenting an overview of findings and contributions from this research analysis. Finally, further research and work stemming from the results of this thesis are outlined. In particular, direction for additional analysis with respect to a nested network seat inventory control system, which is presented in this thesis, is outlined.

Chapter 2

Seat Inventory Control

2.1 Definition of the Problem

Airline seat inventory control is the practice of limiting the number of seats made available to different fare classes that share a common cabin on an aircraft. The objective of seat inventory control is to balance the number of passengers in each fare class in order to maximize total flight revenues. By offering more seats at discounted fares, an airline can capture extra passengers who otherwise would not travel, in turn providing additional revenue. Too many seats offered at lower fares will cause a diversion of high-fare passengers to the available low fares and may also displace some high-fare passengers altogether, therefore lowering total revenues.

Airlines use differential pricing to increase total revenues, as well as to be competitive. By offering a limited number of low-fare tickets, an airline can appear to be competitive with other carriers that offer deeply discounted fares. At the same time, it may also be able to fill otherwise empty seats by stimulating demand. Given the differential pricing strategy used by most airlines, the seat inventory problem is to determine the optimal booking limits, the maximum number of reservations that should be accepted, for each fare class for a future scheduled flight departure that will maximize the airline's total revenue.

The need for seat inventory control stems from a basic economic problem: supply does not equal demand. In air transportation, supply and demand seldom match exactly. In the first place, demand for future flights is probabilistic and cannot be forecasted precisely. But the problem is due, to a greater extent, to the actual scheduling of the aircraft. Because of route structure constraints, constraints on the number and size of aircraft, scheduling constraints and the lack of balance in passenger demands over a network, it is not always possible to have the aircraft size equal demand. Therefore, when there are either more seats on an aircraft than demand or more demand than the given number of seats, it is the control over the number of discounted seats made available which can allow the airline to achieve a closer match between supply and demand.

The seat inventory control problem is not simply one of allocating seats to four, five, six, seven or even ten fare classes on a single flight leg. Today, more often than not, a single flight involves passengers with many different origin-destination itineraries, each of which have different revenue contributions. Therefore, seat inventory control decisions are not just the number of seats to allocate to each fare class, but decisions may need to be made as to whether a seat should be sold to a higher-yield fare class on a single leg itiernary, or to a lower-yield fare class, but at a higher total revenue, on a multi-leg or connecting itinerary.

The complexity of the problem has grown tremendously with the development of large hub-and-spoke operations. On a given flight departure into a major hub, there can be passengers heading towards as many as 40 possible destinations. With major U.S. airlines currently offering seven coach cabin fare classes, that makes over 280 possible fare class/destination combinations on a single flight leg, each having a different level of attractiveness, in terms of revenue, for the airline. As carriers continue to expand the number of fare classes offered on a flight and hub operations continue to grow, offering increasing numbers of connecting possibilities, they can benefit more and more from an effective seat inventory control system.

The seat inventory problem can be approached from a variety of perspectives. Seat inventories can be controlled over individual flight legs, over the entire network or over separate sub-sets of the network. Most airlines currently manage seat inventories by flight leg. It is by far the simplest method to use and can be implemented into the airline reservations system without major revisions to current practices.

Using a leg-based seat inventory control method, efforts are made to maximize revenue on each flight leg. This does not necessarily mean total system revenues are maximized. For example, consider a simple linear two flight leg network from Boston to Atlanta and from Atlanta to Miami with low priced Q-class fares of $69, $89, and $59 for BOSATL, BOSMIA, and ATLMIA, respectively. With a leg-based inventory system, passengers on either a BOSATL or BOSMIA itinerary can reserve a Q seat on the BOS-ATL flight leg if one is available. This makes it possible to book all Q seats to the BOSATL passengers while denying higher revenue BOSMIA passengers. If demand for local travel from Atlanta to Miami is low, seats could go unsold on the ATL-MIA flight leg, and a reduced total revenue for the two flight legs combined could be obtained.

In the above example the exact opposite can also happen. The Q seats on the BOS-ATL flight leg can all be booked to the higher itinerary revenue passengers going from Boston to Miami. If short-haul demand is high for both the BOS-ATL flight and the ATL-MIA flight, total revenues can be increased by selling the Q seats on the BOS-ATL flight leg to BOSATL local passengers. With high demand on the ATL-MIA flight for local travel, the total revenue for a Q seat will be the sum of $69 and $59, or $128, from Boston to Atlanta and Atlanta to Miami, versus $89 which would be received from a BOSMIA through passenger.

In the first example, by protecting some Q-class seats on the first flight leg, BOS-ATL, for the longer haul Miami passengers, total revenues could have been increased. But when short-haul demands are high, by limiting seats to multi-leg passengers, higher revenues can be obtained. In order to maximize revenue over an entire route network, a seat inventory control system must be based on origin-destination (O-D) itineraries rather than flight leg. In an O-D method, seats are allocated to the fare class/passenger itinerary combinations which generate the greatest revenue and maximize total system revenues. Solutions from such an approach involve making decisions about which fare class/O-D combinations are the most desirable. For a network in which there are a large number of connections and flights with a multitude of passenger options, an O-D method can become very complex.

Mathematical algorithms for O-D seat inventory control are usually based on network formulations. For each O-D/fare class combination, the expected revenue of each additional seat sold must be determined. Seats are then allocated according the expected revenue of a single O-D/fare class or combination of O-D/fare classes. The solution from such network formulations is based on distinct, separate inventories. Once the number of seats which should be allocated to each fare class/passenger itinerary is determined, an important question is how to use these results in the leg-based, nested fare class structures of current airline reservations systems without totally reconfiguring them.

An additional problem in seat inventory control is that air transportation demand is probabilistic. Demand for a future flight has both cyclical and stochastic variations. Both may be forecasted, but stochastic variations are less predictable. There will always be some uncertainty as to the number of requests for a future flight and fare class. An optimal seat inventory control model needs to take into account the uncertainty associated with stochastic variations by incorporating the variances of estimated demand, along with the revenue values and expected levels of demand for each fare class. A decision model that fails to consider the probabilistic nature of demand will overestimate expected revenues and may not allocate seats optimally. As the actual variations in demand increase, the greater the differences will be between the expected revenues and recommended seat allocation levels of a deterministic decision model and those of the optimal solution.

Demand for a future flight is also dynamic. From one day to the next the total number of requests changes continuously due to new demand as well as cancellations. As time passes and the departure day of the flight approaches, the number of bookings changes and the estimates of demand for each fare class and passenger itinerary also change. These changes can affect the optimal allocation of remaining seats for the flight. Therefore, it is important to be able to monitor the flight and make adjustments in seat allocations and booking limits when needed. In order to do this, the mathematical algorithms used in a seat inventory control system should not be overly complicated and take too much time in running in order to make frequent revisions possible.

Another complication involved in optimally solving the seat inventory control problem is the nested fare class reservations systems which many airlines use. Nested fare class inventories are structured so that as long as there is a seat available on the plane, a high-fare class request will not be denied. Each discounted, low-fare class inventory is nested within the next higher fare class. For example, take a four fare class structure—Y, M, B and Q—with Y being the highest fare class and Q being the lowest. The Q-class seat inventory is nested within B-class, and in turn B-class is nested within M, and M within Y. If there were 25 seats allocated to Y-class, 30 to M, 25 to B and 20 to Q-class, there would actually be a maximum of 100 seats available for Y-class requests, while a maximum of 75 seats and 45 seats would be available to M and B classes, respectively. Q-class availability will remain at 20 seats.

In a nested system such as this, if there is a demand for 50 seats in the highest fare class, Y-class, the passengers would be allowed to book on the particular flight as long as there were 50 seats still available. The passengers would not be turned away because only 25 seats were actually allocated to Y-class. On the other hand, in a distinct fare class system, only 25 seats could be sold in Y-class regardless of the extra demand at the high-fare level. Once a seat has been allocated to a distinct fare class inventory, it can be booked only in that fare class or remain unsold. That is why airlines prefer a nested system. If there are requests for the highest fare, and seats are available, these requests will be accepted and not turned down becaues of expected lower-fare demand. The problem with a nested system is that most traditional mathematical optimization techniques determine solutions for distinct classes. Such distinct inventory class solutions may not be the optimal seat allocations for a nested system.

Besides the problem with a nested system, the size and complexity of a method which finds an "optimal" network solution makes it unrealistic for use in current airline seat inventory control and reservations processes. However, simpler leg-based methods, which allocate seats by fare class alone and maximize revenue on individual flight legs, do not take into account the interaction between flight legs across an entire network. A seat inventory control approach is needed which is somewhere between the two extremes. It needs to be aggregated enough to make the problem manageable since making seat allocation decisions for each individual seat on every flight across an entire network is impractical. At the same time, the approach needs to be disaggregated enough to allow control of passenger itineraries over the route network and not just flight leg by flight leg.

Airlines have begun to see the importance of revenue management, which involves pricing as well as seat inventory control. Since prices are almost entirely dependent on the pricing strategy of an airline's competitors, emphasis is put on the seat inventory control component of revenue management with the hope of increasing total expected revenues. There is a great interest through-out the airline industry in seat inventory control methods. Current practices vary in sophistication and the area of seat inventory control is evolving fast, but overall there is still a strong emphasis on human judgement in determining seat allocations.

2.2 Current Practices

Initially, airline practices in seat inventory control were based exclusively on human judgement rather than systematic analysis. Over the past several years seat inventory control has entered into a transitional stage throughout the industry, with some airlines further ahead than others. Carriers are moving away from controlling seat inventories on the basis of human expertise alone and moving towards automated systems which use mathematical techniques and algorithms, as well as data management systems. Analysts who are responsible for controlling seat inventories have been able to achieve increased control over revenues. However, as networks become larger and more complicated due to hub-and-spoke operations, and as competition continues to dictate fare levels, making the margin between bottom-line revenues and operating expenses minimal, methods are needed which are more rigourous, consistent and comprehensive.

The simplest approach to seat inventory control is a one-time setting of booking limits based on booking histories. More complicated approaches use historical data, competitors' actions and current trends to set initial booking limits and to make adjustments. As actual bookings are made, past data and future forecasts are used to adjust the booking limits. Many airlines have the beginnings of such a computer-based automated system. Yet, for the most part the capabilities of these systems extend as far as setting initial booking limits, while making adjustments to the initial limits as reservations are accepted is done by humans.

Airlines today employ seat inventory control analysts who are responsible for monitoring and adjusting booking limits throughout the reservations process [1]. The number of analysts and the extent of their responsibilities vary from airline to airline, and the number of analysts is not necessarily proportional to the size of the airline itself. It also is not proportional to the number of flights per day. However, the higher the number of analysts, proportionally, the more effective the seat inventory control system seems to be.

Airlines with relatively few analysts use an *ad hoc* process of seat inventory control. Only certain flights and markets are selected for detailed review. These are usually markets which are highly competitive or are specific flights which operate during peak demand periods. Airlines with more analysts use a more systematic process. Teams of analysts are assigned to groups of markets and flight legs for which they are made responsible. Carriers with the most analysts, proportionally, allot to each analyst all the flights that serve a particular market or a set of routes. These analysts are totally responsible for the mix of passengers and the revenue achieved on their flights.

Most carriers use some type of decision support system in the form of statistical data management and analysis functions for seat inventory control. A data management system collects and stores historical data from reservations systems and can then estimates demand based on historical patterns and forecasting models. These systems can provide data in a form which can help an analyst to respond to changes in booking patterns as departure time approaches.

Airlines owning larger computer systems have developed, or are in the process of developing, their own decision support and data management systems. Such systems are tied into and used along with their reservations systems. There are also a small number of computer packages which have been made available by software companies particularly for seat inventory control. Most of these software packages are strictly data management packages. Carriers with limited computer facilities are interested in such data management capabilities and are either investing in an existing package or developing their own version for the purpose of decision support in seat inventory control.

There are a few new developments in software packages which actually allocate seats and determine booking limits. However, seat allocation solutions from such packages are not always easy to implement and use in a current airline reservations system. Also, the algorithms used in these systems are not necessarily "optimal". It may be easier for an airline to develop its own automated system which can be structured to fit into its existing system.

The seat inventory control systems of most airlines limit fare class bookings by flight leg. As mentioned before, maximizing revenue on individual flight legs is not the same as maximizing total revenues on an entire system. Some carriers have advanced as far as being able to limit local passenger sales in favor of through and connecting passengers with higher total revenue. For example, American Airlines' seat inventory control system is based on itinerary revenues as well as fare classes. However, no airline controls seat inventories on the basis of a passenger's origin and destination over their entire network system.

Practices of setting initial fare class limits, monitoring actual reservations and making adjustments to booking limits vary throughout the industry. The first step in seat inventory control is setting initial booking limits. The simplest approach involves setting initial booking limits with default values across every flight. While such a system does not take much effort, developing a more complex system which differentiates initial booking limits according to markets and flights can reduce the amount of intervention required later on in the reservations process. A more complex method in setting initial booking limits is to set lower fare class limits based on market, day of the week and time of the day. Such a method requires substantial effort but is better than not differentiating initial booking limits at all. Most airlines use a combination of the two methods, depending on the degree of competition and the level of demand for a given flight.

Rather than improving initial booking limit accuracy, greater benefits can be obtained from developing reservations monitoring systems and adjusting booking limits. A more sophisticated monitoring system can offset weaknesses in a simple initial booking limit method. Major carriers have realized this and have advanced beyond merely setting initial booking limits. Nonetheless, the different approaches used in monitoring actual bookings relative to the pre-set initial limits vary in sophistication.

The simplest approach used in the monitoring step of seat inventory control is merely listing flights for which reservations approach the booking limit for any given fare class. All major carriers have monitoring systems which can at least do this. Some carriers have systems that are a little more advanced where flights are selected and listed on the basis of several parameters. Computer routines in these systems will flag flights in which actual reservations meet or approach a number of different booking limit criteria.

Once reservations monitoring systems have flagged a flight, booking limit adjustment decisions must be made. This is the most important aspect of seat inventory control but is the least sophisticated. Decisions must be made whether to increase the number of seats allocated to a fare class and make it available for additional bookings or to leave the booking limit as is and allow the fare class to close down. In the past these decisions, for the most part, have been made by analysts on the basis of experience and judgement. Airlines have lacked practical models to calculate optimal booking limits.

If optimization algorithms and models are developed and used, they need to consider the probabilistic nature of demand. They also need to make use of forecasts based on historical data. Still, human judgement cannot be eliminated totally.

Optimization algorithms cannot take into account such things as occurrences of unexpected events and changes in competition and competitive strategies. Models can only be used to help analysts and to make seat inventory control more systematic.

Few airlines are well advanced in the redesign of their reservations system and seat inventory control system. Changes in reservations systems are hindered by the need to remain consistent with the rest of the industry. An airlines' reservations system must be in keeping with the standards of the industry because of carriers dependence on other airlines and travel agents to make bookings. For example, Delta Air Lines accepts 150,000 to 180,000 reservations a day, but 70% of these bookings come from other airlines' reservations systems.

Existing reservations systems do not differentiate between passengers on the same flight leg and within the same fare class, and there are many different origin-destination itineraries on most flights today. An ultimate goal for many airlines is to develop an O-D based reservations system. Still, an upgrade such as this must remain compatible with other reservations systems. The current objective for most airlines is to continue making improvements to decision support tools in order to make more useful infomation available to analysts who make booking limit decisions. Some airlines have advanced to the stage of researching and implementing automated systems to make the booking limit revision process less *ad hoc*.

Overall, there is a wide range of approaches used by airlines to control seat inventories. There are still many improvements which could be made in the seat inventory control system, but the industry as a whole is progressing in the sophistication of its methods and tools. With the increased importance of revenue management to airline profitability, emphasis is being placed on better and more effective seat inventory control systems.

Chapter 3

Overview of Past Research

The main reason behind the low sophistication of seat inventory control systems is the fairly recent realization by airlines of its importance to revenues and profits. At the same time, there is also a lack of practical optimization models for determining the number of seats to allocate to each origin-destination and fare class itinerary in a network. There has been a substantial amount of theoretical research done in the field, but such research has been devoted, for the most part, to large-scale optimization techniques which solve a simplified version of the seat inventory control problem.

The following is an overview of mathematical concepts and models relating to the seat inventory control problem. The discussion is based on past work which started in the early 1970's. The models reviewed range from simple two-class, single flight leg seat inventory control methods to multi-class multi-leg network optimization methods.

An approach based on equating marginal seat revenues was used by Littlewood [2] in 1972. Taking into account the probabilistic nature of demand, Littlewood developed a method to control low yield fares, in a two fare pricing structue, based on the objective of maximizing revenues by flight leg. He suggests that low fare passengers, paying a mean revenue of r, should be accepted on a flight as long as:

$$r \geq P \cdot R \tag{3.1}$$

where R is the higher yield revenue and P the maximum risk that acceptance of a low fare passenger will result in the subsequent rejection of a high yield passenger. In other words, total flight revenue will be maximized by accepting low yield passengers up to the point where the probability of selling all remaining seats to high yield passengers is equal to the ratio of the mean revenues of low yield and high yield passengers, r/R.

Bhatia and Parekh [3] of TWA and Richter [4] of Lufthansa expanded on Littlewood's model in 1973 and 1982, respectively. In each case, the formulas derived where in essence equivalent to Littlewood's. Through a rather lengthy differentiation and transformation process, Bahatia and Parekh were able to derive the formula:

$$\frac{F_1}{F_2} = \int_{C-T}^{\infty} f_2(y)dy \tag{3.2}$$

where F_1 and F_2 are the average low and high fare revenues, respectively, C is the aircraft capacity, $f_2(y)$ is the high-fare demand distribution and T is the optimal allotment for low-fare passengers.

Richter, on the other hand, approaches the problem by looking at changes in the expected total revenue of the flight as additional seats are offered to low-fare passengers. He derives an equality for what he calls differential revenue, defined as the additional low-fare (LF) revenue obtained from offering an extra low-fare seat minus the high-fare (HF) revenue lost:

$$DR = \text{(additional LF revenue)} - \text{(HF revenue lost)} \quad (3.3)$$

$$= ARP_L \cdot \text{Prob[1 additional LF seat]} - ARP_H \cdot$$

$$\text{Prob[1 additional LF seat displacing 1 HF passenger]} \quad (3.4)$$

where ARP_L and ARP_H are Average Revenue per Passenger, low-fare and high-fare, respectively. By equating DR to zero, Richter's formula for the optimal low-fare seat allotment, A_{LO}, becomes:

$$A_{LO} = C - H(ARP_L/ARP_H) \quad (3.5)$$

where $H(x)$ is the high-fare demand value which is exceeded with risk probability of x. This is equivalent to formulas 3.1 and 3.2. Through Richter's formulation, it can easily be seen that the low-fare seat allotment, A_{LO}, is a function of the fare ratio, capacity, and high-fare demand distribution, but is not influenced by the low-fare demand distribution. However, the low-fare demand distribution does have an influence on the total expected revenue of a flight.

In 1982, Buhr [5] of Lufthansa extended the seat allocation problem to a two-leg flight, where decisions as to whether seats should be allocated to local versus through passengers come into play. Buhr considers only one fare class on a linear A to B to C flight. He defines expected residual revenue, E, as being the probability of getting additional passengers, P, multiplied by the corresponding revenue per passenger, R:

$$E_{AC} = P_{AC}(x) \cdot R_{AC} \quad (3.6)$$

Local and through passenger demand is assumed to be independent, and the number of seats allocated to segment AB is equal to that of segment BC. Under these assumptions, Buhr states that total revenue is maximized when:

$$|\, E_{AC}(x) - [E_{AB}(y) + E_{BC}(y)]\,| \rightarrow \text{minimum} \tag{3.7}$$

subject to the capacity constraint. An iterative solution method is used to find the optimal seat allotments. These seat allotments are based on distinct buckets.

For multiple class situations, Buhr suggests that strict O-D itinerary booking limits be determined first, based on average O-D revenues. Once these limits are known they can be divided and allocated among different fare classes offered for an O-D itinerary. In the process of determining low fare seat allocations versus high fare allocations for an O-D itinerary, the average revenue used in determining the itinerary seat allotments will change, affecting expected revenue levels. Buhr recognizes this problem but does not address it in his paper.

Wang [6] of Cathay Pacific Airways addressed the problem of optimizing seat allocations for multi-leg flights with multiple fare types in 1983. He develops a model based on expected marginal revenue which can handle up to six fare types for each O-D pair on flights of up to four legs. The model determines the O-D combination which gives the highest expected revenue and allocates a seat to that combination. It then computes the expected revenue for the next seat of each O-D/fare combination and allocates the seat to the highest. This process continues until all seats have been allocated.

The expected revenue for each combination is computed by multiplying the marginal probability of obtaining a given passenger by the fare of the respective O-D/fare combination. That is:

$$E(R_i) = \sum \Pr(x_{jk} \geq s_{jk}) \cdot Y_{jk} \tag{3.8}$$

where Y_{jk} is the O-D/fare combination revenue. The marginal probability,$\Pr(x_{jk} \geq s_{jk})$, is actually the probability of receiving more than s_{jk} requests.

In his model, Wang assumes independence of market and fare class demands and seat inventories. His approach is to rank O-D/fare combinations by expected revenues and allocate seats one by one. This is feasible for six fare classes and four flight legs, giving as many as ten O-D itineraries and 60 O-D/fare class combinations, but for a typical multi-leg multi-class seat inventory problem faced by major airlines today, where a large number or flights are being fed into and out of a hub, each of which can have as many as 35 or more different O-D passenger itineraries aboard, this model is not very efficient. Network optimization and mathematical programming techniques which find optimal seat allocation solutions, also based on distinct, non-nested inventories, are more pratical.

In 1982 Glover, Glover, Lorenzo and McMillan [7] worked on the O-D/fare class seat inventory control problem using a network flow formulation. Their method was designed to find the "optimum" passenger mix, that is the number of passengers for each fare class/O-D itinerary on each flight segment that would optimize revenue. They formulated the problem as a minimum cost/maximum profit network flow problem with special side constraints. Forward arcs represented the aircraft capacity on a flight leg, while backward arcs represented the number of passengers for each O-D/fare class itinerary.

An actual system was built using this network flow model for Frontier Airlines. The network contained 600 flights, 30,000 passenger itineraries, and 5 fare classes. Running times were brief compared to a linear programming formulation of the problem which would take several hours. The main disadvantage to this method is that it is based on demand estimates which are entirely deterministic. It is also a non-nested system which is not compatiable with nested airline reservations systems.

Wollmer [8] of McDonnell Douglas Corporation developed a mathematical programming technique which takes into account probabilistic demands for the multi-leg multi-class problem. In his formulation, Wollmer uses an x_{jk} variable to represent each O-D/fare class combination j and seat k on a flight leg. Associated with each x_{jk} is a value $m_j(k)$ which is the expected marginal revenue of the kth booking request for the O-D/fare class combination j. The objective in his model is to maximize total network revenue by choosing the right combination of x_{jk} values to set to one, and all others to zero, such that the sum of all x_{jk} values associated with individual flight leg is equal to the aircraft capacity of the leg. Total network revenue is given as the sum of all $[x_{jk} \cdot m_j(k)]$ products.

In maximizing revenue, Wollmer's algorithm computes the total network expected revenue without a given O-D/fare class reservation and then computes this revenue quantity for the reduced seating capacity that results from accepting the reservation. By comparing the difference of these two expected revenue values with the actual revenue of the O-D/fare class itinerary under consideration, the model determines whether the O-D/fare class should be closed to further bookings. The drawback of this model, like the other models, is that the optimization solution is based on distinct, non-nested O-D/fare class inventories. Another drawback is the size of the formulation itself. An x_{jk} variable is needed for every seat available to every O-D and fare class combination for every flight in a given network.

In 1983, Boeing did a study of seat inventory management at the itinerary level [9]. Boeing's method uses a non-linear integer program to find optimal seat assignments for O-D itineraries based on fares and demand distributions. These seat assignments are then aggregated into buckets by flight leg to conform to the current structure of reservations systems in the airline industry. For each leg, the different O-D itineraries which use the leg are partitioned into buckets based on fare value. The sum of the seat allocations for each O-D itinerary in a bucket is the total seat allocation for the nested bucket. Any number of buckets may be defined, and buckets may have different fare ranges for each leg. The drawback of this method, as Boeing itself admits, is the fact that although a sophisticated assignment process is used, seats are then clumped together and nested in a somewhat *ad hoc* manner.

The optimal solutions found by these mathematical programming and network formulations are based on non-nested systems. Such solutions are not necessarily optimal for a nested system. Optimization models, which produce seat allocations for distinct inventories, may be re-run frequently to take into account a nested type of system. However, an additional problem with multi-class multi-leg optimization models is the size of their formulations, especially for models which incorporate probabilistic demand behavior. Therefore, simple nested leg-based models may be more pratical than huge probabilistic network formulations which generate non-nested solutions. The best solution, though, may be some type of compromise approach.

None of the models discussed here include overbooking, up-grade potential or correlation in demand. Such considerations can have an effect on the optimality of determined seat allocations. A realistic seat inventory control system should consider these factors. It also needs to conform to the practical constraints of the seat inventory control problem, such as reservations system capabilities, data availablities, and airline competition.

Chapter 4

Techniques Evaluated For Seat Inventory Control

This chapter presents six alternative techniques for origin-destination seat inventory control. Seat inventory control is the allocation of seats among different passenger itineraries and/or fare classes in order to maximize the expected revenues of future scheduled flights. The rationale behind seat inventory control is not to limit seats for low-fare passengers, but rather to protect seats for higher-fare passengers. Effective seat inventory control results in additional revenue being obtained by selling otherwise empty seats to low-fare passengers. At the same time, revenue is not lost from displacing higher-fare passengers if enough seats have been protected for these passengers.

The six different approaches which are discussed in this chapter and later evaluated and compared are:

1. Leg Based Expected Marginal Seat Revenue (EMSR)
2. Prorated EMSR
3. Virtual Nesting EMSR
4. Deterministic Linear Program (LP)

5. Probabilistic Linear Program
6. Deterministic LP Nested on Shadow Prices.

Each of these is an alternate methodology for a seat inventory control system. The approaches range from simple leg-based models to more complex origin-destination techniques. They also include both probabilistic and deterministic optimization algorithms.

Leg-based methods allocate seats on a flight leg by fare class in an effort to maximize revenue on each individual flight leg, independent of other flight legs. On the other hand, origin-destination methods take into account the revenue contribution of different passenger itineraries. Seats are allocated not only on the basis of fare class, but also on the basis of different passenger itineraries on the flight leg. In a complete O-D optimization approach, revenue is maximized over the entire system, not by individual flight legs. An origin-destination approach takes into account the interaction of the many flight legs in a network.

Both probabilistic and deterministic methods are discussed and considered in this evaluation. The probabilistic techniques take into account the uncertainty in air traffic demand, which is necessary to model the the seat inventory control problem accurately. However, with the introduction of origin-destination approaches, the size and complexity of the problem grows almost exponentially. Therefore, a deterministic representation makes the problem much more manageable. Although the probabilistic nature of demand is not included, the interaction between flight legs is considered and a system-wide revenue maximization solution can be obtained.

In this chapter, the six techniques to be evaluated are described in detail. Results from the application of each technique to a small hypothetical hub connecting bank are presented in Chapter 5. The methods, with their various attributes of leg-based versus O-D based and probabilistic versus deterministic, are evaluated to give a comparison of the revenue potential and booking limits obtained from each. The different techniques have been chosen for this analysis to determine which are "better", given the complexity of each method and its compatibility with current reservations systems.

4.1 EMSR Model

The three EMSR methods are based on the Expected Marginal Seat Revenue model developed by Belobaba [10] in conjunction with Western Airlines. It is a probabilistic revenue optimization model which can be used to set and revise fare class booking limits for a future flight leg departure. It is a leg-based approach to seat inventory control which maximizes expected revenue by flight leg for a nested fare class reservations system. The EMSR model determines the number of seats which should be authorized for sale in each fare class by using historical demand data, average fares, and current bookings.

4.1.1 Probabilistic Demand and Expected Marginal Revenue

As the name suggests, the EMSR model is based on expected marginal seat revenues. Seats for a given fare class are protected over lower fare classes by equating the expected marginal revenue of protecting an additional seat in the higher fare class with the expected marginal revenue of not protecting the seat and selling it in the lower fare class. The expected marginal revenue of allocating, or protecting, a seat to a fare class inventory is simply the probability of being able to fill the seat

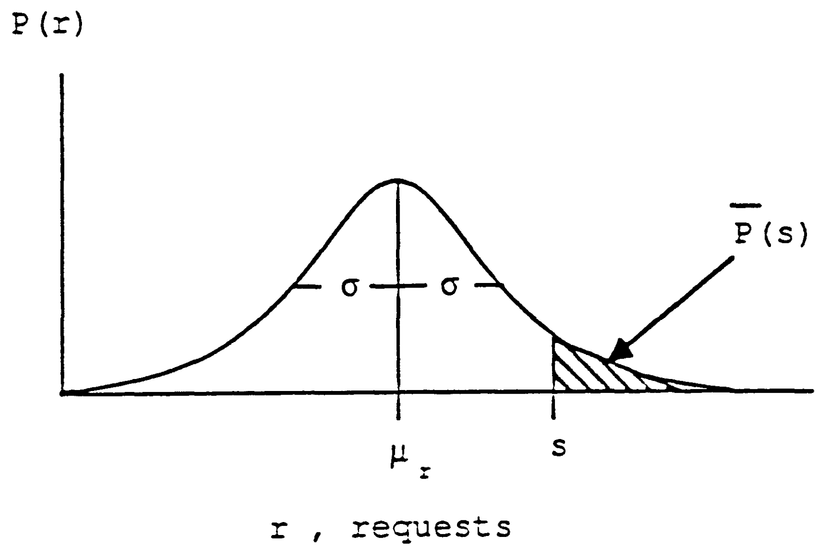


Figure 4.1: Normal Probability Density Function of Demand

multiplied by the average revenue which would be obtained by selling the seat. The average revenue of selling the seat is the same as the average fare of the respective fare class.

As mentioned before, total demand for a particular flight, and for a fare class on that flight, is probabilistic. From past analysis it has been found that this demand can be assumed to have a normal distribution [11]. Therefore, the probability density function, $p(r)$, for tne total number of requests, r, rece ved by an airline, and the demand for a given fare class, is a normal curve as shown in Figure 4.1. Given that demand is normally distributed, the probability function of demand for a fare class can be derived. From a sample of historical data of the same or similar flights, the average, or mean, expected future demand, μ, and the standard deviation of the expected demand for the flight, σ, can be calculated. From this the probability distribution of the demand can easily be found.

In order to sell S seats in a given fare class, the number of requests for seats in the particular fare class must be greater than or equal to S. Therefore, the probability of selling S seats is the probability of having S or more requests, that is $P[r \geq S]$. In a continuous probability distribution such as the normal distribution, the probability of having S or more requests is:

$$P[r \geq S] = \int_{S}^{\infty} p(r)dr = 1 - P(S) = \overline{P}(S) \tag{4.1}$$

$P(S)$ in the above equation is the cumulative probability of having S requests. $P(S)$ is equivalent to $P[r \leq s]$, the probability that the number of requests will be less than or equal to S.

The probability of having S or more requests, $\overline{P}(S)$, is equivalent to the probability of selling at least S seats, as mentioned before. This probability is equal to the area under the probability distribution curve for requests, as shown in Figure 4.1. From the fundamental property of a probability distribution function:

$$\int_{-\infty}^{\infty} f(y)dy = 1 \tag{4.2}$$

we know the area under the probability distribution is 1. We also know that the probability distribution curve will lie completely above $r = 0$ since it is impossible to have a negative number of requests.

Because of this, the probability of selling the first seat in a particular fare class is approximately equal to 1. This can be found from either determining the probability of having 1 or more requests or from finding the area under the probability curve from 1 to ∞. The probability of selling μ seats, the mean expected demand, is equal to 0.50. As the number of seats increases, the probability of selling them decreases. This decreasing probability curve of selling the Sth seat is shown in Figure 4.2.

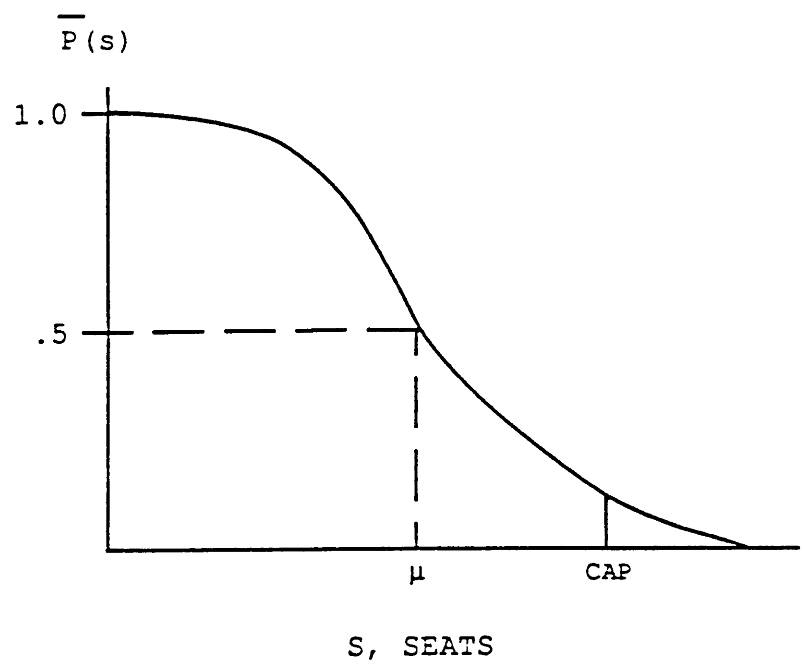


Figure 4.2: Probability Distribution Function of Selling the *Sth* Seat

Once the probability of selling the *Sth* seat, $\overline{P}(S)$, is known, the expected marginal revenue of the seat is simply:

$$EMSR(S) = f \times \overline{P}(S) \tag{4.3}$$

That is, the expected marginal revenue of the *Sth* seat is the average fare level of the seat, f, multiplied by the probability of selling the *Sth* seat. $EMSR(S)$ is directly dependent on $\overline{P}(S)$. Thus, the expected marginal revenue curve has the same shape as the probability distrubution function of selling the *Sth* seat, but the curve is scaled up by the constant f, the fare. (Figure 4.3)

4.1.2 Leg Based EMSR

The EMSR model was developed as a leg based model. This model is being evaluated as an "origin-destination" seat inventory control system in order to see how

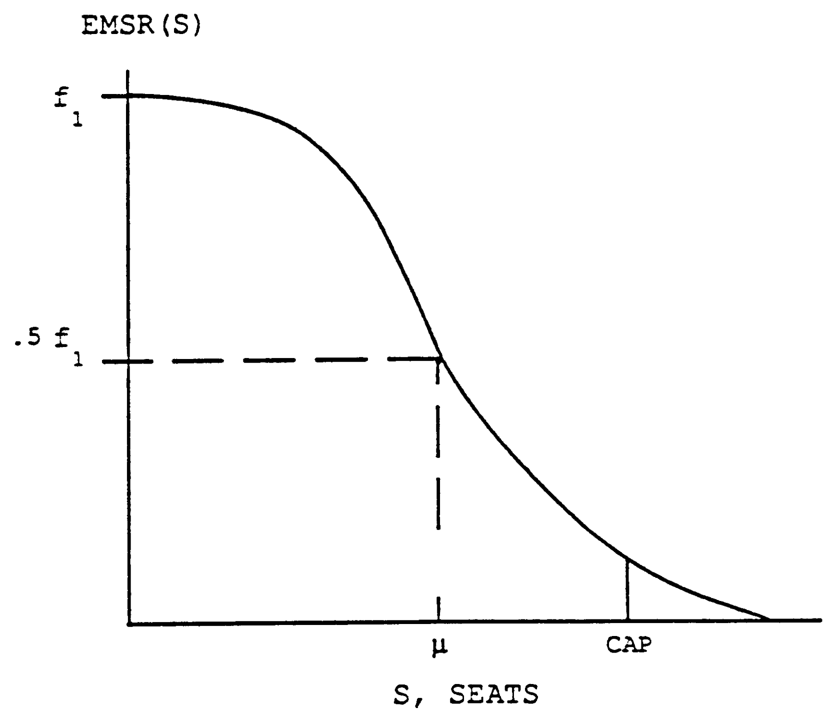


Figure 4.3: Expected Marginal Seat Revenue

a basic leg-based solution compares to network-based solutions. The idea behind the EMSR model is to protect seats for a higher fare class as long as the expected marginal revenue of the seats is greater than the expected marginal revenue of the seats at a lower fare class. As formulated by Belobaba [12], the expected marginal revenue for the Sth seat made available to class i is:

$$EMSR_i(S_i) = f_i \times \overline{P}_i(S_i) \tag{4.4}$$

The number of seats which should be protected for class 1, a higher fare class, over class 2 is S_2^1. S_2^1 is found by equating the expected marginal revenue of the S_2^1th seat in class 1 with the expected marginal revenue of the first seat made available in class 2,

$$EMSR_1(S_2^1) = EMSR_2(1) \tag{4.5}$$

The expected marginal revenue of selling the first seat in class i is simply f_i, the fare in class i, since the probability of selling the first seat in a particular fare class is approximately equal to 1. Therefore, the number of seats which should be protected, S_2^1, can be derived from the relationship:

$$f_1 \times \overline{P}_1(S_2^1) = f_2 \tag{4.6}$$

This is shown graphically in Figure 4.4. At the point S_2^1, the airline is indifferent between the revenue f_2 and the expected revenue $EMSR_1(S_2^1)$ for the S_2^1th seat made available in class 1.

In order to determine the booking limits for each fare class, the EMSR model requires inputs of the probability distribution function of expected future demand, for each class, from the point in time when the model is run to flight departure time. The probability distribution of demand for each class can be determined simply from

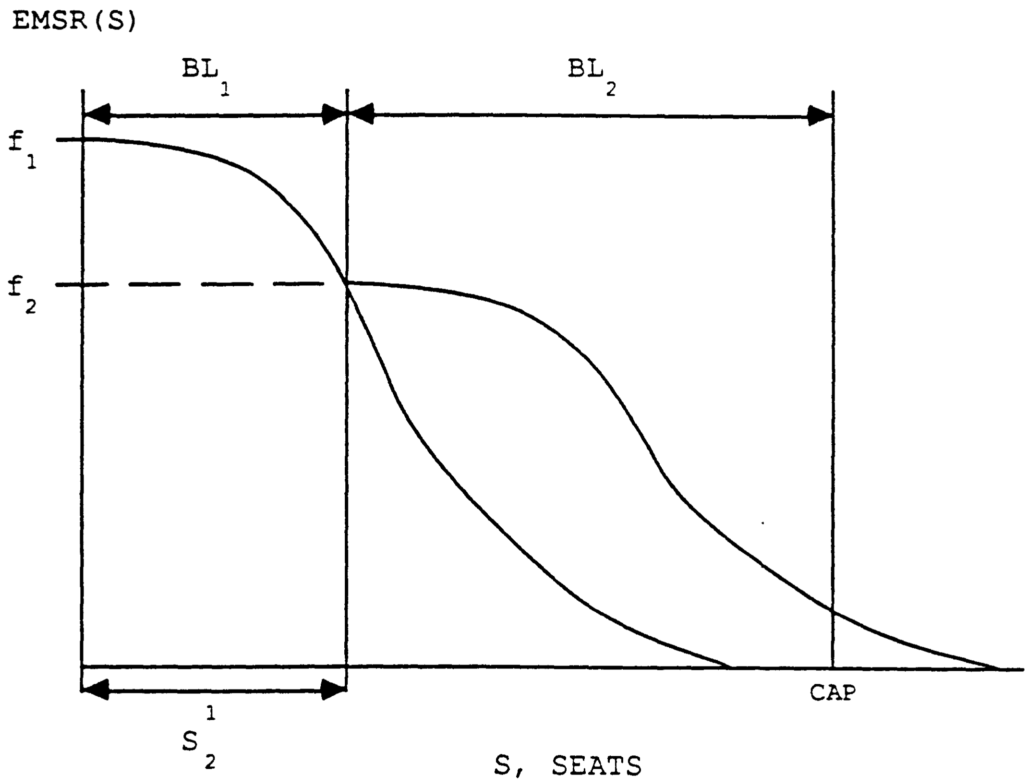


Figure 4.4: EMSR Protection Level For Two Classes

the mean (average) expected future demand and the standard deviation of expected future demand. Also needed is the average revenue in each class and the capacity (the number of seats) in the coach cabin which must be shared among the fare classes.The EMSR model can also be used for making adjustments in booking limits by taking into account the number of seats booked in each class at the time the model is run. For a complete discussion of this process see [13].

Once the above information is inputted into the EMSR model, the model determines protection levels. In a four fare class system—Y, M, B, and Q—six protection limits are calculated. Protection limits for Y class over M class, M over B, and B over Q are calculated. Also needed are the number of seats to protect for Y class over B and Q, and M class over Q class. These protection levels are calculated as in equations 4.5 and 4.6. Once the protection levels are determined, the booking limits for each class i are simply the capacity of the cabin minus the total number of seats protected for higher fare classes over the given class. That is:

$$BL_i = CAP - \sum_{j=1}^{i-1} S_i^j \tag{4.7}$$

where Y class is equivalent to class 1, M class to class 2, etc. The optimal protection levels and booking limits for a four class system are illustrated in Figure 4.5.

Note that for each class i, the seats protected in each higher fare class over class i are individually summed and protected from class i. For example, in B class the booking limit is dependent on the number of seats which are protected for Y class over B and the number of seats protected for M class over B. In determining the B class booking limit, the problem involves first protecting seats for Y class and then protecting seats for M class separately. These protected seats are then used in a nested system. The seats which were separately protected for Y class over B

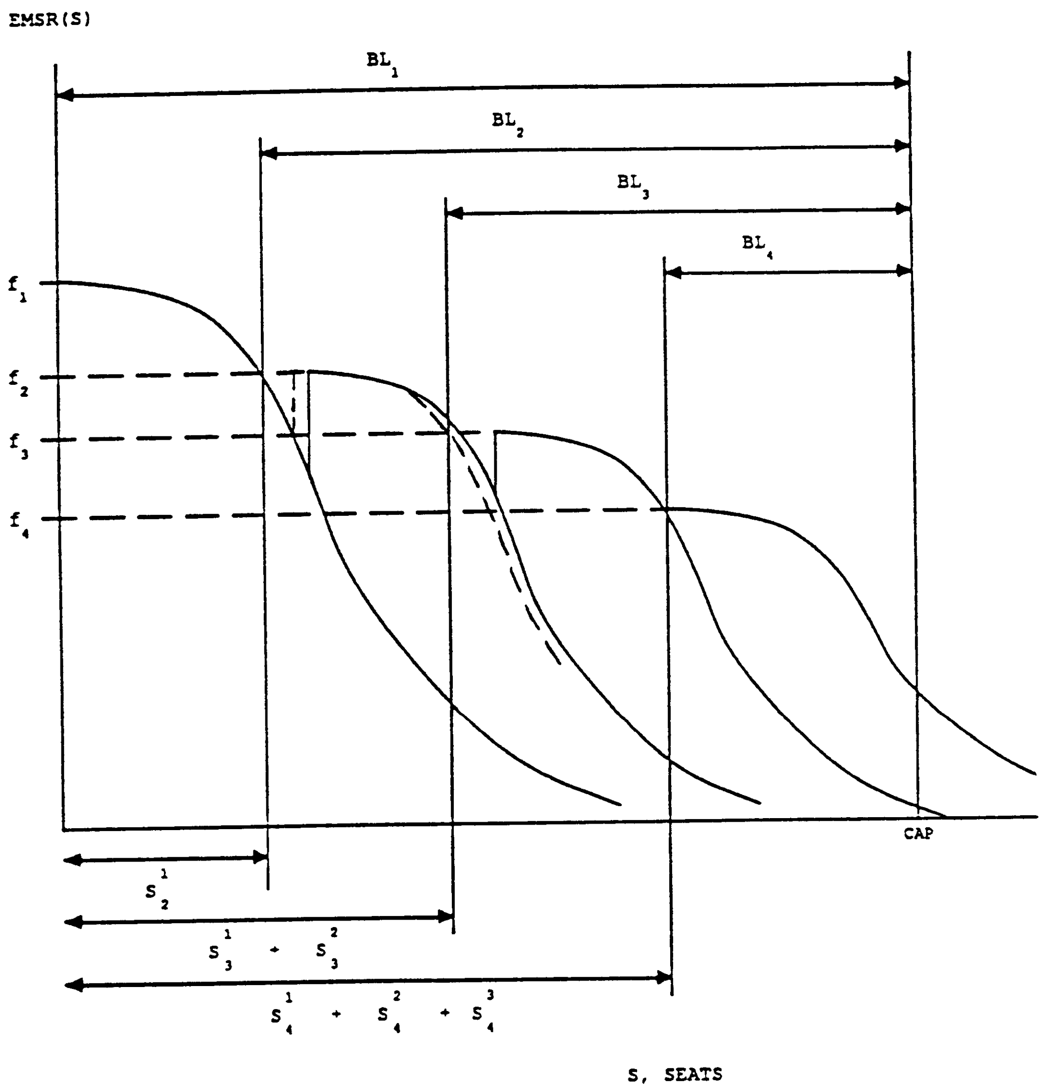


Figure 4.5: EMSR Protection Levels and Booking Limits - Four Fare Classes

class and then M class over B are put together and made available to Y and M class jointly, with no distinction made between them (except for those seats previously protected for Y only).

When applying the leg based EMSR model as an origin-destination seat inventory control technique, protection levels and booking limits are determined by fare class on each individual flight leg. On a flight leg there may be many different passenger itineraries in each fare class, but there is no distinction made between these different itineraries. The itineraries are aggregated together according to fare class. Mean expected demands and standard deviations are combined and fares are averaged in order to obtain joint fare class demands and fares for the EMSR model. In this model, the fare values for a particular itinerary on any given flight leg are based on total non-prorated ticket revenue value.

Once the fare class mean demands, standard deviations of demand and average fares have been determined, the EMSR model is applied seperately to each individual flight leg. Expected marginal revenues are equated to obtain the seat allocations and booking limits for each fare class. These allocations and limits are based on fare classes alone. Differences in origin-destination itinerary do not matter. If there is a seat available in a given fare class on a flight, it can be booked by any passenger, regardless of destination. In addition to being leg-based, the optimal solutions obtained are based on a nested reservations system and take into account the stochastic behavior of demand.

4.1.3 Prorated EMSR

The prorated EMSR approach is a slight variation of the standard leg-based EMSR method described above. The model itself is not changed and is applied to the network leg by leg, as above. The only difference is in the fares used in the model. Instead of averaging fares for the four fare classes of each flight leg based on the total origin-destination fares, average fares are based on the prorated fares for the flight leg.

A previous study done on prorated versus non-prorated fares in October 1987 found that ratios between fare classes of prorated and non-prorated fares appeared to be significantly different in many cases. The prorated EMSR method is being evaluated here to compare overall network results in revenues and seat allocations with that of the non-prorated seat inventory control approach, as well as network-based approaches. Since the basic methodology behind the prorated technique is the same as that of the non-prorated technique, any differences in the network solution should be due to the assumed fare structure inputs.

The fares in the prorated EMSR approach are prorated by flight leg. The total origin-destination fares are allocated to each appropriate flight leg according to the proportion of total itinerary miles the flight leg contributes. For example, the Y class fare for a Boston to Miami itinerary of 1541 miles is $403.00. If the passenger connects through Atlanta on his way from Boston to Miami, the O-D itinerary is made up of two flight legs, the first being the BOS-ATL leg at 946 miles and the second being the ATL-MIA leg at 595 miles. In a prorated fare structure, the fare for the BOSMIA passenger on the BOS-ATL flight leg will be 946/1541 of $403, or $246.40. At the same time, the Y fare for the same BOSMIA passenger on the ATL-MIA flight leg is 595/1541 of $403, or $155.60. In a non-prorated fare structure, the Y fare used for a BOSMIA passenger on both the BOS-ATL flight leg and the ATL-MIA flight leg is $403.00.

Once the fares have been prorated by leg, these prorated fares are averaged according to the mix of passenger O-D's on each leg in order to find the respective flight leg fares. The averaged prorated fares, along with the aggregate mean demands and standard deviations for each fare class, are used as inputs into the EMSR model. The model is run on each flight leg as before, and booking limits are obtained for each class on every flight leg in the network. Like the standard EMSR approach, this approach does not account for the interaction of traffic between flight legs in the system. By using prorated fares, higher

yield fares are distinguished from lower yield, but higher total revenue, fares.

4.2 Virtual Nesting EMSR

The virtual nesting EMSR method is a modified EMSR approach which controls seat inventories on the basis of passengers' itinerary revenues. Total itinerary ticket revenues are "virtually nested" by flight leg and the EMSR model is then applied to these "virtual classes" on each flight leg. The objective of a virtually nested system is to allow the airlines to take into account requests for different fare class/passenger itinerary combinations on an individual flight leg which generate different revenue levels. The method is a leg-based approach, as are the other EMSR approaches. Booking limits are determined and expected revenue is maximized by individual flight leg, but seat allocations are controlled on the basis of fare class and passenger itinerary. Although this is not an optimal system-wide solution, it is a more sophisticated approach to seat inventory control.

In a virtual seat inventory system, each origin-destination/fare class combination is identified with a virtual, or hidden, seat inventory class based on total itinerary ticket value. The virtual inventory classes are defined by a dollar range, with the dollar range corresponding to total passenger itinerary ticket revenue. The approach is labeled "virtual" because the inventory classes themselves are classes in essence, but they are not formally recognized classes in that the virtual fare classes are not offered by the airline. The airlines offer the standard fare classes of service. In a four fare class system those classes might be Y, M, B, and Q. Each of these fare classes are in turn assigned to a virtual inventory class according to the respective origin-destination itinerary and ticket revenue. Booking limits are then set for each of the virtual classes on each flight leg. The seat availabilities for the virtual classes are then related back to the standard fare classes offered in the O-D markets. Fare class availabilities are displayed by O-D itinerary and flight leg, as opposed to flight leg only, as in the other EMSR methods. The virtual classes exist only within the seat inventory system itself, they are not apparent to the users of the system.

The virtual inventories are defined on the basis of actual fares, no proration is involved. Each possible O-D itinerary and fare class combination that can use a given flight leg is assigned a virtual inventory class. From previous work done on fare basis distributions and ticket usage, a virtual inventory of eight classes was defined on the basis of O-D ticket fare ranges. The fare ranges for the virtual inventory classes are listed in Table 4.1. These fare ranges were derived based on Delta Air Lines fare quota data and ticket sales data from a sample period in May 1987. Ticket counts by fare basis and fare level were used to determine dollar intervals of equal frequency in terms of ticket usage. Clustering of ticket sales at the lower fare levels requires much narrower fare range definitions at low dollar values than at high dollar values.

Once the fare ranges for the virtual inventory classes have been defined, each origin-destination and fare class combination is assigned to a virtual class. Figure 4.6

VIRTUAL CLASS	FARE RANGE
8	0.00 - 74.99
7	75.00 - 94.99
6	95.00 - 109.99
5	110.00 - 129.99
4	130.00 - 159.99
3	160.00 - 254.99
2	255.00 - 399.99
1	350.00 - ∞

Table 4.1: Fare Ranges for the Virtual Inventory Classes

demonstrates how these assignments can be made for a Boston-Atlanta flight leg. Note how different Q class fare/itinerary possibilities can fall into different virtual inventory classes depending on their respective O-D ticket revenue. The BOSATL and BOSSAV Q class itineraries fall into the lowest virtual class while the BOSMIA Q class itinerary is assigned to the $7th$ virtual class and the BOSLAX Q class itinerary is as high as the $4th$ virtual class. This makes it possible to close down the Q class for local BOSATL passengers while Q class seats for BOSMIA passengers on the same initial flight leg remain available.

Although O-D itineraries are distinguished on a given flight leg, the overall method is leg-based. Therefore, availabilities for multi-flight itineraries depend on the relevant virtual inventories being available for all flight legs involved in the O-D itinerary. In our example and analysis, the defined fare ranges for each virtual inventory are the same across the different flight legs in a network. If the BOSMIA Q class itinerary falls into the $7th$ virtual inventory class on the BOS-ATL flight leg, it will also fall into the $7th$ virtual class on the ATL-MIA flight leg. In order to have

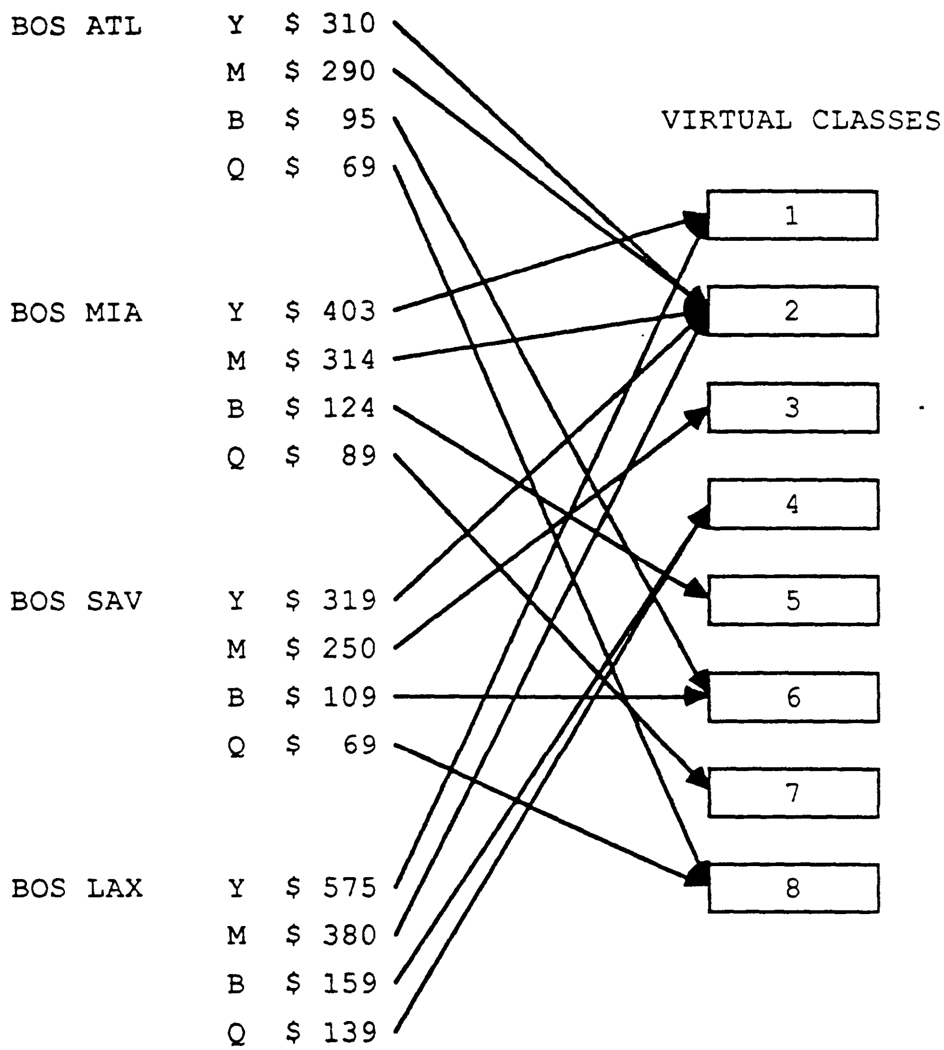


Figure 4.6: Virtual Inventory Assignments

a seat available for a BOSMIA passenger in the Q class, the $7th$ virtual inventory class on both the BOS-ATL and the ATL-MIA flight legs must be open.

The application of the probabilistic EMSR decision model to set booking limits on the virtual inventories is straightforward. The mean expected demands, standard deviations and average fares for each *virtual* class are derived as they were for each fare class before. An eight class EMSR model is then run on each flight leg. The EMSR model protects seats for each upper-ranked virtual inventory class from lower virtual inventories on a flight leg. Booking limits are determined on the premise of maximizing total expected revenue on each flight leg, but on the basis of O-D itinerary revenues.

Once booking limits are found for each virtual inventory class, the virtual classes are disaggregated back to the specific O-D itineraries. The booking limit for each O-D itinerary/fare class is simply the booking limit of its corresponding virtual class. The process of assigning these booking limits is done internally within the seat inventory system, such that reservations and travel agents only have access to the fare class/O-D itinerary booking limits.

The application of the EMSR model to virtual inventory classes represents an attempt to take into account total revenue contribution of different passenger itineraries on a flight leg. It does not take into account interaction between passenger flows on different flight legs. The algorithm represents a "greedy" approach to seat inventory control producing potentially non-optimal solutions. A single long-haul passenger will be favored over combinations of short-haul passengers, in spite of the possibility that taking the latter could result in a higher total revenue.

The virtual nesting concept used in this way comes from our knowledge of how at least two major carriers have designed their seat inventory control systems. The virtual nesting EMSR approach to seat inventory control is used in this evaluation to compare this "more sophisticated" approach with that of the purely leg based methods. Also of interest is the question of whether the "greediness" of this method on each flight leg has a significant impact over an entire network.

4.3 Deterministic Linear Program

The first three seat inventory control techniques described have been leg based optimization algorithms which maximize revenue over individual flight legs. The next three techniques are network based. These techniques are true origin-destination seat inventory control methods which optimize entire system-wide revenues. Although a network optimization problem can be formulated in a variety of ways, a linear programming formulation is used in each of the following network seat inventory control techniques, making them solvable by the linear programming computer software package, LINDO [14].

The deterministic linear programming method is a traditional mathematical programming technique. It is a constrained optimization method which can deal with problems involving thousands of variables and thousands of constraints. The deterministic linear programming method is a network formulation of the seat inventory control problem based on origin-destination combinations. As the name suggests, this method is deterministic, it does not incorporate probabilistic demand. Unlike leg-based methods, interaction between flight legs is taken into account.

The standard linear program consists of two parts, an objective function and constraints. The objective of the linear programming method is to maximize total revenue over the entire network. That is, the number of seats allocated to each fare class/O-D itinerary multipied by the respective itinerary revenue is to be maximized. This objective is subject to certain constraints. The first set of constraints are based on capacity, with a capacity constraint on each flight leg. The second set are the demand constraints. For each fare class/passenger itinerary, there is a demand constraint which is based on a deterministic estimate of demand, usually the mean expected demand.

The variables in the deterministic linear programming seat inventory control problem are individual seats, or groups of seats, which are to be allocated among the many different possible fare class/O-D combinations on a flight leg. The problem is actually a linear integer-programming problem. Each decision variable is required to be an integer value, a fractional number of seats is unrealistic. In the formulation, the integer decision variable for the number of seats allocated to a given fare class i and O-D combination is designated by $x_{i,\text{O-D}}$. The fare associated with the respective fare class and origin-destination is $f_{i,\text{O-D}}$, making the objective function:

$$\text{Maximize} \quad \sum_{\text{O-D}} \sum_{i} f_{i,\text{O-D}} \cdot x_{i,\text{O-D}} \tag{4.8}$$

This maximization is constrained by two types of constraints. The total number of seats allocated to the different fare classes and O-D combinations on each flight leg j must be less than the aircraft capacity, CAP_j. Also, the number of seats allocated to each O-D/fare class must be less than or equal to a fixed expected demand which is chosen to be the mean demand, $\mu_{i,\text{O-D}}$. Therefore, the second part of the linear programming formulation is:

subject to:

$$\sum_{\text{O-D}} \sum_{i} x_{i,\text{O-D}} \leq CAP_j \quad \text{for all O-D itineraries and } i \text{ fare classes on flight } j, \text{ for all flights } j.$$

$$x_{i,\text{O-D}} \leq \mu_{i,\text{O-D}} \quad \text{for all O-D itineraries and } i \text{ fare classes.}$$

The solution from this deterministic LP formulation is a set of integer values assigned to each $x_{i,\text{O-D}}$, which correspond to the optimal number of seats to be allocated to the respective O-D/fare class. Seats are allocated on the basis of total revenue contribution to the system-wide network, taking into consideration the interaction of traffic flow over different flight legs. In such a network approach, if a seat is made available on one flight leg for a specific multi-leg passenger, a seat has also been allocated for that passenger on other flights included in the passenger's itinerary. In leg-based methods, availability for a multi-leg passenger is dependent on the separate availabilities on each flight leg. There is no guarantee that if a passenger can book a seat on the first leg of his itinerary, the second will also be available.

The deterministic LP solution is an optimal network solution which requires demand to be forecasted exactly, with no provision for uncertainty. Since demand is probabilistic, there will be many times when demand will not reach the expected deterministic value and other times when it will exceed this value. The solution obtained from the LP formulation is optimal only when actual demand is equal to the expected deterministic value used in the formulation's demand constraints. The chances of actual demand being exactly as forecasted is very small, therefore the recommended seat allocations will not be optimal most of the time. Nonetheless, the LP approach is based on an airline's entire network and the interaction of flights within the network. The interesting question is how such "non-optimal" network solutions compare to leg-based solutions.

An additional problem with the deterministic LP approach is that seat allocations are based on distinct inventories. Each fare class and O-D combination is treated as separate. Since most reservations systems today are both leg-based and nested, this creates a major problem in implementing the non-nested O-D solution obtained from the LP model. Once the airline industry overcomes the initial hurdle of converting reservations systems from leg-based to O-D based, there is still the need to find a way to nest the different fare class/O-D inventories effectively. In the process of nesting fare class and O-D combinations, the accuracy of an O-D seat inventory control method does not want to be lost. Also of consideration is the fact that "optimal" seat allocations for a distinct system are not necessarily optimal for a nested system.

4.4 Probabilistic Linear Program

The probabilistic linear programming method is a traditional mathematical programming technique, like the deterministic LP method, for an entire network. The difference is that the probabilisitic LP approach takes into account the uncertainty of demand, producing a mathematically optimal solution to the multi-leg origin-destination problem. The downfall of this method is that the required network formulation rapidly becomes very large.

The probabilistic LP approach is a binary integer programming problem, where the decision variables, $x_{i,\text{O-D},j}$, represent every possible seat j which could potentially be available on a flight leg for each fare class i and itinerary O-D combination. The decision variables are taken to be $x_{i,\text{O-D},j} = 0$ or 1. These (0,1) decision variables indicate whether the jth seat is rejected or accepted for allotment to fare class i and itinerary O-D. The total number of decision variables which are set equal 1 for a given fare class and O-D itinerary will represent the number of seats to allocate to the O-D/fare class inventory.

In the probabilistic LP formulation, total system-wide expected revenue is maximized subject to certain capacity constraints. In the objective function each decision variable, $x_{i,\text{O-D},j}$, is multiplied by its own expected marginal revenue coefficient to reflect the probability of selling seat j to the specific fare class/O-D combination. The expected marginal revenue for the jth seat in fare class i and itinerary O-D is:

$$EMSR(j_{i,\text{O-D}}) = f_{i,\text{O-D}} \times \overline{P}(j_{i,\text{O-D}}) \tag{4.9}$$

as derived in Section 4.1.1. The only difference here is that the probability of selling the jth seat, $\overline{P}(j_{i,\text{O-D}})$, is dependent on the expected demand of the respective fare class/O-D itinerary, and not the fare class alone. The probability density function of demand for fare class i and itinerary O-D is also assumed to be a normal distribution. Based on the mean demand and standard deviation of historical data, the normal probability curve for each fare class/O-D combination can be found.

In the probabilistic LP formulation, the objective function is limited by only one true set of constraints. The total number of seats alloted to each fare class and itinerary combination on a given flight leg k must be less than or equal to the capacity of the aircraft servicing the flight leg, CAP_k. The demand constraints, which are found in the deterministic LP problem, are incorporated within the objective function itself and are probabilistic in nature. The other constraints associated with the problem are the zero/one constraints on each of the individual decision variables.

The probabilistic optimization problem formally stated is:

$$\text{Maximize} \quad \sum_{\text{O-D}} \sum_{i} \sum_{j=1}^{CAP_k} EMSR(j_{i,\text{O-D}}) \cdot x_{i,\text{O-D},j} \tag{4.10}$$

subject to:

$$\sum_{\text{O-D}} \sum_{i} \sum_{j=1}^{CAP_k} x_{i,\text{O-D},j} \leq CAP_k \quad \text{for all O-D itineraries and } i \text{ classes on flight } k,$$

$$\text{for all flights } k$$

$$x_{i,\text{O-D},j} = 0 \text{ or } 1 \quad \text{for all O-D itineraries, } i \text{ classes,}$$

$$\text{and } j = 1, 2, \ldots, CAP_k.$$

As can be seen, the number of decision variables expands exponentially with the size of the network involved. Certain cutoff points can be included to keep the size of the problem down, but the problem is still very large.

The probabilistic LP approach chooses those decision variables, or combination of decision variables, which are associated with the greatest expected revenues, and sets them equal to 1. Note that it is impossible to have a $j + 1th$ seat allocated to a specific fare class/O-D itinerary without the allocation of the jth seat because of the monotonically decreasing nature of the EMSR curve. Protection levels and booking limits are found on the basis of distinct seat inventories, such that the $EMSR(j_{i,\text{O-D}})$ values for different fare class and O-D combinations are equated based on the last seat allocated to each inventory. This is illustrated in Figure 4.7 for a two fare class/O-D inventory example. Notice how the seat allocations in the probablistic LP approach differs from the protection levels of the EMSR approach shown in Figure 4.4.

The solution to such an approach incorporates probabilistic demand and is a network based solution, as opposed to leg-based. It is an optimal solution which is not nested. The solution is the "correct" allocation of seats for the given network and should not be nested. The problem is to determine how this optimal system-wide revenue solution can be implemented into nested reservations systems. Although it is the "correct" solution, it is not a very realistic approach to seat inventory control

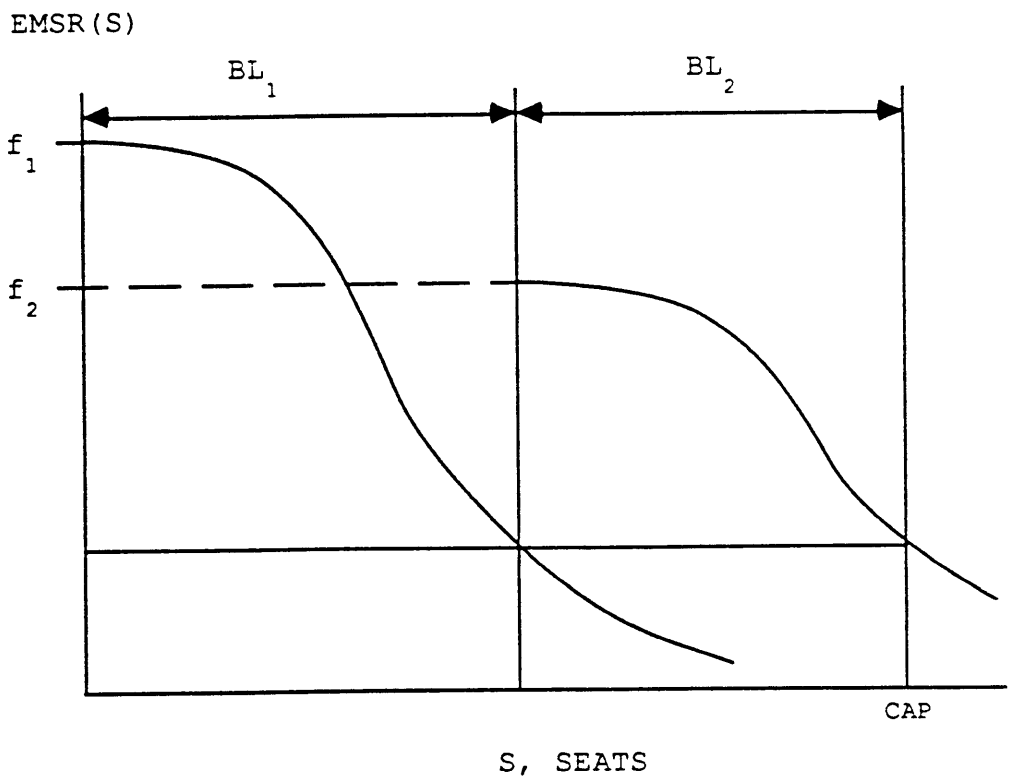


Figure 4.7: Probabilistic LP Protection Levels for Two Inventories

because of the size of the probabilistic LP formulation. Setting initial booking limits with such a method itself takes huge amounts of computer power. Even if an airline had the computer capabilities to accomplish this, continually making adjustments to these limits as reservations are accepted would be almost impossible.

4.5 Deterministic LP Nested on Shadow Prices

The solution from the deterministic LP is for a distinct, non-nested fare class structure. Most reservations systems in the airline industry today are nested, making it difficult to use the deterministic LP solution directly. If the results from this approach were nested, the deterministic LP technique could be a viable alternative for origin-destination seat inventory control. The problem is how to nest the fare class/O-D inventories. There are many simple ways in which this might be done, such as nesting the inventories by fare class or ticket revenue but these methods do not distinguish which specific fare class/O-D combinations are preferred in terms of generating revenue.

After disaggregating and solving the problem on the basis of O-D and fare class, reaggregating the solution into the original fare class inventories seems contradictive. A given fare class is a conglomeration of a range of different revenues obtained from long-haul and short-haul itineraries, and within the fare class there is no distinction between these itineraries. Therefore, using the less complicated leg-based EMSR method, which does not differentiate between origin-destination itineraries, but determines optimal booking limits for a nested system on the basis of fare class alone, taking into account probabilistic demand, is as good an approach, if not better, than the aggregation of the deterministic LP network solution.

Nesting the fare class/O-D inventories by ticket revenue leads to the same type of situation as in the virtual inventory technique. Individual ticket revenue does not take into account the network revenue generated by a combination of low-fare single-leg passengers versus a higher fare multi-leg passenger. A nesting order based on maximum ticket revenue is not the same as maximum system-wide revenue.

Another approach to nesting the O-D/fare class seat allocations derived from this deterministic network optimization technique is to use marginal revenue information, such as shadow prices and reduced costs. The shadow price or reduced cost associated with each decision variable gives the amount the optimal system revenue would change if one more unit of the variable were used in the solution. For a variable that is limited by a demand constraint, the shadow price is the amount the objective function will increase in value if the demand constraint were increased by one unit. A reduced cost is affixed to a variable which is not being used in the optimal solution. This reduced cost value gives the amount the optimal value of the objective function would decrease if the variable were forced into the solution.

Using shadow prices and reduced costs as nesting variables allows the fare class/O-D combination that will increase the overall network revenue the most (if increased by one unit) to be nested above variables with lower marginal revenue impacts. By using such nesting variables, problems associated with other nesting techniques can be avoided. The disaggregate solution of the deterministic LP method is not reaggregated. Each fare class/O-D inventory is kept separate in the process of nesting and not grouped together. The nesting order is also based on maximum network revenue, not individual ticket revenue. The deterministic LP approach nested on shadow prices takes into account situations in which two local passengers should be ranked higher, in revenue terms, than a single through or connecting passenger.

With the use of shadow prices, it might be possible for the deterministic linear programming solution to be implemented in a nested system without undermining the overall objective of this seat inventory control approach, which is to find an optimal origin-destination network solution. Nesting the optimal fare class/O-D allocations provides the airline with potentially higher expected revenues than using the same allocations in a non-nested fare class structure. Nesting also makes this solution compatible with nested reservations systems. Yet, the practical problem of a limited number of nested fare classes in the reservations system still remains.

When the different inventories, with their respective deterministic LP seat allotments, are nested, the results are not necessarily optimal. First, the optimal non-nested seat allocations are not usually the optimal nested seat allocations. Secondly, the shadow prices give information about the initial increases in network revenue based on one additional seat alloted to a fare class/O-D combination, but not for seat allocation changes beyond that. Even though the end result may not be "optimal", the deterministic LP approach nested on shadow prices may provide a better overall solution to the origin-destination seat inventory contol problem than the leg-based and non-nested network approaches.

Chapter 5

Analysis and Comparisons

This chapter contains an analysis of the six different alternatives described in Chapter 4 which are being evaluated for origin-destination seat inventory control. The techniques have been applied to a small network with assumed demand levels and fares for a four class coach system. From each alternative and demand scenario, seat allocations, booking limits, nesting orders and system revenues were determined. On the basis of these attributes, the different techniques are compared and the results are discussed in this chapter.

5.1 Network Model and Data

The network used for the evaluation is a small hypothetical hub connecting bank involving eight different flights. It is an Atlanta-based hub with service to four airports: Boston, Los Angeles, Miami, and Savannah (Figure 5.1). In this network there are 10 possible city pairs: ATLBOS, ATLSAV, BOSMIA, BOSLAX, etc.). Traffic flows in both directions, for example BOS-ATL and ATL-BOS, making a total of 20 directional O-D markets (itineraries). In each of these O-D markets there are four fare classes offered, Y, M, B, and Q, in descending order of fare level,

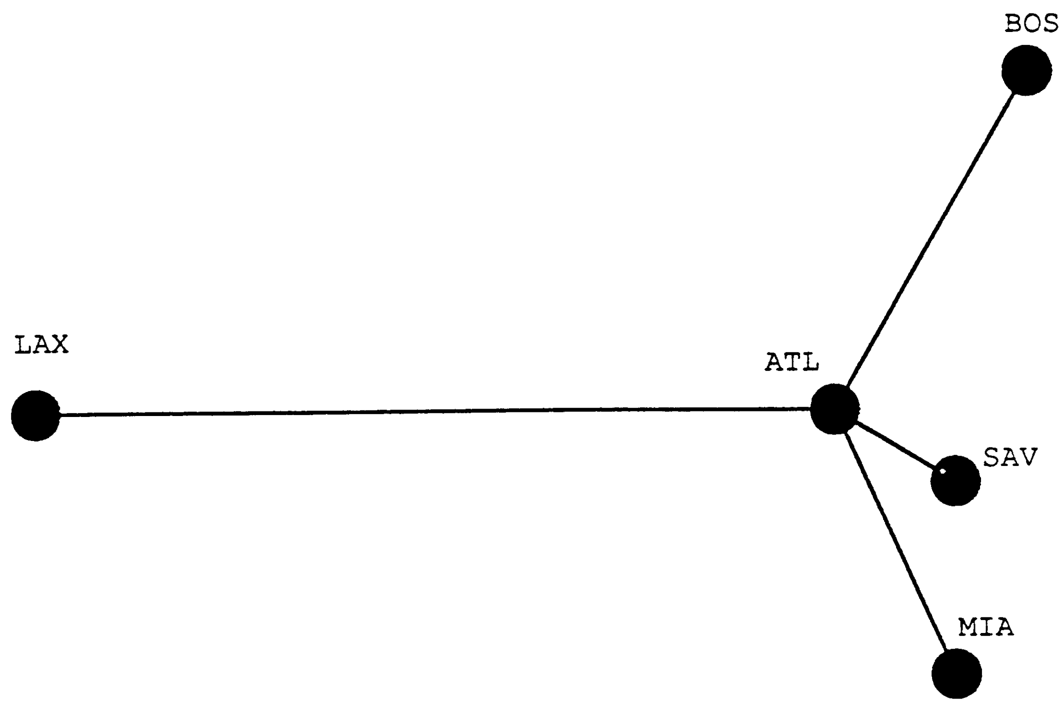


Figure 5.1: Atlanta-Based Hub Network

which results in a total of 80 different fare class/O-D combinations for the given network.

For each fare class and O-D itinerary, a fare, mean demand and standard deviation of demand are assumed. The different techniques have been evaluated under three different scenarios of demand—low, medium, and high. The medium demand assumption is a total mean demand per leg of approximately 150. For example, on the BOS-ATL leg, the mean demand for all four classes, Y, M, B, and Q, of all origin-destination pairs BOSATL, BOSSAV, BOSMIA, and BOSLAX must sum to 150. This represents an average demand of 37 or 38 per O-D pair. The low demand is set at about 67% of the medium demand, or a total of 100, and the high demand is 133% of the medium level, or about 200. The fares for each O-D/fare class combination reflect actual published fares in November 1987. Tables 5.1, 5.2, and 5.3 give the mean demand, the standard deviation of demand and the fare for each fare class/O-D itinerary for the low, medium, and high demand scenarios, respectively.

In this analysis it has been assumed that each leg of the network, ATL-BOS, ATL-SAV, ATL-MIA, and ATL-LAX, is served with one flight, making this situation similar to a single connecting bank at a hub airport. To keep the model simple, each flight is operated by the same size aircraft, which is assumed to have a coach cabin capacity of 150 seats. This is true for both directions of each link in the the network. Therefore, the capacity constraint on the ATL-BOS flight leg is 150, and the constraint on the BOS-ATL flight leg is also 150.

	Y	M	B	Q
ATLBOS/BOSATL	6 2 $310	5 1 $290	5 2 $95	9 2 $69
ATLSAV/SAVATL	13 2 $159	3 1 $140	3 1 $64	7 2 $49
ATLMIA/MIAATL	10 2 $280	5 2 $209	3 1 $94	7 2 $59
ATLLAX/LAXATL	4 1 $455	2 2 $391	5 2 $142	13 3 $122
BOSSAV/SAVBOS	3 1 $319	3 2 $250	5 2 $109	13 4 $69
BOSMIA/MIABOS	5 2 $403	3 1 $314	7 2 $124	10 2 $89
BOSLAX/LAXBOS	5 1 $575	3 2 $380	5 2 $159	12 2 $139
MIASAV/SAVMIA	7 3 $226	2 1 $168	3 1 $84	13 3 $59
MIALAX/LAXMIA	9 2 $477	5 2 $239	3 1 $139	7 2 $119
LAXSAV/SAVLAX	3 2 $502	3 2 $450	3 1 $154	16 5 $134

Table 5.1: Fares, Means, and Standard Deviations for Low Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	9	7	8	13
	2	2	3	3
	$310	$290	$95	$69
ATLSAV/SAVATL	19	5	4	10
	3	2	1	3
	$159	$140	$64	$49
ATLMIA/MIAATL	15	7	5	11
	3	3	2	2
	$280	$209	$94	$59
ATLLAX/LAXATL	6	3	8	20
	2	2	2	5
	$455	$391	$142	$122
BOSSAV/SAVBOS	4	5	8	20
	2	2	4	6
	$319	$250	$109	$69
BOSMIA/MIABOS	8	4	11	15
	3	2	3	3
	$403	$314	$124	$89
BOSLAX/LAXBOS	7	5	8	18
	2	3	3	4
	$575	$380	$159	$139
MIASAV/SAVMIA	10	3	5	19
	4	2	2	5
	$226	$168	$84	$59
MIALAX/LAXMIA	13	8	5	11
	3	3	1	2
	$477	$239	$139	$119
LAXSAV/SAVLAX	4	5	5	24
	2	3	2	7
	$502	$450	$154	$134

Table 5.2: Fares, Means, and Standard Deviations for Medium Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	12	9	11	17
	3	3	4	4
	$310	$290	$95	$69
ATLSAV/SAVATL	25	7	5	13
	5	2	2	3
	$159	$140	$64	$49
ATLMIA/MIAATL	20	9	7	14
	4	4	2	3
	$280	$209	$94	$59
ATLLAX/LAXATL	8	4	11	27
	2	3	3	7
	$455	$391	$142	$122
BOSSAV/SAVBOS	5	7	11	27
	3	3	5	8
	$319	$250	$109	$69
BOSMIA/MIABOS	11	5	14	20
	3	2	4	4
	$403	$314	$124	$89
BOSLAX/LAXBOS	9	7	11	24
	3	3	3	5
	$575	$380	$159	$139
MIASAV/SAVMIA	13	4	7	25
	6	2	3	7
	$226	$168	$84	$59
MIALAX/LAXMIA	17	11	7	14
	4	4	2	3
	$477	$239	$139	$119
LAXSAV/SAVLAX	5	7	7	32
	3	3	2	9
	$502	$450	$154	$134

Table 5.3: Fares, Means, and Standard Deviations for High Demand Level

5.2 Applications and Results

The six techniques were applied to the above network at the three different levels of demand. The first step in applying the different models to the network was to determine the necessary inputs from the data given for the fare class/O-D combinations in the network. The basic steps needed in the application of each technique are explained below along with sample origin-destination results from the six techniques. A complete set of nesting and booking limit results for each origin-destination seat inventory control technique is provided in Appendix A.

5.2.1 Leg Based EMSR

In the leg based EMSR approach, the model is applied to individual flight legs. For each leg, the EMSR model requires a mean demand, standard deviation, and a fare estimate for each class. In order to obtain this information, all origin-destination itineraries which flow over a particular flight leg must be identified, and aggregate means, standard deviations and fares derived by fare class. For example, the Y class demand information from the ATLBOS, SAVBOS, MIABOS, and LAXBOS origin-destination pairs is used to obtain the Y class ATL-BOS flight leg EMSR inputs. Demand information of the same O-D pairs for other fare classes is also combined to find the ATL-BOS flight leg M class, B class, and Q class demand quantities.

The mean expected demand for each fare class on a flight leg is simply the sum of the means from the appropriate O-D itineraries. The calculation of the standard deviations of demand for a flight leg is not as straightforward. An aggregate standard deviation is determined from the sum of the O-D variations. Each O-D variance is simply found by squaring its respective standard deviation. Variances of independent variables, like means, can be summed to obtain an aggregate variance. Therefore, to determine the fare class demand variance for a given flight leg, the O-D variances are summed. By taking the square root of this total variance, the standard devation of demand for the given fare class on the flight leg is estimated.

The final fare class input that needs to be derived for the EMSR flight leg analysis is the average fare of each class. These flight leg fares are not a simple average of O-D itinerary fares, but rather a weighted average. The weighted average is based on the magnitude of the mean expected demands of the O-D itineraries. If the demand for a given fare class on a specific flight leg consists primarily of long-haul high ticket revenue O-D demand rather than short-haul low revenue demand, the associated fare level of the class should be typical of the long-haul revenue amount more than the short-haul revenue. For example, a particular flight leg is made up of two sets of passengers, the first set traveling from A to B and the second set from A to F. On the flight leg AB there are 50 AB passengers paying a fare of $75 and 100 more passengers who are continuing on to point F at a fare of $150. The average fare on this flight is $[(50 \times \$75) + (100 \times \$150)] \div 150$, or $125. Since more passengers are traveling on AB at a ticket revenue of $150, the average flight leg fare is weighted more heavily towards this higher revenue. When deriving average flight leg fares using mean expected demands, the O-D fares are averaged in this same way to reflect the overall fare class averages for the flight leg on the basis of total (non-prorated) ticket revenue.

The flight leg mean, standard deviation and fare were determined as described above for each leg in the network. Table 5.4 lists these inputs for the Y, M, B, and Q fare classes on the ATL-BOS and ATL-MIA flight legs. These leg based EMSR input values were derived from the respective medium demand O-D passenger segment

LEG	CLASS	SEGMENT	O - D MEAN	SD	FARE	FLIGHT LEG MEAN	SD	FARE
ATL-BOS	Y	ATLBOS	9	2	$310	28	4.58	$404.11
		SAVBOS	4	2	$319			
		MIABOS	8	3	$403			
		LAXBOS	7	2	$575			
	M	ATLBOS	7	2	$290	21	4.58	$306.48
		SAVBOS	5	2	$250			
		MIABOS	4	2	$314			
		LAXBOS	5	3	$380			
	B	ATLBOS	8	3	$95	35	6.56	$121.94
		SAVBOS	8	4	$109			
		MIABOS	11	3	$124			
		LAXBOS	8	3	$159			
	Q	ATLBOS	13	3	$69	66	8.37	$92.64
		SAVBOS	20	6	$69			
		MIABOS	15	3	$89			
		LAXBOS	18	4	$139			
ATL-MIA	Y	ATLMIA	15	3	$280	46	6.56	$345.33
		BOSMIA	8	3	$403			
		SAVMIA	10	4	$226			
		LAXMIA	13	3	$477			
	M	ATLMIA	7	3	$209	22	5.10	$233.41
		BOSMIA	4	2	$314			
		SAVMIA	3	2	$168			
		LAXMIA	8	3	$239			
	B	ATLMIA	5	2	$94	26	4.24	$113.42
		BOSMIA	11	3	$124			
		SAVMIA	5	2	$84			
		LAXMIA	5	1	$139			
	Q	ATLMIA	11	2	$59	56	6.48	$78.82
		BOSMIA	15	3	$89			
		SAVMIA	19	5	$59			
		LAXMIA	11	2	$119			

Table 5.4: Leg Based EMSR Inputs - Medium Demand Level

LEG	CLASS	SEGMENT	O - D MEAN	SD	FARE	FLIGHT LEG MEAN	SD	FARE
ATL-BOS	Y	ATLBOS	12	3	$310	37	6.00	$403.32
		SAVBOS	5	3	$319			
		MIABOS	11	3	$403			
		LAXBOS	9	3	$575			
	M	ATLBOS	9	3	$290	28	5.57	$306.79
		SAVBOS	7	3	$250			
		MIABOS	5	2	$314			
		LAXBOS	7	3	$380			
	B	ATLBOS	11	4	$95	47	8.12	$121.89
		SAVBOS	11	5	$109			
		MIABOS	14	4	$124			
		LAXBOS	11	3	$159			
	Q	ATLBOS	17	4	$69	88	11.00	$92.64
		SAVBOS	27	8	$69			
		MIABOS	20	4	$89			
		LAXBOS	24	5	$139			

Table 5.5: Leg Base EMSR Inputs - High Demand Level

data shown. Table 5.5 shows the inputs for an example in which the demand for the ATL-BOS flight leg is high. Notice how the average flight leg fares vary slightly from those in the medium demand example (Table 5.4). This is due to the slightly different ratios of the O-D mean demands between the two demand levels. In actual practice, different combinations of long-haul versus short-haul demand could result in greater differences.

Once the flight legs' demand and fare quantities were derived for all classes, the EMSR model was applied to each flight leg. The EMSR model determines booking limits for the flight leg classes based on a completely nested fare class structure, with Q class on bottom and Y class on top. The individual booking limit for a Q-class origin-destination itinerary on a given flight leg is the total Q-class limit. Also, in order to book a seat from Miami to Boston in a particular class, say B-class, a B seat must be available on the MIA-ATL leg as well as the ATL-BOS leg. Examples of the booking limits at the medium demand level for three legs of the network (ATL-BOS, ATL-MIA and MIA-ATL) and the implied O-D segment limits are shown in Table 5.6. Since the EMSR model is leg based and does not take into account interaction between flight legs in the network, the booking limits for flight legs which have the same O-D demands and fare values will be the same. In this particular Atlanta-based network, where demand is equivalent in both directions, the booking limits for opposite direction flight legs are the same, as is the case for the ATL-MIA and MIA-ATL legs. Note also how the booking limits from the EMSR method are grouped and nested entirely by fare class.

Tables 5.7 and 5.8 show the leg based EMSR booking limits for ATL-BOS under low and high demand levels, respectively. In cases where demand is low, all extra seats are made available to the lowest fare class. However, through nesting, these seats are not only available to the lowest fare class, but are also available to all other fare class passengers. When demands are high, the EMSR model protects additional seats for the top classes, Y and M, and fewer seats are allocated to the lower priced classes, B and Q. This can be seen particularly well from the high demand booking limits for the ATL-MIA flight leg, also shown in Table 5.8.

5.2.2 Prorated EMSR

The application of the prorated EMSR method to the small Atlanta hub network is the same as the standard leg based EMSR method with one exception. The first step in determining the flight leg fares is to calculate the *prorated* fares for each origin-destination/fare class combination on each of its respective flight legs. As described in section 4.1.3, the prorated fare for a O-D/fare class combination on a

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	7	2	$575	150
	MIABOS	Y	8	3	$403	150
	SAVBOS	Y	4	2	$319	150
	ATLBOS	Y	9	2	$310	150
	LAXBOS	M	5	3	$380	125
	MIABOS	M	4	2	$314	125
	ATLBOS	M	7	2	$290	125
	SAVBOS	M	5	2	$250	125
	LAXBOS	B	8	3	$159	96
	MIABOS	B	11	3	$124	96
	SAVBOS	B	8	4	$109	96
	ATLBOS	B	8	3	$95	96
	LAXBOS	Q	18	4	$139	63
	MIABOS	Q	15	3	$89	63
	ATLBOS	Q	13	3	$69	63
	SAVBOS	Q	20	6	$69	63
ATL-MIA	LAXMIA	Y	13	3	$477	150
	BOSMIA	Y	8	3	$403	150
	ATLMIA	Y	15	3	$280	150
	SAVMIA	Y	10	4	$226	150
	BOSMIA	M	4	2	$314	106
	LAXMIA	M	8	3	$239	106
	ATLMIA	M	7	3	$209	106
	SAVMIA	M	3	2	$168	106
	LAXMIA	B	5	1	$139	78
	BOSMIA	B	11	3	$124	78
	ATLMIA	B	5	2	$94	78
	SAVMIA	B	5	2	$84	78
	LAXMIA	Q	11	2	$119	50
	BOSMIA	Q	15	3	$89	50
	ATLMIA	Q	11	2	$59	50
	SAVMIA	Q	19	5	$59	50
MIA-ATL	MIALAX	Y	13	3	$477	150
	MIABOS	Y	8	3	$403	150
	MIAATL	Y	15	3	$280	150
	MIASAV	Y	10	4	$226	150
	MIABOS	M	4	2	$314	106
	MIALAX	M	8	3	$239	106
	MIAATL	M	7	3	$209	106
	MIASAV	M	3	2	$168	106
	MIALAX	B	5	1	$139	78
	MIABOS	B	11	3	$124	78
	MIAATL	B	5	2	$94	78
	MIASAV	B	5	2	$84	78
	MIALAX	Q	11	2	$119	50
	MIABOS	Q	15	3	$89	50
	MIAATL	Q	11	2	$59	50
	MIASAV	Q	19	5	$59	50

Table 5.6: Leg Based EMSR Booking Limits - Medium Demand Level

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	5	1	$575	150
	MIABOS	Y	5	2	$403	150
	SAVBOS	Y	3	1	$319	150
	ATLBOS	Y	6	2	$310	150
	LAXBOS	M	3	2	$380	133
	MIABOS	M	3	1	$314	133
	ATLBOS	M	5	1	$290	133
	SAVBOS	M	3	2	$250	133
	LAXBOS	B	5	2	$159	114
	MIABOS	B	7	2	$124	114
	SAVBOS	B	5	2	$109	114
	ATLBOS	B	5	2	$95	114
	LAXBOS	Q	12	2	$139	92
	MIABOS	Q	10	2	$89	92
	ATLBOS	Q	9	2	$69	92
	SAVBOS	Q	13	4	$69	92

Table 5.7: Leg Based EMSR Booking Limits - Low Demand Level

given flight leg is based on the proportion of the flight leg's mileage relative to the total O-D itinerary mileage. The prorated fares for each O-D/fare class by flight leg are given in Table 5.9.

Once the fares have been prorated, these prorated fares are averaged to find the respective leg based fares. The average leg based fares are weighted according to mean expected demands as before, and the flight leg means and standard deviations are calculated as in the standard leg based EMSR application. An example of the prorated fares for the medium demand level, along with the demand means and standard deviations, is provided in Table 5.10. The flight leg means and standard deviations are the same as those in Table 5.4, but the average fares are significantly different.

The EMSR model is applied to each flight leg as in the standard leg based EMSR method and flight leg booking limits are determined by fare class. Table 5.11 gives

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	9	3	$575	150
	MIABOS	Y	11	3	$403	150
	SAVBOS	Y	5	3	$319	150
	ATLBOS	Y	12	3	$310	150
	LAXBOS	M	7	3	$380	117
	MIABOS	M	5	2	$314	117
	ATLBOS	M	9	3	$290	117
	SAVBOS	M	7	3	$250	117
	LAXBOS	B	11	3	$159	79
	MIABOS	B	14	4	$124	79
	SAVBOS	B	11	5	$109	79
	ATLBOS	B	11	4	$95	79
	LAXBOS	Q	24	5	$139	35
	MIABOS	Q	20	4	$89	35
	ATLBOS	Q	17	4	$69	35
	SAVBOS	Q	27	8	$69	35
ATL-MIA	LAXMIA	Y	17	4	$477	150
	BOSMIA	Y	11	3	$403	150
	ATLMIA	Y	20	4	$280	150
	SAVMIA	Y	13	6	$226	150
	BOSMIA	M	5	2	$314	92
	LAXMIA	M	11	4	$239	92
	ATLMIA	M	9	4	$209	92
	SAVMIA	M	4	2	$168	92
	LAXMIA	B	7	2	$139	55
	BOSMIA	B	14	4	$124	55
	ATLMIA	B	7	2	$94	55
	SAVMIA	B	7	3	$84	55
	LAXMIA	Q	14	3	$119	17
	BOSMIA	Q	20	4	$89	17
	ATLMIA	Q	14	3	$59	17
	SAVMIA	Q	25	7	$59	17

Table 5.8: Leg Based EMSR Booking Limits - High Demand Level

ORIG - DEST (MILES)	LEG (MILES)	Y	M	B	Q
ATLBOS/BOSATL 946	ATLBOS 946	$310.00	$290.00	$95.00	$69.00
ATLSAV/SAVATL 215	ATLSAV 215	$159.00	$140.00	$64.00	$49.00
ATLMIA/MIAATL 595	ATLMIA 595	$280.00	$209.00	$94.00	$59.00
ATLLAX/LAXATL 1946	ATLLAX 1946	$455.00	$391.00	$142.00	$122.00
BOSSAV/SAVBOS 1161	ATLBOS 946	$259.93	$203.70	$88.81	$56.22
	ATLSAV 215	$59.07	$46.30	$20.19	$12.78
TOTAL		$319.00	$250.00	$109.00	$69.00
BOSMIA/MIABOS 1541	ATLBOS 946	$247.40	$192.76	$76.12	$54.64
	ATLMIA 595	$155.60	$121.24	$47.88	$34.36
TOTAL		$403.00	$314.00	$124.00	$89.00
BOSLAX/LAXBOS 2892	ATLBOS 946	$188.09	$124.30	$52.01	$45.47
	ATLLAX 1946	$386.91	$255.70	$106.99	$93.53
TOTAL		$575.00	$380.00	$159.00	$139.00
MIASAV/SAVMIA 810	ATLMIA 595	$166.01	$123.41	$61.70	$43.34
	ATLSAV 215	$59.99	$44.59	$22.30	$15.66
TOTAL		$226.00	$168.00	$84.00	$59.00
MIALAX/LAXMIA 2541	ATLMIA 595	$111.69	$55.96	$32.55	$27.86
	ATLLAX 1946	$365.31	$183.04	$106.45	$91.14
TOTAL		$477.00	$239.00	$139.00	$119.00
LAXSAV/SAVLAX 2161	ATLLAX 1946	$452.06	$405.23	$138.68	$120.67
	ATLSAV 215	$49.94	$44.77	$15.32	$13.33
TOTAL		$502.00	$450.00	$154.00	$134.00

Table 5.9: Prorated Fares

LEG	CLASS	SEGMENT	O - D MEAN	O - D SD	O - D FARE	FLIGHT LEG MEAN	FLIGHT LEG SD	FLIGHT LEG FARE
ATL-BOS	Y	ATLBOS	9	2	$310.00	28	4.58	$254.48
		SAVBOS	4	2	$259.93			
		MIABOS	8	3	$247.40			
		LAXBOS	7	2	$188.09			
	M	ATLBOS	7	2	$290.00	21	4.58	$211.48
		SAVBOS	5	2	$203.70			
		MIABOS	4	2	$192.76			
		LAXBOS	5	3	$124.30			
	B	ATLBOS	8	3	$95.00	35	6.56	$77.83
		SAVBOS	8	4	$88.81			
		MIABOS	11	3	$76.12			
		LAXBOS	8	3	$52.01			
	Q	ATLBOS	13	3	$69.00	66	8.37	$55.45
		SAVBOS	20	6	$56.22			
		MIABOS	15	3	$54.64			
		LAXBOS	18	4	$45.47			
ATL-MIA	Y	ATLMIA	15	3	$280.00	46	6.56	$186.02
		BOSMIA	8	3	$155.60			
		SAVMIA	10	4	$166.01			
		LAXMIA	13	3	$111.69			
	M	ATLMIA	7	3	$209.00	22	5.10	$125.72
		BOSMIA	4	2	$121.24			
		SAVMIA	3	2	$123.41			
		LAXMIA	8	3	$55.96			
	B	ATLMIA	5	2	$94.00	26	4.24	$56.46
		BOSMIA	11	3	$47.88			
		SAVMIA	5	2	$61.70			
		LAXMIA	5	1	$32.55			
	Q	ATLMIA	11	2	$59.00	56	6.48	$40.97
		BOSMIA	15	3	$34.36			
		SAVMIA	19	5	$43.34			
		LAXMIA	11	2	$27.86			

Table 5.10: Prorated EMSR Inputs - Medium Demand Level

an example of the recommended booking limits using prorated fares. Comparing these booking limits with the limits found using the standard leg based EMSR method (with non-prorated fares), Table 5.6, there is almost no difference. Since the basic methodology behind the two methods is the same, one would not expect the limits to be significantly different. Yet, to have so little difference is noteworthy since prorated and non-prorated fare class ratios were judged to be significantly different in a previous analysis of fare levels.

From Equation 4.6, we know that the number of seats protected for a higher fare class over a lower fare class is directly proportional to the fare ratio between the two classes. In turn, the booking limit for a lower fare class is dependent on the seats protected for the higher fare classes. Since there is very little difference between booking limits obtained from the leg based EMSR method and the prorated EMSR method, the difference in the ratios between the non-prorated and prorated fares does not appear to have a significant impact on booking limits. From the comparison of the two EMSR methods, we find that by averaging the O-D fares from an overall network application, any significant difference in the prorated and non-prorated fare ratios is lost. Even when the demand is high, corresponding to higher aggregate standard deviations (Table 5.5), the difference between the prorated and non-prorated average fare ratios is still not significant enough to effect the booking limits substantially, as can be seen by comparing the prorated EMSR limits for the high demand scenario given in Table 5.12 with those of the leg based EMSR method in Table 5.8. Using prorated versus non-prorated fares does not appear to make a significant difference in fare class booking limits when averaging fares and equating expected marginal revenues by flight leg for an entire network. This might not be true for sets of O-D itinerary demand levels that have a very high proportion of long-haul or short-haul itineraries.

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	7	2	$575	150
	MIABOS	Y	8	3	$403	150
	SAVBOS	Y	4	2	$319	150
	ATLBOS	Y	9	2	$310	150
	LAXBOS	M	5	3	$380	126
	MIABOS	M	4	2	$314	126
	ATLBOS	M	7	2	$290	126
	SAVBOS	M	5	2	$250	126
	LAXBOS	B	8	3	$159	96
	MIABOS	B	11	3	$124	96
	SAVBOS	B	8	4	$109	96
	ATLBOS	B	8	3	$95	96
	LAXBOS	Q	18	4	$139	62
	MIABOS	Q	15	3	$89	62
	ATLBOS	Q	13	3	$69	62
	SAVBOS	Q	20	6	$69	62
ATL-MIA	LAXMIA	Y	13	3	$477	150
	BOSMIA	Y	8	3	$403	150
	ATLMIA	Y	15	3	$280	150
	SAVMIA	Y	10	4	$226	150
	BOSMIA	M	4	2	$314	106
	LAXMIA	M	8	3	$239	106
	ATLMIA	M	7	3	$209	106
	SAVMIA	M	3	2	$168	106
	LAXMIA	B	5	1	$139	77
	BOSMIA	B	11	3	$124	77
	ATLMIA	B	5	2	$94	77
	SAVMIA	B	5	2	$84	77
	LAXMIA	Q	11	2	$119	49
	BOSMIA	Q	15	3	$89	49
	ATLMIA	Q	11	2	$59	49
	SAVMIA	Q	19	5	$59	49

Table 5.11: Prorated EMSR Booking Limits - Medium Demand Level

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	9	3	$575	150
	MIABOS	Y	11	3	$403	150
	SAVBOS	Y	5	3	$319	150
	ATLBOS	Y	12	3	$310	150
	LAXBOS	M	7	3	$380	118
	MIABOS	M	5	2	$314	118
	ATLBOS	M	9	3	$290	118
	SAVBOS	M	7	3	$250	118
	LAXBOS	B	11	3	$159	79
	MIABOS	B	14	4	$124	79
	SAVBOS	B	11	5	$109	79
	ATLBOS	B	11	4	$95	79
	LAXBOS	Q	24	5	$139	33
	MIABOS	Q	20	4	$89	33
	ATLBOS	Q	17	4	$69	33
	SAVBOS	Q	27	8	$69	33

Table 5.12: Prorated EMSR Booking Limits - High Demand Level

5.2.3 Virtual Nesting EMSR

The virtual nesting EMSR technique, like the other EMSR methods, focuses on the individual flight leg, determining seat allocations to maximize revenue on each separate leg. The difference is that instead of being applied to the fare classes on a flight leg, the EMSR model is applied to a set of virtual inventory classes. These virtual classes are based on dollar ranges corresponding to passenger itinerary ticket revenue. In this analysis, the virtual nesting EMSR model is based on eight virtual inventory classes. These virtual classes were defined in section 4.2, Table 4.1.

Once the virtual class fare ranges are determined, each O-D itinerary and fare class combination is assigned to a virtual class. The O-D/fare class combinations are listed according to their assigned virtual classes for each flight leg in Table 5.13. Each O-D/fare class combination is listed using a three letter code. The three letters correspond to the city origin, the destination, and the fare class code. For example,

VIRTUAL CLASS	FLIGHT LEG ATL-BOS BOS-ATL	ATL-MIA MIA-ATL	ATL-LAX LAX-ATL	ATL-SAV SAV-ATL
1	MBY/BMY LBY/BLY	BMY/MBY LMY/MLY	ALY/LAY BLY/LBY MLY/LMY SLY/LSY SLM/LSM	LSY/SLY LSM/SLM
2	ABY/BAY ABM/BAM SBY/BSY MBM/BMM LBM/BLM	AMY/MAY BMM/MBM	ALM/LAM BLM/LBM	BSY/SBY
3	SBM/BSM	AMM/MAM SMY/MSY SMM/MSM LMM/MLM	MLM/LMM	BSM/SBM MSY/SMY MSM/SMM
4	LBB/BLB LBQ/BLQ	LMB/MLB	ALB/LAB BLB/LBB BLQ/LBQ MLB/LMB SLB/LSB SLQ/LSQ	ASY/SAY ASM/SAM LSB/SLB LSQ/SLQ
5	MBB/BMB	BMB/MBB LMQ/MLQ	ALQ/LAQ MLQ/LMQ	
6	ABB/BAB SBB/BSB			BSB/SBB
7	MBQ/BMQ	AMB/MAB BMQ/MBQ SMB/MSB		MSB/SMB
8	ABQ/BAQ SBQ/BSQ	AMQ/MAQ SMQ/MSQ		ASB/SAB ASQ/SAQ BSQ/SBQ MSQ/SMQ

Table 5.13: O-D/Fare Class Virtual Inventory Assignments

MBY stands for Miami-Boston Y class and SLQ for Savannah-Los Angeles Q class. Note that the defined fare ranges for each virtual inventory class do not change between flight legs, although this could also be possible.

After the O-D/fare classes have been assigned to their respective virtual classes, the actual seat inventory control process begins. The EMSR algorithm is applied to the virtual classes, and recommended booking limits are determined for each virtual class. Before applying the EMSR model, the mean demand, standard deviation, and average fare must be derived for each virtual class. This is done in the same manner as for the leg based EMSR, but instead of combining demand and fare data from the O-D itineraries based on a common fare class, the means, standard deviations, and fares of the different O-D/fare class combinations assigned to a virtual class are aggregated. Table 5.14 gives an example of the make-up of the virtual classes and the derived means, standard deviations, and average fares for the ATL-BOS and BOS-ATL flight legs.

There are some cases in which there is no O-D/fare class associated with a virtual inventory class for a given flight leg, such as in the $6th$, $7th$, and $8th$ virtual classes on the ATL-LAX and LAX-ATL flight legs. When such a situation arises, although there is no demand (i.e. the mean and standard deviation are 0), an average fare must be determined for input into the EMSR model. The EMSR model requires these fares in order to determine seat protection between classes, although it is actually only a formality needed in implementing the EMSR model. In the case were there is no demand in the lowest classes, as on the ATL-LAX and LAX-ATL flight legs, all seats which are not protected for higher virtual classes can simply be "dumped" into the lowest virtual inventory which is offered on the flight leg, which in this example is the $5th$ virtual class. In cases where there is no demand in a

ATL-BOS/BOS-ATL

VIRTUAL CLASS		MEDIUM DEMAND O-D CLASS MEAN	SD	FARE	MEDIUM DEMAND VIRTUAL CLASS MEAN	SD	FARE	LOW DEMAND O-D CLASS MEAN	SD	FARE	LOW DEMAND VIRTUAL CLASS MEAN	SD	FARE	HIGH DEMAND O-D CLASS MEAN	SD	FARE	HIGH DEMAND VIRTUAL CLASS MEAN	SD	FARE
8	ABQ/BAQ	13	3	$69	33	6.71	$69.00	9	2	$69	22	4.47	$69.00	17	4	$69	44	8.94	$69.00
	SBQ/BSQ	20	6	$69				13	4	$69				27	8	$69			
7	MBQ/BMQ	15	3	$89	15	3.00	$89.00	10	2	$89	10	2.00	$89.00	20	4	$89	20	4.00	$89.00
6	ABB/BAB	8	3	$95	16	5.00	$102.00	5	2	$95	10	2.83	$102.00	11	4	$95	22	6.40	$102.00
	SBB/BSB	8	4	$109				5	2	$109				11	5	$109			
5	MBB/BMB	11	3	$124	11	3.00	$124.00	7	2	$124	7	2.00	$124.00	14	4	$124	14	4.00	$124.00
4	LBB/BLB	8	3	$159	26	5.00	$145.15	5	2	$159	17	2.83	$144.88	11	3	$159	35	5.83	$145.29
	LBQ/BLQ	18	4	$139				12	2	$139				24	5	$139			
3	SBM/BSM	5	2	$250	5	2.00	$250.00	3	2	$250	3	2.00	$250.00	7	3	$250	7	3.00	$250.00
2	ABY/BAY	9	2	$310	29	5.00	$319.03	6	2	$310	20	3.32	$317.45	12	3	$310	38	6.32	$319.87
	ABM/BAM	7	2	$290				5	1	$290				9	3	$290			
	SBY/BSY	4	2	$319				3	1	$319				5	3	$319			
	MBM/BMM	4	2	$314				3	1	$314				5	2	$314			
	LBM/BLM	5	3	$380				3	2	$380				7	3	$380			
1	MBY/BMY	8	3	$403	15	3.61	$483.27	5	2	$403	10	2.24	$489.00	11	3	$403	20	4.24	$480.40
	LBY/BLY	7	2	$575				5	1	$575				9	3	$575			

Table 5.14: Virtual Nesting EMSR Inputs For ATL-BOS/BOS-ATL

middle virtual class, such as the $6th$ virtual class on the ALT-MIA and MIA-ATL flight legs, any seats which have not been allocated to the higher virtual classes—1, 2, 3, 4, or 5, but should be protected for these classes over the $7th$ and $8th$ virtual classes, can be added to seats allotted to the $5th$ virtual class. The EMSR model, on the other hand, would allocate such seats to the $6th$ virtual class, which is actually not offered on the flights.

In the EMSR model, the number of seats to protect for a higher class is dependent on the fare ratio between the two classes being evaluated. In cases where there is no O-D itinerary assigned to a given virtual class, the average fare for the virtual finventory class is assumed to be the mean of the respective fare range, except for the lowest virtual class, Virtual Class 8. For this virtual class, the average fare is based on the mean between \$50 and \$74.99. Table 5.15 shows an example of the inputs for the situation where there are no O-D fares to aggregate.

In determining the average fare for the $8th$ virtual class, the range is assumed to have a lower limit of \$50 instead of \$0 when finding the range mean. This is because there are only a few fares below \$50, and at the same time, there are a disproportionate amount of O-D itinerary fares on the higher side of the \$50 to \$74.99 range which counter-balances the fares below \$50. Previous analysis of fare data frequency distributions resulted in an overall mean fare of \$63.83 for the 8th virtual class, very close to the \$62.50 mean fare value assumed.

Based on the virtual class means, standard deviations, and average fares, an eight class EMSR model is run on each leg of the network. The outcome of this procedure is a recommended booking limit for each virtual class on each flight leg. Table 5.16 shows these virtual inventory limits for the medium demand scenario.

ATL-LAX/LAX-ATL

VIRTUAL CLASS		MEDIUM DEMAND O-D CLASS MEAN	SD	FARE	MEDIUM DEMAND VIRTUAL CLASS MEAN	SD	FARE	LOW DEMAND O-D CLASS MEAN	SD	FARE	LOW DEMAND VIRTUAL CLASS MEAN	SD	FARE	HIGH DEMAND O-D CLASS MEAN	SD	FARE	HIGH DEMAND VIRTUAL CLASS MEAN	SD	FARE
8					0	0.00	$62.50				0	0.00	$62.50				0	0.00	$62.50
7					0	0.00	$85.00				0	0.00	$85.00				0	0.00	$85.00
6					0	0.00	$102.50				0	0.00	$102.50				0	0.00	$102.50
5	ALQ/LAQ	20	5	$122	31	5.39	$120.94	13	3	$122	20	3.61	$120.95	27	7	$122	41	7.62	$120.98
	MLQ/LMQ	11	2	$119				7	2	$119				14	3	$119			
4	ALB/LAB	8	2	$142	68	9.11	$141.04	5	2	$142	44	6.24	$140.82	11	3	$142	92	11.49	$141.15
	BLB/LBB	8	3	$159				5	2	$159				11	3	$159			
	BLQ/LBQ	18	4	$139				12	2	$139				24	5	$139			
	MLB/LMB	5	1	$139				3	1	$139				7	2	$139			
	SLB/LSB	5	2	$154				3	1	$154				7	2	$154			
	SLQ/LSQ	24	7	$134				16	5	$134				32	9	$134			
3	MLM/LMM	8	3	$239	8	3.00	$239.00	5	2	$239	5	2.00	$239.00	11	4	$239	11	4.00	$239.00
2	ALM/LAM	3	2	$391	8	3.61	$384.12	2	2	$391	5	2.83	$384.40	4	3	$391	11	4.24	$384.00
	BLM/LBM	5	3	$380				3	2	$380				7	3	$380			
1	ALY/LAY	6	2	$455	35	5.48	$491.83	4	1	$455	24	3.74	$493.50	8	2	$455	46	6.86	$490.96
	BLY/LBY	7	2	$575				5	1	$575				9	3	$575			
	MLY/LMY	13	3	$477				9	2	$477				17	4	$477			
	SLY/LSY	4	2	$502				3	2	$502				5	3	$502			
	SLM/LSM	5	3	$450				3	2	$450				7	3	$450			

Table 5.15: Virtual Nesting EMSR Inputs For ATL-LAX/LAX-ATL

LEG	CAPACITY	VIRTUAL CLASS	DEMAND	STD DEVI	AVERAGE REVENUE	SEATS ALLOC	BK LIMIT
ATL-BOS BOS-ATL	150	1	15	3.61	$483.27	14	150
		2	29	5.00	$319.03	27	136
		3	5	2.00	$250.00	11	109
		4	26	5.00	$145.15	24	98
		5	11	3.00	$124.00	13	74
		6	16	5.00	$102.00	14	61
		7	15	3.00	$89.00	21	47
		8	33	6.71	$69.00	26	26
ATL-MIA MIA-ATL	150	1	21	4.24	$448.81	20	150
		2	19	3.61	$287.16	19	130
		3	28	6.16	$219.25	31	111
		4	5	1.00	$139.00	6	80
		5	22	3.61	$121.50	23	74
		6	0	0.00	$102.50	2	51
		7	25	4.12	$89.00	32	49
		8	30	5.39	$59.00	17	17
ATL-LAX LAX-ATL	150	1	35	5.48	$491.83	31	150
		2	8	3.61	$384.12	12	119
		3	8	3.00	$239.00	14	107
		4	68	9.11	$141.04	59	93
		5	31	5.39	$120.94	33	34
		6	0	0.00	$102.50	1	1
		7	0	0.00	$85.00	0	0
		8	0	0.00	$62.50	0	0
ATL-SAV SAV-ATL	150	1	9	3.61	$473.11	8	150
		2	4	2.00	$319.00	5	142
		3	18	4.90	$223.00	20	137
		4	53	8.12	$145.42	48	117
		5	0	0.00	$120.00	3	69
		6	8	4.00	$109.00	13	66
		7	5	2.00	$84.00	11	53
		8	53	8.43	$61.26	42	42

Table 5.16: "Virtual Class" Booking Limits - Medium Demand Level

Notice how two seats have been allocated to the unused, 6*th* virtual class on the ATL-MIA and MIA-ATL flights. This is a situation in which these two seats have not been allocated to the five highest virtual classes, but should be protected for these classes over the 7*th* and 8*th* virtual classes. Therefore, the seats have been allotted to the 6*th* class, when in actuality they belong to the expected demand for the top five virtual classes, since there is no demand in the 6*th* class.

The booking limits in Table 5.16 are not the limits which are shown in availability displays to reservations agents. Once the virtual class booking limits are determined, they are matched back to the O-D/fare class combinations offered to the public. The booking limit for each O-D itinerary/fare class is simply the booking limit of its corresponding virtual class. Table 5.17 gives an example of the disaggregated passenger itinerary possibilities with their respective booking limits under medium demand levels for the ATL-BOS and ATL-MIA flight legs. Notice how the O-D pairs are nested entirely by fare amount. By applying the EMSR model to virtual classes, long-haul passengers with higher itinerary ticket revenue are always favored over short-haul passengers, in spite of the possibility that the combined revenue of two short-haul passengers could result in a higher total revenue.

Table 5.18 gives O-D/fare class booking limits for the ATL-BOS and SAV-ATL flight legs under high demand conditions. By using the virtual nesting EMSR method, some O-D itineraries within a given fare class can be distinguished between others. Under high demand, two Q class O-D itineraries on the ATL-BOS flight leg, ATLBOS and SAVBOS, are closed to any bookings, while the other Q class O-D itineraries, LAXBOS and MIABOS, are open. At the same time, the booking limit on the LAXBOS Q class itinerary is 81 while the MIABOS Q class

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	7	2	$575	150
	MIABOS	Y	8	3	$403	150
	LAXBOS	M	5	3	$380	136
	SAVBOS	Y	4	2	$319	136
	MIABOS	M	4	2	$314	136
	ATLBOS	Y	9	2	$310	136
	ATLBOS	M	7	2	$290	136
	SAVBOS	M	5	2	$250	109
	LAXBOS	B	8	3	$159	98
	LAXBOS	Q	18	4	$139	98
	MIABOS	B	11	3	$124	74
	SAVBOS	B	8	4	$109	61
	ATLBOS	B	8	3	$95	61
	MIABOS	Q	15	3	$89	47
	ATLBOS	Q	13	3	$69	26
	SAVBOS	Q	20	6	$69	26
ATL-MIA	LAXMIA	Y	13	3	$477	150
	BOSMIA	Y	8	3	$403	150
	BOSMIA	M	4	2	$314	130
	ATLMIA	Y	15	3	$280	130
	LAXMIA	M	8	3	$239	111
	SAVMIA	Y	10	4	$226	111
	ATLMIA	M	7	3	$209	111
	SAVMIA	M	3	2	$168	111
	LAXMIA	B	5	1	$139	80
	BOSMIA	B	11	3	$124	74
	LAXMIA	Q	11	2	$119	74
	ATLMIA	B	5	2	$94	49
	BOSMIA	Q	15	3	$89	49
	SAVMIA	B	5	2	$84	49
	ATLMIA	Q	11	2	$59	17
	SAVMIA	Q	19	5	$59	17

Table 5.17: Virtual Nesting Booking Limits - Medium Demand Level

LEG	SEGMENT	CLASS	MEAN DEMAND	STD DEVI	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	9	3	$575	150
	MIABOS	Y	11	3	$403	150
	LAXBOS	M	7	3	$380	131
	SAVBOS	Y	5	3	$319	131
	MIABOS	M	5	2	$314	131
	ATLBOS	Y	12	3	$310	131
	ATLBOS	M	9	3	$290	131
	SAVBOS	M	7	3	$250	96
	LAXBOS	B	11	3	$159	81
	LAXBOS	Q	24	5	$139	81
	MIABOS	B	14	4	$124	50
	SAVBOS	B	11	5	$109	34
	ATLBOS	B	11	4	$95	34
	MIABOS	Q	20	4	$89	14
	ATLBOS	Q	17	4	$69	0
	SAVBOS	Q	27	8	$69	0
SAV-ATL	SAVLAX	Y	5	3	$502	150
	SAVLAX	M	7	3	$450	150
	SAVBOS	Y	5	3	$319	139
	SAVBOS	M	7	3	$250	133
	SAVMIA	Y	13	6	$226	133
	SAVMIA	M	4	2	$168	133
	SAVATL	Y	25	5	$159	107
	SAVLAX	B	7	2	$154	107
	SAVATL	M	7	2	$140	107
	SAVLAX	Q	32	9	$134	107
	SAVBOS	B	11	5	$109	38
	SAVMIA	B	7	3	$84	23
	SAVBOS	Q	27	8	$69	5
	SAVATL	B	5	2	$64	5
	SAVMIA	Q	25	7	$59	5
	SAVATL	Q	13	3	$49	5

Table 5.18: Virtual Nesting Booking Limits - High Demand Level

limit is only 14. Nonetheless, this is still a leg based method which does not take into consideration interaction between flight legs.

Although this is a leg-based system which is consistent with current airline reservations systems, substantial internal re-programming is needed to use such a virtual inventory system. Internal processes need to be developed which aggregate O-D/fare class combinations by virtual class and then disaggregate the O-D/fare class booking limits from virtual class booking limits. The reservations systems must also be re-programmed to make and cancel requests by O-D itineraries. Currently reservations agents are able to book and cancel pieces of a passenger's travel plans, leg by leg. In a virtual inventory system, availabilities are not based simply on fare classes, but on O-D itinerary fare amounts. If seats are available for a multi-leg O-D itinerary on a given flight, seats may not be available for the single-leg local itinerary on the flight leg. Currently, if a seat is available on a flight leg, any passenger can book the seat regardless of his O-D itinerary.

5.2.4 Deterministic Linear Program

The deterministic linear program is a network based formulation. Since it is deterministic, O-D/fare class demand is based on a single value. The mean expected demand of each fare class itinerary demand distribution is used as the deterministic value in this analysis. Since the expected demands are assumed to be normal, the mean value has the highest probability of occurring, and the probability that the actual demand will be less than the mean is the same as the probability of the actual demand being greater than the mean, each being 50%.

Formulating the deterministic linear programming problem is straightforward. Total network revenue is maximized subject to the capacity and demand constraints as in section 4.3, equation 4.8. At the medium level of demand, this formulation becomes:

Maximize

$$
\begin{aligned}
&310\,ABY + 290\,ABM + \ 95\,ABB + \ 69\,ABQ + \\
&159\,ASY + 140\,ASM + \ 64\,ASB + \ 49\,ASQ + \\
&\vdots \\
&502\,SLY + 450\,SLM + 154\,SLB + 134\,SLQ
\end{aligned}
\qquad (5.1)
$$

subject to:

$$
\begin{aligned}
ABY + SBY + MBY + LBY + ABM + \cdots + LBQ &\le 150 \\
ALY + BLY + MLY + SLY + ALM + \cdots + SLQ &\le 150 \\
&\vdots
\end{aligned}
$$

$$
\begin{aligned}
ABY &\le 9 \\
ABM &\le 7 \\
ABB &\le 8 \\
ABQ &\le 13 \\
BAY &\le 9 \\
&\vdots \\
ASY &\le 19 \\
&\vdots
\end{aligned}
$$

Using a linear programming computer software package to solve this deterministic LP network, the solution obtained is a maximum revenue value based on the best combination of values for each decision variable, ABY, ABM, etc, which comply with the given constraints. By specifying that all decision variables must be integer values, since it is impossible to have a fraction of a seat or a fraction of a passenger, the solution is the optimal number of seats to allocate to each O-D/fare class itinerary on every flight leg in the network.

Table 5.19 gives these seat allocations for each O-D itinerary and fare class under medium demand conditions. Below each seat allocation is the mean demand, standard deviation and fare for each O-D/fare class combination. In this case, where the level of demand on each flight leg equals capacity, the number of seats allocated to each O-D/fare class is the same as the mean demand.

When the demand level is high, the assumed mean demand for each leg is 133% of the medium demand level, or approximately 200 per flight leg. In this situation, the 150 best O-D/fare class itineraries per flight leg, in terms of system-wide revenue, are chosen. These O-D/fare class seat allocations are given in Table 5.20. In order to determine these seat allocations, the same objective function and capacity constraints hold, as described in the medium demand level problem. The only difference in the formulation is the demand constraints, in which the O-D/fare class variables are set less than or equal to the high mean demand values rather than the medium values. In the process of selecting the best O-D/fare class combinations, the interaction between flight legs is taken into account.

In the seat allocations for the high demand level, the BOSSAV and SAVBOS Q class allocations are not the same, even though the network is symmetrical, both in physical routing as well as demand distribution. For the BOSSAV Q class itinerary, 3 seats are allocated, while 8 seats are allocated to the SAVBOS Q class itinerary.

	Y	M	B	Q
ATLBOS/BOSATL	9	7	8	13
	9	7	8	13
	2	2	3	3
	$310	$290	$95	$69
ATLSAV/SAVATL	19	5	4	10
	19	5	4	10
	3	2	1	3
	$159	$140	$64	$49
ATLMIA/MIAATL	15	7	5	11
	15	7	5	11
	3	3	2	2
	$280	$209	$94	$59
ATLLAX/LAXATL	6	3	8	20
	6	3	8	20
	2	2	2	5
	$455	$391	$142	$122
BOSSAV/SAVBOS	4	5	8	20
	4	5	8	20
	2	2	4	6
	$319	$250	$109	$69
BOSMIA/MIABOS	8	4	11	15
	8	4	11	15
	3	2	3	3
	$403	$314	$124	$89
BOSLAX/LAXBOS	7	5	8	18
	7	5	8	18
	2	3	3	4
	$575	$380	$159	$139
MIASAV/SAVMIA	10	3	5	19
	10	3	5	19
	4	2	2	5
	$226	$168	$84	$59
MIALAX/LAXMIA	13	8	5	11
	13	8	5	11
	3	3	1	2
	$477	$239	$139	$119
LAXSAV/SAVLAX	4	5	5	24
	4	5	5	24
	2	3	2	7
	$502	$450	$154	$134

Table 5.19: Deterministic LP Seat Allocations - Medium Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	12	9	11	17
	12	9	11	17
	3	3	4	4
	$310	$290	$95	$69
ATLSAV/SAVATL	25	7	5	13
	25	7	5	13
	5	2	2	3
	$159	$140	$64	$49
ATLMIA/MIAATL	20	9	7	14
	20	9	7	14
	4	4	2	3
	$280	$209	$94	$59
ATLLAX/LAXATL	8	4	11	27
	8	4	11	27
	2	3	3	7
	$455	$391	$142	$122
BOSSAV/SAVBOS	5	7	11	3/8
	5	7	11	27
	3	3	5	8
	$319	$250	$109	$69
BOSMIA/MIABOS	11	5	14	18
	11	5	14	20
	3	2	4	4
	$403	$314	$124	$89
BOSLAX/LAXBOS	9	7	11/6	0
	9	7	11	24
	3	3	3	5
	$575	$380	$159	$139
MIASAV/SAVMIA	13	4	7	0
	13	4	7	25
	6	2	3	7
	$226	$168	$84	$59
MIALAX/LAXMIA	17	11	0	0
	17	11	7	14
	4	4	2	3
	$477	$239	$139	$119
LAXSAV/SAVLAX	5	7	7	31/26
	5	7	7	32
	3	3	2	9
	$502	$450	$154	$134

Table 5.20: Deterministic LP Seat Allocations - High Demand Level

This situation also arises in the LAXSAV/SAVLAX Q class seat allocations and in the BOSLAX/LAXBOS B class allotments. This is because the deterministic LP solution to this network problem is not unique. There are many different sets of seat allocations which give the same optimal revenue value. In the solution shown here, 6 seats are allocated to LAXBOS B class, 11 to BOSLAX B class, 8 to SAVBOS Q class, 3 to BOSSAV Q class, 31 to LAXSAV Q class and 26 to SAVLAX Q class. Another solution which would give the same revenue amount would be to allocate 7 seats to LAXBOS B class, 10 to BOSLAX B class, 7 to SAVBOS Q class, 4 to BOSSAV Q class, 30 to LAXSAV Q class and 27 to SAVLAX Q class. Note how the LAXBOS B class increased from 6 to 7 seats allocated, while the BOSLAX B class decreased from 11 to 10 seats allocated. The opposite situation, LAXBOS B class being allocated 5 seats while BOSLAX B class increasing to 12 seats, is not possible since BOSLAX B class is constrained by a mean demand of 11.

In the latter solution, an extra seat was allocated to three of the O-D/fare class combinations, while one less seat was allocated to the opposite direction O-D/fare class combinations. This could continue happening until LAXBOS B class was allocated 11 seats and BOSLAX B class 6, and then the LAXBOS B class would be constrained from increasing any further because of its demand constraint. All of these different seat allocation solutions give the same optimal revenue. There is also a symmetrical solution for these six O-D/fare class combinations, but this solution is not an integer solution, and therefore is not feasible.

Because of the demand constraints affixed to each O-D/fare class combination, when the mean demand on a flight leg is not as great as the capacity of the leg, the seats in excess of the demand are not allocated to any O-D/fare class combination and are left to go empty. This is the case in the low demand scenario where the mean expected demand is 67% of the medium level. Under this demand scenario, as many seats as the mean demand level, and no more, are allocated to each O-D/fare class itinerary. Even if the actual demand, which is probabilistic in nature, happens to be greater than the mean demand level of 100 per flight leg, only 100 seats will be allocated to specific O-D/fare classes on each flight leg, while the remaining 50 seats will be left empty, In practice, these unallocated seats could be used to accommodate excess demand.

The deterministic LP method is a network based solution in which the optimal seat allocations maximize total system revenue, rather than individual flight leg revenue. The problem is that these seat allocations are not nested. This in turn means that if the demand level low, all seats will not be allocated, and the booking limits associated with an O-D/fare class inventory will actually limit the number of reservations which can be accepted, while seats go empty. Because this method is deterministic, requests for an O-D/fare class will be refused on an average of 50% of the flights since the number of seats which can be made available to an O-D/fare class is constrained by the mean expected demand.

5.2.5 Probabilistic Linear Program

The probabilistic linear programming method is also a network based seat inventory control technique, but unlike the deterministic LP approach, the probabilistic LP method accounts for the uncertainty of demand. The probabilistic LP technique allocates seats by optimizing expected marginal revenue over the entire network. In the most detailed approach to the problem, each individual seat on every flight is examined. The O-D/fare class combinations, which contribute the highest expected revenue to the system based on the probability of selling each given seat to an O-D/fare class, are chosen for the solution.

In formulating the probabilistic LP, an integer zero/one variable is used for every seat which could potentially be sold to a fare class and origin-destination pair. Expected revenue is maximized by finding the optimal combination of (0,1) variables. Each (0,1) variable, which represents the option of allocating or not allocating the seat being considered to a given O-D/fare class, is multiplied by the expected marginal revenue of the seat. The sum of all the expected marginal revenues, which are multiplied by either 0 or 1, is maximized subject to flight leg capacity constraints. This probabilistic LP is formulated in equation 4.10. The shortcoming of this appoach is that the required number of binary decision variables becomes very large, very fast. For this small network which connects five different cities with eight flight legs offering four coach classes, the complete probabilistic LP formulation contains 12,000 variables.

There are a number of ways in which to cut down the number of variables needed in the formulation. One cutoff point can be to have j, the number of seats available to each itinerary and fare class combination, range from 1 to $\mu_{i,\text{O-D}} + 3\sigma_{i,\text{O-D}}$, the mean demand plus 3 times the standard deviation, assuming this is less than the capacity of the aircraft on a given flight leg. The number of decision variables can be limited in this way since the probability of selling more than $\mu + 3\sigma$ seats in a given O-D/fare class is 0.00135, which is negligible. In turn, the expected marginal revenue of a seat which has a probability of 0.00135 of selling is very small.

Another cutoff point which lowers the number of decision variables required in the probabilistic LP formulation can be to eliminate any decision variable with an expected marginal seat revenue below a certain dollar amount. The higher the demand, the greater the dollar amount can be. In using this technique to cut down on the number of variables, one must be careful not to set the dollar amount cutoff level too high. Even when the cutoff amount for the expected marginal revenue of one O-D/fare class combination is quite high, other O-D/fare class cutoff levels may be much lower. For example, under the medium demand scenario, the cutoff point of the expected marginal revenues for the BOSLAX Y class itinerary in the network is somewhere between \$177.42 and \$91.22. The $9th$ seat of the BOSLAX Y class itinerary is accepted with an expected marginal revenue of \$177.42 while the $10th$ seat at \$91.22 is not. At the same time, a seat is allocated to the BOSATL Y class at an expected marginal revenue of \$48.18, at least half of the BOSLAX Y class cutoff level. This is possible because the combined expected revenue of the last seats allocated in the BOSATL Y class and ATLLAX Y class is \$48.18 + \$140.39, or \$188.57, which is above the cutoff level for the BOSLAX Y class inventory.

By using different methods to eliminate decision variables in the probabilistic LP formulation, the 12,000 variables needed for this Atlanta-based hub network evaluation were reduced to between 501 and 684 variables, depending on the demand level assumed. Although the reduction methods made this probabilistic LP problem manageable, the size of a typical major airline's network is far to large to make a full-scale probabilistic LP, with reductions, practical to solve with current airline computer capabilites.

The first step in applying the probabilistic LP to the given network is to find the expected revenue for O-D/fare class seat. The expected marginal revenue for each seat is the probability of selling the seat times the revenue obtained from selling the seat, as derived in section 4.1.1. Table 5.21 shows the probability of selling S seats and the expected marginal revenue associated with selling each of the seats in the ATLBOS Y class inventory.

ATLBOS Y - $310

SEAT	$\overline{P}(S)$	EMSR(S)
1	1.00000	$310.00
2	0.99997	$309.99
3	0.99977	$309.93
4	0.99865	$309.58
5	0.99378	$308.07
6	0.97724	$302.94
7	0.93320	$289.29
8	0.84135	$260.82
9	0.69145	$214.35
10	0.50000	$155.00
11	0.30855	$95.65
12	0.15865	$49.18
13	0.06680	$20.71
14	0.02276	$7.06
15	0.00622	$1.93

Table 5.21: $\overline{P}(S)$ and EMSR(S) For ATLBOS Y Class

Using these expected marginal revenues and those associated with other O-D/fare class itineraries, the reduced probabilistic LP formulation for the medium demand level becomes:

Maximize

$$\begin{gathered}310.00\,ABY01 + 309.99\,ABY02 + \cdots + 7.06\,ABY14 + \\ 289.93\,ABM01 + 289.61\,ABM02 + \cdots + 6.60\,ABM12 + \\ \vdots \\ 133.96\,LSQ01 + 133.93\,LSQ02 + \cdots + 2.15\,LSQ40 \end{gathered} \tag{5.2}$$

subject to:

$$\begin{gathered} ABY01 + \cdots + SBY01 + \cdots + MBY01 + \cdots + LBY01 + \cdots \leq 150 \\ ASY01 + \cdots + BSY01 + \cdots + MSY01 + \cdots + LSY01 + \cdots \leq 150 \\ \vdots \end{gathered}$$

$$ABY01 = 0 \text{ or } 1$$

$$ABY02 = 0 \text{ or } 1$$

$$\vdots$$

$$ABY14 = 0 \text{ or } 1$$

$$ABM01 = 0 \text{ or } 1$$

$$\vdots$$

$$ASY01 = 0 \text{ or } 1$$

$$\vdots$$

The capacity constraints are simply a limit on the total number of decision variables accepted (set equal to one) on each flight leg of 150, the aircraft capacity. Note that there are no demand constraints as in the deterministic LP formulation. This is because the objective function includes the demand estimates through the expected marginal revenue values.

Each five character variable in the formulation designates the origin, the destination, the fare class, and the Sth seat (a two digit number) which may be allocated to the O-D/fare class combination. For example, ABY01 is the first seat which can be allocated to the ATLBOS Y class itinerary, while ABY10 would be the tenth seat which could be allocated to the itinerary. ABY10 cannot be accepted and allocated to the ATLBOS Y class itinerary without ABY01, ABY02, and up through ABY09 being accepted.

The solution to the probabilistic LP problem is simply a list of values, 0 or 1, for each of the decision variables. From this list, the number of seats to be allocated to each O-D/fare class combination can be found. This number is either the sum of all the similar O-D/fare class (0,1) decision variables, or it is the seat number of the last decision variable which is equal to 1 for any set of O-D/fare class variables. For example, if $ABY11 = 1$ but $ABY12 = 0$ and $ABY13 = 0$, etc., then the number of seats allocated to the ATLBOS Y class inventory is 11. The results from the medium demand scenario probabilistic LP seat inventory control method are given in Table 5.22.

The probabilistic LP is a network based formulation which takes into account the uncertainty of demand. However, these results assume a non-nested inventory system. The seat allocations are based on distinct inventories, where lower fare class seat allocations are not open to higher fare class bookings, therefore more seats are protected for higher fare class inventories than are required for a similar nested system. For example, the number of seats protected on the ATL-BOS flight leg for Y class under the leg based EMSR model, which is a nested, probabilistic model, is 25 seats. These 25 seats are available to LAXBOS, MIABOS, SAVBOS, and ATLBOS Y class passengers, which have a combined mean demand of 28. Using the non-nested probabilistic LP method, the combined number of seats protected for the four O-D Y class passenger itineraries is 38. The non-nested solution is more conservative and consistently allocates more seats to higher fare classes since there is no chance for high class demand to be accommodated in lower classes, as there is in a nested system which allows any available seat to be booked by a high fare passenger.

Although the probabilistic LP is non-nested, all seats are allocated. Seats are not left empty as in the situation of a low level of demand in the deterministic LP method. The probabilistic LP does not have strict demand constraints on each O-D/fare class. In low demand conditions, the probability of selling every last seat in the given O-D/fare class inventories becomes small, thus making the expected

	Y	M	B	Q
ATLBOS/BOSATL	12	10	9	12
	9	7	8	13
	2	2	3	3
	$310	$290	$95	$69
ATLSAV/SAVATL	23	8	5	11
	19	5	4	10
	3	2	1	3
	$159	$140	$64	$49
ATLMIA/MIAATL	19	10	6	11
	15	7	5	11
	3	3	2	2
	$280	$209	$94	$59
ATLLAX/LAXATL	8	5	8	19
	6	3	8	20
	2	2	2	5
	$455	$391	$142	$122
BOSSAV/SAVBOS	6	7	8	12
	4	5	8	20
	2	2	4	6
	$319	$250	$109	$69
BOSMIA/MIABOS	11	6	10	11
	8	4	11	15
	3	2	3	3
	$403	$314	$124	$89
BOSLAX/LAXBOS	9	7	6	14
	7	5	8	18
	2	3	3	4
	$575	$380	$159	$139
MIASAV/SAVMIA	13	4	5	9
	10	3	5	19
	4	2	2	5
	$226	$168	$84	$59
MIALAX/LAXMIA	16	9	4	6
	13	8	5	11
	3	3	1	2
	$477	$239	$139	$119
LAXSAV/SAVLAX	6	8	5	20
	4	5	5	24
	2	3	2	7
	$502	$450	$154	$134

Table 5.22: Probabilistic LP Seat Allocations - Medium Demand Level

marginal revenue associated with these seats small. Nonetheless, the seats continue to contribute some revenue to the system and will be allocated to an O-D/fare class inventory. The final results obtained are considered to be the "correct" seat allocations for the given network. The problem is determining how this optimal network solution can be implemented in nested reservations systems.

5.2.6 Deterministic LP Nested on Shadow Prices

The deterministic LP method nested on shadow prices is simply the non-nested deterministic LP solution incorporated into a nested fare class structure. The network is formulated exactly as in the deterministic LP approach. The linear programming formulation is then solved and the integer value assigned to each decision variable is the number of seats allocated to the corresponding O-D/fare class combination. In the output of most software packages for linear programming problems, not only are values, which optimize the network formulation, assigned to the decision variables, but for each decision variable a shadow price or reduced cost is also given.

As decribed in Section 4.5, a shadow price (or reduced cost) associated with a decision variable is the amount the optimal system revenue value would increase (or decrease) if one more unit of the variable is used. Variables with the highest shadow prices will increase the overall network revenue the most if their value is allowed to be incremented by one unit. Therefore, these shadow prices can be used as nesting variables. The O-D/fare class combinations with the highest shadow price nesting variables, based on a network formulation, will rank above other O-D/fare class combinations in the nesting order of a flight leg.

Once the nesting order is determined for a given flight leg, the nested booking limits can be found for each O-D/fare class combination on the flight leg. The booking limit for an O-D/fare class inventory is simply the capacity of the aircraft minus the total number of seats allocated to O-D/fare class combinations nested above the O-D/fare class inventory. Table 5.23 gives examples of the nesting order and booking limits, under the medium demand scenario, for three flight legs of the Atlanta-based network.

Table 5.23 also gives examples of the nesting variables (shadow prices) for the O-D itineraries on the ATL-BOS, BOS-ATL and MIA-ATL flight legs. On the BOS-ATL flight leg, the nesting variable for the BOSSAV Y class itinerary is 250. This means that by increasing the number of seats allocated to the BOSSAV Y class from 4 to 5, the optimal network revenue will increase by $250. Only 4 seats were initially allocated to BOSSAV Y class in this deterministic LP formulation because of the mean demand constraint of 4. Since actual demand is probabilistic in nature, the BOSSAV Y demand may be 5, 6 or even more. In situations when demand is greater than the mean expected value of 4, extra seats can be made available to BOSSAV Y class passengers through nesting, allowing additional bookings to be accepted and a potentially higher network revenue obtained.

Table 5.23 shows an example of the nesting order determined from using shadow prices. The origin-destination segments are listed in order of their booking limits. The fares for each O-D segment are also given. It is evident that the nesting order is not determined solely by fares or classes. For example, on the BOS-ATL flight leg, the BOSATL M class itinerary (with a fare of $290) is ranked above the BOSLAX M class and BOSSAV Y class itineraries with respective fares of $380 and $319. This is because the shadow price of the next seat sold to a BOSATL M class itinerary is

LEG	SEGMENT	CLASS	MEAN DEMAND	SEATS ALLOC	NESTING VARIABLE	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	7	7	446	$575	150
	MIABOS	Y	8	8	324	$403	143
	LAXBOS	M	5	5	251	$380	135
	SAVBOS	Y	4	4	250	$319	130
	ATLBOS	Y	9	9	241	$310	126
	MIABOS	M	4	4	235	$314	117
	ATLBOS	M	7	7	221	$290	113
	SAVBOS	M	5	5	181	$250	106
	MIABOS	B	11	11	45	$124	101
	SAVBOS	B	8	8	40	$109	90
	LAXBOS	B	8	8	30	$159	82
	ATLBOS	B	8	8	26	$95	74
	LAXBOS	Q	18	18	10	$139	66
	MIABOS	Q	15	15	10	$89	48
	SAVBOS	Q	20	20	0	$69	33
	ATLBOS	Q	13	13	0	$69	13
BOS-ATL	BOSLAX	Y	7	7	446	$575	150
	BOSMIA	Y	8	8	324	$403	143
	BOSATL	Y	9	9	290	$310	135
	BOSATL	M	7	7	270	$290	126
	BOSLAX	M	5	5	251	$380	119
	BOSSAV	Y	4	4	250	$319	114
	BOSMIA	M	4	4	235	$314	110
	BOSSAV	M	5	5	181	$250	106
	BOSATL	B	8	8	75	$95	101
	BOSATL	Q	13	13	49	$69	93
	BOSMIA	B	11	11	45	$124	80
	BOSSAV	B	8	8	40	$109	69
	BOSLAX	B	8	8	30	$159	61
	BOSLAX	Q	18	18	10	$139	53
	BOSMIA	Q	15	15	10	$89	53
	BOSSAV	Q	20	20	0	$69	20
MIA-ATL	MIALAX	Y	13	13	358	$477	150
	MIABOS	Y	8	8	324	$403	137
	MIAATL	Y	15	15	270	$280	129
	MIABOS	M	4	4	235	$314	114
	MIAATL	M	7	7	199	$209	110
	MIASAV	Y	10	10	167	$226	103
	MIALAX	M	8	8	120	$239	93
	MIASAV	M	3	3	109	$168	85
	MIAATL	B	5	5	84	$94	82
	MIAATL	Q	11	11	49	$59	77
	MIABOS	B	11	11	45	$124	66
	MIASAV	B	5	5	25	$84	55
	MIALAX	B	5	5	20	$139	50
	MIABOS	Q	15	15	10	$89	45
	MIALAX	Q	11	11	0	$119	30
	MIASAV	Q	19	19	0	$59	30

Table 5.23: Nested Deterministic LP Booking Limits - Medium Demand Level

270, versus 251 and 250 for the next seats sold to a BOSLAX M class itinerary or a BOSSAV Y class itinerary, respectively.

In this deterministic LP method, the entire network's revenue is considered in allocating seats, not just each individual leg's revenue. The nesting order can be different for each leg and direction as can be seen by the difference between the nesting of the ATL-BOS leg and the BOS-ATL leg in Table 5.23. Another important difference in the nesting order of this deterministic LP method is how short-haul markets (which usually have a lower ticket revenue than long-haul markets in the same class) can rank above some long-haul market pairs. For example, on the BOS-ATL leg, the short-haul BOSATL M class is ranked above the BOSSAV Y class as well as the other O-D M class pairs, and the BOSATL B class and Q class are both ranked above all the other O-D B class pairs. The MIA-ATL flight leg shows the same type of nesting order.

Table 5.24 gives an example of the nesting for the MIA-ATL flight leg using the high demand assumptions. With the change of demand level, the nesting order also changes. Ths MIAATL M class is no longer ranked above the MIASAV Y class, but the MIABOS Q class is ranked above the MIALAX B class.

The deterministic LP nested on shadow prices is a network based method. By using shadow prices, the non-nested solution of the deterministic LP method is made more compatible with nested reservations systems. Also, by nesting the different O-D/fare class inventories, the probabilistic behavior of demand can be dealt with more readily. If demand for a one O-D/fare class is lower than expected, the seats allocated to the O-D/fare class will not necessarily go empty, as in a distinct inventory system, but can be booked by excess demand for another O-D/fare class.

LEG	SEGMENT	CLASS	MEAN DEMAND	SEATS ALLOC	NESTING VARIABLE	FARE	BK LIMIT
MIA-ATL	MIALAX	Y	17	17	323	$477	150
	MIABOS	Y	11	11	314	$403	133
	MIAATL	Y	20	20	228	$280	122
	MIABOS	M	5	5	225	$314	102
	MIASAV	Y	13	13	162	$226	97
	MIAATL	M	9	9	157	$209	84
	MIASAV	M	4	4	104	$168	75
	MIALAX	M	11	11	85	$239	71
	MIAATL	B	7	7	42	$94	60
	MIABOS	B	14	14	35	$124	53
	MIASAV	B	7	7	20	$84	39
	MIAATL	Q	14	14	7	$59	32
	MIABOS	Q	20	18	0	$89	18
	MIASAV	Q	25	0	-5	$59	0
	MIALAX	B	7	0	-15	$139	0
	MIALAX	Q	14	0	-35	$119	0

Table 5.24: Nested Deterministic LP Booking Limits - High Demand Level

If demand is higher than expected, extra seats may be obtained from lower nested O-D/fare class itineraries, potentially increasing network revenues. By nesting the O-D/fare class inventories the situation of having seats completely unallocated does not occur. If the level of demand is low, seats that have not been assigned to specific O-D/fare class combinations are added to the lowest nested inventory, making the extra seats available to all O-D/fare classes.

5.3 Network Revenue Comparisons

The major interest of a origin-destination seat inventory control system is the increase in total revenue the system will provide. It is hard to determine the exact amounts of revenue obtained from each of the above techniques because of the stochastic nature of demand, especially in the case of nested reservations systems. Two different methods of approximating revenue were applied to the six techniques to provide some basis for comparison of the different methods.

The first method approximates system revenue based on the mean demand for each O-D/fare class. It is assumed that the lowest class books first, with the highest class booking last. In many cases the total mean demand from the O-D/fare class itineraries in a common booking inventory is greater than the number of seats available. In such situations, the seats are booked proportionally to the O-D/fare class mean demands, as long as space is available on the different legs. A multi-leg itinerary must have seats available on each leg of its itinerary in order to have a seat booked in it. If seats are available on one leg, but not on a second leg of the itinerary, the seats on the first leg are divided up proportionally between the remaining O-D/fare combinations in the particular inventory.

The second revenue approximation method used is based on the number of seats protected for a given class. Nesting is not taken into account in this method, and inventories are treated as distinct buckets. The revenue estimates from this method compare the maximum "exposure" of the airline under high demand conditions. All seats protected for an inventory class over the next lower inventory class are booked in the higher class. Once again, if the total demand of the respective O-D/fare classes is more than the number of seatsprotected, the seats are booked proportionally to the O-D/fare class mean demands. However, the number of seats booked for any O-D/fare class itinerary cannot exceed three standard deviations over its mean demand value. If the total number of seats protected for a class is more than the total maximum possible demand for the class, the excess seats are left empty.

An indication of the differences in these approximations of system revenues is provided in Table 5.25. This table summarizes the potential revenue estimates for

	LOW	MEDIUM	HIGH
LEG BASED EMSR			
MEAN DEMAND	$92,690	$136,870	$159,602
SEATS PROTECTED	$106,688	$136,592	$159,252
PROTECTED & BOOKED UP	$112,730	$145,496	$171,972
PRORATED LEG BASED EMSR			
MEAN DEMAND	$92,690	$136,858	$159,418
SEATS PROTECTED	$112,380	$143,276	$167,624
VIRTUAL NESTING EMSR			
MEAN DEMAND	$92,690	$134,860	$155,280
SEATS PROTECTED	$97,514	$135,242	$154,162
DETERMINISTIC LP			
MEAN DEMAND	$92,690	$139,054	$164,858
SEATS PROTECTED	$92,690	$139,054	$164,858
PROBABILISTIC LP			
MEAN DEMAND	$92,690	$131,140	$155,190
SEATS PROTECTED	$150,276	$163,844	$191,460
EXPECTED REVENUE	$100,561	$131,717	$154,094
DETERMINISTIC LP NESTED ON SP			
MEAN DEMAND	$92,690	$139,054	$164,858
SEATS PROTECTED	$98,960	$139,054	$164,858

Table 5.25: Revenue Summary

each of the alternative seat inventory control approaches under the three demand level assumptions, low, medium, and high.

The system revenue for the virtual nesting EMSR technique is the lowest for both revenue estimation methods. This "greedy" method of favoring a single long-haul passenger on a given leg over a short-haul passenger does not produce maximum revenues when the entire system is considered. One reason revenues are so low for the virtual nesting technique is because quite often multi-leg itineraries do not have seats available on both legs of the itinerary. This is an additional problem with a leg based method. To complicate the matter, many virtual classes do not have a short-haul O-D/fare class combination assigned to it. When the multi-leg itineraries cannot be booked because of lack of seats available for the virtual class on other legs, there are not single-leg itineraries in the virtual class to fill the seats on the flight leg and the seats go empty.

The leg based EMSR and prorated leg based EMSR techniques are very similar for the mean demand method, but not for the seats protected method. The revenue based on seats protected for the leg based EMSR technique is not a good indication of the system revenue. In the process of calculating the weighted average fare for the ATL-SAV leg in the small hub network with the given demand assumptions and fare levels, the M class average fare was higher than the Y fare. Therefore, in the standard EMSR method, no seats are protected for the Y class O-D itineraries over the M class O-D itineraries. This in turn means that by using the "seats protected" method to determine expected revenues, the seats allocated to the Y class on the ATLSAV leg, and protected from the M class, are zero. At the same time, the number of seats available to the M class is much higher than the total maximum demand.

A better indication of the potential revenue for the leg based EMSR technique, based in part on the protected seat method, is to book ATLSAV Y class itineraries in seats left over after completely booking the maximum number of M class seats possible. In a typical nesting environment, this type of booking would occur. Using this extended protected seats method where revenue is based on seats protected, and booked up in the case of the ATL-SAV flight leg, the potential revenue amounts are higher than that of the prorated leg based EMSR seats protected revenue. This concept of booking the maximum number of seats possible under the protected seat method and then booking any remaining seats in the next highest class does not effect the prorated EMSR seat inventory control revenue estimates. This is because seats are protected for the Y class on the ATL-SAV flight leg in the prorated EMSR method and are not left to be allotted to the M class, as in the non-prorated leg based EMSR method. Also, all seats available to the M class can be booked in the class. There are no remaining seats which can be booked up to the Y class. Therefore, the idea of a "protected and booked up" revenue estimate is equivalent to the seat protected estimate.

In the deterministic linear programming technique, the revenue from both the mean demand and the seats protected methods are the same, except in the low demand scenario where all seats are not initially allocated and through nesting, excess seats are amade available to all O-D/fare class combinations through the lowest fare class inventory. The two revenue estimates are the same because the number of seats allotted to each O-D/fare class combination, based on the demand constraints, is equal to or less than the mean demand. Therefore, in any O-D/fare class inventory there are not excess seats which ultimately go empty. Seats are allocated only if there is expected demand and every seat allocated to an O-D/fare class inventory is booked by the respective O-D/fare class demand.

The revenue estimates for the deterministic LP method nested on shadow prices are the same as the non-nested deterministic LP estimates, with one exception. This is true for the same reasons as above. The deterministic LP solution is based on mean demands and these mean demands are determining the revenue potentials. The one exception is the low demand seats protected revenue estimate. In this case, the mean demand does not fill the capacity on each flight leg. In the non-nested deterministic LP method, excess seats are not allocated to any O-D/fare class combinations, and therefore are left empty. In the nested deterministic LP method, excess seats are added to the the lowest O-D/fare class inventory, making them available to all O-D/fare class requests. Estimating revenues according to the seat protection method, the excess seats in the lowest nested inventory can be booked by the respective O-D/fare class up to the limit of three standard deviations over its mean demand. These additional bookings, over the mean demand bookings, account for the additional revenue estimated by the seats protected method.

The deterministic linear programming techniques appear to give the highest system revenue a majority of the time, yet this revenue value is an overestimate of average potential revenue. This occurs because the determined seat allocation solution for the deterministic LP methods is the optimal solution for the given mean demands, and the revenue estimates are based on this mean demand. Actual demand will be lower than the mean demand 50% of the time. A more accurate estimate of system revenue for the deterministic LP seat inventory control methods would be an overall average revenue of a number of estimates based on randomly selected demand values taken from the respective O-D/fare class demand distributions.

We see that the probabilistic linear programming technique gives us a very high system revenue for the method based on seats protected. This stems from the fact that in a non-nested system the number of seats protected for higher fare classes is greater than in a nested system, such as in the EMSR technique. Therefore, more seats have been protected for the higher fare itineraries, giving a high system revenue. A more realistic estimate, as well as an estimate for the overall long-term average revenue for the network system, is the expected revenue given by the probabilistic LP solution. This revenue estimate is much more consistent with the other revenue estimates.

Chapter 6

Conclusion

6.1 Research Findings

The optimal solution for the network seat inventory control problem is given by the probabilistic linear programming technique. This technique incorporates both the stochastic nature of demand as well as the interaction of flight legs which occurs in a network system. The seat protection revenue, which is basically the maximum possible revenue obtainable given the determined seat allotments, is by far the highest for the probablistic linear programming seat allocation solution. Although the mean demand revenue value for this method is the lowest among the alternatives, it is more realistic to use the expected revenue, or overall average revenue, of the system.

Even though the probabilistic linear program gives the "optimal" solution, it is not a practical seat inventory control method. As mentioned before, the size of the network formulation grows extremely large making frequent solution runs impractical to solve on current computer systems. If solutions can be obtained, they will be based on a distinct inventory structure, rather than the nested structure which is common throughout the airline industry today. Also, changes in demand forecasts and incoming bookings would make frequent revisions necessary to ensure optimal seat allocations. An initial solution itself would require excessive computer facilities and long processing times; frequent revision would be impossible.

The three different EMSR techniques also take into account the of uncertainty of demand, as does the probabilistic LP method, yet they are all leg based techniques which do not take into consideration the system-wide network effects when maximizing revenue and allocating seats. The revenue-driven virtual nesting EMSR method shows the lowest revenue estimates out of all the leg-based techniques. It seems to be the worst method overall in this network example for all three assumed demand scenarios, although it is conceived by many to be a more sophisticated leg-based method. When applying the virtual nesting method to a complete network, it is too greedy to maximize system revenue. Yet, this method may be useful on limited segments and sub-networks of a system which is made up of basically short-haul origin-destination itineraries, such as in the situation of a feeder carrier's route structure.

The leg based EMSR and prorated EMSR methods are both very similar in terms of revenue potential, as well as seat allocations and nesting order. The optimal seat allocation solution obtained from these two methods match so closely that the "better" method, in terms of seat inventory control optimality, cannot be determined. Since the prorated EMSR technique is an extended version of the standard EMSR model, the leg based EMSR method can be chosen over the prorated EMSR method due to simplicity alone. Also, the fact that the standard method is based on total ticket revenue means the O-D mix is better repesented.

The drawback of the leg based EMSR model, and leg based models as a whole, is the lack of consideration for the network and the interaction of flight legs within the network when determining seat allocations and booking limits. Revenues are maximized by individual flight leg, and not based on the entire network. Yet the leg based EMSR model is the simplest to implement. Recommended seat allocations are not only based on the probabilistic nature of demand, but they are also based on a nested system. The output solution from this method gives individual flight leg availabilities, making the leg based EMSR method easily implementable in current airline reservations systems.

Like the probabilistic LP technique, the deterministic linear programming technique is a network based method which takes into account the interaction of passenger flows on different flight legs. Network revenue is maximized based on deterministic demand values, taken to be the means of the origin-destination/fare class demand distributions. As in the probabilistic LP approach, seat allocations are determined on the basis of distinct, non-nested inventory buckets. Such solutions are not directly compatible with the nested inventory class structure of airline reservations systems.

A variation of the non-nested deterministic LP method is the deterministic LP nested on shadow prices. It seems as if this method may have the most potential as an efficient origin-destination seat inventory control method. To begin with, it is a nested approach, and it is also a network based optimization method. Although the problem formulation is large, it does not expand exponentially like the probabilistic LP method. A vulnerability of this method is that it does not consider probabilistic demand.

In the formulation of the deterministic LP nested on shadow prices, O-D/fare class seat allocations are limited by the mean demand. Actual demand will be in excess of this mean value 50% of the time and the other 50% of the time demand will be below the value used in the optimization method. Through nesting, the problem of too little or too much demand for any specific O-D/fare class combination can be alleviated. Some O-D/fare class itineraries will have more demand than planned for, while others will have less. In a distinct inventory system, seats must be booked in their respective O-D/fare class inventories or remain empty. However, in a nested system, seats are allowed to be sold to inventories in which the seats were not originially allocated, as long as the O-D/fare class inventory is nested above other inventories. By using shadow prices, these O-D/fare class inventories are nested according to the extra network revenue obtained from selling one additional seat than initially allotted to the respective O-D/fare class combination.

The different revenue estimates presented in the previous chapter are not necessarily a good basis of comparison for the nested deterministic LP method since both the optimization model and the revenue approximation methods are based on mean expected demand. On the other hand, the nesting order of this deterministic LP method shows how O-D/fare class inventories are not nested solely on the basis of fare class or itinerary revenue, as the different leg based EMSR methods are. Since the nesting order is not dictated by fare class and ticket revenue, there are possibilities for multi-leg high revenue itineraries to be nested below a single-leg lower revenue itinerary, which combined with another single-leg itinerary may give a greater combined revenue than the single multi-leg itinerary.

The one major problem with this seat inventory control method is that seat availabilities are based on origin-destination pairs. This is not compatible with existing leg based reservations systems. Current major reservations systems show seat availabilites by flight leg rather than origin-destination. At the same time, agents book and cancel reservations based on flight legs. It is possible to get around this problem by displaying the O-D itinerary availabilities, as opposed to individual flight leg availabilities, for each flight leg in the itinerary. The obstacle of agents canceling individual flight leg reservations versus O-D itinerary reservations could be overcome simply by re-programming reservations systems to prevent this. The barrier to such an O-D/fare class inventory system is the problem of communicating the multitude of seat availabilities to other airline's reservations systems. For an airline such as Delta, there can be 35 to 40 O-D's per flight, 2500 flights per day and 7 different coach fare classes. This makes 612,500 to 700,000 different seat availability values for a single day's worth of flights. With bookings being accepted as far as 11 months in advanced, other airlines do not have storage space for such O-D availabilities in their reservations systems. The only way to communicate these availabilities is through direct access between reservations systems.

6.2 Further Work

There is extensive work which can be done as a continuation of this evaluation. As mentioned above, the virtual nesting EMSR method may be an effective method for short-haul, sub-network types of systems, similar to route structures of feeder carriers. An evaluation such as this one, based on a feeder-type route structure into a major hub, needs to be performed to see if the virtual nesting idea is beneficial in some applications for a seat inventory control system.

Of major interest is an expansion on the analysis of the deterministic linear programming method which is nested on shadow prices. Using shadow prices in the nesting of a network based solution is a new concept and needs to be explored in depth. Shadow prices are a measure of increased revenue potential based on one additional unit allotted to a given decision variable. Once an additional seat is allocated to a given O-D/fare class inventory, the original shadow price is not necessarily valid any longer. Shadow prices are based on one extra unit of allotment, not necessarily a second, third, or fourth. Also, a shadow price associated with a given decision variable is based on all other problem data remaining unchanged. Once an additional seat is allocated to one O-D/fare class combination, all shadow prices are in essence outdated. A sensitivity analysis of what happens to shadow prices as the actual number of seats allocated to different inventories change is of particular interest. Although shadow prices are not fixed, as described here, this does not make the concept of using shadow prices ineffective. As substantial changes occur in bookings, affecting initial seat allotments, revisions should be made to the booking limits (which is necessary for any effective seat inventory control system). This would be done by rerunning the deterministic LP with new demand expectations and available seats, thereby updating optimal seat allocations and shadow prices.

The overall evaluation in this thesis could be expanded in an effort to analyze the different seat inventory control alternatives in greater detail. The results and conclusions found here could be in a large part dependent on the network model and fare and demand data. Different types of hub networks, which could include feeder type networks, regional networks, and larger country-wide connecting hubs, could be evaluated to see if the basic results found with this Atlanta-based network change. Also a variety of different demand assumptions, which vary proportionally within O-D/fare class combinations, as well as in overall level of demand, could be tested with booking limits, nesting order and revenue estimates compared with those found here.

The major interest of different seat inventroy control systems is the increase in potential revenue the system will provide for an airline. Exact levels of revenue obtained from a system are difficult to determine analytically due to the uncertainty of demand, especially when nested inventories are involved. Although lower fare passengers often book first, it is not usually the case that bookings are made completely in the order of inventory classes.

In this evaluation, revenue estimates were based on mean demand. This does not necessarily give realistic results, especially in the deterministic LP seat inventory control techniques in which the network formulation was also based on mean demands. It would be much more interesting to include some sort of probabilistic behavior of demand when determining revenue potential. Although it would be difficult to realistically simulate the behavior of booking orders, the possibility of extending the revenue analysis to include a simulation of randomly generated O-D/fare class demands themselves would make revenue estimates more of a true measure of seat inventory control optimality.

Overall average expected revenues could be found using a Monte Carlo type of simulation. The first step in determining average revenue obtained from a seat inventory control system would be to use a random number generator under a normal probability distribution assumption and arbitrarily determine the demand for each O-D/fare class combination. Once the demands for each O-D/fare class have been randomly selected, the revenues for each of the six seat inventory control methods can be determined for the given demand levels. Repeating this process a number of times, each time with new randomly determined demand levels, and averaging the revenue estimates will give an overall average expected revenue. Such an expected revenue value would be a true estimate for a seat inventory control system's revenue potential.

Another important issue to consider in greater detail is the implementation requirements of each alternative in airline reservations systems. This has been discussed briefly, but not to the extent of its importance. Several practical considerations in a seat inventory control system involve the limitations imposed by existing reservations systems, the importance of data availability, and the need for human intervention. The implementation and compatibility of a seat inventory control system can dictate the actual method which is best for any given airline. Also, ways of incorporating such factors as overbooking, up-grade potential, and loss of denied requests into a seat inventory control system must be looked at. These factors can have a considerable impact on optimal inventory booking limits.

The potential benefits to the airline industry in an effective network-based origin-destination seat inventory control could be quite great, especially in the highly competitive environment of air transportation today. There is an extensive amount of work which could be done in this area of seat inventory control, simply as a continuation of this analysis, or as a completely new extension to it. Regardless, the emphasis in further research should be in accordance with the practical limitations posed by reservations systems, data availabilities and airline competition.

Bibliography

[1] Peter Paul Belobaba, "Air Travel Demand and Airline Seat Inventory Management", Report FTL-R87-8, Flight Transportation Laboratory, Massachusetts Institute of Technology, Cambridge, MA, May 15, 1987, pp 69-79.

[2] Kenneth Littlewood, "Forecasting and Control of Passenger Bookings", AGIFORS Proceedings, October 1972, pp 95-117.

[3] A. V. Bhatia and S. C. Parekh, "Optimal Allocation of Seats by Fare", AGIFORS Reservation Study Group, 1973.

[4] Helmut Richter, "The Differential Revenue Method to Determine Optimal Seat Allotments by Fare Type", AGIFORS Symposium Proceedings, October 1982, pp 339-362.

[5] Jörn Buhr, "Optimal Sales Limits for 2-Sector Flights", AGIFORS Symposium Proceedings, October 1982, pp 291-303.

[6] Ken Wang, "Optimum Seat Allocation for Multi-Leg Flights with Multiple Fare Types", AGIFORS Symposium Proceedings, October 1983, pp 225-237.

[7] Fred Glover, Randy Glover, Joe Lorenzo and Claude McMillan, "The Passenger-Mix Problem in the Scheduled Airlines", *Interfaces*, Vol. 12, No. 3, June 1982, pp 73-79.

[8] Richard D. Wollmer, "An Airline Reservation Model for Opening and Closing Fare Classes", McDonnell Douglas Corporation, Long Beach, CA, pp 1-6.

[9] "A Pilot Study of Seat Inventory Management for a Flight Itinerary", U.S. and Canadian Airline Analysis, Boeing Commercial Airplane Company, Renton, WA, February 24, 1983.

[10] Belobaba, *op cit.*, pp 107-118.

[11] Wollmer, *op cit.*

[12] Belobaba *op cit.*, pp 105, 109, 115.

[13] Belobaba *op cit.*, pp 119-121.

[14] "Linear Interactive aND Discrete Optimizer", Lindo Systems, Inc., 1984 and 1985.

Appendix A

Complete Booking Limit Results

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	SAVBOS	Y	$319	150
	ATLBOS	Y	$310	150
	LAXBOS	M	$380	133
	MIABOS	M	$314	133
	ATLBOS	M	$290	133
	SAVBOS	M	$250	133
	LAXBOS	B	$159	114
	MIABOS	B	$124	114
	SAVBOS	B	$109	114
	ATLBOS	B	$95	114
	LAXBOS	Q	$139	92
	MIABOS	Q	$89	92
	ATLBOS	Q	$69	92
	SAVBOS	Q	$69	92
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSSAV	Y	$319	150
	BOSATL	Y	$310	150
	BOSLAX	M	$380	133
	BOSMIA	M	$314	133
	BOSATL	M	$290	133
	BOSSAV	M	$250	133
	BOSLAX	B	$159	114
	BOSMIA	B	$124	114
	BOSSAV	B	$109	114
	BOSATL	B	$95	114
	BOSLAX	Q	$139	92
	BOSMIA	Q	$89	92
	BOSATL	Q	$69	92
	BOSSAV	Q	$69	92
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	ATLMIA	Y	$280	150
	SAVMIA	Y	$226	150
	BOSMIA	M	$314	121
	LAXMIA	M	$239	121
	ATLMIA	M	$209	121
	SAVMIA	M	$168	121
	LAXMIA	B	$139	100
	BOSMIA	B	$124	100
	ATLMIA	B	$94	100
	SAVMIA	B	$84	100
	LAXMIA	Q	$119	83
	BOSMIA	Q	$89	83
	ATLMIA	Q	$59	83
	SAVMIA	Q	$59	83

Table A.1: Leg Based EMSR Booking Limits - Low Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIAATL	Y	$280	150
	MIASAV	Y	$226	150
	MIABOS	M	$314	121
	MIALAX	M	$239	121
	MIAATL	M	$209	121
	MIASAV	M	$168	121
	MIALAX	B	$139	100
	MIABOS	B	$124	100
	MIAATL	B	$94	100
	MIASAV	B	$84	100
	MIALAX	Q	$119	83
	MIABOS	Q	$89	83
	MIAATL	Q	$59	83
	MIASAV	Q	$59	83
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	130
	ATLLAX	M	$391	130
	BOSLAX	M	$380	130
	MIALAX	M	$239	130
	BOSLAX	B	$159	113
	SAVLAX	B	$154	113
	ATLLAX	B	$142	113
	MIALAX	B	$139	113
	BOSLAX	Q	$139	98
	SAVLAX	Q	$134	98
	ATLLAX	Q	$122	98
	MIALAX	Q	$119	98
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	130
	LAXATL	M	$391	130
	LAXBOS	M	$380	130
	LAXMIA	M	$239	130
	LAXBOS	B	$159	113
	LAXSAV	B	$154	113
	LAXATL	B	$142	113
	LAXMIA	B	$139	113
	LAXBOS	Q	$139	98
	LAXSAV	Q	$134	98
	LAXATL	Q	$122	98
	LAXMIA	Q	$119	98

Table A.1: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	Y	$450	150
	BOSSAV	Y	$319	150
	BOSSAV	Y	$250	150
	MIASAV	M	$226	150
	MIASAV	M	$168	150
	ATLSAV	M	$159	150
	ATLSAV	M	$140	150
	LAXSAV	B	$154	110
	BOSSAV	B	$109	110
	MIASAV	B	$84	110
	ATLSAV	B	$64	110
	LAXSAV	Q	$134	97
	BOSSAV	Q	$69	97
	MIASAV	Q	$59	97
	ATLSAV	Q	$49	97
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	Y	$450	150
	SAVBOS	Y	$319	150
	SAVBOS	Y	$250	150
	SAVMIA	M	$226	150
	SAVMIA	M	$168	150
	SAVATL	M	$159	150
	SAVATL	M	$140	150
	SAVLAX	B	$154	110
	SAVBOS	B	$109	110
	SAVMIA	B	$84	110
	SAVATL	B	$64	110
	SAVLAX	Q	$134	97
	SAVBOS	Q	$69	97
	SAVMIA	Q	$59	97
	SAVATL	Q	$49	97

Table A.1: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	SAVBOS	Y	$319	150
	ATLBOS	Y	$310	150
	LAXBOS	M	$380	125
	MIABOS	M	$314	125
	ATLBOS	M	$290	125
	SAVBOS	M	$250	125
	LAXBOS	B	$159	96
	MIABOS	B	$124	96
	SAVBOS	B	$109	96
	ATLBOS	B	$95	96
	LAXBOS	Q	$139	63
	MIABOS	Q	$89	63
	ATLBOS	Q	$69	63
	SAVBOS	Q	$69	63
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSSAV	Y	$319	150
	BOSATL	Y	$310	150
	BOSLAX	M	$380	125
	BOSMIA	M	$314	125
	BOSATL	M	$290	125
	BOSSAV	M	$250	125
	BOSLAX	B	$159	96
	BOSMIA	B	$124	96
	BOSSAV	B	$109	96
	BOSATL	B	$95	96
	BOSLAX	Q	$139	63
	BOSMIA	Q	$89	63
	BOSATL	Q	$69	63
	BOSSAV	Q	$69	63
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	ATLMIA	Y	$280	150
	SAVMIA	Y	$226	150
	BOSMIA	M	$314	106
	LAXMIA	M	$239	106
	ATLMIA	M	$209	106
	SAVMIA	M	$168	106
	LAXMIA	B	$139	78
	BOSMIA	B	$124	78
	ATLMIA	B	$94	78
	SAVMIA	B	$84	78
	LAXMIA	Q	$119	50
	BOSMIA	Q	$89	50
	ATLMIA	Q	$59	50
	SAVMIA	Q	$59	50

Table A.2: Leg Based EMSR Booking Limits - Medium Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIAATL	Y	$280	150
	MIASAV	Y	$226	150
	MIABOS	M	$314	106
	MIALAX	M	$239	106
	MIAATL	M	$209	106
	MIASAV	M	$168	106
	MIALAX	B	$139	78
	MIABOS	B	$124	78
	MIAATL	B	$94	78
	MIASAV	B	$84	78
	MIALAX	Q	$119	50
	MIABOS	Q	$89	50
	MIAATL	Q	$59	50
	MIASAV	Q	$59	50
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	122
	ATLLAX	M	$391	122
	BOSLAX	M	$380	122
	MIALAX	M	$239	122
	BOSLAX	B	$159	95
	SAVLAX	B	$154	95
	ATLLAX	B	$142	95
	MIALAX	B	$139	95
	BOSLAX	Q	$139	72
	SAVLAX	Q	$134	72
	ATLLAX	Q	$122	72
	MIALAX	Q	$119	72
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	122
	LAXATL	M	$391	122
	LAXBOS	M	$380	122
	LAXMIA	M	$239	122
	LAXBOS	B	$159	95
	LAXSAV	B	$154	95
	LAXATL	B	$142	95
	LAXMIA	B	$139	95
	LAXBOS	Q	$139	72
	LAXSAV	Q	$134	72
	LAXATL	Q	$122	72
	LAXMIA	Q	$119	72

Table A.2: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	Y	$450	150
	BOSSAV	Y	$319	150
	BOSSAV	Y	$250	150
	MIASAV	M	$226	150
	MIASAV	M	$168	150
	ATLSAV	M	$159	150
	ATLSAV	M	$140	150
	LAXSAV	B	$154	91
	BOSSAV	B	$109	91
	MIASAV	B	$84	91
	ATLSAV	B	$64	91
	LAXSAV	Q	$134	71
	BOSSAV	Q	$69	71
	MIASAV	Q	$59	71
	ATLSAV	Q	$49	71
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	Y	$450	150
	SAVBOS	Y	$319	150
	SAVBOS	Y	$250	150
	SAVMIA	M	$226	150
	SAVMIA	M	$168	150
	SAVATL	M	$159	150
	SAVATL	M	$140	150
	SAVLAX	B	$154	91
	SAVBOS	B	$109	91
	SAVMIA	B	$84	91
	SAVATL	B	$64	91
	SAVLAX	Q	$134	71
	SAVBOS	Q	$69	71
	SAVMIA	Q	$59	71
	SAVATL	Q	$49	71

Table A.2: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	SAVBOS	Y	$319	150
	ATLBOS	Y	$310	150
	LAXBOS	M	$380	117
	MIABOS	M	$314	117
	ATLBOS	M	$290	117
	SAVBOS	M	$250	117
	LAXBOS	B	$159	79
	MIABOS	B	$124	79
	SAVBOS	B	$109	79
	ATLBOS	B	$95	79
	LAXBOS	Q	$139	35
	MIABOS	Q	$89	35
	ATLBOS	Q	$69	35
	SAVBOS	Q	$69	35
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSSAV	Y	$319	150
	BOSATL	Y	$310	150
	BOSLAX	M	$380	117
	BOSMIA	M	$314	117
	BOSATL	M	$290	117
	BOSSAV	M	$250	117
	BOSLAX	B	$159	79
	BOSMIA	B	$124	79
	BOSSAV	B	$109	79
	BOSATL	B	$95	79
	BOSLAX	Q	$139	35
	BOSMIA	Q	$89	35
	BOSATL	Q	$69	35
	BOSSAV	Q	$69	35
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	ATLMIA	Y	$280	150
	SAVMIA	Y	$226	150
	BOSMIA	M	$314	92
	LAXMIA	M	$239	92
	ATLMIA	M	$209	92
	SAVMIA	M	$168	92
	LAXMIA	B	$139	55
	BOSMIA	B	$124	55
	ATLMIA	B	$94	55
	SAVMIA	B	$84	55
	LAXMIA	Q	$119	17
	BOSMIA	Q	$89	17
	ATLMIA	Q	$59	17
	SAVMIA	Q	$59	17

Table A.3: Leg Based EMSR Booking Limits - High Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIAATL	Y	$280	150
	MIASAV	Y	$226	150
	MIABOS	M	$314	92
	MIALAX	M	$239	92
	MIAATL	M	$209	92
	MIASAV	M	$168	92
	MIALAX	B	$139	55
	MIABOS	B	$124	55
	MIAATL	B	$94	55
	MIASAV	B	$84	55
	MIALAX	Q	$119	17
	MIABOS	Q	$89	17
	MIAATL	Q	$59	17
	MIASAV	Q	$59	17
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	114
	ATLLAX	M	$391	114
	BOSLAX	M	$380	114
	MIALAX	M	$239	114
	BOSLAX	B	$159	76
	SAVLAX	B	$154	76
	ATLLAX	B	$142	76
	MIALAX	B	$139	76
	BOSLAX	Q	$139	44
	SAVLAX	Q	$134	44
	ATLLAX	Q	$122	44
	MIALAX	Q	$119	44
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	114
	LAXATL	M	$391	114
	LAXBOS	M	$380	114
	LAXMIA	M	$239	114
	LAXBOS	B	$159	76
	LAXSAV	B	$154	76
	LAXATL	B	$142	76
	LAXMIA	B	$139	76
	LAXBOS	Q	$139	44
	LAXSAV	Q	$134	44
	LAXATL	Q	$122	44
	LAXMIA	Q	$119	44

Table A.3: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	Y	$450	150
	BOSSAV	Y	$319	150
	BOSSAV	Y	$250	150
	MIASAV	M	$226	150
	MIASAV	M	$168	150
	ATLSAV	M	$159	150
	ATLSAV	M	$140	150
	LAXSAV	B	$154	72
	BOSSAV	B	$109	72
	MIASAV	B	$84	72
	ATLSAV	B	$64	72
	LAXSAV	Q	$134	44
	BOSSAV	Q	$69	44
	MIASAV	Q	$59	44
	ATLSAV	Q	$49	44
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	Y	$450	150
	SAVBOS	Y	$319	150
	SAVBOS	Y	$250	150
	SAVMIA	M	$226	150
	SAVMIA	M	$168	150
	SAVATL	M	$159	150
	SAVATL	M	$140	150
	SAVLAX	B	$154	72
	SAVBOS	B	$109	72
	SAVMIA	B	$84	72
	SAVATL	B	$64	72
	SAVLAX	Q	$134	44
	SAVBOS	Q	$69	44
	SAVMIA	Q	$59	44
	SAVATL	Q	$49	44

Table A.3: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	SAVBOS	Y	$319	150
	ATLBOS	Y	$310	150
	LAXBOS	M	$380	134
	MIABOS	M	$314	134
	ATLBOS	M	$290	134
	SAVBOS	M	$250	134
	LAXBOS	B	$159	113
	MIABOS	B	$124	113
	SAVBOS	B	$109	113
	ATLBOS	B	$95	113
	LAXBOS	Q	$139	91
	MIABOS	Q	$89	91
	ATLBOS	Q	$69	91
	SAVBOS	Q	$69	91
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSSAV	Y	$319	150
	BOSATL	Y	$310	150
	BOSLAX	M	$380	134
	BOSMIA	M	$314	134
	BOSATL	M	$290	134
	BOSSAV	M	$250	134
	BOSLAX	B	$159	113
	BOSMIA	B	$124	113
	BOSSAV	B	$109	113
	BOSATL	B	$95	113
	BOSLAX	Q	$139	91
	BOSMIA	Q	$89	91
	BOSATL	Q	$69	91
	BOSSAV	Q	$69	91
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	ATLMIA	Y	$280	150
	SAVMIA	Y	$226	150
	BOSMIA	M	$314	121
	LAXMIA	M	$239	121
	ATLMIA	M	$209	121
	SAVMIA	M	$168	121
	LAXMIA	B	$139	100
	BOSMIA	B	$124	100
	ATLMIA	B	$94	100
	SAVMIA	B	$84	100
	LAXMIA	Q	$119	83
	BOSMIA	Q	$89	83
	ATLMIA	Q	$59	83
	SAVMIA	Q	$59	83

Table A.4: Prorated EMSR Booking Limits - Low Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIAATL	Y	$280	150
	MIASAV	Y	$226	150
	MIABOS	M	$314	121
	MIALAX	M	$239	121
	MIAATL	M	$209	121
	MIASAV	M	$168	121
	MIALAX	B	$139	100
	MIABOS	B	$124	100
	MIAATL	B	$94	100
	MIASAV	B	$84	100
	MIALAX	Q	$119	83
	MIABOS	Q	$89	83
	MIAATL	Q	$59	83
	MIASAV	Q	$59	83
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	130
	ATLLAX	M	$391	130
	BOSLAX	M	$380	130
	MIALAX	M	$239	130
	BOSLAX	B	$159	113
	SAVLAX	B	$154	113
	ATLLAX	B	$142	113
	MIALAX	B	$139	113
	BOSLAX	Q	$139	99
	SAVLAX	Q	$134	99
	ATLLAX	Q	$122	99
	MIALAX	Q	$119	99
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	130
	LAXATL	M	$391	130
	LAXBOS	M	$380	130
	LAXMIA	M	$239	130
	LAXBOS	B	$159	113
	LAXSAV	B	$154	113
	LAXATL	B	$142	113
	LAXMIA	B	$139	113
	LAXBOS	Q	$139	99
	LAXSAV	Q	$134	99
	LAXATL	Q	$122	99
	LAXMIA	Q	$119	99

Table A.4: Continued

ATL-SAV	LAXSAV	Y	$502	150
	BOSSAV	Y	$319	150
	MIASAV	Y	$226	150
	ATLSAV	Y	$159	150
	LAXSAV	M	$450	125
	BOSSAV	M	$250	125
	MIASAV	M	$168	125
	ATLSAV	M	$140	125
	LAXSAV	B	$154	109
	BOSSAV	B	$109	109
	MIASAV	B	$84	109
	ATLSAV	B	$64	109
	LAXSAV	Q	$134	94
	BOSSAV	Q	$69	94
	MIASAV	Q	$59	94
	ATLSAV	Q	$49	94
SAV-ATL	SAVLAX	Y	$502	150
	SAVBOS	Y	$319	150
	SAVMIA	Y	$226	150
	SAVATL	Y	$159	150
	SAVLAX	M	$450	125
	SAVBOS	M	$250	125
	SAVMIA	M	$168	125
	SAVATL	M	$140	125
	SAVLAX	B	$154	109
	SAVBOS	B	$109	109
	SAVMIA	B	$84	109
	SAVATL	B	$64	109
	SAVLAX	Q	$134	94
	SAVBOS	Q	$69	94
	SAVMIA	Q	$59	94
	SAVATL	Q	$49	94

Table A.4: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	SAVBOS	Y	$319	150
	ATLBOS	Y	$310	150
	LAXBOS	M	$380	126
	MIABOS	M	$314	126
	ATLBOS	M	$290	126
	SAVBOS	M	$250	126
	LAXBOS	B	$159	96
	MIABOS	B	$124	96
	SAVBOS	B	$109	96
	ATLBOS	B	$95	96
	LAXBOS	Q	$139	62
	MIABOS	Q	$89	62
	ATLBOS	Q	$69	62
	SAVBOS	Q	$69	62
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSSAV	Y	$319	150
	BOSATL	Y	$310	150
	BOSLAX	M	$380	126
	BOSMIA	M	$314	126
	BOSATL	M	$290	126
	BOSSAV	M	$250	126
	BOSLAX	B	$159	96
	BOSMIA	B	$124	96
	BOSSAV	B	$109	96
	BOSATL	B	$95	96
	BOSLAX	Q	$139	62
	BOSMIA	Q	$89	62
	BOSATL	Q	$69	62
	BOSSAV	Q	$69	62
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	ATLMIA	Y	$280	150
	SAVMIA	Y	$226	150
	BOSMIA	M	$314	106
	LAXMIA	M	$239	106
	ATLMIA	M	$209	106
	SAVMIA	M	$168	106
	LAXMIA	B	$139	77
	BOSMIA	B	$124	77
	ATLMIA	B	$94	77
	SAVMIA	B	$84	77
	LAXMIA	Q	$119	49
	BOSMIA	Q	$89	49
	ATLMIA	Q	$59	49
	SAVMIA	Q	$59	49

Table A.5: Prorated EMSR Booking Limits - Medium Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIAATL	Y	$280	150
	MIASAV	Y	$226	150
	MIABOS	M	$314	106
	MIALAX	M	$239	106
	MIAATL	M	$209	106
	MIASAV	M	$168	106
	MIALAX	B	$139	77
	MIABOS	B	$124	77
	MIAATL	B	$94	77
	MIASAV	B	$84	77
	MIALAX	Q	$119	49
	MIABOS	Q	$89	49
	MIAATL	Q	$59	49
	MIASAV	Q	$59	49
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	122
	ATLLAX	M	$391	122
	BOSLAX	M	$380	122
	MIALAX	M	$239	122
	BOSLAX	B	$159	95
	SAVLAX	B	$154	95
	ATLLAX	B	$142	95
	MIALAX	B	$139	95
	BOSLAX	Q	$139	73
	SAVLAX	Q	$134	73
	ATLLAX	Q	$122	73
	MIALAX	Q	$119	73
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	122
	LAXATL	M	$391	122
	LAXBOS	M	$380	122
	LAXMIA	M	$239	122
	LAXBOS	B	$159	95
	LAXSAV	B	$154	95
	LAXATL	B	$142	95
	LAXMIA	B	$139	95
	LAXBOS	Q	$139	73
	LAXSAV	Q	$134	73
	LAXATL	Q	$122	73
	LAXMIA	Q	$119	73

Table A.5: Continued

ATL-SAV	LAXSAV	Y	$502	150
	BOSSAV	Y	$319	150
	MIASAV	Y	$226	150
	ATLSAV	Y	$159	150
	LAXSAV	M	$450	115
	BOSSAV	M	$250	115
	MIASAV	M	$168	115
	ATLSAV	M	$140	115
	LAXSAV	B	$154	89
	BOSSAV	B	$109	89
	MIASAV	B	$84	89
	ATLSAV	B	$64	89
	LAXSAV	Q	$134	66
	BOSSAV	Q	$69	66
	MIASAV	Q	$59	66
	ATLSAV	Q	$49	66
SAV-ATL	SAVLAX	Y	$502	150
	SAVBOS	Y	$319	150
	SAVMIA	Y	$226	150
	SAVATL	Y	$159	150
	SAVLAX	M	$450	115
	SAVBOS	M	$250	115
	SAVMIA	M	$168	115
	SAVATL	M	$140	115
	SAVLAX	B	$154	89
	SAVBOS	B	$109	89
	SAVMIA	B	$84	89
	SAVATL	B	$64	89
	SAVLAX	Q	$134	66
	SAVBOS	Q	$69	66
	SAVMIA	Q	$59	66
	SAVATL	Q	$49	66

Table A.5: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	SAVBOS	Y	$319	150
	ATLBOS	Y	$310	150
	LAXBOS	M	$380	118
	MIABOS	M	$314	118
	ATLBOS	M	$290	118
	SAVBOS	M	$250	118
	LAXBOS	B	$159	79
	MIABOS	B	$124	79
	SAVBOS	B	$109	79
	ATLBOS	B	$95	79
	LAXBOS	Q	$139	33
	MIABOS	Q	$89	33
	ATLBOS	Q	$69	33
	SAVBOS	Q	$69	33
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSSAV	Y	$319	150
	BOSATL	Y	$310	150
	BOSLAX	M	$380	118
	BOSMIA	M	$314	118
	BOSATL	M	$290	118
	BOSSAV	M	$250	118
	BOSLAX	B	$159	79
	BOSMIA	B	$124	79
	BOSSAV	B	$109	79
	BOSATL	B	$95	79
	BOSLAX	Q	$139	33
	BOSMIA	Q	$89	33
	BOSATL	Q	$69	33
	BOSSAV	Q	$69	33
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	ATLMIA	Y	$280	150
	SAVMIA	Y	$226	150
	BOSMIA	M	$314	92
	LAXMIA	M	$239	92
	ATLMIA	M	$209	92
	SAVMIA	M	$168	92
	LAXMIA	B	$139	54
	BOSMIA	B	$124	54
	ATLMIA	B	$94	54
	SAVMIA	B	$84	54
	LAXMIA	Q	$119	18
	BOSMIA	Q	$89	18
	ATLMIA	Q	$59	18
	SAVMIA	Q	$59	18

Table A.6: Prorated EMSR Booking Limits - High Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIAATL	Y	$280	150
	MIASAV	Y	$226	150
	MIABOS	M	$314	92
	MIALAX	M	$239	92
	MIAATL	M	$209	92
	MIASAV	M	$168	92
	MIALAX	B	$139	54
	MIABOS	B	$124	54
	MIAATL	B	$94	54
	MIASAV	B	$84	54
	MIALAX	Q	$119	18
	MIABOS	Q	$89	18
	MIAATL	Q	$59	18
	MIASAV	Q	$59	18
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	114
	ATLLAX	M	$391	114
	BOSLAX	M	$380	114
	MIALAX	M	$239	114
	BOSLAX	B	$159	76
	SAVLAX	B	$154	76
	ATLLAX	B	$142	76
	MIALAX	B	$139	76
	BOSLAX	Q	$139	46
	SAVLAX	Q	$134	46
	ATLLAX	Q	$122	46
	MIALAX	Q	$119	46
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	114
	LAXATL	M	$391	114
	LAXBOS	M	$380	114
	LAXMIA	M	$239	114
	LAXBOS	B	$159	76
	LAXSAV	B	$154	76
	LAXATL	B	$142	76
	LAXMIA	B	$139	76
	LAXBOS	Q	$139	46
	LAXSAV	Q	$134	46
	LAXATL	Q	$122	46
	LAXMIA	Q	$119	46

Table A.6: Continued

ATL-SAV	LAXSAV	Y	$502	150
	BOSSAV	Y	$319	150
	MIASAV	Y	$226	150
	ATLSAV	Y	$159	150
	LAXSAV	M	$450	105
	BOSSAV	M	$250	105
	MIASAV	M	$168	105
	ATLSAV	M	$140	105
	LAXSAV	B	$154	68
	BOSSAV	B	$109	68
	MIASAV	B	$84	68
	ATLSAV	B	$64	68
	LAXSAV	Q	$134	37
	BOSSAV	Q	$69	37
	MIASAV	Q	$59	37
	ATLSAV	Q	$49	37
SAV-ATL	SAVLAX	Y	$502	150
	SAVBOS	Y	$319	150
	SAVMIA	Y	$226	150
	SAVATL	Y	$159	150
	SAVLAX	M	$450	105
	SAVBOS	M	$250	105
	SAVMIA	M	$168	105
	SAVATL	M	$140	105
	SAVLAX	B	$154	68
	SAVBOS	B	$109	68
	SAVMIA	B	$84	68
	SAVATL	B	$64	68
	SAVLAX	Q	$134	37
	SAVBOS	Q	$69	37
	SAVMIA	Q	$59	37
	SAVATL	Q	$49	37

Table A.6: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	LAXBOS	M	$380	140
	SAVBOS	Y	$319	140
	MIABOS	M	$314	140
	ATLBOS	Y	$310	140
	ATLBOS	M	$290	140
	SAVBOS	M	$250	122
	LAXBOS	B	$159	114
	LAXBOS	Q	$139	114
	MIABOS	B	$124	99
	SAVBOS	B	$109	90
	ATLBOS	B	$95	90
	MIABOS	Q	$89	81
	ATLBOS	Q	$69	66
	SAVBOS	Q	$69	66
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSLAX	M	$380	140
	BOSSAV	Y	$319	140
	BOSMIA	M	$314	140
	BOSATL	Y	$310	140
	BOSATL	M	$290	140
	BOSSAV	M	$250	122
	BOSLAX	B	$159	114
	BOSLAX	Q	$139	114
	BOSMIA	B	$124	99
	BOSSAV	B	$109	90
	BOSATL	B	$95	90
	BOSMIA	Q	$89	81
	BOSATL	Q	$69	66
	BOSSAV	Q	$69	66
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	BOSMIA	M	$314	137
	ATLMIA	Y	$280	137
	LAXMIA	M	$239	124
	SAVMIA	Y	$226	124
	ATLMIA	M	$209	124
	SAVMIA	M	$168	124
	LAXMIA	B	$139	102
	BOSMIA	B	$124	99
	LAXMIA	Q	$119	99
	ATLMIA	B	$94	81
	BOSMIA	Q	$89	81
	SAVMIA	B	$84	81
	ATLMIA	Q	$59	61
	SAVMIA	Q	$59	61

Table A.7: Virtual Nesting EMSR Booking Limits - Low Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIABOS	M	$314	137
	MIAATL	Y	$280	137
	MIALAX	M	$239	124
	MIASAV	Y	$226	124
	MIAATL	M	$209	124
	MIASAV	M	$168	124
	MIALAX	B	$139	102
	MIABOS	B	$124	99
	MIALAX	Q	$119	99
	MIAATL	B	$94	81
	MIABOS	Q	$89	81
	MIASAV	B	$84	81
	MIAATL	Q	$59	61
	MIASAV	Q	$59	61
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	150
	ATLLAX	M	$391	128
	BOSLAX	M	$380	128
	MIALAX	M	$239	120
	BOSLAX	B	$159	112
	SAVLAX	B	$154	112
	ATLLAX	B	$142	112
	MIALAX	B	$139	112
	BOSLAX	Q	$139	112
	SAVLAX	Q	$134	112
	ATLLAX	Q	$122	73
	MIALAX	Q	$119	73
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	150
	LAXATL	M	$391	128
	LAXBOS	M	$380	128
	LAXMIA	M	$239	120
	LAXBOS	B	$159	112
	LAXSAV	B	$154	112
	LAXATL	B	$142	112
	LAXMIA	B	$139	112
	LAXBOS	Q	$139	112
	LAXSAV	Q	$134	112
	LAXATL	Q	$122	73
	LAXMIA	Q	$119	73

Table A.7: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	M	$450	150
	BOSSAV	Y	$319	145
	BOSSAV	M	$250	140
	MIASAV	Y	$226	140
	MIASAV	M	$168	140
	ATLSAV	Y	$159	127
	LAXSAV	B	$154	127
	ATLSAV	M	$140	127
	LAXSAV	Q	$134	127
	BOSSAV	B	$109	92
	MIASAV	B	$84	85
	BOSSAV	Q	$69	76
	ATLSAV	B	$64	76
	MIASAV	Q	$59	76
	ATLSAV	Q	$49	76
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	M	$450	150
	SAVBOS	Y	$319	145
	SAVBOS	M	$250	140
	SAVMIA	Y	$226	140
	SAVMIA	M	$168	140
	SAVATL	Y	$159	127
	SAVLAX	B	$154	127
	SAVATL	M	$140	127
	SAVLAX	Q	$134	127
	SAVBOS	B	$109	92
	SAVMIA	B	$84	85
	SAVBOS	Q	$69	76
	SAVATL	B	$64	76
	SAVMIA	Q	$59	76
	SAVATL	Q	$49	76

Table A.7: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	LAXBOS	M	$380	136
	SAVBOS	Y	$319	136
	MIABOS	M	$314	136
	ATLBOS	Y	$310	136
	ATLBOS	M	$290	136
	SAVBOS	M	$250	109
	LAXBOS	B	$159	98
	LAXBOS	Q	$139	98
	MIABOS	B	$124	74
	SAVBOS	B	$109	61
	ATLBOS	B	$95	61
	MIABOS	Q	$89	47
	ATLBOS	Q	$69	26
	SAVBOS	Q	$69	26
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSLAX	M	$380	136
	BOSSAV	Y	$319	136
	BOSMIA	M	$314	136
	BOSATL	Y	$310	136
	BOSATL	M	$290	136
	BOSSAV	M	$250	109
	BOSLAX	B	$159	98
	BOSLAX	Q	$139	98
	BOSMIA	B	$124	74
	BOSSAV	B	$109	61
	BOSATL	B	$95	61
	BOSMIA	Q	$89	47
	BOSATL	Q	$69	26
	BOSSAV	Q	$69	26
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	BOSMIA	M	$314	130
	ATLMIA	Y	$280	130
	LAXMIA	M	$239	111
	SAVMIA	Y	$226	111
	ATLMIA	M	$209	111
	SAVMIA	M	$168	111
	LAXMIA	B	$139	80
	BOSMIA	B	$124	74
	LAXMIA	Q	$119	74
	ATLMIA	B	$94	49
	BOSMIA	Q	$89	49
	SAVMIA	B	$84	49
	ATLMIA	Q	$59	17
	SAVMIA	Q	$59	17

Table A.8: Virtual Nesting EMSR Booking Limits - Medium Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIABOS	M	$314	130
	MIAATL	Y	$280	130
	MIALAX	M	$239	111
	MIASAV	Y	$226	111
	MIAATL	M	$209	111
	MIASAV	M	$168	111
	MIALAX	B	$139	80
	MIABOS	B	$124	74
	MIALAX	Q	$119	74
	MIAATL	B	$94	49
	MIABOS	Q	$89	49
	MIASAV	B	$84	49
	MIAATL	Q	$59	17
	MIASAV	Q	$59	17
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	150
	ATLLAX	M	$391	119
	BOSLAX	M	$380	119
	MIALAX	M	$239	107
	BOSLAX	B	$159	93
	SAVLAX	B	$154	93
	ATLLAX	B	$142	93
	MIALAX	B	$139	93
	BOSLAX	Q	$139	93
	SAVLAX	Q	$134	93
	ATLLAX	Q	$122	34
	MIALAX	Q	$119	34
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	150
	LAXATL	M	$391	119
	LAXBOS	M	$380	119
	LAXMIA	M	$239	107
	LAXBOS	B	$159	93
	LAXSAV	B	$154	93
	LAXATL	B	$142	93
	LAXMIA	B	$139	93
	LAXBOS	Q	$139	93
	LAXSAV	Q	$134	93
	LAXATL	Q	$122	34
	LAXMIA	Q	$119	34

Table A.8: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	M	$450	150
	BOSSAV	Y	$319	142
	BOSSAV	M	$250	137
	MIASAV	Y	$226	137
	MIASAV	M	$168	137
	ATLSAV	Y	$159	117
	LAXSAV	B	$154	117
	ATLSAV	M	$140	117
	LAXSAV	Q	$134	117
	BOSSAV	B	$109	66
	MIASAV	B	$84	53
	BOSSAV	Q	$69	42
	ATLSAV	B	$64	42
	MIASAV	Q	$59	42
	ATLSAV	Q	$49	42
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	M	$450	150
	SAVBOS	Y	$319	142
	SAVBOS	M	$250	137
	SAVMIA	Y	$226	137
	SAVMIA	M	$168	137
	SAVATL	Y	$159	117
	SAVLAX	B	$154	117
	SAVATL	M	$140	117
	SAVLAX	Q	$134	117
	SAVBOS	B	$109	66
	SAVMIA	B	$84	53
	SAVBOS	Q	$69	42
	SAVATL	B	$64	42
	SAVMIA	Q	$59	42
	SAVATL	Q	$49	42

Table A.8: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	150
	LAXBOS	M	$380	131
	SAVBOS	Y	$319	131
	MIABOS	M	$314	131
	ATLBOS	Y	$310	131
	ATLBOS	M	$290	131
	SAVBOS	M	$250	96
	LAXBOS	B	$159	81
	LAXBOS	Q	$139	81
	MIABOS	B	$124	50
	SAVBOS	B	$109	34
	ATLBOS	B	$95	34
	MIABOS	Q	$89	14
	ATLBOS	Q	$69	0
	SAVBOS	Q	$69	0
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	150
	BOSLAX	M	$380	131
	BOSSAV	Y	$319	131
	BOSMIA	M	$314	131
	BOSATL	Y	$310	131
	BOSATL	M	$290	131
	BOSSAV	M	$250	96
	BOSLAX	B	$159	81
	BOSLAX	Q	$139	81
	BOSMIA	B	$124	50
	BOSSAV	B	$109	34
	BOSATL	B	$95	34
	BOSMIA	Q	$89	14
	BOSATL	Q	$69	0
	BOSSAV	Q	$69	0
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	150
	BOSMIA	M	$314	123
	ATLMIA	Y	$280	123
	LAXMIA	M	$239	99
	SAVMIA	Y	$226	99
	ATLMIA	M	$209	99
	SAVMIA	M	$168	99
	LAXMIA	B	$139	58
	BOSMIA	B	$124	51
	LAXMIA	Q	$119	51
	ATLMIA	B	$94	17
	BOSMIA	Q	$89	17
	SAVMIA	B	$84	17
	ATLMIA	Q	$59	0
	SAVMIA	Q	$59	0

Table A.9: Virtual Nesting EMSR Booking Limits - High Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	150
	MIABOS	M	$314	123
	MIAATL	Y	$280	123
	MIALAX	M	$239	99
	MIASAV	Y	$226	99
	MIAATL	M	$209	99
	MIASAV	M	$168	99
	MIALAX	B	$139	58
	MIABOS	B	$124	51
	MIALAX	Q	$119	51
	MIAATL	B	$94	17
	MIABOS	Q	$89	17
	MIASAV	B	$84	17
	MIAATL	Q	$59	0
	MIASAV	Q	$59	0
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	150
	MIALAX	Y	$477	150
	ATLLAX	Y	$455	150
	SAVLAX	M	$450	150
	ATLLAX	M	$391	109
	BOSLAX	M	$380	109
	MIALAX	M	$239	93
	BOSLAX	B	$159	76
	SAVLAX	B	$154	76
	ATLLAX	B	$142	76
	MIALAX	B	$139	76
	BOSLAX	Q	$139	76
	SAVLAX	Q	$134	76
	ATLLAX	Q	$122	0
	MIALAX	Q	$119	0
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	150
	LAXMIA	Y	$477	150
	LAXATL	Y	$455	150
	LAXSAV	M	$450	150
	LAXATL	M	$391	109
	.AXBOS	M	$380	109
	LAXMIA	M	$239	93
	LAXBOS	B	$159	76
	LAXSAV	B	$154	76
	LAXATL	B	$142	76
	LAXMIA	B	$139	76
	LAXBOS	Q	$139	76
	LAXSAV	Q	$134	76
	LAXATL	Q	$122	0
	LAXMIA	Q	$119	0

Table A.9: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	M	$450	150
	BOSSAV	Y	$319	139
	BOSSAV	M	$250	133
	MIASAV	Y	$226	133
	MIASAV	M	$168	133
	ATLSAV	Y	$159	107
	LAXSAV	B	$154	107
	ATLSAV	M	$140	107
	LAXSAV	Q	$134	107
	BOSSAV	B	$109	38
	MIASAV	B	$84	23
	BOSSAV	Q	$69	5
	ATLSAV	B	$64	5
	MIASAV	Q	$59	5
	ATLSAV	Q	$49	5
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	M	$450	150
	SAVBOS	Y	$319	139
	SAVBOS	M	$250	133
	SAVMIA	Y	$226	133
	SAVMIA	M	$168	133
	SAVATL	Y	$159	107
	SAVLAX	B	$154	107
	SAVATL	M	$140	107
	SAVLAX	Q	$134	107
	SAVBOS	B	$109	38
	SAVMIA	B	$84	23
	SAVBOS	Q	$69	5
	SAVATL	B	$64	5
	SAVMIA	Q	$59	5
	SAVATL	Q	$49	5

Table A.9: Continued

	Y	M	B	Q
ATLBOS/BOSATL	6	5	5	9
ATLSAV/SAVATL	13	3	3	7
ATLMIA/MIAATL	10	5	3	7
ATLLAX/LAXATL	4	2	5	13
BOSSAV/SAVBOS	3	3	5	13
BOSMIA/MIABOS	5	3	7	10
BOSLAX/LAXBOS	5	3	5	12
MIASAV/SAVMIA	7	2	3	13
MIALAX/LAXMIA	9	5	3	7
LAXSAV/SAVLAX	3	3	3	16

Table A.10: Deterministic LP Seat Allocations - Low Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	9	7	8	13
ATLSAV/SAVATL	19	5	4	10
ATLMIA/MIAATL	15	7	5	11
ATLLAX/LAXATL	6	3	8	20
BOSSAV/SAVBOS	4	5	8	20
BOSMIA/MIABOS	8	4	11	15
BOSLAX/LAXBOS	7	5	8	18
MIASAV/SAVMIA	10	3	5	19
MIALAX/LAXMIA	13	8	5	11
LAXSAV/SAVLAX	4	5	5	24

Table A.11: Deterministic LP Seat Allocations - Medium Demand Level

	Y	M	B	Q
ATLBOS	12	9	11	17
BOSATL	12	9	11	17
ATLSAV	25	7	5	13
SAVATL	25	7	5	13
ATLMIA	20	9	7	14
MIAATL	20	9	7	14
ATLLAX	8	4	11	27
LAXATL	8	4	11	27
BOSSAV	5	7	11	3
SAVBOS	5	7	11	8
BOSMIA	11	5	14	18
MIABOS	11	5	14	18
BOSLAX	9	7	11	0
LAXBOS	9	7	6	0
MIASAV	13	4	7	0
SAVMIA	13	4	7	0
MIALAX	17	11	0	0
LAXMIA	17	11	0	0
LAXSAV	5	7	7	31
SAVLAX	5	7	7	26

Table A.12: Deterministic LP Seat Allocations - High Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	11	8	8	12
ATLSAV/SAVATL	17	5	5	9
ATLMIA/MIAATL	15	10	5	10
ATLLAX/LAXATL	6	6	8	17
BOSSAV/SAVBOS	5	7	8	17
BOSMIA/MIABOS	9	5	10	13
BOSLAX/LAXBOS	7	7	8	15
MIASAV/SAVMIA	12	4	5	16
MIALAX/LAXMIA	13	8	5	10
LAXSAV/SAVLAX	7	7	5	21

Table A.13: Probabilistic LP Seat Allocations - Low Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	12	10	9	12
ATLSAV/SAVATL	23	8	5	11
ATLMIA/MIAATL	19	10	6	11
ATLLAX/LAXATL	8	5	8	19
BOSSAV/SAVBOS	6	7	8	12
BOSMIA/MIABOS	11	6	10	11
BOSLAX/LAXBOS	9	7	6	14
MIASAV/SAVMIA	13	4	5	9
MIALAX/LAXMIA	16	9	4	6
LAXSAV/SAVLAX	6	8	5	20

Table A.14: Probabilistic LP Seat Allocations - Medium Demand Level

	Y	M	B	Q
ATLBOS/BOSATL	16	13	12	16
ATLSAV/SAVATL	30	9	6	13
ATLMIA/MIAATL	24	13	7	12
ATLLAX/LAXATL	10	7	10	22
BOSSAV/SAVBOS	8	9	9	0
BOSMIA/MIABOS	14	7	12	5
BOSLAX/LAXBOS	12	9	8	0
MIASAV/SAVMIA	16	5	4	0
MIALAX/LAXMIA	20	11	0	0
LAXSAV/SAVLAX	8	9	6	18

Table A.15: Probabilistic LP Seat Allocations - High Demand Level

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	145
	LAXBOS	M	$380	140
	SAVBOS	Y	$319	137
	MIABOS	M	$314	134
	ATLBOS	Y	$310	131
	ATLBOS	M	$290	125
	SAVBOS	M	$250	120
	LAXBOS	B	$159	117
	LAXBOS	Q	$139	112
	MIABOS	B	$124	100
	SAVBOS	B	$109	93
	ATLBOS	B	$95	88
	MIABOS	Q	$89	83
	ATLBOS	Q	$69	73
	SAVBOS	Q	$69	73
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	145
	BOSLAX	M	$380	140
	BOSSAV	Y	$319	137
	BOSMIA	M	$314	134
	BOSATL	Y	$310	131
	BOSATL	M	$290	125
	BOSSAV	M	$250	120
	BOSLAX	B	$159	117
	BOSLAX	Q	$139	112
	BOSMIA	B	$124	100
	BOSSAV	B	$109	93
	BOSATL	B	$95	88
	BOSMIA	Q	$89	83
	BOSATL	Q	$69	73
	BOSSAV	Q	$69	73
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	141
	BOSMIA	M	$314	136
	ATLMIA	Y	$280	133
	LAXMIA	M	$239	123
	SAVMIA	Y	$226	118
	ATLMIA	M	$209	111
	SAVMIA	M	$168	106
	LAXMIA	B	$139	104
	BOSMIA	B	$124	101
	LAXMIA	Q	$119	94
	ATLMIA	B	$94	87
	BOSMIA	Q	$89	84
	SAVMIA	B	$84	74
	ATLMIA	Q	$59	71
	SAVMIA	Q	$59	71

Table A.16: Nested Deterministic LP Booking Limits - Low Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	141
	MIABOS	M	$314	136
	MIAATL	Y	$280	133
	MIALAX	M	$239	123
	MIASAV	Y	$226	118
	MIAATL	M	$209	111
	MIASAV	M	$168	106
	MIALAX	B	$139	104
	MIABOS	B	$124	101
	MIALAX	Q	$119	94
	MIAATL	B	$94	87
	MIABOS	Q	$89	84
	MIASAV	B	$84	74
	MIAATL	Q	$59	71
	MIASAV	Q	$59	71
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	145
	MIALAX	Y	$477	142
	ATLLAX	Y	$455	133
	SAVLAX	M	$450	129
	ATLLAX	M	$391	126
	BOSLAX	M	$380	124
	MIALAX	M	$239	121
	BOSLAX	B	$159	116
	ATLLAX	B	$142	108
	MIALAX	B	$139	103
	BOSLAX	Q	$139	103
	SAVLAX	B	$154	111
	SAVLAX	Q	$134	88
	ATLLAX	Q	$122	72
	MIALAX	Q	$119	59
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	145
	LAXMIA	Y	$477	142
	LAXATL	Y	$455	133
	LAXSAV	M	$450	129
	LAXATL	M	$391	126
	LAXBOS	M	$380	124
	LAXMIA	M	$239	121
	LAXBOS	B	$159	116
	LAXSAV	B	$154	111
	LAXATL	B	$142	108
	LAXMIA	B	$139	103
	LAXBOS	Q	$139	103
	LAXSAV	Q	$134	88
	LAXATL	Q	$122	72
	LAXMIA	Q	$119	59

Table A.16: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	M	$450	147
	BOSSAV	Y	$319	144
	MIASAV	Y	$226	141
	BOSSAV	M	$250	134
	MIASAV	M	$168	131
	ATLSAV	Y	$159	129
	LAXSAV	B	$154	116
	ATLSAV	M	$140	113
	LAXSAV	Q	$134	110
	BOSSAV	B	$109	94
	MIASAV	B	$84	89
	BOSSAV	Q	$69	86
	ATLSAV	B	$64	73
	MIASAV	Q	$59	70
	ATLSAV	Q	$49	57
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	M	$450	147
	SAVBOS	Y	$319	144
	SAVBOS	M	$250	141
	SAVMIA	Y	$226	138
	SAVMIA	M	$168	131
	SAVATL	Y	$159	129
	SAVLAX	B	$154	116
	SAVATL	M	$140	113
	SAVLAX	Q	$134	110
	SAVBOS	B	$109	94
	SAVMIA	B	$84	89
	SAVBOS	Q	$69	86
	SAVATL	B	$64	73
	SAVMIA	Q	$59	70
	SAVATL	Q	$49	57

Table A.16: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	143
	LAXBOS	M	$380	135
	SAVBOS	Y	$319	130
	ATLBOS	Y	$310	126
	MIABOS	M	$314	117
	ATLBOS	M	$290	113
	SAVBOS	M	$250	106
	MIABOS	B	$124	101
	SAVBOS	B	$109	90
	LAXBOS	B	$159	82
	ATLBOS	B	$95	74
	LAXBOS	Q	$139	66
	MIABOS	Q	$89	48
	SAVBOS	Q	$69	33
	ATLBOS	Q	$69	13
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	143
	BOSATL	Y	$310	135
	BOSATL	M	$290	126
	BOSLAX	M	$380	119
	BOSSAV	Y	$319	114
	BOSMIA	M	$314	110
	BOSSAV	M	$250	106
	BOSATL	B	$95	101
	BOSATL	Q	$69	93
	BOSMIA	B	$124	80
	BOSSAV	B	$109	69
	BOSLAX	B	$159	61
	BOSLAX	Q	$139	53
	BOSMIA	Q	$89	53
	BOSSAV	Q	$69	20
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	137
	BOSMIA	M	$314	129
	ATLMIA	Y	$280	125
	SAVMIA	Y	$226	110
	ATLMIA	M	$209	100
	LAXMIA	M	$239	93
	SAVMIA	M	$168	85
	BOSMIA	B	$124	82
	ATLMIA	B	$94	71
	SAVMIA	B	$84	66
	LAXMIA	B	$139	61
	BOSMIA	Q	$89	56
	LAXMIA	Q	$119	41
	ATLMIA	Q	$59	41
	SAVMIA	Q	$59	41

Table A.17: Nested Deterministic LP Booking Limits - Medium Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	137
	MIAATL	Y	$280	129
	MIABOS	M	$314	114
	MIAATL	M	$209	110
	MIASAV	Y	$226	103
	MIALAX	M	$239	93
	MIASAV	M	$168	85
	MIAATL	B	$94	82
	MIAATL	Q	$59	77
	MIABOS	B	$124	66
	MIASAV	B	$84	55
	MIALAX	B	$139	50
	MIABOS	Q	$89	45
	MIALAX	Q	$119	30
	MIASAV	Q	$59	30
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	143
	MIALAX	Y	$477	139
	ATLLAX	Y	$455	126
	SAVLAX	M	$450	120
	ATLLAX	M	$391	115
	BOSLAX	M	$380	112
	MIALAX	M	$239	107
	SAVLAX	B	$154	99
	ATLLAX	B	$142	94
	ATLLAX	Q	$122	49
	BOSLAX	B	$159	86
	SAVLAX	Q	$134	78
	MIALAX	B	$139	54
	BOSLAX	Q	$139	29
	MIALAX	Q	$119	11
LAX-ATL	LAXBOS	Y	$575	150
	LAXATL	Y	$455	143
	LAXSAV	Y	$502	137
	LAXMIA	Y	$477	133
	LAXSAV	M	$450	120
	LAXATL	M	$391	115
	LAXBOS	M	$380	112
	LAXMIA	M	$239	107
	LAXATL	B	$142	99
	LAXATL	Q	$122	91
	LAXSAV	B	$154	71
	LAXBOS	B	$159	66
	LAXSAV	Q	$134	58
	LAXMIA	B	$139	34
	LAXBOS	Q	$139	29
	LAXMIA	Q	$119	11

Table A.17: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	M	$450	146
	BOSSAV	Y	$319	141
	BOSSAV	M	$250	137
	MIASAV	Y	$226	132
	ATLSAV	Y	$159	122
	MIASAV	M	$168	103
	ATLSAV	M	$140	100
	BOSSAV	B	$109	90
	LAXSAV	B	$154	95
	LAXSAV	Q	$134	82
	MIASAV	B	$84	82
	ATLSAV	B	$64	53
	BOSSAV	Q	$69	49
	MIASAV	Q	$59	49
	ATLSAV	Q	$49	49
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	M	$450	146
	SAVBOS	Y	$319	141
	SAVBOS	M	$250	137
	SAVMIA	Y	$226	132
	SAVATL	Y	$159	122
	SAVATL	M	$140	103
	SAVMIA	M	$168	98
	SAVATL	B	$64	95
	SAVATL	Q	$49	91
	SAVLAX	B	$154	81
	SAVBOS	B	$109	76
	SAVLAX	Q	$134	68
	SAVMIA	B	$84	68
	SAVBOS	Q	$69	39
	SAVMIA	Q	$59	39

Table A.17: Continued

LEG	SEGMENT	CLASS	FARE	BK LIMIT
ATL-BOS	LAXBOS	Y	$575	150
	MIABOS	Y	$403	141
	ATLBOS	Y	$310	130
	ATLBOS	M	$290	118
	SAVBOS	Y	$319	109
	MIABOS	M	$314	104
	LAXBOS	M	$380	99
	SAVBOS	M	$250	92
	ATLBOS	B	$95	85
	SAVBOS	B	$109	74
	MIABOS	B	$124	63
	ATLBOS	Q	$69	49
	LAXBOS	B	$159	32
	MIABOS	Q	$89	32
	SAVBOS	Q	$69	32
	LAXBOS	Q	$139	0
BOS-ATL	BOSLAX	Y	$575	150
	BOSMIA	Y	$403	141
	BOSATL	Y	$310	130
	BOSSAV	Y	$319	118
	BOSATL	M	$290	113
	BOSMIA	M	$314	104
	BOSLAX	M	$380	99
	BOSSAV	M	$250	92
	BOSSAV	B	$109	85
	BOSATL	B	$95	74
	BOSMIA	B	$124	63
	BOSATL	Q	$69	49
	BOSLAX	B	$159	32
	BOSMIA	Q	$89	32
	BOSSAV	Q	$69	32
	BOSLAX	Q	$139	0
ATL-MIA	LAXMIA	Y	$477	150
	BOSMIA	Y	$403	133
	ATLMIA	Y	$280	122
	BOSMIA	M	$314	102
	ATLMIA	M	$209	97
	SAVMIA	Y	$226	88
	SAVMIA	M	$168	75
	LAXMIA	M	$239	71
	ATLMIA	B	$94	60
	BOSMIA	B	$124	53
	ATLMIA	Q	$59	39
	SAVMIA	B	$84	25
	BOSMIA	Q	$89	18
	SAVMIA	Q	$59	0
	LAXMIA	B	$139	0
	LAXMIA	Q	$119	0

Table A.18: Nested Deterministic LP Booking Limits - High Demand Level

MIA-ATL	MIALAX	Y	$477	150
	MIABOS	Y	$403	133
	MIAATL	Y	$280	122
	MIABOS	M	$314	102
	MIASAV	Y	$226	97
	MIAATL	M	$209	84
	MIASAV	M	$168	75
	MIALAX	M	$239	71
	MIAATL	B	$94	60
	MIABOS	B	$124	53
	MIASAV	B	$84	39
	MIAATL	Q	$59	32
	MIABOS	Q	$89	18
	MIASAV	Q	$59	0
	MIALAX	B	$139	0
	MIALAX	Q	$119	0
ATL-LAX	BOSLAX	Y	$575	150
	SAVLAX	Y	$502	141
	ATLLAX	Y	$455	136
	MIALAX	Y	$477	128
	SAVLAX	M	$450	111
	ATLLAX	M	$391	104
	BOSLAX	M	$380	100
	MIALAX	M	$239	93
	ATLLAX	B	$142	82
	SAVLAX	B	$154	71
	ATLLAX	Q	$122	71
	BOSLAX	B	$159	37
	SAVLAX	Q	$134	37
	MIALAX	B	$139	0
	BOSLAX	Q	$139	0
	MIALAX	Q	$119	0
LAX-ATL	LAXBOS	Y	$575	150
	LAXSAV	Y	$502	141
	LAXATL	Y	$455	136
	LAXMIA	Y	$477	128
	LAXSAV	M	$450	111
	LAXATL	M	$391	104
	LAXBOS	M	$380	100
	LAXMIA	M	$239	93
	LAXSAV	B	$154	82
	LAXATL	B	$142	82
	LAXBOS	B	$159	64
	LAXSAV	Q	$134	64
	LAXATL	Q	$122	64
	LAXMIA	B	$139	0
	LAXBOS	Q	$139	0
	LAXMIA	Q	$119	0

Table A.18: Continued

ATL-SAV	LAXSAV	Y	$502	150
	LAXSAV	M	$450	145
	BOSSAV	Y	$319	138
	BOSSAV	M	$250	133
	MIASAV	Y	$226	126
	ATLSAV	Y	$159	113
	ATLSAV	M	$140	88
	MIASAV	M	$168	81
	ATLSAV	B	$64	77
	BOSSAV	B	$109	72
	ATLSAV	Q	$49	61
	LAXSAV	B	$154	48
	MIASAV	B	$84	48
	LAXSAV	Q	$134	34
	BOSSAV	Q	$69	34
	MIASAV	Q	$59	0
SAV-ATL	SAVLAX	Y	$502	150
	SAVLAX	M	$450	145
	SAVBOS	Y	$319	138
	SAVBOS	M	$250	133
	SAVMIA	Y	$226	126
	SAVATL	Y	$159	113
	SAVATL	M	$140	88
	SAVMIA	M	$168	81
	SAVBOS	B	$109	77
	SAVATL	B	$64	66
	SAVMIA	B	$84	61
	SAVLAX	B	$15[illegible]	61
	SAVATL	Q	$49	47
	SAVLAX	Q	$134	34
	SAVBOS	Q	$69	34
	SAVMIA	Q	$59	0

Table A.18: Continued