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AIRLINE O-D CONTROL USING NETWORK DISPLACEMENT CONCEPTS

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ABSTRACT

In the airline industry, it is customary for carriers to offer a wide range of fares for any given seat in the same cabin on the same flight. In order to maximize the total network revenue, the airline practices so-called seat inventory control methods.

In this thesis, we first examine several seat inventory control methods which are employed or are being developed by some airlines. Then, based on these methods, we propose three new models to control the seats: the Network Non-greedy Heuristic Bid Price model, the Leg Based Probability Non-greedy Bid Price Model, and the Convergence Model. These models are created in order to find a better way to evaluate the connecting fares, taking into consideration their displacement impacts.

An integrated optimization / booking simulation tool is employed in this research to compare the new models with the other methods in terms of network performance under the same demand circumstances.

Generally, all the three new models improve network performance. The revenue results obtained from the simulation show that using network displacement concepts can provide us with an average of 0.5% revenue gain over the methods that do not explicitly include the displacement impacts at a load factor of 93%. The simulation results also show that under the same demand circumstance and seat control strategies, using a better way to evaluate the displacement impacts can provide 0.05% revenue improvement.

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Chapter 1

Introduction

1.1 Motivation for Revenue Management

Revenue Management is an attempt by airlines to optimize their total revenue by achieving a different passenger mix on each flight departure for those passengers paying full fares, those paying discount fares, and those paying deep discount fares[1]. It includes two processes: Price Differentiation and Seat Inventory Control. In the pricing differentiation process, airlines try to distribute passengers into different groups and make the passengers who are able to pay more spend as much as they can, and make the remaining seats, which otherwise will be empty, available for those passengers who will not travel if the price is too high. Figure 1.1 is a demand curve of a particular flight. We know that the optimal *potential* revenue from this flight is the whole area under the demand curve, which is $25,000.

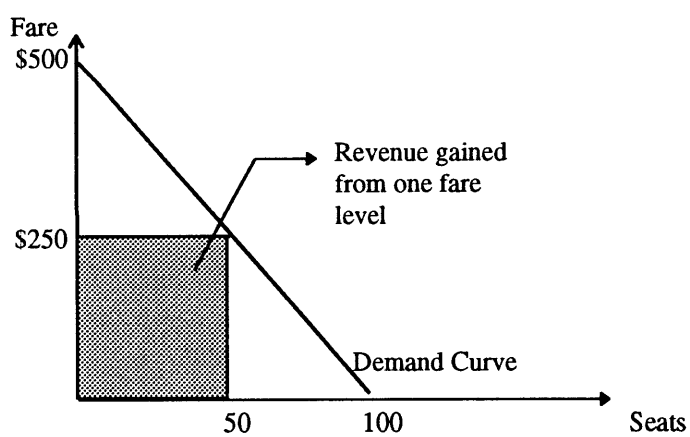

Figure 1.1 Single Fare Product Example

[1] GAO/RCED-90-102, *Fares and Service at Major Airports*

If a single fare level strategy is employed, the revenue the airline can achieve from this flight is only $12,500 (the shaded area). In practice, sometimes, a single fare level will not cover total operating costs[2]. However, under the same demand assumption, if the airline offers seats at three different fare levels and if we assume there are perfect strategies to segment the passengers, then the total revenue the airline can acquire will be $18,750. Figure 1.2 explains how those three fare levels can provide a higher total network revenue than the single fare strategy.

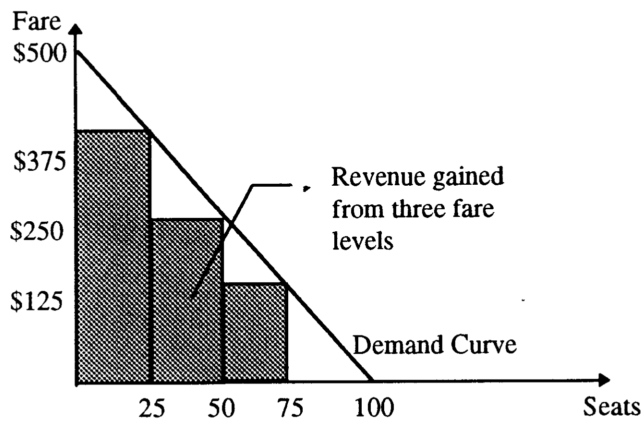

Figure 1.2 Multiple Fare Product Example

Such a fare structure increases the total revenue and makes it easier for an airline to cover total operating costs.

As Williamson (1992) wrote in her thesis "the airlines are not directly discriminating in price between different passengers for the same fare product. The differential in price offered by airlines is usually based on differences in fare products, each of which is uniquely defined by restrictions on their purchase and use for air travel." In order to obtain the benefits of price differentiation, the airlines must manage their fare levels and the restrictions associated with those fare levels effectively.

In practice, several issues need to be considered to achieve revenue maximization. First, how many fare levels should an airline set in its fare structure? Theoretically, there should be as many fare levels as the number of seats on each flight, which means one price for each passenger. One obvious problem of this scheme is that the complicated fare structure will confuse not only the passengers but also the travel agents. It is also difficult to employ this theory in the real world due to a lack of perfect information about the price

[2] E.L.Williamson, *Airline Network Seat Inventory Control: Methodologies and Revenue Impacts*, Flight Transportation Lab Report, R92-3, June 1992.

elasticity for every single passenger. Furthermore, as the number of the different fare prices increases, the cost for advertisement and reservation systems will increase. Considering all those elements, most airlines have no more than 10 different fare products for each market.

Second, how should the airlines prevent the time sensitive business passengers from taking advantage of certain low fare products designed for the price sensitive leisure passengers? Usually, the airlines place some restrictions along with each discounted fare product, such as Saturday night stay, 7/14 days advance purchase, etc. In practice, these restrictions are, at least as, if not more, important than how many fare levels the airlines should offer.

The third issue relating to revenue maximization regards the booking strategy. Usually in the real world operation, the low fare passengers are more likely to request bookings earlier, while the full fare passengers tend to come late and book at the last minute before a flight. Figure 1.3 shows hypothetical booking curves of different fare types.

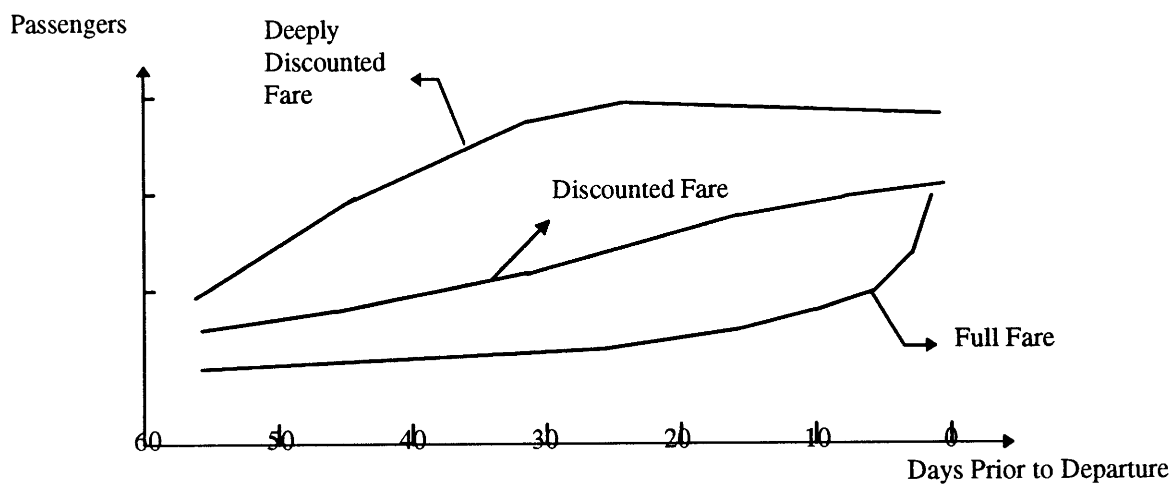

Figure 1.3 Booking Performance of Different Fare Types before Departure

From the figure above we can see that the deep discount requests come much earlier than full fare requests. The airlines then face a trade-off as to whether they should give a seat to a low fare passenger or reserve it for a last minute high fare passenger, but if he/she does not materialize then this seat will go empty. Therefore, it is very important for an airline to control the number of seats available as discounted fare products and to reserve enough seats for last minute high fare passengers. This process is called seat inventory control. It is an attempt for airlines to balance the number of seats sold at each fare level so as to maximize total passenger revenue. Belobaba(1987) said in his doctoral thesis, "The seat inventory control process is a tactical component of revenue management that is entirely under the control of each individual airline and is hidden from consumers and competitors alike.... Nevertheless, seat inventory control has the potential of increasing total revenues expected from flights on a departure-by-departure basis, something that would be far more difficult through pricing actions."[3]

1.2 Goal of The Thesis

In the competition among airlines, price differences are no longer determined by how low an airline's fare levels are. If we look at the published fare prices of the airlines in America, most airlines have the same fare prices. The primary difference among them is how many seats are available to each fare product by each airline on each flight it operates.

Generally speaking, there are five major different seat inventory control methods applied or being developed in the airline industry today.

- Leg Based Fare Class Yield Management.
- Greedy Virtual Nesting and/or Fare Stratification Yield Management.
- Greedy EMSR Heuristic Bid Price.
- Non-greedy Virtual Nesting based on Network Displacement.
- Network Bid Price Control.

One objective of this thesis is to analyze the algorithms of these five methods, see how they are applied in the real world, and identify the major differences among them. Then, a new revenue management method is proposed. This method combines the merits of both the EMSR Heuristic Bid Price method and the non-greedy virtual nesting method, and it shows revenue improvement with the true data from several airlines.

[3] P.P. Belobaba, *Air Travel Demand And Airline Seat Inventory Management*, Flight Transportation Laboratory Report, R87-7, May 1987.

1.3 Structure of the Thesis

This thesis includes five chapters.

Following this introductory chapter, in Chapter 2, the five existing major yield management methods will be briefly discussed. Using a simple example, the major differences among these five methods will be considered, and pros and cons associated with each method will be addressed.

In Chapter 3, a new revenue management method is proposed. In this new method, we will employ network optimization tools to obtain the displacement impact of the connecting passengers. Then, passengers are booked based on the consideration of their whole network revenue contribution. Several implementation issues associated with this new method are also discussed.

In Chapter 4, actual data from two airlines will be used to compare the different revenue performance of these six revenue management methods through simulation. For the new proposed method, sensitivity analyses will be presented to identify the best formula in the bid price calculation.

Finally, Chapter 5 will summarize the research findings and contributions of this work.

Chapter 2

Existing Revenue Management Methods

As a prelude to the extension of existing models in the subsequent chapter, this chapter presents an overview of the mathematical approaches developed previously. There are five major methods introduced here: Leg Based Fare Class Yield Management, Greedy Virtual Nesting, Heuristic Bid Price, Non-greedy Virtual Nesting, and Network Bid Price. A simple example will be used to explain these methods. Also, the booking limits calculated from these five different methods are compared.

2.0 Introduction

Generally speaking, the approaches to seat inventory control can be divided into two major groups, the partitioned network method and the heuristic nesting method. In a partition method, the solution is simply the number of seats allocated to each origin-destination (OD) fare product, while in a heuristic nesting method the solution is the number of seats protected for a group of OD fare products.

2.0.1 Partitioned Network Optimization

One way to solve the seat inventory control problem is by using network optimization tools to solve the following linear program (LP) problem[1]:

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[1] E. L. Williamson, *Airline Network Seat Inventory Control: Methodologies And Revenue Impacts,* Flight Transportation Laboratory Report, R92-3, June 1992.

Max:

$$\mathrm{Re}\,venue = \sum_{i}^{ODs} \sum_{j}^{FareClasses} fare(i,j)x(i,j); \qquad [2.1]$$

Subject to:

$$\sum_{i,j}^{x(i,j)\in Leg(k)} x(i,j) \le Capacity(k)\ , \qquad \text{for all leg k;}$$

$$x(i,j) \le Demand(i,j)\,, \qquad \text{for all OD pairs;}$$

where in the formula, *i* is the number of the OD pairs, and j is the fare class number. This is a partitioned method because the *x(i,j)* is simply the allocation of seats to each OD fare. Once a seat is allocated to an OD fare, it can only be used by that OD fare. The limitation of this method is that it ignores the stochastic character of demand because, in the second set of constraints, the right hand side *Demand(i,j)* is just the mean of each OD demand. So this method assumes perfect demand forecasting and zero demand deviation.

One more drawback of this method is that it is difficult for an airline to collect the data on demand for all OD fares in a large network. Most computer reservation systems do not routinely store information to support such a network optimization process. In the meantime, since the data changes with the booking periods, how often an airline should re-run such a large optimization process becomes critical. Re-running this process too infrequently will make a negative network revenue impact while re-running it too often is very uneconomical.

Another shortcoming of this method is the "small numbers" problem. For a hub network with 25 flights in and 25 flights out of a connecting hub, there can be 26 different OD itineraries on a single flight leg. With 10 fare classes and an overall average aircraft size of 158 seats[2], the number of seats per OD fare is, on average, less than 1[3]. In addition, there will be a great chance that a seat remains empty due to the deviation of demand. These reasons make such partitioned network optimization rarely used in the airline industry[4].

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[2] Airline Economics, Inc., Datagram: *Major U. S. Airline Performance,* Aviation Week & Space Technology, Volume 133, October 8, 1990.

[3] E. L. Williamson, *Airline Network Seat Inventory Control: Methodologies And Revenue Impacts,* Flight Transportation Laboratory Report, R92-3, June 1992.

[4] P.P. Belobaba, *Airline O-D Seat Inventory Control Without Network optimization,* Flight Transportation Laboratory, June 1995.

2.0.2 Heuristic Nesting Method

The airline seat inventory management problem is probabilistic because there exists uncertainty about the ultimate number of requests that an airline will receive for seats on a future flight and, more specifically, for the different fare classes offered on that flight[5]. Nested Heuristics is different from the partitioned method in that it divides all the network OD fares into several groups. Seats are thereby allocated to each group. It is like protecting seats for higher fare products from lower ones. In his doctoral thesis, Belobaba[6] presented a heuristic nesting method called Expected Marginal Seat Revenue (EMSR) method to solve the multiple fare class problem on a single leg. In this method, Belobaba proposed using the expected marginal seat revenue to determine the number of seats that should be protected for each higher fare class i over the lower fare class j. By the definition of this method, the seat will be protected for the higher fare class as long as the following equation holds.

$$EMSR(S_j^i) = Fare_i \times \overline{P_i}(S_j^i) \geq Fare_j \qquad [2.2]$$

where i is the higher fare class, while j is the lower fare class. Then, the seat protection for the highest fare class Π_1 is simply S_2^1. However, the number of seats that should be protected for the two highest fare classes, Π_2, is defined as the sum of S_3^1 and S_3^2, determined separately from the equation above. Therefore the booking limits, BL_i, the number of seats available to each fare class, is defined as the capacity minus the number of seats protected for all higher fare classes.

$$BL_i = Capacity - \Pi_{i-1} \qquad [2.3]$$

This method is known as EMSRa. In 1992, Belobaba suggested another method known as EMSRb. In this method, he proposed to calculate the seat protection levels jointly for all higher fare classes relative to a given lower class, based on a combined demand forecast and a weighted price level for all classes above the one for which a booking limit is being calculated. The weighting is done based on expected demand that class[7]. Such

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[5] P.P. Belobaba, *Air Travel Demand and Airline Seat Inventory Control Management*, Flight Transportation Laboratory Report, R87-7, May 1987.
[6] P.P. Belobaba, *Air Travel Demand and Airline Seat Inventory Control Management*, Flight Transportation Laboratory Report, R87-7, May 1987.
[7] P.P. Belobaba, *Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations*, Decision Sciences, Volume 27, Number 2, Spring 1996.

method has been shown to have higher net revenue in practice of seat inventory control than the partitioned network method.

2.0.3 Some Issues

Before we do any research, we must understand what makes a revenue management method implementable in the airline industry. First, of course, there must be network revenue improvement. Second, which is very important yet easy to ignore, is that the information to support the method must be easy to access. That is why some of the methods we discuss in this chapter cannot be implemented in the real world even though they give very good revenue performance.

Following we will discuss five different revenue management methods that are implemented or being developed in the airline industry in order to find out why some methods perform better than others. Such information provides a direction for our research.

2.1 Leg Based Fare Class YM

In this method, data are collected based on each leg and fare class, and forecasts are made based on each leg. All OD fares that traverse the same legs are divided into several fare groups based on their relative yield and restriction types no matter whether they are local fares or connecting fares. Next, the EMSRb method is used to calculate the number of seats that should be protected for each group of fare products. The fare value, which represents each fare group, used as the input for the EMSRb calculation, could be demand-weighted-mean fares, local fares, or prorated fares.

The following example will be helpful to explain this method. Figure 2.1 shows a very simple network with only two legs.

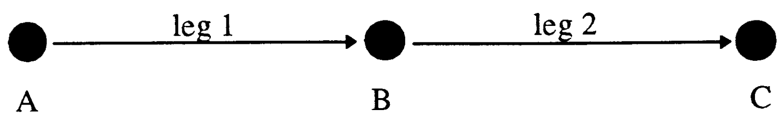

Figure 2.1 A Simple Network with 2 Legs

Suppose we have the following information about the demand and fares.

Leg 1: 500 miles in trip length.

	Fare Class	Fare	Demand	Standard Deviation
Local Passengers	Y	500	10	5
	B	300	20	8
	Q	150	40	15
Connecting Passengers	Y	1000	8	4
	B	625	25	10
	Q	250	45	17

Table 2.1 Fare and Demand Information on Leg 1

Leg 2: 600 miles in trip length.

	Fare Class	Fare	Demand	Standard Deviation
Local Passengers	Y	600	15	6
	B	350	25	10
	Q	200	45	17
Connecting Passengers	Y	1000	8	4
	B	625	25	10
	Q	250	45	17

Table 2.2 Fare and Demand Information on Leg 2

There are three steps to fulfill the inventory control process: grouping fares, generating EMSR inputs, and calculating booking limits.

2.1.1 Process of the Method

1. Grouping of the Fares

The leg-based fare class yield management is a nesting heuristic method, and it involves nesting by fare class. In this method, the fares are grouped into different classes based on their fare classes. In our example here, there are three fare classes, full fare Y class, business B class, and discount Q class. All OD fares that cross one particular leg will be grouped in a class, which is the same as their fare classes, no matter whether they are local fares or connecting fares. Therefore we will have the following fare groups:

Leg 1	Leg 2
Y classes: Y(AB) Y(AC)	Y classes: Y(BC) Y(AC)
B classes: B(AB) B(AC)	B classes: B(BC) B(AC)
Q classes: Q(AB) Q(AC)	Q classes: Q(BC) Q(AC)

Table 2.3 Fare Groups in Leg Based Fare Class YM Method

The seats will be protected according to each of those groups.

2. EMSR Fare Inputs

The fares in each fare group are not the same. For example, fares in Y class on leg 1 are

$$Fare_{Y(AB)} = \$500, \text{ and} \\ Fare_{Y(AC)} = \$1000. \qquad [2.4]$$

Therefore, before calculating the EMSR curve, we need to decide which value of a fare should be used as EMSR input. There are several ways to decide this[8]: Demand Weighted Mean Fare, Local Fare, Mileage Prorated Fare, and Local Fare Prorated Fare. In this thesis, we will use the demand weighted fare as our input.

[8] E.L. Williamson, *Airline Network Seat Inventory Control: Methodologies And Revenue Impacts,* Flight Transportation Laboratory Report, R92-3, June 1992.

• **Demand Weighted Mean Fare**

In this method, the fares are weighted by demand. In our example, for instance, Y class on leg 1, the demand-weighted fare can be calculated as follows:

$$Fare = \frac{Fare(AB) \times Demand(AB) + Fare(AC) \times Demand(AC)}{Demand(AB) + Demand(AC)} \quad [2.5]$$

$$= \$722.22$$

In the same way, we can obtain other input fares as shown in Table 2.4.

	Y class	B class	Q class
Leg 1	$722.22	$462.50	$225.00
Leg 2	$739.13	$480.56	$202.94

Table 2.4 Demand-weighted Mean Fare for EMSR Input

3. Calculating the EMSR Seat Protection for Each Fare Group

The rule of EMSR seat protection is: as long as the Expected Marginal Seat Revenue of the higher fare class is higher than the next lower fare class, the seats should be protected for the higher fare class passengers. The formula for such protection is given in (2.2). From this formula, we can calculate the number of seats that should be protected for each fare level. In this calculation, EMSRb method is implemented to decide the protection. Please refer to Belobaba (1992) for a detailed description. The solutions are listed in the following table.

	Y class	Y and B classes
Seats Protection on Leg 1	16	66
Seats Protection on Leg 2	20	78

Table 2.5 Seats Protection using Fare Class Nesting

Using formula (2.3) we have the following booking limits for each class:

	Y class	B class	Q class
Booking Limits on Leg 1	100	74	34
Booking Limits on Leg 2	100	80	22

Table 2.6 Booking Limits in Leg Based Fare Class YM Method

For local passengers, if the booking limits of their corresponding fare classes are greater than zero, then they are booked. On the other hand, the connecting passengers can be booked if and only if the booking limits of their fare classes are greater than zero on all the legs they traverse.

2.1.2 Summary

This method directly uses the fare classes to group the fares regardless of whether they are local fares or connecting fares. Hence it is easy to implement and normally results in a passenger mix with a high yield. However, this method tends to give priority to short haul high yield passengers; therefore it may take the risk of spilling lower yield but high revenue long haul connecting passengers. This may cause a negative network revenue impact especially when demand is low and there are empty seats remaining on some legs. Under such a circumstance, taking one connecting passenger from a lower fare class may produce more network revenue contribution than one local passenger from a higher fare class.

2.2 Greedy Nesting

First proposed by Boeing[9], in this method, OD fares are assigned to different inventory buckets according to a fare range associated with the bucket. OD fares within each bucket are then controlled as a group. "Greedy" here means the nesting approach is based totally on the itinerary fare value, and it will tend to give higher priority to long haul,

[9]Boeing Commercial Airplane Company, *A Pilot Study of Seat Inventory Management for a Flight Itinerary*, Unpublished Internal Report, U.S. and Canadian Airline Analysis, Renton, WA, Feb. 24, 1983.

connecting passengers. This method can be divided into two different sub-methods: Fare Stratification and Virtual Nesting. They are similar since both of them are "greedy", while they differ in the way they are implemented.

2.2.1 Fare Stratification

This method is distinguished from Fare Class Nesting because it groups all fares on each leg into several classes based on their fare values. The number of those classes is generally the same as the number of the fare classes, but used in a separate way. For example, determined by the number of booking classes in the system, we can choose to cluster the fares into four groups as follows (based on their total fare value):

Leg 1			Leg 2		
Y class	Y(AC)		Y class	Y(AC)	
B class	Y(AB),	B(AC)	B class	Y(AB),	B(AC)
Q class	B(AB),	Q(AC)	Q class	B(AB),	Q(AC)
V class	Q(AB)		V class	Q(AB)	

Table 2.7 Fare Groups in the Fare Stratification Method

Next, we use demand-weighted mean fare or mileage-weighted mean fare to calculate the EMSR input, then calculate the seat protection for each fare class group. Since, in this method, we have put the full fare connecting passengers in the highest fare class, they will have the higher priority to access the seats than full fare local passengers. The local discount fare, which has the lowest fare value, is ranked in V class, and it will be the first class to be rejected (spilled) if the demands exceed the seat capacity.

2.2.2 Greedy Virtual Nesting

In contrast to Fare Stratification, in this method, the fares are no longer assigned to Y, B, Q and V classes. Instead, they are grouped into several so-called "virtual" classes by their fare values. The number of virtual classes can be arbitrarily selected. The maximum number of virtual classes can be equal to the number of different fare values on each leg. Following are the virtual classes for our example in Figure 2.1.

Leg 1:

Virtual Classes	Fare Classes	Fare
Y1	Y(AC)	$1000
Y2	B(AC)	$625
Y3	Y(AB)	$500
Y4	B(AB)	$300
Y5	Q(AC)	$250
Y6	Q(AB)	$150

Table 2.8 Virtual Classes on Leg 1

Leg 2:

Virtual Classes	Fare Classes	Fare
Y1	Y(AC)	$1000
Y2	B(AC)	$625
Y3	Y(BC)	$600
Y4	B(BC)	$350
Y5	Q(AC)	$250
Y6	Q(BC)	$200

Table 2.9 Virtual Classes on Leg 2

Here we have chosen an extreme case, where each fare value is in a different virtual class, and the fares are ranked totally based on their itinerary fares. Next, based on these virtual classes, the EMSR value will be calculated. The calculation is exactly the same as what has been done in leg-based fare class yield management method. The seat protection and booking limits are given in the following tables. For the detailed calculations, please refer to Appendix 1.

	Y1	Y2	Y3	Y4	Y5
Seats Protection on Leg 1	7	27	44	65	116
Seats Protection on Leg 2	7	22	48	75	121

Table 2.10 Seats Protection using Virtual Class Nesting

Assume that the capacity on both legs is 100 seats.

	Y1	Y2	Y3	Y4	Y5	Y6
Booking Limits on Leg 1	100	93	73	56	35	0
Booking Limits on Leg 2	100	93	78	52	25	0

Table 2.11 Booking Limits in the Greedy Virtual Nesting Method

Comparing the results in Tables 2.10 and 2.11 with the results given in Table 2.5 and 2.6, we can see that, unlike the leg-based yield management method, in which both local and connecting Q classes have seats available, the virtual class Y6, local Q class, is not available while virtual class Y5 (connecting Q class) is still available. This is due to the greedy virtual nesting method giving higher priority to the long haul high fare passengers.

2.2.3 Summary

Unlike the leg-based fare class YM method, this method groups the fares according to their total fare values, so it tends to give more favor to the high fare long haul connecting passengers. For a simple network like Figure 2.1, which has only 2 legs, there are four circumstances depending upon the demand: Both legs have low demand, both legs have high demand, Leg 1 has low demand while Leg 2 has high demand, or Leg 1 has high demand while Leg 2 has low demand. In three out of four of these cases, that is, when both legs or at least one of the legs has empty seats, such a method is desirable. However, when the demand is severely high and both legs have spill, this method may have a consequential negative impact. This is because, under such a condition, booking two local passengers can give have higher total network revenue contribution than booking one connecting passenger.

2.3 EMSR Heuristic Bid Price

The greedy virtual nesting method groups the fares in the virtual classes based totally on their fare values. The disadvantage with such a strategy is that, when the demands on both legs are extremely high, loading one connecting passenger while spilling two local passengers, whose total fares are higher than one connecting passenger, will have a negative network fare impact. The idea of the heuristic bid price method (Belobaba 1995) is to capture the displacement impact of a connecting passenger onto the local passengers. It uses the virtual classes to calculate the current EMSR curve, then uses this EMSR value to set a "cut-off" price called "bid price." For the local passengers, this bid price is simply the current EMSR value on that leg. For the connecting passengers, the bid price will be the combination of EMSR values on all legs the fare traverses. Next, we will use the example in Figure 2.1 to explain this method.

2.3.1 Process of the Method

1.EMSR Curve Calculation

As shown in Table 2.8 and 2.9, we have six virtual classes on both Leg 1 and Leg 2. Suppose the capacity on each leg is 100. The EMSR curve is shown below.

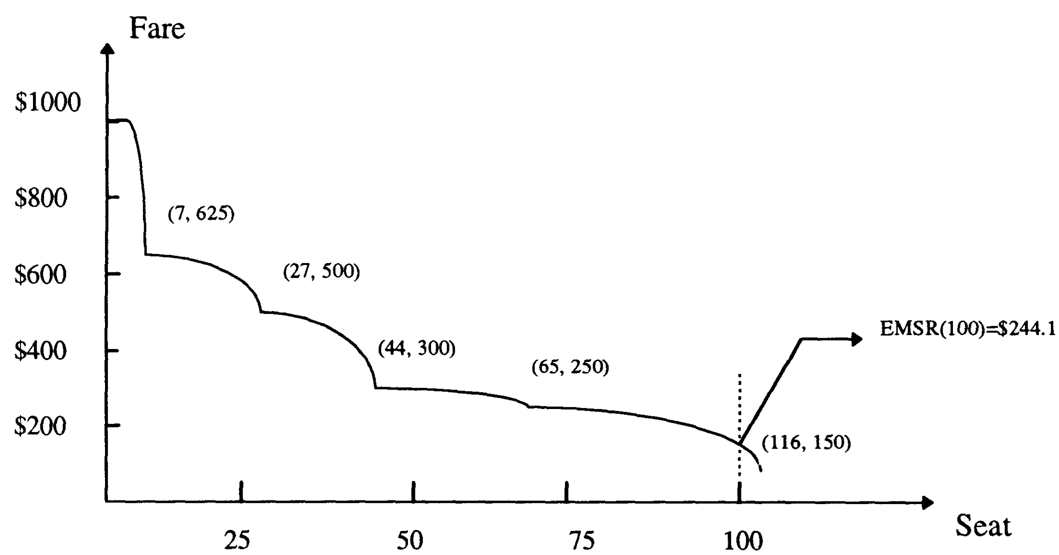

Figure 2.2 EMSR Curve from Greedy Virtual Nesting Method on Leg 1

On leg 2, we can find a similar EMSR Curve:

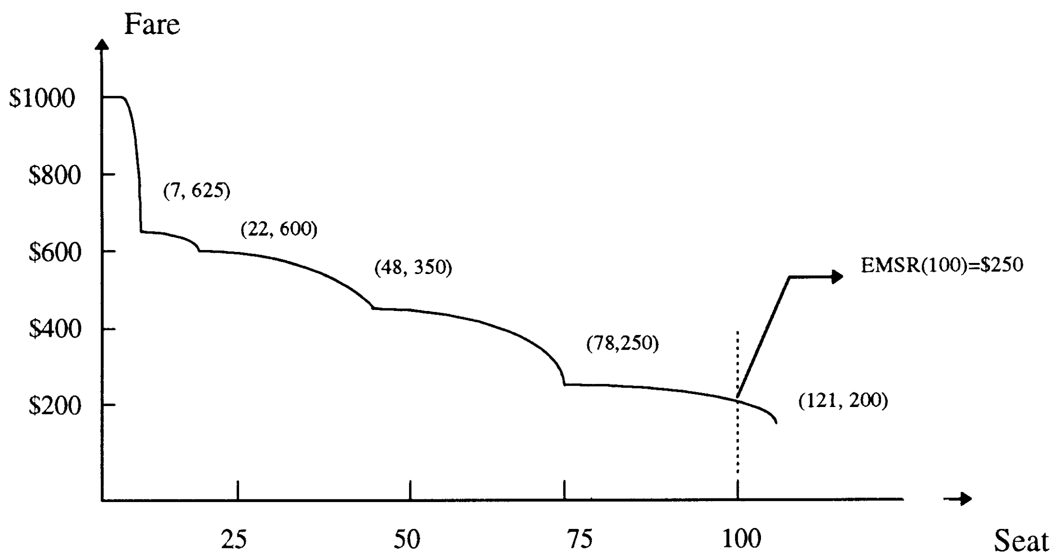

Figure 2.3 EMSR Curve from Greedy Virtual Nesting Method on Leg 2

The detailed calculations of these two curves and the EMSR value of the 100th seat are in Appendix 1.

2. Calculation of the Bid Price

For the local fare, the bid price is simply the EMSR value on that leg. For the connecting fare, the formula of the bid price on leg 1 is given as

$$BP_1 = EMSR_1 + d \times EMSR_2 . \qquad [2.6]$$

While the bid price on leg 2 is given as

$$BP_2 = EMSR_2 + d \times EMSR_1 . \qquad [2.7]$$

The "*d*" is the product of the percentages of the local passengers on both legs, and its range is $0 \leq d \leq 1$[10].

On this basis, we can obtain the following bid price:

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[10] P.P. Belobaba, *Airline O-D Seat Inventory Control Without Network Optimization*, Flight Transportation Laboratory, MIT, June 1995.

	Leg 1	Leg 2
Local	244.1	250
Connecting	306.6	311.0

Table 2.12 EMSR Heuristic Bid Prices

3. Booking Limits

Considering the bid prices, we can decide whether a passenger should be booked or spilled. For local passengers, if their fares are higher than the bid price on the leg they traverse, they can be booked. For connecting passengers, their fares must be higher than the bid prices on all legs they traverse. As a result, we have the following tables showing the airline's booking decision for local and connecting passengers:

Local fares on Leg 1:

	Fare	Bid Price	Book/Spill
Y3	500	244.1	Book
Y4	300	244.1	Book
Y6	150	244.1	Spill

Table 2.13 Booking Decisions for Local Passengers on Leg 1

Local on Leg 2:

	Fare	Bid Price	Book/Spill
Y3	600	250	Book
Y4	350	250	Book
Y6	200	250	Spill

Table 2.14 Booking Decisions for Local Passengers on Leg 2

Connecting Passengers:

	Fare	Bid Price on Leg 1	Bid Price on Leg 2	Book/Spill
Y1	1000	306.6	311.0	Book
Y2	625	306.6	311.0	Book
Y5	250	306.6	311.0	Spill

Table 2.15 Booking Decisions for Connecting Passengers

We notice that the connecting passengers in virtual class Y5 are spilled in this method, while they have seat availability in the greedy virtual nesting method. Refer to Table 2.10 and 2.11. Thus, the EMSR Heuristic Bid Price approach can reject some lower valued connecting passengers by using information from both legs.

2.3.2 Summary

This "leg-based bid price" control employs the flight leg structure of the existing yield management system, and does not require network optimization. In this method, the displacement impact of the connecting passengers has been taken into account: the bid price for the connecting passengers is the combination of the EMSR values on all legs the connecting passengers traverse. One disadvantage of this method is that there are certain heuristic factor in the formula of the bid price: the parameter in the formulation for connecting passengers is empirically related to the proportion of local passengers on each leg. Another limitation of this method is that it is still essentially a greedy approach: The calculation of the EMSR value is based on the total fares.

2.4 Non-greedy Virtual Nesting

As we mentioned previously, a drawback of the Greedy Virtual Nesting method is that it gives more priority to high fare long haul connecting passengers. When demand is high, booking one connecting passenger leads two local passengers to be spilled. This may result in a great loss in network revenue. In this non-greedy virtual nesting method, since the booking of passengers is not based on their total itinerary fare value, when demand is high enough, the long haul connecting passengers may be spilled first. We will use the example in Figure 2.1 to describe how this method works.

2.4.1 Process of the Method

1. Shadow Prices from Linear Programming

Earlier in this chapter, we introduced a partitioned network optimization method for seat inventory control using formula [2.1]. Even though this method is not used in real world operations, it can provide congestion information on each leg through the analysis of the value of dual variables (or shadow prices) associated with each capacity constraint. Dual variables refer to how much the total network revenue can be improved by increasing the capacity of a particular leg by one seat. If there is no congestion on a leg, that is if there are surplus seats on that leg, then the shadow price will be 0. On the contrary, if spill is expected on a leg, then the shadow price will be positive. Such a concept can be used to measure the displacement impact of the connecting passengers.

The first step of this approach is to solve equation [2.1], and obtain the shadow prices on all legs. In our example, we will solve the following LP:

Max:

$$Revenue = 500 \times Y_{l1} + 300 \times B_{l1} + 150 \times Q_{l1} + 600 \times Y_{l2} + 350 \times B_{l2} + 200 \times Q_{l2} + 1000 \times Y_C + 625 \times B_C + 250 \times Q_C \quad [2.8]$$

Subject to:

Capacity Constraints

$Y_{l1} + B_{l1} + Q_{l1} + Y_c + B_c + Q_c \le 100$ for leg 1,

$Y_{l2} + B_{l2} + Q_{l2} + Y_c + B_c + Q_c \le 100$ for leg 2.

Demand Constraints

$Y_{l1} \le 10$;	$B_{l1} \le 20$;	$Q_{l1} \le 40$;
$Y_{l2} \le 15$;	$B_{l2} \le 25$;	$Q_{l2} \le 45$;
$Y_c \le 8$;	$B_c \le 25$;	$Q_c \le 45$;

Y, B, and Q are the numbers of seats allocated to each OD fare. By plugging the above problem into an LP solver, such as Cplex, OSL, or Lindo, we can get the following shadow prices (The detailed solution for this problem is in Appendix 2.):

$SP1=\$150.0$ [2.9]
$SP2=\$200.0$

2. Calculating the Pseudo Fares

The "pseudo fares" are defined as the revenue contribution of the passengers after considering their displacement impact on the network. Therefore, for local passengers, their pseudo fares are equal to their total fares. On the other hand, for the connecting passengers, their pseudo fares on one leg are equal to their total fares minus the shadow prices on the other legs they traverse. So the same connecting OD fare may have different pseudo fares on different legs. The values of the pseudo fares of connecting passengers in our example are given in the following table:

	Leg 1	Leg 2
Y class	\$800	\$850
B class	\$425	\$475
Q class	\$50	\$100

Table 2.16 Pseudo Fares of Connecting Passengers

3. EMSR Seat Inventory Control

After getting the pseudo fares, a method similar to the greedy virtual nesting procedure is applied. This time, instead of using fares to achieve the virtual nesting, we utilize pseudo fares. The results are given in the following tables:

Leg 1 (SP_2=\$200):

Virtual Classes	Fare Classes	Fare	Pseudo Fare
Y1	Y(AC)	\$1000	\$800
Y2	Y(AB)	\$500	\$500
Y3	B(AC)	\$625	\$425
Y4	B(AB)	\$300	\$300
Y5	Q(AB)	\$150	\$150
Y6	Q(AC)	\$250	\$50

Table 2.17 Virtual Classes based on Pseudo Fares on Leg 1

Note here that the fare B(AC) is now ranked in a virtual class lower than fare Y(AB). This is due to the consideration of the displacement impact of the connecting passengers AC on Leg 2.

Leg 2 (SP_1=$150):

Virtual Classes	Fare Classes	Fare	Pseudo Fare
Y1	Y(AC)	$1000	$850
Y2	Y(BC)	$600	$600
Y3	B(AC)	$625	$475
Y4	B(BC)	$350	$350
Y5	Q(BC)	$200	$200
Y6	Q(AC)	$250	$100

Table 2.18 Virtual Classes based on Pseudo Fares on Leg 2

Using the virtual class given above we can **calculate the EMSR** curves on both leg 1 and leg 2 as shown in following two figures.

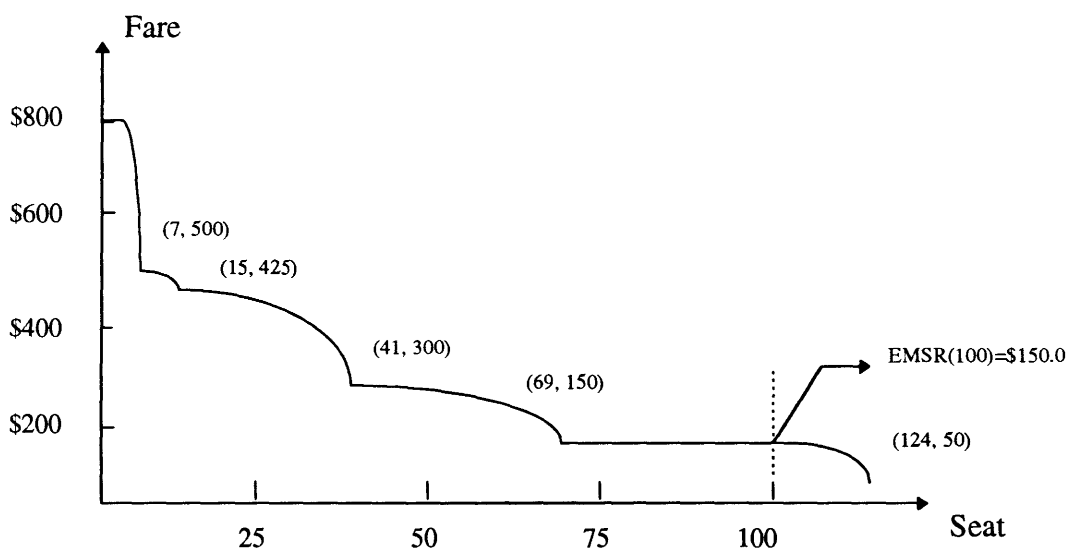

Figure 2.4 EMSR Curve Based on Pseudo Fares on Leg 1

On leg 2, we can get a similar EMSR Curve:

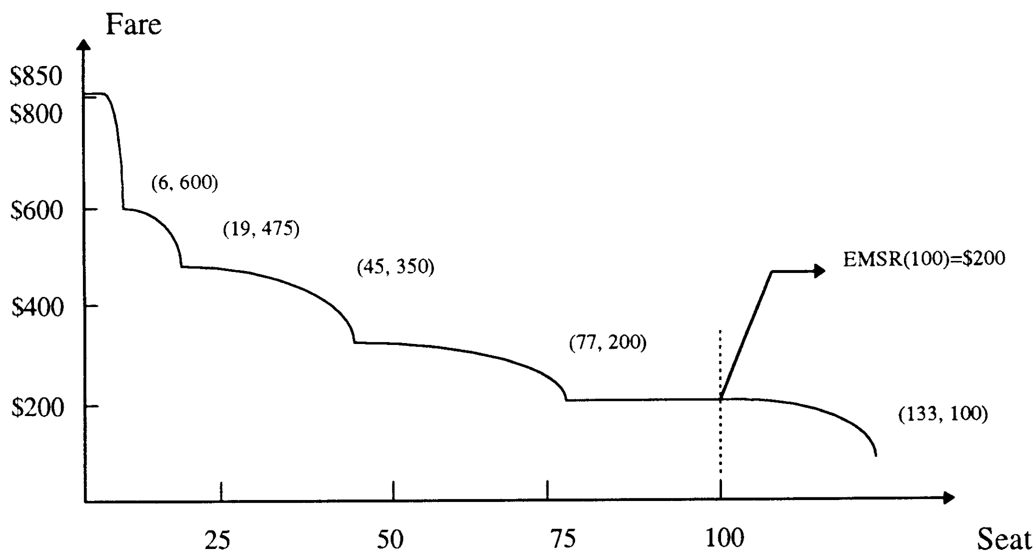

Figure 2.5 EMSR Curve Based on Pseudo Fares on Leg 2

From the two figure above we can easily determine that the seat protection for each virtual class. They are listed in Table 2.19.

	Y1	Y2	Y3	Y4	Y5
Seats Protection on Leg 1	7	15	41	69	124
Seats Protection on Leg 2	6	19	45	77	133

Table 2.19 Seats Protection in the Non-greedy Virtual Nesting Method

If we assume that the capacity on both legs is 100 seats we will have the following booking limits.

	Y1	Y2	Y3	Y4	Y5	Y6
Booking Limits on Leg 1	100	93	85	59	31	0
Booking Limits on Leg 2	100	94	71	55	23	0

Table 2.20 Booking Limits in the Non-greedy Virtual Nesting Method

The critical EMSR value on leg 1 and leg 2 are $150 and $200, respectively.

By comparing the results we achieved here with those obtained from greedy virtual nesting, we find that some of the long haul connecting passengers no longer have a higher priority than the local passengers, and the connecting Q class is ranked in the lowest virtual class without seat availability at all. On the other hand, the local Q class passengers on both legs have seats available at this time. This observation also differs from the results we obtained from the heuristic bid price.

2.4.2 Summary

The non-greedy virtual nesting method employs simple network LP concepts to calculate the shadow prices, and to measure the displacement impact of the connecting passengers. Such a strategy normally performs well when demand is extremely high. However several issues need to be taken into account when we calculate the shadow prices, and we will discuss them in Chapter 3.

2.5 Network Deterministic Bid Price

The four methods introduced above used the concept of heuristic nesting. The method we will address in this section is a pure network optimization process. Its concept is similar to that of the heuristic bid price. The difference is that it uses the shadow prices directly as the bid prices on each leg. We will use the previous example to explain this approach.

2.5.1 Process of the Method

First, solve the LP equation [2.2] as we did in section 2.4.1 and get the shadow prices on each leg. The results are:

$$Sp_{leg1} = \$150 \qquad [2.10]$$
$$Sp_{leg2} = \$200$$

The principles for booking decisions in this method are: for local passengers, if their fare value is higher than the shadow price on the leg they traverse, they are booked; for connecting passengers, they can be booked if and only if their fare values are greater than the sum of the shadow prices on all legs they traverse. So, we have the following three booking decision tables.

Local passengers on leg 1

	Fare	Bid Price	Book/Spill
Y	500	200	Book
B	300	200	Book
Q	150	200	Spill

Table 2.21 Booking Decisions for Local Passengers on Leg 1

Local passengers on leg 2

	Fare	Bid Price	Book/Spill
Y	600	150	Book
B	350	150	Book
Q	200	150	Book

Table 2.22 Booking Decisions for Local Passengers on Leg 2

Connecting passengers

	Fare	Bid Price	Book/Spill
Y	800	200+150	Book
B	500	200+150	Book
Q	250	200+150	Spill

Table 2.23 Booking Decisions for Connecting Passengers

2.5.2 Summary

The network bid price method uses the LP network optimization concepts to establish the bid prices. One problem of utilizing this model is that, in the formulation of the optimization model, the right-hand sides of the constraints require the remaining capacity on each leg and forecast future demands of each OD fare class. This information changes during the booking process for a departure date. Therefore, to use this method requires the airline to run the LP frequently. This is impractical because most airlines do not have the data to support such a dynamic optimization process. Plus, running an LP is very time consuming as the network grows larger and larger.

2.6 Chapter Summary

In this chapter, we introduced five different revenue management methods:

The Leg Based Fare Class Yield Management Method directly utilizes the existing fare classes to calculate the EMSR booking limits. It is simple to employ and therefore is practiced by many airlines. However, this method ignores the fact that the long haul connecting passengers normally provide more revenue than the short haul local passengers.

The Greedy Virtual Nesting Method treats the fares based on their absolute values and re-arranges the fares to the so-called virtual buckets. In such a way, the long haul connecting passengers obtain the highest priority to access the seats. The greedy method performs well when the demand is low, however when demand is high, booking one connecting passenger may cause two or more local passengers to be spilled, that is, the connecting fares have displacement impacts.

The Greedy Heuristic Bid Price Method takes approximates the displacement impacts of the connecting fares by applying higher bid prices for the connecting passengers than the local passengers. However this method is still a greedy method because the critical EMSR values are calculated based on the total fares.

The Non-greedy Virtual Nesting Method uses the shadow prices from the linear programming models as the displacement impacts. The advantage of this method is that it gives high revenue contributions (from the simulation results in Chapter 4), but the disadvantage of this method is that it requires more information in terms of more detailed ODF demand forecasts, at least for the total demand on a network.

The Deterministic Bid Price method is a pure network optimization method, which simply uses the shadow prices from the LP model, therefore it requires more detailed incremental ODF forecasts by booking period.

These five methods, each with its own advantages and disadvantages, suggests a clear research direction: How the connecting fares should be treated under different demand circumstances. Ideally, when the demand is low, the connecting passengers should have higher priority to access the seats than the local passengers, while when the demand is high, enough seats should be reserved for the high yield local passengers. In next chapter, we will follow these ideas and propose three new models. We will also discuss how to avoid over-emphasizing the displacement impacts of the connecting passengers in practice.

Chapter 3

Approaches to Network O-D Control Considering Displacement Impacts

The mathematical approaches reviewed in the previous chapters suggest directions for developing new seat inventory control methods. For example, in the non-greedy virtual nesting method, the displacement impact of the connecting passengers is taken into account by employing an LP model. Are there any better ways to evaluate such effects? Can we take advantages of several good methods to achieve the best revenue performance? In this chapter, the methodologies of several new models are presented. A simple network (Figure 2.1) will be employed to explain how to apply these methods in practice, and several related subjects will be explored.

3.1 Non-greedy Heuristic Bid Price Control Model

3.1.1 Introduction

We propose this new model based on two observations about the previously reviewed airline seat inventory control methods:

1. The greedy virtual nesting method does not perform very well when the demand is extremely high, because under such conditions, booking one connecting passenger may

cause two local passengers to be spilled. We know that, normally, two local passengers have a higher total revenue contribution than one connecting passenger. This observation tells us that the total itinerary network revenue contribution of the connecting passengers should not be their total itinerary fares, if they make any negative revenue impact on the network. In order to evaluate such negative impacts, we have to solve the LP to calculate the shadow prices associated with the capacity constraints.

2. The heuristic bid price method has shown positive revenue performance in practice[1], which we ascribe to the consideration of the displacement impacts of the connecting passengers through setting a higher bid price for them than for local passengers. However, it is still based on a greedy virtual nesting method, and the total itinerary fares are utilized to calculate the critical EMSR values.

The new model, which we name Non-Greedy Heuristic Bid Price method, inherits the merits of both non-greedy virtual nesting and the greedy heuristic bid price methods. In this approach, we calculate the pseudo fares first, then obtain the critical EMSR values and the bid prices.

3.1.2 The Model

The main idea of this new model is that we will incorporate the displacement impacts of the connecting passenger by using the pseudo fares to calculate the critical EMSR values. This is the same as the non-greedy virtual nesting method. Then, in the next step, the heuristic bid prices are utilized to decide the booking limits.

A broad definition of the shadow price is "the maximum amount that a manager should be willing to pay for an additional unit of resource."[2] In our deterministic LP optimization model, the shadow price is the value of the last seat on each leg, and it is positive only when spill is expected to occur.

In the non-greedy virtual nesting method (reviewed in Chapter 2), the pseudo fares, which are defined as fares minus shadow prices, are introduced to reflect the total itinerary revenue contribution of the passengers. For local passengers, their pseudo fares are identical to their

[1] P.P. Belobaba, *Airline O-D Seat Inventory Control Without Network Optimization*, Flight Transportation Laboratory, MIT, Cambridge, MA, June 1995.
[2] W.L. Winston, *Operations Research, Application and Algorithms*, Third Edition, Duxbury Press, Belmont, California.

total fares since they do not make any displacement impacts, while for the connecting passengers, their pseudo fares will be less than their total fares if the shadow prices are positive (or spills are expected).

An alternative way to evaluate this displacement impact of the connecting passengers is to utilize the critical EMSR value on each leg. The critical EMSR value is a marginal value of the last seat on a leg.

$$EMSR(S) = f \cdot \overline{P}(S) \qquad [3.1]$$

where the *EMSR(S)* is the expected marginal seat revenue of the *S*th seat, *f* is the fare value, and $\overline{P}(S)$ is the probability that the *S*th seat can be occupied by the passengers paying fare *f*. The EMSR is a measurement of the value of the last seat on a leg, and we can employ it to gauge the displacement impact of a connecting passenger. Therefore we will be able to define the pseudo fares in another form, that is, fares minus the critical EMSR values.

Since there are two alternatives to evaluate the displacement impacts of the connecting passengers--the shadow prices from network optimization and the critical EMSR values from leg based probability optimization--we can divide our model into two sub-methods: the Network Non-greedy Heuristic Bid Price Method and the Leg-based Probability Non-greedy Heuristic Bid Price Method.

1. Network Non-greedy Bid Price Method

In this method, the shadow prices are calculated from solving a deterministic linear program, then the pseudo fares are employed to evaluate the revenue contribution of the passengers to obtain the critical EMSR value on each leg, and consequently, the bid prices are decided.

For the local fares, the bid prices are simply the EMSR values on the corresponding legs, while for the connecting fares, the bid prices on Leg 1 are:

$$BP_1 = EMSR_1 + d \times EMSR_2, \qquad [3.2]$$

while the bid prices on Leg 2 are:

$$BP_2 = EMSR_2 + d \times EMSR_1. \qquad [3.3]$$

The *"d"* in the formula is simply the product of the percentages of the local passengers on both legs[3].

In our example in Figure 2.1, for instance, we obtain the following bid prices (the calculation of the critical EMSR values is introduced in Chapter 2.4: Non-greedy Virtual Nesting Method):

	Leg 1	Leg 2
Local	150	200
Connecting	200	237.5

Table 3.1 EMSR Heuristic Bid Prices Based on Pseudo Fares

Based on these bid prices, we can decide whether a passenger should be taken or spilled. For local passengers, if their fares are higher than the bid prices on the legs they traverse, they can be booked. For connecting passengers, their fares must be higher than the bid prices on all legs that they travel. The outcomes of the booking decisions are listed in following three tables:

Local fares on Leg 1:

	Fare	Bid Price	Book/Spill
Y2	500	150	Book
Y3	300	150	Book
Y5	150	150	Book

Table 3.2 Booking Decisions for Local Passengers on Leg 1

Local fares on Leg 2:

	Fare	Bid Price	Book/Spill
Y2	600	200	Book
Y4	350	200	Book
Y6	200	200	Book

Table 3.3 Booking Decisions for Local Passengers on Leg 2

Connecting fares:

[3] P.P. Belobaba, *Airline O-D Seat Inventory Control Without Network Optimization*, Flight Transportation Laboratory, MIT, Cambridge, MA, June 1995.

	Fare	Psfare(1)	Bid Price(1)	Psfare(2)	Bid Price(2)	Book/Spill
Y1	800	600	200	650	237.5	Book
Y2	500	300	200	350	237.5	Book
Y4	250	50	200	100	237.5	Spill

Table 3.4 Booking Decisions for Connecting Passengers

The conclusions in Tables 3.2 to 3.4 differ from those outcomes in Tables 2.13 to 2.15 obtained from the non-greedy heuristic bid price method: the local passengers have more seat availability here than they have in the greedy heuristic bid price method. For example, both local Q classes on Leg 1 and Leg 2, which are spilled in the greedy heuristic method, now have seats available.

2. Leg Based EMSR Non-greedy Bid Price Method

We may also apply the critical EMSR values to measure the network displacement impacts. The first step is to calculate the critical EMSR values using the total itinerary fares as the inputs. Then these critical EMSR values are subtracted from the fares to obtain the pseudo fares. Using these pseudo fares, the critical EMSR values are re-calculated, and the bid prices are subsequently acquired based on these new EMSR values.

In our example in Figure 2.1, for instance, we obtain the following pseudo fares:

	Leg 1	Leg 2
Y class	$750.0	$755.1
B class	$375.0	$380.9
Q class	$0.0	$5.9

Table 3.5 Pseudo Fares of Connecting Passengers

The bid prices calculated based on these pseudo fares are:

	Leg 1	Leg 2
Local	150.0	200.0
Connecting	200.0	237.5

Table 3.6 EMSR Heuristic Bid Prices Based on Pseudo Fares

As a result, we have the following three booking tables:

Local fares on Leg 1:

	Fare	Bid Price	Book/Spill
Y2	500	150	Book
Y3	300	150	Book
Y5	150	150	Book

Table 3.7 Booking Decisions for Local Passengers on Leg 1

Local fares on Leg 2:

	Fare	Bid Price	Book/Spill
Y2	500	200	Book
Y4	350	200	Book
Y6	200	200	Book

Table 3.8 Booking Decisions for Local Passengers on Leg 2

Connecting passengers:

	Fare	Psfare(1)	Bid Price(1)	Psfare(2)	Bid Price(2)	Book/Spill
Y1	800	600	200	650	237.5	Book
Y2	500	300	200	350	237.5	Book
Y4	250	50	200	100	237.5	Spill

Table 3.9 Booking Decisions for Connecting Passengers

We recognize that this approach provides us with a similar result compared with the outcomes we derived from the network LP optimization process. However, the advantage of using the EMSR calculation is that it avoids the network optimization process, which requires a lot of data and computer time. On the other hand, the disadvantage is that the critical EMSR value might not be the most proper estimation of the displacement effects on the network optimization. However, this method provide us with a direction of developing a convergence model: re-calculate the EMSR values based on the pseudo fares till the values of the pseudo fares converge. For detail discussion about this convergence, please refer to page 104 and 105 in Chapter 4.

3.1.3 Some Concerns about the Non-greedy Heuristic Bid Price Model

1. Linear Program Debate

a. Dynamic LP vs. Static LP

In the network non-greedy heuristic bid price method, we evaluated the displacement consequences of the connecting passengers using the shadow prices of each leg. To calculate the shadow prices involves solving a traditional Operation Research network optimization model (2.1). In this model, the right-hand sides (RHS) of the capacity constraints are the number of seats available on each flight leg, and the RHS of the demand constraints are the number of bookings still to come in the future. Both of these RHS change with the booking process, because the capacity remaining on each leg decreases, and the demands still to come also decline. If these input data change, so will the outputs of the optimization process. Thereby the airlines face the decision of how often they should re-solve the linear program. One solution is to update the LP results whenever the demand and capacity statistics change. However, practically, most airlines cannot employ such strategies due to the costs of the data collection and storage and the inaccuracy of demand forecasting.

Therefore, we need to evaluate the revenue effects of employing shadow prices from an LP that are out of date. If the network revenue decreases, is the change in an acceptable range? In our experiments, two cases are taken into account to evaluate such effects: Solving LP multiple times and solving LP once. In the dynamic (or multiple) LP case, instead of re-solving LP whenever the data change, we re-solve it at each booking check point, which is more realistic. There are 16 such check points in our demonstrated network (we will introduce this network in Chapter 4). On the other hand, in the static (or single) LP case, the LP is solved at the beginning of each departure, and the shadow prices stay the same in each booking period within that particular departure's booking process.

Generally, the revenue performance from the multiple LP solving is better than that from the single LP solving. This is understandable, because multiple LP solving provides more accurate information about the capacity and demands. However, we also find that the single LP solving provides a comparable revenue improvement to the multiple LP solving. On a few occasions, solving LP once has even better revenue gains. This can be explained by large variations in the demand. For instance, we found that it is possible that during one booking period, there are only one or two passengers who are booked, while during the very next booking period, there are more than 200 passengers who come for booking. If this is the case, then after the first booking period, we will have a lot of seats left, which will cause the shadow prices to be somewhat low, and the pseudo fares for the connecting passengers to be correspondingly high. Therefore, when it comes to the next booking period, in which the demand is high, many connecting passengers with high pseudo fare values and high booking priority will be booked. This will cause some high fare local passengers who will come during the following booking periods to be spilled. On the contrary, if we only solve LP once, those RHS we will apply are the mean demand and capacity over all the booking periods. They are more reliable with less deviation; therefore the booking permission will not be affected by the great changes among the booking periods. Even though such occasions are rare, it did happen in our experiments.

b. Demand Forecasting

Recall that in the LP problem, there are two types of constraints: capacity constraints and demand constraints. The RHS of the demand constraints are simply the forecast mean demands of each OD (Origin-Destination) fare class. Practically, the forecast of demands is done based on the historical data, up to several months before the departures. Such forecasting cannot be exactly accurate, and the errors may bring additional mistakes to the shadow prices and the pseudo fares derived from the LP model. Hence, it is necessary to evaluate how the total network revenue will be affected if the forecast of the demands is imprecise. Two cases need to be considered: when the forecast goes too low and when the forecast goes too high.

In the first case, we will solve the LP at a lower demand level than the actual demand. The shadow prices derived from the LP will be comparatively low, and the pseudo fares of the connecting passengers will be correspondingly high. Such a solution normally has better performance when demand is low, because we know when demand is low it would be better to carry as many passengers as possible than to take the risk of allowing any seats to go empty. Nevertheless, when the demand is high, this oversight may allow those connecting passengers to eat up the seats that should be reserved for the high yield local passengers, and subsequently cause a negative revenue impact on the whole network.

In the second case, we will solve the LP at a higher demand level than the actual demand. The shadow prices will be relatively high. Then the pseudo fares will be relatively low. As a result, the connecting passengers will have lower priority to access the seats than they should have. If demand eventually turns out to be high, such a solution has very good network revenue performance, because some connecting passengers from low fare classes are spilled, and enough seats are reserved for the high yield local passengers. However, if it turns out to be a low demand season, this misjudgment will cause an unnecessary rejection of the connecting passengers. If there are not enough local passengers coming to book, seats perish.

At this point, we are facing the trade-off between solving LP at a higher demand level to have a good network revenue achievement when demand is high (although suffering from an inefficiency when demand is unexpectedly low), and solving LP at a low demand level to guarantee that there will be no negative performance in low demand periods (although during the high demand periods resulting in perhaps too many connecting passengers).

c. Degeneracy of the Linear Program

In practice, almost all airline LP applications are degenerate. We know that whether an LP is degenerate depends on how many constraints of the LP are binding. If the number of the binding constraints exceeds the number of variables, then the problem becomes degenerate. Therefore the more constraints the LP has, the higher the chance it is degenerate. If we look at the LP formula in Equation [2.1], we will find that for every variable there is an upper bound constraint. So the total number of the constraints is equal to the number of variables plus the number of flight segments (capacity constraints).

One problem associated with the degeneracy of the LP is that the LP's dual problem will have multiple solutions. The optimization tools, such as OSL and Cplex, only supply the dual solutions they encounter first. Since the shadow prices will directly affect the pseudo fares and bid prices, the correction of their values can be important. Therefore a non-degenerate solution is always preferred.

There are a couple of procedures to afford a non-degenerate solution, such as perturbation of the RHS of the linear program[4], or instead of finding the corner/extreme point as the optimal primal solution, which will cause a multiple dual solution, utilization of the convex combination of two degenerate bases[5].

In the following, we will address this degeneracy problem through several approaches. Figure 3.1 shows a small network with only two legs:

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[4] D. Bertsimas and J. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, August 1996.
[5] T. Magnanti, Handout for course Introduction to Mathematical Programming, MIT, Fall 1996.

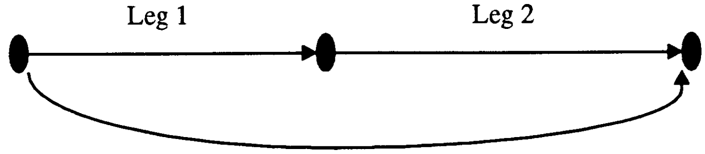

Figure 3.1 A Two-Leg Network

Assume that the capacity on each leg is 15 seats. The demand and fares of local and connecting trips are shown in the figure. We can employ the following LP equations to calculate the dual variables:

Max:

$$200\,X1 + 300\,X2 + 400\,X3 \qquad [3.4]$$

Subject to:

$$\begin{aligned} X1 + X3 &\leq 15 \\ X2 + X3 &\leq 15 \\ X1 &\leq 10 \\ X2 &\leq 10 \\ X3 &\leq 10 \\ Xi &\geq 0, \quad for\ i=1,3 \end{aligned}$$

Through the simplex tableau method we then derive the two optimal solutions:

Solution 1:

Primal	Dual	Objective Value	
$X_1 = 10$	$P_{11} = \$100$	$\$7000$	[3.5]
$X_2 = 10$	$P_{12} = \$300$		
$X_3 = 5$			

Solution 2:

Primal	Dual	Objective Value	
$X_1 = 10$	$P_{21} = \$200$	$\$7000$	[3.6]
$X_2 = 10$	$P_{22} = \$200$		
$X_3 = 5$			

We notice that these two sets of dual variables are different from each other. Which one will be generated by the optimization solver depends on the sequence of the input constraints. Additionally, both of these two solutions are degenerate. To overcome this difficulty, we propose the following three approaches. Each are suitable for different circumstances, and following, we will discuss them one by one.

- True Shadow Prices

The shadow prices are defined as "the amount a manager would like to pay for one more unit of resource."[6] Therefore, we can figure out the correct shadow prices from this definition. Increase the capacity on one leg by one unit and re-solve the LP in Equation [3.4] to obtain a new objective value; compare this new objective value with the original objective value (in Equation [3.5] and [3.6]). The difference is the true shadow price on that particular leg. To obtain the true shadow prices of our example in Figure 3.1, we will need to solve the following two LPs in Equations [3.7] and [3.8].

Max:

$$200\,X_1 + 300\,X_2 + 400\,X_3 \qquad [3.7]$$

Subject to:

$$\begin{aligned} X_1 + X_3 &\le \mathbf{16} \\ X_2 + X_3 &\le 15 \\ X_1 &\le 10 \\ X_2 &\le 10 \\ X_3 &\le 10 \\ X_i &\ge 0, \quad for\ i=1,3 \end{aligned}$$

and

Max:

$$200\,X_1 + 300\,X_2 + 400\,X_3 \qquad [3.8]$$

Subject to:

$$\begin{aligned} X_1 + X_3 &\le 15 \\ X_2 + X_3 &\le \mathbf{16} \\ X_1 &\le 10 \\ X_2 &\le 10 \\ X_3 &\le 10 \\ X_i &\ge 0, \quad for\ i=1,3 \end{aligned}$$

The objective value of Equation [3.7] is \$7100, and of [3.8], \$7200. Therefore, the correct shadow prices on Leg 1 and Leg 2 should be

$$SP_1 = \$100 \qquad [3.9]$$
$$SP_2 = \$200$$

[6] W.L. Winston, *Operations Research, Application and Algorithms*, Third Edition, Duxbury Press, Belmont, California.

subsequently. We find that even in the simple example above, the dual variables obtained from the simplex tableau method are different from the accurate ones (SP_1 and SP_2). In the above example, the dual variables in Solution Set 1 [3.5] are:

$$P_{11} = \$100 \quad [3.10]$$
$$P_{12} = \$300$$

The second dual value ($300) for Leg 2 is higher than the true shadow price (SP_2), while the first value for Leg 1 happens to be correct. In Solution Set 2 [3.6], the dual variables are:

$$P_{21} = \$200 \quad [3.11]$$
$$P_{22} = \$200$$

This time, the dual variable on Leg 2 is correct, while that on Leg 1 is too high (the true shadow price on Leg 1 is $100). By our experience, most of the time, it is better to employ a lower set of shadow prices than a higher one, because the risk of using a lower one is accepting too many low yield connecting passengers, while the risk of using a **higher one is** leaving seats empty.

- Mean of the Dual Variables

To solve the difficulty of choosing the correct dual variables, one alternative is to employ the mean value of all sets of the dual variables, that is to find out all possible dual solutions by re-arranging the sequence of the input constraints. For the applied example, we would then have the shadow prices as follows:

$$SP_1 = \$150 \quad [3.12]$$
$$SP_2 = \$250$$

- Adjusted Dual Variables

Another alternative to avoid implementing a set of high shadow prices is to multiply an empirical parameter, which is less than 1 by any set of dual variables released from the LP solver ([3.10] and [3.11]), for example, 0.8. We will then have either:

$$SP_1 = \$80 \quad [3.13]$$
$$SP_2 = \$240$$

or

$$SP_1 = \$160 \quad [3.14]$$
$$SP_2 = \$160.$$

All three alternatives above try to avoid employing a high set of shadow prices and leaving seats empty at departure. Alternative 1, true shadow price model, requires re-solving the LP once for each leg, so the total number of times that the LPs need to be re-solved is the number of legs plus one. So, for a large network, it may not applicable. Alternative 2, the mean of the dual variables, requires re-arranging the constraints to obtain the different sets of dual variables. The extreme case, which means obtaining all sets of the dual solutions, requires re-solving the LP

$$P_{legs}^{legs} = (\text{Number of Legs})! \qquad [3.15]$$

times, which is also unrealistic when networks are large. The last alternative, the adjusted dual variables, is simple to employ, and gives good revenue performance as we will see in Chapter 4. However it involves a heuristic factor, which we may need to do sensitivity studies to select.

d. Linear Program versus Integer Program

The seat inventory control assignment is in fact an integer program problem, while the optimization process that we practiced to obtain the shadow prices is a linear program model. The shadow prices calculated from the LP are different from those obtained from the IP. Then why don't we treat this task as an IP in the first place? Mainly we have two reasons: First, the optimization procedure to solve an IP requires a much longer time than it does for an LP problem; second, there is a small number problem associated with current input data. Since most demands of the OD fares are less than one, the output of these OD fares through the IP will all be zeros if the requirement of integer solution is imposed. However, to treat this problem as an integer problem, we can obtain the solution in a distinct way. Figure 3.2 is the process of the seat inventory control we are using now in the network displacement adjustment method.

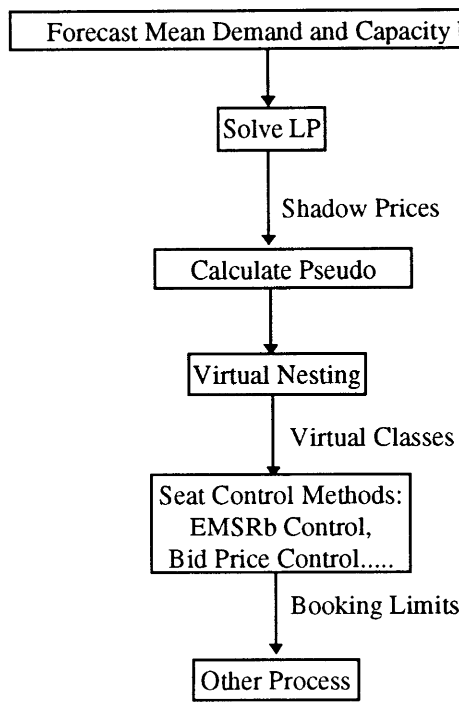

Figure 3.2 Process of the Non-greedy Seat Controls Method

We observe that the mean forecast demand is required by the LP. Since those inputs are real numbers, the output of the LP will also be real. A revised process is proposed in Figure 3.3.

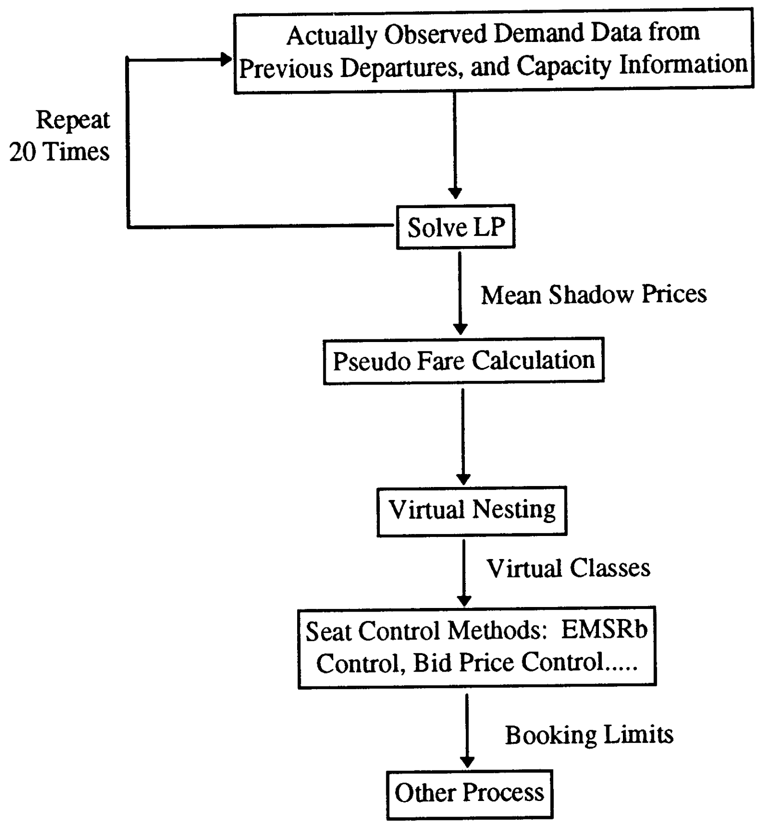

Figure 3.3 Revised Process of the Non-greedy Seat Inventory Control Method

In this process, the airlines first observe the actual demand data (should be unconstrained demand data) from 20 historical departures (Note: those actual demand data are integers). Then these observed demands are employed as the right-hand sides of the demand constraints in the LP solving process. Since this is a network problem, if the RHS and all the parameter in the constraints are integers, the outcomes of the LP will be integers as well. Therefore, an LP problem is transformed into an IP problem. Also, the optimal objective value from this process should be the theoretical upper bound of the revenues from those 20 departures, and the shadow prices associated with each capacity constraints reflect the value of the last seat on each leg more accurately.

e. Additional Thoughts about the Shadow Prices

The 20 sets of the shadow prices calculated from the process above (Figure 3.5) represent a sample with variance. In Figure 3.5, we simply take the mean values to continue the pseudo fare calculation. However, if we draw the distribution of these shadow prices, we can identify that the their values approximate the normal distribution as shown in Figure 3.4.

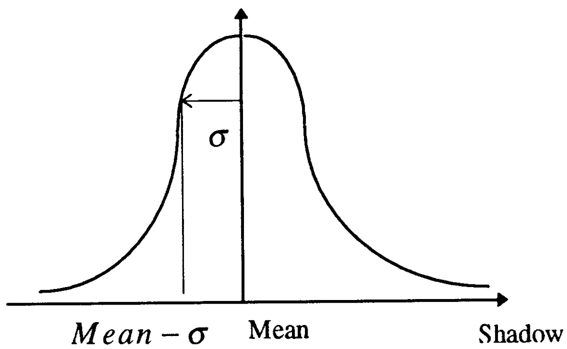

Figure 3.4 Distribution of the Shadow Prices

We can then make a few further adjustments: By definition of the normal distribution, if we choose to exploit the mean of the shadow prices then there is a 50% opportunity that the actual shadow prices are lower than the shadow prices we utilize. We know that too high shadow prices will cause too many connecting passengers to be spilled, which will consequently lead to empty seats. So we decide to employ the value of ($Mean - \sigma$) as the value of the shadow prices (instead of the *Mean*) to calculate the pseudo fares. Therefore the probability that the actual shadow prices will be lower than the implemented shadow prices decreased to 30%. In practice, we employ the following equation to revise the shadow prices.

$$SP_i = MAX\{0.0, (\overline{SP_i} - \sigma_{SP_i})\} \qquad [3.16]$$

The equation above shows that if the deviation of a shadow price is higher than its mean value, the value of this shadow price will be set at zero. Otherwise it will be the difference between the mean value and the standard deviation.

2. Heuristic Subjects

a. Heuristic Factor

As mentioned before, the formula of the bid price contains a heuristic factor "d". Belobaba (1995) suggested in the greedy heuristic method, for a 2-leg itinerary involving a connection between Leg 1 and Leg 2, the probability outcome that results in the displacement of local passengers on both legs is the product of $PLOC_1$ and $PLOC_2$[7], where $PLOC_j$ is the probability that a seat on leg j will be sold to a local passenger. During the development of our non-greedy heuristic method, we start to realize that the previous handling of that parameter may or may not be proper for our case. Therefore sensitivity studies must be carried out to determine the best value of this parameter.

b. Formula of the Bid Price

For a 2-leg-trip passenger, Belobaba gives the booking criterion as:

$$F_{ik} \geq \max\left[EMR_1(A_1) + d \times EMR_2(A_2), EMR_2(A_2) + d \times EMR_1(A_1)\right] \qquad [3.17]$$

where F_{ik} is the fare of a connecting passenger of itinerary i and fare class k. When a connecting passenger traverses three or more legs, the above equation is no longer applicable. We were thinking of imitating the formula above and using d_1, d_2, and d_3 before each *EMSR* value on every leg, but then this idea was dismissed, because first, it would involve too many uncertainty factors, and second, such consideration may place too high restrictions to connecting passengers, which may cause too many of them to be spilled. Based on the experience from our research, we proposed Formula 3.16 for the connecting passages which covers multiple flight legs.

$$BP = Max\{EMSR_i, i=1, 2......\} + d * 2ndMax\{EMSR_i \mid i=1,2......\} \qquad [3.18]$$

In this formula, only the two largest EMSR values have contributions to the bid prices, and it is shown to have better revenue performance in the seat inventory control (Chapter 4).

[7] P.P. Belobaba, *Airline O-D Seat Inventory Control Without Network Optimization*, Flight Transportation Laboratory, MIT, June 1995.

3.2 Convergent EMSR Control Model

3.2.1 Introduction

One limitation associated with the greedy methods is that they treat the connecting passengers as their total itinerary fare values on all legs they traverse. So the revenue contribution of the connecting passengers is double or even triple counted. To overcome this shortcoming, we introduced the concept of the pseudo fares. However, the revenue contribution of the connecting passengers has yet to be properly evaluated, because the sum of the pseudo fares on all legs of a connecting passenger's itinerary may not equal to his/her total itinerary fare value. Therefore the question is: is there a better way to distribute the connecting fare?

3.2.2 An Observation

In Williamson's thesis, she mentions that the connecting fare can be prorated by the mileage, or the local fares etc.[8] Those methods may not be proper. Consider the following example: A passenger travels over 4 flight legs with equal length in distance. His/her total fare is $1000. Suppose due to the difference in local fares and the congestion conditions, the critical EMSR values on these four legs are: $200, $500, $100, and $10. If we utilize the mileage to distribute the connecting fares, and the critical EMSR value is employed as a "cut-off" price to decide the booking, this passenger will be spilled, because his/her distributed fare on Leg 2 is $333, which is less than the "cut-off" price ($500). We also notice that on Leg 4, his distributed fare ($333) is much higher than the "cut-off" price ($10). If we reallocate his fare as $247, $617, $123, and $12 subsequently (the total value of these four numbers is equal to his/her total fare), this is passenger will be booked.

From the example above, we realize that the purpose of the seat inventory control is to try to take as many high fare passengers as possible; not to spill as many low fare passengers as possible. Therefore the proper scheme of distributing the connecting fare should make connecting passengers have the highest ability to compete with local passengers.

---

[8] E. L. Williamson, *Airline Network Seat Inventory Control: Methodologies And Revenue Impacts*, Flight Transportation Laboratory Report, R92-3, June 1992.

3.2.3 The Model

The traditional two-class EMSR problem can be solved using Equation [3.19]

$$\begin{aligned} f_1 P(S_1) &= E \\ f_2 P(S_2) &= E \\ S_1 + S_2 &= C \end{aligned} \qquad [3.19]$$

where S_1 and S_2 are the desired seat allocations for the two fare classes, C is the capacity of the flight, $f1$ and $f2$ are fare values of these two classes, and E is a Langrange undetermined multiplier. We can alternatively write the Equation [3.19] as following:

$$f_1 P(S_1) = f_2 P(S_2) \qquad [3.20]$$

where S_1 and S_2 still need to satisfy the capacity constraint ($S_1+S_2 = C$).

For the simplest two-leg segment problem as shown in Figure 3.5,

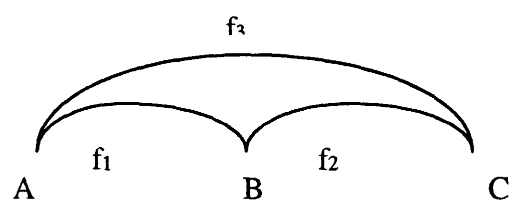

Figure 3.5 A Two-leg Segment Network

to find the optimal seat allocations in the same way as above involves two multipliers, one for each leg. The equations turn out to be:

$$\begin{aligned} f_1 P(S_1) &= E_1 \\ f_2 P(S_2) &= E_2 \\ f_3 P(S_3) &= E_1 + E_2 \end{aligned} \qquad [3.21]$$

We can re-construct Equation [3.21] as following:

$$f_1 P(S_1) = \theta_1 f_3 P(S_3)$$
$$f_2 P(S_2) = \theta_2 f_3 P(S_3) \qquad [3.22]$$
$$\theta_1 + \theta_2 = 1$$

where in Equation [3.22],

$$\theta_1 = \frac{E_1}{E_1 + E_2} \qquad [3.23]$$

and

$$\theta_2 = \frac{E_2}{E_1 + E_2}.$$

If we treat the "$\theta_1 f_3$" as a new fare class f_4, and "$\theta_2 f_3$" as f_5, then Equation [3.22] contains two traditional 2-class EMSR problems, which is similar to Equation [3.20]. Furthermore f_4 and f_5 satisfy the relationship of :

$$f_4 + f_5 = f_3 \qquad [3.24]$$

Such ideas provide us a direction of how the connecting fares should be prorated to the traversed legs, that is to prorate them according to the EMSR values on each leg. The process is as follows:

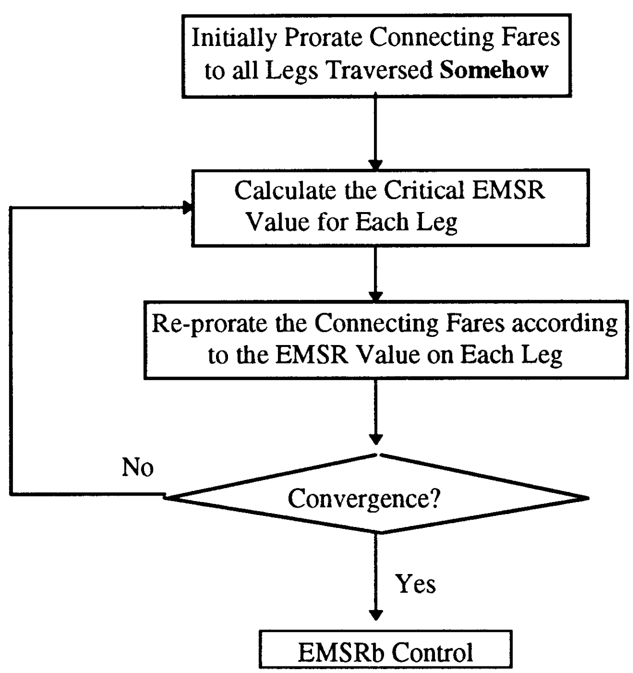

Figure 3.6 Convergence in Fare Distribution

First we can arbitrarily distribute the connecting fares to all legs (for example, equally distribute) to calculate the critical EMSR value, then distribute the connecting fares based on these EMSR values. For example, if it is a two leg trip, then we will define the pseudo fares as follows:

$$PSF_1 = Fare \times \frac{EMSR_1}{EMSR_1 + EMSR_2} \quad [3.25]$$

$$PSF_2 = Fare \times \frac{EMSR_2}{EMSR_1 + EMSR_2} \quad [3.26]$$

Next, we will compare the difference between the new distributed fares with those of the previous iteration. If the difference is within a tolerable range:

$$-x \le PSF_{ij}^{k+1} - PSF_{ij}^{k} \le x \quad [3.27]$$

then the process is stopped. Otherwise we will go back to re-calculate the critical EMSR value based on the new distributed fares, and so on. In Equation [3.27], PSF_{ij}^{k} is the value of OD fare i as prorated to the jth leg in the kth iteration, and the x is a positive value. The value of x is the convergence criterion. The implementation of such a convergence method will be discussed in Chapter 4.

3.3 Summary

One major barrier in seat inventory control is that we do not know how to evaluate the revenue contribution of the connecting passengers on each leg they traverse. In the greedy methods, connecting passengers are evaluated as their total itinerary fares. Such a strategy may cause great negative impacts when demand is high, because two local passengers usually have a higher total revenue contribution than one connecting passenger does. However, we don't want to over-emphasize the displacement effects of the connecting passengers either, because spilling too many of them may lead to empty seats.

In this chapter, we proposed two directions to deal with such a dilemma. One is to employ the shadow prices and/or the critical EMSR values along with the fine adjustment from the heuristic bid prices to obtain a better booking mix among the local and the connecting passengers. The other is trying to find a more proper way to distribute the connecting fares to the traversed legs. Both these directions provides us with very good network performance in practice. We will show some data in Chapter 4.

Chapter 4

Case Studies

In previous chapters, we have explained and compared several seat inventory control methods using a simple example. In this chapter, we will exploit some actual airline data and an integrated optimization/booking process simulation to examine how these methods behave in a real airline network.

In Section 4.1, we will introduce a simulation software developed in the Flight Transportation Laboratory at MIT. Then in Section 4.2, the characteristics of the network we will simulate are presented. Next, the simulation results of the network performance from different methods are compared in Section 4.3. The last section concludes some concerns related to the implementation of these methods.

4.1 The Booking Process Simulation

In the following sections of this chapter, we will examine different seat control methods on a simulated network. The most reasonable comparison is to put all methods under an identical environment to test how they respond to the same situation. Therefore we prefer to have a controlled set of circumstances. Such circumstances are difficult to obtain in the real world because the demand and other factors change from time to time. So a simulation tool is necessary in our research.

A simulation is "a procedure in which a computer-based mathematical model of a physical system is used to perform experiments with that system by generating external demands and observing how the system reacts to the demands over a period of time[1]". With certain reasonable assumptions, the simulation can provide a similar environment to the real world.

The airline booking simulation software we used in our research was developed in the Flight Transportation Laboratory at MIT. The software, first created by Williamson (1992) and then enhanced by several other people, is written in FORTRAN 90. It is an integrated simulation combining both the revenue management optimization process and random demand generating process. This simulation model is a Monte Carlo simulation, which can provide an approach for evaluating different seat inventory control options in a controlled environment that otherwise would be difficult to obtain through the real-time experimentation. Also this simulation allows us to compare different seat inventory control methods under the same demand situation.

There are three kinds of input files in this simulation:

- Demand Input File.

  Includes: Number of OD pairs;
  Number of fare classes on each OD pair;
  Number of flight legs in the network;
  Capacity on each flight leg;
  Number of book (revision) check points for each departure*;
  Path for each OD pair;
  Forecast demand for each ODF at each check point.

- Random Demand Generating File.

  Includes: Number of departures to be simulated under the same control strategy;
  Parameters for the Poisson random number generation.

- Strategy Input File.

  Includes: Seat inventory control method for the booking limits calculation;
  Demand adjustment parameter*;
  Other parameters, such as the heuristic parameter for the bid price formula.

****Booking check points***. The airlines adjust their seat inventory control strategy dynamically according to the updated information about the remaining capacity and demand still to come. In order to reflect this fact, our simulation is designed as a multiple stages, dynamic process: The time is segmented from the present time to the departure date as a set of booking periods,

[1] R.C. Larson and A.R. Odoni, *Urban Operations Research*, Prentice-Hill, Inc., 1981.

with booking check points (or re-optimization points) at the beginning of each period. The booking limits are re-calculated at each of these check points, as shown in the figure below[2]:

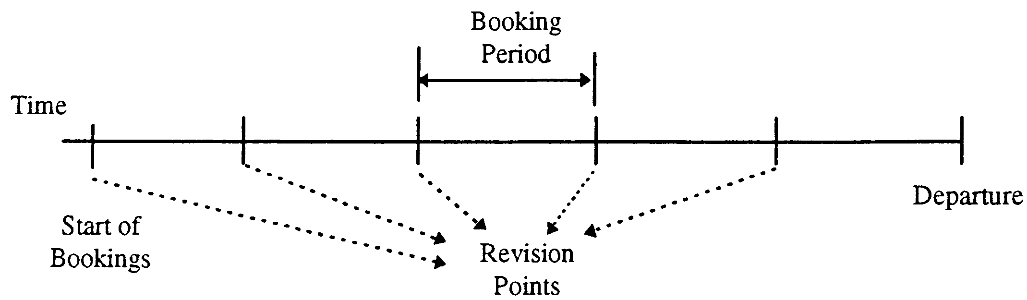

Figure 4.1 Time Line of the Simulation

In the figure above, those points, at which the simulation update the booking limits, are called the booking revision (check) points. The time between two check points is a booking period. To update the booking limits at each check point, the mean and the deviation of the forecast OD demand for each booking period are required in the simulation.

**Demand adjustment parameter*. In order to evaluate the performance of a seat control method, we need to know how consistent this method is for both high demand conditions and low demand conditions. To obtain a high or low demand environment, we do not change the demand input file, instead, we simply apply a multiplier to each ODF demand data. For example, to obtain a low demand environment, we will input a demand adjustment parameter that is less than 1. On the other hand, to obtain a high demand environment, we can input a parameter that are greater than 1. In this simulation model, this parameter can vary from 0 to infinity, but mostly, it alters from 0.8 to 1.2 in our research.

The following is the process of the simulation[3]:

[2] E. L. Williamson, *Airline Network Seat Inventory Control: Methodologies And Revenue Impacts,* Flight Transportation Laboratory Report, R92-3, June 1992.

[3] E. L. Williamson, *Airline Network Seat Inventory Control: Methodologies And Revenue Impacts,* Flight Transportation Laboratory Report, R92-3, June 1992.

For each Departure:

  For each Check Point:

    Demand Still to Come.

    Booking Limits for Each ODFs.

    Poisson Demand Generation.

    Booking Decisions.

  Next

Next

Summary of the Outcomes

Figure 4.2 Simulation Process

The simulation is run on a PC Pentium 166. The simulation time varies with the size of the network and the seat inventory control strategy. For example, a hub-spoken network with 32 legs, 121 OD pairs and 10 fare classes in each OD pair with 16 check points, if the greedy virtual nesting method is implemented, the simulation time for 20 departure days is around 7 minutes. However, if the Deterministic Network Bid Price method is implemented on the same network, we need 15 minutes. For a larger network with 102 legs, 1066 OD pairs, 7 fare classes in each OD pair, the greedy virtual nesting method takes around 45 minutes for 20 departures, and the deterministic network bid price method takes 2 hours and 35 minutes.

For a more detailed description of the underlying simulation concept please refer to Williamson's (1992) doctoral thesis.

4.2 Network Characteristics

The data that we choose to implement to the simulation software is from a sub-network of a European airline. The general characteristics of this network are:

- 32 flight legs.
- 121 origin and destination pairs.
- 10 fare classes on each O-D pair.
- Mix of long-haul and short-haul flights.
- Each OD fare has at most two legs involved.

- Average of 72% local traffic on each leg.
- 16 booking periods.

From the 32 flight legs of the network, we choose two typical legs for more detailed analysis: One (Leg 4) is extremely congested with a 99% load factor, the other (Leg 30) has a reasonable load factor which is around 75%. We will also trace the leg performance of each method to make a detailed comparison.

The demand data of the network is shown in the following table:

Demand Adjustment	0.80	1.00	1.20
Total Demand	5556	6941	8313
Local Demand	4670	5845	699[illegible]
Connecting Demand	886	1096	1315
Average Leg Load Factor	82.51%	90.00%	92.94%

Table 4.1 Demand Scenarios

In Table 4.1, the average leg load factors are obtained from the EMSRa fare class control method.

Figure 4.3 shows the cumulative demand curves of two fare classes on Leg 30 under demand adjustment 1.20.

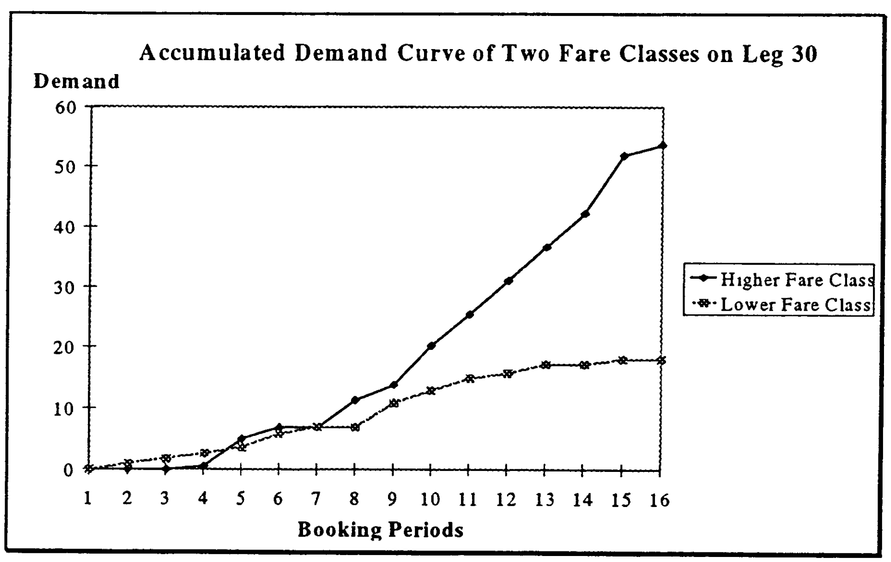

Figure 4.3 Demand Curve for Two Fare Classes on Leg 30

From the demand curves above, we can easily see that the higher fare class passengers tend to book later than the lower fare class passengers. If we go back to Chapter 1, we will find such trends are exactly the same as what we showed in Figure 1.3, and it is the reason why the seat protection for higher fare last-minute passengers is critical in revenue management.

4.3 Strategy Comparison

In this section, different seat control methodologies are compared through the simulation tool. The results from the simulation are presented to provide a comparison of how each individual method responds to the same demand scenarios. Such information is helpful for both research and implementation purposes.

4.3.1 Leg Based Fare Class Yield Management(LBFC)

In this strategy, 10 fare classes, which are defined by fare type, are employed as the input to calculate the EMSR booking limits on each leg, as described in Section 2.1. There are 16 booking periods in this process.

In the following four sections, we present the performance of this method on our example network.

1. Network Revenue Performance

Tables 4.2 and 4.3 are the network revenue performance of the LBFC method. These two tables are similar except that in Table 4.2, we apply EMSRa[4] method, while in Table 4.3 we employ EMSRb[5] method.

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[4] P.P. Belobaba, *Air Travel Demand and Airline Seat Inventory Control Management*, Flight Transportation Laboratory Report, R87-7, May 1987.
[5] P.P. Belobaba, L.R. Weatherford, *Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations*, Decision Sciences, Volume 27, Number 2, Spring 1996.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3584323	4130469	4565648
Local Pax Spilled	370	1140	2088
Connecting Pax Spilled	58	223	449
Avg. Leg Load Factor(%)	82.51	90	92.94
Avg. Rev. Per Pax($/Pax)	699.00	740.52	790.36
Avg. Rev. Per Avail. Seat($/Seat)	509.86	587.55	649.45

Table 4.2 Network Performance of LBFC Method using EMSRa Control

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3588448	4140586	4586011
Local Pax Spilled	321	1051	1944
Connecting Pax Spilled	63	268	530
Avg. Leg Load Factor(%)	83.27	90.14	92.85
Avg. Rev. Per Pax **($/Pax)**	**693.86**	736.38	785.32
Avg. Rev. Per Avail. Seat($/Seat)	510.45	588.99	652.35
Rev. Improvement over EMSRa	**0.12%**	**0.24%**	**0.45%**

Table 4.3 Network Performance of LBFC Method using EMSRb Control

The items listed in above two tables are factors which we can compare among different methods, such as number of local passengers spilled and number of connecting passengers spilled. These numbers can show how a method favors local to connecting passengers, and they can also provide a reference if a method is too strict to a particular kind of passenger.

In the last row of Table 4.3, we observe that the network revenue can be increased by up to 0.45% at demand level 1.20, simply by changing the control method from EMSRa to EMSRb. Mostly, in practice, the EMSRb is better then EMSRa[6]. Therefore in the rest of our simulations, if not otherwise specified, EMSR control always means using EMSRb to calculate the booking limits.

2. Leg Performance

On the two selected legs, the performance of the seat control (using EMSRb method) is shown in the following table.

[6]P.P. Belobaba, *Optimal vs. Heuristic Methods for Nested Seat Allocation*, Presentation to the AGIFORS Yield Management Study Group, Brussels, Belgium, May 1992.

	CAP.	LF	PAX Load	Demand	Spill	%Local Demand	Local SLD
Leg 4	181	.99	180	275	95	67.68	111
Leg 30	142	.72	103	111	8	85.40	95

Table 4.4 Performance of LBFC method on Two Selected Legs at Demand Level 1.20

The columns, in the table above, are sequentially: The capacity on each leg; the average leg load factor; the number of passengers on board; the total demand on each leg; the number of passengers spilled; the percentage of local demand; and the number of seats sold to local demands.

3, Some Booking Problems on Leg 4

The figure below shows the average (over 20 departures) booking performance of the 16 booking periods.

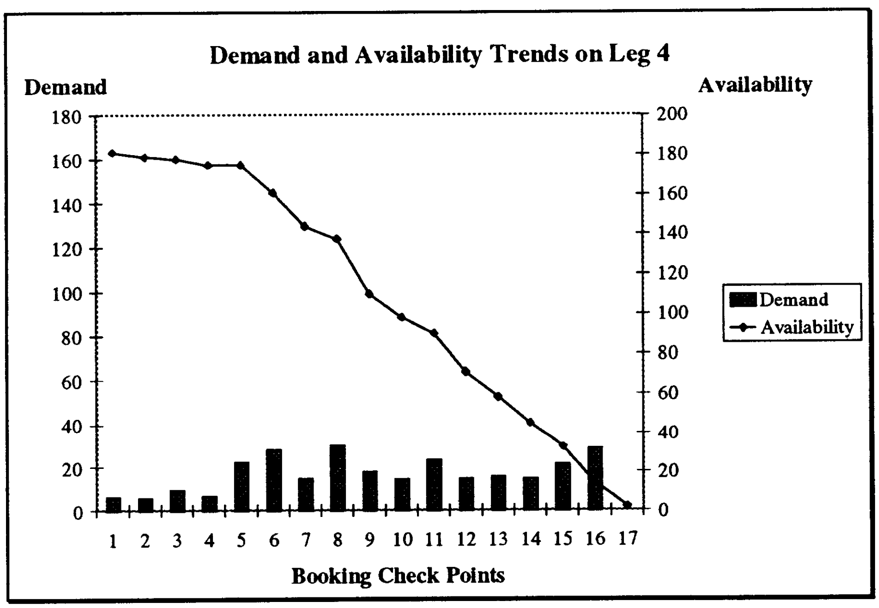


Figure 4.4 Mean Demand and Availability Trends Over 16 Booking Periods on Leg 4

We may notice that the capacity proceeds to almost 0 when it comes to the departure date. This represents a desirable performance because no seats remain empty. The following three tables show the average demand, sold and spill (over 20 departures) of the last five booking periods by fare classes.

Booking Period	12	13	14	15	16
Fare Class 1	1.5	2.75	1.8	0	0
Fare Class 2	0	0.6	0	0	0
Fare Class 3	0.4	1.45	0.25	0.85	0
Fare Class 4	0	0	0	0	0
Fare Class 5	0.95	1.6	2.05	1	0
Fare Class 6	0	0	0	0	0
Fare Class 7	0	0.7	0	0	0
Fare Class 8	10.8	8.4	9.05	17.3	18.6
Fare Class 9	0	0	0	0	0
Fare Class 10	1.25	1.15	2.3	2.25	11

Table 4.5 Demands come During the Last 5 Booking Periods on Leg 4

Booking Period	12	13	14	15	16
Fare Class 1	1.50	2.75	1.80	0	0
Fare Class 2	0	0.60	0	0	0
Fare Class 3	0.40	**1.40**	**0.25**	0.75	0
Fare Class 4	0	0	0	0	0
Fare Class 5	0.45	1.05	1.80	0.35	0
Fare Class 6	0	0	0	0	0
Fare Class 7	0	0	0	0	0
Fare Class 8	10.50	8.35	9.05	16.1	**9.6**
Fare Class 9	0	0	0	0	0
Fare Class 10	0	0	0	**0.3**	**1.1**

Table 4.6 Booking Performance During the Last 5 Booking Periods on Leg 4

Booking Period	12	13	14	15	16
Fare Class 1	0	0	0	0	0
Fare Class 2	0	0	0	0	0
Fare Class 3	0	**0.05**	0	**0.1**	0
Fare Class 4	0	0	0	0	0
Fare Class 5	**0.5**	**0.55**	**0.25**	**0.65**	0
Fare Class 6	0	0	0	0	0
Fare Class 7	0	0.7	0	0	0
Fare Class 8	0.3	0.05	0	1.15	8.95
Fare Class 9	0	0	0	0	0
Fare Class 10	1.25	1.15	2.3	1.95	9.9

Table 4.7 Spill Performance During the Last 5 Booking Periods on Leg 4

These three tables reveal that some demands in higher fare classes are spilled (bold numbers in Table 4.7) while the demand in lower fare classes are not spilled very sufficiently. Then we realize that if we can be more aggressive in controlling the seats from the lower fare passengers in previous booking periods (bold numbers in table 4.6), we could have better network revenue performance.

4. Issues on Leg 30

On Leg 30, there is another story. We know that the average load factor on this leg is around 75%. So generally, the total demand is lower than the total capacity, but passengers are still spilled on this leg. See the following figure:

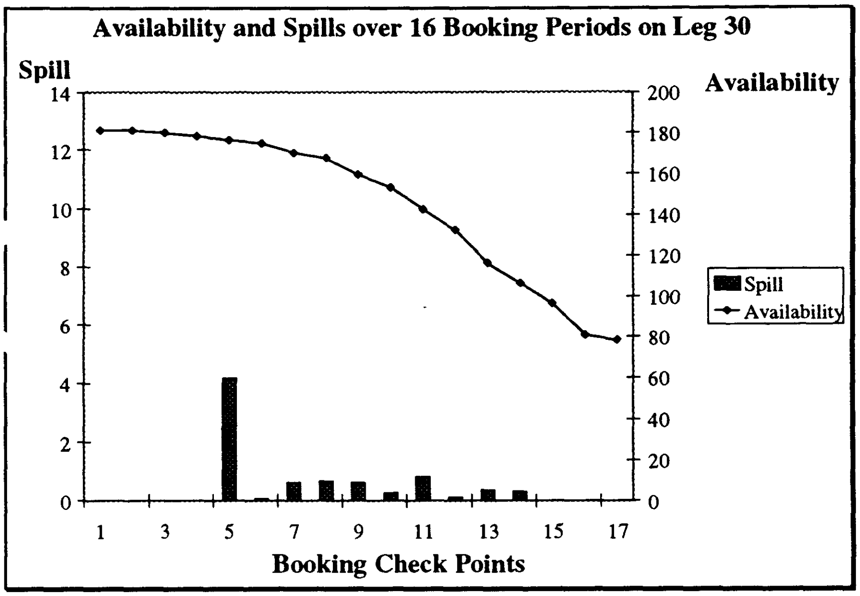


Figure 4.5 Spill Performance on Leg 30

We notice that some passengers are spilled even though there are empty seats available on that leg. Further research shows that the passengers are not spilled because of the wrong protection strategy, but due to the critical seat availability problem on other legs. Actually, if we check in Table 4.4, we will find that all those spilled passengers are connecting passengers, and the spills of the local passengers on this leg are 0.

5. Summary

The LBFC method (EMSRb) utilizes the existing fare class as the inputs to the EMSRb booking limits calculation. This may not be the best way to maximize revenues on a network of flights. Remember on Leg 30, the less congested leg, we find that spills happen, and all the spills are connecting passengers. This represents a problem of this method: Normally, for a two-leg network, if one (or both) of the two legs have empty seats, then connecting passengers should be given higher priority to access the seats. On the congested leg, Leg 4, we also find that some passengers from higher fare classes are spilled because the seats are not protected adequately from the passengers from lower fare classes.

For comparison purpose, we will always use the LBFC (EMSRb) method as the base case to compare with the other methods in the following discussions, and in the following, the LBFC always means LBFC combined with EMSRb method.

4.3.2 Greedy Virtual Nesting Method (GVN)

In this method, the fares on each leg are arrayed into 10 virtual classes based totally on the values of the fares of passengers. This is also known as "Leg Specific Virtual EMSRb" method. It means that the virtual classes are sorted **on a leg basis that** is, each individual leg has a different virtual class range. There is another way to do this, in which the fares of the whole network are grouped into 10 virtual classes together. The latter method is known as "Network Wide Virtual EMSR". Normally, leg specific virtual nesting method has a better network revenue performance, and in this thesis we always employ the leg specific virtual nesting EMSRb control method for comparisons.

1. Virtual Class Ranges (VRANGE)

The virtual ranges are calculated according to the total itinerary fares. In the following table, we list the ranges of those virtual classes on the two selected legs: Leg 4 and Leg 30.

VRANGE on LEG 4	VRANGE on LEG 30
1643.0	398.0
588.0	398.0
558.0	398.0
413.0	358.0
401.0	343.0
401.0	251.0
304.0	242.0
152.0	230.0
61.0	194.0
.0	.0

Table 4.8 Ranges of 10 Virtual Classes on Leg 4 and Leg 30 in GVN Method

From the table above we notice that the different legs have dramatically different virtual class ranges. Generally, there are two factors that may affect the values of the virtual class ranges: values of fares and number of demands in each fare level. Usually, long legs have higher virtual ranges than short ones.

We notice that on Leg 30, the first three virtual class ranges are the same ($398.0). The reason for that is that we sort the fares to a virtual class on a demand base. It means that we divide the total demand by the number of virtual classes and obtain the number of demand in each virtual class. Then from the top virtual class we accumulate the demand, as long as the accumulated demand is less that the number of demand that should be in this virtual class, the corresponding fares are arrayed to this virtual class. Sometimes, there are a huge number of demand in one fare class, then it may take two or more virtual classes to contain this fare class. Therefore these two or more virtual classes will have the same virtual class ranges.

2. Revenue Performance

The network performance of this method at different demand levels is shown in the following tables.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3602209	4159008	4616957
Local Pax Spilled	333	1123	2167
Connecting Pax Spilled	61	238	406
Avg. Leg Load Factor(%)	82.97	89.85	93.02
Avg. Rev. Per Pax($/Pax)	697.87	745.17	804.35
Avg. Rev. Per Avail. Seat($/Seat)	512.41	591.61	656.75
Rev. Imprv. over LBFC	**0.38%**	**0.44%**	**0.67%**

Table 4.9 Performance of GVN Method

Compared with the LBFC method (Table 4.3), the greedy virtual nesting method has a higher revenue performance. The values of revenue per passenger have been increased, so have the values of revenue per seat. Additional comparisons tell us that the number of the local passengers that were spilled was greatly increased, and the spills of connecting passengers are decreased. Such comparisons are shown in the two figures below.

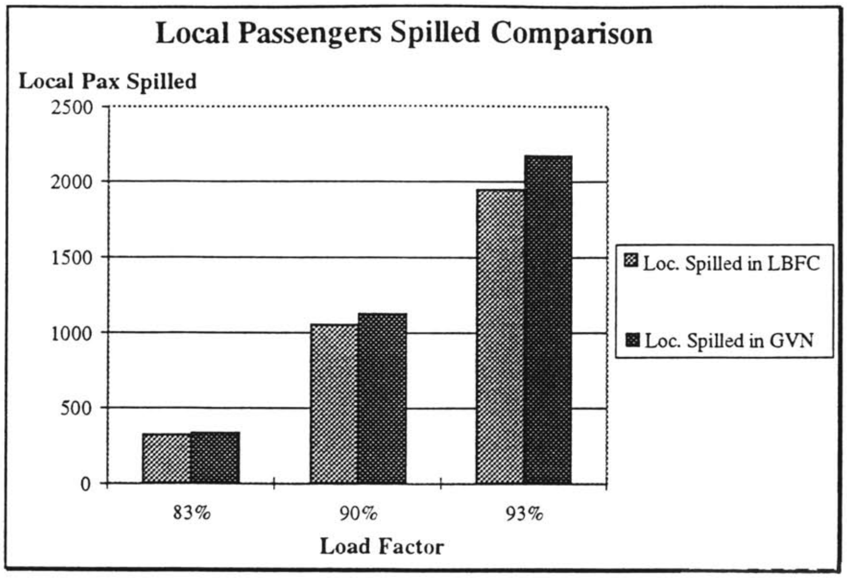


Figure 4.6 Comparison of Number of Local Passengers Spilled

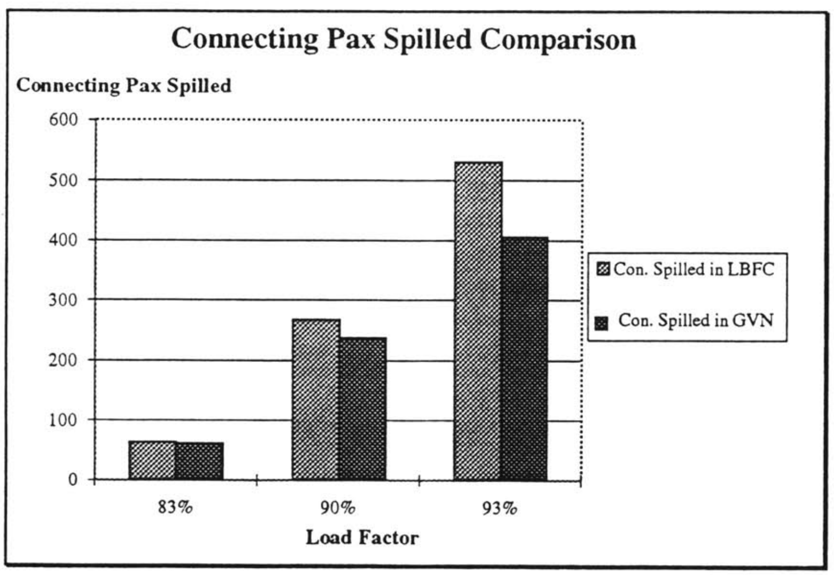


Figure 4.7 Comparison of Number of Connecting Passengers Spilled

Such differences are obviously due to the different strategies of these two methods in treating the connecting passengers. The GVN method ranks the connecting fares to higher virtual classes than the local fares, therefore less connecting passengers are spilled.

3. Leg Performance

The leg performance of the two selected legs is shown in the following table.

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.96	174	275	101	67.68	117
Leg 30	142	.74	105	111	6	85.40	95

Table 4.10 Performance of GVN on Two Selected Legs at Demand Level 1.20

In practice, the GVN method usually has a higher load factor than the LBFC method. If we compare Table 4.10 with Table 4.4, we find that the GVN method does have a higher load factor on Leg 30, however it has a lower load factor on Leg 4. The unusual performance on Leg 4 can be explained by the extreme congestion on this leg and the inter-actions among the legs. Another interesting thing we find is that on Leg 30, even though all of the spills are still connecting passengers, the total number of spills decreases. Those spills are still due to the congestion from other legs, but since the connecting passengers have higher priority to access the seats on those critical legs, therefore more connecting passengers can show up on this less congested leg.

4. Spill Performance

We found that the GVN method controls seats more aggressively than the LBFC method does. In the two figures below we can easily see that the average seat availability over 20 departures of the GVN method on the congested Leg 4 is always greater than that of LBFC method. On the less congested leg (Leg 30), these two methods tend to have similar control performances.

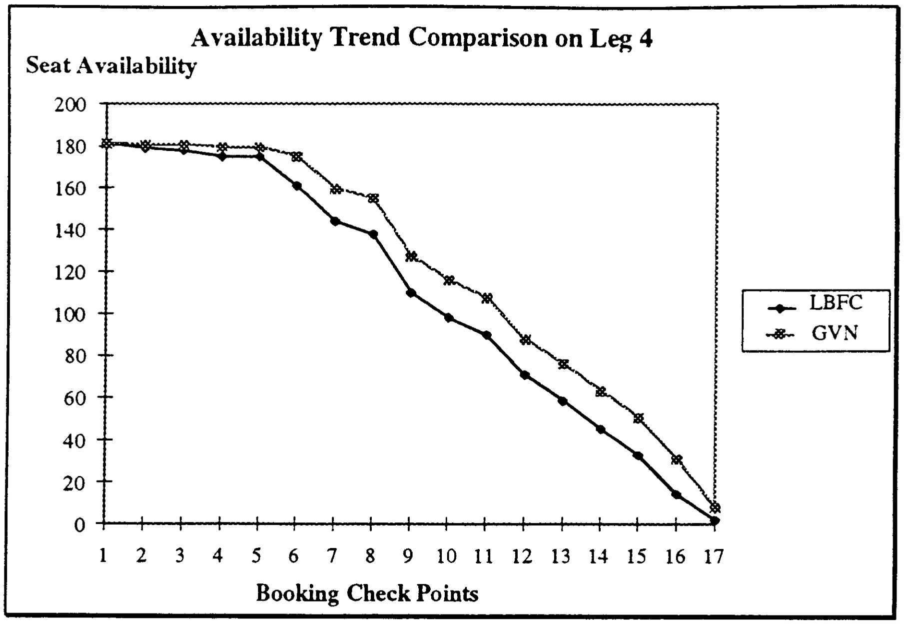


Figure 4.8 Availability Comparison between LBFC and GVN on Leg 4

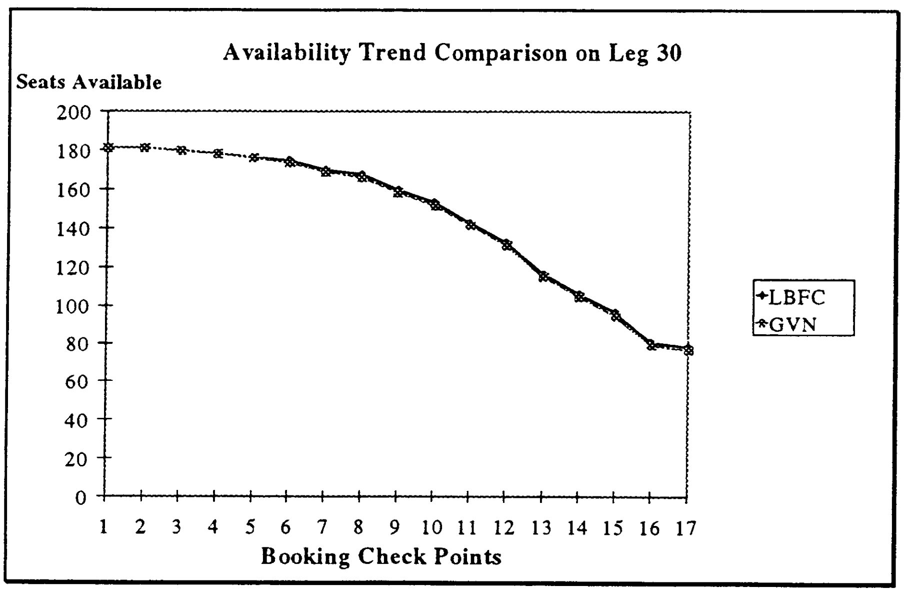


Figure 4.9 Availability Comparison between LBFC and GVN on Leg 30

4.3.3 Greedy Heuristic Bid Price Method (GHBP)

1. Formula of the Bid Prices

Similar to the GVN method, in the GHBP method, the fares on each leg are also grouped into 10 virtual classes based totally on the values of the fares. Then EMSRb method is employed to calculate the critical EMSR value on each leg. These critical EMSR values are then utilized as the "cut-off" prices to decide whether a passenger should be booked. To a local passenger, by definition in Chapter 2, the bid price is the value of the EMSR value on the leg this passenger travels. For a connecting passenger, the bid prices are the combinations of the EMSR values of all legs this passenger traverses.

$$BP_{Connecting} = Max(EMSR1, EMSR2) + d \times Min(EMSR1, EMSR2) \quad [4.1]$$

[7]

However, in practice, we find that it is always better to set the bid price for local passengers to be zero, which means that the local passengers are controlled by EMSR booking limits as before, rather than the EMSR bid price value. The two tables below show the differences in setting the bid price for local passengers. In Table 4.11, the bid price of local passengers is 0, while in Table 4.12, it equals critical EMSR value.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3595406	4156513	4624277
Local Pax Spilled	233	931	1839
Connecting Pax Spilled	147	364	603
Avg. Leg Load Factor(%)	81.90	88.89	92.03
Avg. Rev. Per Pax($/Pax)	694.71	736.15	787.57
Avg. Rev. Per Avail. Seat($/Seat)	511.44	591.25	657.79
Rev. Imprv. over LBFC	**0.19%**	**0.38%**	**0.83%**

Table 4.11 GHBP Control Applied to *Only* Connecting Passengers

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3557461	4102171	4565252
Local Pax Spilled	417	1219	2134
Connecting Pax Spilled	119	312	535
Avg. Leg Load Factor(%)	80.14	86.20	89.78
Avg. Rev. Per Pax($/Pax)	708.71	758.25	808.72
Avg. Rev. Per Avail. Seat($/Seat)	506.04	583.52	649.40
Rev. Imprv. over LBFC	**-0.86%**	**-0.93%**	**-0.45%**

Table 4.12 GHBP Control Applied *Both* Connecting and Local Passengers

[7] P.P. Belobaba. Airline O-D Seat Inventory Control Without Network Optimization, Flight Transportation Laboratory, MIT, Cambridge, June 1995.

In these two tables above, the parameter of the bid prices for connecting passengers has been chosen to be 0.25 (we will explain the reason in the section "Sensitivity Studies"). The performance of the latter method is worse than the former. If we compare the difference we will find that in the latter method, too many local passengers are spilled. The reason for this is that the absolute fare values of local passengers are lower than those of the connecting passengers. When we calculate the critical EMSR value for each leg, we had taken into account all fares that traverse that leg. So the EMSR value is extremely high, and many local passengers cannot meet the cut-off prices, then are spilled.

2. Sensitivity Studies

We may remember that in the formula of the bid prices for the connecting passengers, there is a heuristic parameter in Equation [4.1]. As discussed in Chapter 2, this is a heuristic factor and can be specific to network characteristics. So we need to conduct some sensitivity studies to find out the best value for this parameter. Figure 4.10 shows how the network revenue changes with this parameter at demand level 1.20.

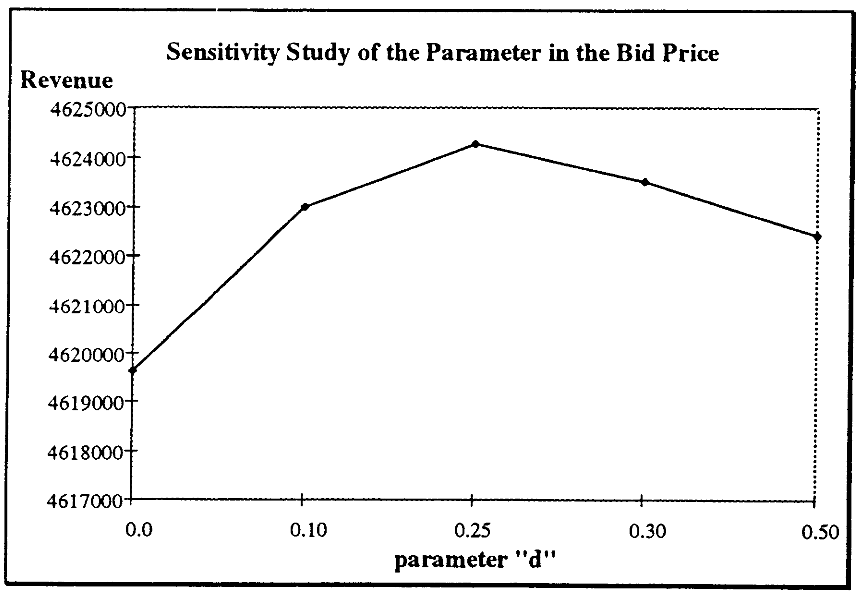


Figure 4.10 Sensitivity Studies of Bid Price Formula for GHBP Method

From the figure above we find that the parameter value of 0.25 gives the best network revenue performance. That is why in the previous section we have chosen 0.25 in our formula.

3. Comparison of the GHBP and the GVN Methods

a. Network Revenue Performance

The following table shows the comparison between the network performance of the GHBP and that of the GVN method using the data from demand level 1.20.

	Revenue	Improvement	Load Factor	Loc Pax Spilled	Con Pax Spilled
GVN	4616957	0.67%	93.02%	2167	406
GHBP	4624277	0.83%	92.03%	1839	603

Table 4.13 Comparison between the GVN and the GHBP Methods

We find from the table above that the GHBP method has a better revenue performance while a lower average leg load factor than the GVN method. Theoretically, at demand level 1.20 (a relatively high demand environment), the GVN method gives too high a priority to high fare, connecting passengers while spilling too many high yield, local passengers. We know that the total revenue of two local passengers is normally higher than that of one connecting passenger. So the GVN method tends to have negative network revenue impact when demand is high. On the contrary, the GHBP method considers the displacement impact of the connecting passengers, and sets a higher bid price to control the load of the connecting passengers and reserve enough seats for local passengers.

b. Leg Performance Comparison

The Leg performance on the two selected legs of the GHBP method is shown in the following table.

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.94	170	275	105	67.68	128
Leg 30	142	.72	102	111	9	85.40	95

Table 4. 14 Leg Performance of the GHBP at Demand Level 1.20

Compared to the leg performance on Leg 4 with that in the GVN method, the results are

	LF	PAX Load	Local Sold
GVN	.96	174	117
GHBP	.94	170	128

Table 4.15 Comparison between the GVN and the GHBP on Leg 4

From the table above we find that the heuristic bid price method has a lower load factor than the GVN method. This is because in the GHBP method, the bid prices set for the connecting passengers are the combination of the EMSR values of all the legs the passengers traverse. So more connecting passengers are spilled. However, the load of local passengers increases dramatically.

c. Spill Performance Comparison

The passengers who are spilled are different in these 2 methods. In the following table, the spills on Leg 4 are listed by virtual classes:

	1	2	3	4	5	6	7	8	9	10	Total
Virtual Nesting	0.00	**29.45**	0.00	2.70	0.00	0.00	0.15	18.05	50.75	0.00	101.10
Heuristic Bid Price	0.00	**44.05**	0.00	2.60	0.00	0.00	0.00	13.26	45.05	0.00	104.95

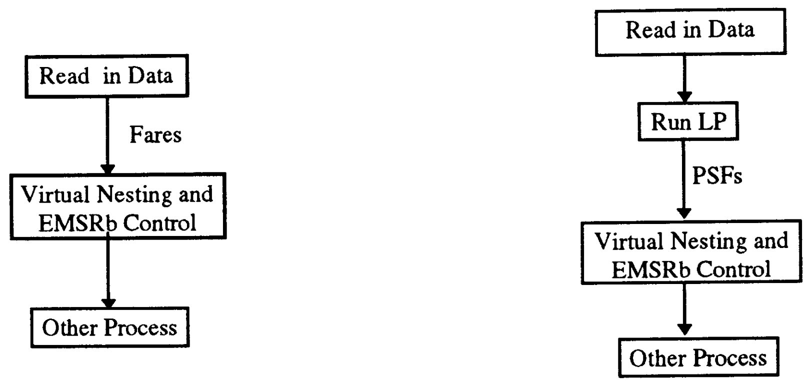
Table 4.16 Average Spill comparison Between GVN and GHBP by Virtual Classes

At the first look, we may not feel confident about the GHBP method because it spills more passengers who belong to the higher virtual classes than the GVN method does (two bold numbers in the table above). However, the reason for this phenomenon is that both methods are greedy methods. So when the demand is sorted into virtual classes, the total fares of passengers are utilized. So those long haul connecting passengers are ranked into a higher class. However this may not be the best approach. Actually, in GHBP method, a higher bid price for connecting passengers is set to overcome the previous mistakes. So most of those passengers spilled in higher virtual classes in GHBP method are connecting passengers.

4.3.4 Non-greedy Virtual Nesting Method (NGVN)

1. Virtual Class Ranges

The only difference between the GVN and the NGVN methods are the inputs to the EMSR calculation. If we draw a process picture, it will be clear.



Greedy Virtual Nesting

Non-greedy Virtual Nesting

Figure 4.11 Process Comparison between GVN and NGVN

From the figure above we see that the NGVN has an additional process: run the LP to calculate the shadow prices; utilize the shadow prices as the measurement of the displacement impacts of the connecting passengers. In our simulation process, the pseudo fares are calculated as follows:

$$PSF(i,j,l) = Max\{0.0,[fare(i,j) - \sum_{l \in TraversedLegs(excludeLegl)} SP(l)]\} \quad [4.2]$$

where i is the OD pairs, j is the fare class, and l is the leg that the passenger traverses. Since the shadow prices are greater than or equal to 0, we know that the pseudo fares are less than or equal to the total fares. That is why, if we compare the ranges of the virtual classes in GVN method (Table 4.8), we will find that the virtual class ranges in the NGVN method tend to be lower (Table 4.17).

VRANGE on LEG 4	VRANGE on LEG 30
890.0	552.0
776.0	448.0
638.0	420.0
565.0	358.0
304.0	251.0
152.0	246.0
73.0	230.0
61.0	194.0
.0	72.0
.0	.0

Table 4.17 16 Virtual Class Ranges on Leg 4 and Leg 30 in NGVN Method

2. Network Revenue Performance

The revenue performance of the NGVN method is listed below.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3602187	4175482	4647918
Local Pax Spilled	160	782	1669
Connecting Pax Spilled	171	408	655
Avg. Leg Load Factor(%)	82.33	89.86	93.10
Avg. Rev. Per Pax($/Pax)	689.42	725.88	775.93
Avg. Rev. Per Avail. Seat($/Seat)	512.40	593.95	661.15
Rev. Imprv. over LBFC	**0.38%**	**0.84%**	**1.35%**

Table 4.18 Revenue Performance of the NGVN Method

Here we run the LP only once at the beginning of the first booking period at a correct demand level. Issues about the combination of running LP at different demand levels and running LP at each booking check point will be discussed later.

3. Comparison between the GVN and the NGVN Methods

a. Network Revenue Performance

The table below shows the comparison between the GVN and the NGVN method using the data at demand levels 1.20 and 0.80.

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
GVN	4616957	0.67%	93.02%	2167	406	804.35	656.75
NGVN	4647918	1.35%	93.10%	1669	655	775.93	661.15

Table 4.19 Comparison between GVN and NGVN at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
GVN	3602209	0.38%	82.97%	333	61	697.87	512.41
NGVN	3602187	0.38%	82.33%	160	171	689.42	512.40

Table 4.20 Comparison between GVN and NGVN at Demand Level 0.80

We find from the two tables above that the NGVN method has much better revenue performance than the GVN method when demand is high, and a revenue performance similar to that of GVN method when demand is low. This is because when demand is high, we want to favor more local passengers than connecting passengers, while when demand is low we want to favor more connecting passengers than local passengers. The strategies of greedy and non-greedy methods are totally different. The greedy method values the connecting passengers as their total fares, so most connecting passengers are arrayed in a higher virtual class than the local passengers. However the non-greedy method considers the displacement impacts of the connecting passengers and values them as their fares minus all shadow prices on the other legs they traverse. This value is lower than the total fares, so the connecting passengers are re-arranged to lower virtual classes based on their revised fares. Therefore the long haul, high fare but low yield connecting passengers no longer have the highest priority to the seats, and the seats are protected for the high yield local passengers. Such strategy is very profitable when the demand is high because two local passengers normally have higher revenue contribution than one connecting passengers. On the low demand side, since the shadow prices are no longer positive (if there is no congestion, the shadow prices are zero), the pseudo fares of the connecting passengers are equal to their total fares and then the non-greedy and greedy virtual nesting will give similar booking decisions.

From the two tables above we also notice that at demand level 1.20, the NGVN method has lower revenue per passenger but higher revenue per seat due to the high load of local passengers. Figures 4.12 and 4.13 show such comparison.

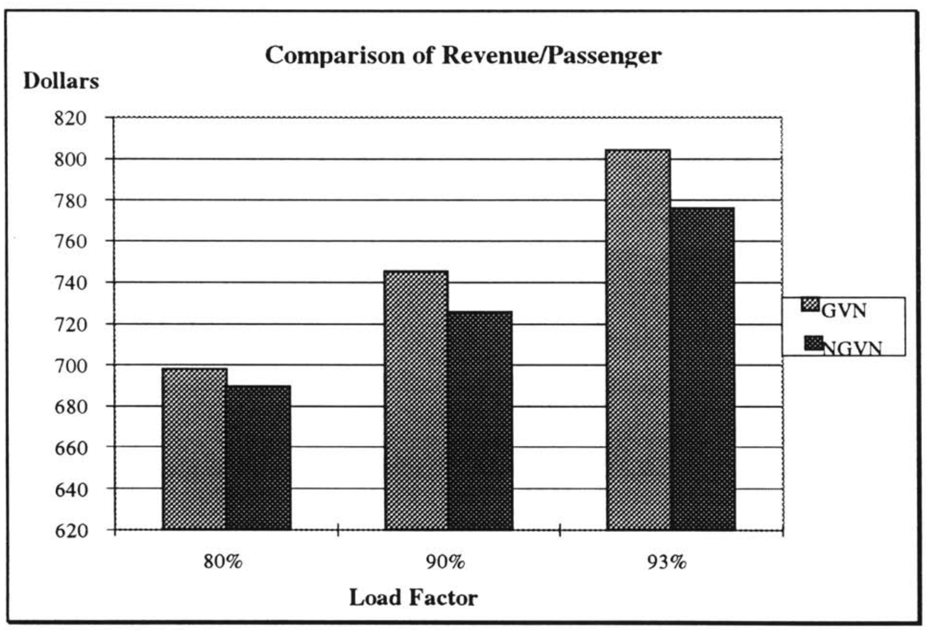


Figure 4.12 Revenue/Passenger Comparison between GVN and NGVN at Demand Level 1.20

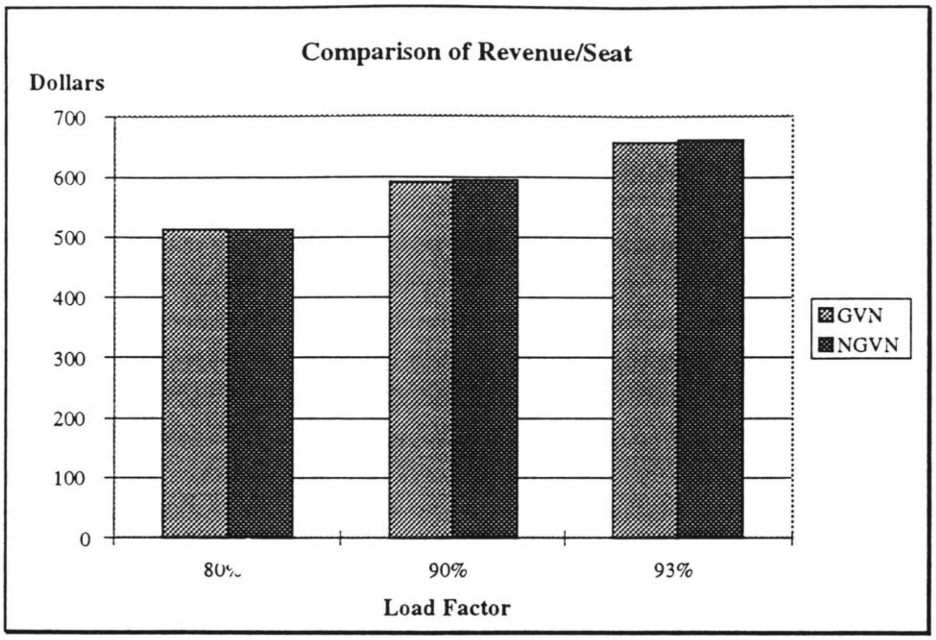


Figure 4.13 Revenue/Seat Comparison between GVN and NGVN at Demand Level 1.20

The reason that the NGVN has a lower value in the revenue per passenger is because the NGVN method has a high load of local passengers. It lowers the revenue contribution from each individual passengers. However, the total revenue contributions of two local passengers are higher than one connecting passenger, therefore the NGVN has a higher value in revenue per seat than the GVN method.

b. Leg Performance

The following table shows the seat control performances of the two selected legs.

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.96	173	275	102	67.68	135
Leg 30	142	.74	105	111	6	85.40	95

Table 4. 21 Leg Performance of NGVN

Compared to Table 4.10, which is the seat control performance of the same two legs in the GVN method, we have the following table:

	LF	PAX Load	Spill	Local Sold
GVN	.96	174	101	117
NGVN	.96	173	102	135

Table 4.22 Comparison of GVN and NGVN on Leg 4

From the table above we find that the NGVN method spills fewer passengers than the GVN method, and sold much more seats to the local passengers than the GVN method does. Such performance is desirable at a high demand level.

4.3.5 Deterministic Network Bid Price Method (DNBP)

This method is similar to the GHBP method in the respect of setting a "cut-off" price to control the booking process, but different from the GHBP method in that DNBP method utilizes the sum of the shadow prices as the cut-off price. So it is a pure network optimization method. The process of this method is shown in the figure below.

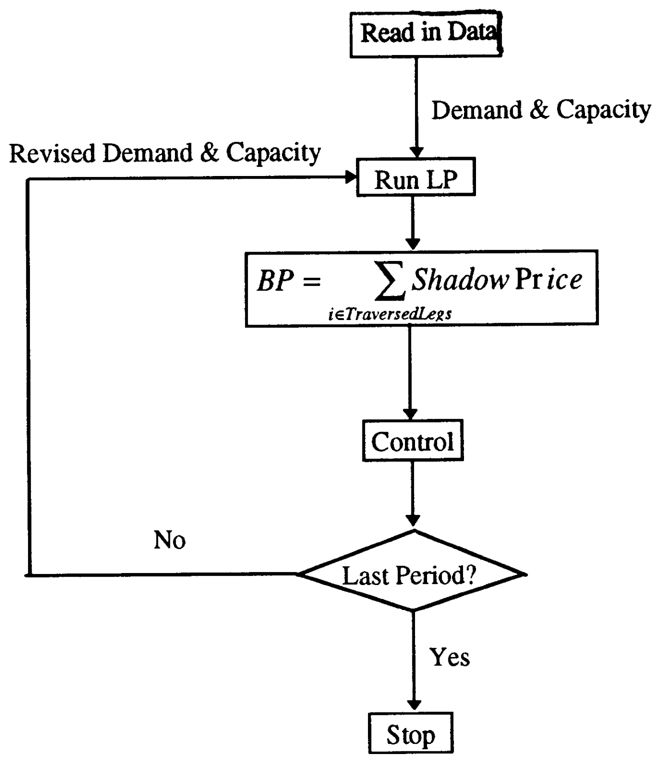


Figure 4.14 Process of the DNBP Control

From the figure above we see that in the deterministic bid price method, an LP needs to be run at each booking period to obtain the most updated shadow prices. Then those shadow prices are applied as the cut-off prices. In the real world, this method is difficult to implement because of the technology and information limitations. Even though we assume that we have the information and time to run LP multiple times, this method may not give a very good performance, because the demand input we use is the forecast mean demands, and the real demands usually deviate from the mean. Therefore the method is very sensitive to the inputs, because the wrong demand input will cause the wrong shadow prices as output and consequently cause the wrong bid prices and spill decisions. As our example shows, the deterministic bid price method gives a very poor network performance (Table 4.23), compared to the other methods (such as NGVN, GHBP, and GVN methods).

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3585530	4141165	4577434
Local Pax Spilled	172	731	1577
Connecting Pax Spilled	109	373	649
Avg. Leg Load Factor(%)	84.03	91.78	94.67
Avg. Rev. Per Pax($/Pax)	679.83	709.39	751.94
Avg. Rev. Per Avail. Seat($/Seat)	510.03	589.07	651.13
Rev. Imprv. over LBFC	**-0.08%**	**0.01%**	**-0.19%**

Table 4.23 Network Performance of DNBP Method

From the table above we find that the revenue performances of the DNBP method are even worse than those of the LBFC method. This is due to several reasons: First, there are only 16 booking periods assumed in this simulation process, and the small number of the revision points causes the shadow prices to be come out of date between re-optimizations. Another reason for the failure of this method is the great deviation in demand from one period to another. Suppose we have a very low demand in the first booking period, then we will have a lot of empty seats after that period. When it comes to the revision time, we will come up with relatively low shadow prices, which will be applied to the second booking period as the bid prices. If the second period turns out to be a high demand period, then a lots of low fare passengers will be booked by mistake, and they will take the seats away from the high fare passengers from later booking periods. Therefore we need to be very careful when we employ the DNBP method. We expect to see (and have simulated) better performance for DNBP with more frequent re-optimization.

4.3.6 New Methods

1. Network Non-greedy Heuristic Bid Price Model (NGHBP)

The only difference between the GHBP (Greedy Heuristic Bid Price) method and the NGHBP method are the inputs to the EMSR calculation. Figure 4.15 shows the control process of the NGHBP method:

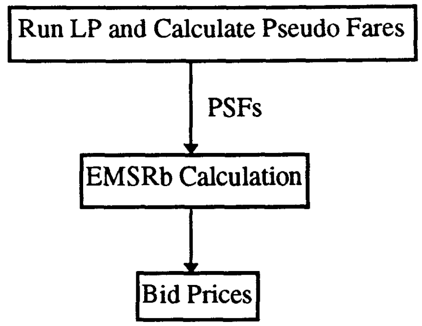


Figure 4.15 Process of the NGHBP Method

As shown in the figure above, the LP is run first to calculate the shadow prices associated with each leg. Then the shadow prices are employed as the measurements of the displacement effects of the connecting passengers. The pure network revenue contribution of each ODF (pseudo fare) is input into the EMSRb calculation. The detailed description about this method can be refereed to Part 1 of Section 3.1.2.

a. Compare with the GHBP Method

1) Network Revenue Performance

The table below shows the network performance of the NGHBP method.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3604315	4176231	4649692
Local Pax Spilled	206	824	1698
Connecting Pax Spilled	135	378	638
Avg. Leg Load Factor(%)	82.61	90.06	93.02
Avg. Rev. Per Pax($/Pax)	691.17	727.60	777.87
Avg. Rev. Per Avail. Seat($/Seat)	512.70	594.06	661.41
Rev. Imprv. over LBFC	**0.44%**	**0.86%**	**1.39%**

Table 4.24 Revenue Performance of the NGHBP Method

Here we run the LP only once at the beginning of the first booking period, and the right-hand sides (RHS) of the demand constraints are chosen to be at the corresponding demand levels. The heuristic parameter in the formula of the bid price is obtained from the sensitivity studies: 0.6 for demand level 1.20; 0.4 for demand level 1.00; and 0.3 for demand level 0.80. If we choose to fix the parameter at 0.50 for all demand levels, then the revenue performance of the NGHBP is as follows:

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3602322	4175123	4649616
Rev. Imprv. over LBFC	**0.39%**	**0.83%**	**1.39%**

Table 4.25 Revenue Performance of the NGHBP Method with fixed Parameter at 0.50

Tables 4.26 and 4.27 compare the network performance of the GHBP method with that of the NGHBP method using the data from demand levels at 1.20 and 0.80.

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
GHBP	4624277	0.83%	92.03%	1839	603	787.57	657.79
NGHBP	4649692	1.39%	93.02%	1698	638	777.87	661.41

Table 4.26 Comparison of Network Performance Between GHBP and NGHBP at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
GHBP	3595406	0.19%	81.90%	233	147	694.71	511.44
NGHBP	3604315	0.44%	82.61%	206	135	691.17	512.70

Table 4.27 Comparison of Network Performance Between GHBP and NGHBP at Demand Level 0.80

We find from the two tables above that the NGHBP method has better revenue performance than the GHBP method does both when demand is high and demand is low. The reason is that, when demand exceeds the capacity, even though the GHBP method tries to overcome the shortcomings from its greedy approach by setting a higher bid price for connecting passengers, this parameter can improve the network revenue performance in only a relatively small range. However, in the network NGHBP method, the displacement impacts are taken into account and connecting passengers are re-arranged into more appropriate virtual classes. With the additional adjustment from the heuristic parameter, we can achieve a very good revenue performance.

From Table 4.26 we also notice that at demand level 1.20, the NGHBP method has a lower revenue per passenger value and a higher revenue per seat value than the GHBP method. Such comparisons are shown in following two figures below (Figure 4.16 and 4.17).

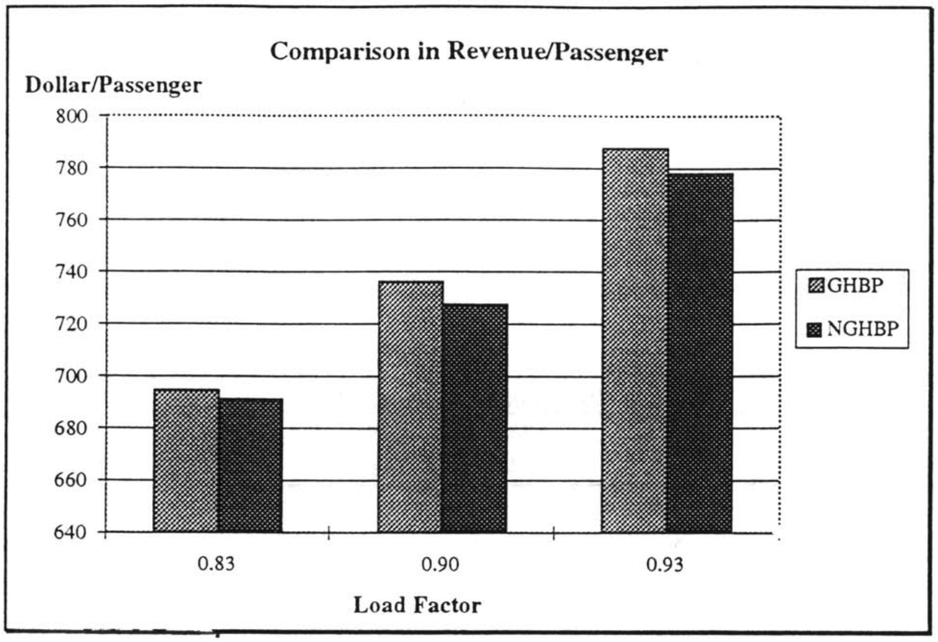


Figure 4.16 Comparison in Revenue/Passenger between GHBP and NGHBP Methods

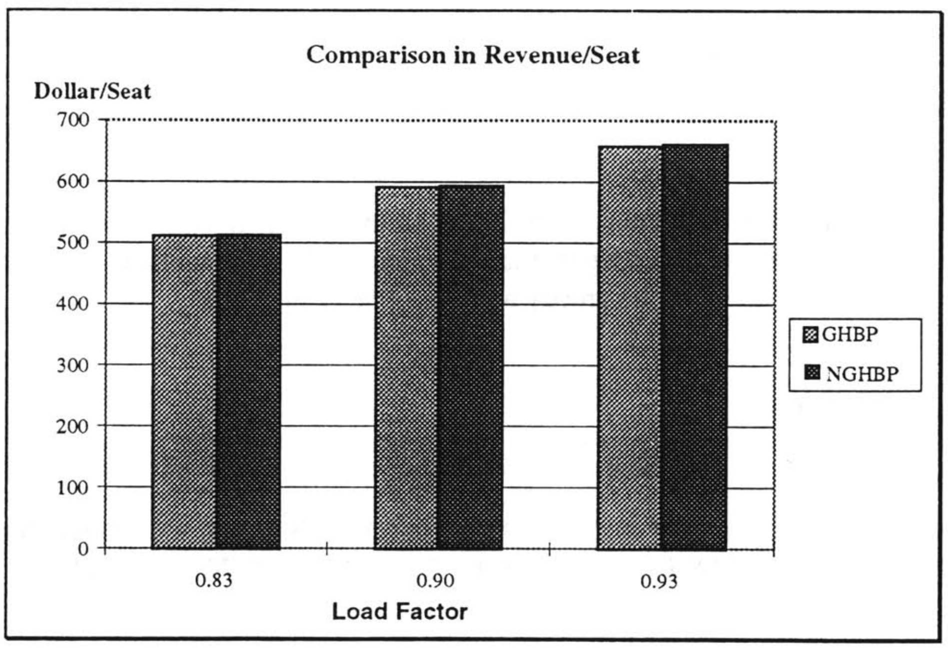


Figure 4.17 Comparison in Revenue/Seat between GHBP and NGHBP Methods

This phenomenon can be explained by the characteristics of the greedy and non-greedy methods. The greedy methods tend to book many connecting passengers; therefore they produce very high values in the revenue per passenger. However, those connecting passengers take two or more seats in the network; therefore the values of the revenue per seat in the greedy methods are lower.

2) Leg Performance

Table 4.28 shows the seat control performance of the two selected legs from the NGHBP method.

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.96	174	275	101	67.68	134
Leg 30	142	.73	104	111	7	85.40	95

Table 4. 28 Seat Control Performance of NGHBP Method on Leg 4 and Leg 30

The comparison of the seat controls on Leg 4 between the GHBP and the NGHBP methods are listed in Table 4.29.

	LF	PAX Load	Spill	Local Sold
GHBP	.94	170	105	128
NGHBP	.96	174	101	134

Table 4.29 Comparison of Seat Controls between GHBP and NGHBP methods on Leg 4

From the table above we find that the NGHBP method tends to spill fewer passengers and sell more seats to the local passengers than the greedy bid price method does. Such a difference is typical between the greedy and non-greedy methods.

3) Sensitivity Studies

To find out the best parameter for the heuristic bid price formula, we conduct the sensitivity analysis. The figure below shows how the revenue changes with the parameter.

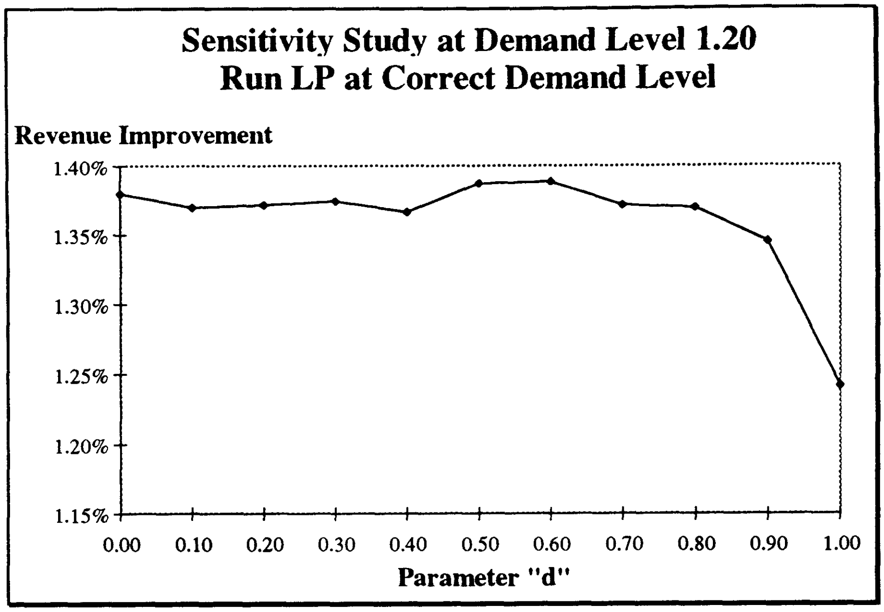


Figure 4.18 Sensitivity Studies of Parameter in Bid Price Formula for NGHBP Method

From the figure above we find that the best parameter (0.60) of the NGHBP method differs from that of the GHBP method (which is 0.25). We know that the critical EMSR values in the NGHBP method are lower than those in the GHBP method. Therefore to obtain the same level of control for the connecting passengers, the value of the parameter "d" in the NGHBP should be larger than that in the GHBP method. Also we find that the network revenue is less sensitive to this parameter in the NGHBP method than it is in the GHBP method. As the value of this parameter changes from 0.0 to 0.8, the maximum difference in the revenue improvement is less than 0.02%. In other words, the revenue changes are not very sensitive to the parameter when this parameter is small, but we can still see that the revenue decreases with the parameter when it grows large (from 0.60 to 1.00).

Such a phenomenon can be explained by the complicated booking criteria of the NGHBP. In the NGHBP control process of our simulation, the fares are not just controlled by the bid prices. Actually, they are controlled by both the bid prices and the EMSR virtual classes; that is, if a fare is greater than the bid price, then the booking limits of this fare class on a leg, instead of being equal to the availability on that leg, is equal to the booking limits of its corresponding virtual class on that leg (decided by virtual EMSRb method). Since we use only the virtual class booking limits for local passengers, such booking criteria are too strict for the connecting passengers. Therefore we add an additional compensation rule: The final booking limits for the connecting passengers are the largest limits of all legs they traverse. The inter-actions of all these criteria weaken the effects of the heuristic parameter, and this is why we find that within a certain range the parameter "d" seems not to affect the total network revenue.

4) Spill Performance Comparison

The differences between GHBP and NGHBP methods are also reflected in their spill performance. The following figure compares the percentage spills of local and connecting passengers in the GHBP and the NGHBP methods at demand level 1.20.

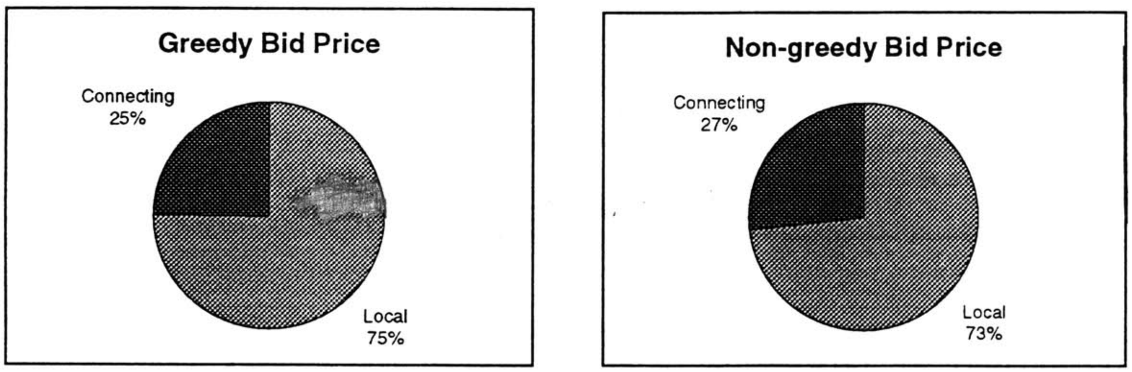


Figure 4.19 Comparisons of Percentage of Spills between GHBP and NGHBP at Demand Level 1.20

We find that of the total spill, the local passengers comprise 75% in the GHBP method, and 73% in the NGHBP method. This verifies once more that non-greedy methods favor local passengers more than the greedy methods do.

5) Additional Comparisons

We also compare the remaining capabilities over 16 booking periods on Leg 4 and 30 (Figure 4.20 and 4.21) from the GHBP and the NGHBP methods.

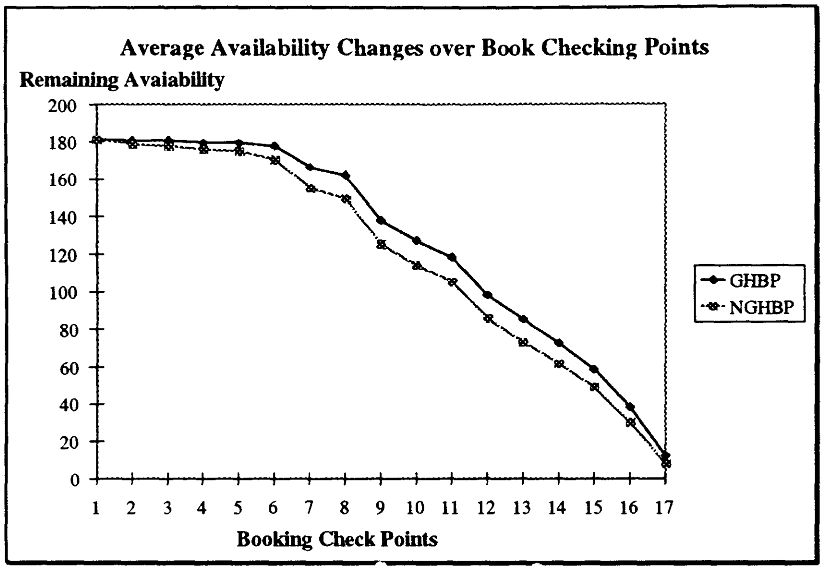


Figure 4.20 Availability Comparison between GHBP and NGHBP on Leg 4

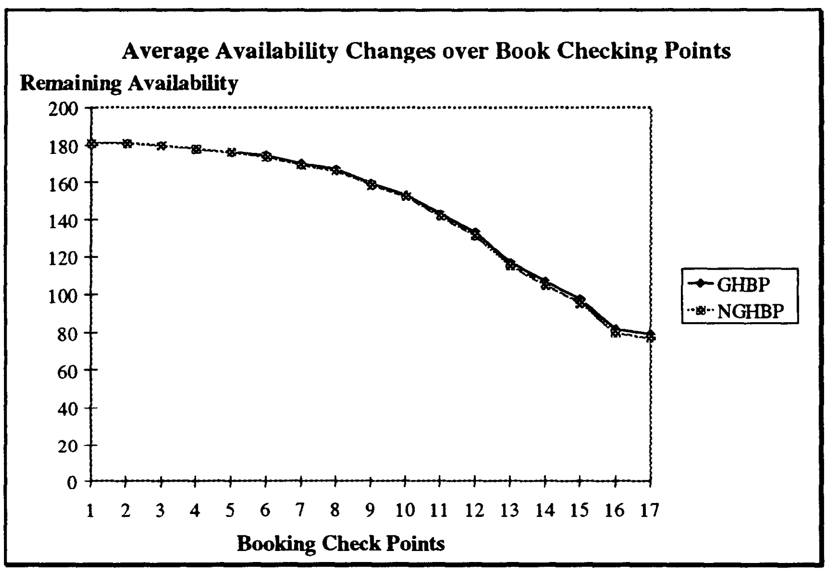


Figure 4.21 Availability Comparison between GHBP and NGHBP on Leg 30

The NGHBP method seems to book more passengers and have fewer seats available than the greedy method does over all the booking periods. In addition, we find that the NGHBP method produces fewer empty seats than the GHBP method.

b. Comparison with Non-greedy Virtual Nesting Method (NGVN)

1) Network Revenue Performance

Tables 4.30 and 4.31 list the comparison in the network performances of the NGHBP method and the NGVN method.

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGVN	4647918	1.35%	93.10%	1669	655	775.93	661.15
NGHBP	4649692	1.39%	93.02%	1698	638	777.87	661.41

Table 4.30 Network Performance Comparison between NGVN and NGHBP at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGVN	3602187	0.38%	82.33%	160	171	689.42	512.40
NGHBP	3604315	0.44%	82.61%	206	135	691.17	512.70

Table 4.31 Network Performance Comparison between NGVN and NGHBP at Demand Level 0.80

Generally, we find from the two tables above that the NGHBP method has a slightly better revenue performance than the NGVN method. Through a more detailed comparison we believe the reason is that, in the NGVN method, the displacement impacts of the connecting passengers are over-emphasized and too many connecting passengers are spilled. Such mistakes in the booking strategy may come from running a deterministic LP. We know in the LP model, the RHS of the demand constraints are fixed. However, in practice, the demand deviates greatly from one booking period to another. Therefore the solution of the shadow prices may be out of date and consequently cause the revenue contribution of the connecting passengers to be mis-interpreted. On the other hand, the NGHBP method applies a heuristic factor in the control process and can make a fine adjustment for such mis-evaluation, therefore achieving a better performance.

2) Leg Performance

Comparing the leg performance of the NGHBP with that of the NGVN method, we have the following tables:

	LF	PAX Load	Spill	Local Sold
NGVN	.96	173	102	135
NGHBP	.96	174	101	134

Table 4.32 Comparison between NGVN and NGHBP on Leg 4

	LF	PAX Load	Spill	Local Sold
NGVN	.74	105	6	95
NGHBP	.73	104	7	95

Table 4.33 Comparison between NGVN and NGHBP on Leg 30

From the tables above we find that the two methods have very similar leg performances on both legs.

3) Spill Performance Comparison

The differences between the NGVN method and the NGHBP method are also reflected in their spill performances. The following two figures compare the percentage spills of local and connecting passengers at demand levels 1.20 and 0.80.

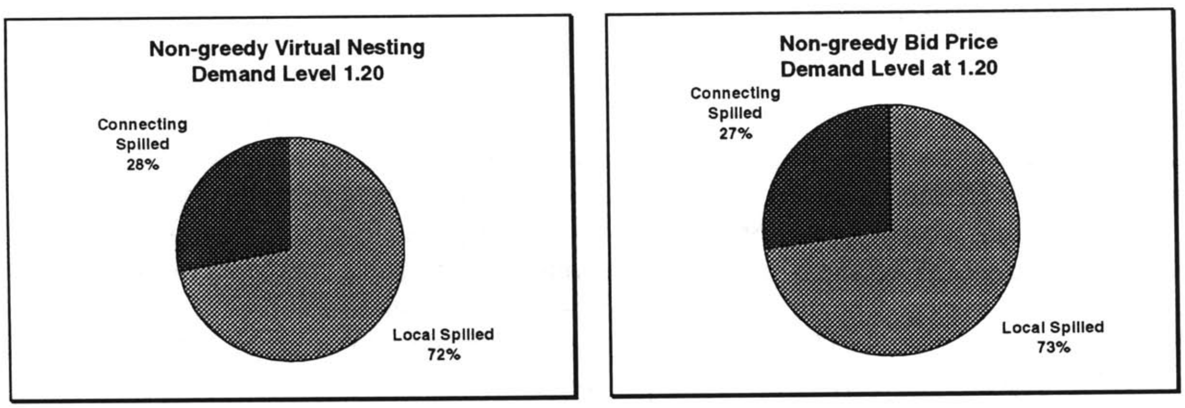


Figure 4.22 Comparison of Percentage of Spills between NGVN and NGHBP at Demand Level 1.20

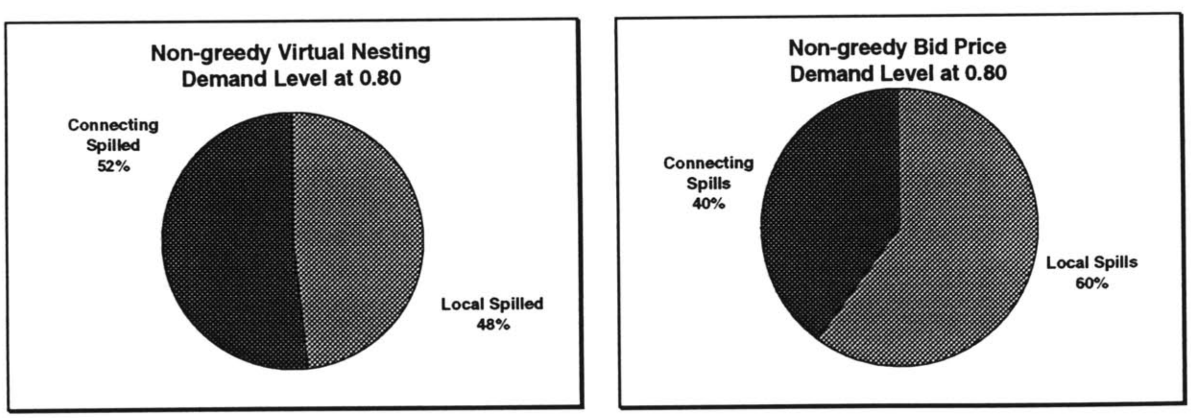


Figure 4.23 Comparison of Percentage of Spills between NGVN and NGHBP at Demand Level 0.80

We find that less connecting passengers are spilled in the NGHBP method at both demand levels 1.20 and 0.80. The difference is especially obvious at demand level 0.80. We know that even though both the NGVN and the NGHBP methods have the strategy of favoring the local passengers when demand is high (using shadow prices to decide the displacement impacts of the connecting passengers), the NGHBP method can adjust such controls more dynamically than the NGVN method can. We know that sometimes, the shadow price may not exactly reflect the displacement impacts of the connecting passengers due to the deviation of the demand. Once the LP is solved and the shadow prices are decided, the NGVN cannot make any adjustments anymore. On the other hand, the NGHBP method still uses more current EMSR values to adjust seat availability within a small range. That is why the two methods come to different booking decisions.

c. Some Issues in the Network Methods

1) Run LP at Different Demand Levels

The question of which demand level we should run the LP is a common issue for both the NGVN method and the NGHBP methods. As stated in Chapter 3, the RHS of the demand constraints in the LP model are the mean forecast demands. Those numbers may not be accurate in the real world. So one of the tasks of this study is to find how using a wrong demand forecast will affect the whole network revenue performance compared to using a correct demand forecast. The different scenarios we have chosen to compare are running the LP at a lower demand level (0.80), at a medium demand level (1.00), and at a higher demand level (1.20). Figure 4.24 shows such a comparison in the NGVN method.

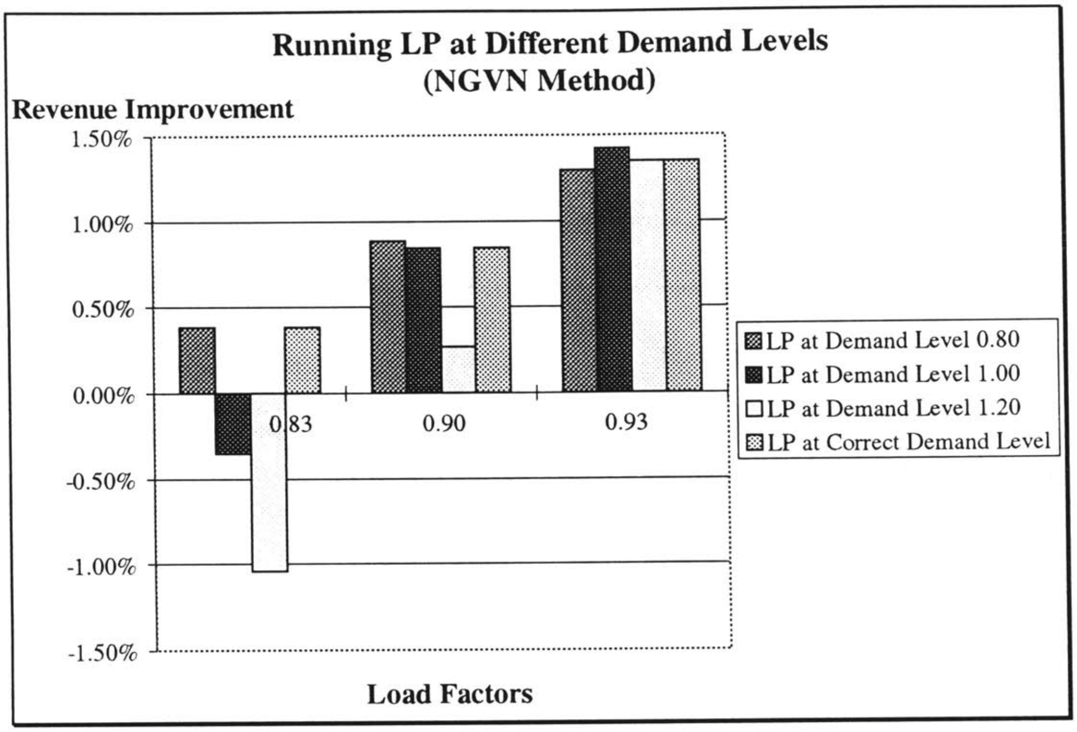


Figure 4.24 Running LP at Different Demand Levels (NGVN Method)

The base for the comparison of revenue improvement is the LBFC (EMSRb). We find that running LP at a high demand level gives us very good revenue achievement on the high demand side, while very poor performance on the low demand side (even worse than the LBFC method). On the other hand, running LP at a lower demand level we can achieve a higher revenue on the low demand side, but it does not give a revenue improvement comparable to running LP at high demand levels. It may be worth pointing out that the best revenue performance at each demand level is not running LP at the correct demand levels, but at a relatively lower demand levels. For example, the best revenue performance at demand level 1.20 occurs when we run LP at demand level 1.00, and the best revenue performance at demand level 1.00 occurs when we run LP at demand level 0.80, and so forth. This phenomenon tells us that running LP at a correct demand level is, sometimes, too strict for the connecting passengers. If we can run LP at a slightly lower demand level than the actual demand level, we will have an even better revenue performance.
Figure 4.24 shows the revenue performance of running LP at different demand levels in the non-greedy bid price method.

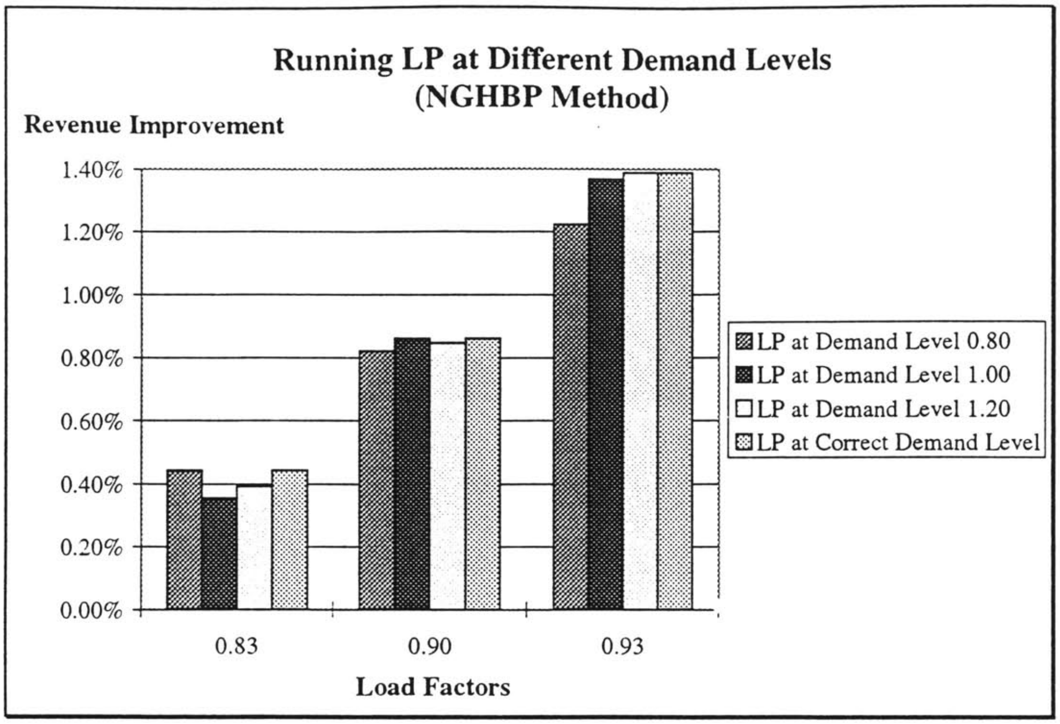


Figure 4.25 Running LP at Different Demand Levels (NGHBP Method)

From the figure above we notice that since the NGHBP introduced a heuristic factor in its formula, it is more robust for the deviation of the demand. The greatest advantage here is that running LP at a higher demand level no longer has the negative revenue impacts on the low demand side, and running LP at a low demand level can provide a more comparable revenue performance on the high demand side. The heuristic factor provides a very good reimbursement for the mistakes from the LP.

Note: We conduct the sensitivity studies for each combination. Therefore the heuristic parameter "d" is different for each bar in the figure above.

2) Multiple LP Runs vs. Single LP Run

Another argument about the linear program is that if we run LP only once at the beginning of the first booking period, is it accurate enough to capture the displacement impact of the connecting passengers? We know that the shadow prices change if the demand and capacity in the network vary. Therefore we need to find out how the multiple LP (once at each check point) can improve the network revenue. The reason we care about this problem is that to run a multiple LP, we will need more information about demand and more computer calculation time, which are very costly. The figure below is the comparison of revenue performance between the multiple LP running and the single LP running.

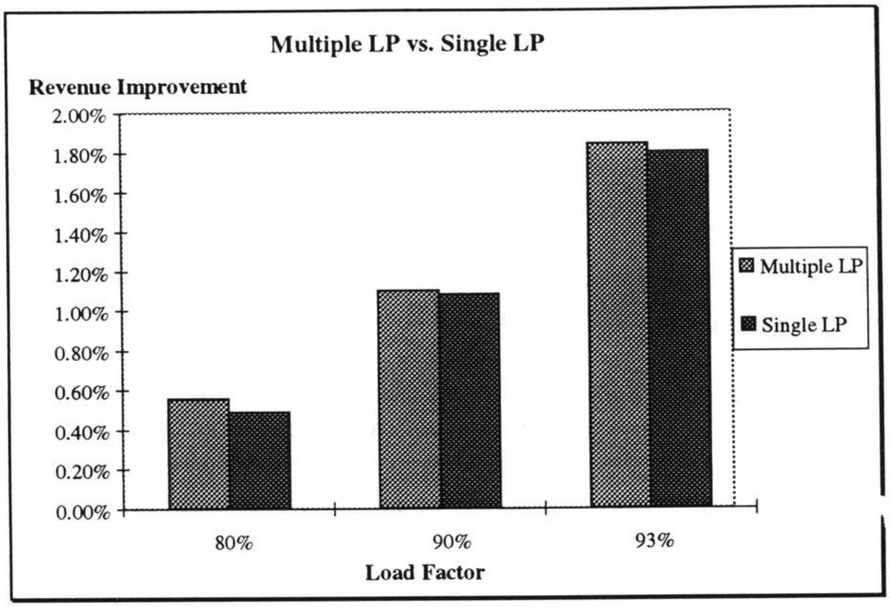


Figure 4.26 Multiple LP vs. Single LP

We find that the multiple LP actually gives similar network revenue contributions to the single LP. As stated in Chapter 3, this fact is due to the great deviation of the demand. However, it is no doubt good news because we can achieve a good revenue improvement with less information and at a lower cost by running LP only once at the beginning of the first booking period.

3) <u>Shadow Prices</u>

As mentioned in Chapter 3, one problem of the previous LP is that it is a degenerate problem. So the dual problem has multiple solutions. The solution obtained from the Cplex and OSL is the first solution generated in the simplex tableau process. The dual variables then may not reflect the true shadow prices. The task of this study is to find a set of better shadow prices, which can evaluate the displacement impacts of the connecting passengers more accurate. In Table 4.34 below, we listed the network performance using five different kinds of shadow prices. The seat control method we employ is simply the non-greedy virtual nesting method, and those shadow prices are all calculated at demand level 1.20.

	First Period SP	0.8*First Period SP	True SP(Cap+1)	True SP(Cap-1)	Upper SP	New UP SPs
Rev.	4647918	4648807	4647712	4647727	4648517	4650493
Improv	**1.35%**	**1.37%**	**1.35%**	**1.35%**	**1.36%**	**1.41%**
Loc Spill	1669	1773	1660	1660	1670	1683
Con Spill	655	607	663	663	659	648

Table 4.34 Shadow Price Studies

The first set of shadow prices is the dual variables calculated from running LP at the first booking period.

The second shadow prices are calculated from multiplying a parameter 0.8 by the first set of shadow prices (in an attempt to arbitrarily reduce the shadow prices).

The third one, the so-called "True Shadow Prices", is obtained from the definition of the shadow prices. That is, the shadow price associated with a capacity constraint in the linear programming problem is the amount that the optimal objective value is improved by increasing the right-hand side of the corresponding capacity constraint for one unit (assuming that the current basis remains optimal).[8]

The fourth one, also called "True Shadow Prices," is similar to the third one. The only difference is that the shadow prices are calculated from decreasing the right-hand side of the capacity constraint by one unit.

The fifth one, which is named "Upper Bound Shadow Prices," is calculated from the upper bound running of the network. The process of obtaining the upper bound is referred to in Figure 3.5.

The sixth shadow prices are from the revision of the upper bound shadow prices. We notice that the values of the upper bound shadow prices follow the Normal Distribution (refer to the fifth part in Section 3.1.3). Therefore, by using the value of ($Mean - \sigma$) as the value of the shadow price (instead of the *Mean*), we reduce the opportunities that the true shadow prices are lower than the value we employ to 30%. This strategy gives us the best revenue performance among the five sets of shadow prices.

In Appendix 3, we list the values of these six different kinds of shadow prices. We find:

- 0.8 * First Period SP $\leq$ First Period SP
- True SP (Cap+1) $\leq$ First Period SP $\leq$ True SP (Cap-1).
- Since the New Upper SP is calculated as ($Mean - \sigma$) , so it always has a lower value than the Upper SP.
- The First Period SP is generally greater than the Upper SP.
- Generally, 0.8 * First Period SP $\leq$ Upper Bound SP (with only one exception: Leg 17).

[8] W.L.Winston, "Operations Research, Applications and Algorithms", Third Edition, Duxbury Press, 1994.

From Table 4.34, we find: First, under the same control method and demand environment, the network revenue performance can be improved by up to 0.05% by simply employing better shadow prices. Second, the degenerate dual solution has a revenue performance comparable to other good shadow prices. Third, in the NGVN method, it is better to employ a lower set of shadow prices than a higher one, all else being equal.

2. Leg Based EMSR Non-greedy Bid Price Model (LNGBP)

As introduced in Section 3.1.2 (Part 2), this method uses a critical EMSR value (these values are calculated based on the total fares) as the measurement of the displacement impacts of the connecting passengers. Then a pseudo fare, which is defined as fare minus the critical EMSR value, is calculated. The critical EMSR value is then re-calculated based on these pseudo fares inputs.

a. Network Performance

The following two tables list the network and the leg performances of the LNGBP method.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3597702	4171245	4637248
Local Pax Spilled	208	790	1618
Connecting Pax Spilled	133	390	677
Avg. Leg Load Factor(%)	82.67	90.15	93.11
Avg. Rev. Per Pax($/Pax)	689.98	724.07	770.42
Avg. Rev. Per Avail. Seat($/Seat)	511.76	593.35	659.64
Rev. Imprv. over EMSRa	**0.26%**	**0.74%**	**1.12%**

Table 4.35 Revenue Performance of the LNGBP method

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.96	174	275	102	67.68	135
Leg 30	142	.75	107	111	4	85.40	95

Table 4.36 Leg Performance of the LNGBP Method

In the following, we will compare these performances with those from the NGHBP and the GHBP methods.

b. Comparison with the NGHBP Method

1) Measurement of the Displacement Impacts

In the LNGBP model, the critical EMSR values are implemented to evaluate the displacement effects of the connecting passengers. Since the EMSR values are (or should be) different from the shadow prices, the pseudo fares should also be different from those from the NGHBP (network non-greedy heuristic bid price). The following table shows the comparison of the shadow prices and the EMSR values on some legs. The detailed comparisons are listed in Appendix 4.

Leg Number	Shadow Prices	Critical EMSR Values	Difference
2	0.00	0.00	0.00
3	61.00	61.00	0.00
4	61.00	179.24	118.24
10	729.00	685.06	-43.94
30	0.00	12.40	12.40

Table 4.37 Comparison between the Shadow Prices and the Critical EMSR Values

We notice that it is very hard to say which one (SP or EMSR) is strictly greater than the other one. However, 16 (out of 32) legs have higher shadow price values; 5 legs have higher EMSR values; and 11 legs have identical shadow prices and EMSR values. If we consider $10 as a tolerable difference, then 16 legs (50%) have similar shadow prices and critical EMSR values.

Another fact we notice is that almost all legs that have identical shadow prices and EMSR values have very high load factors, which is above 96%. The only exception is Leg 2, which has an identical shadow prices with a load factor of 46%. All legs that have higher EMSR values than shadow prices have very high load factors (above 92%), and low percentage of local demand.

2) Virtual Class Range

Since in the probability method we exploit a different way to measure the displacement impact of the connecting passengers (critical EMSR value on each leg), and we know from the previous section that the shadow prices and the EMSR have different values on some legs, we can expect different virtual class ranges from these two different methods. In the following two tables, we list the virtual class ranges obtained from the network method (LP) and probability method (EMSR) on Leg 4 and Leg 30.

VRANGE on LEG 4 From LP Method	VRANGE on LEG 4 From EMSR Method
890.0	**890.0**
776.0	742.0
638.0	539.8
565.0	531.0
304.0	**304.0**
152.0	147.8
73.0	102.0
61.0	**61.0**
.0	**.0**
.0	**.0**

Table 4.38 Comparison of Virtual Class Ranges between NGHBP and LNGBP on Leg 4

VRANGE on LEG 30 from LP Method	VRANGE on LEG 30 from EMSR Method
552.0	526.8
448.0	518.0
420.0	401.5
358.0	**358.0**
251.0	286.0
246.0	251.0
230.0	**230.0**
194.0	**194.0**
72.0	**72.0**
.0	**.0**

Table 4.39 Comparison of Virtual Class Ranges between NGHBP and LNGBP on Leg 30

We notice that those virtual class ranges are very close to each other. Some of them are identical (bold numbers). However, on a high congested leg, such as Leg 4, the VRANGEs from the EMSR method tend to be lower than those from the LP method. On the other hand, on a less congested leg, there are more opportunities that the VRANGEs obtained from the EMSR method are higher than those obtained from the LP method.

3) Network Revenue Performance

Tables 4.40 and 4.41 show the comparison of the network performance between the NGHBP and the LNGBP method at different demand levels (1.20 and 0.80).

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGHBP	4649692	1.39%	93.02%	1698	638	777.87	661.41
LNGBP	4637248	1.12 %	93.11%	1618	677	770.42	659.64

Table 4.40 Comparison of Network Performance Between NGHBP and LNGBP at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGHBP	3604315	0.44%	82.61%	206	135	691.17	512.70
LNGBP	3597702	0.26%	82.67%	208	133	689.98	511.76

Table 4.41 Comparison of Network Performance Between NGHBP and LNGBP at Demand Level 0.80

The network revenue performances of the LNGBP are not as good as those of the NGHBP method both when demand is low and when demand is high. This is because the LNGBP uses the critical EMSR values as the measurements of the displacement impacts of the connecting passengers, and it is not a network optimal solution. We observe that when demand is high, the LNGBP method spills more connecting passengers than the NGHBP method, and when demand is low, the LNGBP method spills less connecting passengers than the NGHBP method.

4) Sensitivity Study

The following figure shows the sensitivity study of the heuristic parameter in the bid price formula.

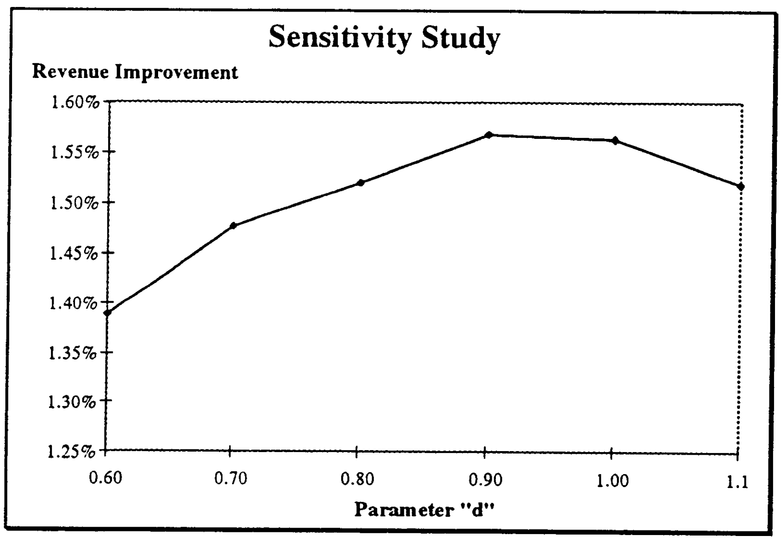


Figure 4.27 Sensitivity Study for LNGBP at Demand Level 1.20

The best value is 0.90, which differs from the value we obtained from the NGHBP method (0.60). This fact can be explained as follows: The critical EMSR values are normally greater than the values of shadow prices obtained from the optimization LP model. Therefore, the pseudo fares calculated from the LNGBP are smaller than those from the NGHBP model.

So the re-calculated critical EMSR values (using the pseudo fares as the inputs) obtained from the LNGBP, which will be used to determine the bid prices, are also smaller than those obtained from the NGHBP method. To achieve the same level of control of the booking of connecting passengers, the LNGBP will need a larger adjustment parameter.

Also we notice that in this method, the revenue is more sensitive to the parameter than it is in the NGHBP method.

c. Comparison with the GHBP Method

In Tables 4.42 and 4.43, we list the comparison of the revenue performance between the GHBP and the LNGBP methods at demand levels 1.20 and 0.80.

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
GHBP	4624277	0.83%	92.03%	1839	603	787.57	657.79
LNGBP	4637248	1.12%	93.11%	1618	677	770.42	659.64

Table 4.42 Comparison of Network Performance Between GHBP and LNGBP at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
GHBP	3595406	0.19%	81.90%	233	147	694.71	511.44
LNGBP	3597702	0.26%	82.67%	208	133	689.98	511.76

Table 4.43 Comparison of Network Performance Between GHBP and LNGBP at Demand Level 0.80

The LNGBP method performs much better in revenue than the GHBP method does, since it takes into account the displacement impacts of the connecting passengers. The critical EMSR values reflect the congestion condition of the flight legs: it has a higher value at high demand levels and a lower value at low demand levels.

d. Conclusions

The network revenue performance of the LNGBP method cannot compete with that of the NGHBP method both when demand is high and when demand is low. The reason is obviously due to the measurement of the displacement impacts of the connecting passengers. The LNGBP method employs the critical EMSR value as the estimation of the displacement effects of the connecting passengers, and it is a leg based method--it does not reflect the optimal network solution. This model still has some of the characteristics of the greedy method: the critical EMSR values are obtained based on the total fares. However this model provides us with a much better revenue performance than GHBP method. One fact that we realize is that this method needs more information than the GHBP method does since it needs to re-calculate the critical EMSR values based on the pseudo fares of each ODF.

This model suggests two directions we can go:

- Utilize as much information as the Greedy Heuristic Bid Price method does; that is, after the calculation of the pseudo fares, we only re-arrange the fares to new virtual classes based on the old virtual class ranges (do not re-calculate the EMSR curve). Then employ the EMSRb method to control the seats. The revenue performance of such a method is shown in the following table.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3597262	4167447	4629500
Local Pax Spilled	228	736	1563
Connecting Pax Spilled	115	418	707
Avg. Leg Load Factor(%)	82.91	90.12	93.05
Avg. Rev. Per Pax($/Pax)	690.21	720.14	766.03
Avg. Rev. Per Avail. Seat($/Seat)	511.70	592.81	658.53
Rev. Imprv. over LBFC	**0.25%**	**0.65%**	**0.95%**

Table 4.44 Revenue Performance of the Revised LNGBP Method

Such a method does not provide as good a performance at high demand levels as the LNGBP method, but it does provide very close performance at low demand levels. Compared to the GHBP and the GVN methods, this method has obvious advantages.

- Try to overcome the greedy respect by doing the following convergence process:

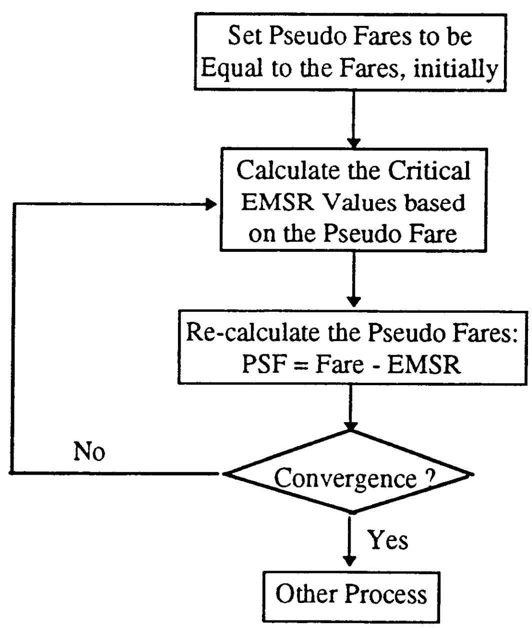


Figure 4.28 A Loop to Find the Converged Pseudo Fares Based on Critical EMSR Values

This is a by-product of the LNGBP model, we call it Convergent Pseudo Fare model (CONPSF). The convergence criterion can be referred to Equation [3.27] in Chapter 3. The following are some performance results of this model.

e. Convergent Pseudo Fare Model (CONPSF)

As mentioned in the previous section we can make a loop as shown in Figure 4.28 to find the best pseudo fares based on the critical EMSR values. At demand level 1.20, it takes about 10 seconds to find the convergence (18 iterations). The converged EMSR values are shown in Appendix 5, together with the shadow prices and EMSR values from the LNGBP method.

1) Virtual Class Ranges

Since the converged pseudo fares differ from the pseudo fares we obtained in the LNGBP method, we can expect the difference in the virtual class ranges (VRANGEs) between the LNGBP method and the CONPSF method. Tables 4.45 and 4.46 show the comparisons of the VRANGEs among the LP method, the LNGBP method, and the CONPSF method on Leg 4 and Leg 30.

VRANGEs on LEG 4 From LP Method	VRANGEs on LEG 4 From EMSR Method	VRANGEs on LEG 4 From Convergence Method
890.0	**890.0**	**890.0**
776.0	742.0	765.8
638.0	539.8	646.2
565.0	531.0	554.8
304.0	**304.0**	**304.0**
152.0	147.8	**152.0**
73.0	102.0	62.8
61.0	**61.0**	**61.0**
.0	**.0**	**.0**
.0	**.0**	**.0**

Table 4.45 Comparison of Virtual Class Ranges among NGHBP, LNGBP, and CONPSF on Leg 4

VRANGEs on LEG 30 From LP Method	VRANGEs on LEG 30 From EMSR Method	VRANGEs on LEG 30 From Convergence Method
552.0	526.8	541.8
448.0	518.0	419.9
420.0	401.5	392.1
358.0	**358.0**	**358.0**
251.0	286.0	254.2
246.0	251.0	251.0
230.0	**230.0**	**230.0**
194.0	**194.0**	**194.0**
72.0	**72.0**	**72.0**
.0	**.0**	**.0**

Table 4.46 Comparison of Virtual Class Ranges among NGHBP, LNGBP, and CONPSF on Leg 30

The bold number in the above two tables means that the VRANGEs are the same as that from the LP method. We find that the CONPSF method has more identical VRANGEs than the LNGBP method. Also for those non-identical values, the VRANGEs from the CONPSF method are closer to those from the LP method than those from the LNGBP method.

b) Revenue Performance

Tables 4.47 and 4.48 show the revenue and leg performance of the CONPSF method.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3600050	4172507	4647204
Local Pax Spilled	213	782	1660
Connecting Pax Spilled	139	404	662
Avg. Leg Load Factor(%)	82.38	89.90	92.84
Avg. Rev. Per Pax($/Pax)	691.82	724.97	775.78
Avg. Rev. Per Avail. Seat($/Seat)	512.10	593.53	661.05
Rev. Imprv. over EMSRa	**0.32%**	**0.77%**	**1.33%**

Table 4.47 Revenue Performance of the CONPSF Method

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.96	173	275	102	67.68	135
Leg 30	142	.73	103	111	7	85.40	95

Table 4.48 Leg Performance of the CONPSF Method

The revenue performance of the CONPSF method is obviously improved compared with that of the LNGBP method (Table 4.49 and 4.50).

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
LNGBP	4637248	1.12%	93.11%	1618	677	770.42	659.64
CONPSF	4647204	1.33%	92.84%	1660	662	775.78	661.05

Table 4.49 Comparison of Network Performance Between LNGBP and CONPSF at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
LNGBP	3597702	0.26%	82.67%	208	133	689.98	511.76
CONPSF	3600050	0.32%	82.38%	213	139	691.82	512.10

Table 4.50 Comparison of Network Performance Between LNGBP and CONPSF at Demand Level 0.80

The load factor of the CONPSF method is lower than that of the LNGBP method both at low demand levels and high demand levels. However, both the revenue per passenger and the revenue per seat from the CONPSF method are higher that those from the LNGBP method. At high demand levels, the CONPSF method spills more local and less connecting passengers than the LNGBP method does. At the low demand levels, the CONPSF method spills both more local and connecting passengers compared to the LNGBP method.

The revenue performance of the CONPSF method is comparable to (slightly worse than) that of the NGHBP method (Table 4.51 and 4.52).

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGHBP	4649692	1.39%	93.02%	1698	638	777.87	661.41
CONPSF	4647204	1.33%	92.84%	1660	662	775.78	661.05

Table 4.51 Comparison of Network Performance Between NGHBP and CONPSF at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGHBP	3604315	0.44%	82.61%	206	135	691.17	512.70
CONPSF	3600050	0.32%	82.38%	213	139	691.82	512.10

Table 4.52 Comparison of Network Performance Between NGHBP and CONPSF at Demand Level 0.80

We notice that the CONPSF method is more strict for connecting passengers than the NGHBP is, and therefore it has a lower load factor than the NGHBP method.

3) Sensitivity Studies

Finally, Figure 4.29 shows the sensitivity study of the parameter in the bid price calculation of CONPSF method.

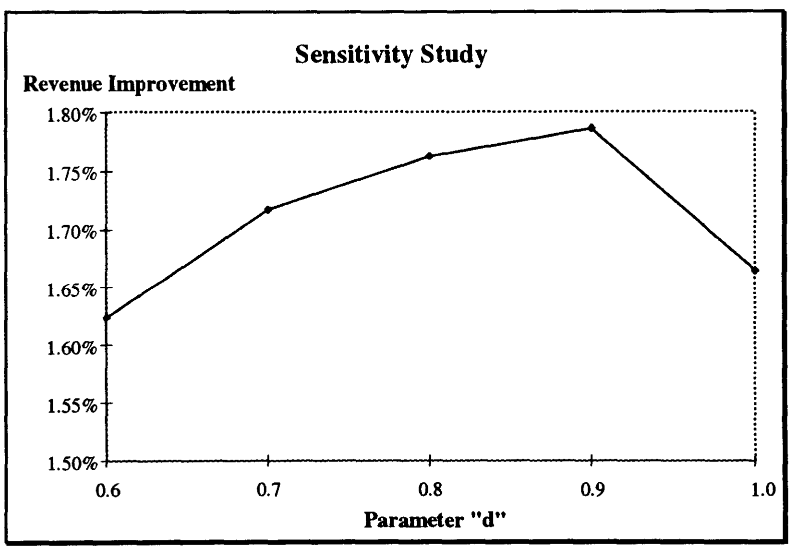


Figure 4.29 Sensitivity Studies in CONPSF at Demand Level 1.20

From the sensitivity study, we obtain the same parameter value (0.9) as what we gains in the LNGBP method.

3. Convergent Proration Model (CONPRT)

In Chapter 3, we discussed how the connecting fares can be optimally distributed to the legs traversed by employing the convergence concepts (refer to the second part of Section 3.2.2). In Figure 3.6, we proposed a convergence process, which can be summarized as: 1) Based on the critical EMSR values on each leg, prorate the connecting fares to each leg; 2) Re-rank the fares according to these prorated fares; 3) Re-calculate the critical EMSR values based on these new virtual classes and prorated fares; 4) Check the convergence criterion. If convergence, then stop. Otherwise go to step 1. For detailed description, please refer to Section 3.2.

For our simulated network, we can easily find the convergence within 10 seconds (21 convergence iterations). Then virtual EMSRb method (using the prorated fares and the new virtual classes) is employed to control the seats.

Figure 4.30 shows the convergence processing of prorating the Fare Class 1 of OD pair 7 to the two legs traversed.

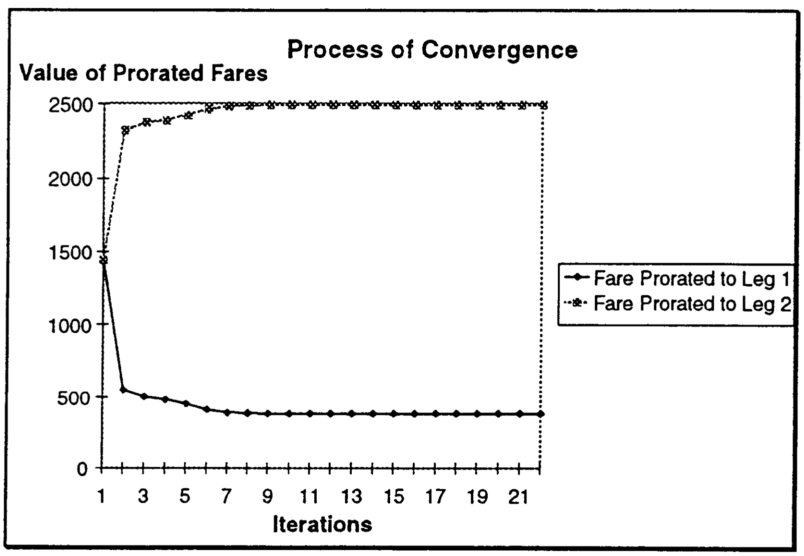


Figure 4.30 Convergence Process of Prorating a 2-Leg Fare (O-D Pair 7, Class 1) using Half Fare to Start

We find that the fare has been converged smoothly and quickly. The critical EMSR values from this method are listed in Appendix 6, together with the shadow prices from LP model and the EMSR values from the LNGBP and CONPRT methods.

a. Network Revenue Performance

Tables 4.53 and 4.54 show the revenue and leg performance of the CONPRT method.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3602298	4173740	4644965
Local Pax Spilled	161	747	1636
Connecting Pax Spilled	165	423	673
Avg. Leg Load Factor(%)	82.47	89.90	93.00
Avg. Rev. Per Pax($/Pax)	688.83	723.09	773.61
Avg. Rev. Per Avail. Seat($/Seat)	512.42	593.70	660.73
Rev. Imprv. over LBFC	**0.39%**	**0.80%**	**1.29%**

Table 4.53 Revenue Performance of the CONPRT Method

	CAP.	LF	PAX Load	Demand	Spill	%Local D	Local SLD
Leg 4	181	.96	174	275	101	67.68	135
Leg 30	142	.72	103	111	8	85.40	95

Table 4.54 Leg Performance of the CONPRT Method

b. Starting Points of the Convergence

Remember that in the convergence processing in Figure 3.6, the first step is "prorate the connecting fares for legs traversed somehow". In the process above, we choose to use the average strategy, which means that we prorated the connecting fare equally to all the legs traversed in the first iteration. Therefore questions arise: is the final convergence stable if we choose to employ different initial prorating points? We examine, in the following, the scheme of starting with the total fares, that is, connecting passengers are evaluated as their total fares on all the legs they traverse initially. The convergence process of the same OD fare (Fare Class 1, OD pair 7) is shown in the following figure:

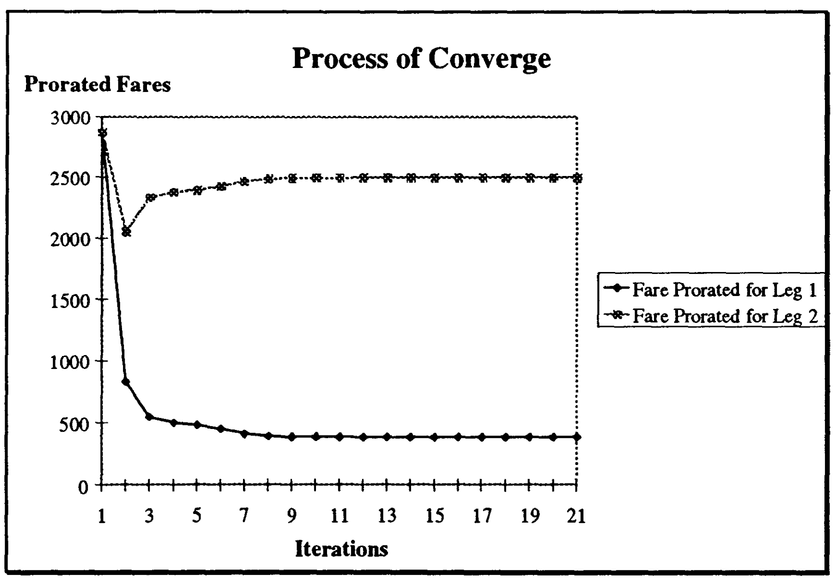


Figure 4.31 Converge Process of Prorating a 2-Leg Fare (O-D Pair 7, Class 1) using Total Fare to Start

Compared to the previous results, they are very close to each other. Additionally, the revenue performance of using the average prorated fares as initial input is similar to that of using total fares as initial input. Table 4.54 shows the comparison.

Demand Adjustment	Half of the Fares	Total Fares
Total Revenue	4644965	4644962
Local Pax Spilled	1636	1636
Connecting Pax Spilled	673	673
Avg. Leg Load Factor(%)	93.00	92.99
Avg. Rev. Per Pax(S/Pax)	773.61	773.63
Avg. Rev. Per Avail. Seat($/Seat)	660.73	660.73
Rev. Imprv. over LBFC	**1.29%**	**1.29%**

Table 4.55 Comparison between the CONPRT methods with Different Starting Points

The revenue performance of these two convergence schemes are almost identical. Therefore it shows that the process of the convergence is not necessarily dependent on the starting point.

c. Comparison between the CONPSF method and the CONPRT method

Tables 4.56 and 4.57 list the revenue performances of the CONPSF and the CONPRT methods at demand levels 1.20 and 0.80.

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
CONPSF	4647204	1.33%	92.84%	1660	662	775.78	661.05
CONPRT	4644965	1.29%	93.00%	1636	673	773.61	660.73

Table 4.56 Network Performance Comparison Between CONPSF and CONPRT at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
CONPSF	3600050	0.32%	82.38%	213	139	691.82	512.10
CONPRT	3602298	0.39%	82.47%	161	165	688.83	512.42

Table 4.57 Network Performance Comparison Between CONPSF and CONPRT at Demand Level 0.80

The CONPRT method tends to perform better at low demand levels, while the CONPSF method tends to perform better on the high demand side. If we pay attention to the spills, we find that the CONPRT method rejects more connecting passengers and less local passengers than the CONPSF method. The reason can be explained as: The strategy of the CONPRT method is to prorate the connecting fares to the legs traversed, therefore the total network contribution of the connecting fares are equal to their total itinerary fares. Such an idea may lead the connecting passengers to having a very low ability to compete with the connecting passengers when demand is very high.

d. Comparison between the NGHBP method and the CONPRT method

Tables 4.58 and 4.59 list the revenue performances of the CONPSF and the CONPRT methods at demand levels 1.20 and 0.80.

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGHBP	4649692	1.39%	93.02%	1698	638	777.87	661.41
CONPRT	4644965	1.29%	93.00%	1636	673	773.61	660.73

Table 4.58 Network Performance Comparison Between NGHBP and CONPRT at Demand Level 1.20

	Revenue	Improvement	LF	Loc. Spilled	Con. Spilled	Rev./Pax	Rev./Seat
NGHBP	3604315	0.44%	82.61%	206	135	691.17	512.70
CONPRT	360[illegible]298	0.39%	82.47%	161	165	688.83	512.42

Table 4.59 Network Performance Comparison Between NGHBP and CONPRT at Demand Level 0.80

Even though the CONPRT method provides a slightly worse revenue performance than the NGHBP method, it does not contain any heuristic factors. Therefore we do not need to conduct the sensitivity studies in this method, and it should be more robust than the NGHBP method.

4.4 Conclusions

1. General Discoveries

Using an integrated optimization/booking simulation tool, we implement six different revenue management methods to the same simulation network under controlled demand assumptions. Figure 4.32 shows the revenue improvement comparison among those methods (LBFC EMSRb as base).

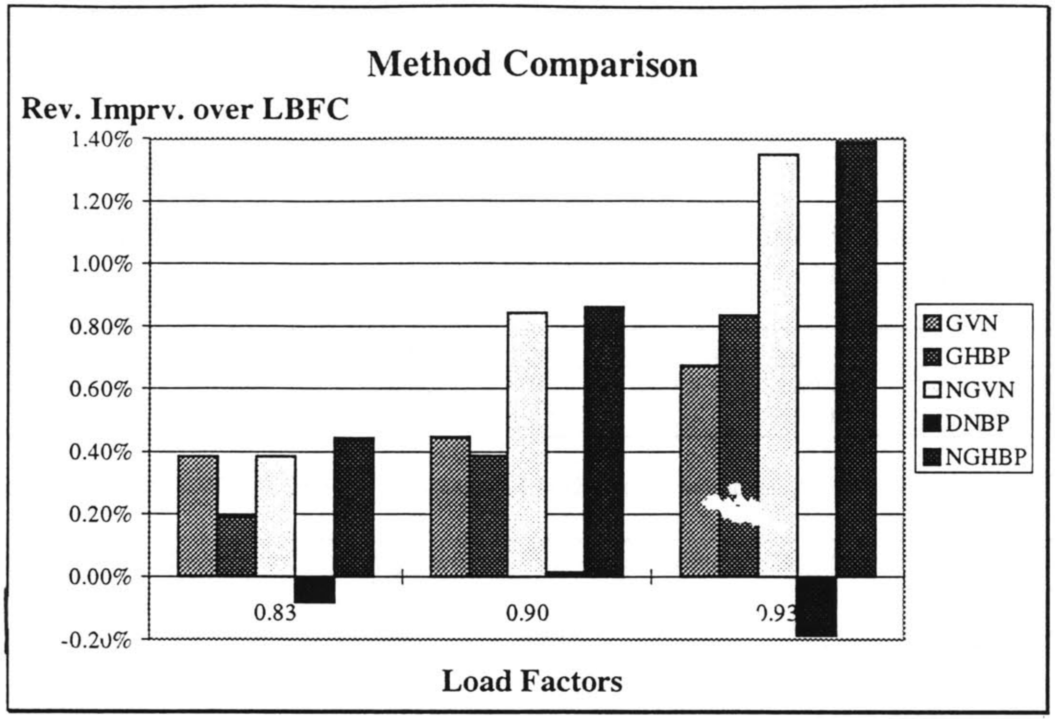


Figure 4.32 Method Comparison

From the figure above, we find that the non-greedy methods generally perform better than the greedy methods. This is due to the consideration of the displacement impacts of the connecting passengers. However the price for the good performance of the non-greedy methods is more information required about the demand. Therefore the non-greedy methods are harder to implement than the greedy methods.

2. Different Network Example

We should mention here that the comparison given in Figure 4.32 is based on the example network we employed. If some assumptions about the network characteristics are changed, the relative network performance may also change. Figure 4.33 shows the method comparison of those six different methods in another network. The base case for the comparison is the LBFC (leg based fare class) method using EMSRb control.

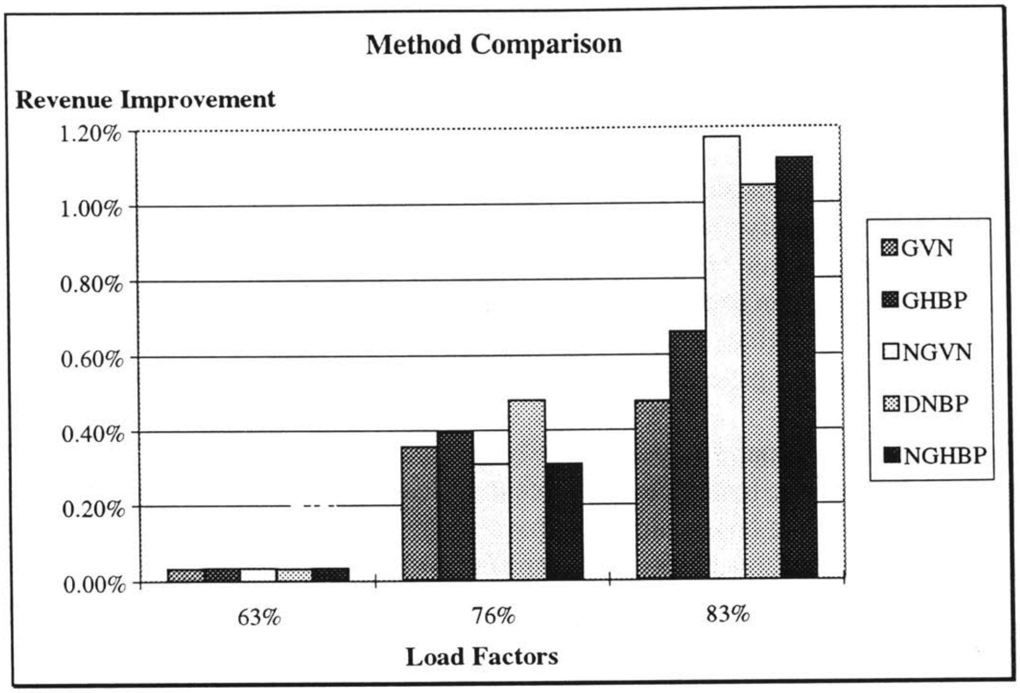


Figure 4.33 Method Comparison in Another Network

Some important characters of this network are:

- 102 flight legs.
- 1066 origin and destination pairs.
- 7 fare classes on each O-D pair, 10 virtual classes are implemented.
- Mix of long-haul and short-haul flights.
- Average 60% local traffic on each leg.
- Booking classes are defined by fare type.
- 18 booking periods.
- A trip with up to 4 legs is allowed.

The demand data of the network is shown in the following table:

Demand Adjustment	0.80	1.00	1.20
Total Demand	5406	6762	8088
Local Demand	3712	4641	5559
Connecting Demand	1694	2121	2529
Average Leg Load Factor	63.08%	75.67%	82.77%

Table 4.60 Demand Scenarios

We may notice that the network deterministic bid price method (DNBP) provides a very good performance in this network while it does not in the previous network. The possible reason may be that there are 18 booking periods in this network, and the demand deviation from period to period are smaller than the previous network.

The non-greedy methods consistently result in better revenue performances. Here, for this network, in all the non-greedy methods, we have run the LP at the correct demand level. That is, we have supposed that we can obtain accurate demand forecasts. One result in this network that is different from the previous network is that the NGHBP method gives slightly less revenue improvements than the NGVN method (remember that in the previous network, the NGHBP method always gave better performances then the NGVN method).

Also we find that at low demand side, the greedy methods tend to give better performance. This can be explained by the load factors. Since this network is relatively less congested. the strategy of favoring the long haul connecting passengers tends to work better. The problem of the non-greedy method comes from the deterministic LP model. Sometimes the shadow prices do not reflect the accurate displacement impacts of the network. Therefore connecting passengers are rejected by mistake.

The different network performance of the seat inventory control methods for two different networks suggests how important it is to understand the characteristics of the network before choosing the seat control method.

3. Implementation Concerns

As we mentioned earlier in Chapter 1, for a seat inventory control method to be implemented in the real world, it must meet certain requirements, such as being easy to employ, and the information required by the method should be accessible. Among the above compared methods, the non-greedy methods (NGVN, NGHBP, and CONPSF CONPRT) normally require more information support than the greedy methods, and therefore are more costly. During the decision making process, such factors should be taken into account.

4. Upper Bound

Finally, we use the process in Figure 3.3 to calculate the upper bound revenue for both networks.

Average Load Factor	83%	90%	93%
Network Revenue	3628841	4217629	4695782
Difference from LBFC	**1.13%**	**1.86%**	**2.39%**

Table 4.61 Upper Bound for the Smaller Network

Average Load Factor	63%	75%	83%
Network Revenue	3078862	3795041	4333306
Difference from LBFC	**0.14%**	**1.50%**	**3.03%**

Table 4.62 Upper Bound for the Larger Network

The theoretical upper bound of the revenue improvement provides us with a reference for maximum revenue potential. From the two tables above we find that the improvement opportunities are very small, and generally, the lower the load factor the less revenue opportunities. We therefore obtain the following relationship between the upper bound and different methods:

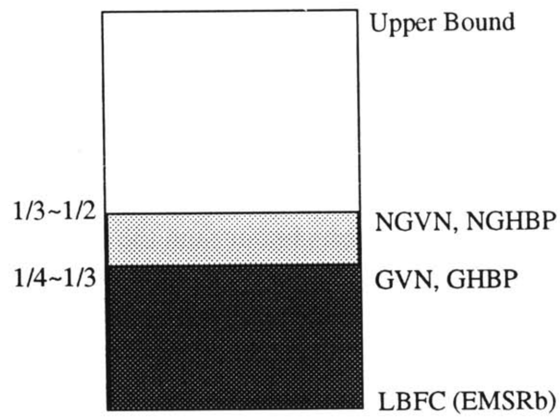


Figure 4.34 Upper Bound Interpretation

In our research, we find that the greedy methods (include GVN and GHBP) can provide a revenue improvement around 1/4 to 1/3 of the upper bound of the revenue gain. The non-greedy methods (include NGVN and NGHBP) can provide an improvement up to 1/3 to 1/2 of the upper bound. For example, in the smaller network, the NGHBP method can provide a revenue improvement up to 1.39% at the load factor of 93%, and 0.44% at an average network load factor of 83%, which are around the half of the upper bound of the revenue opportunities. On the other hand, in the larger network, the NGHBP method can provide a revenue improvement up to 1.12% at average load factor of 83% and with revenue improvement of 0.03% at load factor of 63% level, which is around one third of the revenue upper bound.

Chapter 5

Conclusion

5.1 Summary of Research Findings

In this thesis, we examine several seat inventory control methods. Using an integrated optimization / booking simulation tool, we compare how different methods can achieve different network revenues under the same demand circumstances.

The Greedy Virtual Nesting Method performs better than the Leg Based Fare Class Yield Management method, because the greedy method takes into account the fact that the total revenue of a connecting passenger from a low fare class is higher than the revenue of a local passenger from a high fare class. In this method, airlines re-rank the passengers to virtual classes based completely on the fare values. Therefore the seats are reserved for the high fare long haul connecting passengers. Such a strategy generally has higher load factors and revenues at high demand environments.

However under some conditions--at very high demands-- the greedy method tends to have negative revenue impact. When both flight legs have very high demands, airlines would rather book two local passengers than one connecting passenger because even though one connecting passenger has a high total fare, he/she takes two or more seats of the network, therefore displacement impact occurs. The non-greedy methods (including non-greedy virtual nesting, non-greedy heuristic bid price, etc.) take into account the displacement impacts, and evaluate the connecting passengers as their network contributions--fare minus the displacement impacts. With proper demand information, the non-greedy models always perform better than the greedy models. In our simulated network, the non-greedy method can provide up to a 0.5% revenue improvement over the best greedy method (the greedy heuristic bid price method) at an average load factor of 93%, and up to a 0.2% revenue improvement at an average load factor of 83%.

In the implementation we find that even though we would like the displacement impact to be considered, we do not want it to be over-emphasized. This is because high shadow prices may cause too many connecting passengers to be spilled and consequently result in empty seats (if there are not enough local demands). Therefore we find that it is always better to implement a slightly lower shadow price than a higher one.

The leg-based probability model, which uses the critical EMSR values as the measurement of the displacement impacts, even though it cannot compete with the network methods in terms of revenue gain, uses much less demand data and can achieve a better revenue performance than the greedy methods. This is a valuable characteristics in the airline industry because most airlines cannot provide OD data to support a network optimization process. A by-product of this probability model is a convergence model we called CONPSF model. The idea of this model is as follows: Since the critical EMSR values we used to evaluate the displacement impacts in the probability model are based on the total fares, they may be higher than the actual displacement effects. Therefore we naturally think that we can use the pseudo fares obtained from the probability model to re-calculate the critical EMSR values, and subsequently re-calculate the pseudo fares till the pseudo fares converge. Such a model improves the revenue performance of the probability model greatly.

The last model we proposed is called CONPRT model. It comes from the observation of that the sum of the pseudo fares from the above two models are not equal to the total fares. Therefore the contribution of the connecting fares are still mis-evaluated. Therefore we come up with an idea of using the convergence loop to optimally prorate the connecting fares to the legs traversed based on the critical EMSR value of each leg. Such a method provides us with a revenue performance similar to other methods.

5.2 Contributions

Based on different approaches of how to evaluate the displacement impacts of the connecting passengers, we proposed three new models: the Network Non-greedy Heuristic Bid Price model, the Leg-based Probability Bid Price model (with a by-product of the Convergent Pseudo Fare model), and the Convergent Proration model. All of them provide us with good revenue performances.

In our research, we found that even though the seat allocation results obtained from the deterministic LP model are partitioned solutions, and rarely employed in the real world, we can derive some very useful information about the displacement impacts from this model: the shadow prices. In the network non-greedy heuristic bid price model, we implement the shadow prices to obtain the pure network revenue contribution of the connecting passengers. Then, the critical EMSR values are calculated based on these pure contributions.

We also discuss some implementation issues of the shadow prices. For example, the degeneracy of the LP. Since the degenerate LP model provides multiple dual solutions, we need to consider how different dual solutions will affect the revenue. If it is possible, how can we avoid the degeneracy? Five different shadow prices are proposed to overcome the degeneracy problem: The heuristic model (d*dual variables), the upper bound, the upper bound deviated shadow price, the true shadow price (capacity +1), and the true shadow price (capacity -1). Another implementation issue is how frequently the airlines should update the LP results. We know that the inputs of the LP (demand and capacity) change over time. The frequency of re-solving an LP is limited by the technique and data difficulties. We compared the different updating plans and find that running LP once using the average demand forecasting and total capacity can provide a very good revenue performance. Finally, we consider how the accuracy of the demand forecasting will affect the revenue performance. Different demand right-hand sides are tested. The general findings are that, if we run LP at a too low demand level, the non-greedy method may perform like a greedy method and book too many connecting passengers; if the demand right-hand sides are too high, we may spill too many connecting passengers and cause empty seats. However, running LP at a slightly lower demand level provides a very good revenue performance.

We extend the definition of the pseudo fares as the pure fare contribution of the connecting passengers, and therefore give another form of pseudo fares: fare minus the critical EMSR values. In this model, we try to approach the network optimal solution through a leg-based seat inventory control approach, which represents a desirable research direction.

Even though the shadow prices and the critical EMSR values can evaluate the displacement impact of the connecting passengers, the sum of the pseudo fares (derived as fare minus the shadow prices or critical EMSR values) of the connecting passengers on all the traversed legs are not equal to the total itinerary fares (normally the sum of the pseudo fares are greater than the total fares). Therefore the connecting fares may not be properly allocated to each leg. The convergence model overcomes this fact by finding an optimal way to prorate the connecting fares on all legs traversed based on the critical EMSR value on each leg. This model can provide us with revenue results comparable to the network method.

5.3 Future Research Directions

By the end of this thesis, we can clearly see several directions for future research:

- Improve the LP model to derive more accurate displacement impacts. There are two ways to achieve this goal. One is to employ a stochastic LP model to capture the variable character of the demand. The other is to integrate the LP model by taking into account more information such as passenger recapture, path choice, cancellation, no-shows, and the passenger up-grades, etc.

- Obtain more accurate demand forecasts. Most of the non-greedy methods depend greatly on the accuracy of the demand forecast for each ODF. Today, most airlines are still doing forecasting on a leg base. Without the OD demand forecast, the LP model cannot by implemented and therefore we cannot obtain the shadow prices. One barrier in the OD forecast is the limits of the historical data, because to store these data requires huge data bases. Another problem is OD demands have very small mean values and very large deviations. One possible direction for the forecasting is to derive the OD flows from the link flows.

- Derive optimal "d" values theoretically. In the non-greedy heuristic model, the parameter "d" is obtained by sensitivity studies. Future theoretical works can try to derive the optimal value for this parameter.

- Decrease the dependency on demand information. Since forecasting the OD level demand is very difficult, another research direction could be how we can use as less information as a leg level to achieve a network optimal solution. We proposed a model in this thesis, and we can improve it in future research.

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Appendix 1

Calculation of EMSR Curve

On Leg 1:

• The EMSR Curve

The virtual classes of fares are given as below:

Virtual Classes	Fare Classes	Fare	Mean Demand	Std Error
Y1	Y(AC)	$1000	8	4
Y2	B(AC)	$625	25	10
Y3	Y(AB), B(AC)	$500	10	5
Y4	B(AB)	$300	20	8
Y5	Q(AC)	$250	45	17
Y6	Q(AB)	$150	40	15

Table 1 Virtual Classes on Leg 1

1. Protection for Y1:

Mean Demand = 8 [A1.1]
Standard Deviation = 4

We know that the seats should be protected for class Y1 as long as the following equation holds:

$$EMSR_{Y1}(\pi_{Y1}) = Fare_{Y1} \times \overline{P_{Y1}}(\pi_{Y1}) \geq Fare_{Y2} \quad \text{[A1.2]}$$

That is

$$1000 \times \overline{P_{Y1}}(\pi_{Y1}) \geq 625 \quad \text{[A1.3]}$$
$$\Rightarrow \overline{P_Y}(\pi_Y) \geq 0.625$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.625, \quad \text{[A1.4]}$$

is -0.32. Since our sample follows Normal Distribution -- N(8, 4), we can get that:

$$Z = \frac{\pi_Y - \mu}{\sigma} = -0.32 \quad \text{[A1.5]}$$
$$\pi_Y = 8 - 0.32 \times 4 = 6.7 \approx 7$$

2) Seats protected for Y1 and Y2 classes.

First, we need to calculate the mean fare of Y1 and Y2 together.

$$Fare_{Y12} = \frac{1000 \times 8 + 625 \times 25}{8 + 25} = 716.00 \quad \text{[A1.6]}$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1 and Y2 classes.

$$Mean\ Demand = 8 + 25 = 33 \quad \text{[A1.7]}$$
$$Standard\ Deviation = \sqrt{4^2 + 10^2} = 10.77$$

We know that the seats should be protected for Y1 and Y2 classes as long as the following equation holds:

$$EMSR_{Y12}(\pi_{Y12}) = Fare_{Y12} \times \overline{P_{Y12}}(\pi_{Y12}) \geq Fare_3 \quad \text{[A1.8]}$$

That is

$$716.00 \times \overline{P_{Y12}}(\pi_{Y12}) \geq 500 \quad \text{[A1.9]}$$
$$\Rightarrow \overline{P_{Y12}}(\pi_{Y12}) \geq 0.6983$$

From N(0,1) table, we can find Z satisfies

$$P(x>Z) = 0.6983 \quad \text{[A1.10]}$$

is -0.52. Since our sample follows N(43, 11.87), we can get that:

$$Z = \frac{\pi_{Y12} - \mu}{\sigma} = -0.52 \qquad [A1.11]$$

$$\pi_{Y12} = 33 - 0.52 \times 10.77 = 27.4 \approx 27$$

3) Seats protected for Y1, Y2 and Y3 classes.

First calculate the mean fare for Y1, Y2 and Y3.

$$Fare_{Y123} = \frac{1000 \times 8 + 625 \times 25 + 500 \times 10}{8 + 25 + 10} = 665.70 \qquad [A1.12]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1, Y2, and Y3 classes.

$$Mean\ Demand = 8 + 25 + 10 = 43 \qquad [A1.13]$$

$$Standard\ Deviation = \sqrt{4^2 + 10^2 + 5^2} = 11.87$$

We know the seats should be protected for Y1, Y2 and Y3 class as long as the following equation holds:

$$EMSR_{Y123}(\pi_{Y123}) = Fare_{Y123} \times \overline{P_{Y123}}(\pi_{Y123}) \geq Fare_4 \qquad [A1.14]$$

That is

$$665.70 \times \overline{P_{Y123}}(\pi_{Y123}) \geq 300 \qquad [A1.15]$$

$$\Rightarrow \overline{P_{Y123}}(\pi_{Y123}) \geq 0.4507$$

From N(0,1) table, we can find Z which satisfies

$$P(x>Z) = 0.4507, \qquad [A1.16]$$

is 0.12. Since our sample follows Normal Distribution -- N(63, 14.32), we can get that:

$$Z = \frac{\pi_{Y123} - \mu}{\sigma} = 0.12 \qquad [A1.17]$$

$$\pi_{Y123} = 43 + 0.12 \times 11.87 \approx 44$$

4) Seats protected for Y1, Y2, Y3 and Y4 classes.

First calculates the mean fare for Y1, Y2, Y3 and Y4.

$$Fare_{Y1234} = \frac{1000 \times 8 + 625 \times 25 + 500 \times 10 + 300 \times 20}{8 + 25 + 10 + 20} = 549.6 \qquad [A1.18]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1, Y2, and Y3 classes.

$$Mean\ Demand = 8 + 25 + 10 + 20 = 63 \qquad [A1.19]$$
$$Standard\ Deviation = \sqrt{4^2 + 10^2 + 5^2 + 8^2} = 14.32$$

We know the seats will be protected for Y1, Y2, Y3, and Y4 classes as long as the following equation holds:

$$EMSR_{Y1234}(\pi_{Y1234}) = Fare_{Y1234} \times \overline{P_{Y1234}}(\pi_{Y1234}) \geq Fare_5 \qquad [A1.20]$$

That is

$$549.6 \times \overline{P_{1234}}(\pi_{Y1234}) \geq 250 \qquad [A1.21]$$
$$\Rightarrow \overline{P_{Y1234}}(\pi_{Y1234}) \geq 0.4549$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.4549 \qquad [A1.22]$$

is 0.11. Since our sample follows N(108, 22.23), we can get that:

$$Z = \frac{\pi_{Y1234} - \mu}{\sigma} = 0.11 \qquad [A1.23]$$
$$\pi_{Y1234} = 63 + 0.11 \times 14.32 \approx 65$$

5) Seats protected for Y1, Y2, Y3, Y4 and Y5 classes.

First calculates the mean fare for Y1, Y2, Y3, Y4 and Y5.

$$Fare_{Y12345} = \frac{1000 \times 8 + 625 \times 25 + 500 \times 10 + 300 \times 20 + 250 \times 45}{8 + 25 + 10 + 20 + 45} = 424.8 \qquad [A1.24]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1, Y2, Y3, Y4 and Y5 classes.

$$Mean\ Demand = 8 + 25 + 10 + 20 + 45 = 108 \quad [A1.25]$$
$$Standard\ Deviation = \sqrt{4^2 + 10^2 + 5^2 + 8^2 + 17^2} = 22.23$$

We know the seats will be protected for Y1, Y2, Y3, Y4 and Y5 classes as long as the following equation holds:

$$EMSR_{Y12345}(\pi_{Y12345}) = Fare_{Y12345} \times \overline{P_{Y12345}}(\pi_{Y12345}) \geq Fare_6 \quad [A1.26]$$

That is

$$424.8 \times \overline{P_{12345}}(\pi_{Y12345}) \geq 150 \quad [A1.27]$$
$$\Rightarrow \overline{P_{Y12345}}(\pi_{Y12345}) \geq 0.353$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.353, \quad [A1.28]$$

is 0.38. Since our sample follows N(118, 23.26), we can get that:

$$Z = \frac{\pi_{Y12345} - \mu}{\sigma} = 0.38 \quad [A1.29]$$
$$\pi_{Y1234} = 108 + 0.38 \times 22.23 = 116.4 \approx 116$$

• EMSR Value for Current Seat

Here what we know is that there are 100 seats available, that is

$$\pi = 108 + Z \times 22.23 = 100 \quad [A1.30]$$

So $Z = -0.3599$

Then the probability

$$P(x \geq -0.3599) = 0.6406 \quad [A1.31]$$
$$EMSR(100) = 0.6406 \times 424.8 = \$272.1$$

Since the EMSR value of the 100th seat should not access the lowest fare value in the higher fare group. So

$$EMSR(100) = Max\{EMSR(100), \$250\} = \$250.0 \quad [A1.32]$$

On Leg 2:

• The EMSR Curve

The virtual classes of fares are given as below:

Virtual Classes	Fare Classes	Fare
Y1	Y(AC)	$1000
Y2	B(AC)	$625
Y3	Y(BC)	$600
Y4	B(BC)	$350
Y5	Q(AC)	$250
Y6	Q(BC)	$200

Table 2 Virtual Classes on Leg 2

1. Protection for Y1:

$$Mean\ Demand = 8 \qquad [A1.33]$$
$$Standard\ Deviation = 4$$

We know the seats will be protected for Y1 as long as the following equation:

$$EMSR_{Y1}(\pi_{Y1}) = Fare_{Y1} \times \overline{P_{Y1}}(\pi_{Y1}) \geq Fare_{Y2} \qquad [A1.34]$$

That is

$$1000 \times \overline{P_{Y1}}(\pi_{Y1}) \geq 625 \qquad [A1.35]$$
$$\Rightarrow \overline{P_Y}(\pi_Y) \geq 0.625$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.625, \qquad [A1.36]$$

is -0.32. Since our sample follows N(8, 4), we can get that:

$$Z = \frac{\pi_Y - \mu}{\sigma} = -0.32 \qquad [A1.37]$$
$$\pi_Y = 8 - 0.32 \times 4 = 6.72 \approx 7$$

2) Seats protected for Y1 and Y2 classes.

First calculates the mean fare for Y1 and Y2 together.

$$Fare_{Y12} = \frac{1000 \times 8 + 625 \times 25}{8 + 25} = 716.00 \qquad [A1.38]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1 and Y2 classes.

$$Mean\ Demand = 8 + 25 = 33 \qquad [A1.39]$$
$$Standard\ Deviation = \sqrt{4^2 + 10^2} = 10.77$$

We know the seats will be protected for Y1 and Y2 class as long as the following equation holds:

$$EMSR_{Y12}(\pi_{Y12}) = Fare_{Y12} \times \overline{P_{Y12}}(\pi_{Y12}) \geq Fare_3 \qquad [A1.40]$$

That is

$$716.00 \times \overline{P_{Y12}}(\pi_{Y12}) \geq 600 \qquad [A1.41]$$
$$\Rightarrow \overline{P_{Y12}}(\pi_{Y12}) \geq 0.838$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.838, \qquad [A1.42]$$

is -0.99. Since our sample follows N(23, 7.211), we can get that:

$$Z = \frac{\pi_{Y12} - \mu}{\sigma} = -0.99 \qquad [A1.43]$$
$$\pi_{Y12} = 33 - 0.99 \times 10.77 = 22.3 \approx 22$$

3) Seats protected for Y1, Y2 and Y3 classes.

Calculate the mean fare for Y1, Y2 and Y3 together.

$$Fare_{Y123} = \frac{1000 \times 8 + 625 \times 25 + 600 \times 15}{8 + 25 + 15} = 679 \qquad [A1.44]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1, Y2 and Y3 classes.

$$\begin{aligned} &Mean\ Demand = 8 + 25 + 15 = 48 \\ &Standard\ Deviation = \sqrt{4^2 + 10^2 + 6^2} = 12.33 \end{aligned} \qquad [A1.45]$$

We know the seats will be protected for Y1, Y2 and Y3 class as long as the following equation holds:

$$EMSR_{Y123}(\pi_{Y123}) = Fare_{Y123} \times \overline{P_{Y123}}(\pi_{Y123}) \geq Fare_4 \qquad [A1.46]$$

That is

$$\begin{aligned} &675 \times \overline{P_{Y123}}(\pi_{Y123}) \geq 350 \\ &\Rightarrow \overline{P_{Y123}}(\pi_{Y123}) \geq 0.515 \end{aligned} \qquad [A1.47]$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.515, \qquad [A1.48]$$

is -0.04. Since our sample follows N(48, 12.33), we can get that:

$$\begin{aligned} &Z = \frac{\pi_{Y12} - \mu}{\sigma} = -0.04 \\ &\pi_{Y12} = 48 - 0.04 \times 12.33 = 47.5 \approx 48 \end{aligned} \qquad [A1.49]$$

4) Seats protected for Y1, Y2, Y3 and Y4 classes.

First calculates the mean fare for Y1, Y2, Y3 and Y4.

$$Fare_{Y1234} = \frac{1000 \times 8 + 625 \times 25 + 600 \times 15 + 350 \times 25}{8 + 25 + 15 + 25} = 566.78 \qquad [A1.50]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1, Y2, Y3 and Y4 classes.

$$\begin{aligned} &Mean\ Demand = 8 + 25 + 15 + 25 = 73 \\ &Standard\ Deviation = \sqrt{4^2 + 10^2 + 6^2 + 10^2} = 15.87 \end{aligned} \qquad [A1.51]$$

We know the seats will be protected for Y1, Y2, Y3 and Y4 class as long as the following equation holds:

$$EMSR_{Y1234}(\pi_{Y1234}) = Fare_{Y1234} \times \overline{P_{Y1234}}(\pi_{Y1234}) \geq Fare_5 \qquad [A1.52]$$

That is

$$566.78 \times \overline{P_{Y1234}}(\pi_{Y1234}) \geq 250 \qquad [A1.53]$$
$$\Rightarrow \overline{P_{Y1234}}(\pi_{Y1234}) \geq 0.441$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.441, \qquad [A1.54]$$

is 0.15. Since our sample follows N(73, 15.87), we can get that:

$$Z = \frac{\pi_{Y1234} - \mu}{\sigma} = 0.15 \qquad [A1.55]$$
$$\pi_{Y1234} = 73 + 0.15 \times 15.87 = 75.4 \approx 75$$

5) Seats protected for Y1, Y2, Y3, Y4 and Y5 classes.

First calculates the mean fare for Y1, Y2, Y3, Y4 and Y5.

$$Fare_{Y12345} = \frac{1000 \times 8 + 625 \times 25 + 600 \times 15 + 350 \times 25 + 250 \times 45}{8 + 15 + 25 + 25 + 45} = 446.0 \qquad [A1.56]$$

Then we can get the Mean Demand and Standard Deviation of the combination demand of Y1, Y2, Y3, Y4 and Y5 classes.

$$Mean\ Demand = 8 + 15 + 25 + 25 + 45 = 118 \qquad [A1.57]$$
$$Standard\ Deviation = \sqrt{4^2 + 10^2 + 6^2 + 10^2 + 17^2} = 23.26$$

We know the seats will be protected for Y1, Y2, Y3, Y4 and Y5 classes as long as the following equation holds:

$$EMSR_{Y12345}(\pi_{Y12345}) = Fare_{Y12345} \times \overline{P_{Y12345}}(\pi_{Y12345}) \geq Fare_6 \qquad [A1.58]$$

That is

$$446.0 \times \overline{P_{12345}}(\pi_{Y12345}) \geq 200 \qquad [A1.59]$$
$$\Rightarrow \overline{P_{Y12345}}(\pi_{Y12345}) \geq 0.448$$

From N(0,1) table, we can find Z, which satisfies

$$P(x>Z) = 0.448, \quad [A1.60]$$

is 0.13. Since our sample follows N(118, 23.26), we can get that:

$$Z = \frac{\pi_{Y12345} - \mu}{\sigma} = 0.13 \quad [A1.61]$$
$$\pi_{Y1234} = 118 + 0.13 \times 23.26 \approx 121$$

• EMSR Valuc for Current Seat

Here what we know is that there are 100 seats available, that is

$$\pi = 118 + Z \times 23.26 = 100 \quad [A1.62]$$

So $Z = -0.774$

Then the probability

$$P(x \geq -0.774) = 0.7794 \quad [A1.63]$$
$$EMSR(100) = 0.7794 \times 446.0 = \$347.6$$

Since the EMSR value of the 100th seat should not access the lowest fare value in the higher fare group. So

$$EMSR(100) = Max\{EMSR(100), \$250\} = \$250 \quad [A1.64]$$

Appendix 2

LP Solution Outputs

LP OPTIMUM FOUND AT STEP 8

OBJECTIVE FUNCTION VALUE 63325.00

VARIABLE	VALUE	REDUCED COST
Y_{L1}	10.000000	.000000
B_{L1}	20.000000	.000000
Q_{L1}	37.000000	.000000
Y_{L2}	15.000000	.000000
B_{L2}	25.000000	.000000
Q_{L2}	27.000000	.000000
Y_C	8.000000	.000000
B_C	25.000000	.000000
Q_C	.000000	100.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2	.000000	150.000000
3	.000000	200.000000
4	.000000	350.000000
5	.000000	150.000000
6	3.000000	.000000
7	.000000	400.000000
8	.000000	150.000000
9	18.000000	.000000
10	.000000	650.000000
11	.000000	275.000000
12	45.000000	.000000

NO. ITERATIONS= 8

Sensitivity Analysis

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
YL1	500.000000	INFINITY	350.000000
BL1	300.000000	INFINITY	150.000000
QL1	150.000000	150.000000	100.000000
YL2	600.000000	INFINITY	400.000000
BL2	350.000000	INFINITY	150.000000
QL2	200.000000	150.000000	100.000000
YC	1000.000000	INFINITY	650.000000
BC	625.000000	INFINITY	275.000000
QC	250.000000	100.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	100.000000	3.000000	37.000000
3	100.000000	18.000000	27.000000
4	10.000000	37.000000	3.000000
5	20.000000	37.000000	3.000000
6	40.000000	INFINITY	3.000000
7	15.000000	27.000000	15.000000
8	25.000000	27.000000	18.000000
9	45.000000	INFINITY	18.000000
10	8.000000	27.000000	3.000000
11	25.000000	27.000000	3.000000
12	45.000000	INFINITY	45.000000

Appendix 3

Shadow Prices Comparison

Following are some shadow prices at demand level 1.20

	Leg #	First Period SP	0.8*First Period SP	True SP(Cap+1)	True SP(Cap-1)	Upper SP	New UP SPs
Rev.		4647918	4648807	4647712	4647727	4648517	4650493
Impro.		**1.35**	**1.37**	**1.35**	**1.35**	**1.36**	**1.41**
Loc Spill		1669.00	1773.00	1660.00	1660.00	1670.00	1683.00
Con Spill		655.00	607.00	663.00	663.00	659.00	648.00
SPs	1	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	0.00
	3	61.00	48.80	61.00	61.00	61.00	61.00
	4	61.00	48.80	61.00	61.00	70.10	42.09
	5	301.00	240.80	301.00	301.00	301.00	301.00
	6	301.00	240.80	301.00	301.00	301.00	301.00
	7	322.00	257.60	322.00	336.00	342.50	316.86
	8	322.00	257.60	322.00	322.00	322.00	322.00
	9	0.00	0.00	0.00	0.00	0.00	0.00
	10	729.00	583.20	681.00	681.00	699.00	676.30
	11	331.00	264.80	331.00	331.00	331.00	331.00
	12	309.00	247.20	309.00	309.00	310.10	305.18
	13	753.00	602.40	753.00	753.00	740.35	694.69
	14	753.00	602.40	753.00	753.00	753.00	753.00
	15	583.00	466.40	583.00	583.00	586.25	577.17
	16	583.00	466.40	583.00	583.00	583.00	583.00
	17	323.00	258.40	312.00	312.00	126.15	0.00
	18	505.00	404.00	470.00	470.00	465.55	442.14
	19	385.00	308.00	385.00	385.00	391.00	366.43
	20	462.00	369.60	413.00	413.00	422.80	406.52
	21	72.00	57.60	92.00	149.00	132.10	67.75
	22	167.00	133.60	167.00	167.00	141.90	92.60
	23	72.00	57.60	72.00	72.00	86.25	51.45
	24	88.00	70.40	211.00	211.00	189.70	133.51
	25	68.00	54.40	103.00	103.00	93.60	78.71
	26	103.00	82.40	103.00	103.00	105.25	97.38
	27	157.00	125.60	157.00	157.00	150.05	133.06
	28	89.00	71.20	89.00	89.00	93.35	77.78
	29	72.00	57.60	72.00	72.00	71.70	70.36
	30	0.00	0.00	0.00	0.00	0.00	0.00
	31	72.00	57.60	72.00	72.00	87.00	45.88
	32	0.00	0.00	0.00	0.00	0.00	0.00

Appendix 4

Comparison between Shadow Prices and Critical EMSR values

Demand Level 1.20

Leg Number	Shadow Prices	Critical EMSR Values	Difference	Load Factors	% Local Demand
1	0.00	21.51	21.51	0.8	80.94
.	0.00	0.00	0.00	0.46	69.57
3	61.00	61.00	0.00	0.96	76.33
4	61.00	179.24	118.24	0.96	67.68
5	301.00	301.00	0.00	0.99	66.09
6	301.00	301.20	0.20	0.99	81.75
7	322.00	322.00	0.00	0.99	56 71
8	322.00	322.10	0.10	1	61.34
9	0.00	51.91	51.91	0.79	85.01
10	729.00	685.06	**-43.94**	0.99	83.88
11	331.00	331.00	0.00	0.99	86.15
12	309.00	309.00	0.00	0.99	91.97
13	753.00	753.00	0.00	0.98	61.59
14	753.00	753.00	0.00	0.99	97.37
15	583.00	588.27	5.27	0.99	86.29
16	583.00	583.00	0.00	1	96.99
17	323.00	148.29	**-174.71**	0.92	61.14
18	505.00	403.82	**-101.18**	1	46.98
19	385.00	214.40	**-170.60**	1	61.85
20	462.00	413.43	**-48.57**	0.99	52.98
21	72.00	167.32	95.32	0.97	93.71
22	167.00	172.69	5.69	0.97	98.94
23	72.00	126.95	54.95	0.95	99.73
24	88.00	188.44	100.44	0.98	70.09
25	68.00	97.12	29.12	0.97	62.31
26	103.00	104.43	1.43	0.99	70.49
27	157.00	157.00	0.00	0.99	75.45
28	89.00	110.77	21.77	0.97	50.64
29	72.00	72.00	0.00	0.97	90.86
30	0.00	12.40	12.40	0.72	85.4
31	72.00	117.52	45.52	0.97	77.42
32	0.00	0.26	0.26	0.57	95.03

Demand Level 0.80

Leg Number	Shadow Prices	Critical EMSR Values	Difference	Load Factors	% Local Demand
1	0.00	0.01	0.01	0.54	79.11
2	0.00	0.00	0.00	0.35	70.95
3	0.00	16.90	16.90	0.78	76.88
4	61.00	61.00	0.00	0.93	65.78
5	0.00	241.90	241.90	0.92	64.68
6	301.00	301.61	0.61	0.95	83.01
7	322.00	323.18	1.18	0.98	56.66
8	0.00	234.24	234.24	0.95	61.55
9	0.00	0.00	0.00	0.53	85.40
10	0.00	41.00	41.00	0.75	83.78
11	3[illegible]	331.00	0.00	0.99	86.22
12	0.00	144.02	144.02	0.88	92 36
13	499.00	373.65	**-125.35**	0.97	62 88
14	519.00	513.82	**-5.18**	0.97	97.41
15	0.00	356.79	356.79	0.88	86.78
16	0.00	185.93	185.93	0.86	96.70
17	0.00	0.11	0.11	0.68	60.41
18	401.00	350.83	**-50.17**	1.00	47.22
19	0.00	56.14	56.14	0.86	61.43
20	413.00	348.15	**-64.85**	1.00	52.64
21	0.00	39.66	39.66	0.80	93.31
22	0.00	72.00	72.00	0.89	99.45
23	0.00	18.37	18.37	0.77	99.44
24	88.00	88.00	0.00	0.91	69.59
25	25.00	44.99	19.99	0.89	61.14
26	68.00	85.54	17.54	0.97	70.88
27	18.00	83.02	65.02	0.97	75 58
28	89.00	89.33	0.33	0.96	49 68
29	0.00	18.86	18.86	0.79	92.00
30	0.00	0.01	0.01	0.52	85.33
31	0.00	29.83	29.83	0.84	78.76
32	0.00	0.00	0.00	0.37	94.97

Note: The shaded numbers are the identical shadow prices and EMSR values.

Appendix 5

Comparison between Shadow Prices, EMSR values, and Converged EMSR Values

Demand Level 1.20

Leg Number	Shadow Prices	Critical EMSR Values	Converged EMSR	Load Factors	% Local Demand
1	0.00	21.51	22.02	0.80	80.94
2		0.00	0.00	0.46	69.57
3	61.00	61.00	96.03	0.96	76 33
4	61.00	179.24	194.79	0.96	67.68
5	301.00	301.00	301.05	0.99	66.09
6	301.00	301.20	301.51	0.99	81 75
7	322.00	322.00	378.54	0.99	56 71
8	322.00	322.10	322.00	1.00	61.34
9	0.00	51.91	52.04	0.79	85.01
10	729.00	685.06	685.06	0.99	83.88
11	331.00	331.00	331.00	0.99	86.15
12	309.00	309.00	309.00	0.99	91 97
13	753.00	753.00	753.00	0.98	61.59
14	753.00	753.00	753.00	0.99	97.37
15	583.00	588.27	588.29	0.99	86.29
16	583.00	583.00	583.00	1.00	96.99
17	323.00	148.29	303.91	0.92	61.14
18	505.00	403.82	462.04	1.00	46.98
19	385.00	214.40	395.18	1.00	61.85
20	462.00	413.43	413.65	0.99	52.98
21	72.00	167.32	167.32	0.97	93.71
22	167.00	172.69	174.96	0.97	98.94
23	72.00	126.95	126.95	0.95	99.73
24	88.00	188.44	212.55	0.98	70.09
25	68.00	97.12	103.00	0.97	62.31
26	103.00	104.43	113.98	0.99	70.49
27	157.00	157.00	157.00	0.99	75.45
28	89.00	110.77	117.52	0.97	50.64
29	72.00	72.00	72.00	0.97	90.86
30	0.00	12.40	19.35	0.72	85.4
31	72.00	117.52	122.42	0.97	77.42
32	0.00	0.26	0.27	0.57	95.03

Demand Level 0.80

Leg Number	Shadow Prices	Critical EMSR Values	Converged EMSR	Load Factors	% Local Demand
1	0.00	0.01	0.01	0.54	79.11
2	0.00	0.00	0.00	0.35	70.95
3	0.00	16.90	18.34	0.78	76.88
4	61.00	61.00	61.00	0.93	65.78
5	0.00	241.90	246.03	0.92	64.68
6	301.00	301.61	301.78	0.95	83.01
7	322.00	323.18	325.34	0.98	56.66
8	0.00	234.24	255.52	0.95	61.55
9	0.00	0.00	0.00	0.53	85.40
10	0.00	41.00	41.04	0.75	83.78
11	331.00	331.00	331.00	0.99	86.22
12	0.00	144.02	144.40	0.88	92.36
13	499.00	373.65	498.31	0.97	62.88
14	519.00	513.82	515.18	0.97	97.41
15	0.00	356.79	357.01	0.88	86.78
16	0.00	185.93	185.98	0.86	96.70
17	0.00	0.11	0.12	0.68	60.41
18	401.00	350.83	370.63	1.00	47.22
19	0.00	56.14	59.42	0.86	61.43
20	413.00	348.15	375.66	1.00	52.64
21	0.00	39.66	34.09	0.80	93.31
22	0.00	72.00	72.00	0.89	99 45
23	0.00	18.37	18.37	0.77	99.44
24	88.00	88.00	130.78	0.91	69.59
25	25.00	44.99	62.34	0.89	61.14
26	68.00	85.54	88.86	0.97	70.88
27	18.00	83.02	89.24	0.97	75.58
28	89.00	89.33	89.00	0.96	49 68
29	0.00	18.86	18.86	0.79	92.00
30	0.00	0.01	0.01	0.52	85.33
31	0.00	29.83	39.13	0.84	78.76
32	0.00	0.00	0.00	0.37	94.97

Appendix 6

Shadow Prices and EMSR Values

Demand Level 1.20

Leg Number	Shadow Prices	Critical EMSR Values	Converged Pseudo Fares	Converged Proration
1	0.00	21.51	22.02	31.30
2	0.00	0.00	0.00	0.04
3	61.00	61.00	96.03	81.18
4	61.00	179.24	194.79	115.62
5	301.00	301.00	301.05	301.02
6	301.00	301.20	301.51	301.29
7	322.00	322.00	378.54	378.96
8	322.00	322.10	322.00	322.00
9	0.00	51.91	52.04	48.51
10	729.00	685.06	685.06	684.70
11	331.00	331.00	331.00	331.00
12	309.00	309.00	309.00	309.00
13	753.00	753.00	753.00	753.00
14	753.00	753.00	753.00	753.00
15	583.00	588.27	588.29	588.30
16	583.00	583.00	583.00	583.00
17	323.00	148.29	303.91	321.80
18	505.00	403.82	462.04	461.42
19	385.00	214.40	395.18	388.05
20	462.00	413.43	413.65	414.38
21	72.00	167.32	167.32	167.30
22	167.00	172.69	174.96	173.65
23	72.00	126.95	126.95	165.78
24	88.00	188.44	212.55	209.53
25	68.00	97.12	103.00	103
26	103.00	104.43	113.98	103
27	157.00	157.00	157.00	157
28	89.00	110.77	117.52	97
29	72.00	72.00	72.00	72
30	0.00	12.40	19.35	29.41
31	72.00	117.52	122.42	113.61
32	0.00	0.26	0.27	1.35

Demand Level 0.80

Leg Number	Shadow Prices	Critical EMSR Values	Converged Pseudo Fares	Converged Proration
1	0.00	0.01	0.01	0.20
2	0.00	0.00	0.00	0.00
3	0.00	16.90	18.34	17.74
4	61.00	61.00	61.00	61.00
5	0.00	241.90	246.03	223.23
6	301.00	301.61	301.78	301.58
7	322.00	323.18	325.34	323.49
8	0.00	234.24	255.52	270.43
9	0.00	0.00	0.00	0.00
10	0.00	41.00	41.04	38.20
11	331.00	331.00	331.00	319.39
12	0.00	144.02	144.40	142.55
13	499 00	373.65	498 31	488.57
14	519.00	513.82	515.18	526.52
15	0.00	356.79	357.01	355.52
16	0.00	185.93	185.98	185.03
17	0.00	0.11	0.12	0.17
18	401.00	350.83	370.63	379.76
19	0.00	56.14	59.42	46.64
20	413.00	348.15	375.66	378.79
21	0.00	39.66	34.09	64.25
22	0.00	72.00	72.00	72.00
23	0.00	18.37	18.37	29.49
24	88.00	88.00	130.78	107.95
25	25.00	44.99	62.34	62.44
26	68.00	85.54	88.86	99.61
27	18.00	83.02	89.24	88.37
28	89.00	89.33	89.00	99.61
29	0.00	18.86	18.86	33.28
30	0.00	0.01	0.01	0.14
31	0.00	29.83	39.13	56.68
32	0.00	0.00	0.00	0.00