Multidisciplinary System Design Optimization (MSDO)

Optimization of a Hybrid Satellite Constellation System

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Outline

• Introduction
  – Satellite constellation design

• Simulation
  – Modeling
  – Benchmarking

• Optimization
  – Single objective
    • Gradient based
    • Heuristic: Simulated Annealing
  – Multi-objective

• Conclusions and Future Research
Motivation/Background

Past attempts at mobile satellite communication systems have failed as there has been an inability to match user demand with the provided capacity in a cost-efficient manner (e.g. Iridium & Globalstar)

Two main assumptions:
- Circular orbits and a common altitude for all the satellites in the constellation
- Uniform distribution of customer demand around the globe

Given a non-uniform market model, can the incorporation of elliptical orbits with repeated ground tracks expand the cost-performance trade space favorably?

Aspects of the satellite constellation design problem previously researched:
- M. Parker (MEng Thesis, 2001, MIT)
- O. de Weck and D. Chang (AIAA 2002-1866)
Market Distribution Estimation

GNP PPP Map + Population Map

Market Distribution Map

Demand Distribution Map

Reduced Resolution for Simulation
Problem Formulation

- A circular LEO satellite backbone constellation designed to provide minimum capacity global communication coverage,
- An elliptical (Molniya) satellite constellation engineered to meet high-capacity demand at strategic locations around the globe (in particular, the United States, Europe and East Asia).

Single Objective J: \( \text{min} \) the lifecycle cost of the total hybrid satellite constellation sys.

Constraints:

- the total lifecycle cost must be strictly positive
- the data rate market demand must be met at least 90% of the time
  - the satellites must service 100% of the users 90% of the time
  - data rate provided by the satellites \( \geq \) to the demand
  - all satellites must be deployable from current launch vehicles

Design Vector for Polar Backbone Constellation: <\( C \) [polar/walker], \( \text{emin} \) [deg], MA, ISL [0/1], \( h \) [km], Pt [W], DA [m]>

Design Vector for Elliptical Constellation: <\( T \) [day], \( e \) [-], \( Np \) [-], Pt [W], Da [m]>
Simulation Model
Tradespace Exploration

- An orthogonal array was implemented for the elliptical constellation DOE

- The recommended initial start point for the numerical optimization of the elliptical constellation is
  \( X_{\text{init}} = [T=1/6, e=0.6, \text{NP}=4, \text{Pt}=500, \text{DA}=3]^T \)

- In order to analyze the tradespace of the Polar constellation backbone, a full factorial search was conducted, the Pareto front of non dominated solutions was then defined

- The lowest cost Polar constellation was found to have the following design vector values
  \( X = [C=\text{polar}, \text{emin}=5 \text{ deg}, \text{MA}=\text{QPSK}, \text{ISL}=1, \text{h}=2000, \text{Pt}=0.25, \text{DA}=0.5]^T \)
LEO BACKBONE:

- Simulation created by de Weck and Chang (2002)
  - Code benchmarked against a number of existing satellite systems
    - Outputs within 20% of the benchmark’s values
  - Slight modifications made to suit the broadband market demand
    - # of subscribers, required data rate per user, avg. monthly usage etc…

CODE VALIDATION:

- Orbit and constellation calculations
  - Validated by plotting and visually confirming orbits
**ELLiptical constellation**:

- Simulation benchmarked against Ellipso
- Ellipso
  - Elliptical satellite constellation system proposed to the FCC in 1990
  - \((T = 24, \, NP = 4, \text{phasing of planes } = 90 \text{ degrees apart})\)
- System benchmarked on modular basis

<table>
<thead>
<tr>
<th>System Module</th>
<th>Ellipso</th>
<th>Simulation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Link Budget</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>12</td>
<td>11.93</td>
<td>[dB]</td>
</tr>
<tr>
<td>EIRP</td>
<td>27</td>
<td>24.93</td>
<td>[dBW]</td>
</tr>
<tr>
<td>Data Rate</td>
<td>2.2</td>
<td>1.08</td>
<td>[Mbps]</td>
</tr>
<tr>
<td><strong>Spacecraft</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sat Mass</td>
<td>68</td>
<td>98.68</td>
<td>[Kg]</td>
</tr>
<tr>
<td>Sat Volume</td>
<td>0.0008</td>
<td>0.810</td>
<td>(\text{m}^3)</td>
</tr>
<tr>
<td><strong>Lifecycle Cost</strong></td>
<td>249.6</td>
<td>290.9</td>
<td>[YR2002 $M]</td>
</tr>
</tbody>
</table>

Ellipso didn’t use the same demand model, thus a constraint benchmark process was not conducted.
Gradient-Based Optimization

- Sequential Quadratic Programming (SQP)
  - Simplification => number of planes integer
- Objective: minimize lifecycle cost

**Initial guess:**
- Period (T): 0.5 day
- Eccentricity (e): 0.01
- # Planes (NP): 4
- Transmitter Power (Pt): 4000 W
- Antenna Diameter (DA): 3 m
- \( J: \$6280.5999 \text{ M} \)

**Optimal:**
- Period (T): 0.7 day
- Eccentricity (e): 0
- # Planes (NP): 4
- Transmitter Power (Pt): 3999.7 W
- Antenna Diameter (DA): 1.76 m
- \( J^*: \$6187.8559 \text{ M} \)
Sensitivity Analysis

Optimal Design, \( x^* \):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (T)</td>
<td>0.7 day</td>
</tr>
<tr>
<td>Eccentricity (e)</td>
<td>0</td>
</tr>
<tr>
<td># Planes (NP)</td>
<td>4</td>
</tr>
<tr>
<td>Transmitter Power (Pt)</td>
<td>3999.7 W</td>
</tr>
<tr>
<td>Antenna Diameter (DA)</td>
<td>1.76 m</td>
</tr>
</tbody>
</table>

Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Rate</td>
<td>1000 kbps</td>
</tr>
<tr>
<td>Step Size</td>
<td>10 kbps</td>
</tr>
<tr>
<td># Subscribers</td>
<td>1000 users</td>
</tr>
<tr>
<td>Step Size</td>
<td>10 users</td>
</tr>
</tbody>
</table>

![Normalized Sensitivities of Objective with Respect to the Design Variables](image1.png)

![Sensitivities of Objective with Respect to Two Parameters (using FD)](image2.png)
Heuristic Optimization

• Simulated annealing was used

• Quite sensitive to cooling schedule and starting conditions

• Not very repeatable
  – Low confidence that global optimum was reached

• Total computational cost high

• Abandoned in favor of full-factorial evaluation of the tradespace for the multi-objective case
  – Possibly gain insight into key trends
Simulated Annealing Sample Run

- System Temperature [$k$]
- Lifecycle Cost [$k$]
- Iteration #

Graph showing the relationship between system temperature and lifecycle cost over iterations.
Multi-Objective Optimization

• Minimum cost design tend not to have the possibility for future growth

• Try to simultaneously:
  – Minimize Lifecycle Cost (LCC)
  – Maximize Time Averaged Over Capacity

\[
\text{If } \% \text{ market served } > \text{ min market share}
\]
\[
\text{Over capacity } = \ldots
\]
\[
\text{Total capacity } - \text{ Market served}
\]

\[
\text{Else}
\]
\[
\text{Over capacity } = 0
\]

\[
\text{End}
\]

• Min market share chosen to be 90%
Full Factorial Tradespace

- 1280 designs evaluated
- Interesting trends revealed

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>1,1/2,1/3,1/4,1/5</td>
<td>[days]</td>
</tr>
<tr>
<td>e</td>
<td>0.001, 0.1, 0.3, 0.4</td>
<td>[-]</td>
</tr>
<tr>
<td>NP</td>
<td>2, 3, 4, 6</td>
<td>[-]</td>
</tr>
<tr>
<td>Pt</td>
<td>1, 2, 4, 6</td>
<td>[kW]</td>
</tr>
<tr>
<td>DA</td>
<td>1.5, 2, 2.5, 3</td>
<td>[m]</td>
</tr>
</tbody>
</table>
Unrestricted Pareto Front

• Very high average over capacity

• Seems counterintuitive that high success does not yield high average over capacity

• Look at the design trade to find an explanation
Unrestricted Tradespace

- All high AOC designs have high eccentricity and short period
- Many satellites per planes
  - Very high system capacity
Restricted Pareto Front

- Much smaller AOC when demand constraint is enforced
- Again explore the tradespace by coloring by DV values
Restricted Tradespace

- a) by T
- b) by e
- c) by NP
- d) by Pt
- e) by Da
Some Useful Visualizations

- **Convex Hulls**
  - Smallest convex polygon that contains all points in the tradespace that have a design variable at a particular value
  - Determines regions that are ‘closed off’ when a design choice is made

- **Conditional Pareto Fronts**
  - Pareto optimal set of points given that a particular design choice has been made
  - When compared to the unconditioned front, can determine key characteristics of designs on sections of the Pareto front
Convex Hulls

a) by T

b) by e

c) by NP

d) by Pt

e) by Da
Conditional Pareto Fronts

a) by $T$

b) by $e$

c) by $NP$

d) by $Pt$

e) by $Da$
Conclusions and Future Work

- Historic mismatch between capacity and demand

- Hybrid constellations
  - First provide baseline service
  - Then supplement backbone to cover high demand
  - Allows for staged deployment that adjusts to an unpredictable market

- Pareto analysis
  - ½ day period, ~0 eccentricity
  - Transmitter power key to location on Pareto front
  - Number of planes, antenna gain not as important
Future Work

• Coding for radiation shielding due to van Allen belts
  – Current CER for satellite hardening is taken as 2-5% increment in cost
  – Can compute hardening needed using NASA model – need to translate hardening requirement into cost increment

• Model hand-off problem
  – Transfer of a ‘call’ from one satellite to another
  – Not addressed in current simulation
  – Key component of interconnected network satellite simulations

• Increase the fidelity of the simulation modules with less simplifying assumptions

• Increase fidelity of cost module
  – Include table of available motors for the apogee and geo transfer orbit kick motors
Demand Distribution Map

GNP-PPP

Population

Demand
Example Ground Tracks

Sample Ground Track: $T=1/2$ day; $e=0.5$
Sensitivity Analysis:
Design Variables

- Compute Gradient

\[
\nabla J = \begin{bmatrix}
\frac{\partial J}{\partial T} \\
\frac{\partial J}{\partial \varepsilon} \\
\frac{\partial J}{\partial N_P} \\
\frac{\partial J}{\partial P_t} \\
\frac{\partial J}{\partial D_A}
\end{bmatrix} = \begin{bmatrix}
-102.1317 \\
114.5666 \\
204.0848 \\
0.3328 \\
40.5873
\end{bmatrix}
\]

- Normalize

\[
\nabla J_{\text{normalized}} = \frac{x^*}{J(x^*)} \nabla J = \begin{bmatrix}
0.7 \\
0 \\
4 \\
3999.7 \\
1.8
\end{bmatrix} \begin{bmatrix}
-102.1317 \\
114.5666 \\
204.0848 \\
0.3328 \\
40.5873
\end{bmatrix} = \begin{bmatrix}
0.0116 \\
0.1319 \\
0.0002 \\
0.0118
\end{bmatrix}
\]
Sensitivity Analysis: Parameters

- **Basic Equation**
  - Finite Differencing
  \[
  \frac{\Delta J}{\Delta p} = \frac{J(p^o + \Delta p) - J(p^o)}{\Delta p} = \frac{2003.884M$ - 2008.7703M$}{10} = -0.48863
  \]

- **Data Rate**
  - Step Size: 10 kbps
  \[
  \frac{\Delta J}{\Delta p} = \frac{J(p^o + \Delta p) - J(p^o)}{\Delta p} = \frac{2003.7966M$ - 2008.7703M$}{10} = -0.49737
  \]

- **# Subscribers**
  - Step Size: 10 users
### Simulated Annealing Tuning (I)

<table>
<thead>
<tr>
<th>Nature of Tuning Implemented</th>
<th>$J^*$ [$M$]</th>
<th>$x^*$ $[T, e, NP, Pt, DA]^T$</th>
<th>Improvement from optimal SA cost of 5389 [$M$]?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Geometric progression cooling schedule with a 15% decrease per iteration</td>
<td>$5753.4$ (50 runs)</td>
<td>$[1/7, 0.01, 2, 2918.23, 2.33]^T$</td>
<td>No, optimal cost increased by $364 million dollars</td>
</tr>
<tr>
<td>2. Geometric progression cooling schedule with a 25% decrease per iteration</td>
<td>$5427.9$ (50 runs)</td>
<td>$[1/7, 0.01, 3, 1581.72, 2.23]^T$</td>
<td>No, optimal cost increased by $39 million dollars</td>
</tr>
<tr>
<td>3. Stepwise reduction cooling schedule with a 25% reduction per iteration</td>
<td>$6278.7$ (50 runs)</td>
<td>$[1/2, 0.01, 4, 4000, 3]^T$</td>
<td>No, optimal cost and design vector remained the values they were before optimization</td>
</tr>
<tr>
<td>4. Geometric progression cooling schedule with a 15% decrease per iteration but with the added constraint that the result of each iteration has to be better than the one preceding it.</td>
<td>$5800.1$ (41 runs)</td>
<td>$[1/2, 0.01, 3, 3256.08, 2.17]^T$</td>
<td>No, optimal cost increased by $411 million dollars</td>
</tr>
</tbody>
</table>
## Simulated Annealing Tuning (II)

| Nature of Tuning Implemented | $J^*$ [SM] | $x^*$ [T, e, NP, Pt, DA]$^T$ | Improvement from optimal SA cost of 5389 [SM]?
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5.</strong> Initial Temperature is doubled (i.e., initial temperature changed from 6278.7 [SM] to 12557.4 [SM])</td>
<td>$6278.7$ (50 runs)</td>
<td>$[1/2, 0.01, 4, 4000, 3]^T$</td>
<td>No, optimal cost and design vector remained the values they were before optimization</td>
</tr>
<tr>
<td><strong>6.</strong> Initial Temperature is halved. (i.e., initial temp changed from 6278.7 [SM] to 3139.4 [SM])</td>
<td>$5622.7$ (50 runs)</td>
<td>$[1/2, 0.01, 2, 3658.08, 2.3]^T$</td>
<td>No, optimal cost increased by $234$ million dollars</td>
</tr>
<tr>
<td><strong>7.</strong> Initial design vector is altered such that $x_0 = [1, 0, 3, 3000, 3]^T$</td>
<td>$5719.1$ (50 runs)</td>
<td>$[1, 0, 3, 3000, 3]^T$</td>
<td>No, optimal cost increased by $330$ million dollars</td>
</tr>
<tr>
<td><strong>8.</strong> Initial design vector was altered such that $x_0 = [0.25, 0.5, 5, 3000, 3]^T$</td>
<td>Failed to find a feasible solution</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>